Exception 2

In the last video we discussed how some Mixed Effect models might not be interpretable and how we have a subset of the set of all Random effect models, using the black box approach, which is interpretable, namely making the effect whose individual estimates we may not be worried about, to be the random effect. We also discussed and exception of that, which is a interpretable model outside the aforementioned subset.

Let us discuss another example of such kind. Consider a case where you have a machine with 8 settings and 10 environments in which it can be used and we want to do 3 experiments for each combination. For example, a washing machine has different settings determined by how much water you put, how long it has to spin, etc and the environments here can be the different types of clothes or the different types of detergents one used. If we use a linear model of the form:

$$\boldsymbol{Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{i,j} + \epsilon_{ijk}, \quad n = ijk}$$

We would need to do 8x10x3= 240 experiments with the machine. Had the machine been something more valuable than a washing machine. For ex, a gem stone cutter with different spinning speed settings and gemstones of different variety. Then using the thing for 240 times would probably make it unusable and cost a lot, but we can't neglect the interaction effect too! Thus we settle with the following random effects model:

$$\boldsymbol{Y_{ijk} = \mu + \alpha_i + \beta_j + g_{i,j} + \epsilon_{ijk}, \quad n = ijk}$$

It looks a bit fishy, since the individual effects are assumed to be fixed, but their interaction is taken to be random.

This is how it works:

We first assume that the $\boldsymbol{g_{i,j} \sim N(0, \sigma_g^2\Big)}$, i.e they come from same distribution. Then we choose ,say 10 out of the 80 possible combinations of i,j randomly and do 3 experiments on each. Then although we may not get the estimates of the $\gamma_{i,j}$ since we have not done the experiments for all i,j pairs, but owing to our assumption of similarity of the $g_{i,j}$, we can still get an estimate of σ_g^2. That's better than getting nothing and so we settle with it.