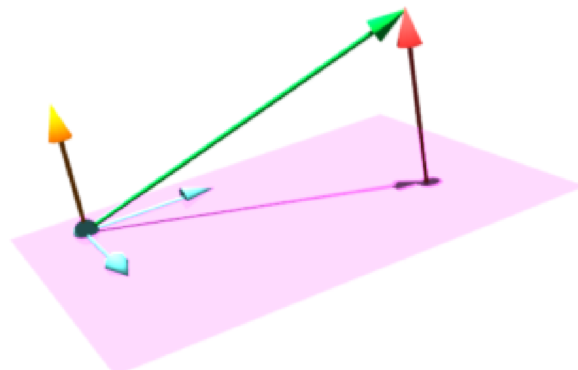
Gauss-Markov Model: Use of ONB

We are going to revisit the concept of orthonormal basis.

Just like the coordinate system done in school, we have presented an orthonormal system. The three arrows coming out from the origin are mutually perpendicular. Each of them are length 1.



We take an orthonormal basis such that the first few elements of the basis will span the column space of x and the remaining will poke up into the orthogonal complement .To put in formally, we start with column space of X We extend the orthonormal basis in the space of X to the entire space.So, in this picture ,we took the column space of the vectors and extend it to the R3.(cyan vectors)

If we do this for n dimensional space and the column space of X is r dimensional ,we will have r arrows along the space and n-r arrows sticking up. If we take our data vector y and express it in terms of this orthonormal basis ,then there will be some component along the directions of the basis vectors.If we look at all the components along these cyan arrows that is the basis of the column space of X, then all these components will together give the projection.

The above is basically given by the sum of all these components of the green vector. When we take an orthonormal basis and we want to find out the components of a vector with respect to that orthonormal basis,there is a very simple formula.

Suppose we have an orthonormal basis [v1,..,vn], and some vector y, and we wish to express y as a linear combination of v1,..,vn. We know this exists as vi's are bases ,but they being orthogonal makes the computation much simpler. $\vec{vi}'\vec{y}$ will give the component of y along vi. vi is the direction along which we are projecting as it is a property of orthonormal basis. If it is some other kind of basis that need not be true so that's one advantage why we work with orthonormal basis so these are the different components.

There exists a very simple example . Suppose we have (1,0,0) ,(0,1,0) and (0,0,1) and a vector A (x, y

,z).Then look at the components of the vector X, we just take the dot product of A with (1,0,0) to get x.

This gives us something useful as if we can find the squared norm of this by just squaring these things and adding. This is what happens when we work with euclidean bases but is true for any orthonormal basis in general.