Linear Statistical Models
Video 143 - Henderson's Equation

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1 Introduction

In video 142, we learnt about predicting a random effect coefficient. We say, $\vec{l}'\vec{y}$ is a Best Linear Unbiased Prediction (BLUP) for $\vec{b}$ if Var($\vec{l}'\vec{y}$) is minimum subject to E($\vec{l}'\vec{y} - \vec{b}$)=0. However, finding BLUP is not easy. Thus, we are introduced to Henderson's Equation method to find BLUP under a special model.

2 Theory

We have a Random Effects model as follows :

$$\vec{y} = X\vec{\beta} + Z\vec{g} + \vec{\epsilon}$$

where, $\vec{g} \sim (\vec{0}, \sigma^2 G)$, $\vec{\epsilon} \sim (\vec{0}, \sigma^2 R)$

G,R are known Positive Definite Matrices.

The Henderson's Equation is given as follows :

$$\begin{bmatrix} X'R^{-1}X & X'R^{-1}Z \\ Z'R^{-1}X & Z'R^{-1}Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\vec{\beta}} \\ \hat{\vec{g}} \end{bmatrix} = \begin{bmatrix} X'R^{-1}\vec{y} \\ Z'R^{-1}\vec{y} \end{bmatrix}$$

On solving the above equation, we get, $\hat{\vec{\beta}}$ which is the Best Linear Unbiased Estimator (BLUE) of $\vec{\beta}$ and $\hat{\vec{g}}$ which is the Best Linear Unbiased Prediction (BLUP) of $\vec{g}$.

3 Conclusion

The Henderson's Equation makes finding BLUP easier but it works under a special model only and the assumption in the model that G,R are known positive definite matrices is a bit unrealistic. The paper "That BLUP is a Good Thing : The Estimation of Random Effects" by G.K.Robinson provides more details about BLUP and its applications in real life like in animal husbandry.