Identifiability

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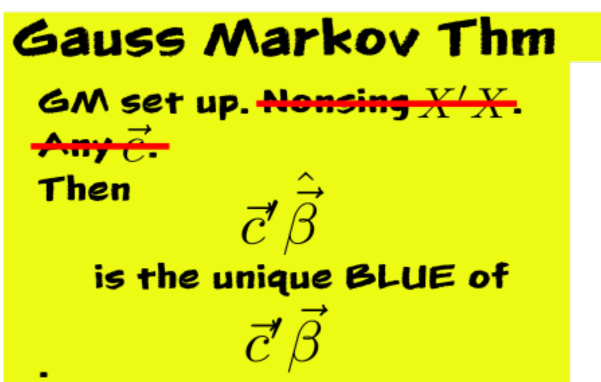
For the general case of Gauss-Markov theorem,i.e, we take only GM setup without non-singular $X'X$ and any $\vec{c}$.



Then $\vec{c}'\hat{\vec{\beta}}$ is the unique BLUE of $\vec{c}'\vec{\beta}$.

Consider a system of equations with GM setup.

$$\begin{aligned} y_1 =& \beta_1 + \beta_2 + \beta_3 + \epsilon_1 \\ &\vdots \\ y_5 =& \beta_1 + \beta_2 + \beta_3 + \epsilon_5 \\ y_6 =& \beta_1 + \beta_2 + 2\beta_3 + \epsilon_6 \\ &\vdots \\ y_9 =& \beta_1 + \beta_2 + 2\beta_3 + \epsilon_9 \end{aligned}$$

So, how do we estimate β_1 and β_3?

We can see that we can't estimate β_1 from this setup because it is bundled with β_2. So, everywhere it occurs as $\beta_1 + \beta_2$. So we can't extricate β_1 separately.

For β_3 we can substract y_1 from y_6 and get an unbiased estimator (not claiming to be the best).

Suppose we have an accurate data such that

$$\begin{aligned} y_1 =& \beta_1 + \beta_2 + \beta_3 \\ &\vdots \\ y_5 =& \beta_1 + \beta_2 + \beta_3 \\ y_6 =& \beta_1 + \beta_2 + 2\beta_3 \\ &\vdots \\ y_9 =& \beta_1 + \beta_2 + 2\beta_3 \end{aligned}$$

From this, we can find β_3 but still can't get the value of β_1.

So we generally say that the term β_1 is not **identifiable** from this setup.

Now to frame this identifiability in a mathematical condition:

Consider the above setup where

$$X = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}}_{Design\ matrix}$$

and

$$\beta_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \qquad \beta_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

So the condition of identifiability in this case reduces to checking whether $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ belongs to row space of X.