Gauss-Markov Model : Geometric Intuition

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Here, we try to give the geometric intuition behind the proof :

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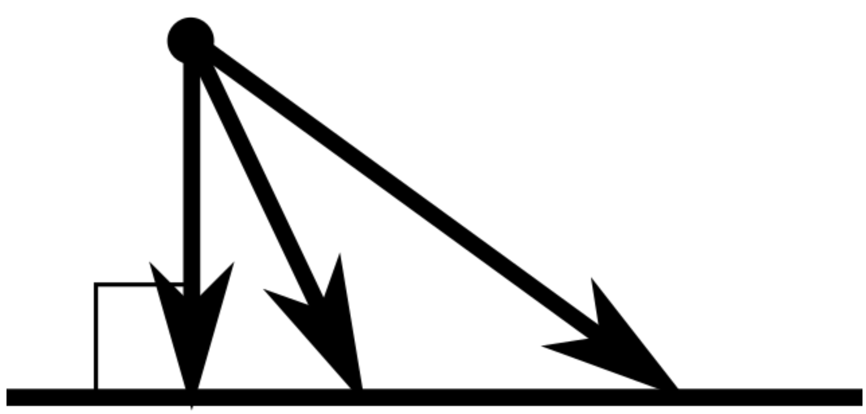

Suppose we have a straight line and a point not on the line.

We want to prove the following two things :

1. The shortest distance from the point to the line is along the perpendicular dropped from the point onto the line.

2. The shortest distance line is unique.

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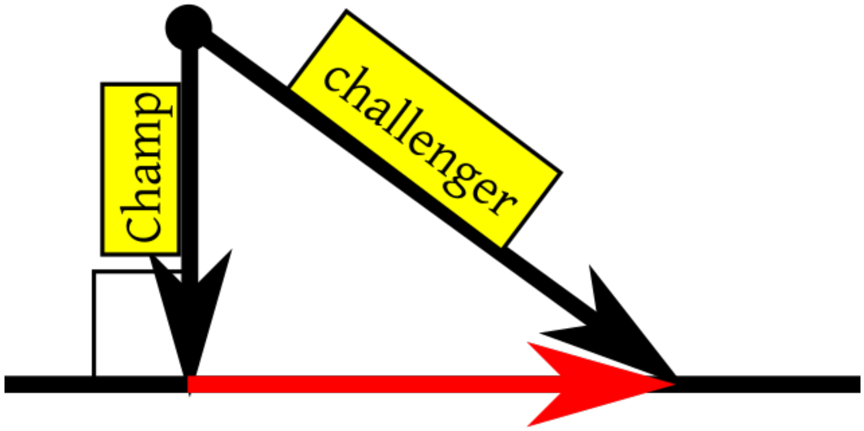

Proof of 1. We claim that, *champ* is the shortest distance to the line. To prove that, we'll show any other line, say *challenger*, is longer than the defending *champ*.

Let the red-coloured ray in the figure above be δ.

We see that in this right- angled triangle, by pythagoras theorem :

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\begin{aligned}
(\text{challenger})^2 &= (\text{champ})^2 + \delta^2 \\
\implies \text{challenger}^2 &\geq (\text{champ})^2 \\
\implies \text{challenger} &\geq \text{champ}
\end{aligned}
$$

Thus, the distance of the perpendicular is the shortest.

Proof of 2. Suppose, on contrary, we have a *challenger* which has the same length as the *champ*. Then our defined δ^2 is zero. Thus, the *challenger* coincides with the *champ* and hence the shortest distance from the point to the line is unique.