

Mixed effects models

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1 Hypothesis testing:

We would now like to perform tests of significance of the different effects present in a model.

Consider the following fixed effect model :

$$y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}$$

Let us recall how to test the significance of the input corresponding to the j index (β parameter, in this case).

$$H_0 : \beta_j \text{'s are all same.}$$

Since under the null hypothesis , all β_j 's are same , we assume them to take a constant value (say k) which is then absorbed into μ by the model.

We then fit both the models (the one with β_j 's and the one without) and compare them using the Log-likelihood test !

Now consider the following Random effect model :-

$$y_{ijk} = \mu + \alpha_i + b_j + \epsilon_{ijk}$$
$$b_j \sim N(0, \sigma_b^2)$$

In this case however , testing whether input corresponding to j index (b's in this case) is not so simple. The prime reason : b_j 's are not parameters. (A null hypothesis must always be couched using only parameters.)

We have to test something involving σ_b^2 . Thus,

$$H_0 : \sigma_b^2 = 0$$

Note :

This is only in case of b's. In case of α 's (in case we want to test); we still have:

$H_0 : \alpha$'s are all same

Note :

Thus, tests are no longer as basic as in case of fixed effect model.
(even for α 's which is a fixed effect in the mixed effect model)

Reason :

The null distribution gets far more complicate due to the presence of random effect.

We shall now see how to deal with the tests in these cases.