One Way ANOVA: Different Identifiability Constraints

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

In this model we have seen that we have four parameters and in no way we can estimate them from given data as rank of the design matrix remains 3. So we add some additional constraints by which we can say these α_i's and $\mu's$ have the interpretation that we wanted to have. For that we impose a condition on α_i's which is

$$\sum \alpha_i = 0.$$

Now this condition is what make that interpretation possible i.e., μ is now average yield of crop irrespective of varieties, so when we are averaging over all the α_i's we get zero. So out of the three α_i's only two remains free and third one can be obtained from other two. So under this condition we can uniquely obtain μ, α_1 and α_2, and the interpretations here are μ is the overall mean effect and α_i is the additional effect of variety i. In the first version of the model(where there was no μ) α_i was the mean effect for i^{th} variety but now in this model it is the additional effect for that variety.

Sometimes however we want to write it in a slightly different form and then we use this formulation; same model but the condition changes to α_1 equal to zero. We'll do this if due to some reason we believe that variety 1 is something like a benchmark variety. Suppose variety 1 is something that the farmers already use, and the second and third variety are new varieties that are going to be introduced by the govt. and the study is about to estimate if the other two varieties are better or not.
In this case taking $\alpha_1 = 0$ is a good idea but now the interpretations will change. Now the estimated value of μ will be the benchmark value to which farmers are already accustomed with as $\mu + \alpha_1 = \mu$, and now α_2 and α_3 gives the additional effects of two new varieties over that benchmark value.

So all of these formulations are essentially same, we can shift from one to another easily by manipulating coefficients. We have to use them depending on which interpretations we want to give to the various coefficients. For example if we have in mind that α_1 should reflect the average yield of variety 1 and by mistake we impose the condition where $\sum \alpha_i = 0$ we may get negative estimate of α_1 which will be meaningless.

Practically in a software you do not see that you will fit this version or that version, most soft-wares will fit one of the versions of their favourite choice and you have to recast it in your form of parametrization.