# Mixed Effects Models: BLUP

Tarun Agarwal

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In this video, we want to discuss whether we can *predict* a random effect coefficient. Notice, we said *predict* instead of *estimate* as it is not a parameter, it is a random quantity, and therefore we cannot *estimate* it. Instead, we can *predict* it.

Notice, we are using random effects because we are not interested in knowing any specific value of these particular coefficients. These are like a random sample that we generated from a large population of similar such coefficients. For example, if we had many villages out of which we had to randomly choose 3 villages, the three villages chosen are not significant in particular.

**Example.** In animal husbandry, people who care about how to improve milk production of cows, they like to have predictions of various things. This is because let us say they have different oxen and cows and they want to know whether the resulting calves, when you mate them, are good with respect to some measure of health and they want to link that up with the various properties of the ox that they used. Notice, these oxen are chosen randomly (so random effects) however each oxen is chosen multiple times in future as well. So, it is important to know which ox is capable of producing better calves. Thus people in animal husbandry care to predict these random effects.

Now, in case of fixed effects, we had BLUE. Here we have a similar concept called **BLUP**, i.e, **Best Linear Unbiased Predictor**.

**Definition.** Suppose $b$ is a random coefficient that we are trying to predict. Then $\vec{l}'\vec{y}$, a linear function of $\vec{y}$ ($l$ is fixed) will be called a **BLUP** (Best Linear Unbiased Predictor) for $b$ if

$$V(\vec{l}'\vec{y}) \text{ is minimum subject to the } E(\vec{l}'\vec{y} - b) \equiv 0.$$

Notice the identity is used to refer that whatever parameter values are chosen, $E(\vec{l}'\vec{y} - b)$ will *always* equal to 0.