Home Assignment 4

Problem 1

```
Pi=[0.1 0.3 0.4 0.2;
    0.2 0.1 0.3 0.4;
    0.4 0.2 0.1 0.3;
    0.3 0.4 0.2 0.1];
[u,v]=eig(Pi')
```

```
u = 4×4 complex
   0.5000 + 0.0000i   0.5000 + 0.0000i   0.0000 + 0.5000i   0.0000 - 0.5000i
   0.5000 + 0.0000i  -0.5000 + 0.0000i   0.5000 + 0.0000i   0.5000 + 0.0000i
   0.5000 + 0.0000i   0.5000 + 0.0000i  -0.0000 - 0.5000i  -0.0000 + 0.5000i
   0.5000 + 0.0000i  -0.5000 + 0.0000i  -0.5000 + 0.0000i  -0.5000 - 0.0000i
v = 4×4 complex
   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3000 + 0.1000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3000 - 0.1000i
```

Staionary Probablity vector

```
P=u(:,1)/sum(u(:,1))
```

```
P = 4×1
    0.2500
    0.2500
    0.2500
    0.2500
```

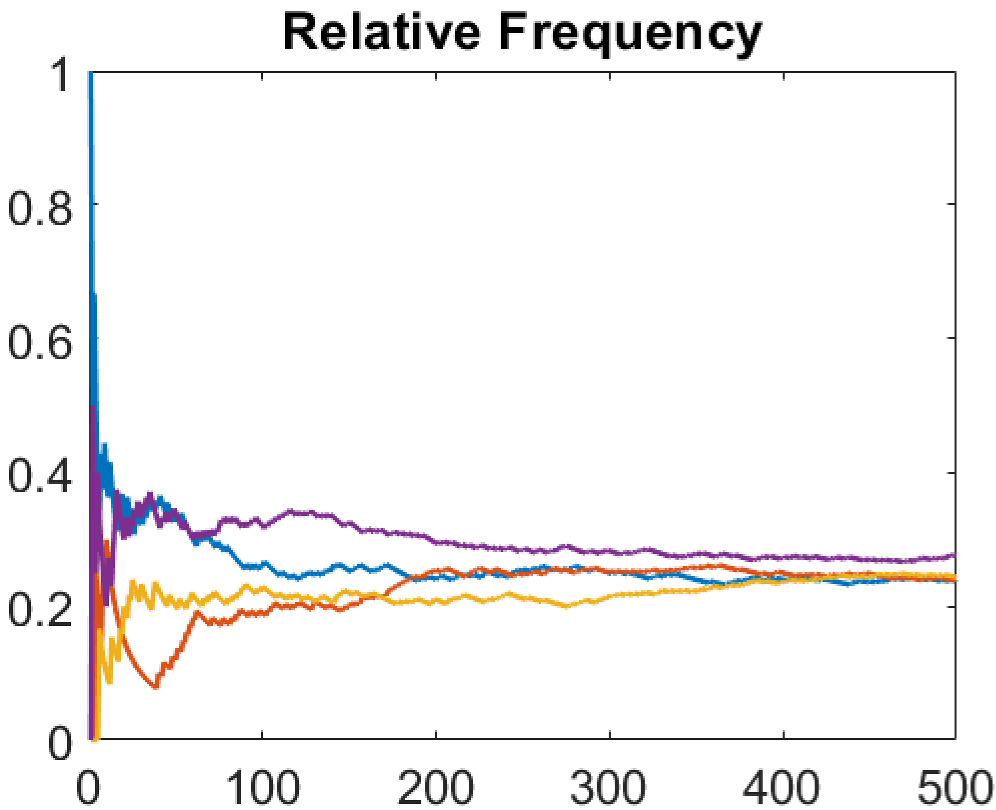
```
T = 500;
n=4;
X(1) = 1;

for t = 2:T
    p = cumsum(Pi(X(t-1),:));
    X(t) = min(find(rand<p));
end

for t = 1:T
   for i = 1:n
```

```
        freq(i,t) = length(find(X(1:t)==i))/t;
    end
end


plot([1:T],freq,'linewidth',2);
title("Relative Frequency")
set(gca,'fontsize',18);
```




```
sum1 = 0;
sum2 = 0;
sum3 = 0;
sum4 = 0;
for i = 1:T
if X(i)==1
sum1 = sum1+2;
end
if X(i)==2
sum2 = sum2+1;
end
if X(i)==3
sum3 = sum3+2.5;
end
if X(i)==3
```

```
sum4 = sum4-1;
end
end
```

Average along sample path

```
average = (sum1+sum2+sum3+sum4)/T
```

average = 1.0860

```
f=[2 1 2.5 -1]';
```

Limit of Average

```
limit=(P')*f
```

limit = 1.1250

Problem 2

- The markov chain is irreducible since all the element except (4,4) is non zero which means one can travel from any state i to j in finite no of steps
- The markov chain is aperiodic as it is irreducible and the period of 1st state is 1.

```
Pi=[0.1 0.3 0.4 0.2;
    0.2 0.4 0.0 0.4;
    0.0 0.3 0.5 0.2;
    0.5 0.3 0.2 0.0];
[u,v]=eig(Pi')
```

```
u = 4×4 complex
  -0.3870 + 0.0000i  -0.7071 + 0.0000i  -0.7071 + 0.0000i  -0.8111 + 0.0000i
  -0.6531 + 0.0000i   0.0000 - 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i
  -0.4838 + 0.0000i   0.7071 + 0.0000i   0.7071 - 0.0000i   0.3244 + 0.0000i
  -0.4354 + 0.0000i  -0.0000 - 0.0000i  -0.0000 + 0.0000i   0.4867 + 0.0000i
v = 4×4 complex
   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.1000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.1000 - 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.2000 + 0.0000i
```

Staionary Probablity vector

```
P=u(:,1)/sum(u(:,1))
```

P = 4×1

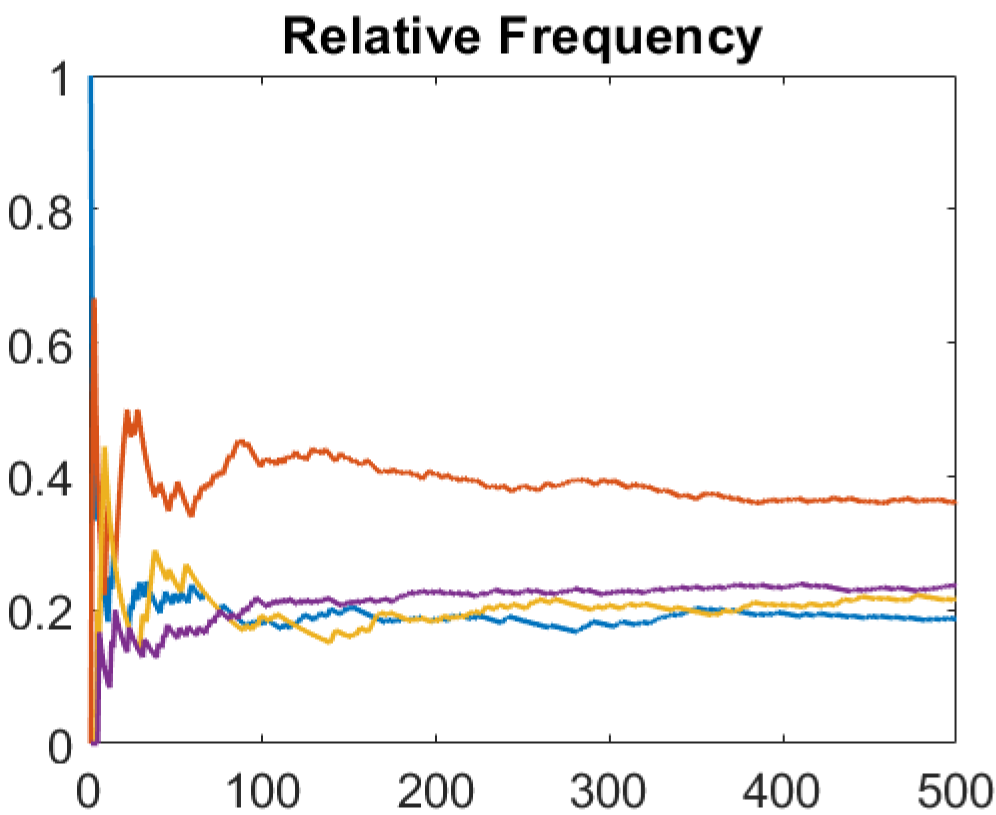
0.1975
0.3333
0.2469
0.2222

```
T = 500;
n=4;
X(1) = 1;


for t = 2:T
    p = cumsum(Pi(X(t-1),:));
    X(t) = min(find(rand<p));
end


for t = 1:T
   for i = 1:n
       freq(i,t) = length(find(X(1:t)==i))/t;
   end
end


plot([1:T],freq,'linewidth',2);
title("Relative Frequency")
set(gca,'fontsize',18);
```




```
sum1 = 0;
sum2 = 0;
sum3 = 0;
sum4 = 0;
for i = 1:T
if X(i)==1
sum1 = sum1+2;
end
if X(i)==2
sum2 = sum2+1;
end
if X(i)==3
sum3 = sum3+2.5;
end
if X(i)==3
sum4 = sum4-1;
end
end
```

Average along sample path

```
average = (sum1+sum2+sum3+sum4)/T
```

average = 1.0580

```
f=[2 1 2.5 -1]';
```

Limit of Average

```
limit=(P')*f
```

limit = 1.1235

Problem 3

3) Let Π be a transition matrix of order p of a irreducible Markov Chain

$\Rightarrow \exists\ n>0$ such that $\Pi_{ij}^{(n)} > 0 \quad \forall (i,j)$

Since Π is idempotent $\Pi^{m} = \Pi \quad \forall\ n>0$

$\therefore\ \Pi_{ij} = \Pi_{ij}^{(n)} > 0 \quad \forall\ i \neq j$

Then by chapman kolmogorov theorem

$$\Pi_{ii} = \Pi_{ii}^{2} \geq \Pi_{ij}\Pi_{ji} > 0 \quad \forall\ i$$

hence every state is aperiodic

Now we have the MC is irreducible & aperiodic

$$\pi_j = \lim_{n\to\infty} \Pi_{ij}^{(n)} = \lim_{n\to\infty} \Pi_{ij} = \Pi_{ij}$$

(Proved earlier)

similarly

$$\pi_j = \lim_{n\to\infty} \Pi_{jj}^{(n)} = \Pi_{jj}$$

$\therefore$ All rows are identical

Problem 4

```
T = 1000;
lamb = 0.85;
```

proposal density: Exponential distribution with mean 0.85

```
q = @(x,lamb)(1/lamb)*exp(-x./lamb);
```

target density

```
f=@(x) x.^2.*abs(sin(pi.*x)).*exp(-abs(x).^3);
```

```
X(1) = 1;
c = 0;
for t = 1:T
Y = -lamb*log(rand);
x = X(t);
r = (f(Y)*q(x,lamb))/(f(x)*q(Y,lamb));
rho = min(r,1);
if rho > 0.5
X(t+1) = Y;
c = c+1;
else
X(t+1) = X(t);
end
mu(t) = mean(X);
variance(t) = var(X);
end
```

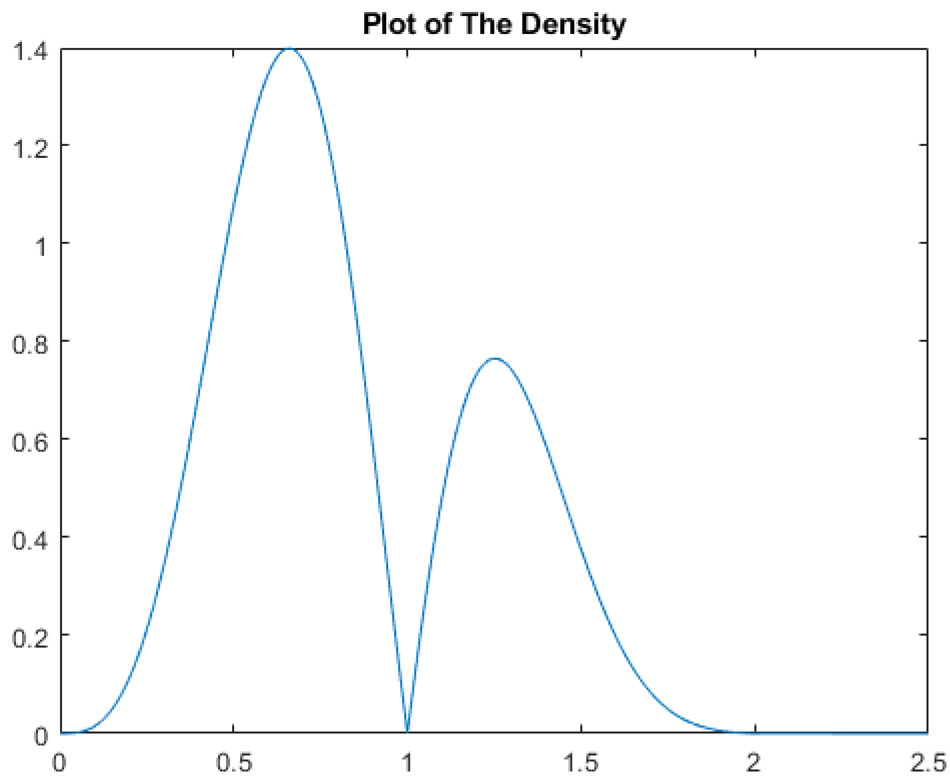
Acceptance Rate of sample

```
(c/T)*100
```

```
ans = 40.8000
```

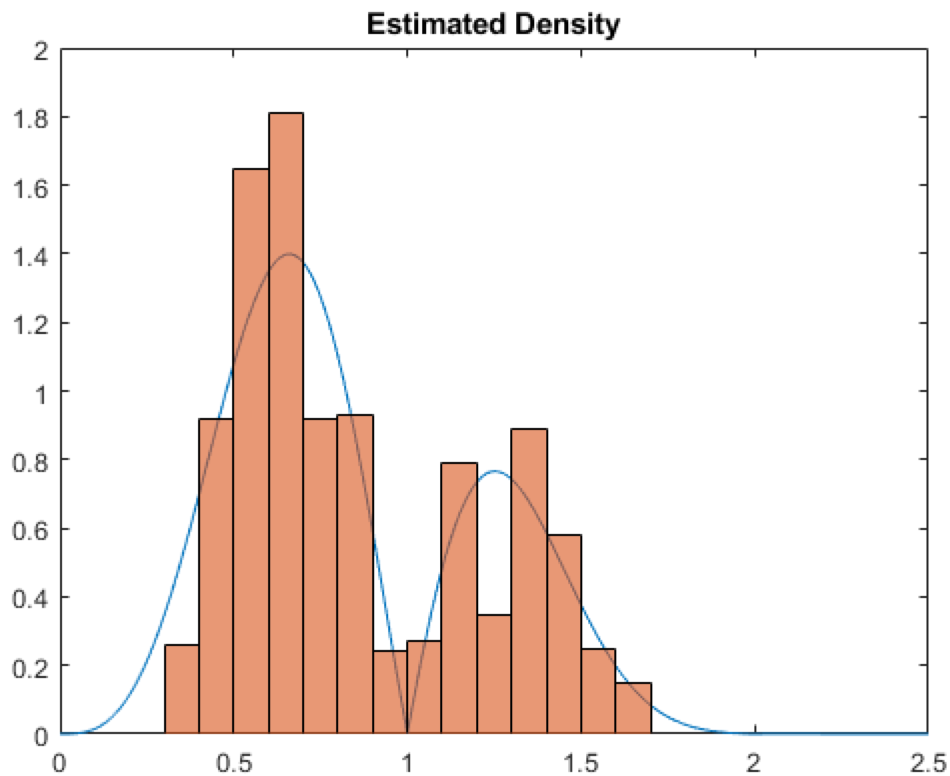
part a

```
dnom = integral(f,0,Inf);
figure(1);
plot(0:0.01:2.5, f(0:0.01:2.5)/dnom);
title('Plot of The Density');
```



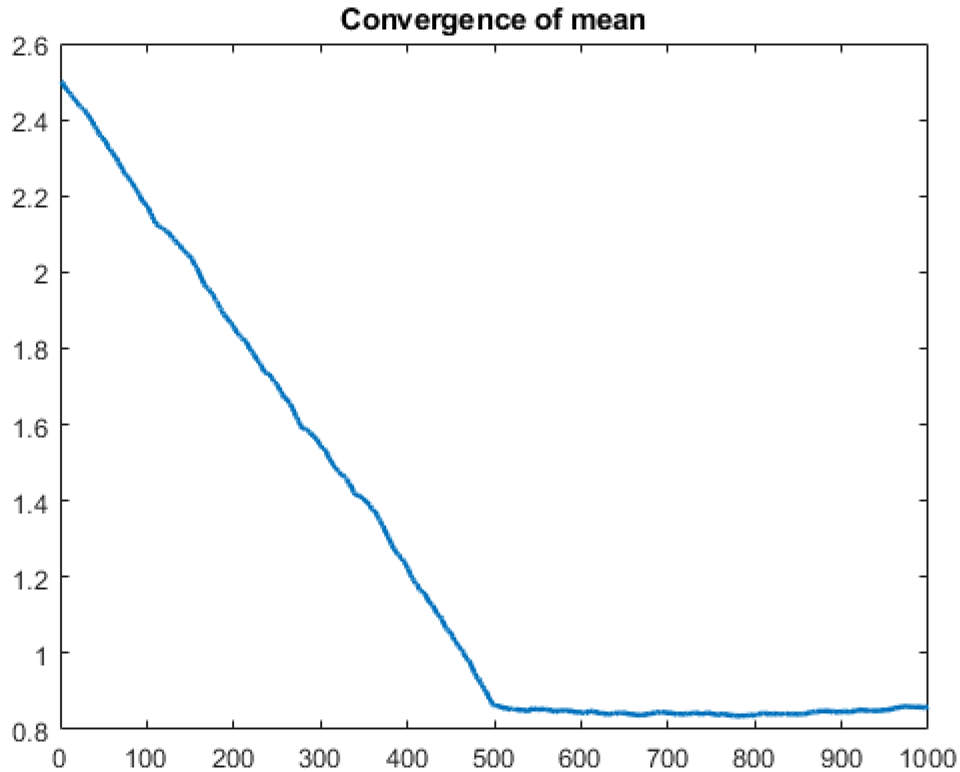
part b

```
figure(2);
plot(0:0.01:2.5, f(0:0.01:2.5)/dnom);
hold on;
histogram(X,'Normalization','pdf');
title('Estimated Density');
```



part c

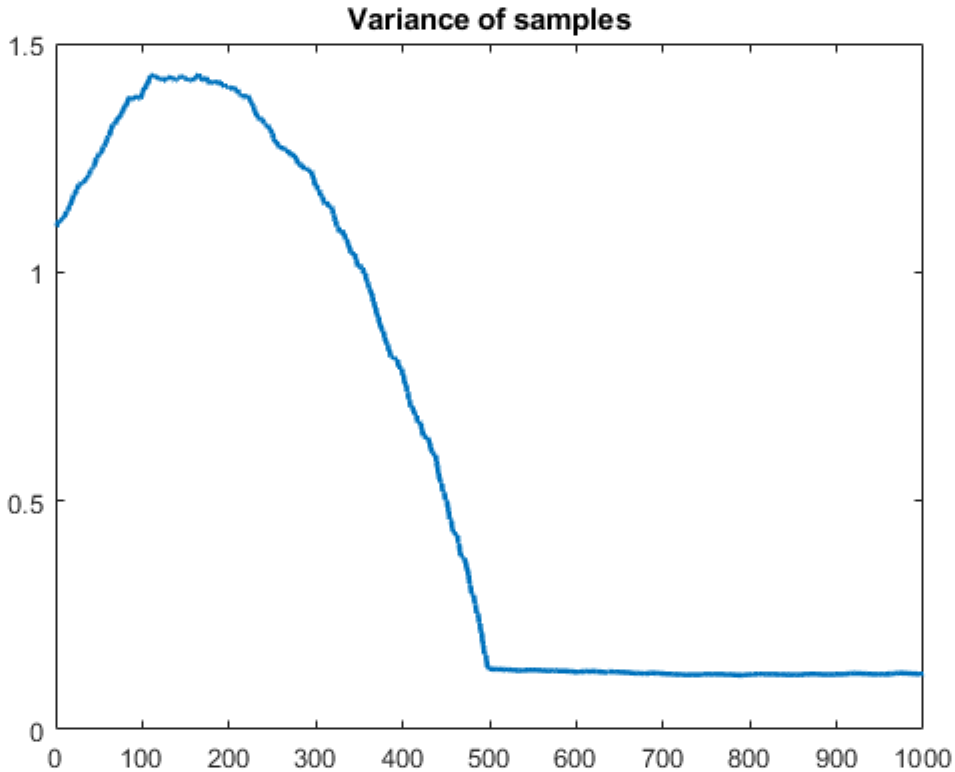
```
figure(3);
plot(mu, 'Linewidth',2);
set(gca, 'fontsize', 10);
title('Convergence of mean');
```



mean of the sample:

```
mean(X)
ans = 0.8598
```

```
figure(4);
plot(variance, 'Linewidth',2);
set(gca, 'fontsize', 10);
title('Variance of samples');
```



Variance of the sample

```
var(X)
ans = 0.1221
```