Casting Into Linear Algebra

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In the last video we saw the three equations for the masses of atoms A and B as obtained experimentally from the masses of the 3 compounds were as follows:

$$3A + 4B \approx 9.8 \tag{1}$$

$$4A + B \approx 9.1 \tag{2}$$

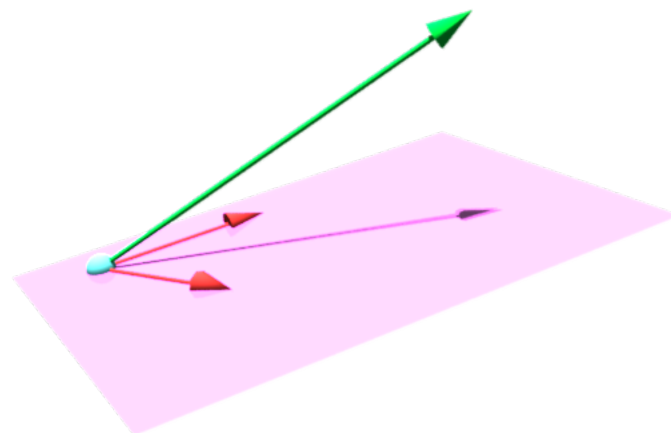
$$2A + 3B \approx 7.0 \tag{3}$$

Or, writing these in matrix form,we get

$$\begin{pmatrix} 3 & 4 \\ 4 & 1 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} A \\ B \end{pmatrix} \approx \begin{pmatrix} 9.8 \\ 9.1 \\ 7.0 \end{pmatrix} \tag{4}$$

We see that the above system of equations is inconsistent, i.e., no value of A and B satisfies all 3 equations simultaneously.

What are we supposed to do,then?



The red vectors in the above picture refer to the vectors $\begin{pmatrix}3\\4\\2\end{pmatrix}$ and $\begin{pmatrix}4\\1\\3\end{pmatrix}$ respectively.The green vector is the vector $\begin{pmatrix}9.8\\9.1\\7.0\end{pmatrix}$ Clearly,the green vector does not belong to the span of the 2 red vectors,so it lies out of the plane containing the two red vectors. Next,we look for the vector in the plane which is nearest to the green vector.Clearly,it is the shadow,or the orthogonal projection of the green vector on the plane,as every point on the green vector is nearest to every point perpendicularly downwards on the plane. We therefore look for the orthogonal projection of the green vector on the plane in the coming videos.