STA 5107/4013: Home Assignment # 5

Spring 2021/Due Date: February 25th

1. You have written a program to use Metropolis-Hastings algorithm to sample a random variable X with the density

$$f(x) = \frac{x^2|sin(\pi x)|e^{-|x|^3}}{\int_{\mathbb{R}_+} x^2|sin(\pi x)|e^{-|x|^3}dx} \ , \quad x > 0 \ .$$

Compare the acceptance rates of the following proposal densities:

- $q(x) \propto \exp(-x)$, $x > 0$.
- $q(x) \propto \exp(-2x)$, $x > 0$.
- $q(x) \propto \exp(-\frac{1}{2}x^2)$, $x \in \mathbb{R}$.
- $q(x) \propto \exp(-\frac{1}{2}x^2)$, $x > 0$.

2. Consider the problem of sampling from a posterior density when it is easy to sample from the prior density. For two continuous random variables X and Y, the Bayes' rule states that:

$$f_{X|Y}(x|y) = \frac{f_{Y|X}(y|x)f_X(x)}{f_Y(y)} \ .$$

$f_{X|Y}(x|y)$ is the *posterior density*, $f_X(x)$ is the prior density, $f_{Y|X}(y|x)$ is called the likelihood function (as a function of x), and $f_Y(y)$ is a normalizer (which is fixed for a given y). Let X be a normal random variable with mean 5 and variance 4, i.e. $X \ \sim \mathcal{N}(5, 4)$and the conditional density of Y given $X = x$ is given by:

$$f_{Y|X}(y|x) = \frac{1}{Z}e^{-\lambda|y-x|^{0.5}} \ .$$

Choose the prior $f_X(x)$ as the proposal density and the posterior $f_{X|Y}(x|y)$ as the target density. Set $\lambda = 1$.

(a) Write a matlab program implementing Metropolis-Hastings algorithm to sample from the posterior for $y = 4$. Show multiple evolutions of this chain for several starting points.

(b) Use the Markov chain to estimate the posterior mean and variance .