T-503-AFLE: Class Test 2 - Solution

- **Duration:** 1.5 hours
- **Total Marks:** 20 marks

1: Example 1.2.4 (Profit of a Written Covered Call)(2 marks)

An investor purchases a nondividend-paying stock and writes a t-year, European call option for this stock, with call premium C. The stock price at time of purchase and strike price are both K. Assume that there are no transaction costs. The continuously compounded risk-free rate is r. Let S represent the stock price at time t and assume $S>K$.
Determine an algebraic expression for the investor's profit at expiration.

Solution:
The time-0 investment is $S(0)-C=K-C$ and the time- t payoff is $S-(S-K)_{+}=S-(S-K)=K$ because $S>K$. The profit at expiration is $K-(K-C)\mathrm{e}^{rt}=C\mathrm{e}^{rt}+K\left(1-\mathrm{e}^{rt}\right)$.

2: Problem 2.5.4 (Price of Continuous Random Dividends) (2 marks)

The current price of a stock is 100. The stock pays dividends continuously at a rate proportional to its price. The dividend yields is 3%.
The continuously compounded risk-free interest rate is 7%.
Calculate the price of the stream of dividends to be paid in the next 5 years.

Solution:

- If you pay $F_{0,5}^{P}=S(0)\mathrm{e}^{-5\delta}$ for the prepaid forward at time 0 , you will get 1 unit of the stock at time 5 .
- If you pay $S(0)$ for 1 unit of stock at time 0 , you will get 1 unit of the stock, together with the accumulation of dividends received over the past 5 years. The difference between what you pay under the two methods accounts for the price (or current value) of the stream of (random) dividends and equals $S(0)\left(1-\mathrm{e}^{-5\delta}\right)=100\left(1-\mathrm{e}^{-5(0.03)}\right)=13.93$.

3: Example 3.2.9 (Parity Arbitrage) (4 marks)

You are given:
(i) The price of a nondividend-paying stock is $\$31$.
(ii) The continuously compounded risk-free interest rate is 10%.
(iii) The price of a 3-month 30-strike European call option is $\$3$.
(iv) The price of a 3-month 30-strike European put option is $\$2.25$.
Construct a trading strategy that will generate risk-free arbitrage profits at time 0 and compute the resulting profit.

Solution. It is easy to see that the call and put prices violate put-call parity:

- LHS:

$$C(30, 0.25) - P(30, 0.25) = 3 - 2.25 = 0.75,$$

- RHS:

$$F_{0,T}^{P}(S) - F_{0,T}^{P}(K) = S(0) - K\mathrm{e}^{-rT} = 31 - 30\mathrm{e}^{-(0.1)(0.25)} = 1.7407.$$

To exploit arbitrage profits, we "buy the LHS" ("low") and "sell the RHS" ("high") by engaging in the following transactions:

	Cash Flows	
Transaction	**Time 0**	**Time 0.25**
Buy a 3-month 30-strike call	-3	$(S(0.25) - 30)_+$
Sell a 3-month 30-strike put	$+2.25$	$(30 - S(0.25))_+$
Short sell one share of the stock	$+31$	$-S(0.25)$
Lend $30\mathrm{e}^{-(0.1)(0.25)} = 29.2593$	-29.2593	30
Total	0.9907	0

4: Problem 3.5.12. (Profit comparison) (4 marks)

Problem 3.5.12. (Profit comparison) The current price of stock ABC is 40. Stock ABC pays dividends continuously at a rate proportional to its price. The dividend yield is 2%
.
You are given the following premiums of one-year European call options on stock ABC for various strike prices:

Strike	Call premium
35	7.24
40	4.16
45	2.62

The effective annual risk-free interest rate is 8%.
Let $S(1)$ be the price of the stock one year from now.
Determine the range for $S(1)$ such that a 35-strike short put produces a higher profit than a 45-strike short put, but a lower profit than a 40-strike short put.
(Note: All put positions being compared are short.)

Solution:
Step 1. By put-call parity, the required put premiums are:

Strike	Put premium $(= C - 40\mathrm{e}^{-0.02(1)} + K/1.08)$
35	0.44
40	1.99
45	5.08

Step 2. Compute the future value of the put premiums:

Strike	FV of put premium
35	$0.44 \times 1.08 = 0.48$
40	$1.99 \times 1.08 = 2.15$
45	$5.08 \times 1.08 = 5.49$

Step 3. The profit diagrams of the three short puts are sketched in Figure 1.

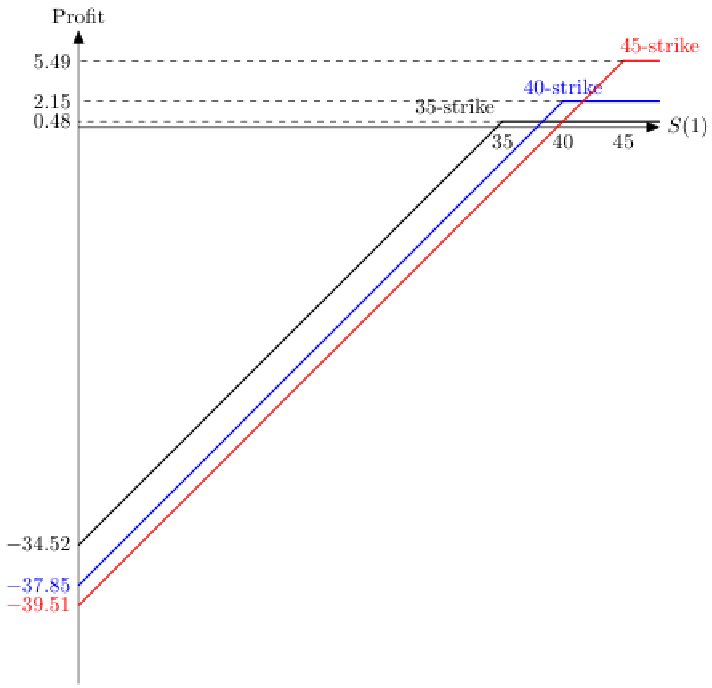

Figure 1: Profit diagrams of the three puts in Problem 3.5.12.

Step 4. The 35-strike line crosses the 40-strike and 45-strike lines respectively at $40-(2.15-0.48)=38.33$ and $45-(5.49-0.48)=39.99$. Visually inspecting the above diagram, we conclude that the 35 -strike put produces a higher profit than the 45 -strike put, but a lower profit than the 40 -strike put, when $38.33<S(1)<39.99$.

5: Problem 3.5.21. (Does the option box permit arbitrage?) (5 marks)

You are given the following information about four European options on the same underlying asset:
(i) The price of a 25-strike 1-year call option is 6.85 .
(ii) The price of a 35 -strike 1-year call option is 1.77 .
(iii) The price of a 25-strike 1-year put option is 0.63 .
(iv) The price of a 35-strike 1-year put option is 5.06 .
The continuously compounded risk-free interest rate is 6%.
Describe actions you could take at time 0 using only appropriate bull/bear spread(s) and/or zero-coupon bond(s) to earn arbitrage profits at time 0. Specify the contractual details of the bull/bear spread(s) and zero-coupon bond(s) you use clearly.

Solution. A (long) box spread is created by buying a 25 -strike call, selling a 25 -strike put, selling a 35-strike call and buying a 35-strike put. The investment required is

$$C(25)-P(25)-C(35)+P(25)=9.51$$

and the payoff at expiration is $35-25=10$. The implicit 1-year accumulation factor is $10/9.51=1.0515$, whereas the 1-year accumulation factor in the market is $\mathrm{e}^{0.06}=1.0618$. In other words, the long box spread is worse than a risk-free investment in the market and should be short:

Strike	Position in Call	Position in Put
25	Short	Long
35	Long	Short

In the language of bull/bear spread(s), we should:

- Buy a 25-35 call bear spread (or equivalently sell a 25-35 call bull spread)

- Buy a 25-35 put bull spread (or equivalently sell a 25-35 put bear spread) Together with a long 1-year zero-coupon bond with a face value of 10, the resulting profit at time 0 is $9.51-10\mathrm{e}^{-0.06}=0.0924$. (At time 1 , our payoff is constant at $\underbrace{-10}_{\text{box spread}} \underbrace{+10}_{\text{from bond}}=0$)

6: Example 4.1.3 (Valuing a Straddle) (3 marks)

For a one-year straddle on a nondividend-paying stock, you are given:
(i) The straddle can only be exercised at the end of one year.
(ii) The payoff of the straddle is the absolute value of the difference between the strike price and the stock price at expiration date.
(iii) The stock currently sells for $\$60$.
(iv) The continuously compounded risk-free interest rate is 8%.
(v) In one year, the stock will either sell for $\$70$ or $\$45$.
(vi) The option has a strike price of $\$50$.

Calculate the current price of the straddle.

Solution:
Consider a replicating portfolio consisting of Δ shares of stock and B amount of cash. Matching the payoffs in the up and down scenarios, we solve

$$\begin{cases}70\Delta+\mathrm{e}^{0.08}B=|50-70|=20\\45\Delta+\mathrm{e}^{0.08}B=|50-45|=5\end{cases}$$

which gives $\Delta=0.6$ and $B=-20.3086$. The current price of the derivative is the same as the portfolio value at time 0 , which is

$$60(0.6)+(-20.3086)=15.6914. \quad (\text{Answer: } (\mathbf{E}))$$