Topics in Structural Operational Semantics

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Overview

Practical

Abstract

0. Introduction to SOS
1. Using SOS to design semantics of authorization framework
2. Decompositional reasoning for a process calculus with history
3. SOS rule formats for determinism and idempotency

Structural Operational Semantics

- Syntax of languages is specified e.g. with EBNF or similar
- Tells us that

  ```
  print(“Hello”); print(“World!”)
  ```

  is a legal program.
- ...but not what it *does*.

Structural Operational Semantics

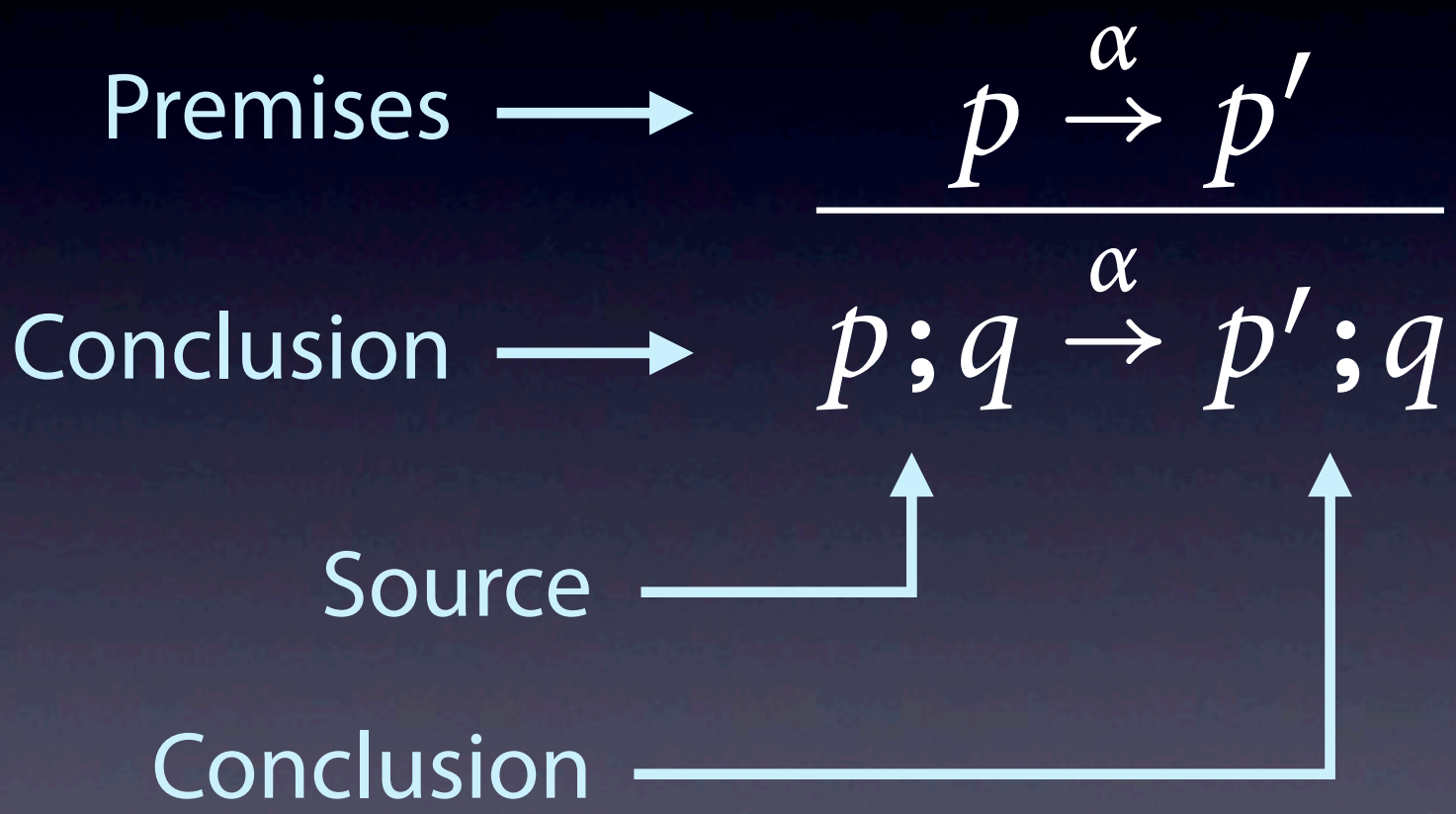
- SOS specifies the semantics of a language with a collection of rules
- Rules specify the operations of programs,

$$\texttt{print}(s) \xrightarrow{s} \varepsilon$$

- ... in terms of their structure

$$\frac{p \xrightarrow{\alpha} p'}{p;q \xrightarrow{\alpha} p';q}$$

Anatomy of an SOS rule



Example

```
print("Hi"); print("Mom!")
```

$$\texttt{print}(s) \xrightarrow{s} \varepsilon$$

$$\frac{p \xrightarrow{\alpha} p' \quad p' \neq \varepsilon}{p;q \xrightarrow{\alpha} p';q}$$

$$\frac{p \xrightarrow{\alpha} \varepsilon}{p;q \xrightarrow{\alpha} q}$$

Example

```
print(“Hi”); print(“Mom!”)
```

$$\texttt{print}(s) \xrightarrow{s} \varepsilon$$

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print("Hi"); print("Mom!")
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print("Hi"); print("Mom!")
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```
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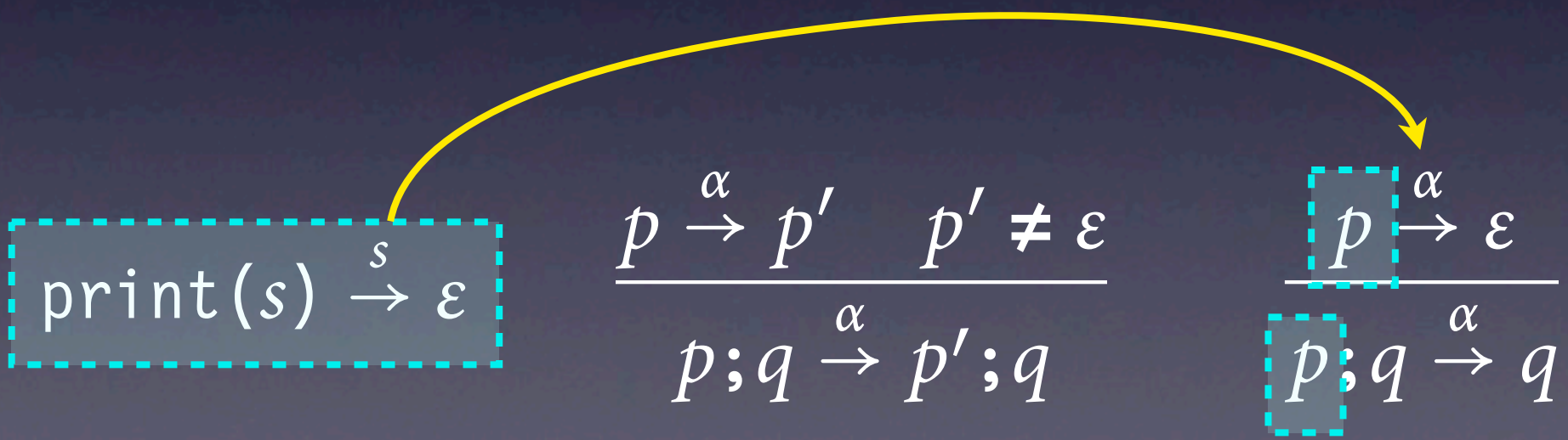
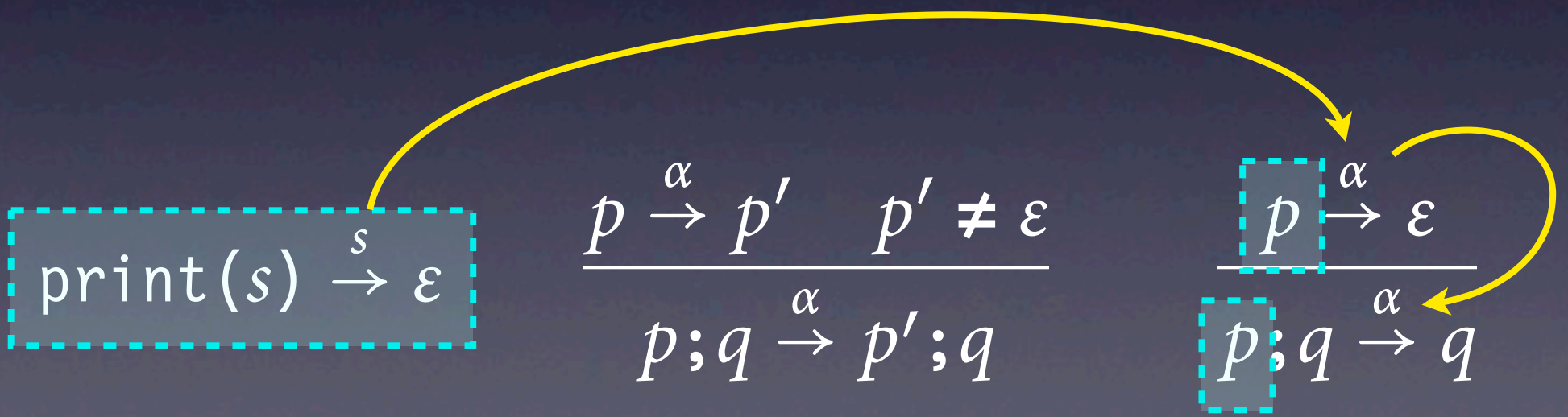


Example

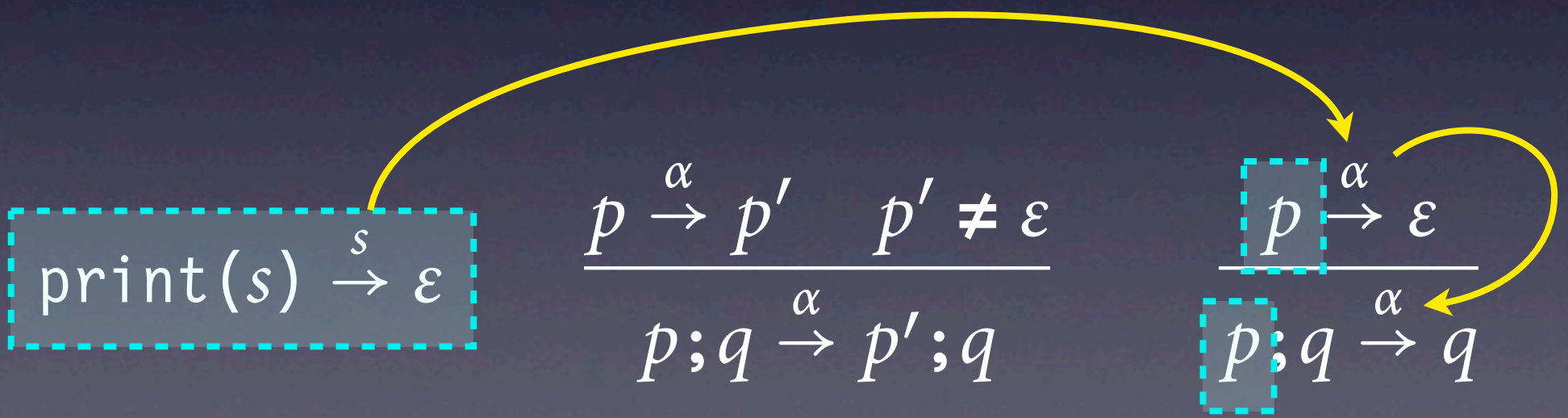
```
print("Hi"); print("Mom!")
```



Example

```
print("Hi"); print("Mom!")
```

$$\xrightarrow{\text{"Hi"}} \texttt{print("Mom!");}$$



Example

```
print("Hi"); print("Mom!")
```

$$\xrightarrow{\text{"Hi"}} \texttt{print("Mom!");}$$

$$\texttt{print}(s) \xrightarrow{s} \varepsilon$$

$$\frac{p \xrightarrow{\alpha} p' \quad p' \neq \varepsilon}{p;q \xrightarrow{\alpha} p';q}$$

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Example

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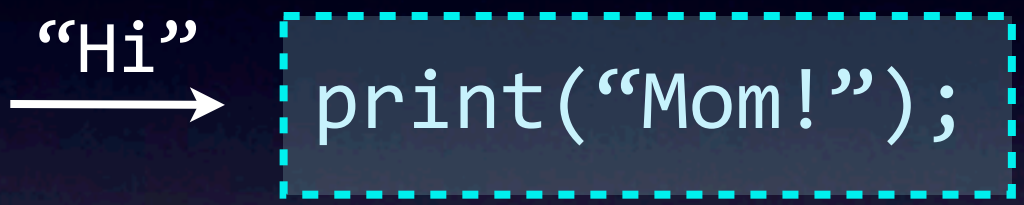

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```

$$\xrightarrow{\text{"Hi"}} \texttt{print("Mom!");}$$

$$\xrightarrow{\text{"Mom!"}} \varepsilon$$

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$$\frac{p \xrightarrow{\alpha} p' \quad p' \neq \varepsilon}{p;q \xrightarrow{\alpha} p';q}$$

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Uses of SOS

- Rigourous specification of semantics (Part I)
- Reasoning about program behaviour (Part II)
- Proving properties of languages (Part III)
- Proving two programs are equivalent, *e.g.*
  `parallel(f,g)` *behaves like* `parallel(g,f)`
- .... and more

Part I

Transactional Memory Introspection

Joint work with Úlfar Erlingsson

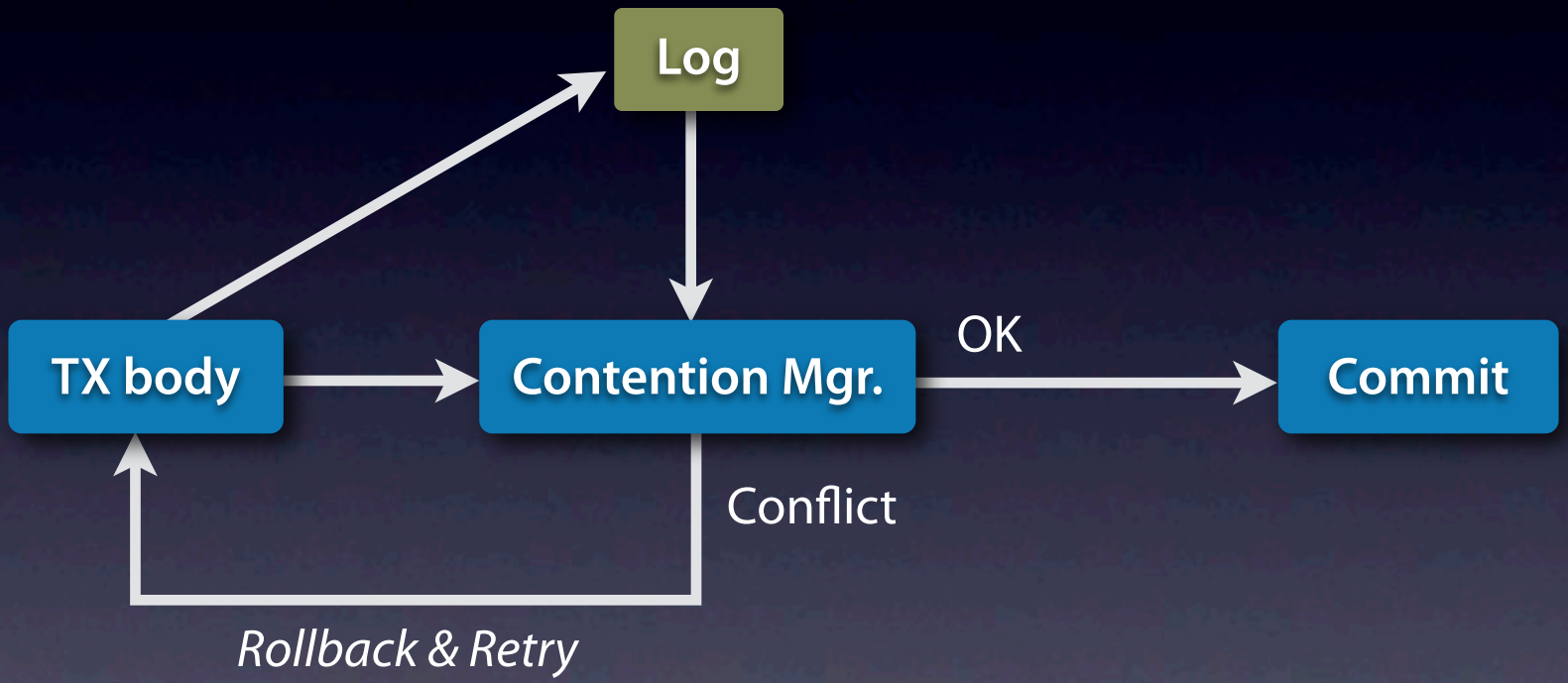
Reykjavik University

(published in PLAS'09 in June '09)

Software Transactional Memory

- STM provides synchronization for threads
- “Atomic” blocks of code are executed in parallel, optimistically but isolated
- The STM system monitors execution to detect conflicting memory accesses
- In case of conflict, an atomic block is rolled back and retried at a later time

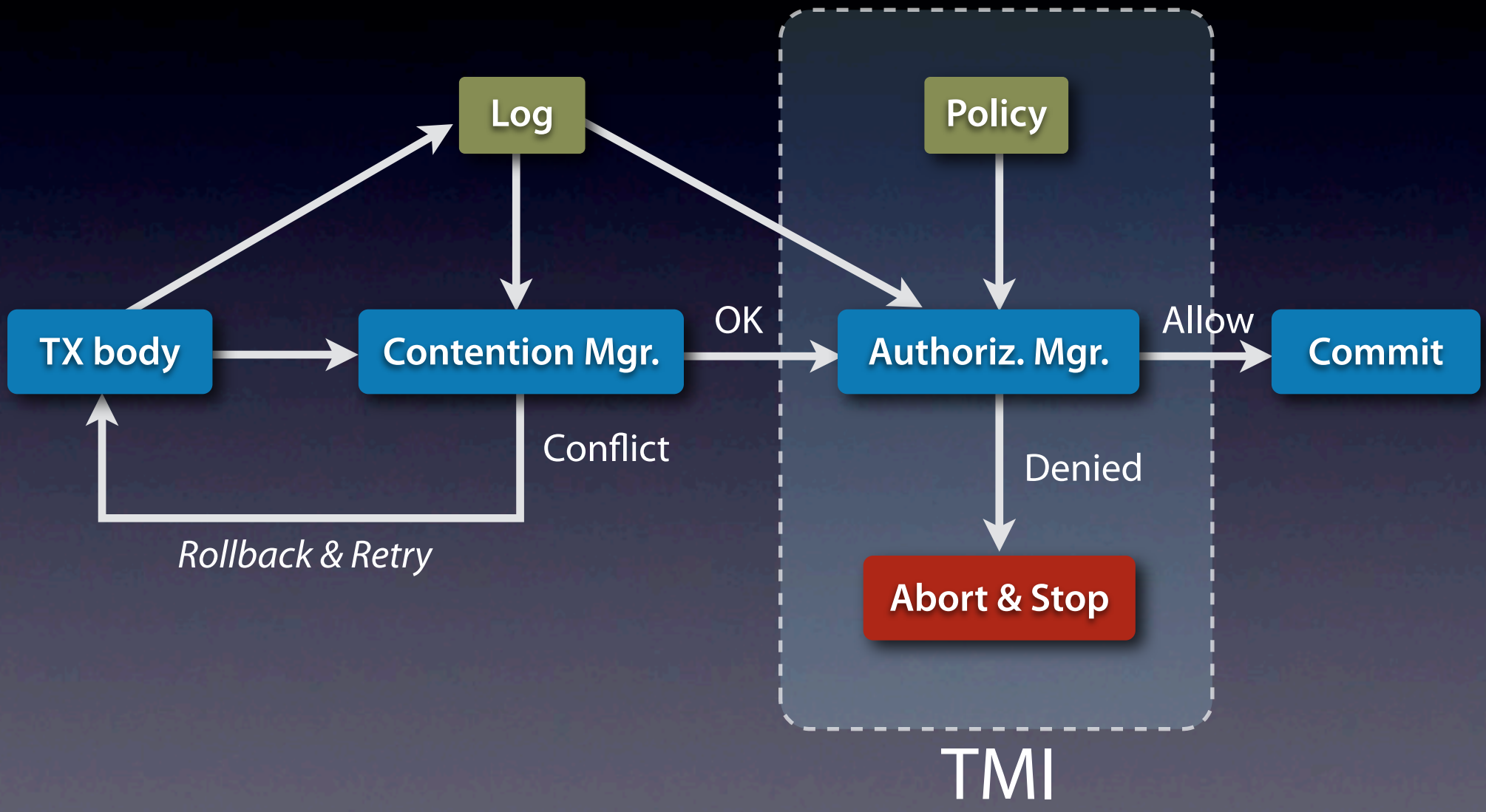
Software Transactional Memory



Transactional Memory Introspection

- A policy enforcement architecture
- Uses the existing mechanisms of STM
- Code, possibly violating policy, lives inside isolated transactions
- STM monitoring used to detect policy violations as well as race conditions
- Transaction containing violation is rolled back, but not retried

Where TMI fits in with STM



Benefits of TMI

- Prevents time of check to time of use bugs
- Ensures complete mediation
- Easier error handling for the programmer
- Decouples policy enforcement code from application code → easier to verify policy enforcement

STM Haskell

```
inc :: TVar Int -> STM ()
inc var = do value <- readTVar var
             writeTVar var (value + 1)

main :: IO ()
main = do x <- newTVarIO 0
          forkIO (atomically (inc x))
          forkIO (atomically (inc x))
          forkIO (atomically (inc x))
```

Value of x is guaranteed to be 3

STM Haskell Semantics

- Main execution context (IO) and STM context handled in separate transition systems
- Creating/reading/writing TVars only possible in the STM transition system
- The "atomically" operator lifts a sequence of STM operations to a single IO operation

$$\frac{M; \Theta \rightarrow^{*}_{STM} \texttt{return}\, N; \Theta'}{\texttt{atomically}\, M; \Theta \rightarrow_{IO} \texttt{return}\, N; \Theta'}$$

TMI Haskell

```
data Label = High | Low

copy :: TMIVar Label t -> TMIVar Label t -> TMI ()
copy x y = do tmp <- readTMIVar x  ;; logged
              writeTMIVar y tmp    ;; logged

policy :: TMILog Label -> Bool
policy log = <(read, High) never precedes
              (write, Low) in log>

main :: IO ()
main = do h <- newTMIVarIO High 10
          l <- newTMIVarIO Low  0
          atomically $ do
             auth policy (copy l h)  ;; OK
             auth policy (copy h l)  ;; fail, TX aborts!
```

TMI semantics

- Yet another context, for TMI protected code
- Creating/reading/writing security sensitive variables only allowed in the TMI context
- “auth” operator lifts TMI operations to STM operations, but clears it with the policy first
- TMI context maintains log of operations, this is the input to the policy decision term

TMI semantics (simplified)

$$\frac{M; \Theta, [] \rightarrow^{*}_{TMI} \texttt{return}\, N; \Theta', \Sigma \qquad \Sigma \vdash P \Rightarrow \mathrm{OK}}{\texttt{auth}\, P\, M; \Theta \rightarrow_{STM} \texttt{return}\, N; \Theta'}$$

$$\frac{M; \Theta, [] \rightarrow^{*}_{TMI} \texttt{return}\, N; \Theta', \Sigma \qquad \Sigma \vdash P \Rightarrow \mathrm{FAIL}}{\texttt{auth}\, P\, M; \Theta \rightarrow_{STM} \texttt{raise AuthError}; \Theta}$$

...not simplified

I/O transitions $P; \Theta \xrightarrow{a} Q; \Theta'$

$$\mathsf{P}[\texttt{putChar}\ c]; \Theta \xrightarrow{!c} \mathsf{P}[\texttt{return}\ ()]; \Theta \quad \textit{(PUTC)}$$
$$\mathsf{P}[\texttt{getChar}\ ]; \Theta \xrightarrow{?c} \mathsf{P}[\texttt{return}\ c]; \Theta \quad \textit{(GETC)}$$
$$\mathsf{P}[\texttt{forkIO}\ M]; \Theta \to (\mathsf{P}[\texttt{return}\ t] \mid M_t); \Theta \quad t \notin \mathsf{P}, \Theta, M \quad \textit{(FORK)}$$
$$\mathsf{P}[\texttt{catch}\ (\texttt{return}\ M)\ N]; \Theta \to \mathsf{P}[\texttt{return}\ M]; \Theta \quad \textit{(CATCH1)}$$
$$\mathsf{P}[\texttt{catch}\ (\texttt{throw}\ M)\ N]; \Theta \to \mathsf{P}[N\ M]; \Theta \quad \textit{(CATCH2)}$$

$$\frac{M \to N}{\mathsf{P}[M]; \Theta \to \mathsf{P}[N]; \Theta} \quad \textit{(ADMIN)}$$

$$\frac{M; \Theta, \{\} \overset{*}{\Rightarrow} \texttt{return}\ N; \Theta', \Delta'}{\mathsf{P}[\texttt{atomic}\ M]; \Theta \to \mathsf{P}[\texttt{return}\ N]; \Theta'} \quad \textit{(ARET1)} \qquad \frac{M; \Theta, \{\} \overset{*}{\Rightarrow} \texttt{throw}\ N; \Theta', \Delta'}{\mathsf{P}[\texttt{atomic}\ M]; \Theta \to \mathsf{P}[\texttt{throw}\ N]; \Theta \cup \Delta'} \quad \textit{(ATHROW1)}$$

Administrative transitions $M \to N$

$$M \to V \quad \text{if } \mathcal{V}[\![M]\!] = V \text{ and } M \notin V \quad \textit{(EVAL)}$$
$$\texttt{return}\ N \gg\!= M \to M\ N \quad \textit{(BIND)}$$
$$\texttt{throw}\ N \gg\!= M \to \texttt{throw}\ N \quad \textit{(THROW)}$$
$$\texttt{retry} \gg\!= M \to \texttt{retry} \quad \textit{(RETRY)}$$

STM transitions $M; \Theta, \Delta, \Sigma \Rightarrow N; \Theta', \Delta', \Sigma'$

$$\mathsf{S}[\texttt{readTVar}\ r]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{return}\ \Theta_1(r)]; \Theta, \Delta, \Sigma \quad \text{if } r \in \mathrm{dom}(\Theta) \text{ and } \Theta_2(s) = \bot \quad \textit{(READ)}$$
$$\mathsf{S}[\texttt{writeTVar}\ r\ M]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{return}\ ()]; \Theta[r \mapsto (M, \bot)], \Delta, \Sigma \quad \text{if } r \in \mathrm{dom}(\Theta) \text{ and } \Theta_2(s) = \bot \quad \textit{(WRITE)}$$
$$\mathsf{S}[\texttt{newTVar}\ M]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{return}\ r]; \Theta[r \mapsto (M, \bot)], \Delta[r \mapsto (M, \bot)], \Sigma \quad r \notin \mathrm{dom}(\Theta) \quad \textit{(NEW)}$$

$$\frac{M \to N}{\mathsf{S}[M]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[N]; \Theta, \Delta, \Sigma} \quad \textit{(AADMIN)}$$

$$\frac{M_1; \Theta, \Delta, \Sigma \overset{*}{\Rightarrow} \texttt{return}\ N; \Theta', \Delta', \Sigma'}{\mathsf{S}[M_1\ \texttt{`orElse`}\ M_2]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{return}\ N]; \Theta', \Delta', \Sigma'} \quad \textit{(OR1)} \qquad \frac{M_1; \Theta, \Delta, \Sigma \overset{*}{\Rightarrow} \texttt{throw}\ N; \Theta', \Delta', \Sigma'}{\mathsf{S}[M_1\ \texttt{`orElse`}\ M_2]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{throw}\ N]; \Theta', \Delta', \Sigma'} \quad \textit{(OR2)}$$

$$\frac{M_1; \Theta, \Delta, \Sigma \overset{*}{\Rightarrow} \texttt{retry}; \Theta', \Delta', \Sigma'}{\mathsf{S}[M_1\ \texttt{`orElse`}\ M_2]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[M_2]; \Theta, \Delta, \Sigma} \quad \textit{(OR3)} \qquad \frac{M; \Theta, \{\}, [] \overset{*}{\Rightarrow} \texttt{return}\ M'; \Theta', \Delta', \Sigma'}{\mathsf{S}[\texttt{catch}\ M\ N]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{return}\ M']; \Theta', \Delta \cup \Delta', \Sigma \oplus \Sigma'} \quad \textit{(XSTM1)}$$

$$\frac{M; \Theta, \{\}, [] \overset{*}{\Rightarrow} \texttt{throw}\ M'; \Theta', \Delta', \Sigma'}{\mathsf{S}[\texttt{catch}\ M\ N]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[N\ M']; \Theta \cup \Delta', \Delta \cup \Delta', \Sigma \oplus (\Sigma'|_{\Delta'})} \quad \textit{(XSTM2)} \qquad \frac{M; \Theta, \{\}, [] \overset{*}{\Rightarrow} \texttt{retry}; \Theta', \Delta', \Sigma'}{\mathsf{S}[\texttt{catch}\ M\ N]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{retry}]; \Theta, \Delta, \Sigma} \quad \textit{(XSTM3)}$$

$$\frac{M; \Theta, \{\}, [] \overset{*}{\Rightarrow} \texttt{return}\ M'; \Theta', \Delta', \Sigma' \qquad \Sigma' \vdash N \overset{*}{\rightsquigarrow} \texttt{return}\ N'}{\mathsf{S}[\texttt{authorized}\ N\ M]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{return}\ M']; \Theta', \Delta', \Sigma} \quad \textit{(AURET1)}$$

$$\frac{M; \Theta, \{\}, [] \overset{*}{\Rightarrow} \texttt{throw}\ M'; \Theta', \Delta', \Sigma' \qquad \Sigma' \vdash N \overset{*}{\rightsquigarrow} \texttt{return}\ N'}{\mathsf{S}[\texttt{authorized}\ N\ M]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{throw}\ M']; \Theta', \Delta', \Sigma} \quad \textit{(AUTHROW1)}$$

$$\frac{M; \Theta, \{\}, [] \overset{*}{\Rightarrow} \texttt{return}\ M'; \Theta', \Delta', \Sigma' \qquad \Sigma' \vdash N \overset{*}{\rightsquigarrow} \texttt{throw}\ N'}{\mathsf{S}[\texttt{authorized}\ N\ M]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{throw UnathorizedError}]; \Theta', \Delta', \Sigma} \quad \textit{(AURET2)}$$

$$\frac{M; \Theta, \{\}, [] \overset{*}{\Rightarrow} \texttt{throw}\ M'; \Theta', \Delta', \Sigma' \qquad \Sigma' \vdash N \overset{*}{\rightsquigarrow} \texttt{throw}\ N'}{\mathsf{S}[\texttt{authorized}\ N\ M]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{throw UnauthorizedError}]; \Theta', \Delta', \Sigma} \quad \textit{(AUTHROW1)}$$

$$\frac{M; \Theta, \{\}, [] \overset{*}{\Rightarrow} \texttt{retry}; \Theta', \Delta', \Sigma'}{\mathsf{S}[\texttt{authorized}\ N\ M]; \Theta, \Delta, \Sigma \Rightarrow \mathsf{S}[\texttt{retry}]; \Theta', \Delta' \Sigma} \quad \textit{(AURETRY)}$$

Figure 2: Operational semantics (part 1)

TMI transitions $M; \Theta, \Delta, \Sigma \rightharpoonup N; \Theta', \Delta', \Sigma'$

$$\mathbb{S}[\texttt{readTMIVar}\ r]; \Theta, \Delta, \Sigma \rightharpoonup \mathbb{S}[\texttt{return}\ \Theta_1(r)]; \Theta, \Delta, \Sigma \oplus [(\textsc{read}, \Theta_2(r))] \quad \text{if } r \in \mathrm{dom}(\Theta) \text{ and } \Theta_2(r) \neq \bot \quad \textit{(TMIREAD)}$$
$$\mathbb{S}[\texttt{writeTMIVar}\ r\ M]; \Theta, \Delta, \Sigma \rightharpoonup \mathbb{S}[\texttt{return}\ ()]; \Theta[r \mapsto (M, \Theta_2(r))], \Delta, \Sigma \oplus [(\textsc{write}, \Theta_2(r))] \quad \text{if } r \in \mathrm{dom}(\Theta) \text{ and } \Theta_2(r) \neq \bot \quad \textit{(TMIWRITE)}$$
$$\mathbb{S}[\texttt{newTMIVar}\ N\ M]; \Theta, \Delta, \Sigma \rightharpoonup \mathbb{S}[\texttt{return}\ r]; \Theta[r \mapsto (M, N)], \Delta[r \mapsto (M, N)], \Sigma \oplus [(\textsc{create}, N)] \quad r \notin \mathrm{dom}(\Theta) \quad \textit{(TMINEW)}$$

$$\frac{M \to N}{\mathbb{S}[M]; \Theta, \Delta, \Sigma \rightharpoonup \mathbb{S}[N]; \Theta, \Delta, \Sigma} \quad \textit{(TADMIN)} \qquad \frac{M; \Theta, \Delta, \Sigma \overset{*}{\Rightarrow} N; \Theta', \Delta', \Sigma'}{\mathbb{S}[\texttt{liftSTM}\ M]; \Theta, \Delta, \Sigma \rightharpoonup \mathbb{S}[N]; \Theta', \Delta', \Sigma'} \quad \textit{(LIFTSTM)}$$

$$\frac{M_1; \Theta, \Delta, \Sigma \overset{*}{\rightharpoonup} \texttt{return}\ N; \Theta', \Delta', \Sigma'}{\mathbb{S}[M_1\ \texttt{`orElse`}\ M_2]; \Theta, \Delta, \Sigma \rightharpoonup \mathbb{S}[\texttt{return}\ N]; \Theta', \Delta', \Sigma'} \quad \textit{(TOR1)} \qquad \frac{M_1; \Theta, \Delta, \Sigma \overset{*}{\rightharpoonup} \texttt{throw}\ N; \Theta', \Delta', \Sigma'}{\mathbb{S}[M_1\ \texttt{`orElse`}\ M_2]; \Theta, \Delta, \Sigma \rightharpoonup \mathbb{S}[\texttt{throw}\ N]; \Theta', \Delta', \Sigma'} \quad \textit{(TOR2)}$$

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$$\frac{M; \Theta, \{\}, [] \overset{*}{\rightharpoonup} \texttt{throw}\ M'; \Theta', \Delta', \Sigma'}{\mathbb{S}[\texttt{catch}\ M\ N]; \Theta, \Delta, \Sigma \rightharpoonup \mathbb{S}[N\ M']; \Theta \cup \Delta', \Delta \cup \Delta', \Sigma \oplus (\Sigma'|_{\Delta'})} \quad \textit{(XTMI2)} \qquad \frac{M; \Theta, \{\}, [] \overset{*}{\rightharpoonup} \texttt{retry}; \Theta', \Delta', \Sigma'}{\mathbb{S}[\texttt{catch}\ M\ N]; \Theta, \Delta, \Sigma \rightharpoonup \mathbb{S}[\texttt{retry}]; \Theta, \Delta, \Sigma} \quad \textit{(XTMI3)}$$

Authorization transitions $\Sigma \vdash M \rightsquigarrow N$

$$\frac{M \to N}{\Sigma \vdash \mathbb{E}[M] \rightsquigarrow \mathbb{E}[N]} \quad \textit{(AUADMIN)} \qquad \Sigma \vdash \mathbb{E}[\texttt{getlog}] \rightsquigarrow \mathbb{E}[\texttt{return}\ hs(\Sigma)] \quad \textit{(GETLOG)}$$

$$\Sigma \vdash \mathbb{E}[\texttt{catch}\ (\texttt{return}\ M)\ N] \rightsquigarrow \mathbb{E}[\texttt{return}\ M] \quad \textit{(ACATCH1)}$$
$$\Sigma \vdash \mathbb{E}[\texttt{catch}\ (\texttt{throw}\ M)\ N] \rightsquigarrow \mathbb{E}[N\ M] \quad \textit{(ACATCH2)}$$

Figure 3: Operational semantics (part 2)

Gain of using SOS

- We had already defined TMI, implemented it in Java and published the results
- Despite this, it took several attempts to design acceptable semantics with SOS
- Revealed ill-understood edge cases
- Basis for more formal future work

Part II

Decompositional reasoning about the past

Joint, ongoing work with Luca Aceto,
Anna Ingólfsdóttir and MohammadReza Mousavi

Reykjavik University &
TU/e - Eindhoven Technical University

Decompositional reasoning

- Model checking has to deal with state space explosion, esp. for parallel processes
- Transforming global properties into local properties can potentially help
- Check local properties on the components instead
- Dates back to early ‘90s (Larsen and Xinxin)

Example

CM = <unknown specification>

CS = $\overline{coin}.coffe.\overline{pub}.CS$

Uni = (CM | CS) \ coin, coffee

We want to verify that Uni satisfies $\langle\overline{pub}\rangle tt$

By looking at CS, we see this holds if and only if

CM satisfies $\langle\overline{pub}\rangle tt \vee \langle coin\rangle\langle\overline{coffe}\rangle tt$

Example

CM = <unknown specification>

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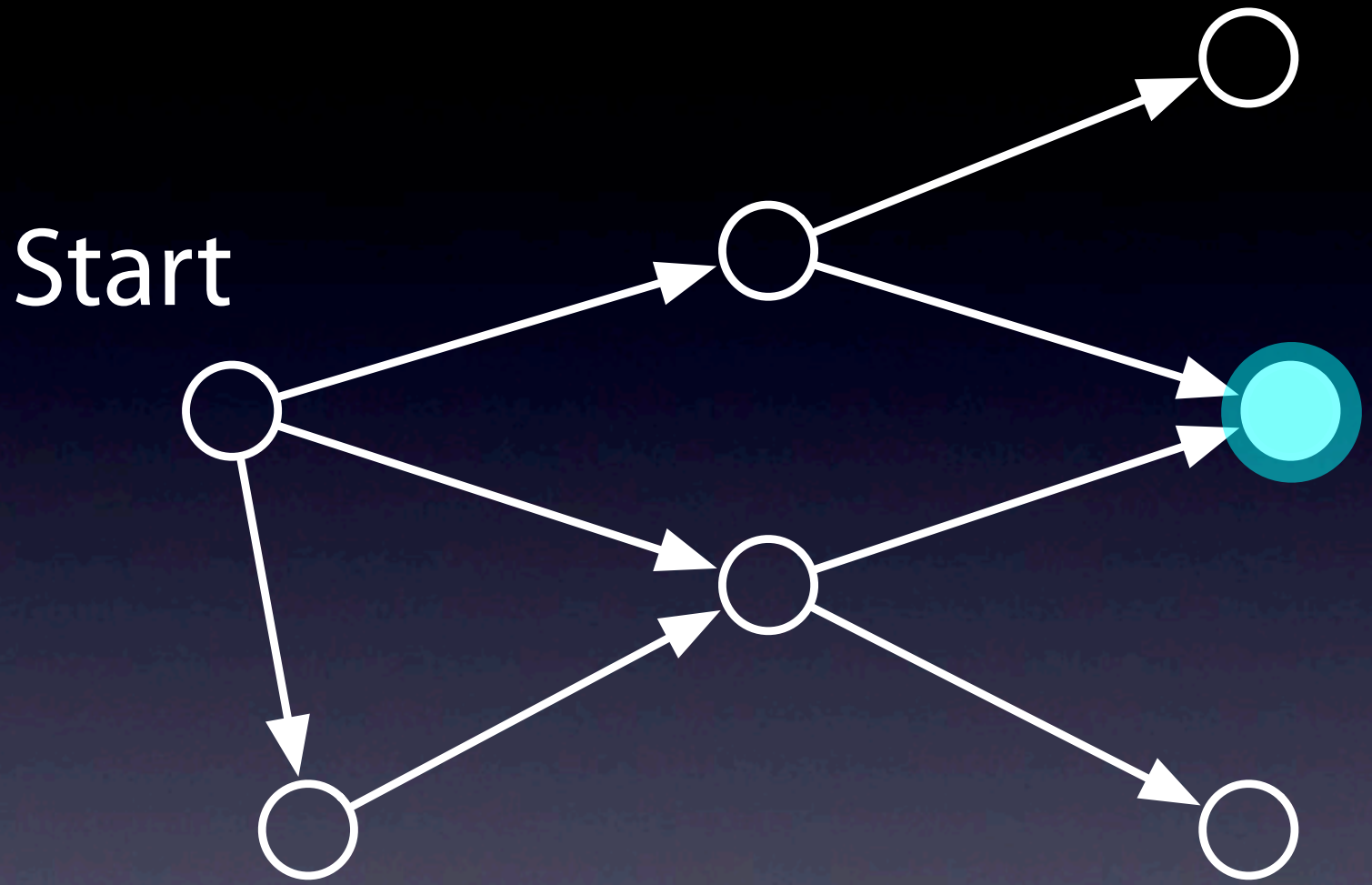
$\langle\overline{pub}\rangle tt$ / CS

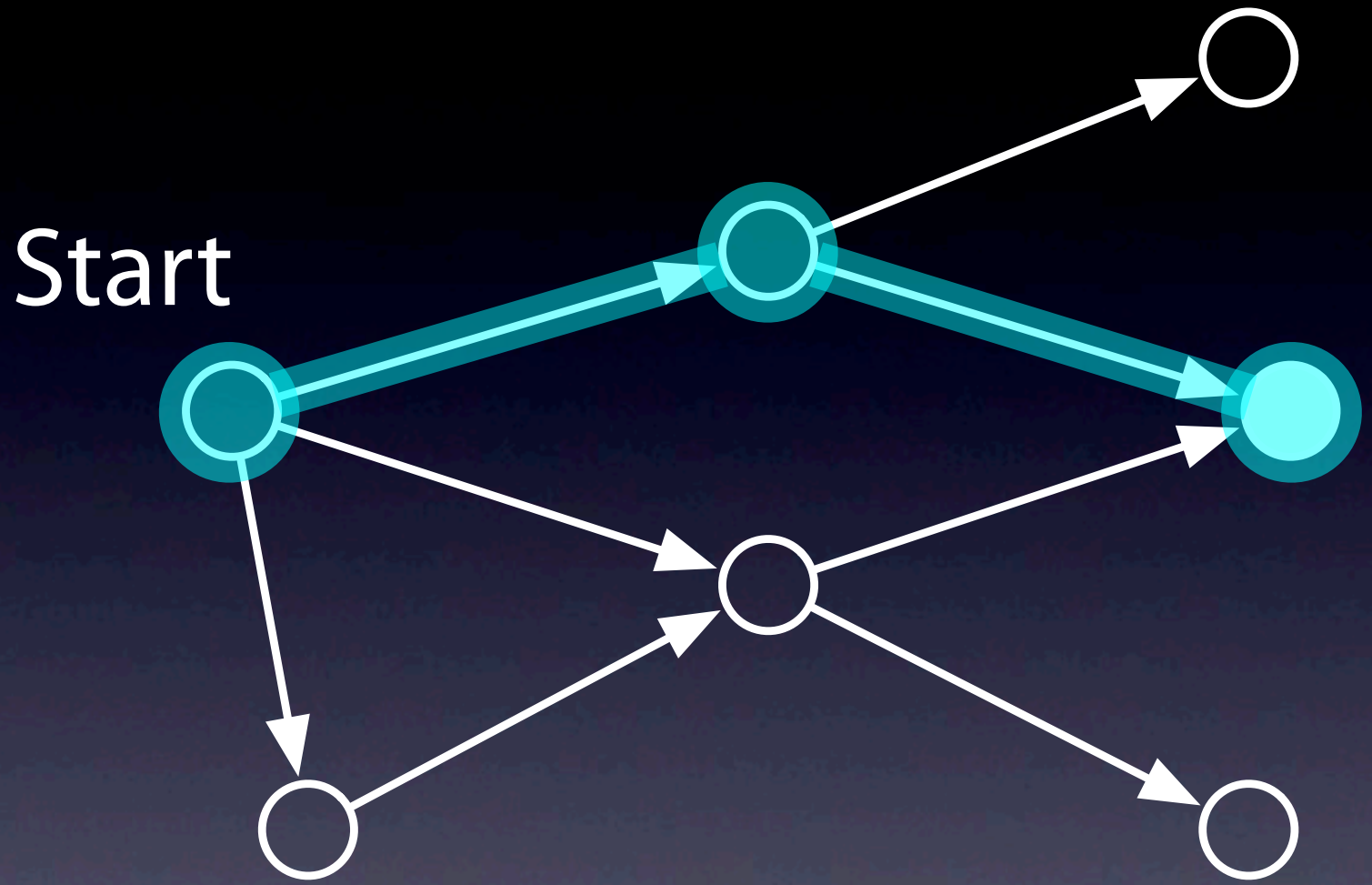
Process algebra with history

- Instead of states being processes, states are computations
- Computations contain the past history of a process

$$(a.b.0 \xrightarrow{a} b.0) \xrightarrow{b} (a.b.0 \xrightarrow{a} b.0 \xrightarrow{b} 0)$$

- Applications in verification of security protocols, epistemic properties


Start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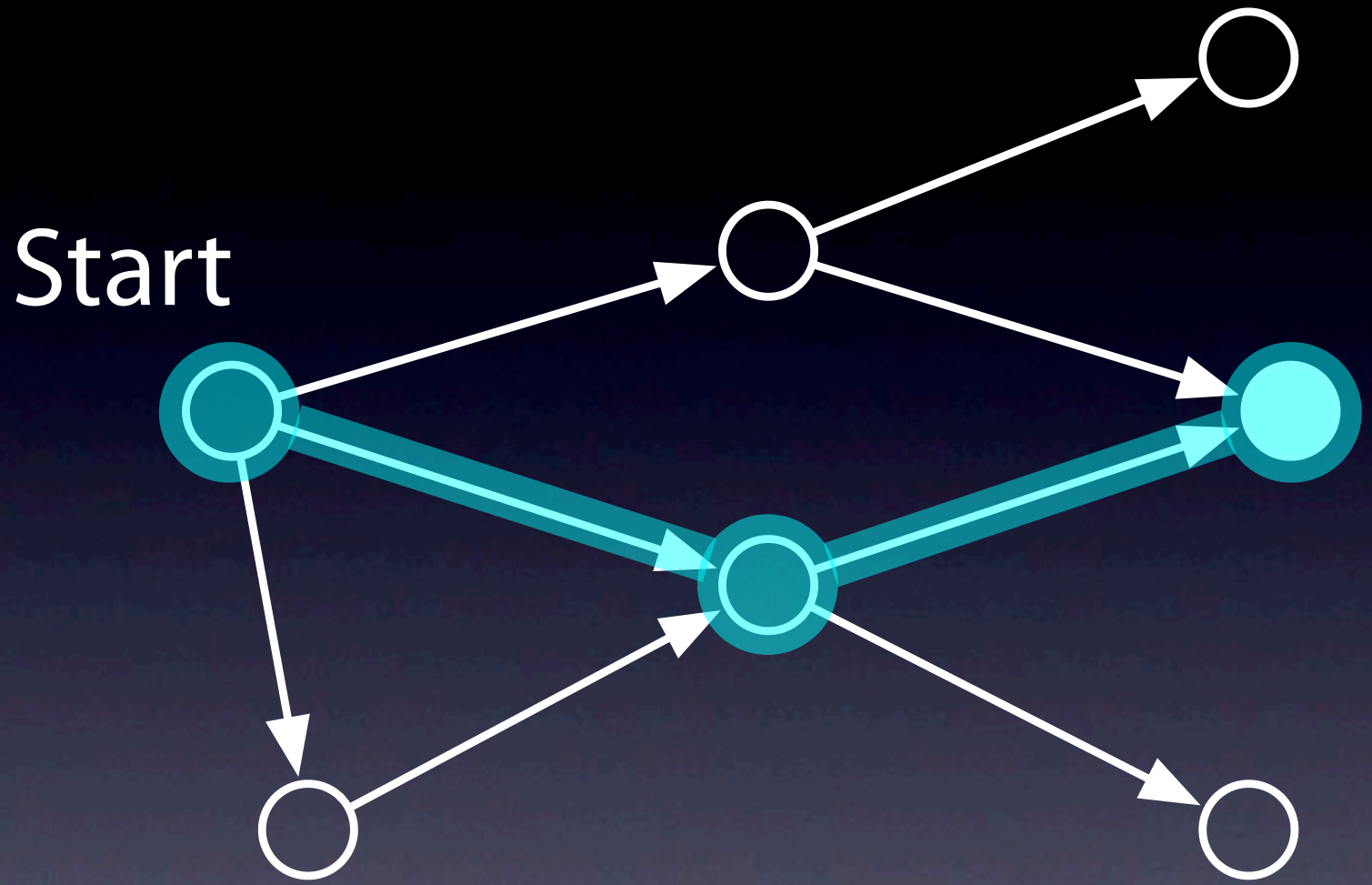
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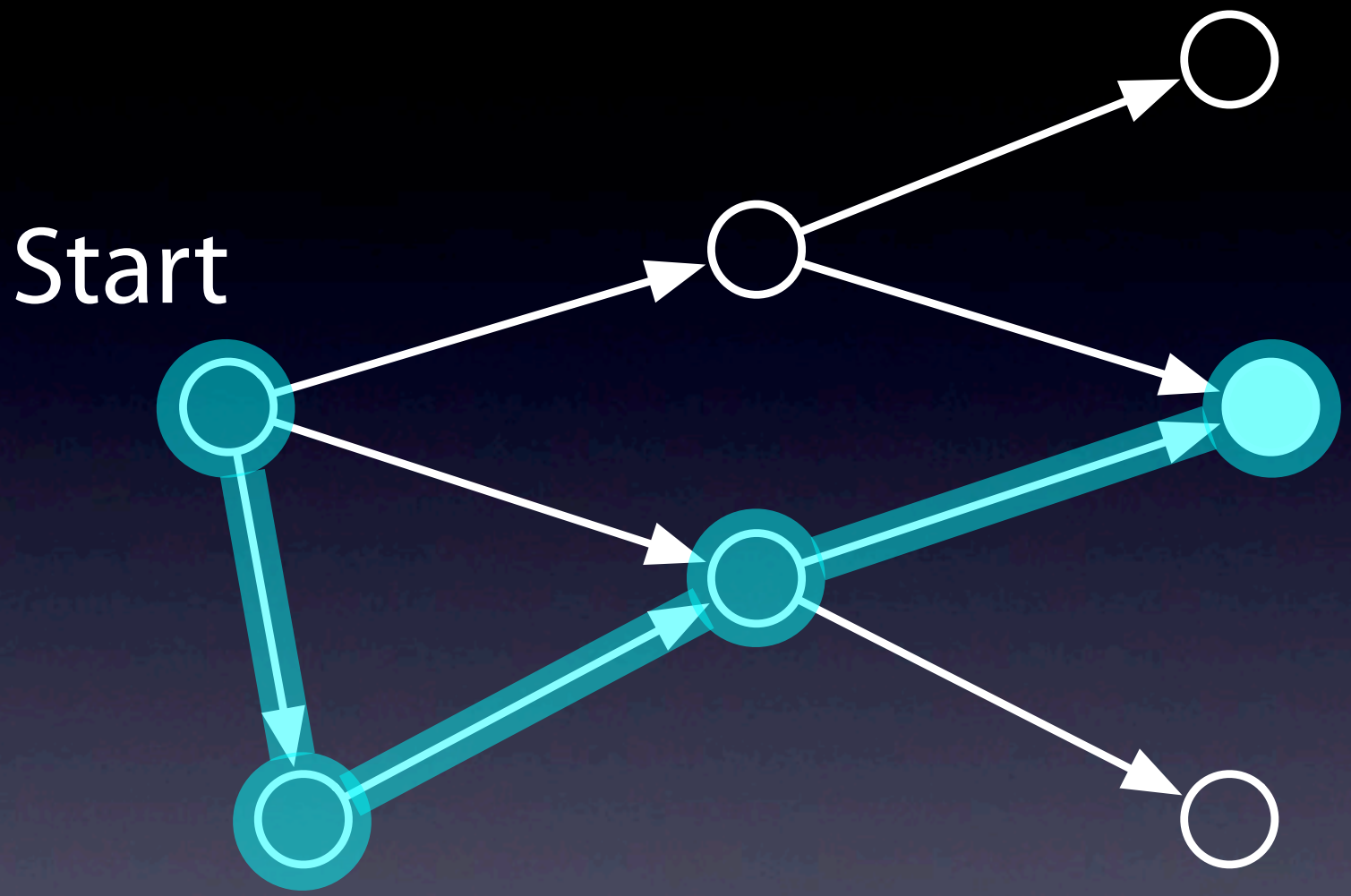
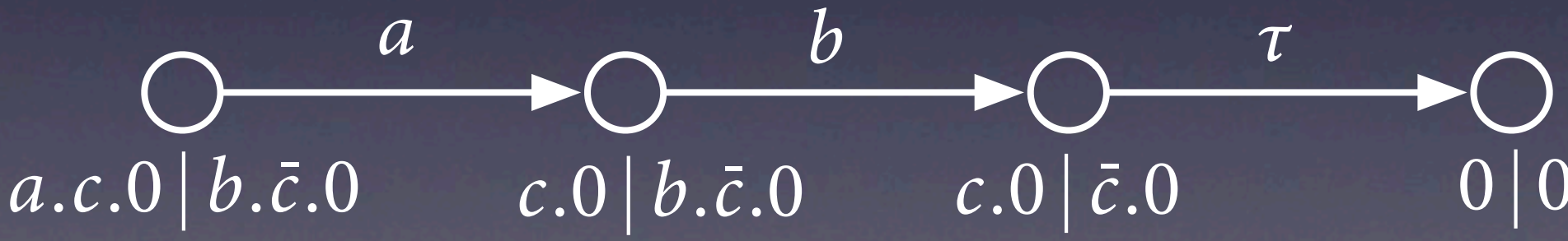

Start

HML with past

- Truth, conjunction, negation, diamond and backwards diamond
- $C \models \langle \leftarrow a \rangle \varphi$ means that the last step of the computation C was labelled with a and the computation with that last step removed satisfies φ

Decomposing a parallel computation

- Each step in a composed computation is either
  - a step of one of the components, or
  - a communication between two components



Decomposing a parallel computation

- Each step in a composed computation is either
  - a step of one of the components, or
  - a communication between two components

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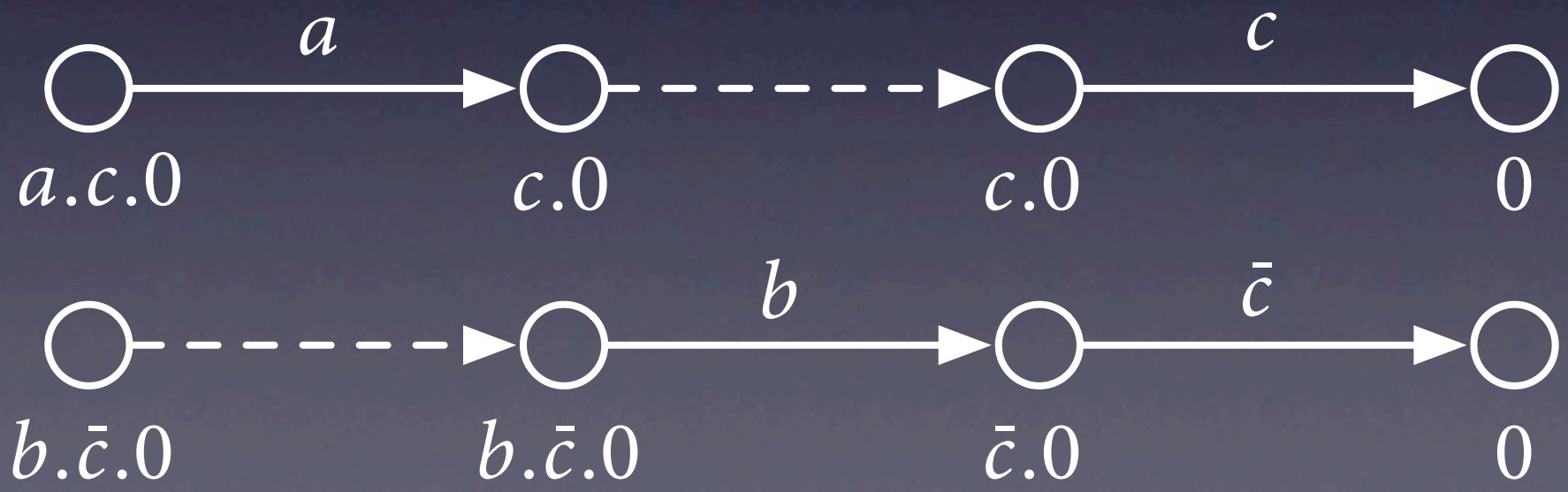

Decomposition

- Decomposition is not unique, e.g.

$$(a.0 + b.0) \mid (\bar{a}.0 + \bar{b}.0) \xrightarrow{\tau} 0 \mid 0$$

- For a computation C of two parallel components, $D(C)$ is the set of decompositions of C, where each decomposition is a pair of computations.

Decomposition theorem

- We define a transformation of HML+past formula φ w.r.t. a computation C, written φ/C

Theorem: For a computation C and formula φ

$$C \vDash \varphi \quad \Rightarrow \quad \forall (C', C'') \in D(C) : C' \vDash \varphi/C$$

$$C \vDash \varphi \quad \Leftarrow \quad \exists (C', C'') \in D(C) : C' \vDash \varphi/C$$

Future (and current) work

- Extend the theorem to handle fixed points
- Generalize with rule formats (currently based on CCS specific syntax and semantics)
- Extend to include epistemic operators in the logic, i.e. knowledge, common knowledge

Part III

SOS rule formats for determinism and idempotency

Joint work with Luca Aceto, Anna Ingólfsdóttir,
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Reykjavik University &
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(published in FSEN'09 in April '09)

Determinism

- In a transition system, a transition relation $\xrightarrow{l}$ is said to be deterministic if

$$p \xrightarrow{l} p' \text{ and } p \xrightarrow{l} p'' \quad \Rightarrow \quad p' \equiv p''$$

- Holds for sublanguages of many process calculi and programming languages
- Important, e.g. in timed systems where passage of time must not resolve any choice.

Idempotency

- An operator binary f is idempotent w.r.t. $\sim$, if for all terms x it holds that

$$f(x, x) \sim x$$

  where $\sim$ is some suitable equivalence.

- Example: The non-deterministic choice operator + (e.g. in CCS) is idempotent w.r.t. to bisimulation:

$$\forall p : p + p \approx p$$

SOS rule formats

- Semantic properties such as determinism and idempotency had to be proven for each transition system specification
- Rule formats are meta-theorems, properties can be inferred from the syntactic structure of rules - no need for complex proofs
- Rule formats exist for many properties

Source dependency

- A variable in a rule is *source dependent* via a set of labels L if

  1. It appears in the source of the conclusion, or
  2. It appears in the target of a premise, whose source variables are all source dependent via L

- Example:

$$\frac{x \xrightarrow{a} z \quad z \xrightarrow{b} y}{f(x) \xrightarrow{c} y}$$

y is source dependent via $\{a, b\}$

Determinism format

- A transition system specification T is in the *determinism format* w.r.t. a set of labels L if for each l in L

  1. In each rule defining an l-transition, all variables in the conclusion are source dependent via (a subset of) L, and
  2. In each two, distinct rules defining l-transitions with the *same source*, either the conclusion is also the same or their predicates contradict

Determinism theorem

If a TSS is in determinism format with respect to a set of labels L,
then for all of the transition relations with labels in L are deterministic.

Example

$$\frac{}{\epsilon \downarrow} \qquad \frac{}{a.x \xrightarrow{a} x} \qquad \frac{x \downarrow}{x \mp y \downarrow} \qquad \frac{y \downarrow}{x \mp y \downarrow}$$

$$\frac{x \xrightarrow{a} x' \qquad y \xrightarrow{a} y'}{x \mp y \xrightarrow{a} x' \mp y'} \qquad \frac{x \xrightarrow{a} x' \qquad y \overset{a}{\nrightarrow}}{x \mp y \xrightarrow{a} x'} \qquad \frac{x \overset{a}{\nrightarrow} \qquad y \xrightarrow{a} y'}{x \mp y \xrightarrow{a} y'}$$

Semantic vs. syntactic

- The reason for quoting *"same"*: there are subtleties in abstracting from variable names
- The general format quantifies over substitutions, i.e. not syntactic and hard to check
- Restricting rules to be *normalized* gives a completely syntactic, but less general format

Normalized rules

- Non-normalized, not covered by syntactic format, but deterministic:

$$\frac{x \xrightarrow{a} x'}{f(x,y) \xrightarrow{a} x'} \qquad \frac{y \overset{a}{\not\to} \quad x \xrightarrow{b} x'}{f(y,x) \xrightarrow{a} x'}$$

- Normalized, covered by syntactic format:

$$\frac{x \xrightarrow{a} x'}{f(x,y) \xrightarrow{a} x'} \qquad \frac{x \overset{a}{\not\to} \quad y \xrightarrow{b} y'}{f(x,y) \xrightarrow{a} y'}$$

Idempotency format

- The idempotency format describes certain types of rules which are allowed
- Relies on the determinism format
- Example: ATP time-deterministic choice, idempotency of $\oplus$ depends on χ being deterministic

$$\frac{x_0 \xrightarrow{a} x_0'}{x_0 \oplus x_1 \xrightarrow{a} x_0'} \qquad \frac{x_1 \xrightarrow{a} x_1'}{x_0 \oplus x_1 \xrightarrow{a} x_1'} \qquad \frac{x_0 \xrightarrow{\chi} x_0' \quad x_1 \xrightarrow{\chi} x_1'}{x_0 \oplus x_1 \xrightarrow{\chi} x_0' \oplus x_1'}$$

Meta

- Computer science is about abstractions
- Formal methods are the way to reason rigorously about those abstractions
- Sound abstractions are key to building reliable, correct systems - which are becoming ever more important
- Theory is nothing without practical applications and vice versa

Thanks!