4.3. (Sec. 4.2.1) Suppose a sample correlation of 0.65 is observed in a sample of 10. Test the hypothesis of independence against the alternatives of positive correlation at significance level 0.05.

4.4. (Sec. 4.2.2) Suppose a sample correlation of 0.65 is observed in a sample of 20. Test the hypothesis that the population correlation is 0.4 against the alternatives that the population correlation is greater than 0.4 at significance level 0.05.

4.5. (Sec. 4.2.1) Find the significance points for testing $\rho = 0$ at the 0.01 level with $N = 15$ observations against alternatives **(a)** $\rho \neq 0$, **(b)** $\rho > 0$, and **(c)** $\rho < 0$.

4.13. (Sec. 4.2.3) Use Fisher's z to estimate ρ based on sample correlations of -0.7 ($N = 30$) and of -0.6 ($N = 40$).

4.14. (Sec. 4.2.3) Use Fisher's z to obtain a confidence interval for ρ with confidence 0.95 based on a sample correlation of 0.65 and a sample size of 25.

4.15. (Sec. 4.2.2). Prove that when $N = 2$ and $\rho = 0$, $\Pr\{r = 1\} = \Pr\{r = -1\} = \frac{1}{2}$.

4.33. (Sec. 4.3) *Invariance of the sample partial correlation coefficient.* Prove that $r_{12\cdot3,\dots,p}$ is invariant under the transformations $x^*_{i\alpha} = a_i x_{i\alpha} + \boldsymbol{b}'_i \boldsymbol{x}^{(3)}_\alpha + c_i$, $a_i > 0$, $i = 1, 2$, $\boldsymbol{x}^{(3)*}_\alpha = \boldsymbol{C}\boldsymbol{x}^{(3)}_\alpha + \boldsymbol{b}$, $\alpha = 1, \dots, N$, where $\boldsymbol{x}^{(3)}_\alpha = (x_{3\alpha}, \dots, x_{p\alpha})'$, and that any function of $\bar{\boldsymbol{x}}$ and $\hat{\boldsymbol{\Sigma}}$ that is invariant under these transformations is a function of $r_{12\cdot3,\dots,p}$.

4.34. (Sec. 4.4) *Invariance of the sample multiple correlation coefficient.* Prove that R is a function of the sufficient statistics $\bar{\boldsymbol{x}}$ and $\boldsymbol{S}$ that is invariant under changes of location and scale of $x_{1\alpha}$ and nonsingular linear transformations of $\boldsymbol{x}^{(2)}_\alpha$ (that is, $x^*_{1\alpha} = c x_{1\alpha} + d$, $\boldsymbol{x}^{(2)*}_\alpha = \boldsymbol{C}\boldsymbol{x}^{(2)}_\alpha + \boldsymbol{d}$, $\alpha = 1, \dots, N$) and that every function of $\bar{\boldsymbol{x}}$ and $\boldsymbol{S}$ that is invariant is a function of R.

4.42. Let the components of X correspond to scores on tests in arithmetic speed (X_1), arithmetic power (X_2), memory for words (X_3), memory for meaningful symbols (X_4), and memory for meaningless symbols (X_5). The observed correla-

tions in a sample of 140 are [Kelley (1928)]

$$\begin{pmatrix} 1.0000 & 0.4248 & 0.0420 & 0.0215 & 0.0573 \\ 0.4248 & 1.0000 & 0.1487 & 0.2489 & 0.2843 \\ 0.0420 & 0.1487 & 1.0000 & 0.6693 & 0.4662 \\ 0.0215 & 0.2489 & 0.6693 & 1.0000 & 0.6915 \\ 0.0573 & 0.2843 & 0.4662 & 0.6915 & 1.0000 \end{pmatrix}.$$

(a) Find the partial correlation between X_4 and X_5, holding X_3 fixed.

(b) Find the partial correlation between X_1 and X_2, holding X_3, X_4, and X_5 fixed.

(c) Find the multiple correlation between X_1 and the set X_3, X_4, and X_5.

(d) Test the hypothesis at the 1% significance level that arithmetic speed is independent of the three memory scores.