Activation of Genuine Multipartite Entanglement

State-Space Structures Beyond the Single-Copy Paradigm

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Quantum Meets
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TECHNISCHE UNIVERSITÄT WIEN
ATOMINSTITUT
VCQ Vienna Center for Quantum Science and Technology
FWF The Austrian Science Fund
Funded by the European Union NextGenerationEU
FFG Promoting Innovation.
Federal Ministry Republic of Austria Education, Science and Research

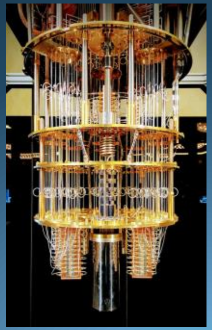
Entanglement

The Entanglement Frontier

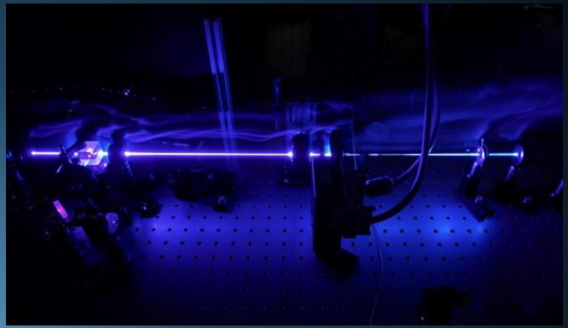
Quantum computation & Quantum simulation

Quantum communication

Ion trap at IQOQI Innsbruck
Image credit: C. Lackner



IBM Q dilution refrigerator
Image credit: Lars Plougmann



Pump laser for SPDC setup, Heriot-Watt University
Image credit: Mehul Malik

"…we are now in the early stages of exploring a new frontier of the physical sciences, what we might call … the **entanglement frontier**."

"Now, for the first time in human history, we are acquiring and perfecting the tools to build and precisely control very complex, highly entangled quantum states of many particles, states so complex that we can't simulate them with our best digital computers or characterize them well using existing theoretical tools."

J. Preskill, *Quantum Computing in the NISQ era and beyond*, Quantum **2**, 79 (2018).

➡ entanglement detection/certification as a benchmarking tool?

➡ New theoretical tools for understanding entanglement/Hilbert-space/network structures?

What is Entanglement?

Pure states: $|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B$ separable

$|\psi\rangle \neq |\phi\rangle_A \otimes |\chi\rangle_B$ entangled

Example: Bell states of 2 qubits

maximally entangled

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle|1\rangle \pm |1\rangle|0\rangle\big)$$

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle|0\rangle \pm |1\rangle|1\rangle\big)$$

Mixed states: $\rho = \sum_i p_i\, \rho_i^A \otimes \rho_i^B$ separable

$\rho \neq \sum_i p_i\, \rho_i^A \otimes \rho_i^B$ entangled

$$0 \leq p_i \leq 1$$

$$\sum_i p_i = 1$$

Entanglement Detection & Certification

- Detection: “Is the quantum system entangled?”
- Certification: “How strongly is the quantum system entangled (at least)?”

Diverse research field with many open theoretical questions and experimental challenges, e.g.,

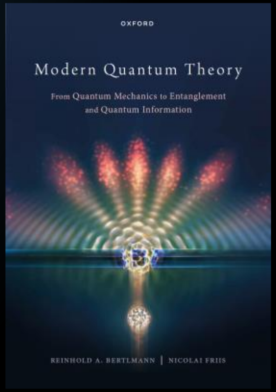
- Necessary and sufficient separability criteria
- Precise relation to “non-locality” (violation of Bell inequalities)
- Feasible, exact & operationally meaningful quantification of entanglement
- Efficient detection/certification across various experimental platforms

Image credit: Christian Murzek

see, e.g., our review:

N. Friis, Giuseppe Vitagliano, Mehul Malik, and Marcus Huber, *Entanglement certification from theory to experiment*, Nat. Rev. Phys. **1**, 72 (2019)

or Chapters 15-18 in our book: R. A. Bertlmann & N. Friis, *Modern Quantum Theory – From Quantum Mechanics to Entanglement and Quantum Information*, Oxford University Press, Oxford, U.K. (2023)



Entanglement Detection

Theoretically: Given $\rho = \sum_{i,j,k,l} \rho_{ij,kl} |j\rangle\langle k| \otimes |k\rangle\langle l|$ (e.g., from full state tomography)

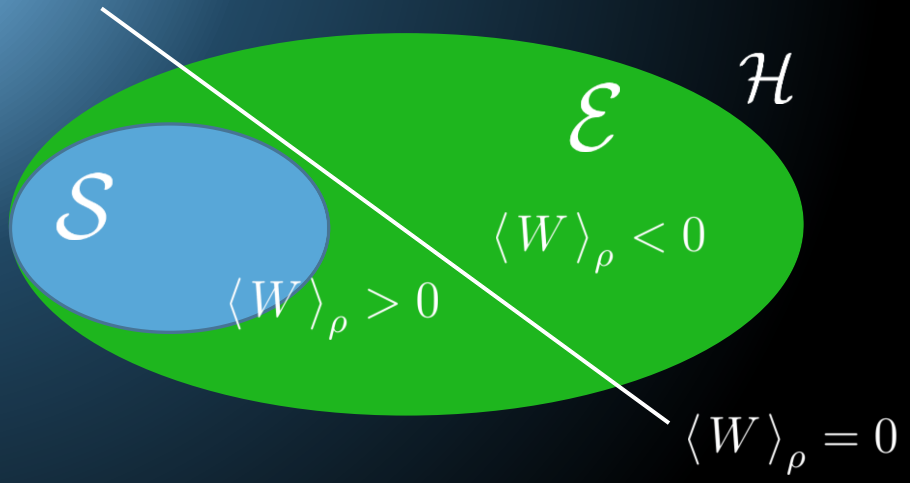
➡ Check necessary (but generally not sufficient) separability criteria

- Positive partial transpose (PPT): $\rho^{T_B} \geq 0$ (or based on other PnCP):
- Postive conditional entropy: $S(A|B) = S(\rho_{AB}) - S(\rho_B) \geq 0$

Separability problem: generally no known necessary & sufficient condition (that can be practically used)

Directly measurable ways to detect entanglement:

- Violation of Bell-type inequalities
- Entanglement Witnesses W



For references, see, e.g.,
[1] N. F., G. Vitagliano, M. Malik, and M. Huber, Nat. Rev. Phys. **1**, 72 (2019) [arXiv:1906.10929].

Entanglement Quantification & Certification

Exact quantification: Several entanglement monotones/entanglement measures available

For instance:

- (Logarithmic) Negativity: $\mathcal{N}(\rho) = \log_2 \|\rho^{T_B}\|_{\mathrm{tr}}$ (easily computable monotone but not a measure)
- Entanglement of Formation $\mathcal{E}_{\mathrm{oF}}(\rho) = \inf_{\mathcal{D}(\rho)} \sum_i p_i\, S\big(\mathrm{Tr}_B\, |\psi_i\rangle\langle\psi_i|\big)$

  where $\mathcal{D}(\rho)$ is the set of all decompositions $\{(p_i, |\psi_i\rangle)\}_i$ of ρ s.t. $\rho = \sum_i p_i\, |\psi_i\rangle\langle\psi_i|$

... and many more Entanglement measures difficult (if not impossible) to compute in general

Entanglement certification:

Experimentally accessible ways to provide lower bounds on entanglement measures

Genuine Multipartite Entanglement

Genuine Multipartite Entanglement

Pure state $|\Phi^{(k)}\rangle$ separable w.r.t. to k-partition $\mathcal{A}_1|\mathcal{A}_2|\dots|\mathcal{A}_k$ if

$$|\Phi^{(k)}\rangle = \bigotimes_{i=1}^{k} |\phi_{\mathcal{A}_i}\rangle$$

Mixed state $\rho^{(k)}$ is called k-separable if it can be written as a convex combination of pure states that are separable w.r.t. to some k-partition

$$\rho^{(k)} = \sum_i p_i |\Phi_i^{(k)}\rangle\langle\Phi_i^{(k)}|$$

➡ k-separability does not imply separabilty w.r.t. any specific partition

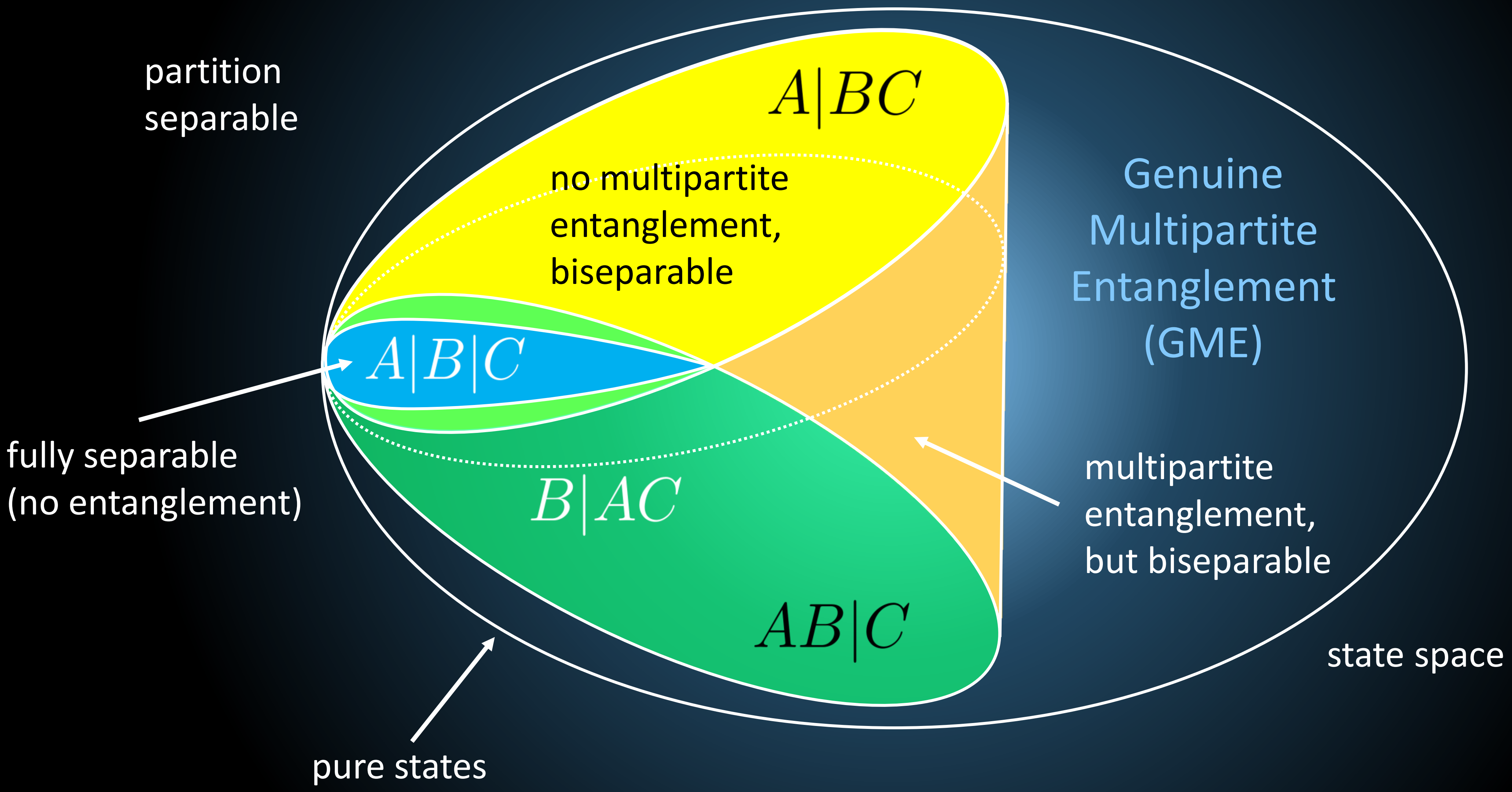
except when $\rho^{(k)}$ is pure or when $k = N$ (fully separable)

Here: States that are separable w.r.t. any partition: partition separable

For $k = 2$: biseparable

For $k = 1$: genuinely N-partite entangled (here, just „GME“)

Multipartite Entanglement for Three Parties

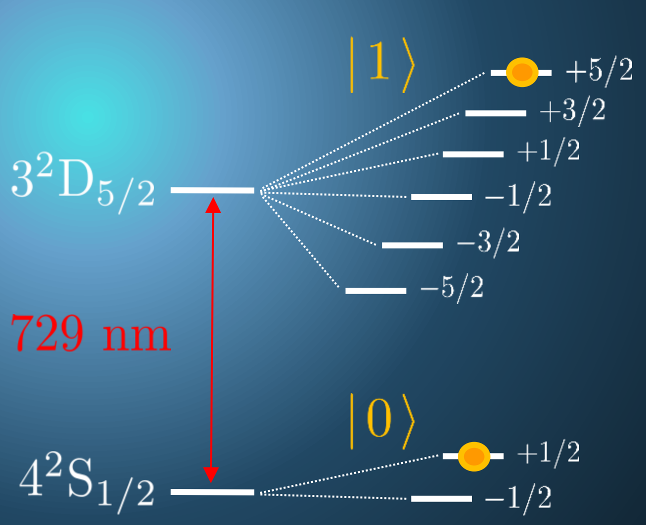


Example: Detecting Genuine Multipartite Entanglement in Ion Traps

N. Friis, O. Marty, C. Maier, C. Hempel, M. Holzäpfel, P. Jurcevic, M.B. Plenio, M. Huber, C. Roos, R. Blatt, B. Lanyon,
Observation of Entangled States of a Fully Controlled 20-Qubit System,
Physical Review X **8**, 021012 (2018)

Overview of the Setup

- 20 qubits realized using string of $^{40}\mathrm{Ca}^{+}$ ions
- Confined in a linear Paul trap 
- Qubit encoded in 2 Zeemann states of different levels (optical transition)
- Measurements via resonance fluorescence (extra levels not shown) and detection on an EMCCD camera



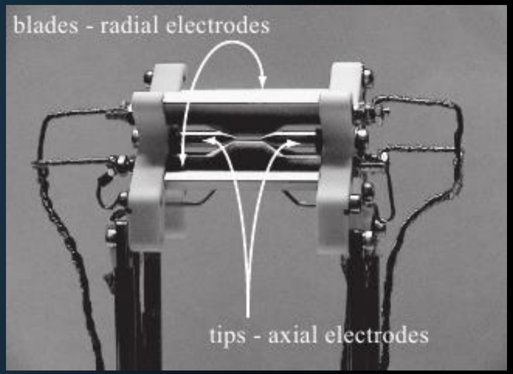


Image from: Schindler et al., New J. Phys. **15** 123012 (2013)

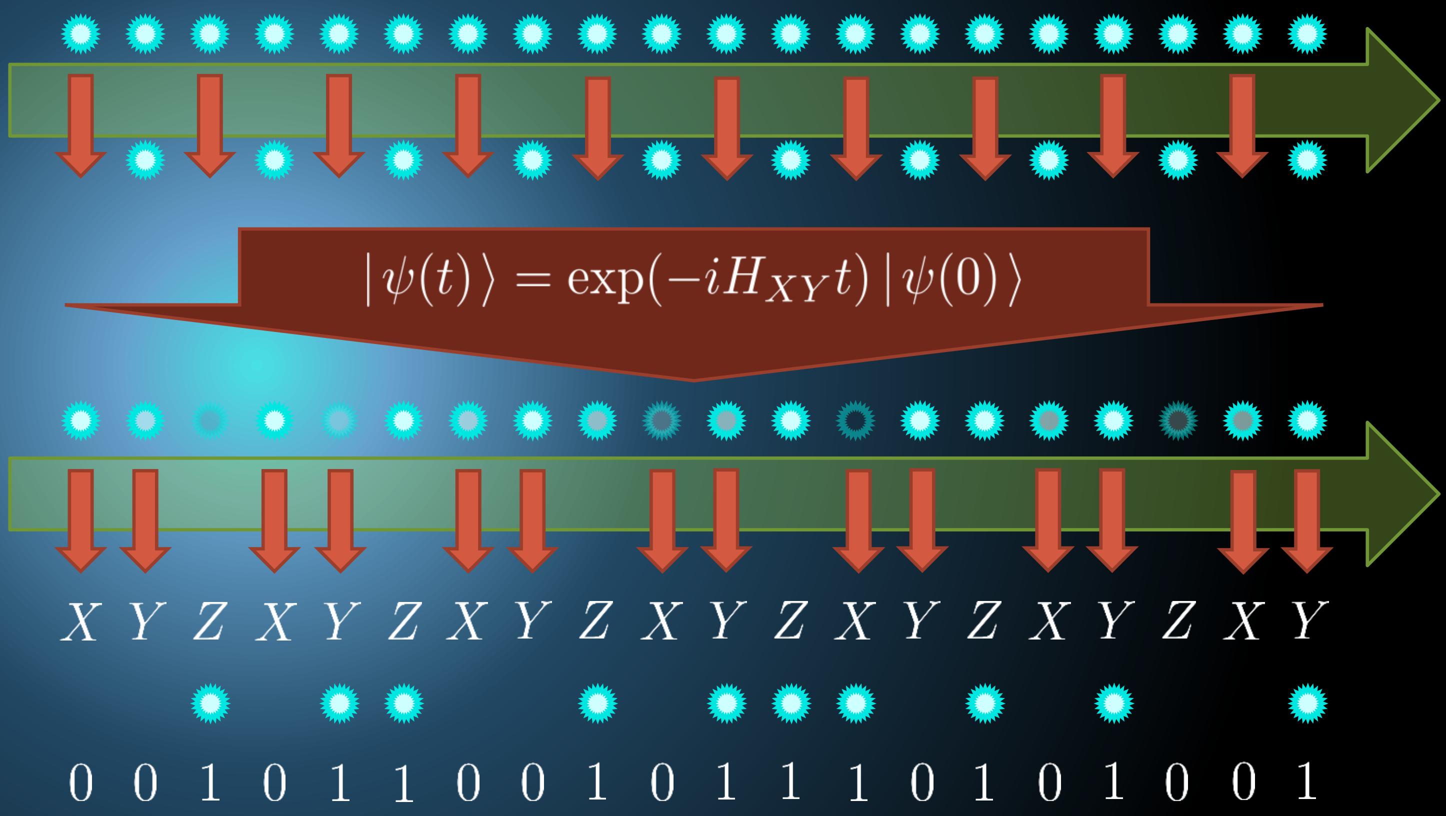
Overview of the Experiment – Digital Quantum Simulation

String of 20 ions/qubits

- Cooling to ground state $|0000\ldots00\rangle$
- Initialization: Néel state $|1010\ldots10\rangle$
- Time evolution: XY Hamiltonian

$$H_{XY} \propto \sum_{i<j} J_{ij}(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+) + B\sum_i \sigma_i^z$$

- Rotation to measurement basis
- Fluorescence measurement
- Collect data



& Repeat: 1000 runs each for 27 product observables

& for 8 evolution times (0.0ms – 3.5ms)

→ Entanglement Dynamics?

Entanglement Detection Technique

Consider measurement:

X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z	X	Y
0	0	1	0	1	1	0	0	1	0	1	1	1	0	1	0	1	0	0	1

Data useful for calculating $\langle X_1 X_4 \rangle, \langle Z_6 Z_9 \rangle$ etc.

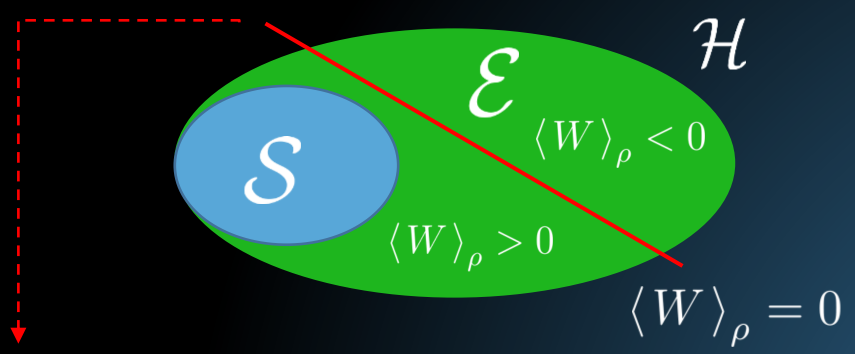
$$\boxed{\mathcal{F}^{\max}_{\mathrm{Bell};\, i,j} := \frac{1}{4}\Big(1 + |\langle X_i X_j \rangle| + |\langle Y_i Y_j \rangle| + |\langle Z_i Z_j \rangle|\Big) > \frac{1}{2} \;\Longrightarrow\; \text{Qubits } i \,\&\, j \text{ entangled}}$$

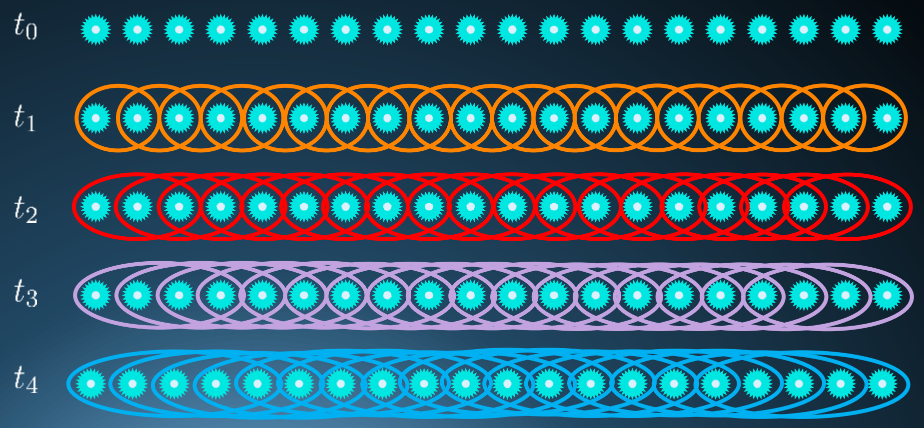
Extension to multi-qubit systems: (k-qubit symmetric) average Bell fidelity

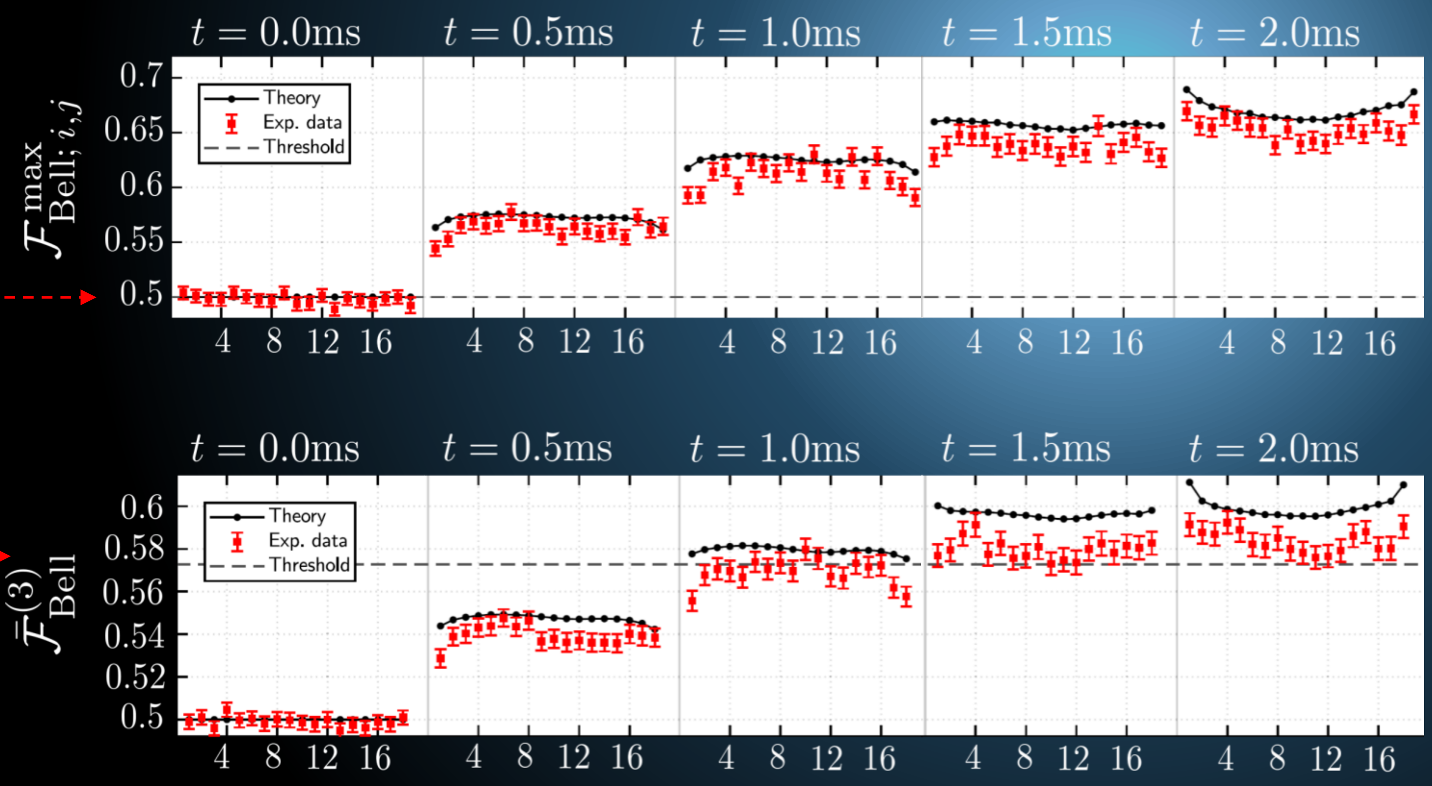
$$\bar{\mathcal{F}}^{(k)}_{\mathrm{Bell}} := \frac{1}{b_k} \sum_{\substack{i,j=1 \\ i<j}}^{k} \mathcal{F}^{\max}_{\mathrm{Bell};\, i,j} = \frac{1}{4 b_k}\Big(b_k + \sum_{\substack{i,j=1 \\ i<j}}^{k} \big(|\langle X_i X_j \rangle| + |\langle Y_i Y_j \rangle| + |\langle Z_i Z_j \rangle|\big)\Big) \qquad b_k = \binom{k}{2} = \frac{1}{2}\frac{k!}{(k-2)!}$$

Example for 3 qubits: Biseparable states satisfy $\boxed{\bar{\mathcal{F}}^{\mathrm{bisep}}_{\mathrm{Bell}} \leq \frac{1}{12}\left(3 + \sqrt{15}\right) \approx 0.572749}$

GME Detection in Ion Trap







Conclusion:

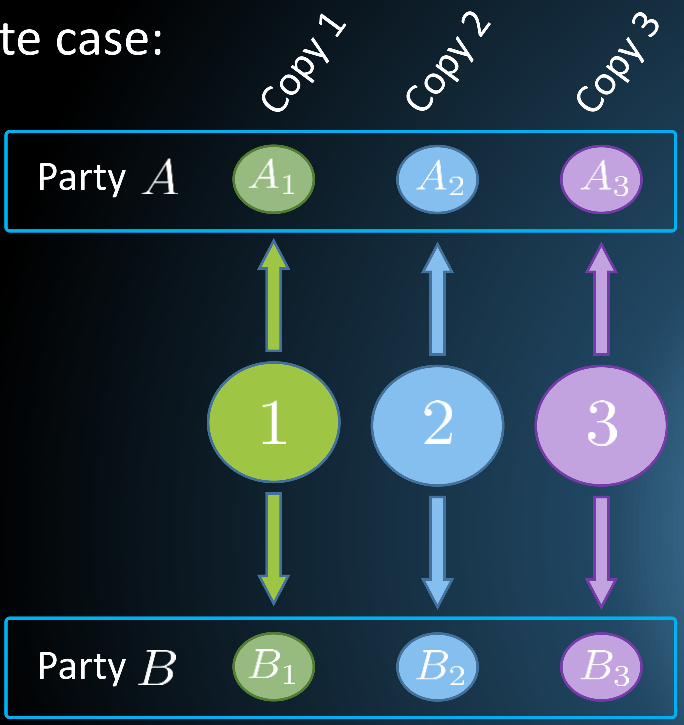
Tracking of emerging entanglement structure through time evolution of many-body system

Benchmarking task?

Activation of Genuine Multipartite Entanglement

Multi-Copy Scenarios

Bipartite case:



If single copy separable w.r.t. $A_1|B_1$

any number of copies separable

w.r.t. $A_1A_2A_3\ldots|B_1B_2B_3\ldots$

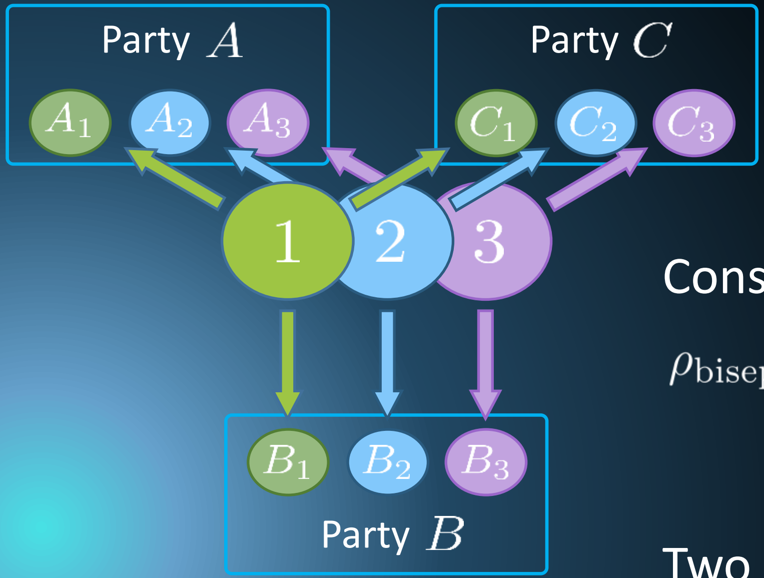
$$A|B$$

$$\rho^{A_1B_1} = \sum_i p_i\, \rho_i^{A_1} \otimes \rho_i^{B_1}$$

$$\rho^{A_2B_2} = \sum_i p_i\, \rho_i^{A_2} \otimes \rho_i^{B_2}$$

$$\rho^{A_3B_3} = \sum_i p_i\, \rho_i^{A_3} \otimes \rho_i^{B_3}$$

Multipartite case: characterization of quantum networks



Consider biseparable state

$$\rho_{\text{bisep}} = p\,\rho^{A_1} \otimes \rho^{B_1C_1} + (1-p)\,\rho^{A_1B_1} \otimes \rho^{C_1}$$

Two copies:

$$\begin{aligned}\rho_{\text{bisep}} \otimes \rho_{\text{bisep}} = {} & p^2\, \rho^{A_1} \otimes \rho^{B_1C_1} \otimes \rho^{A_2} \otimes \rho^{B_2C_2} \\ & +(1-p)^2\, \rho^{A_1B_1} \otimes \rho^{C_1} \otimes \rho^{A_2B_2} \otimes \rho^{C_2} \\ & + p(1-p)\left[\rho^{A_1} \otimes \rho^{B_1C_1} \otimes \rho^{A_2B_2} \otimes \rho^{C_2}\right. \\ & \left. +\ \rho^{A_1B_1} \otimes \rho^{C_1} \otimes \rho^{A_2} \otimes \rho^{B_2C_2}\right]\end{aligned}$$

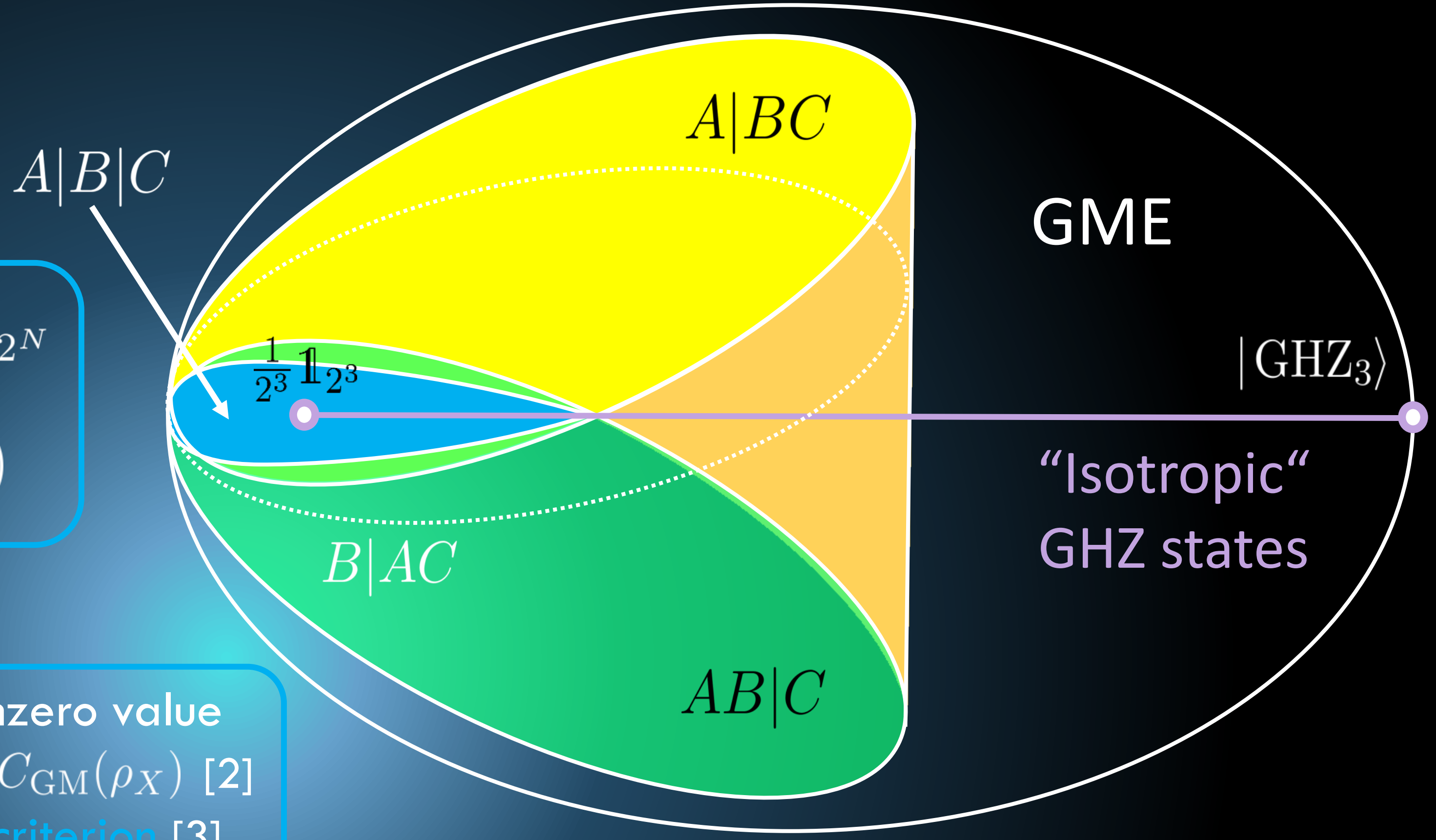
some terms not necessarily separable w.r.t. $A|BC$, $AB|C$, or $B|AC$

Approach (illustrated for $N = 3$)

One-parameter family of states:

$$\rho(p) = p\,|\mathrm{GHZ}_N\rangle\langle \mathrm{GHZ}_N| + (1-p)\,\frac{1}{2^N}\mathbb{1}_{2^N}$$

with $|\mathrm{GHZ}_N\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right)$



We can then leverage two results:

(i) For any N-qubit state ρ_X in X-form, a nonzero value of the genuinely multipartite concurrence $C_{\mathrm{GM}}(\rho_X)$ [2] provides a necessary and sufficient GME criterion [3].

(ii) For any two states ρ and σ in $\mathcal{H}$, the Hadamard map $\mathcal{E}_\circ[\rho\otimes\sigma] = \frac{\rho\circ\sigma}{\mathrm{Tr}(\rho\circ\sigma)} \in \mathcal{H}$, can be implemented via SLOCC [4].

[2] Rafsanjani, Huber, Broadbent, and Eberly, *Phys. Rev. A* **86**, 062303 (2012) [arXiv:1208.2706].

[3] Ma, Chen, Chen, Spengler, Gabriel, and Huber, *Phys. Rev. A* **83**, 062325 (2011) [arXiv:1101.2001].

[4] Lami and Huber, *J. Math. Phys.* **57**, 092201 (2016) [arXiv:1603.02158].

Approach (illustrated for $N=3$)

We observe:

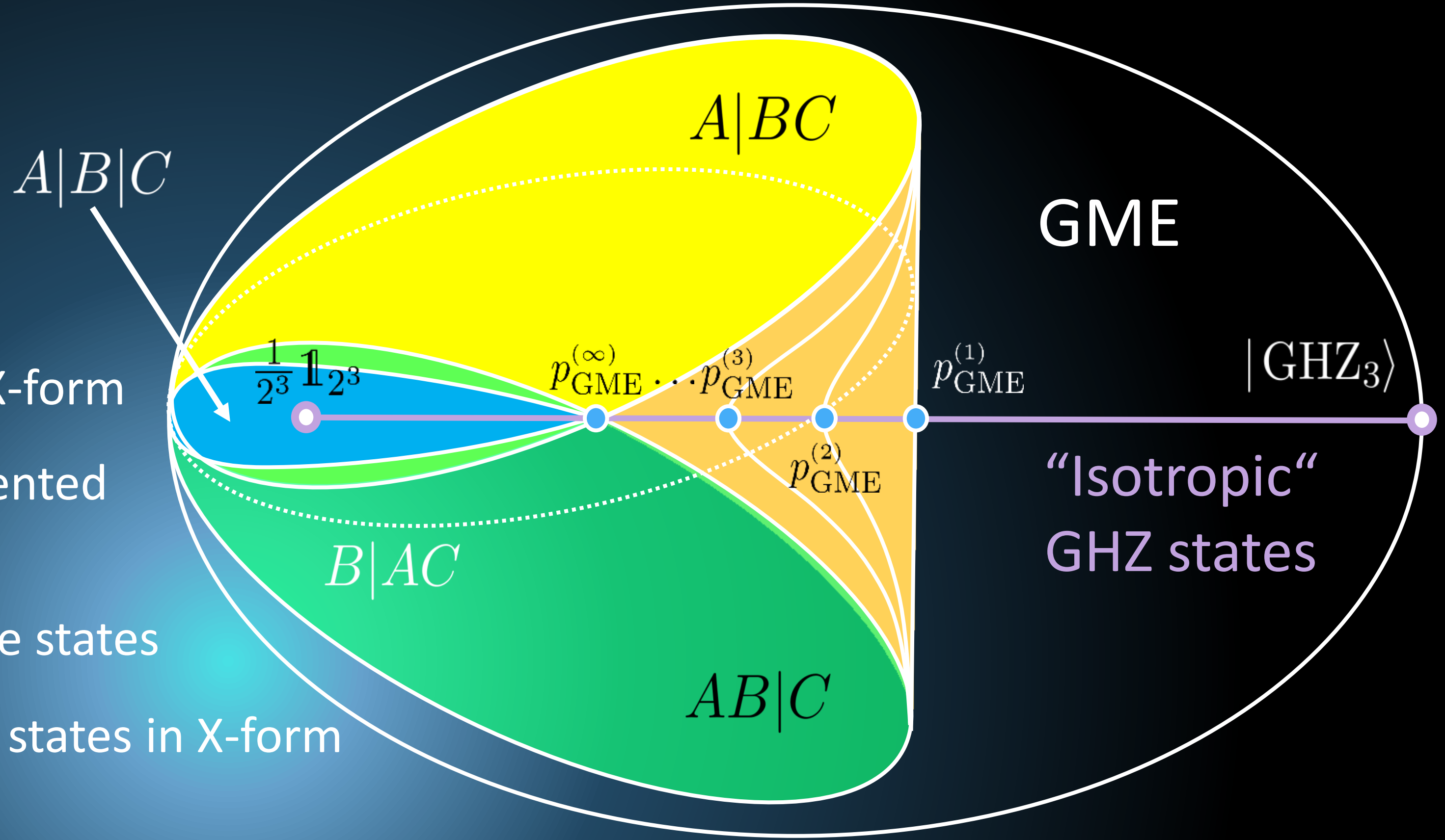
(1) isotropic N-qubit GHZ states are in X-form

(2) Hadamard (Schur) product preserves this X-form

(3) Hadamard (Schur) product can be implemented via SLOCC

(4) SLOCC cannot create GME from biseparable states

(5) Nonzero GM concurrence detects GME for states in X-form



For one copy: If $C_{\mathrm{GM}}\big(\rho(p)\big) > 0 \Longleftrightarrow \rho(p)$ GME $\Longleftrightarrow p > p_{\mathrm{GME}}^{(1)}(N) := \frac{2^{N-1}-1}{2^N-1}$

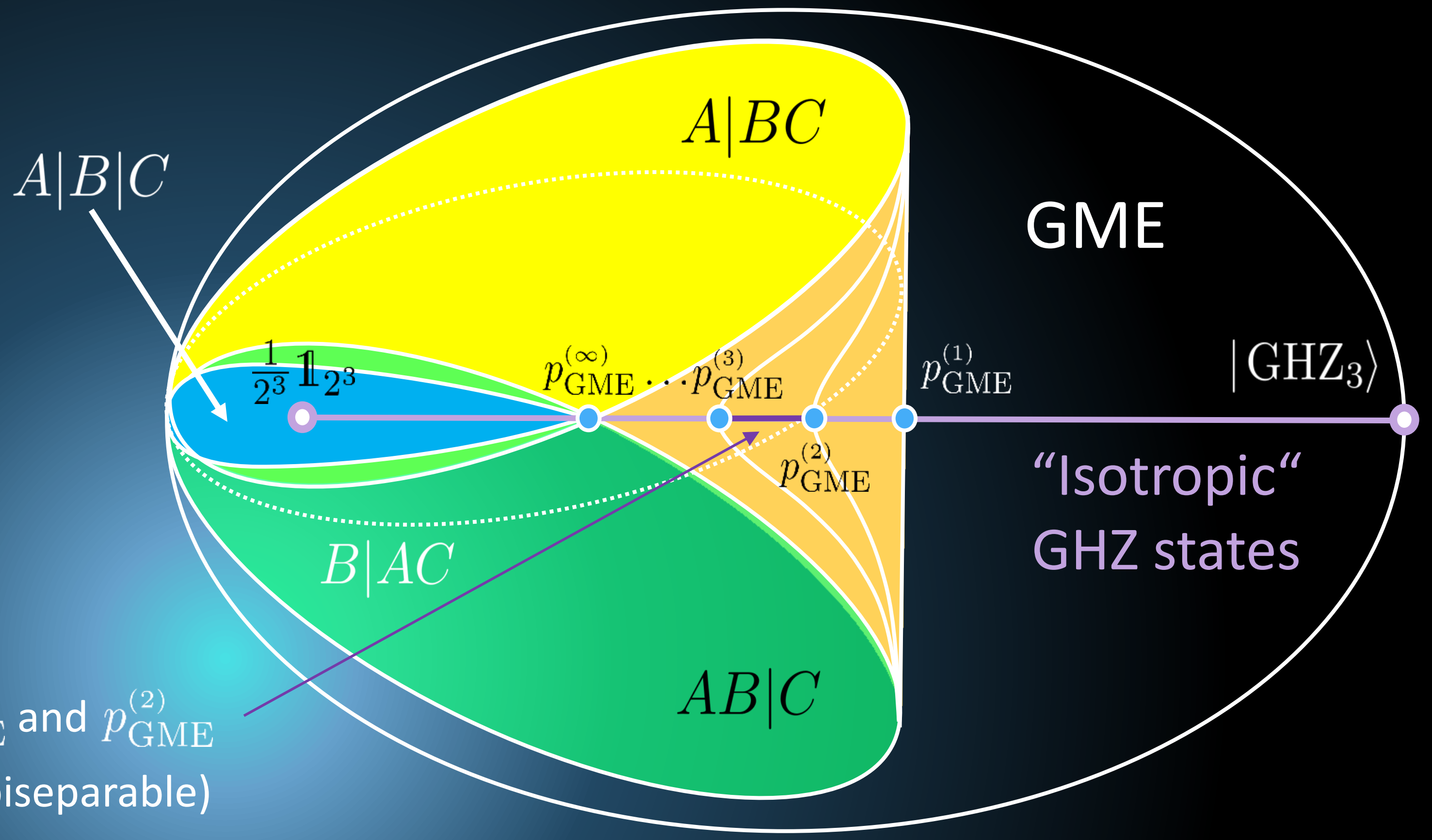
For two copies: $\rho(p)^{\otimes 2} \longmapsto \mathcal{E}_\circ[\rho(p)\otimes\rho(p)]$ If $C_{\mathrm{GM}}\big(\mathcal{E}_\circ[\rho(p)^{\otimes 2}]\big) > 0 \Longrightarrow \rho(p)^{\otimes 2}$ GME

For k copies: If $C_{\mathrm{GM}}\big(\mathcal{E}_\circ^{\circ(k-1)}[\rho(p)^{\otimes k}]\big) > 0 \Longrightarrow \rho(p)^{\otimes k}$ GME

The Results (illustrated for $N = 3$)

Family of k-copy GME thresholds:

$$p > p_{\mathrm{GME}}^{(k)}(N) := \frac{\sqrt[k]{2^{N-1}-1}}{2^{N-1} + \sqrt[k]{2^{N-1}-1}}$$



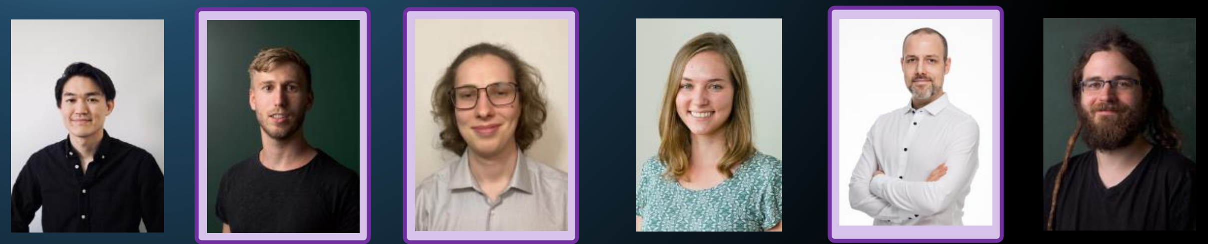
(i) All biseparable isotropic N-qubit GHZ states are activatable

(ii) All isotropic 3-qubit GHZ states between $p_{\mathrm{GME}}^{(3)}$ and $p_{\mathrm{GME}}^{(2)}$ require 3 copies for GME activation (2 copies biseparable)

(iii) We consider a family of biseparable three-qutrit states with no distillable bipartite entanglement across any cut and show that three copies can become GME

Activation of genuine multipartite entanglement: beyond the single-copy paradigm of entanglement characterization, Quantum **6**, 695 (2022)

Collaboration between my team and Marcus Huber´s team



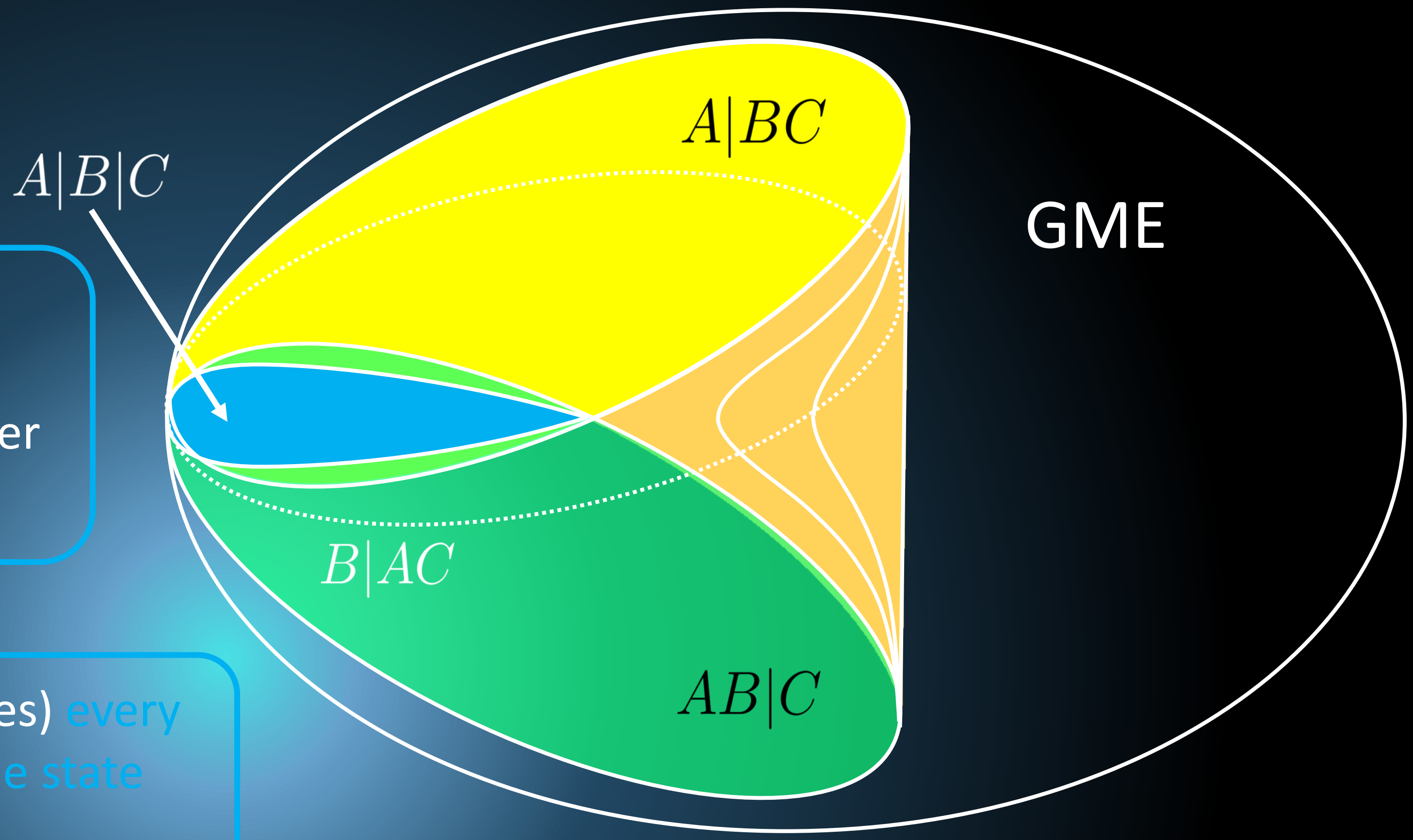
H. Yamasaki (now in Tokyo) | S. Morelli (my team) | M. Miethlinger (now in Geneva) | J. Bavaresco (now in Paris) | N. Friis | M. Huber

Further work

(I) **Hierarchy** of activatable GME:

For any number of copies there are states that require at least this number for activation.

(II) Asymptotically (in the number of copies) every biseparable but not partition-separable state is GME activatable.



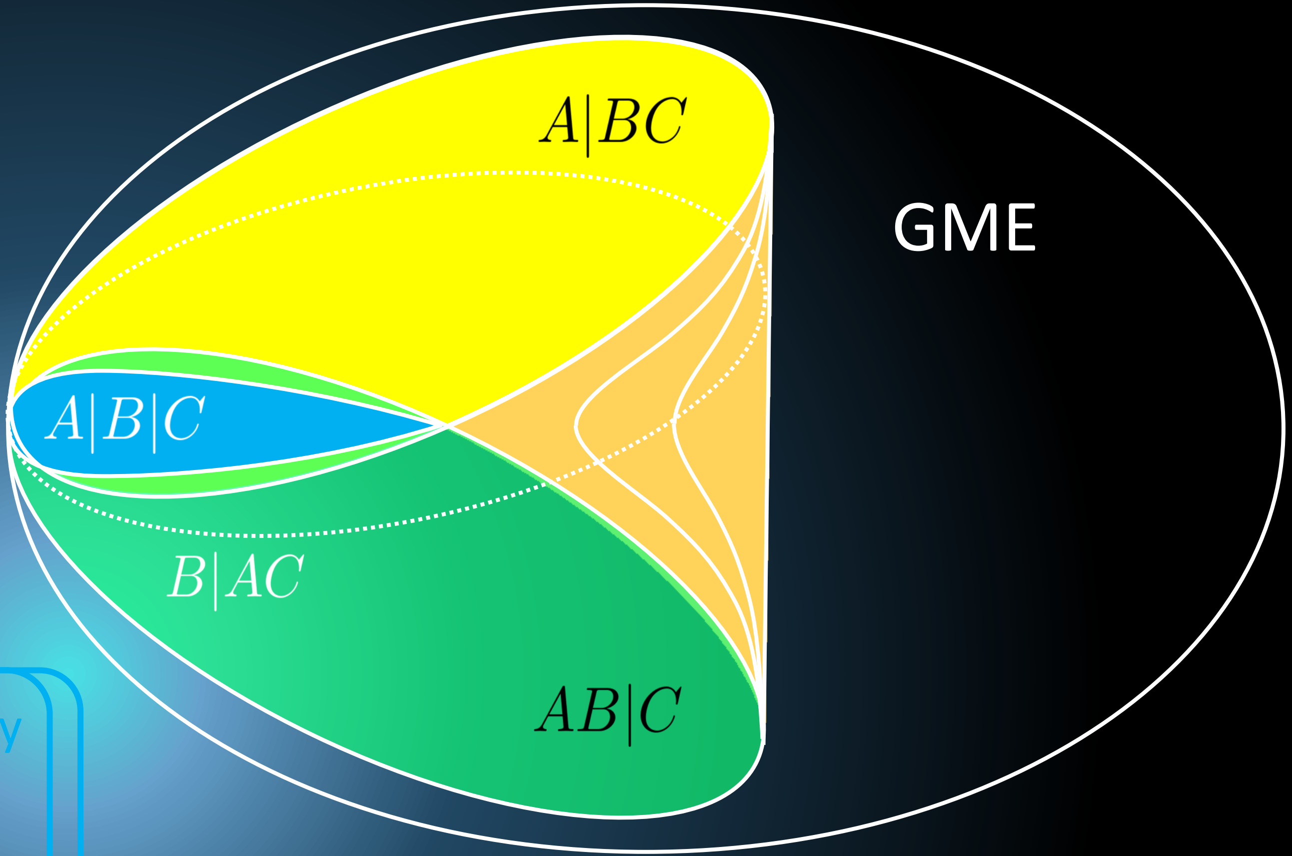
Genuine multipartite entanglement of quantum states in the multiple-copy scenario,
C. Palazuelos and J. I. de Vicente, Quantum **6**, 735 (2022)

→ At least for all finite dimensions

Infinite dimensions

continuous variables $[\hat{x}_j, \hat{p}_k] = i\delta_{jk}$

harmonic oscillators $\hat{a}_j = \frac{1}{\sqrt{2}}(x_j + ip_j), \quad \hat{a}_j^\dagger = \frac{1}{\sqrt{2}}(x_j - ip_j)$



But: Covariance matrix biseparability criterion

$$\Gamma_{ABC} - (p_A\Gamma_{A|BC} + p_B\Gamma_{B|AC} + p_C\Gamma_{AB|C}) \geq 0,$$

not necessary & sufficient for Gaussian states.

Asymptotically (in the number of copies) every biseparable but not partition-separable state is GME activatable.

Multi-copy activation of genuine multipartite entanglement in continuous-variable systems, Quantum **9**, 1699 (2025)

Gaussian states with the same covariance matrix as a biseparable non-Gaussian state can be GME

Olga Leskovjanová (Olomouc)

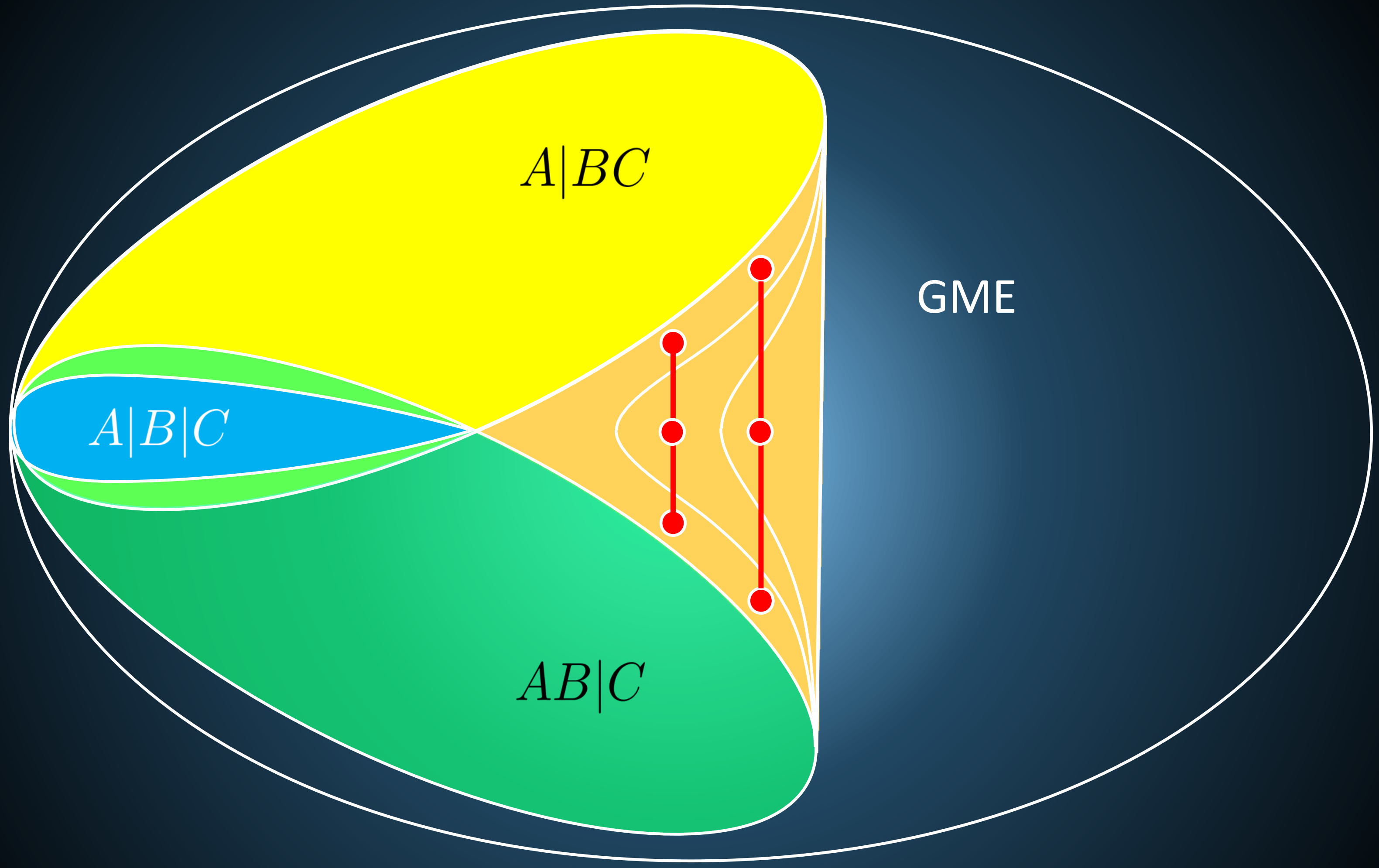
Klára Baksová (my team)

Eliza Agudelo (TU Wien)

Ladislav Mišta (Olomouc)

NF

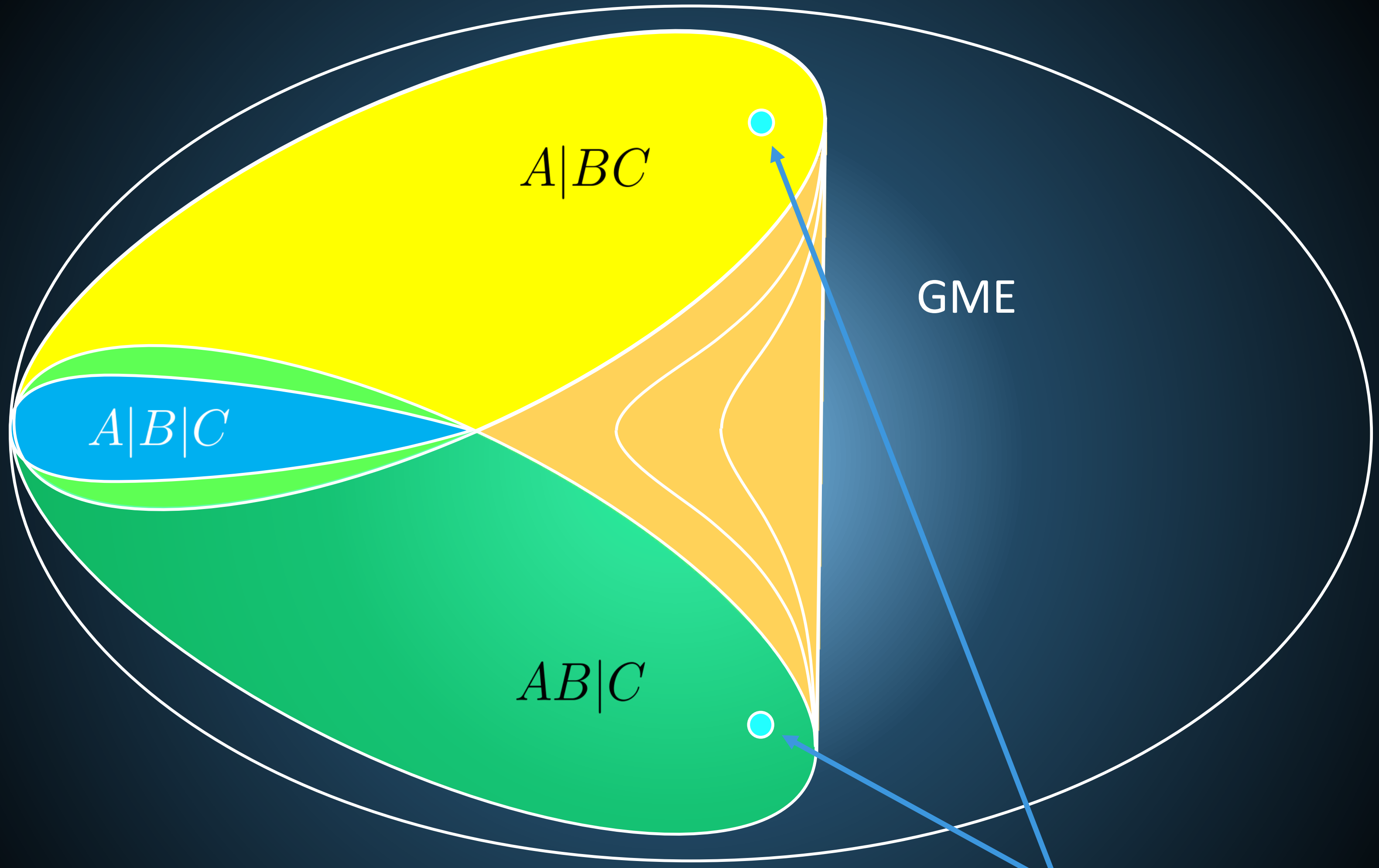
~~Convexity~~ & Shared Randomness in Multi-Copy Scenarios



Shared randomness: incoherent mixture of k-copy activatable states

$(k-1)$ -copy activatable state for $k' < k$

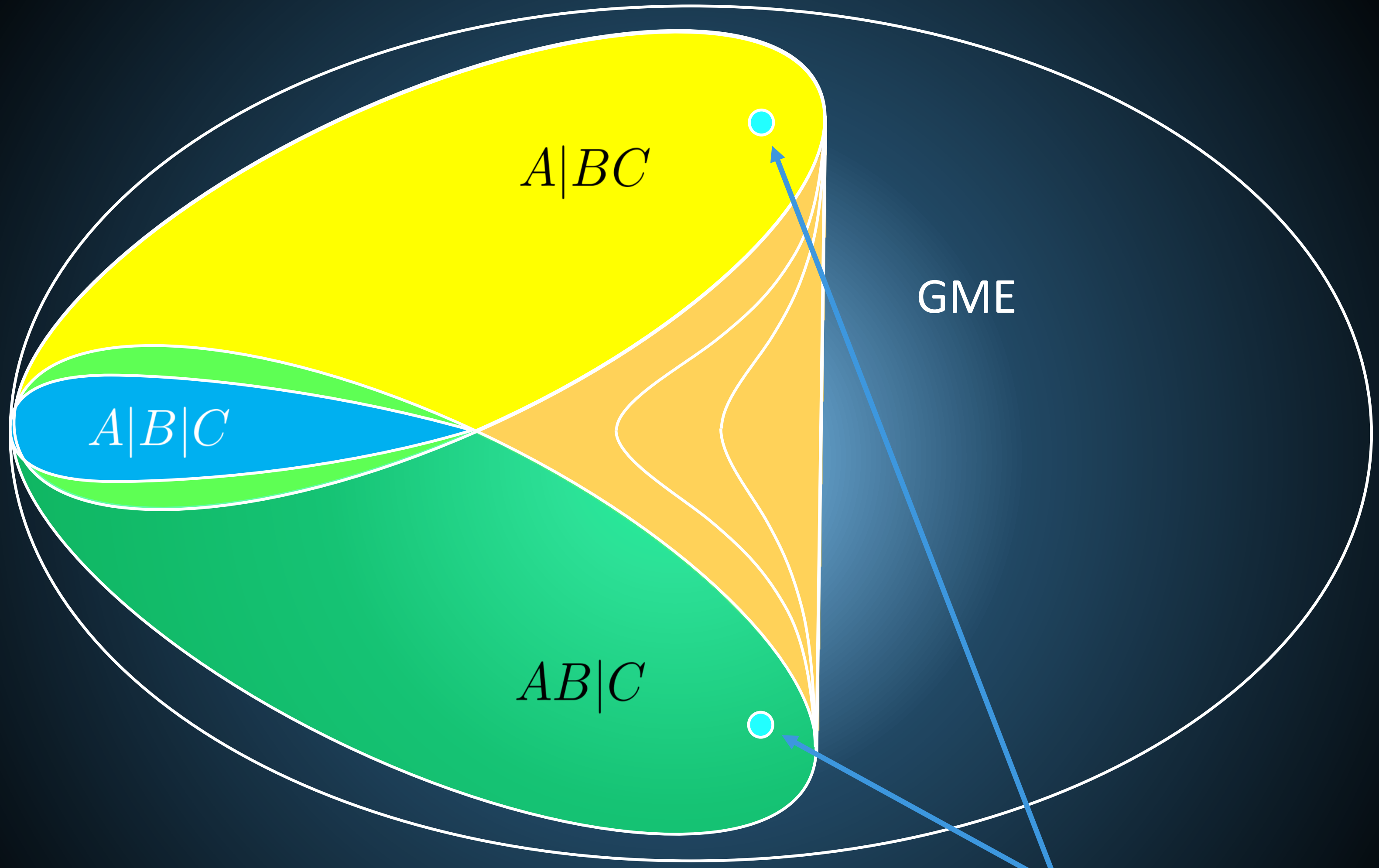
~~Convexity~~ & Shared Randomness in Multi-Copy Scenarios



Perhaps not so shocking: If given two systems in partition-separable but entangled states

$\rho_{\mathcal{A}_1\mathcal{A}_2|\mathcal{A}_3}$ and $\rho_{\mathcal{A}_1|\mathcal{A}_2\mathcal{A}_3}$ $\Rightarrow$ $\rho_{\mathcal{A}_1\mathcal{A}_2|\mathcal{A}_3} \otimes \rho_{\mathcal{A}_1|\mathcal{A}_2\mathcal{A}_3}$ is GME

~~Convexity~~ & Shared Randomness in Multi-Copy Scenarios



"Forgetting" which system is in which state → 2 copies of incoherent mixture

which might now be 2-copy activatable → no net gain in GME via randomness

Ongoing and Future Research

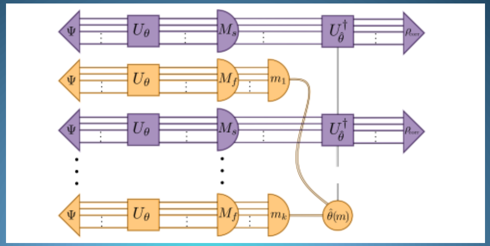
Ongoing work & future directions

Quantum Networks, Multipartite Entanglement & Quantum Metrology

Network Producibility Witnesses

Markus Miethlinger (now Geneva) · Tomasz Andrzejewski (my team) · Tamás Kriváchy (now ICFO) · Moha Mehboudi · Marcus Huber (TU Wien)

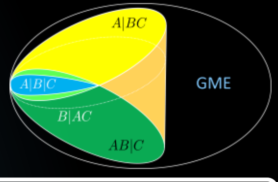
Metrology-Assisted Entanglement Distribution



Quantum 6, 722 (2022)

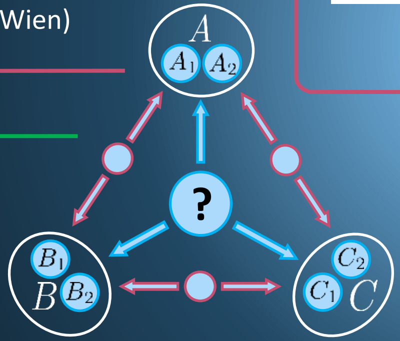
Simon Morelli · Klára Baksová (my team)

GME Activation Witnesses & Compression

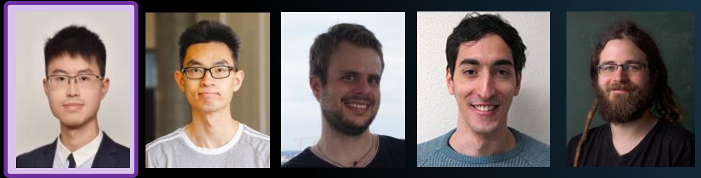


Otfried Gühne · Sophia Denker · Lisa Weinbrenner · Xiao-Dong Wu (Siegen)

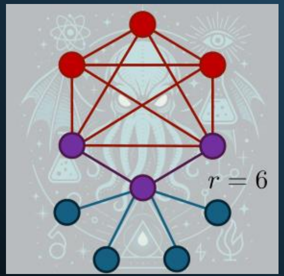
Superactivation and Incompressibility of Genuine Multipartite Entanglement, arXiv:2412.18331 [quant-ph] (2024)



GME Detection with Restricted Measurements



Nicky Li (my team) · Xi Dai · Kevin Reuer (ETH Zurich) · Manuel H. Muñoz-Arias (Shebrooke) · Marcus Huber (TU Wien)



Detecting genuine multipartite entanglement in multi-qubit devices with restricted measurements, preprint arXiv:2504.21076 [quant-ph] (2025).

GME Activation Experiment

Robert Stárek · Olga Leskovjanová · Ladislav Mišta (Olomouc) · Tim Gollerthan · Martin Ringbauer (Innsbruck)

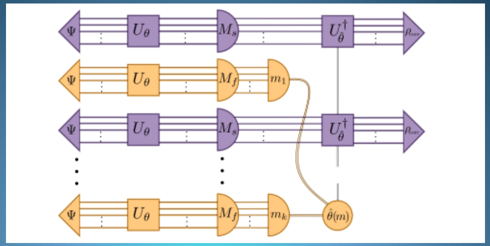
Ongoing work & future directions

Quantum Networks, Multipartite Entanglement & Quantum Metrology

Network Producibility Witnesses

Markus Miethlinger (now Geneva) | Tomasz Andrzejewski (my team) | Tamás Kriváchy (now ICFO) | Moha Mehboudi | Marcus Huber (TU Wien)

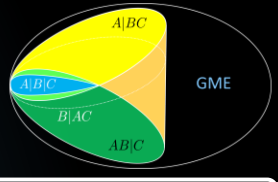
Metrology-Assisted Entanglement Distribution



Quantum 6, 722 (2022)

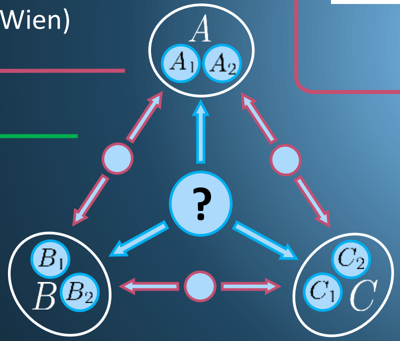
Simon Morelli | Klára Baksová (my team)

GME Activation Witnesses & Compression

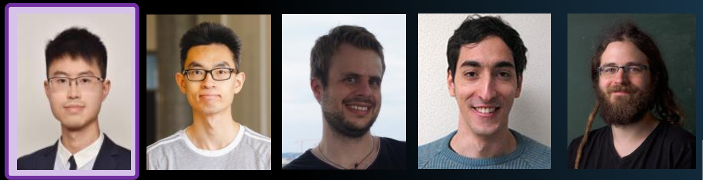


Otfried Gühne | Sophia Denker | Lisa Weinbrenner | Xiao-Dong Wu (Siegen)

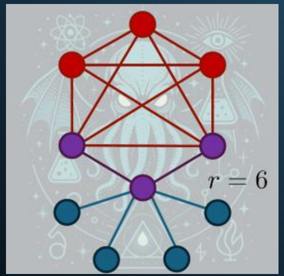
Superactivation and Incompressibility of Genuine Multipartite Entanglement, arXiv:2412.18331 [quant-ph] (2024)



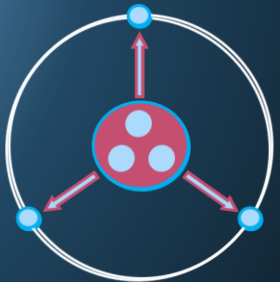
GME Detection with Restricted Measurements



Nicky Li (my team) | Xi Dai | Kevin Reuer (ETH Zurich) | Manuel H. Muñoz-Arias (Shebrooke) | Marcus Huber (TU Wien)



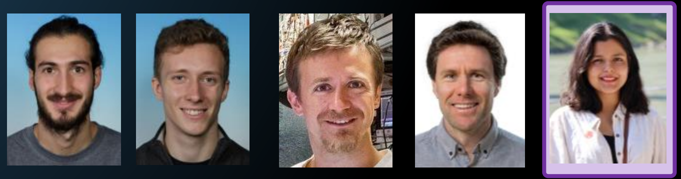
Detecting genuine multipartite entanglement in multi-qubit devices with restricted measurements, preprint arXiv:2504.21076 [quant-ph] (2025).



GME Activation Experiment

Robert Stárek | Olga Leskovjanová | Ladislav Mišta (Olomouc) | Tim Gollerthan | Martin Ringbauer (Innsbruck)

Generation & Verification of GME in Quantum Networks

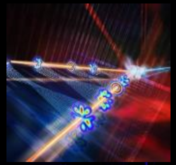


Marco Canteri | James Bate | Victor Krutyanskiy | Ben P. Lanyon (Innsbruck) | Ida Mishra (my team)

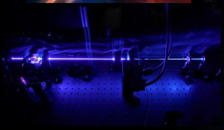
Ongoing work & future directions

Previous Work

High-Dimensional Entanglement



Measurements in two bases are sufficient for certifying high-dimensional entanglement
J. Bavaresco, N. Herrera Valencia, C. Klöckl, M. Pivoluska, P. Erker, N. Friis, M. Malik, and M. Huber
Nature Physics **14**, 1032 (2018) **Theory + Experiment**



High-Dimensional Pixel Entanglement: Efficient Generation and Certification
N. Herrera Valencia, V. Srivastav, M. Pivoluska, M. Huber, N. Friis, W. McCutcheon, and M. Malik
Quantum **4**, 376 (2020) **Theory + Experiment**

Entanglement Quantification in Atomic Ensembles
M. Fadel, A. Usui, M. Huber, N. Friis, and G. Vitagliano
Physical Review Letters **127**, 010401 (2021) **Theory**

Certifying High-Dimensional Entanglement in Continuous-Variable Systems




Ida Mishra | Phila Rembold | Simon Morelli | Klára Baksová
(my team)

Entanglement Detection in Many-Body Systems


Julia Mathé | Giuseppe Vitagliano
(TU Wien)

Certifying Schmidt number


Nicky Li (my team) | Marcus Huber

High-dimensional entanglement witnessed by correlations in arbitrary bases,
npj Quantum Inf. **11**, 50 (2025)

Entanglement Between Electrons and Cherenkov Photons in Electron Microscopes



+ Team

Phila Rembold (my team) | Eliza Agudelo | Philipp Haslinger (TU Wien) | Alexander Preimesberger

State-Agnostic Approach to Certifying Electron-Photon Entanglement in Electron Microscopy,
Quantum Sci. Technol. **10**, 045003 (2025)

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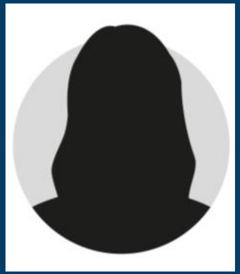

Sophia Baumschlager

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Thank you for your attention