Neural Network Training with Approximate Logarithmic Computations

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- Enabling Neural Network training on edge-devices.

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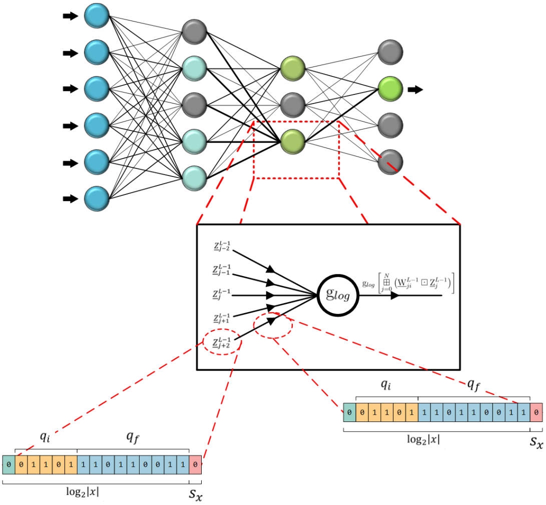
  - In LNS multiplications are cheap but addition are computationally expensive
  - Resort to **Approximate Logarithmic Fixed-Point Computations**

LNS Neural Network Pipeline

Neural Network Training with Approximate Logarithmic Computations



ICASSP2020 Barcelona

UNIVERSITY OF SOUTHERN CALIFORNIA 1880

Equivalence

$$v \longleftrightarrow \underline{V} = (V, s_v)$$
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$$s_v = \mathrm{sign}(v)$$

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Multiplication

$$u = xy \longleftrightarrow \underline{U} = \underline{X} \boxdot \underline{Y}$$
$$U = X + Y$$
$$s_u = \overline{(s_x \veebar s_y)}$$

Addition

$$z = x + y \longleftrightarrow \underline{Z} = \underline{X} \boxplus \underline{Y}$$

$$Z = \begin{cases} \max(X, Y) + \Delta_{+}(|X - Y|) & s_x = s_y \\ \max(X, Y) + \Delta_{-}(|X - Y|) & s_x \neq s_y \end{cases}$$

$$s_z = \begin{cases} s_x & X > Y \\ s_y & X \leq Y \end{cases}$$

Mathematical Operations

Neural Network Training with Approximate Logarithmic Computations

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$$\Delta_+(d) = \log_2 \left(1 + 2^{-d}\right) \qquad d \geq 0$$

$$\Delta_-(d) = \log_2 \left(1 - 2^{-d}\right) \qquad d \geq 0$$

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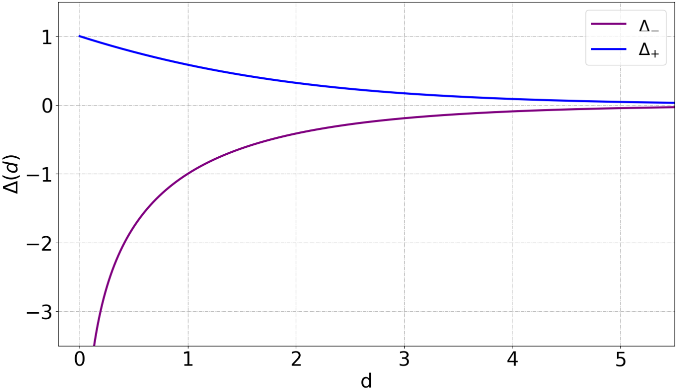
  - Look-Up Table (LUT) based approximations
  - Bit-shift (BS) based approximations

Visualizing Δ from LNS $\boxplus$ Additions

Neural Network Training with Approximate Logarithmic Computations



Look-Up Table Based Approximations

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    - Fixed-point width is a hyper-parameter

Look-Up Table Based Approximations

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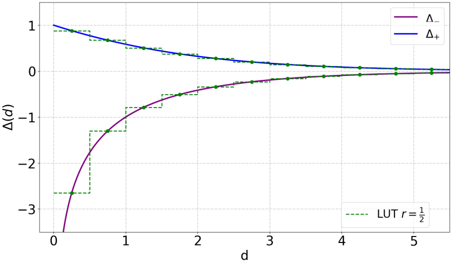

LUT Resolution $r = \frac{1}{2}$

Look-Up Table Based Approximations

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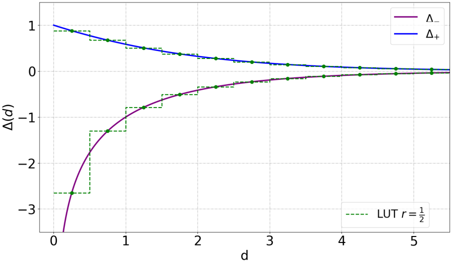

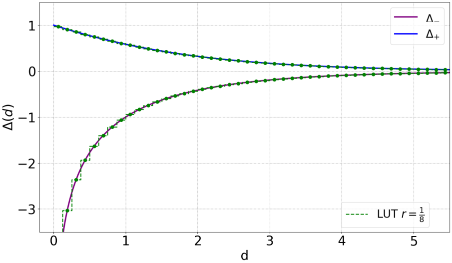

LUT Resolution $r = \frac{1}{2}$

LUT Resolution $r = \frac{1}{8}$

Taylor Series approximation

$$\log_e(1 \pm x) \approx \pm x \qquad 0 \leq x \ll 1$$

$$\begin{aligned}\Delta_{\pm}(d) &= \log_2\left(1 \pm 2^{-d}\right) \\ &\approx \pm \log_2 e \times 2^{-d}\end{aligned}$$

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Further Simplification

$$\begin{aligned}\log_2 e = 1.442695\cdots &\approx 1.4375 = 2^0 + 2^{-1} - 2^{-4}\\ &\approx 1.5 = 2^0 + 2^{-1}\\ &\approx 1 = 2^0\end{aligned}$$

Bit-Shift Based Approximations

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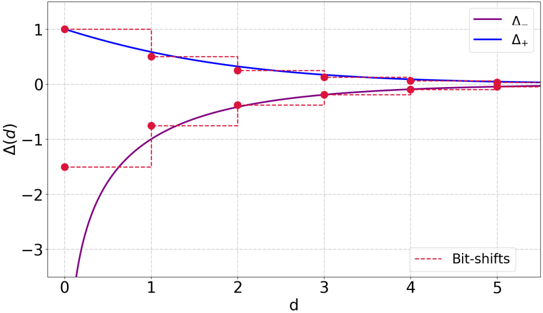

$$\Delta_+(d) \approx \mathbb{BS}(1, -d)$$
$$\Delta_-(d) \approx -\mathbb{BS}(1.5, -d)$$

Can be thought of as a LUT with $r = 1$

Multiply accumulate
(MAC)

The LNS Neuron

Neural Network Training with Approximate Logarithmic Computations

Multiply accumulate (MAC)

$$z_i = \sum_j w_{ij} x_j + b_i \longleftrightarrow \underline{Z}_i = \boxplus_j \underline{W}_{ij} \boxdot \underline{X}_j \boxplus \underline{B}_i$$

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LNS bit-width Constraint

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LNS bit-width Constraint

$$W_{\log} \geq 1 + \max\left(\lceil \log_2 \left(b_i + 1\right) \rceil, \lceil \log_2 b_f \rceil\right) + W_{\mathrm{lin}}$$

Experiments suggest that $W_{\log} \approx W_{\mathrm{lin}}$ suffices in practice. In this work, set $W_{\log} = W_{\mathrm{lin}}$

LNS Weight Initialization

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- Weights generally initialized from symmetric distributions. Hence the sign parameter can be initialized randomly and independently of the magnitude from a *Bernoulli*$(\frac{1}{2})$ distribution.
- The magnitude distribution for weights reduce to,

$$f_W(y) = 2^{y+1} \times \log_e 2 \times f_w(2^y)$$

The Log-Leaky ReLU Activation

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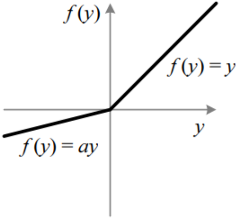
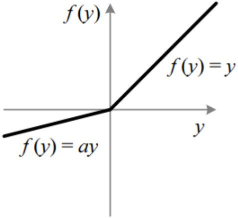


Parametric ReLU



Parametric ReLU

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The Log-Leaky ReLU Activation

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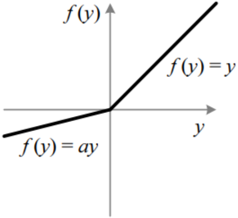

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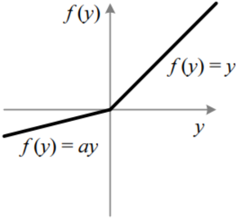

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- This brings us to *Log-Leaky ReLU*

$$g_{\text{llReLU}}\left((X, s_x) \,\middle|\, \beta\right) = \begin{cases} (X, s_x) & s_x = 1 \\ (X + \beta, s_x) & s_x = 0 \end{cases}$$

Gradient calculation

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$$p_{ij} = \frac{e^{a_{ij}}}{\sum_{j=1}^{N} e^{a_{ij}}}$$

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LNS Soft-Max

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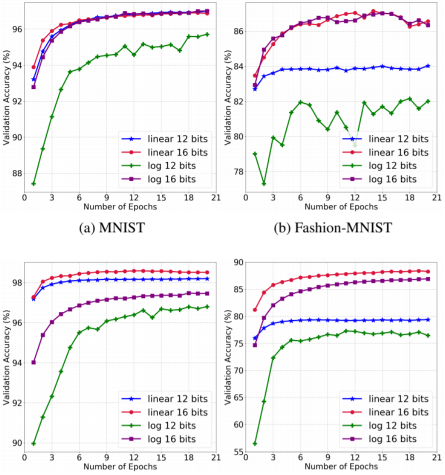
LNS gradients

$$\left(\log_2 |\delta_{ij}|, s_{\delta_{ij}}\right) = \underline{P}_{ij} \boxminus \left(\log_2 |y_{ij}|, s_{y_{ij}}\right)$$

Numerical Results

Neural Network Training with Approximate Logarithmic Computations



(a) MNIST

(b) Fashion-MNIST

(c) EMNIST-digits

(d) EMNIST-letters

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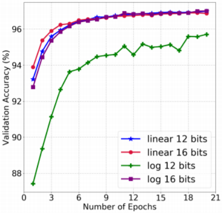

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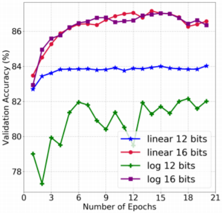

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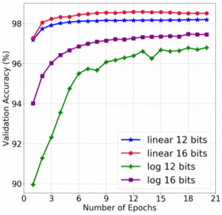

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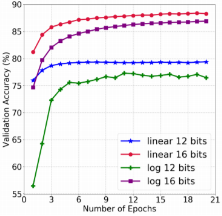

(d) EMNIST-letters

Datasets	Float	**Linear-domain** fixed-point		**Log-domain** fixed-point look-up tables		**Log-domain** fixed-point bit-shifts	
		12b	16b	12b	16b	12b	16b
MNIST	97.4	97.3	96.9	96.0	97.2	95.5	96.5
FMNIST	87.1	82.8	88.0	80.5	87.1	79.3	85.7
EMNISTD	98.6	98.3	98.7	96.9	97.5	96.2	97.4
EMNISTL	88.1	79.7	88.7	76.4	86.7	73.7	82.5

Number of epochs trained $= 20$
Size of tables $= 20(r = \frac{1}{2})$; soft-max uses 640 element tables$(r = \frac{1}{64})$
Table shows test-set accuracy

Future Works

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  - Weight-Activation Relation mapping for LNS-neurons using Supervised Learning (LDA, QDA)

Acknowledgements

This work is supported in part by the National Science Foundation (CCF-1763747)