Probability-3 Lecture-17

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Definition:

$X_n \xrightarrow{d} X$ (also denoted by $X_n \xrightarrow{w} X$ ← weak convergence)

if $F_n(x) \longrightarrow F(x) \quad \forall\, x \in C_F$ ← (set of all continuity points of F.)

recall, **Theorem:**

$X_n \xrightarrow{d} X$ <u>iff</u> $E(f(X_n)) \longrightarrow E f(X)$, $\forall\, f \in C_B(\mathbb{R})$ → [set of all bounded, continuous f^ns.]

for complete proof, refer prev. Lec. (L-16)

Corollary: (Continuous Mapping Theorem)

$X_n \xrightarrow{d} X \Rightarrow g(X_n) \xrightarrow{d} g(X) \quad \forall\, g \in C(\mathbb{R})$ → set of all continuous f^ns over $\mathbb{R}$.

to show: $E(f(g(X_n))) \longrightarrow E(f(g(X)))$

If $f \in C_b(\mathbb{R})$,

& $g \in C(\mathbb{R})$,

then $f \circ g \in C_b(\mathbb{R})$ &, $X_n \xrightarrow{d} X$

then, just simply apply the above characterization of conv. of distribution.

$\therefore E(f(g(X_n))) \longrightarrow E(f(g(X)))$.

$\Leftrightarrow \quad f(g(X_n)) \xrightarrow{d} f(g(X))$.

Corollary:

Suppose K is a compact interval.
If $X_n, n \geq 1$ and X are r.vs, all taking values in K, then

$$X_n \xrightarrow{d} X \iff E(X_n^j) \longrightarrow E(X^j) \quad \forall j = 1, 2, \ldots$$

$(\Rightarrow)$ Here, $f(x) = x^j$. On a compact interval, this is bounded.

$(\Leftarrow)$ for any polynomial p,

$$E(p(X_n)) \longrightarrow E(p(X))$$

Exercise: This thm. can be made "Stronger".

Show that: 1 $X_n \xrightarrow{d} X$ iff. $E(f(X_n)) \longrightarrow E(f(X))$ $\forall f \in C_K(\mathbb{R})$.

$C_K(\mathbb{R})$: Set of all continuous f^ns with compact support.

2 &, $X_n \xrightarrow{d} X$ iff. $E(f(X_n)) \longrightarrow E(f(X))$ $\forall f \in C_\infty(\mathbb{R})$,

$C_\infty(\mathbb{R})$: infinitely diff^ble f^ns.