2.7. (Sec. 2.3) Find $\boldsymbol{b}$ and $\boldsymbol{A}$ so that the following densities can be written in the form of (23). Also find μ_x, μ_y, σ_x, σ_y and ρ_{xy}.

(a) $\dfrac{1}{2\pi}\exp\{-\tfrac{1}{2}[(x-1)^2+(y-2)^2]\}$.

(b) $\dfrac{1}{2.4\pi}\exp\left(-\dfrac{x^2/4-1.6xy/2+y^2}{0.72}\right)$.

(c) $\dfrac{1}{2\pi}\exp[-\tfrac{1}{2}(x^2+y^2+4x-6y+13)]$.

(d) $\dfrac{1}{2\pi}\exp[-\tfrac{1}{2}(2x^2+y^2+2xy-22x-14y+65)]$.

The normal density function is

$$\frac{\sqrt{|A|}}{(2\pi)^{\frac{1}{2}p}}e^{-\frac{1}{2}(x-b)'A(x-b)}. \tag{23}$$

2.8. (Sec. 2.3) For each matrix $\boldsymbol{A}$ in Problem 2.7 find $\boldsymbol{C}$ so that $\boldsymbol{C'AC}=\boldsymbol{I}$.

2.9. (Sec. 2.3) Let $\boldsymbol{b}=\boldsymbol{0}$,

$$A=\begin{pmatrix}7&3&2\\3&4&1\\2&1&2\end{pmatrix}.$$

(a) Write the density (23).

(b) Find $\boldsymbol{\Sigma}$.

2.11. (Sec. 2.3) Suppose the scalar random variables $X_1,\ldots,X_n$ are independent and have a density which is a function only of $x_1^2+\cdots+x_n^2$. Prove that the X_i are normally distributed with mean 0 and common variance. Indicate the mildest conditions on the density for your proof.

2.12. (Sec. 2.3) Show that if $\Pr\{X \geq 0, Y \geq 0\} = \alpha$ for the distribution

$$N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right],$$

then $\rho = \cos(1 - 2\alpha)\pi$. [*Hint:* Let $X = U, Y = \rho U + \sqrt{1-\rho^2}\,V$ and verify $\rho = \cos 2\pi(\frac{1}{2} - \alpha)$ geometrically.]

2.13. (Sec. 2.3) Prove that if $\rho_{ij} = \rho$, $i \neq j$, $i, j = 1, \ldots, p$, then $\rho \geq -1/(p-1)$.

2.17. (Sec. 2.4) Which densities in Problem 2.7 define distributions in which X and Y are independent?

2.6. (Sec. 2.3) Sketch the ellipses $f(x, y) = 0.06$, where $f(x, y)$ is the bivariate normal density with

(a) $\mu_x = 1,\ \mu_y = 2,\ \sigma_x^2 = 1,\ \sigma_y^2 = 1,\ \rho_{xy} = 0.$
(b) $\mu_x = 0,\ \mu_y = 0,\ \sigma_x^2 = 1,\ \sigma_y^2 = 1,\ \rho_{xy} = 0.$
(c) $\mu_x = 0,\ \mu_y = 0,\ \sigma_x^2 = 1,\ \sigma_y^2 = 1,\ \rho_{xy} = 0.2.$
(d) $\mu_x = 0,\ \mu_y = 0,\ \sigma_x^2 = 1,\ \sigma_y^2 = 1,\ \rho_{xy} = 0.8.$
(e) $\mu_x = 0,\ \mu_y = 0,\ \sigma_x^2 = 4,\ \sigma_y^2 = 1,\ \rho_{xy} = 0.8.$

2.18. (Sec. 2.4)

(a) Write the marginal density of X for each case in Problem 2.6.
(b) Indicate the marginal distribution of X for each case in Problem 2.7 by the notation $N(a, b)$.
(c) Write the marginal density of X_1 and X_2 in Problem 2.9.

2.23. (Sec. 2.4) Let $X_1, \ldots, X_N$ be independently distributed with X_i having distribution $N(\beta + \gamma z_i, \sigma^2)$, where z_i is a given number, $i = 1, \ldots, N$, and $\sum_i z_i = 0$.

(a) Find the distribution of $(X_1, \ldots, X_N)'$.
(b) Find the distribution of $\overline{X}$ and $g = \sum X_i z_i / \sum z_i^2$ for $\sum z_i^2 > 0$.

2.24. (Sec. 2.4) Let $(X_1, Y_1)', (X_2, Y_2)', (X_3, Y_3)'$ be independently distributed, $(X_i, Y_i)'$ according to

$$N\left[\begin{pmatrix} \mu \\ \nu \end{pmatrix}, \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix}\right], \qquad i = 1, 2, 3.$$

(a) Find the distribution of the six variables.
(b) Find the distribution of $(\overline{X}, \overline{Y})'$.