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October 2022

1 Introduction

We shall now understand the concept of mixed effect models, which we shall introduce with reference of the following interactive example:

Suppose you are preparing for a particular examination. In general, we would want to go through the previous years' exam question papers to enhance your preparation. But what is the logic behind going through previous years' papers if you know that is not the examination paper that you will be provided with during your exam?

The answer is pretty obvious. There is a certain relationship between the previous paper and the question paper you will get in the exam hall. (In easy English, generally the same person finalises the questions in either paper.) We thus try to make a rough estimate of the standard of the questions by going through all the past papers.

Looking at things from this specific point of view, we can also understand that only the standard of questions is considered, meaning only that aspect of the past is considered that can give us some meaningful and relevant results when used in the future.

Something quite similar is expected during data analysis. Statisticians analyse past data to make predictions for the future. The relationship is explained by the standard procedure of data generation (which we assume to be non-changing). Thus, in such cases, we also have to keep in mind to consider only

those aspects that are meaningful and relevant in the future

To understand this better, consider the standard example of tossing a coin, say 10 times. Suppose we think that the coin is faulty, and the probability of appearing heads when tossed is not 0.5. We try to estimate the natural probability of appearing heads of the coin by counting the number of heads out of the 10 times tossed. Clearly, the order in which the heads and tails occur is irrelevant in the above process. Thus, this aspect of the data is not concentrated upon.

2 BUT WHAT DOES THIS HAVE TO DO WITH LINEAR MODELS?

We shall once again consider our favourite example – the plots. We shall consider only two factor inputs, variety of seeds and the village in which the plot is located. To understand this example better, follow the underlying backstory:

Suppose a certain variety of seeds is used by all the farmers. The government, however, wishes to introduce a new variety of seeds, say a certain kind of hybrid, which it thinks will be beneficial for the state

Thus, the study was done and the observations were noted, over 2 variety of seeds, and 3 different villages. We do not generally expect an interaction in a model like this, so we consider the following linear model:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}$$
$i = 1, 2$
$j = 1, 2, 3$
$k = 1, 2, 3, 4, 5$

The natural way to proceed from here is to firstly consider the identifiability conditions desired, and estimate all the parameters accordingly.

Suppose our identifiability conditions are:

$$\Sigma\alpha_i = 0$$
$$\Sigma\beta_j = 0$$

Let us assume the analysis is carried out. The parameters are estimated and a report is made. Now suppose the government (of any state, the same as the one that performed the study, or a different one) wants to know the result of this study a little while later

Without any doubt, we can state that our identifiability conditions are appropriate for this study. But while the report is being read, how much will be relevance of the numerical values of b_1,b_2 and b_3, the effect of 3 villages in the state that were chosen as randomly as possible?

The β parameter is of much less importance, and does not give any interesting or useful information later. However, the same cannot be said about the α parameter. Thus, at times, we can find fundamental asymmetries between both the inputs (varieties and villages in this case) that is not easily reflected in the linear models when considered blindly. The interpretation is thus of a greater importance in such cases.

So the question we now pose to ourselves is:

Can we construct a linear model which directly reflects the asymmetry in our interpretation of the model?