HW 2

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Problem 1.1

The air density is given by:

$$\rho = \frac{(28.97)10^{-3}(1.013 \times 10^5)P_{atm}}{(8.314)(273 + T_c)}, \tag{1}$$

where, P_{atm} is atmospheric pressure (unit atm), T_c is temperature in $°C$.

Therefore, from (1), at 1 atm and 30°C the air density is 1.165 kg/m^3.

Problem 1.2

P_{atm} can be calculated as follows:

$$P_{atm} = P_0 e^{-\frac{(28.97)10^{-3} gz}{(8.314)(273+T_c)}}, \tag{2}$$

where, P_0 is atmospheric pressure at the sea level (1 atm), T_c is temperature in $°C$, $g = 9.81$ m/s^2, z is the elevation from sea level in meter.

Therefore, from (2), at an elevation of $z = 2000$ m and at $T_c = 15$°C the air pressure is $P_{atm} = 0.79$ atm. Therefore, from (1), the air density is 0.968 kg/m^3.

Problem 1.3

From (2), at an elevation of $z = 2000$ m and at $T_c = 5$°C the air pressure is $P_{atm} = 0.782$ atm. Therefore, from (1), the air density is 0.993 kg/m^3.

Problem 2

If the wind velocity, V, follows Rayleigh distribution, then the PDF is given by:

$$f_V(v) = \frac{2v}{c^2} e^{-\frac{v^2}{c^2}},$$

where $c = \frac{2\mathbb{E}[v]}{\sqrt{\pi}}$. In this question, $\bar{V} = \mathbb{E}[v] = 8$ m/s. Therefore, it can be calculated that the CDF is given by:

$$F_V(v) = 1 - e^{-\frac{\pi}{4}\left(\frac{v}{\bar{V}}\right)^2} = 1 - e^{-\frac{\pi}{4}\left(\frac{v}{8}\right)^2}.$$

Therefore, the probability that the wind speed are between 6.5 and 7.5 m/s is:

$$\begin{aligned} P(6.5 \leq v \leq 7.5) &= \int_{6.5}^{7.5} f_V(v) \mathrm{d}v \\ &= F_V(7.5) - F_V(6.5) \\ &= 0.094. \end{aligned}$$

Problem 3.1

Assuming that the time to failure random variable, T, follows exponential distribution, the PDF of T is:

$$f_T(t) = \begin{cases} \lambda e^{-\lambda t}, \ t \geq 0 \\ 0, \ t < 0, \end{cases}$$

where, λ is the failure rate. The CDF is given by:

$$F_T(t) = \int_{-\infty}^{t} f_T(t)\mathrm{d}t = 1 - e^{-\lambda t}.$$

In this question, $\lambda = 4.28 \times 10^{-4}$ hr^{-1}.

Therefore, the probability that the turbine survives one month of continuous operation is given by:

$$P(t > 30 \times 24) = 1 - F_T(720) = 0.735.$$

Problem 3.2

The mean time to failure is given by:

$$\mathbb{E}[T] = \frac{1}{\lambda} = 2336.45 \text{ hr}.$$

Problem 3.3

The probability that the turbine will fail between third and second month is:

$$P(2 \times 720 \leq t \leq 3 \times 720) = F_T(2160) - F_T(1440) = 0.143.$$

Problem 4

The power from the wind turbine is given by:

$$P = \frac{1}{2}\rho A V^3.$$

Now, it is given that the PDF of the random variable of wind speed, V is:

$$f_V(v) = \frac{2v}{c^2} e^{-\frac{v^2}{c^2}},$$

where, $c = \frac{2\mathbb{E}[V]}{\sqrt{\pi}}$ and in this question, $\mathbb{E}[V] = 6$ m/s. We need to find out the average power, i.e. $\mathbb{E}[P]$. Now,

$$\mathbb{E}[P] = \frac{1}{2}\rho A \mathbb{E}[V^3].$$

Let us find, $\mathbb{E}[V^3]$ first. For this I will use moment generating function of $f_V(v)$ which is given as:

$$M_V(s) = \mathbb{E}[e^s V] = 1 + \sigma s e^{\sigma^2 s^2/2} \sqrt{\frac{\pi}{2}} \left(erf\left(\frac{\sigma s}{\sqrt{2}} + 1 \right) \right),$$

where, $\sigma = \frac{c}{\sqrt{2}}$, $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t$ is the Gauss error function.

Now, we know that

$$\mathbb{E}[V^3] = \frac{\mathrm{d}^3}{\mathrm{d}s^3} M_V(s)|_{s=0}.$$

Carrying out the derivatives (skipping here to conserve space) we can find that

$$\mathbb{E}[P] = 1.92 \frac{1}{2} \rho A (\mathbb{E}[V])^3.$$

Therefore, the average power, normalized by area is

$$P_{avg} = \frac{\mathbb{E}[P]}{A} = 1.92 \frac{1}{2} \rho (\mathbb{E}[V])^3 = 254.016 \text{ W}.$$

Problem 5

It is given that the wind power random variable V follows uniform distribution as follows

$$V \sim U(5, 20).$$

Therefore the PDF and CDF is given by

$$f_V(v) = \begin{cases} \frac{1}{15}, & v \in [5, 20] \\ 0, & \text{otherwise} \end{cases}$$

$$F_V(v) = \begin{cases} 0, & v < 5, \\ \frac{v-5}{15}, & v \in [5, 20], \\ 1, & v > 20. \end{cases}$$

Now, it is given that

$$\begin{aligned} V_c &= 0 \\ V_r &= 5 \\ V_F &= 15 \\ P_{rated} &= 1000 \text{ W}. \end{aligned}$$

Therefore,

$$\begin{aligned} P(v \geq 5) &= 1 - F_V(5) = 1 \\ P(v \geq 15) &= 1 - F_V(15) = 0.33. \end{aligned}$$

It is not clear if the question asks for annual energy yield at rated power. I assume that the question asks for annual energy yield at rated power. Therefore, annual energy that the wind turbine would generate is

$$8670(1 - 0.33)(1000) = 5.87 \times 10^6 \text{ W-h}.$$