Economic Dispatch Formulations

Decision Variable	Description
P_t^{DG}	power produced by a diesel generator at time t
$P_t^{MG,b}$	power bought from the main grid at time t
$P_t^{MG,s}$	power sold to the main grid at time t
$P_t^{B,ch}$	power to charge the battery at time t
$P_t^{B,dch}$	power discharged from the battery at time t
$S_t^{B,ch}$	Charging status of the battery
$S_t^{B,dch}$	Discharging status of the battery
$S_t^{MG,b}$	Buying status of the main grid
$S_t^{MG,s}$	Selling status of the main grid
SOC_t	State of charge of the Battery at time t

Constants	Description
T	Number of time slots
α	Cost coefficient of the diesel generator
η_c & η_d	Charging and discharging efficiencies of the Battery
ρ^b & ρ^s	Main grid buying and selling price
γ^b	Depreciation cost of the Battery
SOC_0 & SOC_T	Initial and final state of charge of the Battery
E^{max}	Maximum capacity of the Battery in KWh
$P^{DG,min}$ & $P^{DG,max}$	Minimum and maximum power generation by Diesel Generator
$P^{MG,min}$ & $P^{MG,max}$	Minimum and maximum power import or export of power from the main grid
RU & RD	Diesel generator's ramp up and down limits

I. Basic Economic Dispatch ILP Formulation

A. Constraints

1) Power balance Constraint:

$$\Big(P_t^{DG} + P_t^{MG,b} + P_t^{PV} + P_t^{WT} + P_t^{B,dch}\Big) \geq P^{B,ch} + P^{L} + P_t^{MG,s} \ \forall t \tag{1}$$

2) Ramp rate limits of Diesel Generator:

$$P_t^{DG} - P_{t-1}^{DG} \leq RU \ \forall t \tag{2}$$

$$P_{t-1}^{DG} - P_t^{DG} \leq RD \ \forall t \tag{3}$$

3) Battery State of Charge constraint:

$$SOC_t = SOC_{t-1} - \frac{P^{B,dch} * \Delta t}{\eta_d * E^{max}} + \frac{P^{B,ch} * \Delta t * \eta_c}{E^{max}} \ \forall t \tag{4}$$

4) Power and Battery SOC limits:

$$P^{DG,min} \leq P_t^{DG} \leq P^{DG,max} \ \forall t \tag{5}$$

$$P^{MG,min} \leq P_t^{MG,b}, P_t^{MG,s} \leq P^{MG,max} \ \forall t \tag{6}$$

$$P^{B,min} \leq P_t^{B,ch}, P_t^{B,dch} \leq P^{B,max} \ \forall t \tag{7}$$

$$SOC^{min} \leq SOC_t \leq SOC^{max} \ \forall t \tag{8}$$

B. Objective Function

$$\textbf{Minimize} \sum_{t=1}^{T} \Big(\alpha * P_t^{DG} + \rho_t^b * P_t^{MG,b} - \rho_t^s * P_t^{MG,s} + \gamma^b * P^{B,ch} + \gamma^b * P^{B,dch}\Big) \tag{9}$$

II. Extended Economic Dispatch ILP Formulation

1) Power balance Constraint:

$$\Big(P_t^{DG} + S_t^{MG,b} * P_t^{MG,b} + P_t^{PV} + P_t^{WT} + S_t^{B,dch} * P_t^{B,dch}\Big) \geq S_t^{B,ch} * P^{B,ch} + P^{L} + S_t^{MG,s} * P_t^{MG,s} \ \forall t \tag{10}$$

2) Battery State of Charge constraint:

$$SOC_t = SOC_{t-1} - \frac{S^{B,dch} * P^{B,dch}}{\eta_d * E^{max}} + \frac{S^{B,ch} * P^{B,ch} * \eta_c}{E^{max}} \ \forall t \tag{11}$$

$$S_t^{B,ch} + S_t^{B,dch} \leq 1 \ \forall t \tag{12}$$

$$S_t^{B,ch} \ \& \ S_t^{B,dch} \in \{0,1\} \ \forall t \tag{13}$$

3) Power and Battery SOC limits:

$$S^{MG,b} * P^{MG,min} \leq P_t^{MG,b} \leq, S_t^{MG,b} * P^{MG,max} \ \forall t \tag{14}$$

$$S^{MG,s} * P^{MG,min} \leq P_t^{MG,s} \leq, S_t^{MG,s} * P^{MG,max} \ \forall t \tag{15}$$

$$S_t^{B,ch} * P^{B,min} \leq P_t^{B,ch} \leq S_t^{B,ch} * P^{B,max} \ \forall t \tag{16}$$

$$S_t^{B,dch} * P^{B,min} \leq P_t^{B,dch} \leq S_t^{B,dch} * P^{B,max} \ \forall t \tag{17}$$

4) Main grid constraints:

$$S_t^{MG,b} + S_t^{MG,s} \leq 1 \ \forall t \tag{18}$$

$$S_t^{MG,b} \ \& \ S_t^{MG,s} \in \{0,1\} \ \forall t \tag{19}$$

5) Linearizing constraints: Let x be a binary decision variable and y be a continuous/integer decision variable within the range $\{y^{min}, y^{max}\}$, then the product xy is a nonlinear term. In order to linearize, the term xy (for example, $S^{B,ch} * P^{B,ch}$) is replaced with another decision variable say z and the following constraints are need to be added to the system of equations.

$$y^{min} * x \leq z \leq y^{max} * x \tag{20}$$

$$y - (1-x)y^{max} \leq z \leq y + (1-x)y^{max} \tag{21}$$