Heading: Multiple Comparison

When we are doing hypothesis testing the general set up is this we have two hypothesis namely H_0 (i.e. null hypothesis) and H_1 (i.e. alternative hypothesis)mutually exclusive and exhaustive we want to know which one is true situation more specifically we want to know whether we will reject or accept the null hypothesis using the data we have. It is kind of a binary decision.

Now,when we are doing multiple hypothesis testing here we are doing the same thing multiple times . Let's say we have k many hypothesis namely

$$H_0^1, H_1^1$$

$$H_0^2, H_1^2$$

$$H_0^3, H_1^3$$

$$\vdots, \vdots$$

$$H_0^k, H_1^k$$

Whenever we will come up with a data we will check those k hypothesis and for each of them either I will reject or accept. Hence our final output will be a vector of yes ,no. (i.e. yes for if we accept and no if we reject the null)

False Discovery

Like every other hypothesis test multiple hypothesis taste also follow some goodness criteria like probability of type -1 and type -2 error are small. But there are other important criteria about multiple hypothesis one of them is False Discovery .

False Discovery denotes rejecting a true null hypothesis Suppose our H_0^i is true but your verdicts says it is false when that happens,for example suppose the null says nothing new happened and alternate hypothesis says something new happen and if you rejects the null based on the sample you have ,but it is true in reality this is called False Discovery.

False Discovery Rate

Naturally we will want to keep a bound on how much false discovery one can made (which is analogue of type -1 error in this case).If we consider the probability of false discovery individually for all K hypothesis then it will be nothing but doing k hypothesis testing parallels. But we do not want that we want to put restriction on over-all family. There are various ways to do that.

Based on your random data count the no of false discover ,since your data is random your no of false discovery will also be random.Let us define N **to be No of False Discovery made out of** k **test of**

hypothesis Obviously we will want to keep N as small as we can by giving an bound on that. Now, there are various ways to do that.

Familywise Error Rate

Familywise Error Rate $= P(N \geq 1)$ Even if you made at-least one false discovery that is bad. You want to keep some bound on that discovery. For example I will not allow to be value more than 0.05. Notice, here we are not putting a restriction on each individual hypothesis test we put restriction on overall family.

False Discovery Rate $= E(\frac{N}{k})$

$\frac{N}{k}$ is rate of how many false discovery we are making , and we are taking expectation of that.

When you have lots of hypothesis testing (i.e. your k is 1000 or higher value like that) then False Discovery Rate is more useful. If k is small something like order of 10 then it is better to use Familywise Error Rate And in case of Linear models our no hypothesis testing are mostly order of 10 hence normally we will use Familywise Discovery Rate more often.