HW 4

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Problem 1.i

The swing equations and simplified turbine-governor model are given by:

$$\begin{aligned}\dot{\delta}_g &= \omega_g - \omega_s \\ M_g\dot{\omega}_g &= P_g^m - P_g^e \\ \tau_g \dot{P}_g^m &= P_g^r - P_g^m - \frac{1}{R_g\omega_s}(\omega_g - \omega_s).\end{aligned} \tag{1}$$

The generator reference power input is given by:

$$P_g^r = P_g^* + \alpha_g\left(\xi - \sum_{j\in\mathcal{G}} P_j^*\right), \tag{2}$$

where α_g is the AGC participation factor and $\sum_{g\in\mathcal{G}}\alpha_g = 1$. ξ is the AGC state whose evolution is given by:

$$\begin{aligned}\dot{\xi} &= -\xi - \text{ACE} + \sum_{g\in\mathcal{G}} P_g^e \\ \text{ACE} &= b\left((1/G)\sum_{g\in\mathcal{G}}(w_g) - w_s\right),\end{aligned}$$

and $b > 0$ and there are a total of G generators in the area. Economic dispatch problem is formulated as follows:

$$\begin{aligned}&\min_{P_g, g\in\mathcal{G}} \sum_{g\in\mathcal{G}} C_g(P_g) \\ &\text{s.t. } \sum_{g\in\mathcal{G}} P_g = P_{load} + P_{loss}(P_{\mathcal{G}}).\end{aligned} \tag{3}$$

It is given in the question that after a load change in the system, the new load is given by $\overline{P}_{load} = P_{load} + \Delta P_{load}$ and let us denote the corresponding changes in the loss as $\overline{P}_{loss} = P_{loss} + \Delta P_{loss}$. In steady state, after the load change, we get the following:

$$\begin{aligned}M_g\dot{\overline{\omega}}_g = 0 &\implies \overline{P}_g^m = \overline{P}_g^e \\ \tau_g\dot{\overline{P}}_g^m = 0 &\implies \overline{P}_g^r - \overline{P}_g^m - \frac{1}{R_g\omega_s}(\overline{\omega}_g - \omega_s) \\ \dot{\overline{\xi}} = 0 &\implies \overline{\xi} = -\frac{b}{G}(\overline{\omega}_g - \omega_s) + \sum_{g\in\mathcal{G}}\overline{P}_g^e.\end{aligned} \tag{4}$$

From 2, summing over all generators in the area, we get,

$$\begin{aligned}&\sum_{j\in\mathcal{G}}\overline{P}_j^r = \sum_{j\in\mathcal{G}}P_j^* + \sum_{j\in\mathcal{G}}(\alpha_g\overline{\xi}) - \sum_{j\in\mathcal{G}}\alpha_g\sum_{j\in\mathcal{G}}P_j^* \\ &\sum_{j\in\mathcal{G}}\overline{P}_j^r = \overline{\xi},\end{aligned}$$

as $\sum_{j\in\mathcal{G}} \alpha_j = 1$. Therefore,

$$\begin{aligned}
&\sum_{j\in\mathcal{G}} \overline{P}^r_j = -\frac{b}{G}(\overline{\omega}_g - \omega_s) + \sum_{j\in\mathcal{G}} \overline{P}^e_g \\
\implies &\sum_{j\in\mathcal{G}} \overline{P}^r_j - \sum_{j\in\mathcal{G}} \overline{P}^e_g = -\frac{b}{G}(\overline{\omega}_g - \omega_s) \\
\implies &\sum_{j\in\mathcal{G}} \overline{P}^r_j - \sum_{j\in\mathcal{G}} \overline{P}^m_g = -\frac{b}{G}(\overline{\omega}_g - \omega_s) \\
\implies &\frac{1}{R_g\omega_s}(\overline{\omega}_g - \omega_s) + \frac{b}{G}(\overline{\omega}_g - \omega_s) = 0 \\
\implies &\overline{\omega}_g = \omega_s.
\end{aligned}$$

Therefore, in steady-state, $\overline{\mathrm{ACE}} = 0$. Thus,

$$\overline{\xi} = \sum_{j\in\mathcal{G}} \overline{P}^e_g = \overline{P}_{load} + \overline{P}_{loss} = P_{load} + \Delta P_{load} + P_{loss} + \Delta P_{loss}.$$

Now, as P^*_g is the outcome of economic dispatch optimization problem, it has to satisfy the constraint, $\sum_{j\in\mathcal{G}} P^*_g = P_{load} + P_{loss}$. Hence, from 4,

$$\begin{aligned}
\overline{P}^m_g &= \overline{P}^r_g \\
&= P^*_g + \alpha_g \left(\overline{\xi} - \sum_{j\in\mathcal{G}} P^*_j\right) \\
&= P^*_g + \alpha_g \left(P_{load} + \Delta P_{load} + P_{loss} + \Delta P_{loss} - P_{load} - P_{loss}\right) \\
&= P^*_g + \alpha_g \left(\Delta P_{load} + \Delta P_{loss}\right).
\end{aligned}$$

Problem 1.ii

From 4,

$$\begin{aligned}
\overline{P}^e_g &= \overline{P}^m_g \\
&= P^*_g + \alpha_g \left(\Delta P_{load} + \Delta P_{loss}\right).
\end{aligned}$$

Problem 1.iii

Using Lagrange multiplier λ and KKT conditions for the given economic dispatch optimization problem, we get the dual objective as follows:

$$\left(\sum_{g\in\mathcal{G}} C_g(P_g)\right) + \lambda \left(P_{load} + P_{loss}(P_{\mathcal{G}}) - \sum_{g\in\mathcal{G}} P_g\right)$$

Denoting the optimal variables as P^*_g and λ^*, we get,

$$\frac{\partial}{\partial P_g}\left(\sum_{g\in\mathcal{G}} C_g(P_g)\right) + \frac{\partial}{\partial P_g}\lambda \left(P_{load} + P_{loss}(P_{\mathcal{G}}) - \sum_{g\in\mathcal{G}} P_g\right) = 0.$$

Carrying out the derivative, and denoting as the optimal values, we get:

$$\begin{aligned}
&C'(P^*_g) - \lambda^* \left(1 - \frac{\partial}{\partial P_g} P_{loss}(P_{\mathcal{G}^*})\right) = 0 \\
\implies &C'(P^*_g) - \frac{\lambda^*}{\Lambda^*_g} = 0, \qquad (5)
\end{aligned}$$

where, $\Lambda^*_g = \left(1 - \frac{\partial}{\partial P_g} P_{loss}(P_{\mathcal{G}^*})\right)^{-1}$.

Problem 1.iv

AGC participation factor is given by:

$$\alpha_g = \frac{(C_g''(P_g^*))^{-1}}{\sum_{j\in\mathcal{G}}(C_j''(P_j^*))^{-1}}.$$

It is given that,

$$\overline{\Lambda}_g^* C_g'(\overline{P}_g^*) - \Lambda_g^* C_g'(P_g^*) = (\overline{P}_g^* - P_g^*)C_g''(P_g^*). \tag{6}$$

Now, from 5 and 6, we get,

$$\begin{aligned} \overline{\lambda}^* - \lambda^* &= (\overline{P}_g^* - P_g^*)C_g''(P_g^*) \\ \implies \frac{\overline{\lambda}^* - \lambda^*}{C_g''(P_g^*)} &= (\overline{P}_g^* - P_g^*). \end{aligned}$$

where $\overline{\lambda}^*$ is the optimal value of Lagrange multiplier of the optimization problem solution after the load change. Summing over all the generators, we get:

$$(\overline{\lambda}^* - \lambda^*)\sum_{j\in\mathcal{G}}(C_j''(P_j^*))^{-1} = \sum_{j\in\mathcal{G}}(\overline{P}_j^* - P_j^*).$$

From the derivation in part (ii),

$$\begin{aligned} \overline{P}_g^e &= P_g^* + \alpha_g\left(\Delta P_{load} + \Delta P_{loss}\right) \\ &= P_g^* + \alpha_g\left(\sum_{j\in\mathcal{G}}\overline{P}_j^* - \sum_{j\in\mathcal{G}}P_j^*\right) \\ &= P_g^* + \frac{(C_g''(P_g^*))^{-1}}{\sum_{j\in\mathcal{G}}(C_j''(P_j^*))^{-1}}\left(\sum_{j\in\mathcal{G}}\overline{P}_j^* - \sum_{j\in\mathcal{G}}P_j^*\right) \\ &= P_g^* + \frac{(C_g''(P_g^*))^{-1}}{\sum_{j\in\mathcal{G}}(C_j''(P_j^*))^{-1}}\sum_{j\in\mathcal{G}}(\overline{P}_j^* - P_j^*) \\ &= P_g^* + \frac{(C_g''(P_g^*))^{-1}}{\sum_{j\in\mathcal{G}}(C_j''(P_j^*))^{-1}}(\overline{\lambda}^* - \lambda^*)\sum_{j\in\mathcal{G}}(C_j''(P_j^*))^{-1} \\ &= P_g^* + (C_g''(P_g^*))^{-1}(\overline{\lambda}^* - \lambda^*) \\ &= P_g^* + (C_g''(P_g^*))^{-1}(\overline{P}_g^* - P_g^*)C_g''(P_g^*) \\ &= P_g^* + \overline{P}_g^* - P_g^* \\ &= \overline{P}_g^*. \end{aligned}$$

Therefore, under the given conditions [A1]-[A2], the setpoints as derived from economic dispatch solution becomes exactly equal to th electrical output power of the generators.

Problem 1.v

Convex cost function.