Multiple Comparisons Hypothesis Testing

Bonferroni Method

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About This File

This file was created for the benefit of all teachers and students wanting to learn about Multiple Comparisons Hypothesis Testing

SECTION 1

Bonferroni Solution to Multiple Comparisons Problem

1.1 Problem

As discussed in the previous part, there lies an issue with conducting multiple hypothesis testing at a particular level say 5% as then because of considering 5% error there will be a risk of getting 5 false discoveries out of 100 hypothesis tests.

1.2 Bonferroni Method

The simplest solution to deal with the problem on reducing the false discovery problem is to rectify the significance level at which we carry each of the test to a smaller value such that the above problem does not creep in. This idea gives rise to the Bonferroni Method. If we want the overall family wise error rate of α then we will conduct each of the k hypothesis tests at a significance level of $\frac{\alpha}{k}$. If we do then, then let $A_1, A_2, \dots A_k$ be such that the A_i represents if the i^{th} test resulted in a rejection. In this case, we see

$$P(FWER) = P(\bigcup_{i=1}^{i=k} A_i) \leq \sum_{i=1}^{i=k} P(A_i) = k.\frac{\alpha}{k} = \alpha$$

where $FWER$ is Family-wise Error Rate. We take $FWER = \bigcup_{i=1}^{i=k} A_i$ as it is the event of rejecting atleast one Hypothesis Test. We used the Bonferroni Inequality above and hence the name.

SECTION

2 Problem with Bonferroni Method

There is a major issue with the Bonferroni Method is we are only caring about the $FWER$ and hence we try to guard against that by decreasing the significance level of all the k hypothesis tests to a very small value that it becomes practically impossible to reject the null hypothesis. So, in the trade-off between not rejecting null hypothesis and making a false discovery Bonferroni Method lies to the left extreme. So, whatever be the null hypothesis we simply bypass committing a false discovery by accepting all the null hypothesis. But this is not desired.
The Bonferroni Method posses the above issue in a disguised form. Let's see an example to get a better grasp on the problem. If we are taking 100 coins and are wanting to test whether all the coins are unbiased or not. In this case we have 100 null hypothesis for each of the coins to be unbiased. Suppose, we want to bound the $FWER \leq 5\%$. Then the significance level of each of the test, by Bonferroni Method, will turn out to be $\frac{0.05}{100} = 0.0005$. If we carry out each of the test at this level, it will be nearly impossible to ever reject the null hypothesis except in the extreme case of all tails or all heads. Hence, we not only want to guard against the type I error but also have considerable power. We will see some more sophisticated method in the next portions.