Linear Statistical Models
Video 106 - Fisher's LSD

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1   Introduction

In Bonferroni's method, each test in the multiple hypotheses testing was performed individually at level $\frac{\alpha}{k}$, where α lies between 0 and 1. However, in general $\frac{\alpha}{k}$ becomes very small and in all most cases except the extreme ones, the hypotheses are accepted. In order to handle this problem, R.A.Fisher introduced the Least Significant Difference (LSD) test.

2   Theory

We shall look at the Fisher's LSD test with the help of an example.
We take the model,
$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$
where, $\vec{\epsilon} \sim N_n(\vec{0}, \sigma^2 I)$, $\sigma^2 > 0$ and unknown.

Each test of the multiple hypotheses test is as follows,
$$H_0^{(s,t)} : \alpha_s = \alpha_t$$

Number of tests in the multiple hypotheses test $= \mathrm{k} = \binom{p}{2}$

In Fisher's LSD method, we go by the following steps:

1. Test whether all α_i's are same. If this is accepted, then all the individual tests are accepted and we are done. In case this is rejected, we go to the next step.

2. Test each pair using a two sample t-test with $\hat{\sigma}^2$ from the entire data. Note that here we are not using the pooled estimator of σ^2 from the two samples being considered. Then we accept and reject accordingly.

If there is a false discovery it cannot happen in step 1. The process has to enter step 2. In case of a false discovery, we have already rejected the overall test in step 1 at the same level as the FWER. Thus, the value of FWER in step 2 can never exceed the value of FWER used in the previous step and hence, the value of FWER is controlled.

3   Conclusion

The method described above is the Fisher's LSD test and is the simplest way to incorporate all the hypotheses simultaneously. However, it has some drawbacks:

1. The power of the Fisher's LSD tet is not very good.

2. Sometimes it provides counter intuitive results. For example, it may happen that the overall test in step 1 is rejected but all the individual tests in step 2 are accepted. This can occur as we are using $\hat{\sigma}^2$ from the entire data. These happen mostly in borderline cases.

In general Fisher's LSD method is very simple and intuitive way of addressing the problem.