HW 3

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Problem 1

In case of a balanced three-phase capacitive circuit with effective series resistance R, we can get the following dynamics from KVL in time domain:

$$C\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}v_a\\v_b\\v_c\end{bmatrix} - RC\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}i_a\\i_b\\i_c\end{bmatrix} = \begin{bmatrix}i_a\\i_b\\i_c\end{bmatrix}. \tag{1}$$

We know that, for a vector $[x_a\ x_b\ x_c]^T$ represented in 3-phase domain can be transformed to d-q axis (at an angle θ_d) with amplitude preservation in the following way:

$$\begin{bmatrix}x_d\\x_q\end{bmatrix} = \frac{2}{3}\Gamma_{dq}\begin{bmatrix}x_a\\x_b\\x_c\end{bmatrix},$$

where,

$$\Gamma_{dq} = \begin{bmatrix}\cos\theta_d & \cos(\theta_d - \frac{2\pi}{3}) & \cos(\theta_d + \frac{2\pi}{3})\\ -\sin\theta_d & -\sin(\theta_d - \frac{2\pi}{3}) & -\sin(\theta_d + \frac{2\pi}{3})\end{bmatrix}.$$

The inverse transform can be shown to be:

$$\begin{bmatrix}x_a\\x_b\\x_c\end{bmatrix} = \Gamma_{dq}^T\begin{bmatrix}x_d\\x_q\end{bmatrix}.$$

Let us derive $\frac{\mathrm{d}}{\mathrm{d}t}\Gamma_{dq}^T$ first as it will be required later.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\Gamma_{dq}^T &= \dot{\theta}_d\begin{bmatrix}-\sin\theta_d & \cos\theta_d\\ -\sin(\theta_d - \frac{2\pi}{3}) & -\cos(\theta_d - \frac{2\pi}{3})\\ -\sin(\theta_d + \frac{2\pi}{3}) & -\cos(\theta_d + \frac{2\pi}{3})\end{bmatrix}\\ &= \dot{\theta}_d\Gamma_{dq}^T\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}.\end{aligned}$$

Therefore, from Eq. 1,

$$\begin{aligned}&C\frac{\mathrm{d}}{\mathrm{d}t}\left(\Gamma_{dq}^T\begin{bmatrix}v_d\\v_q\end{bmatrix}\right) - RC\frac{\mathrm{d}}{\mathrm{d}t}\left(\Gamma_{dq}^T\begin{bmatrix}i_d\\i_q\end{bmatrix}\right) = \Gamma_{dq}^T\begin{bmatrix}i_d\\i_q\end{bmatrix}\\ \implies &C\dot{\theta}_d\Gamma_{dq}^T\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}v_d\\v_q\end{bmatrix} + C\Gamma_{dq}^T\begin{bmatrix}\dot{v}_d\\\dot{v}_q\end{bmatrix} - RC\dot{\theta}_d\Gamma_{dq}^T\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}i_d\\i_q\end{bmatrix} - RC\Gamma_{dq}^T\begin{bmatrix}\dot{i}_d\\\dot{i}_q\end{bmatrix} = \Gamma_{dq}^T\begin{bmatrix}i_d\\i_q\end{bmatrix}\\ \implies &C\dot{\theta}_d\begin{bmatrix}-v_q\\v_d\end{bmatrix} + C\begin{bmatrix}\dot{v}_d\\\dot{v}_q\end{bmatrix} - RC\dot{\theta}_d\begin{bmatrix}-i_q\\i_d\end{bmatrix} - RC\begin{bmatrix}\dot{i}_d\\\dot{i}_q\end{bmatrix} = \begin{bmatrix}i_d\\i_q\end{bmatrix}\\ \implies &C\left(\begin{bmatrix}\dot{v}_d\\\dot{v}_q\end{bmatrix} - R\begin{bmatrix}\dot{i}_d\\\dot{i}_q\end{bmatrix}\right) + C\dot{\theta}_d\begin{bmatrix}-v_q\\v_d\end{bmatrix} - RC\dot{\theta}_d\begin{bmatrix}-i_q\\i_d\end{bmatrix} = \begin{bmatrix}i_d\\i_q\end{bmatrix}.\end{aligned}$$

Therefore, the dynamics in dq domain can be written as:

$$\begin{aligned}C\frac{\mathrm{d}}{\mathrm{d}t}(v_d - i_dR) - C\dot{\theta}_dv_q + RC\dot{\theta}_di_q = i_d,\\ C\frac{\mathrm{d}}{\mathrm{d}t}(v_q - i_qR) + C\dot{\theta}_dv_d - RC\dot{\theta}_di_d = i_q.\end{aligned}$$

Problem 2

We know that for any vector $[x_a \; x_b \; x_c]^T$ in 3-phase time domain can be transformed to $\alpha - \beta$ domain, with amplitude preservation, as follows:

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = T \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix},$$

where,

$$T = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}.$$

The inverse transform can be obtained using pseudo-inverse of T. We can use singular value decomposition (SVD) to derive the pseudo-inverse of T. Using MATLAB, we get the following SVD components:

$$T = USV^T,$$

where,

$$\begin{aligned} U &= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \\ S &= \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ 0 & \frac{\sqrt{2}}{\sqrt{3}} \end{bmatrix}, \\ V &= \begin{bmatrix} 0 & -\frac{\sqrt{2}}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{2\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{2\sqrt{3}} \end{bmatrix}. \end{aligned}$$

Therefore, pseudo-inverse of T is given by:

$$\begin{aligned} T^+ &= VS^{-1}U^T \\ &= \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}. \end{aligned}$$

Therefore,

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = T^+ \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}.$$

Now, the dynamics given in the question is:

$$\begin{aligned} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} &= (L - M)\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \\ \implies T\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} &= (L - M)\frac{\mathrm{d}}{\mathrm{d}t}\left(T\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}\right) \\ \implies T^+T\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} &= (L - M)T^+T\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \\ \implies \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} &= (L - M)\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}. \end{aligned}$$

This is the corresponding dynamics in the abc-domain.

Problem 3

In the $\alpha - \beta$ reference frame, the network and the inverter terminal voltages are given as:

$$\begin{aligned} e_\alpha &= \sqrt{2}E\cos\omega_e t \\ e_\beta &= \sqrt{2}E\sin\omega_e t, \end{aligned} \tag{2}$$

and

$$\begin{aligned} v_\alpha &= \sqrt{2}V\cos\omega_i t \\ v_\beta &= \sqrt{2}V\sin\omega_i t. \end{aligned} \tag{3}$$

The reference angle difference is given by:

$$\delta = \theta_i - \theta_e, \tag{4}$$

where, $\theta_i = \omega_i t$ and $\theta_e = \omega_e t$. The droop control equations are given by:

$$\begin{aligned} V &= V_{nom} - m_q(\overline{Q} - Q^*), \\ \omega_i &= \omega_{nom} - m_p(\overline{P} - P^*), \end{aligned} \tag{5}$$

where the $\overline{P}, \overline{Q}$ are the filtered active and reactive powers and their dynamics are given by:

$$\begin{aligned} \dot{\overline{P}} &= \omega_c(\overline{P} - P), \\ \dot{\overline{Q}} &= \omega_c(\overline{Q} - Q), \end{aligned} \tag{6}$$

where P, Q are instantaneous active and reactive power at the inverter terminal. Therefore, from Eq. 5 and Eq. 6, we can derive that,

$$\begin{aligned} \dot{V} &= -m_q(\dot{\overline{Q}}) \\ &= -m_q\omega_c(\overline{Q} - Q), \\ \dot{\theta}_i &= \omega_i = \omega_{nom} - m_p(\overline{P} - P^*), \\ \dot{\delta} &= \omega_{nom} - \omega_e - m_p(\overline{P} - P^*). \end{aligned} \tag{7}$$

Problem 3.i

In the d-q reference frame, as shown in Fig. (1) in the question,

$$\begin{aligned} \begin{bmatrix} e_d \\ e_q \end{bmatrix} &= \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix} \begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} \\ &= \sqrt{2}E \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix} \begin{bmatrix} \cos\theta_e \\ \sin\theta_e \end{bmatrix} \\ &= \sqrt{2}E \begin{bmatrix} \cos\theta_i\cos\theta_e + \sin\theta_i\sin\theta_e \\ -\cos\theta_e\sin\theta_i + \sin\theta_e\cos\theta_i \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{2}E\cos\delta \\ -\sqrt{2}E\sin\delta \end{bmatrix}. \end{aligned} \tag{8}$$

Before proceeding, let us derive v_d, v_q also, which will be required later.

$$\begin{aligned} \begin{bmatrix} v_d \\ v_q \end{bmatrix} &= \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \\ &= \sqrt{2}V \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix} \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{2}V \\ 0 \end{bmatrix}. \end{aligned} \tag{9}$$

Problem 3.ii

We know that, for a vector $[x_a\ x_b\ x_c]^T$ represented in 3-phase domain can be transformed to d-q axis (at an angle θ_i) with amplitude preservation in the following way:

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \frac{2}{3}\Gamma_{dq} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix},$$

where,

$$\Gamma_{dq} = \begin{bmatrix} \cos\theta_i & \cos(\theta_i - \frac{2\pi}{3}) & \cos(\theta_i + \frac{2\pi}{3}) \\ -\sin\theta_i & -\sin(\theta_i - \frac{2\pi}{3}) & -\sin(\theta_i + \frac{2\pi}{3}) \end{bmatrix}.$$

The inverse transform can be shown to be:

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \Gamma_{dq}^T \begin{bmatrix} x_d \\ x_q \end{bmatrix}.$$

Let us derive $\frac{\mathrm{d}}{\mathrm{d}t}\Gamma_{dq}^T$ first as it will be required later.

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\Gamma_{dq}^T &= \dot{\theta}_i \begin{bmatrix} -\sin\theta_i & \cos\theta_i \\ -\sin(\theta_i - \frac{2\pi}{3}) & -\cos(\theta_i - \frac{2\pi}{3}) \\ -\sin(\theta_i + \frac{2\pi}{3}) & -\cos(\theta_i + \frac{2\pi}{3}) \end{bmatrix} \\ &= \dot{\theta}_i \Gamma_{dq}^T \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \end{aligned}$$

Now, using KVL in the inverter, RL, and network model, we get:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R_f \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + L_f \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}.$$

Transforming all the vectors to d-q axis, we get:

$$\begin{aligned} \Gamma_{dq}^T \begin{bmatrix} v_d \\ v_q \end{bmatrix} &= R_f \Gamma_{dq}^T \begin{bmatrix} i_d \\ i_q \end{bmatrix} + L_f \frac{\mathrm{d}}{\mathrm{d}t}\left(\Gamma_{dq}^T \begin{bmatrix} i_d \\ i_q \end{bmatrix}\right) + \Gamma_{dq}^T \begin{bmatrix} e_d \\ e_q \end{bmatrix} \\ \implies \begin{bmatrix} v_d \\ v_q \end{bmatrix} &= R_f \begin{bmatrix} i_d \\ i_q \end{bmatrix} + L_f \dot{\theta}_i \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + L_f \begin{bmatrix} \dot{i}_d \\ \dot{i}_q \end{bmatrix} + \begin{bmatrix} e_d \\ e_q \end{bmatrix} \\ \implies \begin{bmatrix} \dot{i}_d \\ \dot{i}_q \end{bmatrix} &= -\frac{R_f}{L_f} \begin{bmatrix} i_d \\ i_q \end{bmatrix} - \dot{\theta}_i \begin{bmatrix} -i_q \\ i_d \end{bmatrix} + \frac{1}{L_f} \begin{bmatrix} v_d \\ v_q \end{bmatrix} - \frac{1}{L_f} \begin{bmatrix} e_d \\ e_q \end{bmatrix} \\ \implies \begin{bmatrix} \dot{i}_d \\ \dot{i}_q \end{bmatrix} &= \begin{bmatrix} -\frac{R_f}{L_f} & \dot{\theta}_i \\ -\dot{\theta}_i & -\frac{R_f}{L_f} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{1}{L_f} \begin{bmatrix} \sqrt{2}V - e_d \\ -e_q \end{bmatrix}. \end{aligned} \tag{10}$$

Problem 3.iii

Using 9, we can derive the instantaneous active power at the inverter terminal as follows:

$$\begin{aligned} P &= \frac{3}{2}(v_d i_d + v_q i_q) \\ &= \frac{3}{2}(\sqrt{2}V i_d). \end{aligned} \tag{11}$$

Similarly the instantaneous reactive power is:

$$\begin{aligned} Q &= \frac{3}{2}(-v_d i_q + v_q i_d) \\ &= -\frac{3}{2}\sqrt{2}V i_q. \end{aligned}$$

Problem 3.iv

$$
\begin{aligned}
\dot{\delta} &= f_\delta = \omega_{nom} - \omega_e - m_p(\overline{P} - P^*) \\
\dot{V} &= f_v = -m_q\omega_c(\overline{Q} - Q) \\
&= -m_q\omega_c\left(\frac{V_{nom} - V}{m_q} + Q^*\right) + m_q\omega_c Q \\
&= \omega_c V - \omega_c V_{nom} - m_q\omega_c\left(\frac{3}{2}\sqrt{2}V i_q + Q^*\right) \\
\dot{i_d} &= f_{i_d} = -\frac{R_f}{L_f}i_d + \dot{\theta}_i i_q + \frac{1}{L_f}(\sqrt{2}V - e_d) \\
&= -\frac{R_f}{L_f}i_d + \omega_i i_q + \frac{\sqrt{2}}{L_f}(V - E\cos\delta) \\
\dot{i_q} &= f_{i_q} = -\dot{\theta}_i i_d - \frac{R_f}{L_f}i_q + \frac{1}{L_f}(-e_q) \\
&= -\omega_i i_d - \frac{R_f}{L_f}i_q + \frac{\sqrt{2}}{L_f}(E\sin\delta) \\
\dot{\overline{P}} &= f_{\overline{P}} = \omega_c(\overline{P} - P) = \omega_c\left(\overline{P} - \frac{3}{2}\sqrt{2}V i_d\right)
\end{aligned} \tag{12}
$$

Problem 3.v

In equilibria,

$$
\begin{aligned}
\dot{\delta} &= 0 \\
\dot{V} &= 0 \\
\dot{i_d} &= 0 \\
\dot{i_q} &= 0 \\
\dot{\overline{P}} &= 0.
\end{aligned}
$$

Therefore, from 12, we get:

$$
\begin{aligned}
\omega_{nom} - \omega_c - m_p(\overline{P}_{eq} - P^*) &= 0, \\
V_{eq} - V_{nom} - m_q\left(\frac{3}{2}\sqrt{2}V_{eq}i_{q,eq} + Q^*\right) &= 0, \\
-\frac{R_f}{L_f}i_{d,eq} + \omega_i i_{q,eq} + \frac{\sqrt{2}}{L_f}(V_{eq} - E\cos\delta_{eq}) &= 0, \\
-\omega_i i_{d,eq} - \frac{R_f}{L_f}i_{q,eq} + \frac{\sqrt{2}}{L_f}(E\sin\delta_{eq}) &= 0, \\
\overline{P}_{eq} - \frac{3}{2}\sqrt{2}V_{eq}i_{d,eq} &= 0.
\end{aligned}
$$

Problem 3.vi

From Eq. 12 and Eq. 7, we get:

$$
\begin{aligned}
\dot{i_d} &= f_{i_d} = -\frac{R_f}{L_f}i_d + \left(\omega_{nom} - m_p(\overline{P} - P^*)\right)i_q + \frac{\sqrt{2}}{L_f}(V - E\cos\delta) \\
\dot{i_q} &= f_{i_q} = -\left(\omega_{nom} - m_p(\overline{P} - P^*)\right)i_d - \frac{R_f}{L_f}i_q + \frac{\sqrt{2}}{L_f}(E\sin\delta)
\end{aligned}
$$

Therefore, the linearized model of Eq. 12 is given as follows:

$$
\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{V} \\ \Delta\dot{i_d} \\ \Delta\dot{i_q} \\ \Delta\dot{\overline{P}} \end{bmatrix} = \begin{bmatrix}
\frac{\partial f_\delta}{\partial \delta}\Big|_{eq} & \frac{\partial f_\delta}{\partial V}\Big|_{eq} & \frac{\partial f_\delta}{\partial i_d}\Big|_{eq} & \frac{\partial f_\delta}{\partial i_q}\Big|_{eq} & \frac{\partial f_\delta}{\partial \overline{P}}\Big|_{eq} \\
\frac{\partial f_V}{\partial \delta}\Big|_{eq} & \frac{\partial f_V}{\partial V}\Big|_{eq} & \frac{\partial f_V}{\partial i_d}\Big|_{eq} & \frac{\partial f_V}{\partial i_q}\Big|_{eq} & \frac{\partial f_V}{\partial \overline{P}}\Big|_{eq} \\
\frac{\partial f_{i_d}}{\partial \delta}\Big|_{eq} & \frac{\partial f_{i_d}}{\partial V}\Big|_{eq} & \frac{\partial f_{i_d}}{\partial i_d}\Big|_{eq} & \frac{\partial f_{i_d}}{\partial i_q}\Big|_{eq} & \frac{\partial f_{i_d}}{\partial \overline{P}}\Big|_{eq} \\
\frac{\partial f_{i_q}}{\partial \delta}\Big|_{eq} & \frac{\partial f_{i_q}}{\partial V}\Big|_{eq} & \frac{\partial f_{i_q}}{\partial i_d}\Big|_{eq} & \frac{\partial f_{i_q}}{\partial i_q}\Big|_{eq} & \frac{\partial f_{i_q}}{\partial \overline{P}}\Big|_{eq} \\
\frac{\partial f_{\overline{P}}}{\partial \delta}\Big|_{eq} & \frac{\partial f_{\overline{P}}}{\partial V}\Big|_{eq} & \frac{\partial f_{\overline{P}}}{\partial i_d}\Big|_{eq} & \frac{\partial f_{\overline{P}}}{\partial i_q}\Big|_{eq} & \frac{\partial f_{\overline{P}}}{\partial \overline{P}}\Big|_{eq}
\end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \\ \Delta i_d \\ \Delta i_q \\ \Delta\overline{P} \end{bmatrix}
$$

$$
= \begin{bmatrix}
0 & 0 & 0 & 0 & -m_p \\
0 & \omega_c - m_q\omega_c\frac{3}{2}\sqrt{2}i_{q,eq} & 0 & -m_q\omega_c\frac{3}{2}\sqrt{2}V_{eq} & 0 \\
\frac{\sqrt{2}}{L_f}E\sin\delta_{eq} & \frac{\sqrt{2}}{L_f} & -\frac{R_f}{L_f} & \omega_{nom} - m_p(\overline{P}_{eq} - P^*) & -m_p i_{q,eq} \\
\frac{\sqrt{2}}{L_f}E\cos\delta_{eq} & 0 & -\left(\omega_{nom} - m_p(\overline{P}_{eq} - P^*)\right) & -\frac{R_f}{L_f} & m_p i_{d,eq} \\
0 & -\frac{3}{2}\sqrt{2}\omega_c i_{d,eq} & -\frac{3}{2}\sqrt{2}V_{eq} & 0 & \omega_c
\end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \\ \Delta i_d \\ \Delta i_q \\ \Delta\overline{P} \end{bmatrix}
$$

$$
= A \begin{bmatrix} \Delta\delta \\ \Delta V \\ \Delta i_d \\ \Delta i_q \\ \Delta\overline{P} \end{bmatrix}
$$

Problem 4.i

The time domain dynamics of the circuit is given by:

$$
\begin{bmatrix} v_t^a \\ v_t^b \\ v_t^c \end{bmatrix} = L\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + R\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} v_0^a \\ v_0^b \\ v_0^c \end{bmatrix}.
$$

Converting them to d-q domain (at an angle $\theta_d = \omega_d t$) as illustrated in Q3, we get:

$$
\begin{aligned}
\Gamma_{dq}^T \begin{bmatrix} v_t^d \\ v_t^q \end{bmatrix} &= L\frac{\mathrm{d}}{\mathrm{d}t}\left(\Gamma_{dq}^T \begin{bmatrix} i_d \\ i_q \end{bmatrix}\right) + R\Gamma_{dq}^T \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \Gamma_{dq}^T \begin{bmatrix} v_0^d \\ v_0^q \end{bmatrix} \\
\implies \Gamma_{dq}^T \begin{bmatrix} v_t^d \\ v_t^q \end{bmatrix} &= L\dot{\theta}_d\Gamma_{dq}^T \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + L\Gamma_{dq}^T \begin{bmatrix} \dot{i_d} \\ \dot{i_q} \end{bmatrix} + R\Gamma_{dq}^T \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \Gamma_{dq}^T \begin{bmatrix} v_0^d \\ v_0^q \end{bmatrix} \\
\implies \begin{bmatrix} v_t^d \\ v_t^q \end{bmatrix} &= L\omega_d \begin{bmatrix} -i_q \\ i_d \end{bmatrix} + L\begin{bmatrix} \dot{i_d} \\ \dot{i_q} \end{bmatrix} + R\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} v_0^d \\ v_0^q \end{bmatrix}.
\end{aligned}
$$

Converting the above equations in block diagrams along with feedback control and following Fig. 3 in the question for point of feed forward, we get the following block diagram shown in Fig. 1 in dq domain as requested in the question.

Problem 4.ii

To derive the closed-loop transfer functions $\frac{i_{dq}(s)}{i_{dq}^*(s)}$, we need to ignore the cross-coupling terms and v_{dc} in Fig. 3 (in the question). Then we can get the following the block diagram for $i_d(s)$ (similar diagram for $i_q(s)$ too) as shown in Fig. 2:

Considering $G_c(s) = (k_p + k_i/s)$, we can derive the transfer function as follows:

$$
H(s) = \frac{i_d(s)}{i_d^*(s)}
$$

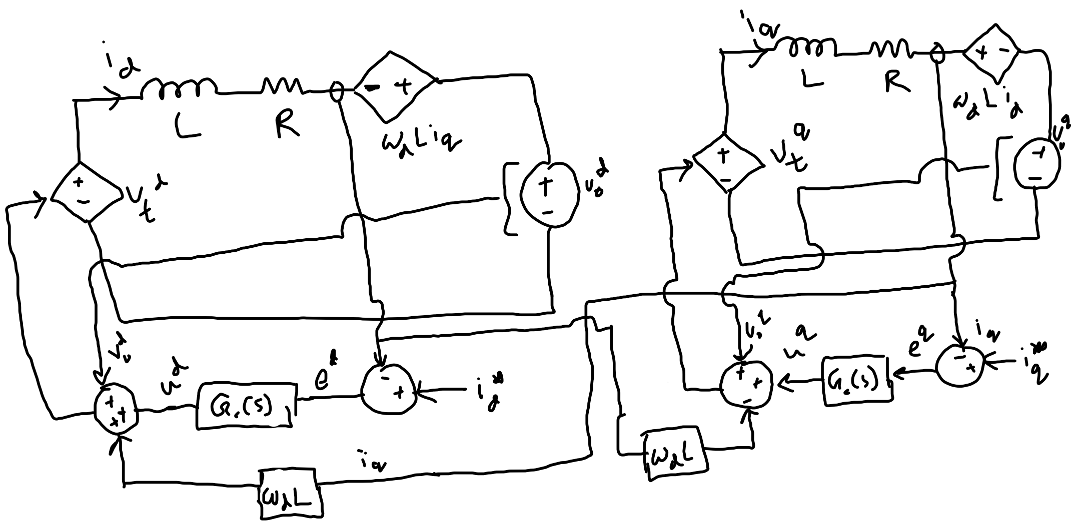

Figure 1: Q4: Inverter feedback control block diagram

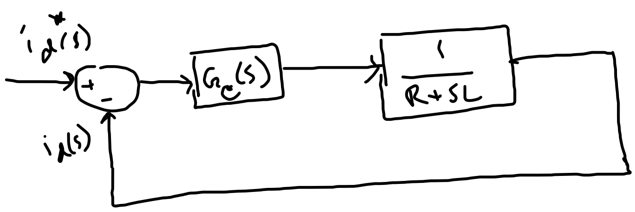

Figure 2: Q4: Block diagram to derive current transfer function

$$\begin{aligned} &= \frac{G_c(s)/(R+sL)}{1+G_c/(R+sL)} \\ &= \frac{G_c(s)}{(R+sL)+G_c(s)} \\ &= \frac{k_p+k_i/s}{(k_p+k_i/s)+(R+sL)}. \end{aligned} \tag{13}$$

Problem 4.iii

From Eq. 13, we get,

$$\begin{aligned} H(s) &= \frac{sk_p+k_i}{s^2L+s(R+k_p)+k_i} \\ &= \frac{1}{\frac{s^2L}{sk_p+k_i}+\frac{s(R+k_p)}{sk_p+k_i}+\frac{k_i}{sk_p+k_i}} \end{aligned} \tag{14}$$

Equating denominator of Eq. 14 with that of desired transfer function $\frac{1}{1+\tau s}$, we get:

$$\begin{aligned} &\frac{s^2L}{sk_p+k_i}+\frac{s(R+k_p)}{sk_p+k_i}+\frac{k_i}{sk_p+k_i} = (1+\tau s) \\ \implies &s^2L+s(R+k_p)+k_i = s^2\tau k_p+s(k_p+\tau k_i)+k_i. \end{aligned}$$

Equating corresponding coefficients of s, we get:

$$L = \tau k_p \implies \tau = \frac{L}{k_p}$$

$$R + k_p = k_p + \tau k_i \implies \tau = \frac{R}{k_i}.$$

Therefore,

$$\frac{L}{k_p} = \frac{R}{k_i}$$
$$\implies \frac{L}{R} = \frac{k_p}{k_i}.$$

Similarly, we can prove the *only if* part as follows.
Let, $\frac{L}{R} = \frac{k_p}{k_i} = c$. Also denote $\tau \coloneqq \frac{R}{k_i}$. Then $\tau = \frac{L}{k_p}$ also. From Eq. 13,

$$\begin{aligned}
H(s) &= \frac{k_p + k_i/s}{(k_p + k_i/s) + (R + sL)} \\
&= \frac{1}{1 + \frac{R+sL}{k_p+k_i/s}} \\
&= \frac{1}{1 + \frac{(R/k_i)+s(L/k_i)}{(k_p/k_i)+(1/s)}} \\
&= \frac{1}{1 + \frac{\tau+s\frac{L}{k_p}\frac{k_p}{k_i}}{c+(1/s)}} \\
&= \frac{1}{1 + \frac{\tau+sc\frac{L}{k_p}}{c+(1/s)}} \\
&= \frac{1}{1 + \frac{s\tau+s^2c\tau}{sc+1}} \\
&= \frac{1}{1 + \tau s}.
\end{aligned}$$