# STA 5107/4013: Home Assignment # 3

Spring 2021/Due Date: February 9th

1. (Classical Monte Carlo Approach): Write a matlab program to estimate the quantity:
$$5 \int_0^{\infty} x \, e^{-5x} dx \ .$$
using a Monte Carlo approach. Choose the distribution you can sample from for this approximation. Show the plot of convergence to the limiting value.

2. In your own words and using mathematical notation used in the class, write down definitions of: (i) a stationary stochastic process, (ii) a Markov chain, and (iii) a homogeneous Markov chain. Is a homogenous Markov chain stationary? Why or why not?

3. Write a matlab program to simulate a discrete-time, finite-state homogenous Markov chain with the following transition matrix:
$$\Pi = \begin{bmatrix} 0.2 & 0.2 & 0.1 & 0.5 \\ 0.1 & 0.3 & 0.4 & 0.2 \\ 0.3 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.1 & 0.5 \end{bmatrix} \ .$$
One way to simulate this Markov chain is the following:
   (a) Set $i = 1$, and choose $X_i$ uniformly among the four states.
   (b) Given $X_i$, select $X_{i+1}$ using the $X_i^{th}$ row of the transition matrix.
   (c) Set $i = i + 1$ and go to Step b.

   Generate 5 sample paths for time interval $[1, 10]$ and display them on the same plot.

4. Using the program written in the last problem, generate a sample path of this Markov chain, and plot the relative frequencies (versus i) with which the path visits the fours states versus $i$ over the interval $[1, 50]$. Repeat this four times and start from a different initial condition each time.

   Compare the vectors of the relative frequencies at $i = 50$ with the dominant eigenvector of the transition matrix. Rescale the dominant eigenvector by the sum of its entries for this comparison.

5. Repeat Problem 4 for a Markov chain having transition matrices:

$$\Pi = \begin{bmatrix} 0.5 & 0.5 & 0.0 & 0.0 \\ 0.1 & 0.9 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.7 \\ 0.0 & 0.0 & 0.2 & 0.8 \end{bmatrix}, \qquad \Pi = \begin{bmatrix} 0 & 0.5 & 0.0 & 0.5 \\ 0.5 & 0 & 0.5 & 0.0 \\ 0.0 & 0.5 & 0 & 0.5 \\ 0.5 & 0.0 & 0.5 & 0 \end{bmatrix}.$$