MIXED EFFECTS MODELS
RANDOM EFFECT

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"THE FUTURE BELONGS TO THOSE WHO BELIEVE IN THE BEAUTY OF THEIR DREAMS."

Eleanor Roosevelt

Hello there, readers ! Yes, you. I need your eyes and attention here in 3 ...... 2 .... 1 ......

Today, what we are interested to discuss is about Random effect in Mixed Effects Models.

Now, the model which we have used so far is of the form :

$$\boxed{y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}} \tag{1}$$

where, i = 1,2 ; j = 1,2,3 ; k = 1,2,...,5; and

$$\epsilon_{ijk} \sim N(0, \sigma_e^2) \tag{2}$$

Here, in this model we are not interested in β_j's as they are not fixed entities and they just represent j-th village. As we need only some random villages and not complete enumeration of all villages, so we consider β_j's as some random quantity, where j is random and β is the corresponding effect of that randomly chosen village. So, we assume that

$$\beta_j \sim N(0, \sigma_b^2) \tag{3}$$

It means β_j 's are i.i.d normal with mean 0 and variance σ_b^2 . So, to distinguish between the two different variances, we used e as a subscript in the earlier variance term. We assume that both of them are independent random variables. Since, it is customary not to use greek letter so instead of β we simply use an english alphabet b. So, this is our new model :

$$\boxed{y_{ijk} = \mu + \alpha_i + b_j + \epsilon_{ijk}} \tag{4}$$

where,

$$\epsilon_{ijk} \sim N(0, \sigma_e^2) \tag{5}$$

and

$$b_j \sim N(0, \sigma_b^2) \tag{6}$$

Here,α_i is an unknown parameter and b_j as well as ϵ_{ijk} are random quantities following given normal distributions.

As the villages are chosen randomly, so their effects are also random but the two varieties are not chosen randomly and are of main imporance for anyone working on the results. So in this case we are allowing our σ_b^2 to be equal to 0 which makes it a special case. It is a kind of degenerate case which conveys the fact that the villages donot have any effect at all. So, the assumption that all β_j's are same now have all analogous b_j's equal to 0, where σ_b^2 measures the variability among the villages. We are not interested in specific effect of villages but in the overall effect of the villages.

It is important to know whether villages have any effect or not as then only we can test whether σ_b^2 is equal to 0 or not.

In certain models like the one above, these (b_j and ϵ_{ijk}) are known as **Random Effects** and we call α_i's our **Fixed Effects**. A linear model which contains both random as well as fixed effects is termed as **Mixed Effects Model**.

If we have only random effects, then we call the model as **Random Effects Model** and if we only have fixed effects, the we call the model as **Fixed Effects Model**.

That's all for today's session.

We shall continue with why these are useful and the extra computational burden when we are working with such models in our next session.

Thank You for reading with patience.