Homework 8

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1 Monte-Carlo Integration

1.1 Integral

$$\int_0^1 x^2 \, dx$$

Analytical Solution:

$$I = [\frac{x^3}{3}]_0^1 = \frac{1}{3} \tag{1}$$

Monte-Carlo Integration algorithm:

The solution got from Monte-Carlo Integration: 0.3334

1.2 Standard Deviation of the Sample Mean

$$\text{Standard Deviation} = \sqrt{\frac{\text{Var}(x^2)}{M}}$$

where M is the number of sample points. The standard deviation value was 0.0011276711

1.3 Importance Sampling Probability Distribution Function

$$W(x) = \frac{x^2 + 2.0}{N}$$

1.4 Sample Mean with Importance Sampling

$$\text{Sample Mean} = \frac{1}{M} \sum_{k=1}^{M} \frac{x_k^2}{x_k^2 + 2.0} \cdot N$$

where M is the number of sample points, and N is the normalization constant.

N is the normalization constant that needs to be determined. It can be determined by integrating the probability distribution function $W(x)$ over the range $[0, 1]$:

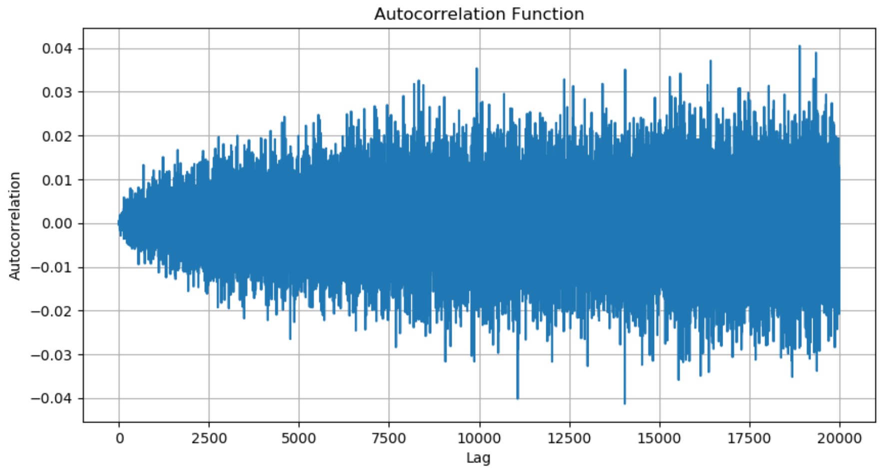

Figure 1: Autocorrelation curve for skipped points

The integral is for skipped samples 55000 is 0.2501967368588373

The integral is for skipped samples 60000 is 0.2654873901342117

The integral is for skipped samples 65000 is 0.28360273847621514

The integral is for skipped samples 70000 is 0.2991448978876265

The integral is for skipped samples 75000 is 0.31702274075374465

The integral is for skipped samples 80000 is 0.3330567531949177

The integral is for skipped samples 85000 is 0.3491850485314857

The integral is for skipped samples 90000 is 0.3659161309431519

The integral is for skipped samples 95000 is 0.38262332360758383

The integral is for skipped samples 100000 is 0.401702426910699

The integral values are changing with the skipped samples.

because the delta value for selecting x in next iteration should also change with the skipped samples to adjust the acceptance rate, hence we should adjust the delta value according to the skipped samples