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Forward Valuation & Pricing

FWD-VAL	**Long payoff**	$V_T^{\text{long}} = S_T - F(t,T) = S_T - K$	*Profit if $S_T > F$*
FWD-SHORT	**Short payoff**	$V_T^{\text{short}} = K - S_T = F(t,T) - S_T$	*Zero-sum with long*
FWD-FAIR	**Fair forward**	$F(t,T) = S(t) \cdot e^{R(t,T)(T-t)}$	$= S/D(t,T)$
FWD-PREP	**Prepaid**	$F_P(t,T) = S(t)$	*No income*
FWD-DIV1	**One dividend**	$F = [S - PV(C_c)]e^{R(T-t)}$	C_c at T_c
FWD-DIVN	**Multiple divs**	$F = [S - \sum PV(D_k)]e^{R(T-t)}$	$F_P = S - \sum PV(D_k)$
FWD-CONT	**Cont yield**	$F = S \cdot e^{(R-\delta)(T-t)}$	*Rate - yield*
FX-FWD	**Currency fwd**	$F_{X_{d,f}} = X_{d,f} \cdot e^{(R_d - R_f)(T-t)}$	*Rate parity*
FRA-LINK	**Zero/fwd**	$e^{R(t,T_c)(T_c-t)} e^{F(t;T_c,T)(T-T_c)} = e^{R(t,T)(T-t)}$	*Bootstrap*
FWD-PREM	**Premium**	$F/S = e^{(R-\delta)(T-t)}$	> 1 *if* $R > \delta$
FUT-SPOT	**Futures parity**	$F_T = S_T$	*At expiry*
FUT-MTM	**Mark-to-market**	Cash flow = $F(T_i) - F(T_{i-1})$	*Daily settle*

Special Cases & Relations

FWD-VALUE	**Value at t**	$f(t,T) = F(t,T) - K$
FWD-PV	**PV of value**	$f(t,T) = [F(t,T) - K]e^{-r(T-t)}$
PREP-REL	**Prepaid link**	$F_{0,T} = F_{0,T}^P \cdot e^{rT}$
CONTANGO	**Contango**	$F > S$ when $r > \delta$
BACKWD	**Backwardation**	Fr

Hedging Strategies

HEDGE-L	**Long hedge**	$\Delta P = -S_T(1-a) + S_0(1-a)e^{rT}$
HEDGE-S	**Short hedge**	$\Delta P = S_T(1-a) - S_0(1-a)e^{rT}$
BASIS	**Basis**	$B(t) = S(t) - F(t,T)$
HEDGE-POS	**Hedged pos**	$\Omega = \Delta S - h\Delta F$
HEDGE-R	**Optimal h**	$h^* = \rho\frac{\sigma_S}{\sigma_F}$
CONTRACTS	**Optimal N**	$N^* = \beta\frac{P}{V_S}$
INDEX-H	**Index hedge**	$N^* = \beta\frac{P}{250F}$
TAILING	**Tailing**	Buy $e^{-\delta T}$ shares

Option Payoffs & Profits

ID	Position	Payoff at T	Profit
CALL-L	**Long Call**	$\max(S_T - K, 0)$	$\max(S_T - K, 0) - C_t e^{r(T-t)}$
CALL-S	**Short Call**	$-\max(S_T - K, 0)$	$C_t e^{r(T-t)} - \max(S_T - K, 0)$
PUT-L	**Long Put**	$\max(K - S_T, 0)$	$\max(K - S_T, 0) - P_t e^{r(T-t)}$
PUT-S	**Short Put**	$-\max(K - S_T, 0)$	$P_t e^{r(T-t)} - \max(K - S_T, 0)$

Black-Scholes Formula

BS-CALL	**Call**	$C = Se^{-\delta T}N(d_1) - Ke^{-rT}N(d_2)$
BS-PUT	**Put**	$P = Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$
BS-D1	**d_1**	$d_1 = \frac{\ln(S/K) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}$
BS-D2	**d_2**	$d_2 = d_1 - \sigma\sqrt{T}$

Greeks - First Order

ID	Greek	Call	Put
DELTA	**Δ**	$e^{-\delta T}N(d_1)$	$-e^{-\delta T}N(-d_1)$
RHO	**ρ**	$KTe^{-rT}N(d_2)$	$-KTe^{-rT}N(-d_2)$
PSI	**Ψ**	$-TSe^{-\delta T}N(d_1)$	$TSe^{-\delta T}N(-d_1)$

Greeks - Second Order & Other

GAMMA	**Γ**	$\frac{e^{-\delta T}\phi(d_1)}{S\sigma\sqrt{T}}$	*C & P same*
VEGA	**ν**	$Se^{-\delta T}\phi(d_1)\sqrt{T}$	*C & P same*
Θ-C	**Θ call**	$-\frac{S\sigma e^{-\delta T}\phi(d_1)}{2\sqrt{T}} + \delta Se^{-\delta T}N(d_1) - rKe^{-rT}N(d_2)$	*Time decay*
Θ-P	**Θ put**	$-\frac{S\sigma e^{-\delta T}\phi(d_1)}{2\sqrt{T}} - \delta Se^{-\delta T}N(-d_1) + rKe^{-rT}N(-d_2)$	*Time decay*
VOL-OPT	**σ_opt**	$\sigma_{option} = \lvert\Omega\rvert\sigma_{stock}$	*Vol amplify*

Delta-Gamma Approximations

Δ-APP	**Delta**	$\Delta V \approx \Delta \cdot \Delta S$
Δ-Γ-APP	**Delta-Gamma**	$\Delta V \approx \Delta \cdot \Delta S + \frac{1}{2}\Gamma(\Delta S)^2$
Δ-Γ-Θ	**With Theta**	Add $+\Theta \cdot \Delta t$

Binomial Model

BIN-U	**Up factor**	$u = e^{(r-\delta)h + \sigma\sqrt{h}}$
BIN-D	**Down**	$d = e^{(r-\delta)h - \sigma\sqrt{h}}$
RN-PROB	**RN prob**	$p^* = \frac{e^{(r-\delta)h} - d}{u - d}$
BIN-VAL	**Value**	$V_0 = e^{-rh}[p^* V_u + (1-p^*)V_d]$
BIN-Δ	**Delta**	$\Delta = \frac{V_u - V_d}{S_u - S_d}$
BIN-AMER	**Amer option**	$V = \max(e^{-rh}[p^* V_u + (1-p^*)V_d], \text{Exercise})$

Option Bounds

Type	American	European
Call ≤	$C \leq S$	$c \leq S$
Call ≥	$C \geq S - K$	$c \geq S - Ke^{-r(T-t)}$
Put ≤	$P \leq K$	$p \leq Ke^{-r(T-t)}$
Put ≥	$P \geq K - S$	$p \geq Ke^{-r(T-t)} - S$

Put-Call Parity

PCP-BASE	**No divs**	$c + Ke^{-r(T-t)} = p + S$
PCP-DIV1	**One div**	$c - p = S - De^{-r(t_d - t)} - Ke^{-r(T-t)}$
PCP-DIVN	**Multiple**	$c - p = S - \sum_i D_i e^{-r(t_i - t)} - Ke^{-r(T-t)}$
PCP-AMER	**American**	$S - K \leq C - P \leq S - Ke^{-r(T-t)}$
CREATE-C	**Create call**	$C = P + S - Ke^{-rT}$
CREATE-P	**Create put**	$P = C - S + Ke^{-rT}$
CREATE-S	**Create stock**	$S = C - P + Ke^{-rT}$
ZERO-COST	**Zero-cost**	$C = P$ when $K = F_{0,T}$

Futures Mechanics

MTM	**MTM**	$D_t = M(F_{t,n} - F_{t-1,n})$
MARGIN-B	**Balance**	$B_t = B_{t-1}e^{r/365} + D_t$
INIT-M	**Initial**	$B_0 = \%M \times M \times F_{0,n}$
MAINT-M	**Maint**	$M_{maint} = k \times B_0$
M-CALL	**Call**	$B_t < M_{maint}$

Key Concepts

Parallel Payoffs	*If* $Payoff_A = Payoff_B + c$ *then* $Profit_A = Profit_B$
Intrinsic	*Call:* $(S-K)^+$, *Put:* $(K-S)^+$
Time Value	*Premium - Intrinsic ≥ 0*
Short Hedge	*Own asset → Short futures*
Long Hedge	*Need asset → Long futures*
Cash-&-Carry	$F_{obs} > F_{fair}$: *Buy S, short F*
Reverse C&C	$F_{obs} < F_{fair}$: *Short S, long F*

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Common Strategies

Strategy	Components	Max Profit
Covered Call	S + Short C	$K - S_0 + C$
Protective Put	S + Long P	Unlimited
Bull Call	Long K_1, Short K_2	$K_2 - K_1 - debit$
Bear Put	Long K_2, Short K_1	$K_2 - K_1 - debit$
Straddle	Long C + Long P	Unlimited
Strangle	OTM C + OTM P	Unlimited
Iron Condor	Bull P + Bear C	Net credit
Butterfly	K_1, -2×K_2, K_3	$K_2 - K_1 - debit$

Strategy Payoffs

BULL-S	Bull call	$\max(0, \min(S_T - K_1, K_2 - K_1))$
BEAR-S	Bear put	$\max(0, \min(K_2 - S_T, K_2 - K_1))$
STRAD	Straddle	$\|S_T - K\| = (S_T - K)^+ + (K - S_T)^+$
STRANG	Strangle	$(S_T - K_2)^+ + (K_1 - S_T)^+$
BUTTER	Butterfly	$\max(0, \min(S_T - K_1, K_3 - S_T))$
COLLAR	Collar	$\max(\min(S_T, K_2), K_1)$

Covered & Insurance

COV-C	Covered call	$\min(S_T, K)$
FLOOR	Floor	$\max(S_T, K)$
CAP	Cap	$-\min(S_T, K)$
COV-P	Covered put	$-\max(S_T, K)$

Statistics Fundamentals

Var(X)	$E[X^2] - (E[X])^2 = E[(X-\mu)^2]$
σ	$\sqrt{\mathrm{Var}(X)}$
Cov(X,Y)	$E[XY] - E[X]E[Y]$
ρ	$\frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y}$, $-1 \leq \rho \leq 1$
Var(X+Y)	$\mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X,Y)$
Var(aX+b)	$a^2 \mathrm{Var}(X)$

Itô's Lemma & Stochastic Calculus

ITO-PROC	Itô process	$dX = \mu(t,X)dt + \sigma(t,X)dW$
ITO-LEM	Itô's Lemma	$df = \left(\frac{\partial f}{\partial t} + \mu\frac{\partial f}{\partial X} + \frac{\sigma^2}{2}\frac{\partial^2 f}{\partial X^2}\right)dt + \sigma\frac{\partial f}{\partial X}dW$
GBM-SDE	GBM SDE	$dS = \mu S dt + \sigma S dW$
GBM-SOL	GBM solution	$S_t = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W(t)\right]$
LOG-S	Log stock dist	$\ln(S_T/S_0) \sim N\left[\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right]$

GBM - Expected Values & Variance

GBM-MEAN	Expected S_t	$E[S_t] = S_0 e^{\mu t}$
GBM-VAR	Variance of S	$\mathrm{Var}(S_t) = S_0^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$
LNORM-MEAN	Lognormal mean	If $\ln Y \sim N(m, v^2)$, then $E[Y] = e^{m + v^2/2}$
LNORM-VAR	Lognormal var	$\mathrm{Var}(Y) = e^{2m+v^2}(e^{v^2} - 1)$

Lognormal Distribution & Probabilistic Quantities

LN-DEF	LN definition	If $\ln(Y) \sim N(m, v^2)$ then $Y \sim LN(m, v^2)$
LN-PDF	LN PDF	$f_Y(y) = \frac{1}{\sqrt{2\pi v^2}y}\exp\left[-\frac{(\ln y - m)^2}{2v^2}\right]$, $y > 0$
MGF-N	Normal MGF	If $X \sim N(m, v^2)$: $M_X(t) = e^{mt + \frac{1}{2}v^2 t^2}$
LN-MOM-K	LN k-th moment	$E[Y^k] = \exp(km + \frac{1}{2}k^2 v^2)$ for any real k
S-MODEL	Stock price model	$S(t) = S(0)\exp[(\alpha - \delta - \frac{\sigma^2}{2})t + \sigma\sqrt{t}Z]$, $Z \sim N(0,1)$
S-EXPECT	Expected S(t)	$E[S(t)] = S(0)e^{(\alpha-\delta)t}$, α = expected return
S-VAR-LOG	Var of log return	$\mathrm{Var}[\ln(S(t)/S(0))] = \sigma^2 t$
DHAT2-PHYS	$\hat{d}_2$ (physical)	$\hat{d}_2 = \frac{\ln(S_0/K) + (\alpha - \delta - \sigma^2/2)T}{\sigma\sqrt{T}}$ (uses α)
DHAT1-PHYS	$\hat{d}_1$ (physical)	$\hat{d}_1 = \hat{d}_2 + \sigma\sqrt{T}$
EXER-CALL	Call exercise prob	$P(S_T > K) = N(\hat{d}_2)$ (physical measure)
EXER-PUT	Put exercise prob	$P(S_T < K) = N(-\hat{d}_2)$
COND-E-C	Cond E (call ITM)	$E[S_T \| S_T > K] = S_0 e^{(\alpha-\delta)T}\frac{N(\hat{d}_1)}{N(\hat{d}_2)}$
COND-E-P	Cond E (put ITM)	$E[S_T|S_T
PRED-UP	Pred interval upper	$S_U = S_0 \exp[(\alpha - \delta - \frac{\sigma^2}{2})t + \sigma\sqrt{t}N^{-1}(1 - p/2)]$
PRED-LOW	Pred interval lower	$S_L = S_0 \exp[(\alpha - \delta - \frac{\sigma^2}{2})t + \sigma\sqrt{t}N^{-1}(p/2)]$

Normal Distribution Properties

N-PDF	Std normal PDF	$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$
N-CDF	Std normal CDF	$N(x) = \int_{-\infty}^{x}\phi(u)du = \frac{1}{2}[1 + \mathrm{erf}(x/\sqrt{2})]$
N-SYM	Symmetry	$N(-x) = 1 - N(x)$; $\phi(-x) = \phi(x)$
N-TRANS	Standardization	If $X \sim N(m, v^2)$: $Z = \frac{X-m}{v} \sim N(0,1)$
N-QUANT	Key quantiles	$N^{-1}(0.90) = 1.28$, $N^{-1}(0.95) = 1.65$, $N^{-1}(0.975) = 1.96$, $N^{-1}(0.99) = 2.33$
N-VAL	Select N values	$N(0) = 0.5$, $N(1) = 0.84$, $N(1.65) = 0.95$, $N(1.96) = 0.975$, $N(2.33) = 0.99$
ERF-DEF	Error function	$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$
CONF-LVL	Confidence interval	$P(\mu - z_{\alpha/2}\sigma < X < \mu + z_{\alpha/2}\sigma) = 1 - \alpha$

American Options - Early Exercise

AMER-C-ND	Amer call (no div)	Never early exercise; $C_{Amer} = C_{Euro}$
AMER-P-EX	Amer put criterion	Exercise when $(K - S) >$ Hold value
AMER-VAL	Amer value	$V_{Amer} = \max(\mathrm{Hold}, \mathrm{Exercise})$
AMER-INEQ	Amer bound	$V_{Amer} \geq V_{Euro}$

Option Bounds - Upper & Lower

C-LOWER	Call lower	$C \geq \max(0, S_0 e^{-\delta T} - K e^{-rT})$
C-UPPER	Call upper	$C \leq S_0 e^{-\delta T}$
P-LOWER	Put lower	$P \geq \max(0, K e^{-rT} - S_0 e^{-\delta T})$
P-UPPER	Put upper	$P \leq K e^{-rT}$ (Euro); $P \leq K$ (Amer)

Advanced Spreads & Strategies

BULL-C	Bull spread (calls)	Long K_1-call + Short K_2-call ($K_1 < K_2$); Max: $K_2 - K_1$
BULL-P	Bull spread (puts)	Long K_1-put + Short K_2-put ($K_1 < K_2$); Same payoff
BEAR-SPR	Bear spread	Short K_1 + Long K_2 (same type); Max: $K_2 - K_1$
BOX	Box spread	Long K_1 synth fwd + Short K_2 synth fwd; Payoff: $K_2 - K_1$
ZERO-COL	Zero-cost collar	$P(K_1) = C(K_2)$ where $K_1 < F_{0,T} < K_2$

Elasticity & Additional Relationships

ELAS-C	Call Elasticity	$\Omega_C = \frac{S_0 e^{-\delta T}N(d_1)}{C}$
ELAS-P	Put Elasticity	$\Omega_P = -\frac{S_0 e^{-\delta T}N(-d_1)}{P}$
PCP-GK	PCP Greeks	$\Delta_C - \Delta_P = e^{-\delta T}$, $\Gamma_C = \Gamma_P$
PORT-GK	Portfolio Gk	$Greek_{port} = \sum Greek_i$

Quick Reference

Symbol	Meaning	Symbol	Meaning
$\phi(x)$	Std N PDF	$N(x)$	Std N CDF
S_0, S_T	Stock price	K	Strike
r	Risk-free	δ	Div yield
σ	Volatility	T	Maturity
p^*	RN prob	$F_{0,T}$	Forward
N(0.0)	0.500	N(1.0)	0.841
N(0.5)	0.691	N(1.65)	0.950
N(1.96)	0.975	N(2.33)	0.990
e	2.71828	ln(2)	0.69315
√2π	2.50663	Trading	252 days
1bp	0.0001	ACT/365	Money mkt

Synthetic Positions

SYN-FWD-L	Synth long fwd	Long C + Short P (same K, T)
SYN-FWD-S	Synth short fwd	Short C + Long P (same K, T)
SYN-CALL	Synth call	Long S + Long P - Borrow $PV(K)$
SYN-PUT	Synth put	Short S + Long C + Lend $PV(K)$
SYN-STK	Synth stock	Long C + Short P + Borrow $PV(K)$

Hedging Strategies

Δ-HEDGE	Delta-neutral	Hold $-\Delta$ shares per option
Γ-HEDGE	Gamma-neutral	$\Gamma_{portfolio} = 0$, use options
ν-HEDGE	Vega-neutral	$\nu_{portfolio} = 0$, vol hedge
H-PROFIT	Hedge profit	$\Pi = \frac{1}{2}\Gamma S^2(\sigma_{real}^2 - \sigma_{impl}^2)\Delta t$

Special Cases - BS Applications

DISC-DIV	Discrete dividends	Use $S_0 - PV(Div)$ for S_0
FX-OPT	Currency options	$S_0 \to X_0$, $\delta \to r_f$, $r \to r_d$
FUT-OPT	Futures options	$C = e^{-rT}[F_0 N(d_1) - K N(d_2)]$

Lognormal Distribution

LN-MEAN	Expected S_T	$E[S_T] = S_0 e^{(\alpha-\delta)T}$
LN-VAR	Var of ln(S)	$Var[\ln(S_T)] = \sigma^2 T$
RN-DRIFT	RN drift	$\ln(S_T) \sim N(\ln(S_0) + (r - \delta - \frac{\sigma^2}{2})T, \sigma^2 T)$
PROB-ITM	Prob ITM (RN)	Call: $N(d_2)$, Put: $N(-d_2)$

Volatility & Implied Vol

HIST-VOL	Historical vol	$\sigma = \sqrt{\frac{252}{n}\sum(r_i - \bar{r})^2}$
IMP-VOL	Implied vol	Solve $C_{mkt} = BS(S, K, r, T, \sigma_{IV})$
ATM-APPR	ATM approx	$C \approx 0.4 S_0 \sigma\sqrt{T}$

Exotic Options - Barriers

DN-IN-C	Down-in call	$(S_T - K)^+ \cdot \mathbf{1}_{m_T \leq H}$, $H < S_0$
UP-OUT-C	Up-out call	$(S_T - K)^+ \cdot \mathbf{1}_{M_T < H}$, $H > S_0$
BAR-PAR	Barrier parity	Knock-in + Knock-out = Vanilla

Exotic Options - Path-Dependent

ASIAN-AR	Arithmetic Asian	$(\frac{1}{n}\sum S_{t_i} - K)^+$
ASIAN-GE	Geometric Asian	$((\prod S_{t_i})^{1/n} - K)^+$
LOOK-C	Lookback call	$S_T - \min_{0 \leq t \leq T} S_t$
LOOK-P	Lookback put	$\max_{0 \leq t \leq T} S_t - S_T$

Exotic Options - Other

GAP-C	Gap call	$(S_T - K_1) \cdot \mathbf{1}_{S_T > K_2}$
CASH-DN	Cash-or-nothing	$B \cdot \mathbf{1}_{S_T > K}$, Price: $Be^{-rT}N(d_2)$
ASSET-DN	Asset-or-nothing	$S_T \cdot \mathbf{1}_{S_T > K}$, Price: $S_0 e^{-\delta T}N(d_1)$
CHOOSER	Chooser option	At t_c: $\max(C_{t_c}, P_{t_c})$
EXCHANGE	Exchange option	$(S_{1,T} - S_{2,T})^+$ (Margrabe)

Important Notes

BS PDE: $\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} = rV$

Time conventions: *30/360 (Bond), ACT/365 (Money), ACT/ACT (Treasury)*

Settlement: *T+2 standard, 252 trading days/year*

Moneyness: *Call ITM: $S_0 > K$; ATM: $S_0 \approx K$; OTM: $S_0 < K$ (opposite for puts)*

Wiener process: *$W(t)$ is standard Brownian motion; $dW \sim N(0, dt)$; $E[dW] = 0$*

Normal PDF: *$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$; $N(x)$ is standard normal CDF*

Physical vs RN: *$\hat{d}_1, \hat{d}_2$ use α (physical prob); d_1, d_2 use r (risk-neutral pricing)*

Key Principles & Concepts

Parallel Payoffs: *If $\mathrm{Payoff}_A = \mathrm{Payoff}_B + c$, then $\mathrm{Profit}_A = \mathrm{Profit}_B$*

Intrinsic Value: *Value if exercised NOW; Call: $(S-K)^+$, Put: $(K-S)^+$*

Time Value: *Premium - Intrinsic Value; Always ≥ 0 for options*

Cash-and-Carry: *When $F_{obs} > F_{fair}$: Buy asset, borrow, short forward (arbitrage)*

Reverse C&C: *When $F_{obs} < F_{fair}$: Short asset, lend, long forward (arbitrage)*