# Mixed Effect Models: A real life examples

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## Abstract

We will take a look at an example where our linear Mixed model will give us a benefit over the other available options.

## Example setup

We will be working with tablets. We can measure the total amount of active content in a batch of tablets with the help of chemical processes. HPLC and NIR are two such chemical processes for estimating the amount of active content in a sample Our aim is to compare these two methods of estimation.

## Data Collection

We select n tablets from our batch, say 10 tablets, split each tablet into two halfs, and we test for the amount for active content in each half with HPLC and NIR. As a result we get a pair of measurements for each tablet.

We get data ( $y_{1,1}$ , $y_{2,1}$ ) for the first tablet, where 1 denotes mesurement from HPLC method and 2 denotes mesurement form NIR method.

So we have Dataset ( $y_{1,1}$ , $y_{2,1}$ ), ( $y_{1,2}$ , $y_{2,2}$ ),....,( $y_{1,10}$ , $y_{2,10}$ )

We want to test whether with two test give apporximately the same results or they differ significantly.

## Testing with paired t-test

We have already studied paired t-test for such type of problem. In paired t-test we assume that ( $y_{1,j}, y_{2,j}) \sim Bivariate\ Normal\ Distribution$, so we can take the difference $y_{1,j} - y_{2,j} \sim Normal\ Distribution$, on which we can carry out the t test on this difference.

However we will show that using a linear mixed effects model can actually improve upon this assumption.

## Testing with Mixed effects Model

We have the Model,

$$y_{i,j} = \mu + \alpha_i + b_j + \epsilon_{i,j} \tag{1}$$

Here we have $\alpha_i$ denoting fixed effect due to methods HPCL and NIR. Now $b_j$ stands for effect of the tablet, but here we have selected the tablets at random form a huge batch,so this tablet effect is a random effect and therefore instead of $\beta_j$ we have $b_j$.

Hence we have an extra assumption on the distribution of $b_j \sim N(0, \tau^2)$. With the usual assumption on $\epsilon_{i,j} \sim N(0, \sigma^2)$ and independence between tablets which is very similar for to the assumption on paired t-test where $y_{1,j}$ and $y_{2,j}$ are jointly Bivariate Normal and they are independent between tablets.

## Comparison

We compute the covariance between $y_{1,j}\ and\ y_{2,j}$ and we expect it to be non-negative as these two methods are applied on the same tablet, that is,

$$Cov(y_{1,j}, y_{2,j}) \geq Var(b_j)$$
$$Cov(y_{1,j}, y_{2,j}) \geq \tau^2 \geq 0$$

So this inherent feature of our model is incorporated in our mixed effect model but not in traditional paired t-test. Although this may not make a huge difference in the result for this set-up. We must note that even in such a simple set-up the salient features are included in the Mixed Effects Model.