Residuals: R^2

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1 Introduction

we have seen the contribution of a extra item in estimation and method to measure it.We take the relative change after fitting an item.The formula is:

$$\frac{RSS_1 - RSS}{RSS_1}$$

here RSS_1 is the model we want to check for fit and RSS is the model fitted against. We often say this quantity as goodness of fit as it track the relative changes in different model. Our model was

$$\boldsymbol{\vec{y} = X\vec{\beta} + \vec{\epsilon}, \quad \vec{\epsilon} \sim \left(\overrightarrow{0}, \sigma^2 I_n\right)}$$

here X_1 is another matrix with $\vec{\beta_1}$ we want to check the model to fit $\vec{\beta_1}$ and $\vec{\beta}$.Here $X = [X_1, X_2]$ and $\vec{\beta} = (\vec{\beta_1}, \vec{\beta_2})$.

2 Example:

Suppose we have this situation where our model is

$$\boldsymbol{\vec{y} = \vec{\alpha} + \vec{\beta}x + \vec{\epsilon}}$$

now here we want to check the usefulness of this model.By this we means that how much importance does x_i plays to predict the y_i.By this we means that not to use x_i so we drop the $\vec{\beta}$ term not $\vec{\alpha}$ as it does not contain x_i term. Now our matrix previously was

$$\begin{bmatrix} 1 & x_1 \\ 2 & x_2 \\ 3 & x_3 \\ . & . \\ . & . \\ . & . \\ . & . \\ n & x_n \end{bmatrix}$$

and our model after is there is only (1,1,...,1) coloumn in the matrix.Our X_1 is the only $\vec{1}$ and X_2 is the $(x_1, x_2, .., x_n)$ coloumn.So now we will calculate RSS_1 and RSS. Where $RSS = ||\vec{y} - P_X\vec{y}||$ and $RSS_1 = ||\vec{y} - P_{X_1}\vec{y}||$.then the value is called R^2 for X to given X_1. suppose our new original model is without intercept term that means our model is

$$\boldsymbol{\vec{y} = \vec{\beta}x + \vec{\epsilon}}$$

so there if you have to check the importance of x_i in this model.So there should be no $\vec{1}$ at all.So X have only X_2 and P_{X_1} became $\vec{0}$.after that again we will calculate the R^2 again.Now using the standard software like R it will carefully take wheather intercept term is there or not and take the correct form to calculate R^2.