STA 5107/4013: Home Assignment # 7

Spring 2021/Due Date: March 16th

1. **Kalman Filtering, Example 1**:

   (a) Write a matlab program to simulate processes $\{x_k\}$ and $\{y_k\}$, $k = 1, 2, \ldots,$ defined as follows:

   $$\begin{aligned} x_{k+1} &= Ax_k + \Gamma u_k \quad \in \mathbb{R}^2 \\ y_{k+1} &= Bx_{k+1} + w_k \quad \in \mathbb{R} \ , \end{aligned}$$

   where

   $$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \Gamma = 0.1 * I_2 \ .$$

   Also, $u_k \sim \mathcal{N}(0, I_2)$ and $w_k \sim \mathcal{N}(0, 0.1)$. Use $x_0 = zeros(2, 1)$ as the initial condition. Showplots of three different realizations of x_k, for $k = 1, \ldots, 100$.

   (b) Write a matlab program to implement the Kalman filter for estimating the mean and the covariance under the posterior $f(x_k|y_1, \ldots, y_k)$. Use $\{y_k, k = 1, \ldots, 100\}$ from the previous item to compute the estimates. Use $\hat{x}_0 = x_0$ and P_0 as identity matrix to initialize the filter.

      i. Show the plot of the original path x and the posterior mean path μ overlaid in different colors.

      ii. Plot the relative estimation error $\frac{\|x_k - \hat{x}_k\|}{\|x_k\|}$ versus k.

2. **Kalman Filtering, Example 2**: Repeat Problem 1 with $x_k \in \mathbb{R}^4$, $y_k \in \mathbb{R}^2$, and

   $$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \Gamma = I_4 \ .$$

   Use $u_k \sim \mathcal{N}(0, I_4)$ and $x_0 = zeros(4, 1)$ as the initial condition. Study the relative errors for the cases: (i) $w_k \sim \mathcal{N}(0, 10 * I_2)$, (ii) $w_k \sim \mathcal{N}(0, 100 * I_2)$, and (iii) $w_k \sim \mathcal{N}(0, 1000 * I_2)$. Show only the relative error plots.