lsm

Yakshith naidu

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1 Quadratic Regression

The variant we'll observe is based on the black box with the system being an elastic spring that only examines a single input (w) and a single output (l).

Note that, the same black box we have used to fit our linear regression may be used to fit some other type of linear model as well.The linear model we are now going to review is the quadratic regression :

$$\ell_i = \beta_1 + \beta_2 w_i + \beta_3 w_i^2 + \epsilon_i \quad \text{for } i = 1, \ldots, n.$$

In this case, we consider the same process as before. We suspend many different weights (one at a time) through the spring, noting down both, the weight suspended and the length of the spring during the suspension.

Note that, for each of these trials, we record the weight applied w_i (no randomness,we know it precisely) ; and also the final length of the spring l_i. However, we try to relate them by a slightly altered equation:

$$l = \beta_1 + \beta_2\omega + \beta_3\omega^2 + \varepsilon \tag{1}$$

Keep in mind that this is linear quadratic regression.Though this is quadratic in "w"; it is still linear equation in the coefficients considered .For R, the unknown quantities are $\beta_1, \beta_2, \beta_3, ...$ Representing as a linear model :

$$\begin{bmatrix} \ell_1 \\ \ell_2 \\ \vdots \\ \ell_n \end{bmatrix} = \begin{bmatrix} 1 & w_1 & w_1^2 \\ 1 & w_2 & w_2^2 \\ \vdots & & \\ 1 & w_n & w_n^2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

But how do we tell this to a computer ?
The command to be used:

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lm(l ~ w+I(w^2))
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The lm() command will change slightly ; then l(response); $\sim$ (approximate quantity) ; w (the linear term) ; and then w^2 ,but it needs to be placed in a wrapper (the I function). This preserves the w^2 term.If it is not applied , w^2 will be treated as w.