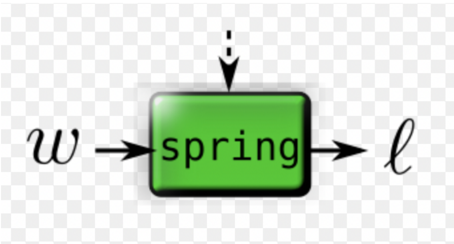
Assignment 1

Aytijhya Saha
Roll No. BS2002

Indian Statistical Institute, Kolkata

August 17, 2022

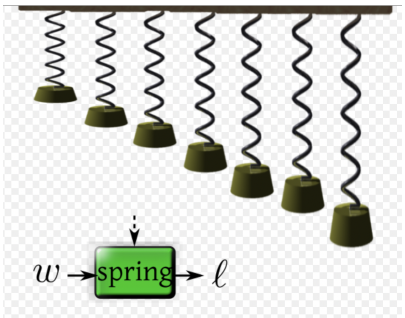
Here, we shall give a detailed example to illustrate how to construct a linear model, get the data and how to fit the model. We consider the system, where a known weight is hanged from a spring. We are interested to construct a linear model, with the weight as input and the the resulting length of the spring as output. So, we have the following black-box diagram.



In this example, both input and output are continuous variables. We expect the length, l to be the function of the weight, w and the random error, ε.

✻ How to get a data in order to fit a model:

We take multiple copies of n identical springs, use different weights for different springs. Then, we measure the final length of the springs. The measurement errors, slight variation among the springs, etc are incorporated into random error, ε.



✻ Model

One way to construct a linear model is the following:

$$l = \beta_1 + \beta_2 w + \varepsilon,$$

where β_1 and β_2 are unknown parameters. Using the n observations in our data, we get n equations:

$$l_i = \beta_1 + \beta_2 w_i + \varepsilon_i, \text{ for } i = 1, \cdots n$$

Equivalently,

$$\begin{pmatrix} l_1 \\ l_2 \\ \cdot \\ \cdot \\ l_n \end{pmatrix} = \begin{pmatrix} 1 & w_1 \\ 1 & w_2 \\ . & . \\ . & . \\ 1 & w_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \varepsilon_n \end{pmatrix}$$

Let, $\mathbf{Y} = \begin{pmatrix} l_1 \\ l_2 \\ \cdot \\ \cdot \\ l_n \end{pmatrix}$, $X = \begin{pmatrix} 1 & w_1 \\ 1 & w_2 \\ . & . \\ . & . \\ 1 & w_n \end{pmatrix}$, $\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \varepsilon_n \end{pmatrix}$ and $\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$. So, we can write our model in the form $\mathbf{Y} = X\beta + \varepsilon$.
The matrix X is called the design matrix. There is no randomness in the design matrix. This is completely known to us.