Exam

Problem 1. [10 points] Consider a series connection of N PV cells. The model for cell ℓ includes a parallel combination of a current source, I_{SC} (that captures the optically excited drift current), a diode (that models the intrinsic PN junction) and series resistance, R_ℓ. Answer the following questions:

- Assuming uniform insolation, derive an expression for the terminal current-versus-voltage characteristic of the series combination. Recall that the current through the PN junction diode, I_{D} is given by

$$I_{\mathrm{D}} = I_o \left(\exp\left(\frac{qV_{\mathrm{D}}}{kT} \right) - 1 \right), \tag{1}$$

  where, the forward voltage across the diode is denoted by V_{D} and all other quantities have their usual meaning.

- Prove that the power-versus-voltage characterisitic of the series combination is a concave function.

Problem 2. [5 points] As illustrated in Fig. 1, the air flowing in and out of a wind turbine is contained in a tube, where A_u is the tube cross-section upwind the turbine through which air enters, A_r is the tube cross-section where the turbine is located, and A_d is the tube cross-section downwind the turbine through which air exits. Similarly, v_u is the windspeed at the tube cross-section upwind the turbine, v_r is the windspeed at the tube cross-section where the turbine is located, and v_d is the windspeed at the tube cross-section downwind the turbine. Let C_P denote the so-called Betz's efficiency. Recall that the maximum power that can be extracted from the wind is given by: $P_r = (1/2)\rho A_r v_u^3 C_P$. Among the choices below, which one describes an alternate expression for the maximum amount of power that can be extracted from the wind? Explain your choice and why the others are wrong.

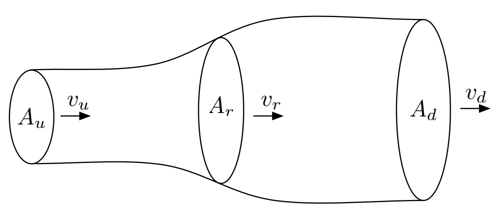

Figure 1: Cross-section of planes relevant to wind turbine.

1. $P_r = \frac{1}{2}\rho A_r v_r^3 C_P$
2. $P_r = \frac{1}{2}\rho A_u v_u^3 C_P$
3. $P_r = \frac{1}{2}\rho A_r v_d^3 (9C_P)$
4. $P_r = \frac{1}{2}\rho A_d v_d^3 \left(\frac{27}{2} C_P\right)$
5. None of the above

Problem 3. [15 points] Obtain the d and q reference-frame dynamics for the currents i_l, i_o and voltage v_o in the LCL filter sketched in Fig. 2. (One leg of the three-phase filter is depicted.)

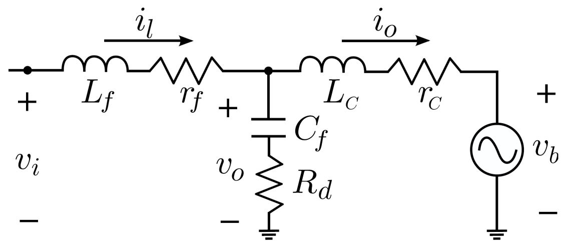

Figure 2: One leg of three-phase LCL filter.

Problem 4. [15 points] Consider a collection of $\mathcal{G}$ turbine-based synchronous generators connected in a lossless electrical power network. We model the dynamics of angular position, frequency, and mechanical-power input for the generators in the network. Pertinent dynamics of the $g \in \mathcal{G}$ generator are:

$$\dot{\theta}_g = \omega_g - \omega_{\mathrm{s}}, \tag{2a}$$
$$M_{\mathcal{G},g}\dot{\omega}_g = P^{\mathrm{m}}_{\mathcal{G},g} - D_{\mathcal{G},g}(\omega_g - \omega_{\mathrm{s}}) - P^{\mathrm{e}}_{\mathcal{G},g}, \tag{2b}$$
$$\tau_g \dot{P}^{\mathrm{m}}_{\mathcal{G},g} = -P^{\mathrm{m}}_{\mathcal{G},g} + P^{\mathrm{r}}_{\mathcal{G},g} - R_{\mathcal{G},g}(\omega_g - \omega_{\mathrm{s}}). \tag{2c}$$

Above, θ_g, ω_g, $P^{\mathrm{m}}_{\mathcal{G},g}$, and $P^{\mathrm{e}}_{\mathcal{G},g}$ are the rotor electrical angular position, generator frequency, turbine mechanical power, and electrical output power respectively, and ω_{s} is the synchronous frequency. Furthermore, $M_{\mathcal{G},g}$ is the inertia constant, $D_{\mathcal{G},g}$ is the load-damping coefficient, $R_{\mathcal{G},g}$ is the inverse of the frequency-power speed-droop regulation constant,[1] τ_g is the turbine time constant, and $P^{\mathrm{r}}_{\mathcal{G},g}$ denotes the reference-power setting computed from secondary control and is assumed to be fixed in this setting. Denote the total system load up to time $t = 0$ by P_{load}. In particular, we have $\forall t < 0$:

$$\sum_{g\in\mathcal{G}} P^{\mathrm{r}}_{\mathcal{G},g} = P_{\mathrm{load}}, \quad \sum_{g\in\mathcal{G}} P^{\mathrm{e}}_{\mathcal{G},g}(t) = P_{\mathrm{load}}. \tag{3}$$

There is a generation-load mismatch at time $t = 0$, and the load increases by an amount ΔP_{load}, but the generator references remain unchanged. Precisely, for time $t \geq 0$, we have

$$\sum_{g\in\mathcal{G}} P^{\mathrm{r}}_{\mathcal{G},g} = P_{\mathrm{load}}, \quad \sum_{g\in\mathcal{G}} P^{\mathrm{e}}_{\mathcal{G},g}(t) = P_{\mathrm{load}} + \Delta P_{\mathrm{load}}. \tag{4}$$

Consider the following assumptions:

- The system operates synchronously post the disturbance, i.e., $\omega_g(t) = \omega_\ell(t) =: \omega(t)$, $\forall g, \ell \in \mathcal{G}$.
- The acceleration of all generators is the same at time $t = 0$ and denoted by $\dot{\omega}(0)$.

Answer the following questions:

1. Derive an expression that relates the acceleration of all generators at time $t = 0$, $\dot{\omega}(0)$ and ΔP_{load}.
2. Derive an expression that relates the post-disturbance steady-state frequency, $\omega(\infty) := \lim_{t\to\infty} \omega(t)$ to the net load change, ΔP_{load}.

[1] We abuse notation here slightly. Typically, the frequency-power speed-droop regulation constant—and not the inverse—is denoted by $R_{\mathcal{G},g}$.

3. If all the time constants of the governors are the same, i.e., $\tau_g = \tau \ \forall g \in \mathcal{G}$, derive the transfer function in analytical closed form from a net-load change, $\Delta P_{\text{load}}(s)$ to the frequency deviation, $\Delta\omega(s)$. (Hint: This is a second-order transfer function.)

Problem 5. [10 points] Suppose that the real- and reactive-power outputs of a given renewable energy system are denoted by P and Q, and further assume that P and Q are independent, standard-normal random variables. Obtain the joint distribution of the apparent power, $S = \sqrt{P^2 + Q^2}$, and the power-factor angle, $\theta = \tan^{-1}(Q/P)$.

Problem 6. [10 points] *Notation.* The matrix transpose will be denoted by $(\cdot)^{\mathrm{T}}$, complex conjugate by $(\cdot)^*$, real and imaginary parts of a complex number by $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$, respectively, magnitude of a complex scalar by $|\cdot|$, and $\mathrm{j} := \sqrt{-1}$. A diagonal matrix formed with entries of the vector x is denoted by $\mathrm{diag}(x)$; $\mathrm{diag}(x/y)$ forms a diagonal matrix with the ℓth entry given by x_ℓ/y_ℓ, where x_ℓ and y_ℓ are the ℓth entries of vectors x and y, respectively; and $\mathrm{diag}(1/x)$ forms a diagonal matrix with the ℓth entry given by x_ℓ^{-1}. For a vector $x = [x_1, \ldots, x_N]^{\mathrm{T}}$, $\cos(x) := [\cos(x_1), \ldots, \cos(x_N)]^{\mathrm{T}}$ and $\sin(x) := [\sin(x_1), \ldots, \sin(x_N)]^{\mathrm{T}}$. We will routinely decompose the complex-valued vector $x \in \mathbf{C}^N$ (complex-valued matrix $X \in \mathbf{C}^{N\times N}$) into its real and imaginary parts as follows: $x = x_{\mathrm{re}} + \mathrm{j}x_{\mathrm{im}}$ ($X = X_{\mathrm{re}} + \mathrm{j}X_{\mathrm{im}}$, respectively). The spaces of $N \times 1$ real-valued and complex-valued vectors are denoted by $\mathbf{R}^N$ and $\mathbf{C}^N$, respectively; $\mathbf{T}^N$ denotes the N-dimensional torus. With 0_N and 1_N, we denote N-dimensional column vectors with all entries equal to 0 and 1, respectively. The $N \times N$ identity matrix is denoted by $I_{N\times N}$ and the $N \times N$ matrix with 0 entries is denoted by $0_{N\times N}$.
Power System Model. Consider a power system with $N+1$ buses collected in the set $\mathcal{N}$. We model loads as the parallel interconnection of constant impedance and a constant power component. Without loss of generality, the slack bus is fixed to be the $N+1$ bus, and its voltage is denoted by $V_\circ \mathrm{e}^{\mathrm{j}\theta_\circ}$. Let $V = [V_1, \ldots, V_N]^{\mathrm{T}} \in \mathbf{C}^N$, where $V_\ell = |V|_\ell \angle \theta_\ell \in \mathbf{C}$ represents the voltage phasor at bus ℓ. In subsequent developments, we will find it useful to define the vectors $|V| = [|V_1|, \ldots, |V_N|]^{\mathrm{T}} \in \mathbf{R}^N_{>0}$ and $\theta = [\theta_1, \ldots, \theta_N]^{\mathrm{T}} \in \mathbf{T}^N$. Given our focus on rectangular coordinates, we will also routinely express $V = V_{\mathrm{re}} + \mathrm{j}V_{\mathrm{im}}$, where $V_{\mathrm{re}}, V_{\mathrm{im}} \in \mathbf{R}^N$ denote the real and imaginary components of V.

Let $I = [I_1, \ldots, I_N]^{\mathrm{T}}$, where $I_\ell \in \mathbf{C}$ denotes the current injected into bus ℓ. Kirchhoff's current law for the buses in the power system can be compactly represented in matrix-vector form as follows:

$$\begin{bmatrix} I \\ I_{N+1} \end{bmatrix} = \begin{bmatrix} Y & \overline{Y} \\ \overline{Y}^{\mathrm{T}} & y \end{bmatrix} \begin{bmatrix} V \\ V_\circ \mathrm{e}^{\mathrm{j}\theta_\circ} \end{bmatrix}, \tag{5}$$

where $V_\circ \mathrm{e}^{\mathrm{j}\theta_\circ}$ is the slack-bus voltage, I_{N+1} denotes the current injected into the slack bus, and the entries of the admittance matrix have the following dimensions: $Y \in \mathbf{C}^{N\times N}$, $\overline{Y} \in \mathbf{C}^N$, and $y \in \mathbf{C}\setminus\{0\}$. We will decompose the matrix Y and its inverse Y^{-1} as follows:

$$Y = G + \mathrm{j}B, \quad Y^{-1} = R + \mathrm{j}X. \tag{6}$$

Denote the vector of complex-power bus injections by $S = [S_1, \ldots, S_N]^{\mathrm{T}}$, where $S_\ell = P_\ell + \mathrm{j}Q_\ell$. By convention, P_ℓ and Q_ℓ are positive for generators and negative for loads.

Answer the following questions for the case when there are no shunt elements in the network and the reference-bus voltage is assumed to be $V_\circ \mathrm{e}^{\mathrm{j}\theta_\circ} = 1\angle 0$:

1. We will denote the voltage profile across the network when the vector of current injections, $I = 0_N$ by $V^{\mathrm{nom}} \in \mathbf{C}^N$. Show that V^{nom} is given by:

$$V^{\mathrm{nom}} = 1_N. \tag{7}$$

2. We will linearize the power-flow expressions around V^{nom}. Leveraging appropriate approximations for the magnitude and phase of complex vectors with small perturbations, show that the voltage magnitudes and phases across the network are approximately given by:

$$|V| \approx 1_N + [I_{N\times N} \quad 0_{N\times N}] \begin{bmatrix} G & -B \\ -B & -G \end{bmatrix}^{-1} \begin{bmatrix} P \\ Q \end{bmatrix}, \tag{8}$$

$$\theta \approx [0_{N\times N} \quad I_{N\times N}] \begin{bmatrix} G & -B \\ -B & -G \end{bmatrix}^{-1} \begin{bmatrix} P \\ Q \end{bmatrix}. \tag{9}$$

Problem 7. [10 points] Consider the nonlinear dynamical system

$$\dot{r} = \varepsilon\sigma\omega_0 \frac{r}{2}\left(1 - \frac{\alpha}{\sigma}r^2\right). \tag{10}$$

Answer the following questions:

1. Write down all equilibrium points of the above nonlinear dynamical system.

2. Indicate which of the equilibria are small-signal stable and which are not.

3. Let us attempt to quantify the dynamic performance of the above model. Focus on a notion of rise time, t_{rise}, defined to be the time for r to build up from 10% to 90% of the stable equilibrium point you found previously. Show that

$$t_{\mathrm{rise}} = \frac{6}{\varepsilon\sigma\omega_0}. \tag{11}$$

Problem 8. [10 points] Indicate whether the following statements are true (T) or false (F). No need to provide justification.

1. Given a one-area system with generators regulated with governor control, if a load is suddenly disconnected, the new steady-state frequency is less than the nominal.

2. The admittance matrix of a connected linear electrical network is always invertible.

3. If an equilibrium point of a nonlinear dynamical system is stable, it means that most of the eigenvalues of the corresponding linearized system have negative real parts.

4. Automatic generation control is the primary means of ensuring economic operation in the power system. As a side effect, it facilitates regulating frequencies across the network.

5. There is always a unique maximum power point in photovoltaic panel power-voltage characteristics.

6. Grid-forming inverters can be readily modeled as fixed and negative injections of active and reactive power in bulk system studies.

7. Droop control typically involves trading active power versus voltage magnitude, and reactive power versus frequency.

8. The base value for power in a three phase system is three times of that in a single phase system.

9. It is not challenging to operate a power-system with 100% grid-following phase-locked loop based inverters.

10. In order to solve the power-flow equations we always need a single slack bus that sets the angle reference.