

২নং প্রঃউঃ

(ক)

দেওয়া আছে,

$$P(a) = a^3 + 5a^2 + 6a + 8$$

$$P(-1) = (-1)^3 + 5 \times (-1)^2 + 6 \times (-1) + 8$$

$$= -1 + 5 - 6 + 8$$

$$= 13 - 7$$

$$= 6$$

$$P(-3) = (-3)^3 + 5 \times (-3)^2 + 6 \times (-3) + 8$$

$$= -27 + 45 - 18 + 8$$

$$= 8$$

(খ)

দেওয়া আছে,

$$g(x) = \frac{1}{1-x^3}$$

$$g(x) = \frac{1}{(1)^3 - (x)^3}$$

$$= \frac{1}{(1-x)(1+x+x^2)}$$

$$\therefore \frac{1}{(1-x)(x^2+x+1)} = \frac{A}{1-x} + \frac{B}{x^2+x+1} \quad \text{--- (i)}$$

(i) নং সমীকরণের উভয়পক্ষে $(1-x)(x^2+x+1)$ গুণন করে দিই,

$$1 = A(x^2+x+1) + B(1-x) \quad \text{--- (ii)}$$

(ii) নং সমীকরণে $x=1$ বসিয়ে দাও,

$$1 = A(1^2 + 1 + 1) + B(1 - 1)$$

$$\text{বা } 1 = A(1+2) + B \times 0$$

$$\text{বা } 1 = 3A$$

$$\text{এ } A = \frac{1}{3}$$

আবার (ii) নং এ $x=-1$ বসিয়ে দাও,

$$1 = A((-1)^2 - 1 + 1) + B(-1 - (-1))$$

$$\text{এ } 1 = A + 2B$$

$$\text{এ } 1 = \frac{1}{3} + 2B \quad \left[A = \frac{1}{3} \right]$$

$$\text{বা } 2B + \frac{1}{3} = 1$$

$$\text{এ } 2B = 1 - \frac{1}{3}$$

$$\text{বা } 2B = \frac{3-1}{3}$$

$$\text{এ } B = \frac{2}{3} \times \frac{1}{2}$$

$$\text{এ } B = \frac{1}{3}$$

A ও B এর মান ১ নং এ বসিয়ে দাও,

$$\frac{1}{x(1-x)(x^2+x+1)} = \frac{\frac{1}{3}}{1-x} + \frac{\frac{4}{3}}{x^2+x+1}$$
$$= \frac{1}{3(1-x)} + \frac{4}{3(x^2+x+1)}$$

(১৭)

দেওয়া আছে,

$$P(a) = a^3 + 5a^2 + 6a + 8$$

$$P(x) = x^3 + 5x^2 + 6x + 8$$

আবার, $a = x$ হলে

$$P(x) = x^3 + 5x^2 + 6x + 8$$

প্রদত্ত,

$$x^3 + 5x^2 + 6x + 8 = y^3 + 5y^2 + 6y + 8$$

$$\text{বা } x^3 - y^3 + 5x^2 - 5y^2 + 6x - 6y + 8 - 8 = 0$$

$$\text{বা } (x-y)(x^2 + xy + y^2) + 5(x^2 - y^2) + 6(x-y) = 0$$

$$\text{বা } (x-y)(x^2 + xy + y^2) + 5(x+y)(x-y) + 6(x-y) = 0$$

$$\text{বা } (x-y) \{ x^2 + xy + y^2 + 5(x+y) + 6 \} = 0$$

$$\text{বা } (x-y) \{ x^2 + xy + y^2 + 5x + 5y + 6 \} = 0$$

$$\text{বা } x^2 + y^2 + xy + 5x + 5y + 6 = 0 \quad [\text{প্রদত্ত } x \neq y]$$

(প্রত্যাখ্যাত)

২ নং প্রঃ উঃ

প্রঃ

দেওয়া আছে,

$$P(x) = x^3 - x^2 - 10x - 8$$

$P(x)$ কে $x+4$ দ্বারা ভাগ করলে,

অর্থাৎ $x = -4$

$$P(-4) = (-4)^3 - (-4)^2 - 10 \times (-4) - 8$$

$$= -64 - 16 + 40 - 8$$

$$= -48$$

প্রঃ

দেওয়া আছে ~~প্রঃ~~

$$P(x) = x^3 - x^2 - 10x - 8$$

$x = -1$ হলে,

$$P(-1) = (-1)^3 - (-1)^2 - 10 \times (-1) - 8$$

$$A = -1 - 1 + 10 - 8$$

$$= -10 + 10$$

$$= 0$$

$\therefore (y+1)$; $P(y)$ ଏବଂ ଏକାଠି ଡେଇଁବା,

ଅର୍ଥାତ୍,

$$\begin{aligned} P(y) &= y^3 + y^2 - 10y - 8 \\ &= y^3 + y^2 - 2y^2 - 2y - 8y - 8 \\ &= y^2(y+1) - 2y(y+1) - 8(y+1) \\ &= (y+1)(y^2 - 2y - 8) \\ &= (y+1)(y^2 - 4y + 2y - 8) \\ &= (y+1) \{ y(y-4) + 2(y-4) \} \\ &= (y+1)(y-4)(y+2) \end{aligned}$$

(৬)

দেওয়া আছে,

$$P(x) = x^3 - x^2 - 10x - 8$$

$$P(a) = a^3 - a^2 - 10a - 8$$

$$P(b) = b^3 - b^2 - 10b - 8$$

প্রদত্ত,

$$P(a) = P(b)$$

$$a^3 - a^2 - 10a - 8 = b^3 - b^2 - 10b - 8$$

$$\text{বা } a^3 - b^3 - a^2 + b^2 - 10a + 10b - 8 + 8 = 0$$

$$\text{বা } (a-b)(a^2 + ab + b^2) - 1(a^2 - b^2) - 10(a-b) = 0$$

$$\text{বা } (a-b)(a^2 + ab + b^2) - 1(a+b)(a-b) - 10(a-b) = 0$$

$$\text{বা } (a-b)(a^2 + ab + b^2 - 1(a+b) - 10) = 0$$

$$\text{বা } (a-b)(a^2 + ab + b^2 - a - b - 10) = 0$$

$$\text{বা } a^2 + ab + b^2 - a - b - 10 = 0 \quad [a \neq b]$$

$$\text{বা } a^2 + b^2 + ab - a - b = 10$$

প্রমাণিত

৩ নং প্রঃ উঃ

ক)

দেওয়া আছে,

$$P(x) = x^3 - x^2 - 10x - 8$$

$x = -1$ বসিয়ে পাই,

$$P(-1) = (-1)^3 - (-1)^2 - 10 \times (-1) - 8$$

$$= -1 - 1 + 10 - 8$$

$$= -10 + 10$$

$$= 0$$

\therefore ~~কি~~ ১), $P(x)$ এর একটি উৎপাদক,

এখন,

$$P(x) = x^3 - x^2 - 10x - 8$$

$$= x^3 + x^2 - 2x^2 - 2x - 8x - 8$$

$$= x^2(x+1) - 2x(x+1) - 8(x+1)$$

$$= (x+1)(x^2 - 2x - 8)$$

$$\begin{aligned}
 &= (x+1)(x^2 - 4x + 2x - 8) \\
 &= (x+1) \{ x(x-4) + 2(x-4) \} \\
 &= (x+1)(x-4)(x+2)
 \end{aligned}$$

(ex)

$$\frac{x}{P(x)} = \frac{x}{x^3 - x^2 - 10x - 8}$$

କିନ୍ତୁ ଯାହା $P(x)$ କର ଉଠିବାର
 $(x+1)(x-4)(x+2)$

$$\frac{x}{x^3 - x^2 - 10x - 8} = \frac{x}{(x+1)(x-4)(x+2)}$$

$$\frac{x}{(x+1)(x-4)(x+2)} = \frac{A}{x+1} + \frac{B}{x-4} + \frac{C}{x+2}$$

i) নং ৬ ডিফারেন্স $(n+1)(n-1)(n+2)$
সুপ করে দাও,

$$n \equiv A(n-1)(n+2) + B(n+1)(n+2) \\ + C(n+1)(n-1) \dots \text{পূর্ণ}$$

ii) নং ৬. $n = -1$ বসিয়ে দাও

$$-1 = A(-1-1)(-1+2)$$

$$\text{বা } A(-2) \times 1 = -1$$

$$\text{বা } -2A = -1$$

$$\text{বা } -2A = -1 \quad [\text{ডিফারেন্স } -1 \text{ দ্বারা সুপ করলে}]$$

$$\text{বা } A = \frac{1}{2}$$

আবার (i) নং $x = 4$ বসিয়ে পাই

~~২০~~

$$4 = B(4+1)(4+2)$$

$$\text{এ } B \times 5 \times 6 = 4$$

$$\text{এ } 30 B = 4$$

$$\text{এ } B = \frac{4}{30}$$

$$\text{এ } B = \frac{2}{15}$$

আবার (ii) নং $x = -2$ বসিয়ে পাই

$$-2 = C(-2+1) \times (-2-4)$$

$$\text{এ } C(-1) \times (-6) = -2$$

$$\text{এ } 6C = -2$$

$$\text{এ } C = -\frac{2}{6} \quad \text{এ } C = -\frac{1}{3}$$

A, B, C এর ক্ষেত্রে (i) নং এর বসিয়ে পাই

$$\frac{n}{(n+1)(n-1)(n+2)}$$

$$= \frac{1}{5} + \frac{2}{15} + \frac{(-\frac{1}{6})}{n+2}$$

$$= \frac{1}{5(n+1)} + \frac{2}{15(n-1)} - \frac{1}{6(n+2)}$$

(৩)

কোন ক্ষেত্রে আছে,

$$\frac{x^2 - yz}{a} = \frac{y^2 - zx}{b} = \frac{z^2 - xy}{c} \neq 0$$

যদি,

$$\frac{x^2 - yz}{a} = \frac{y^2 - zx}{b} = \frac{z^2 - xy}{c} = k$$

তাহলে,

$$\frac{x^2 - yz}{a} = k$$

$$\text{বা } x^2 - yz = ak$$

$$\text{বা } x^3 - x yz = axk \quad \text{--- (i)}$$

$$\frac{y^2 - zx}{b} = k$$

$$\text{বা } y^2 - zx = bk$$

$$\text{বা } y^3 - x yz = byk \quad \text{--- (ii)}$$

আবার,

$$\frac{z^2 - xy}{c} = k$$

$$\text{বা } z^2 - xy = ck$$

$$\text{বা } z^3 - x yz = czk \quad \text{--- (iii)}$$

(i) + (ii) + (iii) કરવા પર,

$$x^3 - xyz + y^3 - xyz + z^3 - xyz$$

$$= axk + byk + czk$$

$$\therefore x^3 + y^3 + z^3 - 3xyz = k(ax + by + cz)$$

$$\frac{1}{2}(x+y+z) \{ (x-y)^2 + (y-z)^2 + (z-x)^2 \}$$

$$= k(ax + by + cz)$$

$$\frac{1}{2}(x+y+z) \{ x^2 - 2xy + y^2 + y^2 - 2yz + z^2 + z^2 - 2zx + x^2 \}$$

$$= k(ax + by + cz)$$

જ

$$\therefore \frac{1}{2}(x+y+z) (2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$$

$$= k(ax + by + cz)$$

$$\text{বা } \frac{1}{2} \times 2 (x+y+z) (x^2+y^2+z^2-xy-yz-zx) \\ = k(ax+by+cz)$$

$$\text{বা } (x+y+z) \{ (x^2-xy) + (y^2-yz) + (z^2-zx) \} \\ = k(ax+by+cz)$$

$$\text{বা } (x+y+z) (xk+yk+zk) = k(ax+by+cz)$$

$$\text{বা } (x+y+z) (a+b+c) = ax+by+cz \\ \text{(প্রমাণিত)}$$