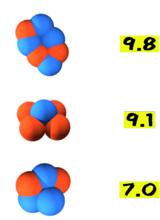
Example : Weighing Atoms

Suppose we have a number of molecules which are made of only two atoms, one is red atom and one is blue. Now we want to measure the masses of these two atoms. So what we can do is we can take each atom separately and weigh them, but it is a very difficult process and they are only available as compounds. So we measure a molecule of a compound. Then solving some linear equations we can find out the masses we need. Suppose by doing some kind of experiment we measure the masses of all molecules of their compounds.

Suppose, we have three compounds available, X,Y and Z, and masses of its molecules are measured 9.8, 9.1 and 7.0 respectively. From these we want to find out the masses of these two atoms. So, that shouldn't be so difficult, let's see the X molecule, it has three red and four blue atoms. Suppose each red atom has got mass A and each blue atom has mass B. So if we try to make an equation out of it, we will get

$$3A + 4B = 9.8 \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Similarly, in molecule Y it has four red and one blue atoms in it and so we get



$$4A + B = 9.1 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

and in molecule Z it has two red and three blue atoms, so we get

$$2A + 3B = 7.0 \ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

So, we have a system of linear equations, three equations with two unknowns. Now we know that it is enough to have two equations to solve this system. But as we have three measurements, we are usually tempted to solve all of them simultaneously i.e., we try to find a solution of A and B which will satisfy all the three equations. Now if we try to do this we realise that this system of equations is inconsistent i.e., we cannot find real values of A and B for which all of them holds simultaneously. But how is that possible? This occurs because our measured quantities are all approximates because of limitations of measurement. So ideally we shouldn't write (i), (ii) and (iii) as equalities, rather we should write it as approximations like

$$3A + 4B \approx 9.8,$$

$$4A + B \approx 9.1,$$

$$2A + 3B \approx 7.0.$$

Now the point is how do we solve this approximate system? We have seen that if we ignore approximations, we can't solve this. Now you may try to ignore any of these three equations (i), (ii), (iii) and solve the other two. But remember, in all these three equations there is some amount of error involved and so our calculated answers aren't exact. And also answers we get from solving three different pairs may also be different from each other, so how do you decide which one to take i.e., how do you decide which equation you have to throw away? So throwing away one equation is not a good idea. What we want is to take all the equations under consideration.

Now this artificial example presents a situation to you that is faced by all working scientists.You have some theory, the theory predicts that you should observe this, then you measure it and there is a little bit of random measurement error, so the equations that all you get will always be or almost always be inconsistent. There must be some way by which you are able to solve an inconsistent system of equation and that is exactly what linear models do. It's a way of solving inconsistent system of equation or something that we should more precisely call approximately consistent system of equation and this will take us to some fascinating application of linear algebra.