Hypothesis Testing More Recap

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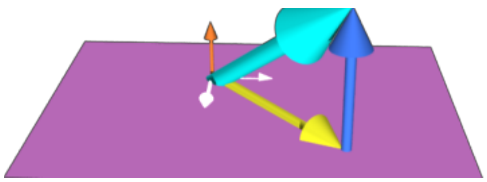
We know, $\vec{X} \sim N_n(\vec{\mu}, \sigma^2 I)$
Let R be any square matrix of order n. $R\vec{X} \sim N_n(R\vec{\mu}, R(\sigma^2 I)R')$
If R is an orthogonal, then $R'R = RR' = I$
$R\vec{X} \sim N_n(R\vec{\mu}, \sigma^2 I)$



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}$$

Figure 1:

If you take a particular normal vector and express it in terms of a different $orthonormal - basis$ other than $Euclidean$, the components are still $independent$ and $homoscedastic$.
If $\vec{\mu} = \vec{0}$ then, $R\vec{X} \sim N_n(\vec{0}, \sigma^2 I)$

Let $\vec{e_1}, \vec{e_2}, \vec{e_3}$ represent X, Y Z components of $\mathbb{R}^3$ respectively, then any vector (x,y,z) in $\mathbb{R}^3$ can be represented as $(x.\vec{e_1} + y.\vec{e_2}) + z.\vec{e_3} = (x, y, 0) + (0, 0, z)$
i.e., the vector can be always represented as the sum of $XY - component$ on which the vector is projected it's orthogonal $(Z-)component$ of the $projection$. Precisely, the same thing work even if we are working with something else which is not $XY - plane$. The only idea is that we start with the vector and then take our $orthonormal - basis$ in a way such that the first few elements of the vectors are along the space onto which we are projecting and the remaining are sticking out perpendicularly from it.
Basic idea: We start with an $orthonormal - basis$ of the space onto which we are going to project. There may be number of them. We then extend that to an $orthonormal - basis$ of the entire space. So we append with some extra stuffs so that the entire bunch is an $orthonormal - basis$ of entire $\mathbb{R}^n$.

We find the components of each $orthonormal - basis$ along each of the directions and stack them row wise as shown in figure. 2. Multiply the original vector and pre-multiply with the matrix formed by the above $ONB\ basis$. The

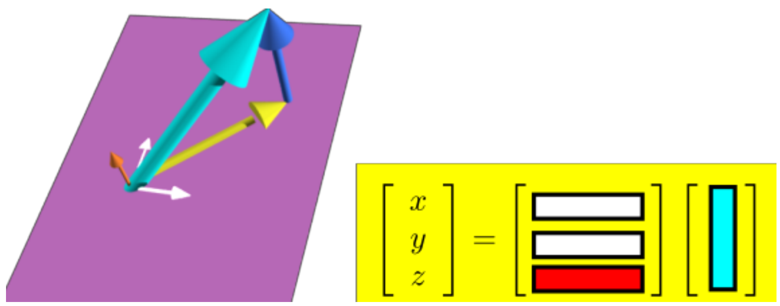


Figure 2:

entries of this result is the inner product of each *ONB basis vector* with the component vector (i.e., the vertical blue vector as shown in figure. 2) which give the length of component along each vectors in our example. Then we keep part corresponding to the projection onto that space and the part corresponding to the extended part will give projection onto the orthogonal space.