Gauss-Markov Model:
Gauss-Markov assumptions for the bar example

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Well, the first condition that comes to our mind is to explain a layman, why $\bar{y}$ is a good thing. So when we are measuring β using our scale y, of-course we are making mistakes and it is quite likely to be happened. So, when we are making any mistake either it is overshooting or undershooting it. And the idea is sometimes we are overshooting it or undershooting it and on an average this mistake should cancel out. But if we think we are overshooting it every time then taking average might not be a good idea. For example if we take the time shown by various people on their watches at a certain instant and take the average of that to estimate the true time that will be a bad way of doing it because most of the people will keep their watch faster.(We are not talking about mobile watches because those are just exact).Usually most of the wrist watches, wall clock we use are usually faster than slower.So, there taking simple average will not be a wise thing.

Or suppose you suddenly figured out the scale you are using to measure is not marked from 0 but from 1.In this case and all these cases the errors will be approximately equal to 1.You have made at least 1cm error. In that case also you will not use simple average. So when we suggest simple average is a good way of combining all the numbers , we are tacitly assuming that we are as much likely to overshoot as we are to undershoot. That is why we take a lot of random measurements and average them out , they likely to cancel each other out. This is just from layman's viewpoint.

Now if we want to write this as a more statistical founding condition by saying that $\boldsymbol{E(\epsilon_i) = 0}$

$$E(\epsilon_i) = 0$$

It might overshoot or it might undershoot but on an average they should be zero. Expected values of $\boldsymbol{\epsilon_i = 0}$. So, when we are suggesting , let's take average we are tacitly making that assumption. If that assumption is not valid, we will not be willing to use average.

Now that's not the only assumption.Suppose we are going to measure that β ten times. Now suppose one person is a bit lazy and he takes five measurements and he asks his friend to make another five measurements . Now suppose our friend is rater a skilled engineer and he uses not just a simple scale but a slide caliper and carefully opening the caliper he measures the β and get some very precise measurements.So we have got five measurements which are taken by the ordinary scale and five measurements using the slide caliper.So if the person averages his five and his friend's five , we will get most likely two different numbers. **Should we take just averages of these two ?**That is not reasonable . Because for the ordinary scale we have to use lots of eye-estimation and gives me a result closest up to certain millimeters , even then we have to a lot of squinting of our eyes. But when we fit with a vernier scale we can go on up to 0.1 cm , that is one-tenth of a centimeter. So, the caliper is more precise. In this case taking simple average does not look like the best way to do. So, if we have the person's average and his friend's average , we would rather prefer a weighted average where we give more weight to the measurement taken by the higher

precision instrument and less weight to the measurement taken by the lower precision instrument. We are going to discuss hoe we choose those weights but the main thing is that when we are using simple average , we are making the tacit assumption that all the measurements are of the same precision level . Statistically this means we are assuming variance of all ϵ_i's are same. We do not know what that variance is , some σ^2, a positive number. If variance is zero, we are in a precise measurement setup which is not feasible in practice. So, we are allowing that the σ^2 is unknown , but all the ϵ_i have the same σ^2. This is referred to as homoskedastic setup.

$$E(\epsilon_i) = 0$$
$$V(\epsilon_i) = \sigma^2, \ (> 0, \text{ unknown})$$

Ok, that's lot all. We put another assumption, suppose we are using the regular scale and we will be measuring all the things by ourselves . So we know that-$E(\epsilon_i) = 0$ and variance is the same but as we have said the person is lazy. So when we are asked to do this what we do is : we will take one measurement very carefully and then we will repeat the same measurement nine times and will claim that we have already measured them ten times and we did all the things.So, we are not violating my assumptions. But will you think that taking average of these ten numbers is really going to act to the precision ?

Now, remember when we take average we take this idea that we have taken ten measurements and average of them.So the entire thing is more precise than the original thing. But here the average of these so called ten measurements are all happened to be the same number which is the same number itself but will you put any greater faith in this average number than the first number ? **NO!**.Because this so called average is not an average at all, it is just the first number itself.So, our intuition that taking average of ten numbers will give us a more reliable figure does not work in this case.So, we are assuming that all this measurements is shading some fresh light on these. So, all the errors are at least uncorrelated otherwise taking repeated measurements is not really a good idea.

$$E(\epsilon_i) = 0$$
$$V(\epsilon_i) = \sigma^2, \ (> 0, \text{ unknown})$$
$$\epsilon_i\text{'s uncorrelated}$$

Suppose we have some lab assistants and they have bad habit of back calculating at every step. So, the moment they will see one measurement they will workout the back calculation what the value supposed to be according to physics theory and if they have overshooted in the very next measurement they will write something that is undershooted such that the average remains same. So, they are making all these measurements negatively correlated in their case ,such that the average is indeed the value given in the text book. Obviously such an experiment is meaningless because even though you have taken say hundred measurements since they are correlated they nearly reflect the value what you believe the value should be and not what nature says the value should be. So when we are taking simple average, we are tacitly making these three sets of assumptions

1. The first is on the mean that all the errors must have zero mean
2. Secondly they must have the same variance (unknown)
3. And the last thing is that they are uncorrelated

These set up is called the **The Gauss-Markov Setup**