T-503-AFLE Derivatives

Forwards

The fair forward price, $F_{0,T}$, for a forward contract on the underlying $S(t)$, struck at time 0 with expiry T, is given by

$$F_{0,T} = S(0)e^{rT} - \underbrace{\sum_{i=1}^{n} d_i e^{r(T-t_i)}}_{\mathrm{FV}_{0,T}(\mathrm{Div})}$$

in the case the underlying pays discrete deterministic dividends of size d_i at times t_i, with $i = 1, \ldots, n$, and $0 \le t_1 < t_2 < \cdots < t_n \le T$. In the case the underlying pays a continuous proportional dividend yield of δ that is fully re-invested, the fair forward price is

$$F_{0,T} = S(0)e^{(r-\delta)T}.$$

Put-call Parity

The prices of a maturity-T European call with strike K and an otherwise identical European put are related by

$$C(K,T) - P(K,T) = F^P_{0,T} - \mathrm{PV}_{0,T}(K) = \mathrm{PV}_{0,T}\left(F_{0,T} - K\right)$$

where

- $F^P_{0,T}$ the price of a maturity-T prepaid forward on the same underlying stock, and
- $\mathrm{PV}_{0,T}(x)$ represents the generalized present-value of time-T cash-flow x.

Option Strategies

Option Strategy	**Cost**
Floor	$S + P(K)$
Cap	$-S + C(K)$
Short Covered Call	$S - C(K)$
Short Covered Put	$-S - P(K)$
Synthetic Forward	$C(K) - P(K)$
Call Bull Spread	$C(K_1) - C(K_2)$
Put Bull Spread	$P(K_1) - P(K_2)$
Call Bear Spread	$-C(K_1) + C(K_2)$
Put Bear Spread	$-P(K_1) + P(K_2)$
Call Ratio Spread	$\pm mC(K_1) \pm nC(K_2)$
Put Ratio Spread	$\pm mP(K_1) \pm nP(K_2)$
Collar	$P(K_1) - C(K_2)$
Straddle	$P(K) + C(K)$
Strangle	$P(K_1) + C(K_2)$
Butterfly Spread	$P(K_1) + C(K_3) - (P(K_2) + C(K_2))$
Box Spread	$C(K_1) - P(K_1) - (C(K_2) - P(K_2))$

Table 1: Fundamental option trading strategies. All options are written on the same underlying, S, for the same maturity, T. Furthermore, $K_1 < K_2 < K_3$, and $m \neq n$. Note that the initial cashflow is the negative of the cost.

Binomial Option Pricing Models

Consider a one-period binomial model. The underlying, S, has a continuously reinvested proportional dividend yield of δ. It begins at S_0 and can move to either $S_u = uS_0$ or $S_d = dS_0$, in one time period of length h. The continuously compounded risk-free rate is r.
The replicating portfolio for a derivative, V, written on S, is given by

$$\Delta = e^{-\delta h}\left(\frac{V_u - V_d}{S_u - S_d}\right)$$

shares of the stock and

$$B = e^{-rh}\left(\frac{uV_d - dV_u}{u - d}\right)$$

dollars in the bank account, where V_u is the payoff of the derivative in the event S attains S_u and V_d is the payoff of the derivative in the event S attains S_d. Thus,

$$V_0 = \Delta S_0 + B.$$