5.5. (Sec. 5.2.2) Let $T^2 = N\bar{x}'S^{-1}\bar{x}$, where $\bar{x}$ and S are the mean vector and covariance matrix of a sample of N from $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Show that T^2 is distributed the same when $\boldsymbol{\mu}$ is replaced by $\lambda = (\tau, 0, \ldots, 0)'$, where $\tau^2 = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$, and $\boldsymbol{\Sigma}$ is replaced by I.

5.19. (Sec. 5.3) Let $\bar{x}$ and S be based on N observations from $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, and let x be an additional observation from $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Show that $x - \bar{x}$ is distributed according to

$$N[\mathbf{0}, (1 + 1/N)\boldsymbol{\Sigma}].$$

Verify that $[N/(N+1)](x - \bar{x})'S^{-1}(x - \bar{x})$ has the T^2-distribution with $N - 1$ degrees of freedom. Show how this statistic can be used to give a prediction region for x based on $\bar{x}$ and S (i.e., a region such that one has a given confidence that the next observation will fall into it).

5.20. (Sec. 5.3) Let $x_\alpha^{(i)}$ be observations from $N(\boldsymbol{\mu}^{(i)}, \boldsymbol{\Sigma}_i)$, $\alpha = 1, \ldots, N_i$, $i = 1, 2$. Find the likelihood ratio criterion for testing the hypothesis $\boldsymbol{\mu}^{(1)} = \boldsymbol{\mu}^{(2)}$.