Admin

- Today's topics
  - Recursive backtracking
- Reading
  - Reader ch. 6 (today)
  - Next: pointers! 2.2-2.3, linked lists 9.5(sort of)
- Terman cafe after class

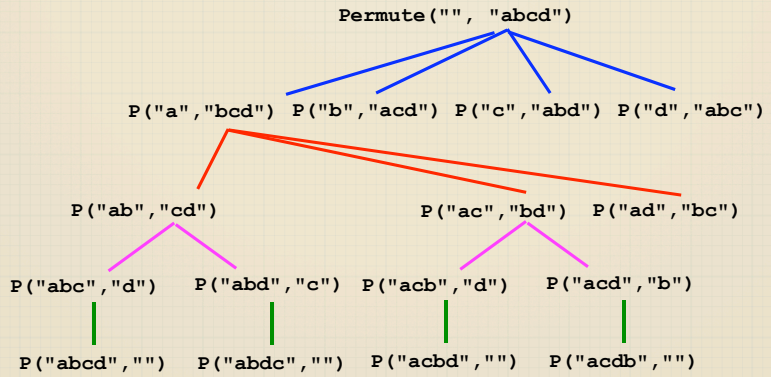
✓

Refresh: permute code

```
void RecPermute(string soFar, string rest)
{
    if (rest == "") {
        cout << soFar << endl;
    } else {
        for (int i = 0; i < rest.length(); i++) {
            string next = soFar + rest[i];
            string remaining = rest.substr(0, i)
                             + rest.substr(i+1);
            RecPermute(next, remaining);
        }
    }
}


 // "wrapper" function
void ListPermutations(string s)
{
    RecPermute("", s);
}
```

Tree of recursive calls



Subsets

- Enumerate all subsets of input
  - "abc" has subsets "a", "b", "ab", "ac", …
  - Order doesn't matter, "ab" is same as "ba"
- Solving recursively
  - Separate one element from input
  - Can either include in current subset or not
  - Recursively form subsets including it
  - Recursively form subsets not including it
  - What is the base case?
- Remind you of any other problem you've seen?
  - Same patterns often resurface!

Subset strategy

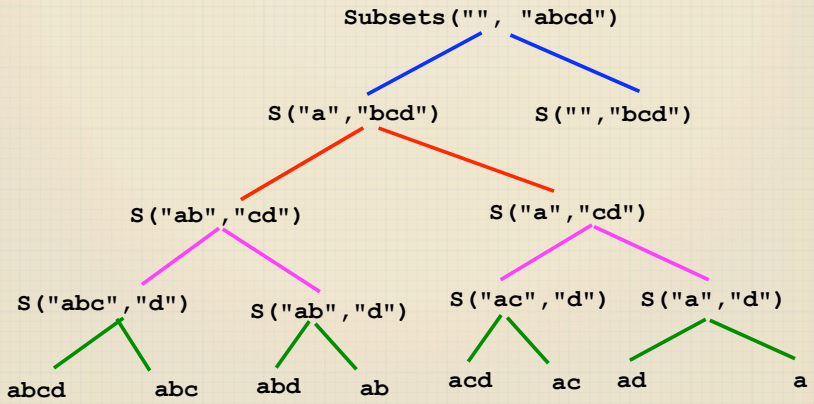
- Result is empty, starting input is "abcd"
- Consider first element: "a"
- Add to subset, remaining input is "bcd"
- Recursively find all subsets from here
- Repeat recursion without including "a"

Subsets code

```
void RecSubsets(string soFar, string rest)
{
   if (rest == "")
       cout << soFar << endl;
   else {
            // add to subset, remove from rest, recur
     RecSubsets(soFar + rest[0], rest.substr(1));
            // don't add to subset, remove from rest,recur
     RecSubsets(soFar, rest.substr(1));
   }
}


void ListSubsets(string str)
{
   RecSubsets("", str);
}
```

Tree of recursive calls



Exhaustive recursion

- Permutations/subsets are about *choice*
  - Both have deep/wide tree of recursive calls
  - Depth represents total number of decisions made
  - Width of branching represents number of available options per decision
- Exhaustive recursion is, well, exhaustive
  - Explores every possible option at every decision point
  - Typically very expensive
    - N! permutations, 2^N subsets
    - (Recursion isn't the problem, there just is a huge space to explore)
- Consider partial exploration of exhaustive space
  - Similar exhaustive structure, but stop at first "satisfactory" outcome

Recursive backtracking

- Cast problem in terms of decision points
  - Identify what decisions need to be made
  - Identify what options are available for each decision
  - A recursive call makes one decision, and recurs on remaining decisions
- Backtracking approach
  - Design recursion function to return success/failure
  - At each call, choose one option and go with it
  - Recursively proceed and see what happens
  - If it works out, great, otherwise unmake choice and try again
  - If no option worked, return fail result which triggers backtracking (i.e. un-making earlier decisions)
- Heuristics may help efficiency
  - Eliminate dead ends early by pruning
  - Pursue most likely choice(s) first

Backtracking pseudocode

```
bool Solve(configuration conf)
{
   if (no more choices)     // BASE CASE
         return (conf is goal state);

   for (all available choices) {
      try one choice c;
        // solve from here, if works out, you're done
      if (Solve(conf with choice c made)) return true;
       unmake choice c;
    }

    return false;   // tried all choices, no soln found
}
```

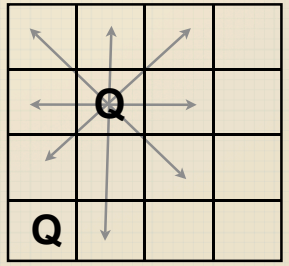
Permute -> anagram finder

```
void RecPermute(string soFar, string rest)
{
   if (rest == "") {
      cout << soFar << endl;
   } else {
      for (int i = 0; i < rest.length(); i++) {
         RecPermute(soFar+rest[i], rest.substr(0,i)+rest.substr(i+1));
      }
   }
}
```

```
bool IsAnagram(string soFar, string rest, Lexicon &lex)
{
  if (rest == "") {
     return lex.containsWord(soFar);
  } else {
    for (int i = 0; i < rest.length(); i++) {
      if (IsAnagram(soFar+rest[i], rest.substr(0,i)+rest.substr(i+1), lex))
          return true;
    }
  }
  return false;
}
```

8 Queens

- Goal: place N queens on board so none threatened
  - Queen can attack in any straight line (horizontally, vertically, diagonally)
- Cast as in terms of decision
  - Each call will make one decision and recur on rest
  - How many decisions do you have to make?
  - What options do you have for each?



N queens code

```
bool Solve(Grid<bool> &board, int col)
{
    if (col >= board.numCols()) return true; // base case

    for (int rowToTry=0; rowToTry<board.numRows(); rowToTry++) {
      if (IsSafe(board,rowToTry,col)) {
          PlaceQueen(board, rowToTry, col);
          if (Solve(board, col + 1)) return true;
          RemoveQueen(board, rowToTry, col);
       }
    }
    return false;
}
```