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Deterministic Finite Automata

Arnab Chakraborty

Indian Statistical Institute

Sep 15, 2023

0+1=1 Don't worry...

...this is not a G-pay QR code:



...just a link to a Youtube playlist containing this lecture.

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Deterministic Finite Automaton (DFA)

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Monoids Transfer mx metho = Finite Automaton

= Finite state machine

= State machine

= Internal state machine

Application areas

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Computer science:

- Robotics
- Digital electronics
- Compilers for languages like C, Java etc
- Embedded systems (e.g., IoT)

Mathematics:

- Algebraic structures
- 2 Combinatorics

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7+3=10 **Definition**

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Three sets:

- $A = \mathbf{set} \ \mathbf{of} \ \mathbf{inputs} = \{\mathbf{smile}, \mathbf{Punch}\}$
- B = set of outputs =

 $\Big\{ ext{Normal, Smile back, Raise eyebrow, Scowl, Growl, Grimace} \Big\}$

• S = set of states =

 $\Big\{ {
m Ordinary,\ Happy,\ Surprised,\ Angry,\ Furious} \Big\}$

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Three sets:

- $A = \mathbf{set} \ \mathbf{of} \ \mathbf{inputs} = \{\mathsf{smile}, \mathsf{Punch}\}$
- B = set of outputs =
 {Normal, Smile back, Raise eyebrow, Scowl, Growl, Grimace}
- S =set of states = {Ordinary, Happy, Surprised, Angry, Furious}

Two functions:

- $f: A \times S \rightarrow B$, the output function.
- $g: A \times S \rightarrow S$, the next state function.

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Three sets:

- $A = \mathbf{set} \ \mathbf{of} \ \mathbf{inputs} = \{\mathsf{smile}, \mathsf{Punch}\}$
- B = set of outputs =
 {Normal, Smile back, Raise eyebrow, Scowl, Growl, Grimace}
- $S = \text{set of states} = \{ \text{Ordinary, Happy, Surprised, Angry, Furious} \}$

Two functions:

- $f: A \times S \rightarrow B$, the output function.
- $g: A \times S \rightarrow S$, the next state function.

One state:

• $s_0 \in S$: the initial state = Ordinary.

10+3=13 Transition diagram

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13+1=14 ____ Why care?

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13+1=14Why care?

C comments

Any machine with a memory must be a state machine!

```
14 + 2 = 16
        C comments
```

C comments

```
void main() {
 printf("Hello world!"); /* Correct this.*/
```

16+1=17 Look for /* ... */

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16+1=17Look for /* ... */

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16+1=17 Look for /* ... */

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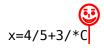
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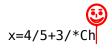
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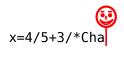
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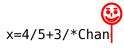
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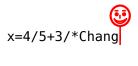
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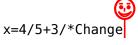
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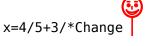
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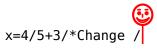
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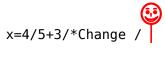
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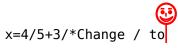
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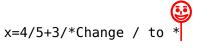
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$$17 + 3 = 20$$



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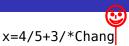
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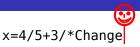
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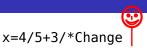
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$$x=4/5+3/*Change /$$

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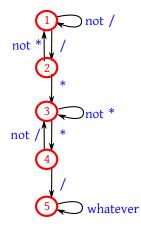
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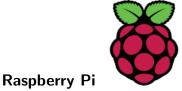
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...a state machine is much like a ...

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...a state machine is much like a monoid.

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Mathematically...

...a state machine is much like a monoid.

A monoid is (M, +) where M is a set and + is an associative binary operation on M having identity.

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Math

Monoids

...a state machine is much like a monoid.

A monoid is (M, +) where M is a set and + is an associative binary operation on M having identity.

Example:

$$({0,1,2,...},+).$$

Mathematically...

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Math

...a state machine is much like a monoid.

A monoid is (M, +) where M is a set and + is an associative binary operation on M having identity.

Example:

$$({0,1,2,...},+)$$
. Identity = 0.

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Math Monoids ...a state machine is much like a monoid.

A monoid is (M, +) where M is a set and + is an associative binary operation on M having identity.

Example:

All finite length words using the letters a, b, c under the concatenation operator: "abc" \bullet "ab" = "abcab".

```
33+2=35
Mathematically...
```

ntroduction

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...a state machine is much like a monoid.

A monoid is (M, +) where M is a set and + is an associative binary operation on M having identity.

Example:

All finite length words using the letters a, b, c under the concatenation operator: "abc" \bullet "ab" = "abcab". Identity = "".

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Certain input words are equivalent for the machine.

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Certain input words are equivalent for the machine.

Club them together. You will get finitely many classes.

Certain input words are equivalent for the machine.

Club them together. You will get finitely many classes.

 $w_3 = w_1 \bullet w_2$.

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Monoids

Monoids Transfer mx me

Certain input words are equivalent for the machine.

Club them together. You will get finitely many classes.

$$[w_3] = [w_1] \bullet [w_2].$$

Certain input words are equivalent for the machine.

Club them together. You will get finitely many classes.

$$[w_3] = [w_1] \bullet [w_2].$$

This defines a new operation on the equivalence classes, under which they form a monoid.

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Certain input words are equivalent for the machine.

Club them together. You will get finitely many classes.

$$[w_3] = [w_1] \bullet [w_2].$$

This defines a new operation on the equivalence classes, under which they form a monoid. This monoid captures the "underlying structure" of the finite machine.

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Math Monoid Certain input words are equivalent for the machine.

Club them together. You will get finitely many classes.

$$[w_3] = [w_1] \bullet [w_2].$$

This defines a new operation on the equivalence classes, under which they form a monoid. This monoid captures the "underlying structure" of the finite machine.

The monoid is always finite.

From monoid to finite state machine

Start with a finite monoid, (M, +). Then we can define a FSM with state set and input set both equal to M. For any state $s \in M$ and any input $i \in M$ we define the next state as m+i.

40+2=42

Some interesting questions

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Monoid to FSM to monoid.

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- Monoid to FSM to monoid.
- FSM to monoid to FSM.

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- Monoid to FSM to monoid.
- FSM to monoid to FSM.
- Size of the monoid for a given FSM.

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Monoids

- Monoid to FSM to monoid.
- FSM to monoid to FSM.
- Size of the monoid for a given FSM.
- Same monoid ⇒ same transition diagram?

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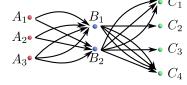
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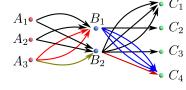
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$$2 \times 3 + 1 \times 1 = 7$$
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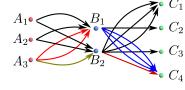
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$$2\times 3+1\times 1=7.$$

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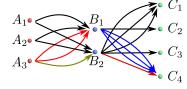
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$$2 \times 3 + 1 \times 1 = 7$$
.

$$\begin{bmatrix} \begin{smallmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \times \begin{smallmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} \begin{smallmatrix} C_1 & C_2 & C_3 & C_4 \\ 0 & 1 & 0 & \mathbf{3} \\ 3 & 0 & 1 & \mathbf{1} \end{bmatrix} \ = \begin{bmatrix} \begin{smallmatrix} C_1 & C_2 & C_3 & C_4 \\ A_2 \\ 3 & 1 & 1 & 4 \\ 3 & 2 & 1 & \mathbf{7} \end{bmatrix}$$

47 + 5 = 52

Same set

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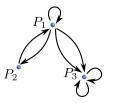
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$$\begin{array}{c|cccc}
P_1 & P_2 & P_3 \\
P_1 & 1 & 2 \\
P_2 & 1 & 0 & 0 \\
P_3 & 0 & 0 & 2
\end{array}$$

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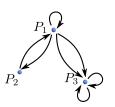
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$$\begin{bmatrix} P_1 & P_2 & P_3 \\ P_2 & 1 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} = A, \text{ say.}$$

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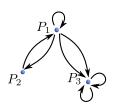
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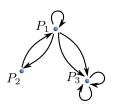
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$$\left[egin{smallmatrix} P_1 & P_2 & P_3 & P_3 \\ P_1 & 1 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{array} \right] = A, ext{ say.}$$

$$A^2 = \left[\begin{array}{ccc} 2 & 1 & 6 \\ 1 & 1 & 2 \\ 0 & 0 & 4 \end{array} \right].$$

Transfer mx method



$$\left[egin{smallmatrix} P_1 & P_2 & P_3 & P_3 \ P_2 & P_3 & 0 & 0 \ 0 & 0 & 2 \ \end{bmatrix} = A, ext{ say.}
ight.$$

$$A^2 = \left[\begin{array}{ccc} 2 & 1 & 6 \\ 1 & 1 & 2 \\ 0 & 0 & 4 \end{array} \right].$$

The (i,j)-th entry of A^k gives the number of paths of length k from P_i to P_i .

Example: a counting problem

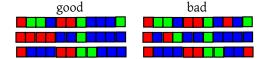
Transfer mx method

Want to count the number (b_n) of sequences with \blacksquare , \blacksquare and \blacksquare . Total number of squares is n. Must have two consecutive 's somewhere before three consecutive 's.

Example: a counting problem

Transfer mx method

Want to count the number (b_n) of sequences with \blacksquare , \blacksquare and \blacksquare . Total number of squares is n. Must have two consecutive 's somewhere before three consecutive 's.



55 + 5 = 60

Solution







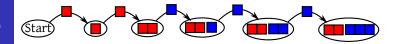






55 + 5 = 60

Solution



55+5=60

Solution

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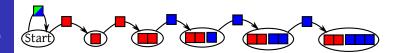
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Monoic



55+5=60

Solution

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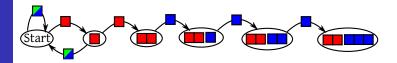
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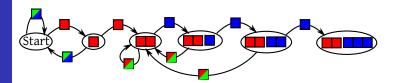
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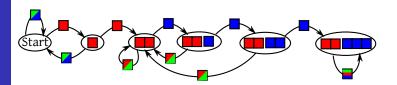
55 + 5 = 60

Solution



55 + 5 = 60

Solution



55+5=60

Solution

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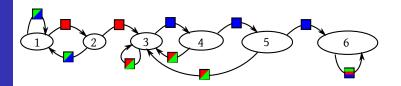
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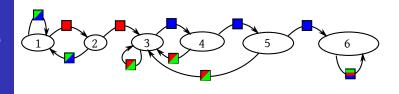
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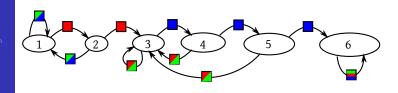
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Transfer mx method



$$A = \begin{bmatrix} \begin{smallmatrix} \mathsf{To} \\ 1 \\ 2 \\ 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} \begin{smallmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

The (1,6)-th entry of A^n gives the number of such patterns of length n, i.e. b_n .

60+2=62 _____ Example

http://maxima.cesga.es

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```
60+2=62
```

Example

```
http://maxima.cesga.es
```

```
A: matrix([2, 1, 0, 0, 0, 0], [2, 0, 1, 0, 0, 0], [0, 0, 2, 1, 0, 0], [0, 0, 2, 0, 1, 0], [0, 0, 2, 0, 0, 1], [0, 0, 0, 0, 0, 0, 3]);
```

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```
60+2=62
```

Example

 A^{45} ;

```
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```
A: matrix([2, 1, 0, 0, 0, 0], [2, 0, 1, 0, 0, 0], [0, 0, 2, 1, 0, 0], [0, 0, 2, 0, 1, 0], [0, 0, 2, 0, 0, 1], [0, 0, 0, 0, 0, 0, 3]);
```

http://maxima.cesga.es

Example

Transfer mx method

```
http://maxima.cesga.es
```

```
A: matrix([2, 1, 0, 0, 0, 0],
           [2, 0, 1, 0, 0, 0],
           [0, 0, 2, 1, 0, 0],
           [0, 0, 2, 0, 1, 0],
          [0, 0, 2, 0, 0, 1],
          [0, 0, 0, 0, 0, 3]);
```

A^{45} ;

1648298555265532254035

Transfer mx method

The generating function of $\{b_n\}_n$ is the formal power series

$$b_0 + b_1 t + b_2 t^2 + b_3 t^3 + \cdots$$

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Transfer mx method

The generating function of $\{b_n\}_n$ is the formal power series

$$b_0 + b_1 t + b_2 t^2 + b_3 t^3 + \cdots$$

This is the (1,6)-th entry of

$$I + At + A^2t^2 + \cdots$$

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Transfer mx method

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$$I + At + A^2t^2 + \cdots = (I - tA)^{-1}.$$

Transfer mx method

The generating function of $\{b_n\}_n$ is the formal power series

$$b_0 + b_1 t + b_2 t^2 + b_3 t^3 + \cdots$$

This is the (1,6)-th entry of

$$I + At + A^2t^2 + \cdots = (I - tA)^{-1}.$$

Just like

$$1 + x + x^2 + \dots = \frac{1}{1 - x}$$
 if $|x| < 1$.

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(1-t*A);

```
66+3=69
<u>Inverse</u>
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Math

Monoid:

```
((I-t*A)^^-1)[1,6];
```

<u>Inverse</u>

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$$((I-t*A)^{-1})[1,6];$$

$$\frac{t^5}{-12 t^6 - 20 t^5 - 10 t^4 + 6 t^3 + 12 t^2 - 7 t + 1}$$

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Transfer mx method

$$((I-t*A)^{-1})[1,6];$$

$$\frac{t^5}{-12 t^6 - 20 t^5 - 10 t^4 + 6 t^3 + 12 t^2 - 7 t + 1}$$

From this we get

$$b_{n+5} = 7b_{n+4} - 12b_{n+3} - 6b_{n+2} + 10b_{n+1} + 20b_n + 12b_{n-1}$$

Inverse

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Transfer mx method

((I-t*A)^^-1)[1,6];

$$\frac{t^5}{-12 t^6 - 20 t^5 - 10 t^4 + 6 t^3 + 12 t^2 - 7 t + 1}$$

From this we get

$$b_{n+5} = 7b_{n+4} - \frac{12}{5}b_{n+3} - \frac{6}{5}b_{n+2} + \frac{10}{5}b_{n+1} + \frac{20}{5}b_n + \frac{12}{5}b_{n-1}$$

How?

69 + 2 = 71

Reference

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