Linear Statistical Models
Video 88 - Adjusted R^2

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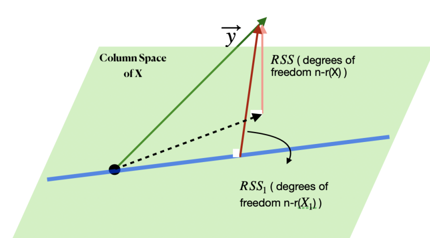
1 Introduction

When fitting a model to a data, we may add too many terms to it i.e, higher degree polynomials are used as models in order to get higher values of R^2. This can often over-fit the data and the misleading high R^2 can lead to misleading projections. Thus, an adjusted value of R^2 is used.

2 Theory

RSS is the residual sum of squares when the entire model is present. RSS_1 is the residual sum of squares when only a part of the model (say X_1) is present. R^2 is defined as follows,

$$R^2 = 1 - \frac{RSS}{RSS_1}$$



RSS lies in an orthogonal complement of the $\mathcal{C}(\mathrm{X})$ i.e, it lies in a space of dimension $n-r(X)$. In RSS, only $n-r(X)$ many terms are free. Similarly, RSS_1 lies in a space of dimension $n-r(X_1)$ i.e, RSS_1 has degrees of freedom $n-r(X_1)$.

Here, $r(X_1)$ is 1 when we are considering the intercept case and 0 when we are working with the no intercept case.

In order to handle the problem of over-fitting the data, we define adjusted R^2 as follows,

$$R^2_{adj} = 1 - \frac{RSS/(n-r(X))}{RSS_1/(n-r(X_1))}$$

As the number of terms increase, the value of $n - r(X)$ decreases. Also, RSS decreases. Thus, the overall value of $\frac{RSS}{n-r(X)}$ remains somewhat maintained, balancing out the effect of increase in degree and decrease in RSS.

3 Conclusion

R^2 tells us how well the model fits the data. Adjusted R^2 also tells the same, but adjusts the number of terms in the model. If more useful terms are added to the model the adjusted R^2 increases while if terms that are not required are added the adjusted R^2 decreases.