# The future environmental impact of irrigated agriculture is grossly underestimated

Arnald Puy, Samuele Lo Piano and Andrea Saltelli

# Contents

1	Preliminary steps	2
2	Scaling relationships  2.1 Irrigated area versus population size	6
3	Uncertainty in irrigated areas	13
4	Presence of outliers	18
5	Estimation of the model parameters  5.1 Irrigated area baseline values $(Y_0)$ }	27 34
6	Creation of the sample matrix	35
7	The model	40
8	Uncertainty analysis	<b>4</b> 1
9	Sensitivity analysis 9.1 Scatterplots	47 47 54

# 1 Preliminary steps

```
# PRELIMINARY STEPS ----
# Before starting the analysis, we define a function to load all R
# packages required in one go, and load them. We then create a function
# to read in all the spreadsheets of the excel file with the data
# needed for the analysis, and read in the data. Finally, we cast a
# function to define the theme of the plots that will be created in
# this work.
# Define function to read in all required libraries in one go:
loadPackages <- function(x) {</pre>
 for(i in x) {
    if(!require(i, character.only = TRUE)) {
      install.packages(i, dependencies = TRUE)
      library(i, character.only = TRUE)
 }
}
# Load all required libraries:
loadPackages(c("data.table", "fitdistrplus", "fGarch", "readxl", "countrycode",
               "scales", "tidyverse", "cowplot", "mvoutlier", "complmrob",
               "randtoolbox", "robustbase", "parallel", "smatr", "boot",
               "doParallel", "sensitivity", "wesanderson", "sensobol",
               "grid", "gridExtra", "NbClust"))
# Set checkpoint
dir.create(".checkpoint")
library("checkpoint")
checkpoint("2019-08-28",
           R.version ="3.6.1",
           checkpointLocation = getwd())
# Define function to read in all excel spreadsheets in one go:
readAll <- function(name, tibble = FALSE) {</pre>
  sheets <- excel_sheets(name)</pre>
 df <- lapply(sheets, function(y) read_excel(name,</pre>
                                              sheet = y))
  if(!tibble) df <- lapply(df, as.data.frame)</pre>
 names(df) <- sheets</pre>
  df
}
```

```
# Read in all excel spreadsheets:
df <- readAll("full.dataset2.xlsx") %>%
  lapply(., function(x) mutate_if(x, is.character, as.factor))
# Redefine column names for population.estimate spreadsheet
colnames(df$population.estimate) <- c("Estimate", "Continent", "Codes",</pre>
                                       paste0("Year.", 2015:2100))
# Create function for custom plot themes
theme AP <- function() {</pre>
  theme bw() +
    theme(aspect.ratio = 1,
          panel.grid.major = element_blank(),
          panel.grid.minor = element_blank(),
          legend.background = element_rect(fill = "transparent",
                                            color = NA),
          legend.key = element_rect(fill = "transparent",
                                     color = NA))
}
```

# 2 Scaling relationships

### 2.1 Irrigated area versus population size

```
# PREPARE DATASETS -----
# Prepare population dataset
pop <- df$pop %>%
  # Select population between 1999-2012 to
  # get the mean and concur with (mean) values
  # for irrigated areas attested by Meier et al.
  # between 1999-2012
  select(Continent, Country, Year.1999:Year.2012) %>%
  gather (Year, Population, Year. 1999: Year. 2012) %>%
  # Multiply to get actual population
 mutate_at(vars(Population), funs(.*1000)) %>%
  separate(., Year, into = c("dummy", "Year")) %>%
 mutate_at(vars(Year), funs(as.numeric)) %>%
  select(-dummy)
# Obtain number codes for each country to ease merging
# with irrigated areas at the country level
pop$Codes <- countrycode(pop$Country, origin = "country.name",</pre>
                         destination = "un")
# Create temporal list of data frame splitted by Dataset
```

```
temp <- df$meier %>%
  gather(Dataset, Area.irrigated, Meier.et.al.2018:Thenkabail.et.al.2009) %>%
  split(., .$Dataset)
# MERGE EACH DATASET WITH CORRESPONDING POPULATION VALUES ---
df.meier <- list()</pre>
for(i in names(temp)) {
  if(i == "Thenkabail.et.al.2009") {
    # Merge with population from 1999
    df.meier[[i]] <- pop %>%
      filter(Year == 1999) %>%
      inner_join(., temp[[i]],
                 by = c("Continent", "Country", "Codes"))
 }
  if(i == "Salmon.et.al.2015") {
    # Merge with population from 2005
    df.meier[[i]] <- pop %>%
      filter(Year == 2005) %>%
      inner_join(., temp[[i]],
                 by = c("Continent", "Country", "Codes"))
 }
  if(i == "Siebert.et.al.2013") {
    df.meier[[i]] <- pop %>%
      # Merge with mean population values 2000-2008
      spread(., Year, Population) %>%
      select(Continent, Country, `2000`: 2008`) %>%
      gather(Year, Population, `2000`: `2008`) %>%
      group_by(Country, Continent) %>%
      summarise(Population = mean(Population)) %>%
      mutate(Year = "2000.2008") %>%
      inner_join(., temp[[i]],
                 by = c("Continent", "Country"))
 } else {
    # For Meier et al.2018, Aquastat and FAOSTAT
    df.meier[[i]] <- pop %>%
      # Merge with mean population values 1999-2012
      group_by(Country, Continent) %>%
      summarise(Population = mean(Population)) %>%
      # Add dummy year column
      mutate(Year = "1999.2012") %>%
      inner_join(., temp[[i]],
                 by = c("Continent", "Country"))
 }
}
```

```
# Create dataset

cols <- c("Population", "Area.irrigated")

df.meier <- df.meier %>%
    rbindlist() %>%
    .[, .(Country, Continent, Codes,
        Area.irrigated, Population, Dataset)] %>%

# Drop countries with no irrigated area
.[!Area.irrigated == 0] %>%
# Drop Oceania due to small sample size
.[!Continent == "Oceania"] %>%
.[, (cols):= .SD / 10^6, .SDcols = (cols)]
```

#### 2.2 Irrigated area versus population density

```
# PLOT IRRIGATED AREAS AGAINST POPULATION DENSITY MEASURES
# Read in dataset
density.population <- fread("density_population.csv")</pre>
# Rename variables
density.population <- density.population[, Variable:= ifelse(Variable %in%
                                                                 "Total area of the country (exc
# Reduce dataset
density.population <- density.population[, c(2, 4, 5):= NULL]</pre>
# Spread
dens <- spread(density.population, Variable, Value)
cols <- c("Population", "Area.irrigated")</pre>
# Plot
inner_join(df.meier, dens, by = "Country") %>%
  data.table() %>%
  .[, `Area cultivable`:= Population / ((Area.cultivated * 1000) / 10^6)] %>%
  .[, `Area country`:= Population / ((Area.country * 1000) / 10^6)] %>%
  gather(parameter, value, `Area cultivable`:`Area country`) %>%
  ggplot(., aes(value, Area.irrigated,
                color = Continent)) +
 geom_point() +
 facet_grid(Dataset~parameter) +
  labs(x = "Population density (M/Mha)",
       y = "Irrigated area (Mha)") +
  scale_x_log10(labels = trans_format("log10", math_format(10^.x))) +
  scale_y_log10(labels = trans_format("log10", math_format(10^.x))) +
```

# 2.3 Plot irrigated area versus population size / population density

#### 2.4 Irrigated area versus water withdrawal / water requirement

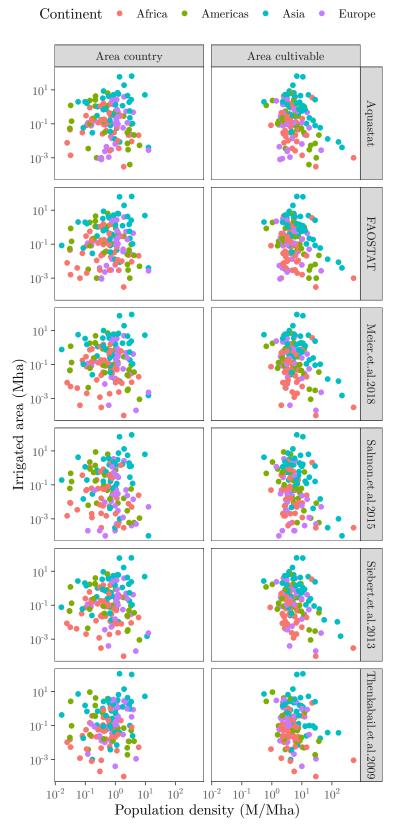


Figure 1: Scatter plots of measures of population density against irrigated areas.

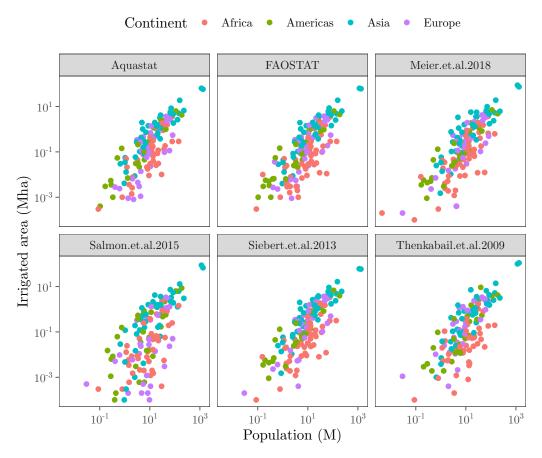


Figure 2: Scatter plots of irrigated areas against population. The strip label indicates the data set used to plot the values for irrigated areas [@FAO2016b; @FAO2017a; @Meier2018; @Salmon2015; @Siebert2013; @Thenkabail2009]. All those data sets have been compiled and studied by @Meier2018. The population data was retrieved from the @UnitedNations2017c.

```
# FUNCTIONS TO CODE -------
getCodes <- function(x) countrycode(x, origin = "country.name", destination = "un")</pre>
getContinent <- function(x) countrycode(x, origin = "country.name", destination = "continent")</pre>
getCountry <- function(x) countrycode(x, origin = "un", destination = "country.name")</pre>
addAll <- function(dt, dataset) {</pre>
  if(is.data.table(dt) == FALSE) {
   setDT(dt)
 }
 dt[, Codes:= lapply(.SD, getCodes), .SDcols = "Country"] %>%
    .[, Continent:= lapply(.SD, getContinent), .SDcols = "Country"] %>%
    .[, Country:= lapply(.SD, getCountry), .SDcols = "Codes"] %>%
   .[, Dataset:= dataset]
}
# CODE COUNTRY AND CONTINENT -----
aquastat <- addAll(aquastat, "Aquastat")</pre>
aquastat.dt <- spread(aquastat, Variable, Value)[
 !c(Year <= 1999 | Year >= 2012) # Retain only years 1999-2012
# READ IN TABLE 4 DATA SET -----
table4 <- fread("table_4.csv",</pre>
               skip = 3,
               nrows = 167) %>%
  [, c(2, 5, 7) := NULL]
# CODE COUNTRY AND CONTINENT -----
table4.dt <- addAll(table4, "Table.4") %>%
  .[!Continent == "Oceania"] %>%
  .[, .(Country, Year, Codes, Continent, Dataset, Water.requirement, Water.withdrawal)] %>%
  .[!c(Year <= 1999 | Year >= 2012)] # Retain only years 1999-2012
# MERGE AQUASTAT AND TABLE 4 DATASETS ------
water.dt <- rbind(aquastat.dt, table4.dt) %>%
 melt(., measure.vars = c("Water.requirement", "Water.withdrawal")) %>%
  .[, .(Max = max(value, na.rm = TRUE),
       Min = min(value, na.rm = TRUE)),
   by = .(Continent, variable, Country)]
## Warning in gmax(value, na.rm = TRUE): No non-missing values found in at
```

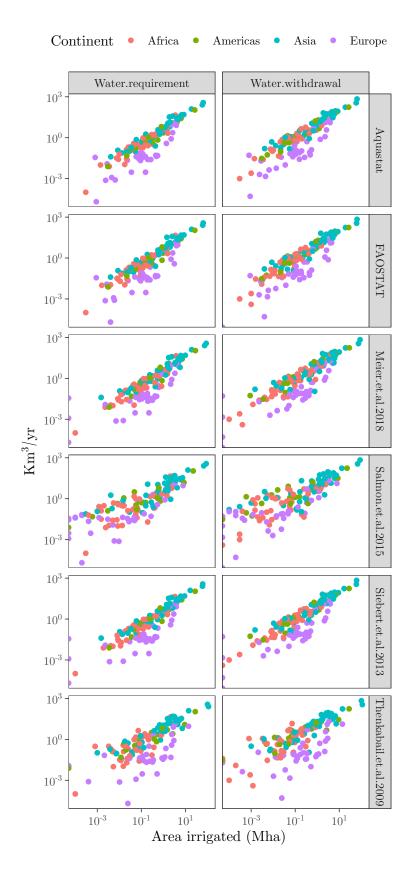
## least one group. Returning '-Inf' for such groups to be consistent with

```
## base
## Warning in gmin(value, na.rm = TRUE): No non-missing values found in at
## least one group. Returning 'Inf' for such groups to be consistent with base
# Transform Inf values in NA
is.na(water.dt) <- do.call(cbind,lapply(water.dt,</pre>
                                      is.infinite))
# READ IN MEIER ET AL. DATASET -----
meier.dt <- df$meier %>%
  data.table() %>%
  .[!Continent == "Oceania"] %>%
  [, (4:9):= lapply(.SD, function(x) x / 10^6), .SDcols = (4:9)]
# CODE COUNTRY AND CONTINENT -----
meier.dt <- addAll(meier.dt, "Meier")[, Dataset:= NULL]</pre>
## Warning in countrycode(x, origin = "country.name", destination = "un"): Some values were no
# CREATE FINAL DATASET ------
dt_water <- melt(water.dt, measure.vars = c("Max", "Min"),</pre>
                variable.name = "Stat") %>%
  .[, .(Value = mean(value, na.rm = TRUE)),
   by = .(Continent, variable, Country)] %>%
  .[meier.dt, on = c("Country", "Continent")] %>%
  .[!variable %in% NA] %>% # Remove rows in variable with NA
 melt(., measure.vars = c(6:11),
      variable.name = "Dataset",
      value.name = "Area.irrigated") %>%
  .[, Dataset:= factor(Dataset, levels = c("Aquastat", "FAOSTAT", "Meier.et.al.2018",
                                         "Salmon.et.al.2015", "Siebert.et.al.2013",
                                         "Thenkabail.et.al.2009"))]
dt.full <- dt_water[variable == "Water.requirement"] %>%
  .[df.meier, on = c("Continent", "Country", "Dataset", "Codes", "Area.irrigated")] %>%
  setnames(., "Value", "Water") %>%
  .[, .(Continent, Country, Codes, Dataset, Area.irrigated, Population, Water)]
```

#### Plot irrigated area versus water withdrawal / water requirement

```
# PLOT WATER VARIABLES AGAINST IRRIGATED AREAS ------
ggplot(dt_water, aes(Area.irrigated, Value,
              color = Continent)) +
 geom_point() +
```

## Warning: Transformation introduced infinite values in continuous x-axis
## Warning: Transformation introduced infinite values in continuous y-axis
## Warning: Removed 113 rows containing missing values (geom\_point).



# 3 Uncertainty in irrigated areas

```
# PLOT DIFFERENCES IN THE MEASUREMENT OF IRRIGATED AREAS
# AS A FUNCTION OF DATASET (CONTINENTAL) (Figure 2) ----
# Create data frame with total irrigated areas per dataset
total <- df$meier %>%
  filter(!Continent == "Oceania") %>%
 gather(Dataset, Value, Meier.et.al.2018:Thenkabail.et.al.2009) %>%
 group_by(Dataset) %>%
 summarise(Total = sum(Value, na.rm = T) / 10^6) %>%
 data.frame()
# Bar plot with continental and total irrigated areas per dataset
df$meier %>%
 gather(Dataset, Value, Meier.et.al.2018:Thenkabail.et.al.2009) %>%
 filter(!Continent == "Oceania") %>%
  group by(Continent, Dataset) %>%
  summarise(Total = sum(Value, na.rm = T) / 10^6) %>%
 ggplot(., aes(Continent, Total)) +
 geom_bar(stat = "identity") +
 geom_text(data = total, aes(label = paste("Total: ",
                                            round(Total, digits = 2),
                                            " Mha",
                                            sep = ""),
                              group = Dataset),
           x = 4
           y = 160,
            inherit.aes = FALSE,
            size = 3) +
  scale_y_continuous(breaks = pretty_breaks(n = 2)) +
 coord_flip() +
 labs(x = "Continent",
       y = "Irrigated area (Mha)") +
 facet wrap(~Dataset) +
 theme bw() +
 theme(aspect.ratio = 1,
       panel.grid.major = element_blank(),
       panel.grid.minor = element_blank())
# DIFFERENCES IN THE MEASUREMENT OF IRRIGATED AREAS
# AS A FUNCTION OF DATASET (COUNTRY) ----
temp <- df$meier %>%
  gather(Dataset, Value, 4:ncol(.)) %>%
 filter(!c(Value == 0 |
              Continent == "Oceania")) %>%
 mutate(Country = fct_recode(Country,
```

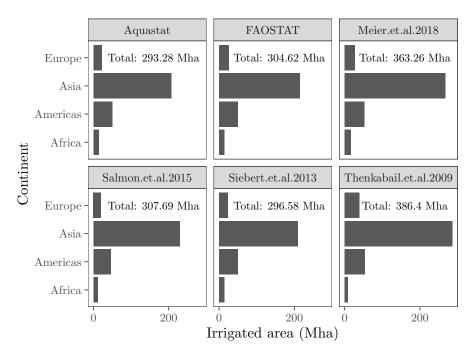


Figure 3: The extension of irrigation documented by different authors and institutions. FAO (2016) and FAOSTAT (FAO 2017) provide official values based on national surveys, census and statistics. Siebert et al. (2013) merges FAOSTAT and Aquastat values with independent maps and remote sensing imagery. Salmon et al. (2015) integrates national and subnational surveys with remote sensing and gridded climate data sets. Thenkabail et al. (2009) relies on remote sensing, Google Earth, and ground control points. Meier, Zabel, and Mauser (2018) downscales the map by Siebert et al. (2013) and uses multi-temporal normalized difference vegetation indexes with agricultural suitability data. The data was retrieved from Meier, Zabel, and Mauser (2018).

```
"Congo" = "Democratic Republic of the Congo",
                              "Tanzania" = "United Republic of Tanzania",
                              "Iran" = "Iran (Islamic Republic of)",
                              "Korea (DPR)" = "Korea, Democratic People's Republic of",
                              "Lao" = "Lao People's Democratic Republic",
                              "Macedonia" = "The former Yugoslav Republic of Macedonia")) %>%
 mutate_at(vars(Value), funs(. / 10^6)) %>%
  droplevels() %>%
  split(., .$Continent)
gg <- list()
for(i in seq(temp)) {
  gg[[i]] <- ggplot(temp[[i]], aes(reorder(Country, Value), Value)) +
    geom_point(stat = "identity", aes(color = Dataset)) +
    scale_y_log10( breaks = trans_breaks("log10", function(x) 10^x),
                    labels = trans_format("log10", math_format(10^.x))) +
    coord_flip() +
    scale_color_manual(name = "Dataset",
                       labels = c("Aquastat", "FAOSTAT", "Thenkabail et al. 2009",
                                  "Siebert et al. 2013", "Salmon et al. 2015",
                                  "Meier et al. 2018"),
                       values = c("yellowgreen", "seagreen4", "magenta3",
                                  "sienna3", "turquoise2", "khaki3")) +
    labs(y = "Irrigated area (Mha)",
         x = "") +
    facet_wrap(~Continent,
              scales = "free y") +
    theme_bw() +
    theme(panel.grid.major = element_blank(),
          panel.grid.minor = element_blank(),
          legend.position = "none",
          plot.margin = margin(t = -4.5,
                               unit = "cm"))
}
# Extract legend
legend <- get_legend(gg[[1]] + theme(legend.position = "top"))</pre>
# PLOT FOR AFRICA AND THE AMERICAS -----
first <- plot_grid(gg[[1]], gg[[2]], ncol = 2)
plot_grid(legend, first, ncol = 1)
# PLOT FOR ASIA AND EUROPE -----
second \leftarrow plot_grid(gg[[3]], gg[[4]], ncol = 2)
plot_grid(legend, second, ncol = 1)
```

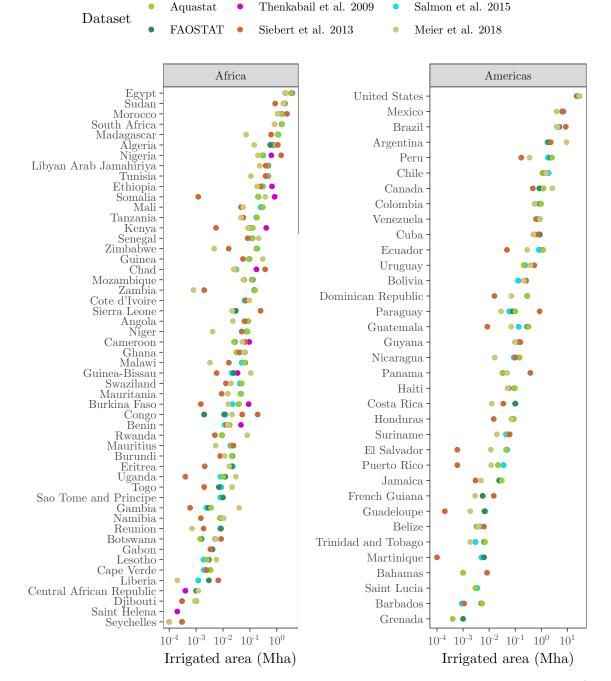


Figure 4: Extension of irrigation at the country level. The data was retrieved from Meier, Zabel, and Mauser (2018).

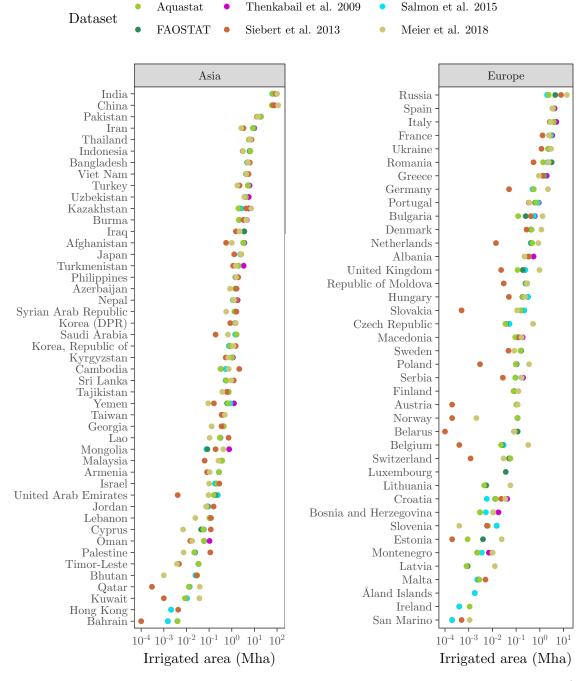


Figure 5: Extension of irrigation at the country level. The data was retrieved from Meier, Zabel, and Mauser (2018).

#### 4 Presence of outliers

```
# CHECK WHETHER THERE ARE OUTLIERS IN AREA.IRRIGATED VS. POPULATION
# AND AREA.IRRIGATED VS WATER REQUIRED FOR IRRIGATION ------
# Create datasets
temp <- dt_water[variable == "Water.requirement"] %>%
  .[df.meier, on = c("Continent", "Country", "Dataset", "Codes", "Area.irrigated")] %>%
 mutate_at(vars(Area.irrigated, Population, Value), funs(log10)) %>%
 data.table()
# Calculate Mahalabobis distances (robust and classic) for each
# continent and dataset
cols <- c("Population", "Area.irrigated")</pre>
temp1 <- temp[, dd.plot(.SD), .SDcols = cols, by = .(Continent, Dataset)] %>%
  .[, Class:= "Population"]
temp2 <- temp %>%
 na.omit() %>%
  .[, dd.plot(.SD), .SDcols = c("Value", "Area.irrigated"),
                 by = .(Continent, Dataset)] %>%
  .[, Class:= "Water"]
out <- rbind(temp1, temp2)</pre>
# Extract number of outliers per continent and dataset
out.n <- out[, .(Outliers = sum(outliers == TRUE)),
             by = .(Continent, Dataset, Class)] %>%
  .[order(Continent)]
# Extract maximum values for Mahalanobis distances
# (robust and classic)
out.md <- out[, .(max.Mahalanobis.classic = max(md.cla),
                  max.Mahalanobis.robust = max(md.rob)),
              by = .(Dataset, Continent, Class)]
# Merge both datasets
out.df <- out.n[out.md, on = c("Dataset", "Continent", "Class")] %>%
  .[order(Continent)]
# EXPORT OUTLIERS DATASET -----
fwrite(out.df, "out.df.csv")
# ARRANGE TO PLOT RESULTS -----
```

```
temp <- out %>%
  split(., .$Class)
gg <- list()
for(i in names(temp)) {
  gg[[i]] <-
              ggplot(temp[[i]], aes(md.cla, md.rob,
                                     color = outliers)) +
   geom point() +
    scale_colour_manual(name = "Outlier",
                        values = setNames(c("black", "red"),
                                          c(FALSE, TRUE))) +
   facet_grid(Dataset~Continent) +
   labs(x = "Mahalanobis distance",
        y = "Robust distance") +
   theme_AP() +
    theme(legend.position = "top")
# PLOT MAHALANOBIS DISTANCES FOR AREA IRRIGATED VERSUS POPULATION ------
plot(gg[["Population"]])
# PLOT MAHALANOBIS DISTANCES FOR AREA IRRIGATED VERSUS
# IRRIGATION WATER REQUIREMENT ----
plot(gg[["Water"]])
```

# 5 Estimation of the model parameters

#### 5.1 Irrigated area baseline values $(Y_0)$

```
# CALCULATE FOR EACH CONTINENT THE MAXIMUM AND MINIMUM
# EXTENSION OF IRRIGATED AREAS -----

total.area.irrigated <- df$meier %>%
  data.table() %>%
  melt(., measure.vars = c(4:9),
     variable.name = "Dataset",
     value.name = "Value") %>%
  .[, Value:= Value / 10 ^6] %>%
  .[, .(Total = sum(Value, na.rm = T)), .(Dataset, Continent)] %>%
  .[, .(min = min(Total), max = max(Total)), Continent] %>%
  .[!Continent == "Oceania"] %>%
  split(., .$Continent, drop = TRUE)
```

# Outlier • FALSE • TRUE

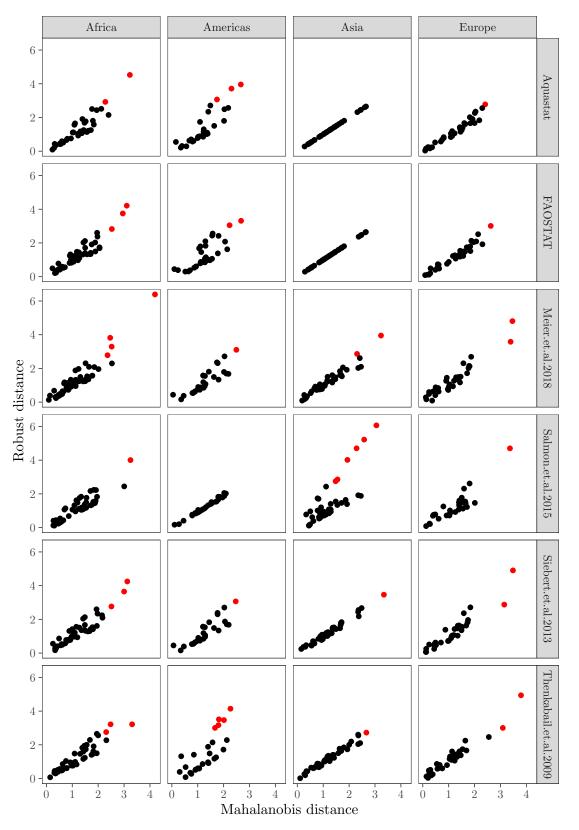


Figure 6: Scatter plot of Mahalanobis vs Robust distances.

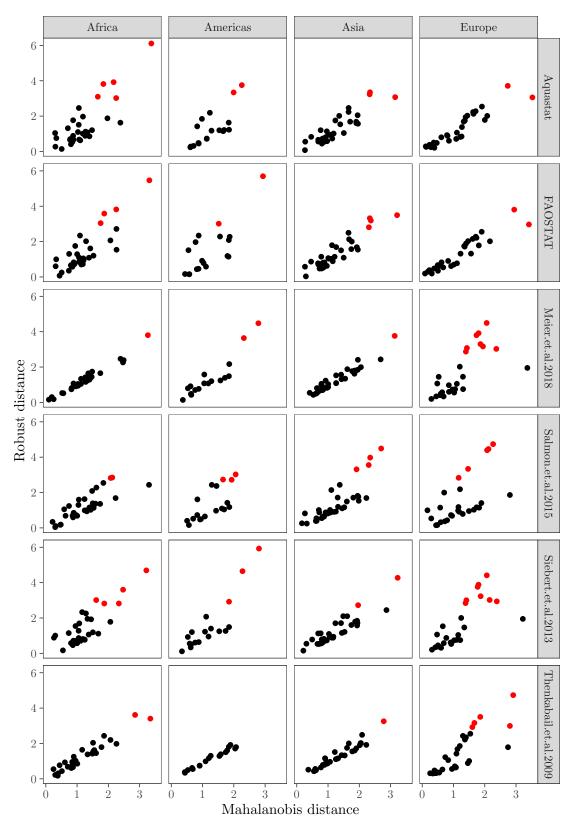


Figure 7: Scatter plot of Mahalanobis vs Robust distances.

5.2 Growth rate between population and irrigated area ( $\beta$ ), irrigated area and water required for irrigation ( $\phi$ ,  $\delta$ ), and noise ( $\varepsilon$ )

```
# DEFINE NUMBER OF BOOTSTRAP REPLICAS -----
R < -10^4
# OLS REGRESSIONS: ALPHA, BETA AND DELTA -----
# Create bootstrap function for non-robust OLS
boot.ols <- function(formula, x, i) {</pre>
  d \leftarrow x[i,]
  # Bootstrap slope
  fit <-lm(formula, data = d)</pre>
  out <-coef(fit)</pre>
  return(out)
}
# t1: alpha ols nonrob
# t2: beta ols nonrob
# Bootstrap alpha and beta
dt.regressions <- dt.full %>%
  .[, -2] %>%
  mutate_at(vars(Population, Area.irrigated, Water), funs(log10)) %>%
  data.table()
# Regular/Robust regressions
olsB <- dt.regressions[, # Regular OLS regressions
                      .(pop = list(boot(.SD,
                                         statistic = boot.ols,
                                         R = R
                                         formula = Area.irrigated ~ Population,
                                         parallel = "multicore",
                                         ncpus = floor(detectCores() * 0.75))),
                        water = list(boot(.SD,
                                           statistic = boot.ols,
                                           R = R.
                                           formula = Water ~ Area.irrigated,
                                           parallel = "multicore",
                                           ncpus = floor(detectCores() * 0.75))),
                        # Robust OLS regressions
                        popR = list(lmrob(Area.irrigated ~ Population)),
                        waterR = list(lmrob(Water ~ Area.irrigated))),
                      by = .(Continent, Dataset)]
# Extract alpha and beta for OLS non robust (population)
ols.nonrobust.pop <- olsB[, "All":= list(lapply(.SD, function(x)</pre>
  map(x, "t"))), .SDcols = "pop", .(Continent, Dataset)] %>%
```

```
.[, .("Alpha" = lapply(All, function(x) lapply(x, function(y) y[, 1])),
       "Beta" = lapply(All, function(x) lapply(x, function(y) y[, 2]))),
   .(Continent, Dataset)] %>%
  .[, lapply(.SD, unlist), .SDcols = c("Alpha", "Beta"), .(Continent, Dataset)] %>%
  .[, Regression:= "OLS"] %>%
  .[, Robust:= "NO"]
# Extract phi and delta for OLS non robust (water)
ols.nonrobust.water <- olsB[, "All":= list(lapply(.SD, function(x)</pre>
  map(x, "t"))), .SDcols = "water", .(Continent, Dataset)] %>%
  .[, .("Phi" = lapply(All, function(x) lapply(x, function(y) y[, 1])),
        "Delta" = lapply(All, function(x) lapply(x, function(y) y[, 2]))),
    .(Continent, Dataset)] %>%
  .[, lapply(.SD, unlist), .SDcols = c("Phi", "Delta"), .(Continent, Dataset)] %>%
  .[, Regression:= "OLS"] %>%
  .[, Robust:= "NO"]
# OLS REGRESSIONS ROBUST: ALPHA, BETA AND DELTA ------
# Create cluster of 4 CPUS to speed up the bootstrapping
# Bootstrap OLS robust
olsR <- olsB[, .(popRob = lapply(popR, function(x) bootcoefs(x,
                                                             method = "frb",
                                                             ncpus = floor(detectCores() * 0.79
                 waterRob = lapply(waterR, function(x) bootcoefs(x,
                                                                  R = R
                                                                  method = "frb",
                                                                  ncpus = floor(detectCores() *
             .(Continent, Dataset)]
# Extract alpha and beta for OLS robust (population)
ols.robust.pop <- olsR[, "Allpop":= list(lapply(.SD, function(x)</pre>
  lapply(x, function(y) y[["bootres"]]))),
           .SDcols = "popRob", .(Continent, Dataset)] %>%
  .[, "temp":= list(lapply(.SD, function(x)
    lapply(x, function(y) lapply(y, function(z) z[["t"]])))),
    .SDcols = "Allpop", .(Continent, Dataset)] %>%
  .[, .("Alpha" = lapply(temp, function(x)
    lapply(x, function(y) lapply(y, function(z) z[, 1]))),
   "Beta" = lapply(temp, function(x)
      lapply(x, function(y) lapply(y, function(z) z[, 2])))) ,
    .(Continent, Dataset)] %>%
  .[, lapply(.SD, unlist), .SDcols = c("Alpha", "Beta"), .(Continent, Dataset)] %>%
  .[, Regression:= "OLS"] %>%
  .[, Robust:= "YES"]
```

```
# Extract phi and delta for OLS robust (water)
ols.robust.water <- olsR[, "Allwater":= list(lapply(.SD, function(x)
  lapply(x, function(y) y[["bootres"]]))),
  .SDcols = "waterRob", .(Continent, Dataset)] %>%
  .[, "temp":= list(lapply(.SD, function(x)
    lapply(x, function(y) lapply(y, function(z) z[["t"]])))),
    .SDcols = "Allwater", .(Continent, Dataset)] %>%
  .[, .("Phi" = lapply(temp, function(x)
    lapply(x, function(y) lapply(y, function(z) z[, 1]))),
    "Delta" = lapply(temp, function(x)
      lapply(x, function(y) lapply(y, function(z) z[, 2])))) ,
    .(Continent, Dataset)] %>%
  .[, lapply(.SD, unlist), .SDcols = c("Phi", "Delta"), .(Continent, Dataset)] %>%
  .[, Regression:= "OLS"] %>%
  .[, Robust:= "YES"]
# SMA REGRESSIONS: ALPHA AND BETA -----
# Create bootstrap function for robust and non-robust SMA
boot.sma <- function(formula, x, i) {</pre>
  d \leftarrow x[i,]
  # Bootstrap coefficients (non-robust)
 fit1 <- sma(formula, data = d, method = "SMA")
  # Bootstrap coefficients (robust)
 fit2 <- sma(formula, data = d, method = "SMA", robust = TRUE)
 coef1 <- coef(fit1)</pre>
 coef2 <- coef(fit2)</pre>
 all <- c(coef1, coef2)
 return(all)
# t1: alpha sma nonrob
# t2: beta sma nonrob
# t3: alpha sma rob
# t4: beta sma rob
# Bootstrap alpha and beta
smaB <- dt.regressions[, list(list(boot(.SD,</pre>
                                         statistic = boot.sma,
                                         R = R
                                         formula = Area.irrigated ~ Population,
                                         parallel = "multicore",
                                         ncpus = floor(detectCores() * 0.75)))),
                        .(Continent, Dataset)]
# EXTRACT ALPHA AND BETA ROBUST SMA ----
# Extract alpha and beta SMA (non-robust)
```

```
sma.nonrobust.pop <- smaB[, "All":= list(lapply(V1, function(x) x["t"]))] %>%
  .[, list("Alpha" = lapply(All, function(x) lapply(x, function(y) y[, 1])),
           "Beta" = lapply(All, function(x) lapply(x, function(y) y[, 2]))),
      .(Continent, Dataset)] %>%
  .[, lapply(.SD, unlist), .SDcols = c("Alpha", "Beta"), .(Continent, Dataset)] %>%
  .[, Regression:= "SMA"] %>%
  .[, Robust:= "NO"]
# Extract alpha and beta SMA (robust)
sma.robust.pop <- smaB[, "All":= list(lapply(V1, function(x) x["t"]))] %>%
  .[, list("Alpha" = lapply(All, function(x) lapply(x, function(y) y[, 3])),
           "Beta" = lapply(All, function(x) lapply(x, function(y) y[, 4]))),
      .(Continent, Dataset)] %>%
  .[, lapply(.SD, unlist), .SDcols = c("Alpha", "Beta"), .(Continent, Dataset)] %>%
  .[, Regression:= "SMA"] %>%
  .[, Robust:= "YES"]
fwrite(sma.nonrobust.pop, "sma.nonrobust.pop.csv")
fwrite(sma.robust.pop, "sma.robust.pop.csv")
# CREATE FINAL DATA SETS WITH ALL BOOTSTRAP SAMPLES
# For beta and alpha
boot.samples.pop <- rbind(ols.nonrobust.pop,</pre>
                          ols.robust.pop,
                          sma.nonrobust.pop,
                          sma.robust.pop) %>%
  .[order(Continent, Dataset)]
# For delta
boot.samples.water <- rbind(ols.nonrobust.water,</pre>
                            ols.robust.water) %>%
  .[order(Continent, Dataset)]
# EXPORT BOOTSTRAP SAMPLES -----
fwrite(boot.samples.pop, "boot.samples.pop.csv")
fwrite(boot.samples.water, "boot.samples.water.csv")
# PREDICT ALPHA FROM BETA -----
# Predict Alpha from Beta
summary.beta.alpha <- boot.samples.pop %>%
 lm(.$Alpha ~ .$Beta,
     data = .)
# Get intercept, slope and epsilon values
Intercept <- coef(summary.beta.alpha)[[1]]</pre>
```

```
Slope <- coef(summary.beta.alpha)[[2]]</pre>
Epsilon <- summary(summary.beta.alpha)$sigma %>%
# CREATE THE LOOKUP TABLE -----
# Create vector to change columns
col_names <- c("Continent", "Dataset", "Regression", "Robust")</pre>
col_names2 <- col_names[!col_names %in% "Regression"]</pre>
# Create lookup table: population
lookup.pop <- boot.samples.pop[order(Beta), .SD, col_names] %>%
  .[, ID:= 1:.N, col_names] %>%
  .[, index:= paste(Continent, Dataset, Regression, Robust, ID, sep = "_")]
setkey(lookup.pop, index)
# Create lookup table: water
lookup.water <- boot.samples.water[order(Delta), .SD, col_names2] %>%
  .[, ID:= 1:.N, col_names2] %>%
  .[, index:= paste(Continent, Dataset, Robust, ID, sep = "_")]
setkey(lookup.water, index)
# EXPORT BOOTSTRAP SAMPLES TO CSV -
fwrite(lookup.pop, "lookup.pop.csv")
fwrite(lookup.water, "lookup.water.csv")
```

#### 5.3 Population baseline values (N)

```
# CREATE DATA FRAME WITH POPULATION BASELINE VALUES -
# Prepare population values between 1999-2015
temp <- df$population %>%
  # Filter out Oceania
 filter(!Continent == "Oceania") %>%
  select(Continent, Codes, Estimate, Year.1999:Year.2012) %>%
  gather(Year, Population, Year.1999:Year.2012) %>%
 mutate(Continent = fct_recode(Continent,
                                "Americas" = "S.America",
                                "Americas" = "N.America")) %>%
 group_by(Continent, Year, Estimate) %>%
  summarise(Population = sum(Population)) %>%
  # Multiply by 1000 to get original population values
 mutate_at(vars(Population), funs(.*10^3)) %>%
  separate(., Year,
           into = c("dummy", "Year")) %>%
```

# 5.4 Population growth rates (r)

```
# DEFINE POPULATION GROWTH RATES DISTRIBUTIONS
# Prepare data frame with growth rates 2015-2050
df.rate1 <- df$growth.rate.estimate %>%
  select(Continent, Estimate, Year.2015.2020:Year.2045.2050) %>%
 gather(Period, Value, Year.2015.2020:Year.2045.2050) %>%
  # Unite population growth rates for N. America and S. America,
  # and consider both regions as one (Americas)
 mutate(Continent = fct_recode(Continent,
                                "Americas" = "S.America",
                                "Americas" = "N.America")) %>%
  # Exclude Oceania
 filter(!Continent == "Oceania")
# Prepare data frame with growth rates 2000-2015
df.rate2 <- df$growth.rate %>%
  select(Continent, Year.2000.2005:Year.2010.2015) %>%
 gather(Period, Value, Year.2000.2005:Year.2010.2015) %>%
 mutate(Continent = fct_recode(Continent,
                                "Americas" = "S.America",
                                "Americas" = "N.America")) %>%
  # Exclude Oceania
 filter(!Continent == "Oceania")
# Create population growth rates data frame
df.rate <- bind_rows(df.rate1, df.rate2) %>%
  # Add constant (+5) to growth rate values to
  # allow fitting distributions later on
 mutate_at(vars(Value), funs(. + 5)) %>%
```

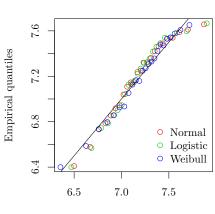
```
split(., .$Continent, drop = TRUE)
# Describe growth rates of continents with a distribution
# Fit possible distributions according to histograms
distr.norm <- lapply(df.rate, function(x) fitdist(x$Value, "norm"))</pre>
distr.logis <- lapply(df.rate, function(x) fitdist(x$Value, "logis",</pre>
                                                      method = "mme"))
distr.weib <- lapply(df.rate, function(x) fitdist(x$Value, "weibull"))</pre>
# Define function to plot
plotDistr <- function(x,...) {</pre>
  funs <- c(denscomp, qqcomp, cdfcomp, ppcomp)</pre>
  lapply(funs, function(f) f(list(x,...),
                              legendtext = plot.legend))
}
# Plot distributions and fits (Figures 9-12)
par(mfrow = c(2, 2),
    oma = c(0, 0, 2, 0)
plot.legend <- c("Normal", "Logistic", "Weibull")</pre>
for(i in names(distr.norm)) {
  gg <- plotDistr(distr.norm[[i]],</pre>
                   distr.logis[[i]],
                   distr.weib[[i]])
  title(names(distr.norm[i]), outer = TRUE)
  print(gg)
}
```

#### Africa

# Histogram and theoretical densitie

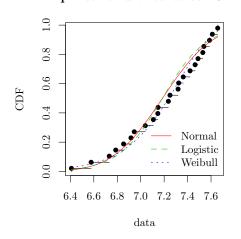
# Normal Logistic Veibull 6.4 6.6 6.8 7.0 7.2 7.4 7.6 data

# Q-Q plot

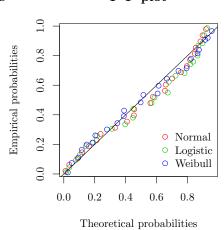


Theoretical quantiles

### Empirical and theoretical CDFs



# P-P plot



# [[1]]

## NULL

##

## [[2]]

## NULL

##

## [[3]]

## NULL

##

## [[4]]

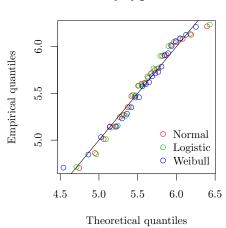
## NULL

Asia

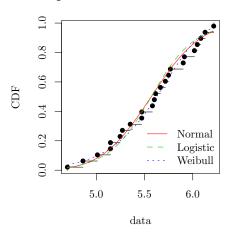
# Histogram and theoretical densitie

# Normal Logistic Weibull 5.0 5.5 6.0 data

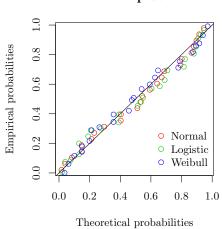
# Q-Q plot



# Empirical and theoretical CDFs



# P-P plot



## [[1]]
## NULL
##
## [[2]]

## [[2]] ## NULL

##

## [[3]]

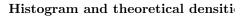
## NULL

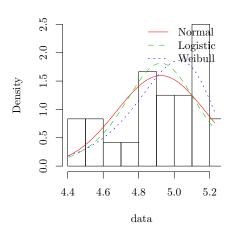
##

## [[4]]

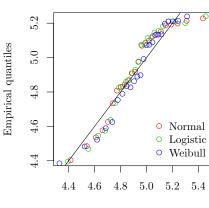
## NULL

# ${\bf Europe}$



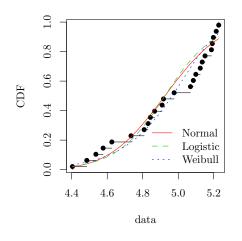


# Q-Q plot

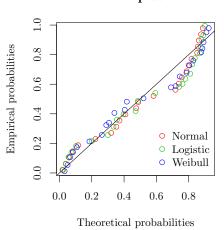


#### Theoretical quantiles

# **Empirical and theoretical CDFs**



### P-P plot



## [[1]]

## NULL

##

## [[2]]

## NULL

##

## [[3]]

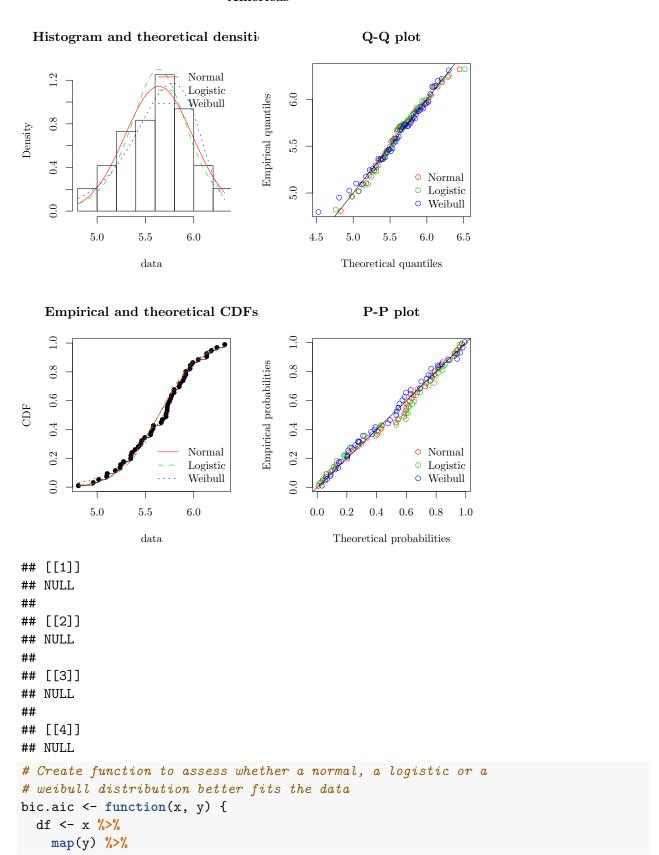
## NULL

##

## [[4]]

## NULL

#### Americas



```
data.frame() %>%
    t() %>%
    data.frame() %>%
    rownames_to_column(., var = "Continent")
 return(df)
}
# Assess via BIC
bic.aic(x = distr.norm,
        y = "bic") %>%
 rename(Normal = ".") %>%
  inner_join(., bic.aic(x = distr.logis,
                        y = "bic"),
             by = "Continent") %>%
 rename(Logistic = ".") %>%
  inner_join(., bic.aic(x = distr.weib,
                        y = "bic"),
             by = "Continent") %>%
 rename(Weibull = ".") %>%
 mutate(Model = "BIC")
##
     Continent
                  Normal Logistic
                                     Weibull Model
        Africa 22.176234 23.640107 19.532196
## 1
## 2
          Asia 31.092193 32.688151 30.187821
                                               BIC
## 3
        Europe 7.928691 9.946661 4.671153
                                               BIC
## 4 Americas 42.877046 45.162335 42.951730
                                               BIC
# The distributions that better fit the data are the following:
# Africa: Weibull distribution
# Asia: Weibull distribution
# Europe: Weibull distribution
# Americas: Normal distribution
# Create data frame with original growth rate values
df.rate.nrm <- bind_rows(df.rate1, df.rate2) %>%
  # Divide growth rates per 100 because it is in percentage
 mutate_at(vars(Value), funs(. / 100)) %>%
  split(., .$Continent, drop = TRUE)
# Fit a normal distribution as we will need the fit for the Americas
distr.norm2 <- lapply(df.rate.nrm, function(x) fitdist(x$Value, "norm"))</pre>
# List with the parameters for the distribution of growth rates
growth.rate.distr <- list(distr.weib$Africa$estimate,</pre>
                          distr.weib$Asia$estimate,
                          distr.weib$Europe$estimate,
                          distr.norm2$Americas$estimate)
```

```
# Name the slots of the list
names(growth.rate.distr) <- c("Africa", "Asia", "Europe", "Americas")</pre>
```

## 5.5 Cropland available (K)

```
# INTEGRATE CROPLAND AVAILABLE WITH MODEL OUTPUT UNCERTAINTY ----
# Prepare dataset by Zhang: create continental frame
df.zhang <- df$zhang.land.available %>%
  # Transform km2 to ha
 mutate_at(vars(Mkm2, Baseline), funs(.*100)) %>%
  group_by(Continent, Estimation, Baseline) %>%
  summarise(Min = min(Mkm2),
            Max = max(Mkm2)) \%>\%
  # Conditional mutation: create column filled with
  # either the minimum value (if both projections decrease)
  # or the maximum value (if both projections increase)
 mutate(Value = ifelse(Max < Baseline, Min, Max)) %>%
  data.frame() %>%
 filter(!Continent == "World") %>%
 mutate(Continent = fct_recode(Continent,
                                "Asia" = "China",
                                "Asia" = "India",
                                "Americas" = "US",
                                "Americas" = "S.America",
                                "Europe" = "Russia"))
# Prepare dataset by Zhang: calculate min and max values
temp <- df.zhang %>%
  select(Continent, Estimation, Min, Max) %>%
  split(.,list(.$Continent, .$Estimation),
        drop = TRUE) %>%
 lapply(., function(x) {
   x$min <- sum(x$Min)
    x$max <- sum(x$Max)
   return(x[1, 5:6])
 })
# Create final data set by Zhang to plot
cropland.1 <- temp %>%
 map(data.frame) %>%
 rbindlist(., idcol = "Continent") %>%
  separate(., col = Continent,
           into = c("Continent", "Estimation"))
cropland <- cropland.1 %>%
```

### 5.6 Water available $(W_a)$

# 6 Creation of the sample matrix

```
AB <- lapply(Continents, function(Continents) sobol_matrices(n = n,
                                                               k = k) \%
               data.table())
# Name the slots, each is a continent
names(AB) <- Continents</pre>
# Name the columns
AB <- lapply(AB, setnames, parameters)
# CREATE THE SAMPLE MATRICES FOR THE CLUSTERED PARAMETERS -----
# Create vectors with the name of the parameters within each cluster
irrigation <- match(c("X1", "Y0", "W1", "W_a", "eta"), parameters)</pre>
population2 <- match(c("r", "gamma"), parameters)</pre>
model <- match(c("X2", "X3", "X4", "W3", "W4", "epsilon"), parameters)</pre>
# Create an A, B and AB matrices for the clustered parameters; retrieve
# only the AB
AB.cluster <- lapply(Continents, function(Continents)
  sobol_matrices(n = n,
                 k = k
                 cluster = list(irrigation, population2, model))) %>%
  lapply(., function(x) x[((2*n) + 1):nrow(x), ]) \%
 lapply(., data.table)
# Name the slots, each is a continent
names(AB.cluster) <- Continents</pre>
# Name the columns
AB.cluster <- lapply(AB.cluster, setnames, parameters)
# Merge the sample matrix and the sample matrix of the
# clustered parameters
for(i in names(AB)) {
  AB[[i]] <- rbind(AB[[i]], AB.cluster[[i]])
# CHECK NUMBER OF BOOTSTRAP SAMPLES OF BETA, DELTA, ETC. -----
N.boot \leftarrow boot.samples.pop[, .(N = .N),
                            .(Continent, Dataset, Regression, Robust)] %>%
  .[, N] %>%
  .[1]
```

```
print(N.boot)
## [1] 10000
# TRANSFORM THE SAMPLE MATRIX -----
# Create function to transform the parameters that
# have the same distribution in all continents
transform.sobol <- function(X) {</pre>
 ifelse(X1 == 2, "FAOSTAT",
                                                     ifelse(X1 == 3, "Siebert.et.al.2013",
                                                           ifelse(X1 == 4, "Meier.et.al.2
                                                                  ifelse(X1 == 5, "Salmon
                                                                        "Thenkabail.et.a
 X[, X2:=floor(X2 * (2-1+1)) + 1][, X2:=ifelse(X2==1, "OLS", "SMA")]
 X[, X3:=floor(X3 * (2-1+1)) + 1][, X3:=ifelse(X3==1, "YES", "NO")]
 X[, X4:= floor(X4 * (N.boot - 1)) + 1]
 ifelse(W1 == 2, "FAOSTAT",
                                                     ifelse(W1 == 3, "Siebert.et.al.2013",
                                                           ifelse(W1 == 4, "Meier.et.al.2
                                                                  ifelse(W1 == 5, "Salmon
                                                                        "Thenkabail.et.a
 X[, W3:=floor(W3 * (2-1+1)) + 1][, W3:=ifelse(W3==1, "YES", "NO")]
 X[, W4:= floor(W4 * (N.boot - 1)) + 1]
 X[, gamma := 0.02 * qnorm(gamma) + 1]
 X[, epsilon:= Epsilon * qnorm(epsilon) + 0]
 X[, t:= floor(t * (51-38 + 1)) + 38]
 X[, eta:= qunif(eta, min = 0.2, max = 0.5)]
 return(X)
}
AB <- lapply(AB, transform.sobol)
# Transform the parameters with their appropriate distributions
transform.sobol.continents <- function(AB) {</pre>
 for(i in names(AB)) {
   if(i == "Africa") {
     # Weibull distribution, substract the constant and divide by 100
     # because original values were in percentage
     AB[[i]][, r:= (growth.rate.distr$Africa[[2]] *
                     (-log(1 - r)) ^ (1/growth.rate.distr$Africa[[1]])
                   -5) / 100]
     # Uniform distribution
     AB[[i]][, YO:= YO *
               (total.area.irrigated$Africa$max-total.area.irrigated$Africa$min) +
               total.area.irrigated$Africa$min]
```

```
# Unifrom distribution
  AB[[i]][, K:= K * (cropland$Africa$max-cropland$Africa$min) +
            cropland$Africa$min]
  # Uniform distribution
  AB[[i]][, W_a:= qunif(W_a, min = water.availability.dt$Africa$lower,
                        max = water.availability.dt$Africa$upper)]
if(i == "Asia") {
  # Weibull distribution, substract the constant and divide by 100
  # because original values were in percentage
  AB[[i]][, r:= (growth.rate.distr$Asia[[2]] *
                   (-log(1 - r)) ^ (1/growth.rate.distr$Asia[[1]])
  # Uniform distribution
  AB[[i]][, YO:= YO *
            (total.area.irrigated$Asia$max-total.area.irrigated$Asia$min) +
            total.area.irrigated$Asia$min]
  # Unifrom distribution
  AB[[i]][, K:= K * (cropland$Asia$max-cropland$Asia$min) +
            cropland$Asia$min]
  # Uniform distribution
  AB[[i]][, W_a:= qunif(W_a, min = water.availability.dt$Asia$lower,
                        max = water.availability.dt$Asia$upper)]
if(i == "Americas") {
  # Normal distribution
  AB[[i]][, r:= (growth.rate.distr$Americas[[2]] *
                   qnorm(r) + growth.rate.distr$Americas[[1]])]
  # Uniform distribution
  AB[[i]][, YO:= YO *
            (total.area.irrigated$Americas$max-total.area.irrigated$Americas$min) +
            total.area.irrigated$Americas$min]
  # Unifrom distribution
  AB[[i]][, K:= K * (cropland$Americas$max-cropland$Americas$min) +
            cropland$Americas$min]
  # Uniform distribution
  AB[[i]][, W_a:= qunif(W_a, min = water.availability.dt$Americas$lower,
                        max = water.availability.dt$Americas$upper)]
}
if(i == "Europe") {
  # Weibull distribution, substract the constant and divide by 100
  # because original values were in percentage
  AB[[i]][, r:=(growth.rate.distr$Europe[[2]] *
                  (-log(1 - r)) ^ (1/growth.rate.distr$Europe[[1]])
                -5) / 100]
  # Uniform distribution
  AB[[i]][, YO:= YO *
```

```
(total.area.irrigated $Europe $max-total.area.irrigated $Europe $min) +
               total.area.irrigated$Europe$min]
      # Unifrom distribution
      AB[[i]][, K:= K * (cropland$Europe$max-cropland$Europe$min) +
               cropland$Europe$min]
      # Uniform distribution
      AB[[i]][, W_a:= qunif(W_a, min = water.availability.dt$Europe$lower,
                           max = water.availability.dt$Europe$upper)]
   }
  }
 return(AB)
}
AB <- transform.sobol.continents(AB)
# WRITE FINAL DATA TABLE -----
final.dt <- rbindlist(AB, idcol = "Continent")</pre>
# EXPORT FINAL DATA TABLE -----
fwrite(final.dt, "final.dt.csv")
print(final.dt)
##
            Continent
                                         X1 X2 X3
                                                      Х4
                           Meier.et.al.2018 SMA NO 5000
##
               Africa
         1:
##
         2:
               Africa
                          Salmon.et.al.2015 OLS NO 2500
               Africa
                                    FAOSTAT SMA YES 7500
##
         3:
         4:
               Africa
                         Siebert.et.al.2013 OLS NO 1250
##
##
         5:
               Africa Thenkabail.et.al.2009 SMA YES 6250
##
## 10485756:
               Europe
                                     FAOSTAT OLS YES 1891
               Europe
## 10485757:
                          Salmon.et.al.2015 SMA NO 6890
## 10485758:
               Europe
                           Meier.et.al.2018 SMA YES 9390
## 10485759:
               Europe
                                   Aquastat OLS NO 4390
## 10485760:
               Europe
                                   Aquastat SMA YES 2314
##
                               W1 W3
                                        W4
                                                             gamma
                                                                         Y0
                 Meier.et.al.2018 NO 5000 0.0223856397 1.0000000 12.45665
##
         1:
##
         2:
                 Salmon.et.al.2015 YES 7500 0.0200542303 0.9865102 14.41732
##
                          FAOSTAT NO 2500 0.0242778499 1.0134898 10.49597
         4: Thenkabail.et.al.2009 NO 1250 0.0233272313 0.9769930 15.39766
##
##
         5:
               Siebert.et.al.2013 YES 6250 0.0180808545 1.0063728 11.47631
##
## 10485756: Thenkabail.et.al.2009 NO 6621 -0.0011411532 0.9648044 25.41198
## 10485757:
               Siebert.et.al.2013 YES 1622 0.0017884742 1.0019695 34.75228
                          Aquastat YES 9121 -0.0035627490 0.9888869 30.08213
## 10485758:
## 10485759:
                 Meier.et.al.2018 NO 4121 0.0002990401 1.0160745 20.74183
## 10485760:
                 Meier.et.al.2018 YES 4893  0.0000225477  0.9934432  26.06138
```

```
##
                 epsilon t
                                   K
                                          W_a
         1: 0.00000000 45 959.5000 3928.000 0.3500000
##
         2: -0.10666007 48 836.7500 4321.000 0.2750000
##
         3: 0.10666007 41 1082.2500 3535.000 0.4250000
##
##
         4: -0.05038786 46 775.3750 3731.500 0.3875000
         5: 0.18190987 39 1020.8750 4517.500 0.2375000
##
##
## 10485756: -0.07705733 51 552.1640 5857.321 0.4527199
## 10485757: 0.14059799 44 665.6640 7172.321 0.3027199
## 10485758: 0.02508753 41 495.4140 7829.821 0.3777199
## 10485759: -0.24192264 48 608.9140 6514.821 0.2277199
## 10485760: 0.06583090 51 548.0343 6730.212 0.4743229
```

#### 7 The model

```
# DEFINE THE MODEL ----
model <- function(X) {</pre>
  # Extract beta
 Beta <- lookup.pop[.(paste0(X[, 1:5], collapse = "_"))][, Beta]</pre>
  # Select population
 N <- population[.(paste0(X[, c("Continent", "t")], collapse = "_"))] %>%
    .[, N]
  # Extract phi and delta
 tmp <- lookup.water[.(paste0(X[, c(1, 6:8)], collapse = "_"))]
 Phi <- tmp[, Phi]
 Delta <- tmp[, Delta]</pre>
  # COMPUTE THE MODEL -----
 Y \leftarrow X[, YO] +
   10 ^ (Intercept + Slope * Beta + X[, epsilon]) * Beta * X[, r] *
    (N ^ (1 - X[, gamma]) + X[, r] * X[, t] * (1 - X[, gamma])) ^ 
    ((Beta + X[, gamma] - 1) / (1 - X[, gamma]))
  # Compute how much water will we need to irrigate Y
 w <- (10 ^ Phi) * Y ^ Delta
  # Compute how much water we have
  # available for irrigation -----
 w_i <- X[, W_a] * X[, eta]
  # Compute the total extension that can
  # be irrigated with the water
  # we have available for irrigation -----
 Y.max <- (w_i / 10 ^ Phi) ^ (1 / Delta)
  # Constrain
 if(Y > X[, K]) {
   Y \leftarrow X[, K]
 if(Y > Y.max) {
  Y <- Y.max
```

```
if(Y < X[, Y0]) {</pre>
     Y \leftarrow X[, Y0]
 }
 return(c(Beta, N, Phi, Delta, w, w_i, Y.max, Y))
# RUN MODEL USING PARALLEL COMPUTING -----
# Define parallel computing
cl <- makeCluster(floor(detectCores() * 0.75))</pre>
registerDoParallel(cl)
# Run model in parallel
Y <- foreach(i=1:nrow(final.dt),
             .packages = c("dplyr", "data.table")) %dopar%
               model(final.dt[i])
# Stop parallel cluster
stopCluster(cl)
# ADD MODEL OUTPUT -----
model.output <- c("Beta", "N", "Phi", "Delta", "w", "w_i", "Y.max", "Y")</pre>
full.dt <- cbind(final.dt, data.table(do.call(rbind, Y))) %>%
  setnames(., paste("V", 1:length(model.output), sep = ""), model.output)
# Select the A and B matrix only (for uncertainty analysis)
AB.dt <- full.dt[, .SD[1:(n * 2)], Continent]
# EXPORT MODEL OUTPUT -----
fwrite(full.dt, "full.dt.csv")
# EXPORT AB MATRICES -----
fwrite(AB.dt, "AB.dt.csv")
```

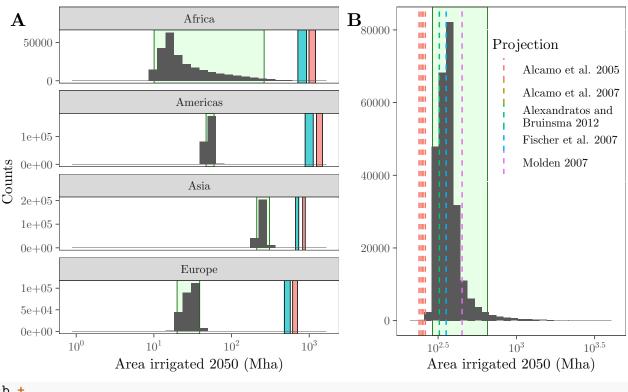
## 8 Uncertainty analysis

```
Continent]
      Continent negative.runs
                                proportion
## 1:
         Africa
                            0 0.000000e+00
## 2: Americas
                            1 3.814697e-06
## 3:
           Asia
                           17 6.484985e-05
## 4:
                         2506 9.559631e-03
         Europe
# Compute quantiles
quant <- AB.dt[Y > 0] %>%
  group_by(Continent) %>%
  do(data.frame(t(quantile(.$Y,
                           probs = c(0.005, 0.01, 0.025, 0.975, 0.99, 0.995, 1),
                           na.rm = TRUE))))
# Print the quantiles
print(quant)
## # A tibble: 4 x 8
## # Groups:
               Continent [4]
##
     Continent X0.5.
                         X1. X2.5. X97.5. X99. X99.5. X100.
     <chr>>
##
                <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
## 1 Africa
                 9.25
                      9.52 10.1
                                   263. 439.
                                                  639.
                                                       1204.
## 2 Americas
                46.7
                       46.8
                              47.0
                                     59.5 71.2
                                                  87.3 1451.
## 3 Asia
                                    307.
                                                  501.
               208.
                      209.
                             211.
                                          385.
                                                         889.
                              19.9
## 4 Europe
                11.4
                       16.2
                                     38.9 51.3
                                                  88.2 704.
# COMPUTE QUANTILES AT THE GLOBAL LEVEL ------
projections <- df$projections %>%
  select(Study, `2050`, Group) %>%
 na.omit()
# Assess uncertainty at the global level
global.uncertainty <- AB.dt %>%
  .[, .(Y, Continent)] %>%
  split(., .$Continent) %>%
  lapply(., function(x) x[, Y]) %>%
  do.call("cbind", .) %>%
  data.table() %>%
  .[, Total:= rowSums(.)]
# Calculate the 2.5 and the 97.5 quantiles
global.quantile <- quantile(global.uncertainty$Total,</pre>
                            probs = c(0.005, 0.01, 0.025, 0.975, 0.99, 0.995, 1),
                            na.rm = TRUE) %>%
  t() %>%
  data.frame()
```

```
# Print quantiles of the global uncertainty
print(global.quantile)
                         X2.5.
                                 X97.5.
                                            X99.
                                                   X99.5.
       X0.5.
                  X1.
                                                             X100.
## 1 285.2141 287.0871 291.0066 652.8831 913.6009 1177.971 4202.648
# PLOT UNCERTAINTY ------
# Continental level
a <- AB.dt %>%
 .[Y > 10^0] \%
 ggplot(., aes(Y)) +
 geom_rect(data = quant,
           aes(xmin = X2.5.,
               xmax = X97.5.,
               ymin = -Inf,
               ymax = Inf,
               group = Continent),
           fill = "green",
           color = "darkgreen",
           alpha = 0.1,
           inherit.aes = FALSE) +
  geom_rect(data = cropland.1,
           aes(xmin = min,
               xmax = max,
               ymin = -Inf,
               ymax = Inf,
               group = Continent,
               fill = Estimation),
           color = "black",
           alpha = 0.7,
           inherit.aes = FALSE) +
  geom_histogram() +
  scale_fill_discrete(name = "Cropland in 2050",
                     labels = c("Gross", "Net")) +
 labs(x = "Area irrigated 2050 (Mha)",
      y = "Counts") +
 facet_wrap(~Continent,
            ncol = 1,
            scales = "free_y") +
  scale_x_log10(labels = trans_format("log10", math_format(10^.x))) +
  scale_y_continuous(breaks = pretty_breaks(n = 2)) +
 theme bw() +
  theme(legend.position = "none",
       panel.grid.major = element_blank(),
       panel.grid.minor = element_blank(),
       legend.background = element_rect(fill = "transparent",
                                        color = NA),
```

```
legend.key = element_rect(fill = "transparent",
                                  color = NA),
        plot.margin = margin(r = -1.5,
                             unit = "cm"))
# Global level
b <- global.uncertainty %>%
  ggplot(., aes(Total)) +
  geom_rect(data = global.quantile,
            aes(xmin = X2.5.,
                xmax = X97.5.,
                ymin = -Inf,
                ymax = Inf),
            fill = "green",
            color = "darkgreen",
            alpha = 0.1,
            inherit.aes = FALSE) +
  geom_histogram() +
  geom_vline(data = projections,
             aes(xintercept = `2050`,
                colour = Study),
             lty = 2,
             size = 1) +
  scale_color_discrete(labels = c("Alcamo et al. 2005",
                                  "Alcamo et al. 2007",
                                  "Alexandratos and \n Bruinsma 2012",
                                  "Fischer et al. 2007",
                                  "Molden 2007")) +
 labs(x = "Area irrigated 2050 (Mha)",
       y = "",
       color = "Projection") +
  scale_x_log10(labels = trans_format("log10", math_format(10^.x)),
                breaks = trans_breaks("log10", function(x) 10^x),
                # Limit the x axis for better visualization
                limits = c(200, 4300)) +
 theme_bw() +
  theme(legend.position = c(0.7, 0.7),
        panel.grid.major = element_blank(),
        plot.margin = margin(l = -1.5,
                             unit = "cm"),
        panel.grid.minor = element_blank(),
        legend.background = element_rect(fill = "transparent",
                                         color = "white"),
        legend.key = element_rect(fill = "transparent",
                                  color = NA))
```

- ## `stat\_bin()` using `bins = 30`. Pick better value with `binwidth`.
- ## Warning in self\$trans\$transform(x): NaNs produced
- ## Warning: Transformation introduced infinite values in continuous x-axis
- ## `stat\_bin()` using `bins = 30`. Pick better value with `binwidth`.
- ## Warning: Removed 289 rows containing non-finite values (stat\_bin).
- ## Warning: Removed 2 rows containing missing values (geom\_bar).
- ## Warning: Graphs cannot be horizontally aligned unless the axis parameter is ## set. Placing graphs unaligned.



```
b +
theme(plot.margin = margin(1 = 0, unit = "cm"))
```

- ## Warning in self\$trans\$transform(x): NaNs produced
- ## Warning: Transformation introduced infinite values in continuous x-axis
- ## `stat\_bin()` using `bins = 30`. Pick better value with `binwidth`.
- ## Warning: Removed 289 rows containing non-finite values (stat\_bin).

```
## Warning: Removed 2 rows containing missing values (geom_bar).
```

```
Projection
80000
           Ш
                                  Alcamo et al. 2005
           Ш
           Ш
                                  Alcamo et al. 2007
           Ш
60000
           Ш
                                  Alexandratos and
           Ш
                                  Bruinsma 2012
           ш
                                  Fischer et al. 2007
           Ш
40000
           Ш
                                  Molden 2007
           Ш
           Ш
           Ш
20000
           Ш
           ш
           100
           ш
                                               10^{3.5}
             10^{2.5}
                               10^{3}
               Area irrigated 2050 (Mha)
```

Africa 0.255203247

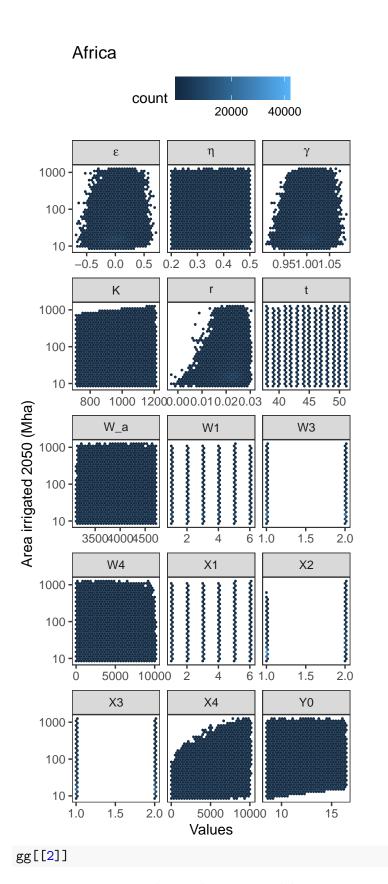
## 1:

```
## N larger.Alcamo.2007 larger.FAO larger.Molden much.larger.Molden
## 1: 262144 99.86649 78.80936 9.84993 2.258301
## twice.Molden less.100
## 1: 1.044846 0.08659363
```

### 9 Sensitivity analysis

#### 9.1 Scatterplots

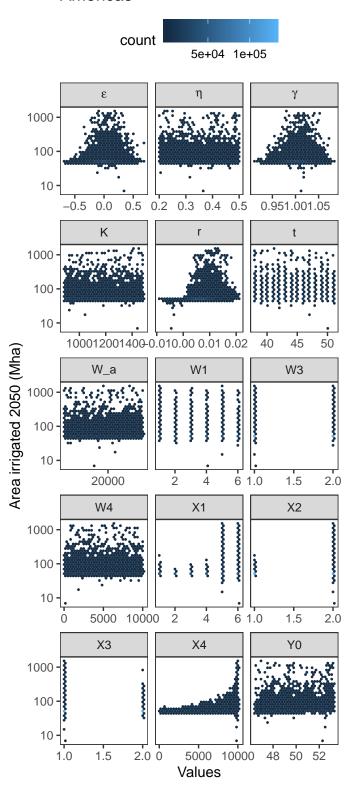
```
# ARRANGE SCATTERPLOTS OF PARAMETERS VS MODEL OUTPUT -----
# Function to recode some parameters to allow plotting
code_columns <- function(x) {</pre>
  dt <- ifelse(x == "Aquastat", 1,
               ifelse(x == "FAOSTAT", 2,
                       ifelse(x == "Siebert.et.al.2013", 3,
                              ifelse(x == "Meier.et.al.2018", 4,
                                      ifelse(x == "Salmon.et.al.2015", 5, 6)))))
 return(dt)
}
code_columns2 <- function(x) ifelse(x == "YES", 1, 2)</pre>
# Vector with renamed parameters for better plotting
parameters.renamed <- c("X1", "X2", "X3", "X4", "W1", "W3", "W4", "r", "$\\gamma$",
                         "YO", "$\\epsilon$", "t", "K", "Wa", "$\\eta$")
# Create temporary data table to plot
tmp <- cbind(AB.dt[, .(Continent)], AB.dt[, ..parameters], AB.dt[, .(Y)])</pre>
# Update columns and arrange
# Update columns to allow plotting of scatterplots
col_names <- c("X1", "W1")</pre>
col_names2 <- c("X3", "W3")</pre>
tmp <- tmp[, (col_names):= lapply(.SD, code_columns), .SDcols = col_names]</pre>
tmp <- tmp[, (col_names2):= lapply(.SD, code_columns2), .SDcols = col_names2]</pre>
tmp <- tmp[, X2:= ifelse(X2 == "OLS", 1, 2)]</pre>
tmp2 <- gather(tmp, Parameters, Values, X1:eta) %>%
 split(., .$Continent)
# PLOT SCATTERPLOTS OF PARAMETERS VS MODEL OUTPUT -----
gg <- list()</pre>
for(i in names(tmp2)) {
  gg[[i]] <- ggplot(tmp2[[i]], aes(Values, Y)) +</pre>
    scale_x_continuous(breaks = pretty_breaks(n = 3)) +
    scale_fill_gradient(breaks = pretty_breaks(n = 2)) +
    scale_y_log10() +
    scale_alpha(guide = "none") +
    labs(x = "Values",
         y = "Area irrigated 2050 (Mha)") +
```



## Warning in self\$trans\$transform(x): NaNs produced

- ## Warning: Transformation introduced infinite values in continuous y-axis
- ## Warning: Removed 15 rows containing non-finite values (stat\_binhex).

## **Americas**

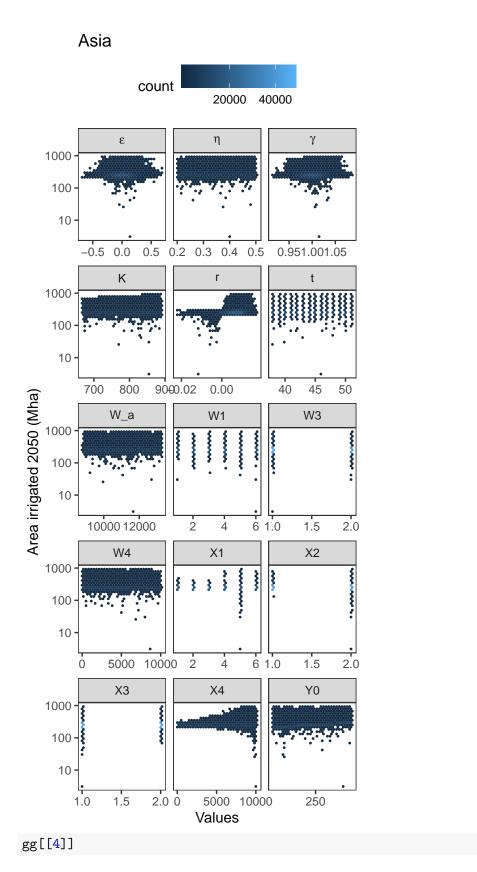


# gg[[3]]

```
## Warning in self$trans$transform(x): NaNs produced
```

## Warning: Transformation introduced infinite values in continuous y-axis

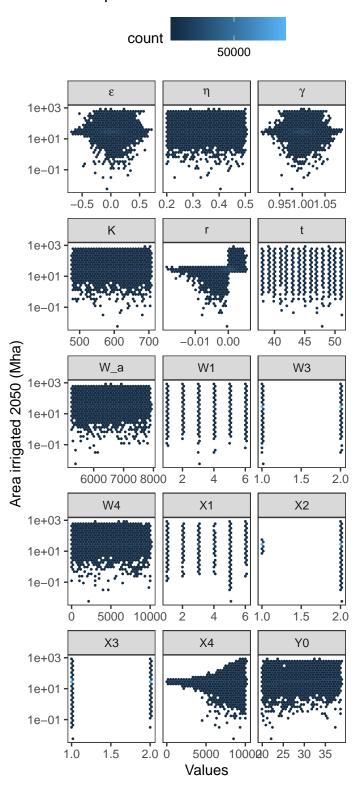
 $\hbox{\tt \#\# Warning: Removed 255 rows containing non-finite values (stat\_binhex).}$ 



## Warning in self\$trans\$transform(x): NaNs produced

- ## Warning: Transformation introduced infinite values in continuous y-axis
- ## Warning: Removed 37590 rows containing non-finite values (stat\_binhex).

## Europe



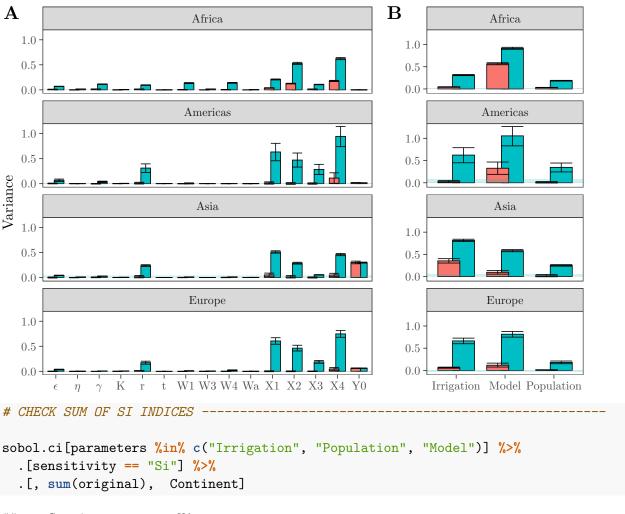
#### 9.2 Sobol' indices

```
# SETTING FOR SOBOL' INDICES
# Set the number of bootstraps
R <- 1000
# Set the confidence interval method
type <- "norm"
# Set the confidence interval
conf < -0.95
# Create vector with the name of the clusters
cluster <- c("Irrigation", "Population", "Model")</pre>
# COMPUTE SOBOL' INDICES -----
# Compute Sobol' indices
out <- full.dt[, sobol_indices(Y,</pre>
                                 params = c(parameters.renamed, cluster),
                                 type = "saltelli",
                                 R = R,
                                 n = n,
                                 parallel = "multicore",
                                 ncpus = floor(detectCores() * 0.75)),
                by = Continent]
# SOBOL' CONFIDENCE INTERVALS -----
# Compute confidence intervals
tmp <- split(out, out$Continent)</pre>
out.ci <- list()</pre>
for(i in names(tmp)) {
  out.ci[[i]] <- sobol_ci(tmp[[i]],</pre>
                           params = c(parameters.renamed, cluster),
                           type = type,
                           conf = conf)
}
# SOBOL' INDICES OF A DUMMY PARAMETER -----
# For the model parameters
out.dummy <- full.dt[, .SD[1:(n * (k + 2))], Continent] %>%
  .[, sobol_dummy(Y,
                  params = parameters.renamed,
                  R = R,
                  n = n,
```

```
parallel = "multicore",
                   ncpus = floor(detectCores() * 0.5)),
    by = Continent]
# Compute confidence intervals
tmp.dummy <- split(out.dummy, out.dummy$Continent)</pre>
out.dummy.ci <- list()</pre>
for(i in names(tmp.dummy)) {
  out.dummy.ci[[i]] <- sobol_ci_dummy(tmp.dummy[[i]],</pre>
                                        type = type,
                                        conf = conf)
}
out.dummy.ci2 <- rbindlist(out.dummy.ci, idcol = "Continent")</pre>
# For the clusters of parameters
tmp1 <- full.dt[, .SD[1:(2 * n)], Continent] %>%
  split(., .$Continent)
tmp2 <- full.dt[full.dt[, tail(.I, length(cluster) * n), by = Continent]$V1, ] %>%
  split(., .$Continent)
for(i in names(tmp1)) {
  tmp1[[i]] <- rbind(tmp1[[i]], tmp2[[i]])</pre>
}
out.dummy.cluster <- rbindlist(tmp1) %>%
  .[, sobol_dummy(Y,
                   params = cluster,
                   R = R
                   n = n,
                   parallel = "multicore",
                   ncpus = floor(detectCores() * 0.5)),
    by = Continent]
# Compute confidence intervals
tmp.dummy <- split(out.dummy.cluster, out.dummy.cluster$Continent)</pre>
out.dummy.cluster.ci <- list()</pre>
for(i in names(tmp.dummy)) {
  out.dummy.cluster.ci[[i]] <- sobol_ci_dummy(tmp.dummy[[i]],</pre>
                                                 type = type,
                                                 conf = conf)
}
out.dummy.cluster.ci2 <- rbindlist(out.dummy.cluster.ci, idcol = "Continent")</pre>
```

```
# EXPORT SOBOL' INDICES -----
sobol.ci <- rbindlist(out.ci, idcol = "Continent")</pre>
fwrite(sobol.ci, "sobol.ci.csv")
# PREPARE PLOT SOBOL' INDICES -----
# Plot Sobol' indices of parameters
a <- rbindlist(out.ci, idcol = "Continent") %>%
  .[!parameters %in% cluster] %>%
 plot_sobol(., type = 1, dummy = out.dummy.ci2) +
  scale_y_continuous(breaks = pretty_breaks(n = 3)) +
 facet_wrap(~Continent, ncol = 1)
# Plot Sobol' indices of clusters of parameters
b <- rbindlist(out.ci, idcol = "Continent") %>%
  .[parameters %in% cluster] %>%
 plot_sobol(., type = 1, dummy = out.dummy.cluster.ci2) +
 facet_wrap(~Continent, ncol = 1) +
 labs(x = "",
       y = "")
# Merge both plots
bottom <- plot_grid(a + theme(legend.position="none"),</pre>
                    b + theme(legend.position="none"),
                    ncol = 2,
                    labels = "AUTO",
                    rel_widths = c(2, 1.1)
# Extract legend
legend <- get_legend(a + theme(legend.direction = "horizontal",</pre>
                               legend.justification="center" ,
                               legend.box.just = "top"))
# PLOT SOBOL' INDICES -----
# Merge legend and a and b
plot_grid(legend,
          bottom,
          ncol = 1,
          rel_heights = c(0.15, 1))
```





## Continent V1
## 1: Africa 0.6413969
## 2: Americas 0.3683183
## 3: Asia 0.4714599
## 4: Europe 0.1896407

#### 10 Session information

```
# SESSION INFORMATION ----
sessionInfo()

## R version 3.6.1 (2019-07-05)

## Platform: x86_64-apple-darwin15.6.0 (64-bit)

## Running under: macOS Mojave 10.14.6

##
## Matrix products: default
```

```
/Library/Frameworks/R.framework/Versions/3.6/Resources/lib/libRblas.0.dylib
## BLAS:
## LAPACK: /Library/Frameworks/R.framework/Versions/3.6/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
## attached base packages:
## [1] grid
                 parallel
                           stats
                                      graphics grDevices utils
                                                                     datasets
## [8] methods
                 base
##
## other attached packages:
   [1] checkpoint_0.4.7
                             NbClust_3.0
                                                 gridExtra_2.3
   [4] sensobol_0.2.1
##
                             wesanderson_0.3.6
                                                 sensitivity_1.16.1
## [7] doParallel_1.0.15
                             iterators_1.0.12
                                                 foreach_1.4.7
## [10] boot_1.3-23
                             smatr_3.4-8
                                                 robustbase_0.93-5
## [13] randtoolbox_1.30.0
                            rngWELL_0.10-5
                                                 complmrob_0.7.0
## [16] mvoutlier_2.0.9
                                                 cowplot_1.0.0
                             sgeostat_1.0-27
                             stringr_1.4.0
## [19] forcats_0.4.0
                                                 dplyr_0.8.3
## [22] purrr_0.3.2
                             readr_1.3.1
                                                 tidyr_1.0.0
                             ggplot2 3.2.1
## [25] tibble 2.1.3
                                                 tidyverse 1.2.1
## [28] scales_1.0.0
                             countrycode_1.1.0
                                                 readxl_1.3.1
## [31] fGarch_3042.83.1
                             fBasics_3042.89
                                                 timeSeries_3042.102
## [34] timeDate_3043.102
                             fitdistrplus_1.0-14 npsurv_0.4-0
## [37] lsei_1.2-0
                             survival_2.44-1.1
                                                 MASS_7.3-51.4
## [40] data.table_1.12.2
##
## loaded via a namespace (and not attached):
   [1] backports_1.1.4
                               plyr_1.8.4
                                                     lazyeval_0.2.2
## [4] sp_1.3-1
                               splines_3.6.1
                                                     digest_0.6.21
   [7] htmltools_0.3.6
                               magrittr_1.5
                                                     cluster_2.1.0
## [10] openxlsx_4.1.0.1
                               modelr_0.1.5
                                                     tikzDevice_0.12.3
## [13] colorspace_1.4-1
                               rvest_0.3.4
                                                     rrcov_1.4-7
## [16] haven_2.1.1
                               xfun_0.9
                                                     crayon_1.3.4
## [19] jsonlite_1.6
                               zeallot_0.1.0
                                                     zoo_1.8-6
## [22] glue 1.3.1
                               gtable_0.3.0
                                                     car_3.0-3
## [25] kernlab_0.9-27
                               prabclus_2.3-1
                                                     DEoptimR_1.0-8
## [28] abind_1.4-5
                               VIM_4.8.0
                                                     mvtnorm_1.0-11
## [31] GGally_1.4.0
                               bibtex_0.4.2
                                                     Rcpp_1.0.2
## [34] sROC_0.1-2
                               laeken_0.5.0
                                                     foreign_0.8-72
## [37] mclust_5.4.5
                               stats4_3.6.1
                                                     vcd_{1.4-4}
## [40] truncnorm_1.0-8
                               httr_1.4.1
                                                     RColorBrewer_1.1-2
## [43] fpc_2.2-3
                                                      spatial_7.3-11
                               modeltools_0.2-22
## [46] pkgconfig_2.0.3
                               reshape_0.8.8
                                                     NADA_1.6-1
## [49] flexmix_2.3-15
                               nnet_7.3-12
                                                     labeling_0.3
## [52] tidyselect_0.2.5
                               rlang_0.4.0
                                                     munsell_0.5.0
## [55] cellranger_1.1.0
                               tools_3.6.1
                                                     cli_1.1.0
## [58] generics_0.0.2
                               ranger_0.11.2
                                                     pls_2.7-1
## [61] broom_0.5.2
                               evaluate_0.14
                                                     cvTools_0.3.2
```

```
## [64] yaml_2.2.0
                               knitr_1.25
                                                     filehash_2.4-2
  [67] zip_2.0.4
                               nlme_3.1-141
                                                     xml2_1.2.2
## [70] compiler_3.6.1
                               rstudioapi_0.10
                                                     curl_4.1
## [73] e1071_1.7-2
                               zCompositions_1.3.2-1 robCompositions_2.1.0
## [76] pcaPP 1.9-73
                               stringi 1.4.3
                                                     lattice 0.20-38
  [79] Matrix 1.2-17
                               vctrs 0.2.0
                                                     pillar_1.4.2
  [82] lifecycle 0.1.0
                               Rdpack 0.11-0
                                                     1mtest 0.9-37
## [85] gbRd_0.4-11
                               R6_2.4.0
                                                     rio_0.5.16
## [88] codetools 0.2-16
                               assertthat 0.2.1
                                                     withr_2.1.2
## [91] diptest_0.75-7
                               hms_0.5.1
                                                     class_7.3-15
## [94] rmarkdown_1.15
                               carData_3.0-2
                                                     lubridate_1.7.4
## [97] tinytex_0.16
```

FAO. 2016. "AQUASTAT website." Food; Agriculture Organization of the United Nations. http://www.fao.org/nr/water/aquastat/didyouknow/index3.stm.

——. 2017. "FAOSTAT database." Rome. http://www.fao.org/faostat/en/.

Meier, J., F. Zabel, and W. Mauser. 2018. "A global approach to estimate irrigated areas – a comparison between different data and statistics." *Hydrology and Earth System Sciences* 22 (2): 1119–33. https://doi.org/10.5194/hess-22-1119-2018.

Salmon, J.M., M. A. Friedl, S. Frolking, D. Wisser, and E. M. Douglas. 2015. "Global rainfed, irrigated, and paddy croplands: A new high resolution map derived from remote sensing, crop inventories and climate data." *International Journal of Applied Earth Observation and Geoinformation* 38. Elsevier B.V.: 321–34. https://doi.org/10.1016/j.jag.2015.01.014.

Siebert, S., V. Henrich, K. Frenken, and J. Burke. 2013. "Update of the digital global map of irrigation areas to version 5." Rome: Rheinische Friedrich-Wilhelms-University; Food; Agriculture Organization of the United Nations.

Thenkabail, P. S., C. M. Biradar, P. Noojipady, V. Dheeravath, Y. Li, M. Velpuri, M. Gumma, et al. 2009. "Global irrigated area map (GIAM), derived from remote sensing, for the end of the last millennium." *International Journal of Remote Sensing* 30 (14): 3679–3733. https://doi.org/10.1080/01431160802698919.