T-503-AFLE Derivatives

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Problem Set I – Reminders

Q1/ Using your own words, explain why if 2 equally risky portfolios P_1 and P_2 are expected to have the same value at the future time T, they must have the same price at all previous times $t < T$ in order to prevent arbitrage opportunity.

Solution: We can demonstrate this statement by showing the existence of arbitrage if the relation $E[P_1(T)] = E[P_2(T)] => P_1(t) = P_2(t)$ does not hold. Particularly, let's assume we have $P_1(t) > P_2(t)$, in such a case, an arbitrageur could short-sell P_1 (given it is more expensive), and buy P_2 (since it is cheaper). He would then keep the two positions until the maturity, at which point they would cancel out because they will be worth the same. However, the arbitrageur would have secured a risk-free profit, with zero initial investment, the amount is $P_1(t) - P_2(t) > 0$, given $P_1(t) > P_2(t)$. Operating the same process assuming this time that $P_1(t) < P_2(t)$ would lead to a risk-free profit amounting to $P_2(t) - P_1(t) > 0$. Hence, under the assumption of no arbitrage, any two portfolios that have the same future value and the same risk profile must have identical prices at all earlier times.

Q2/ Assuming you can return 4% every month on your investments. You observed that you can return 4% every month on your investments. How long would it take to turn 100,000 USD into 300,000 USD?

Hint: You will assume here periodic compounding where the interest/profit is reinvested.

Solution: We will translate this question into an equation:

$$100{,}000 * (1 + 4\%)^t = 300{,}000$$

From there, we understand that we want to multiply by three our initial investment, and by applying the log function, we find:

$$(1 + 4\%)^t = \frac{300{,}000}{100{,}000}$$

$$\ln((1 + 4\%)^t) = 3$$

$$t * \ln(1.04) = \ln(3)$$

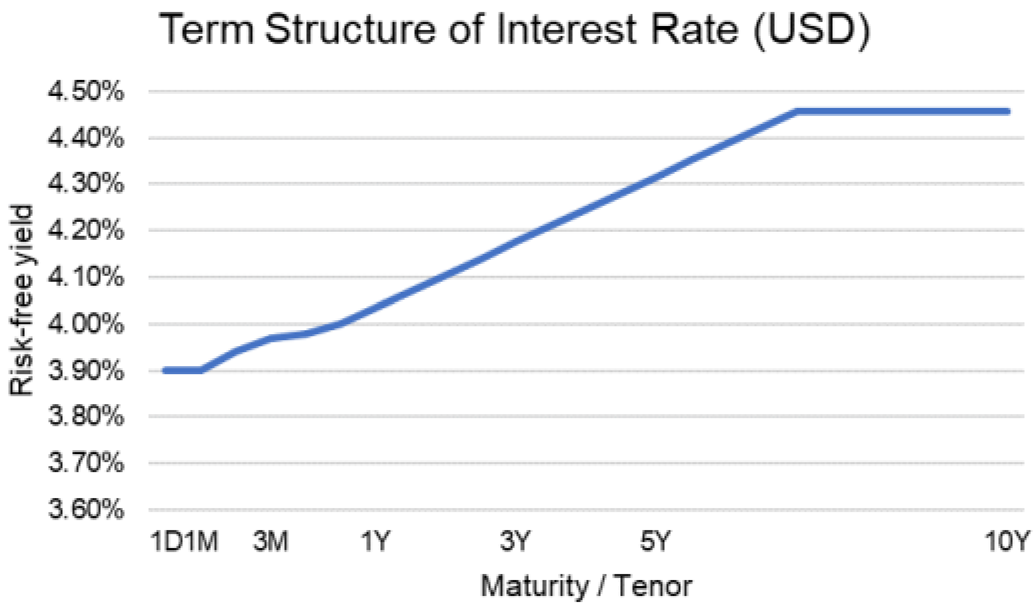
$$t = \frac{\ln(3)}{\ln(1.04)} = 28.011\ months$$

Equivalent to 2 years 4 months 0 days 7 hours 55 minutes 12 seconds.

Q3/ You are a working in the treasury department of an Icelandic bank. As you manage the liquidity portfolio of the Bank, you are looking for investment opportunities in the bond market. Particularly, your intern comes to you with the term sheet for a new bond, issued by Danske Bank A/S.

Danske Bank - US23636BAQ23	
nominal	1000
issue date	12/6/2018
maturity date	12/6/2028
currency	USD
coupon yield	4.375%
nb payment per year	2
time to maturity	10
coupon amount	21.875
callability	No

On your Bloomberg terminal, you find the term structure of interest rate for the U.S Government bonds (called T-bills), it looks like this:



Your intern modelled this by the following function:

From 0.5Y to 7Y included:

$$Interest\ Rate\ (t) = 4\% + 0.000352 * (t - 0.5) * 2$$

From 7Y to 40Y included:

$$Interest\ Rate\ (t) = 4\% + 0.000352 * (7 - 0.5) * 2$$

a) Price this bond.

Hint: In other word, the interest rate is increasing linearly from 0.5Y to 7Y and is flat after that. Example: if you need to evaluate the interest rate at time t=5 years, the IR is 4%+0.000352*(5-0.5)*2 = 4.3168%

b) Assume now that the interest rate is flat at 5% across all maturities. Reprice this bond

c) You want to understand more about your bond's sensitivity to interest rate. In other words, you want to know by how much will the value of the bond change, for a change in interest rate of 1 basis point (0.01%). You will calculate the so-called "effective duration", defined by the following formula:

$$Effective\ Duration = \frac{PV_{-} - PV_{+}}{2 * PV_0 * \Delta r}$$

With:

- $PV-$ the new value of the bond when the interest rate moves by 1 basis point down (0.01%),
- $PV+$ the new value of the bond when the interest rate moves by 1 basis point up (0.01%),
- $PV0$ the initial value of the bond (from subquestion b))
- Δr the change in rate (from $PV0$ to $PV+$ or from $PV-$ to $PV0$).

d) You are working for the Risk department of an Icelandic bank. Notably, you want to measure the risk on the Danske Bank bond we priced on Part III. Under IFRS 9[1], you are required to measure the impairment (=credit losses) on financial assets using the 'expected credit loss (ECL)' approach. A well-known formula for the Expected Credit Loss (ECL) is provided below:

$$ECL = EAD * PD * LGD$$

With EAD, the Exposure at Default, PD the default probability and LGD the loss given default.

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[1] A set of accounting standards and rules specifying how an entity should classify and measure financial assets.

The Bank has an exposure of ISK 8 bn on this bond. Based on a consensus of analysts, the default probability of the bond is estimated at 2.3%. It is assumed that in case of default, 60% of the bond's value would be lost. **What is the ECL on the Danske Bank bond?**

Solution:

a)

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callability	No

Bond Price
998.7236553

t	CF	rate	DF	PV(CF)
0.5	21.875	4.000%	0.980581	21.4502
1	21.875	4.035%	0.961213	21.02654
1.5	21.875	4.070%	0.941909	20.60427
2	21.875	4.106%	0.922682	20.18366
2.5	21.875	4.141%	0.903541	19.76495
3	21.875	4.176%	0.884498	19.3484
3.5	21.875	4.211%	0.865565	18.93423
4	21.875	4.246%	0.846751	18.52268
4.5	21.875	4.282%	0.828067	18.11396
5	21.875	4.317%	0.809522	17.7083
5.5	21.875	4.352%	0.791126	17.30588
6	21.875	4.387%	0.772888	16.90692
6.5	21.875	4.422%	0.754816	16.5116
7	21.875	4.458%	0.736919	16.1201
7.5	21.875	4.458%	0.721024	15.7724
8	21.875	4.458%	0.705472	15.4322
8.5	21.875	4.458%	0.690255	15.09933
9	21.875	4.458%	0.675367	14.77365
9.5	21.875	4.458%	0.660799	14.45499
10	1021.875	4.458%	0.646546	660.6894

b)

Danske Bank - US23636BAQ23	
nominal	1000
issue date	12/6/2018
maturity date	12/6/2028
currency	USD
coupon yield	4.375%
nb payment per year	2
time to maturity	10
coupon amount	21.875
callability	No

Bond Price
955.910475

t	CF	rate	DF	PV(CF)
0.5	21.875	5.000%	0.9759	21.34781
1	21.875	5.000%	0.952381	20.83333
1.5	21.875	5.000%	0.929429	20.33125
2	21.875	5.000%	0.907029	19.84127
2.5	21.875	5.000%	0.88517	19.3631
3	21.875	5.000%	0.863838	18.89645
3.5	21.875	5.000%	0.843019	18.44104
4	21.875	5.000%	0.822702	17.99662
4.5	21.875	5.000%	0.802875	17.5629
5	21.875	5.000%	0.783526	17.13963
5.5	21.875	5.000%	0.764643	16.72657
6	21.875	5.000%	0.746215	16.32346
6.5	21.875	5.000%	0.728232	15.93007
7	21.875	5.000%	0.710681	15.54615
7.5	21.875	5.000%	0.693554	15.17149
8	21.875	5.000%	0.676839	14.80586
8.5	21.875	5.000%	0.660528	14.44904
9	21.875	5.000%	0.644609	14.10082
9.5	21.875	5.000%	0.629074	13.76099
10	1021.875	5.000%	0.613913	627.3426

c) when moving the interest rate to 5.01%, the price of the bond is USD 955.1683175. When moving the interest rate to 4.99%, the price of the bond is USD 956.6533584. Hence, the sensitivity/effective duration is:

$$ED = \frac{956.6533584 - 955.1683175}{2 * 955.910475} = 7.768$$

d) $ECL = 8bn * 2.3\% * 60\% = 110.4m$

Q4/ For this question, you will be required to code in Python, using the yfinance package (yahoo-finance). Make sure you have it installed:

!pip install yfinance

a) Fetch 1 year of daily price data for **AAPL**, compute daily log returns, and estimate the **annualized historical volatility** assuming 252 trading days.

b) Fetch 4 years of historical **daily close prices** for **Microsoft (MSFT)**. Plot i) the stock price ii) the 5-day, 20-day and 40-day moving averages. What can you tell about the lag introduced by a moving average over a longer period? What happened when the 20-day MA is crossing the 40-day MA?

c) Fetch 2 years of daily price data for the S&P 500 (ticker: ^GSPC).

Compute and plot:

- The cumulative return of investing only overnight (buy at close, sell at next open)
- The cumulative return of investing only intraday (buy at open, sell at close)

Which strategy performs better?

Solution:

a)

```
import yfinance as yf

import numpy as np


# Fetch data

data = yf.download("AAPL", period="1y")

prices = data["Close"]


# Compute log returns

log_returns = np.log(prices / prices.shift(1)).dropna()


# Annualized volatility

vol_daily = log_returns.std()

vol_annual = vol_daily * np.sqrt(252)


print(f"\n\nAnnualized volatility: {vol_annual.iloc[0]:.2%}")
```

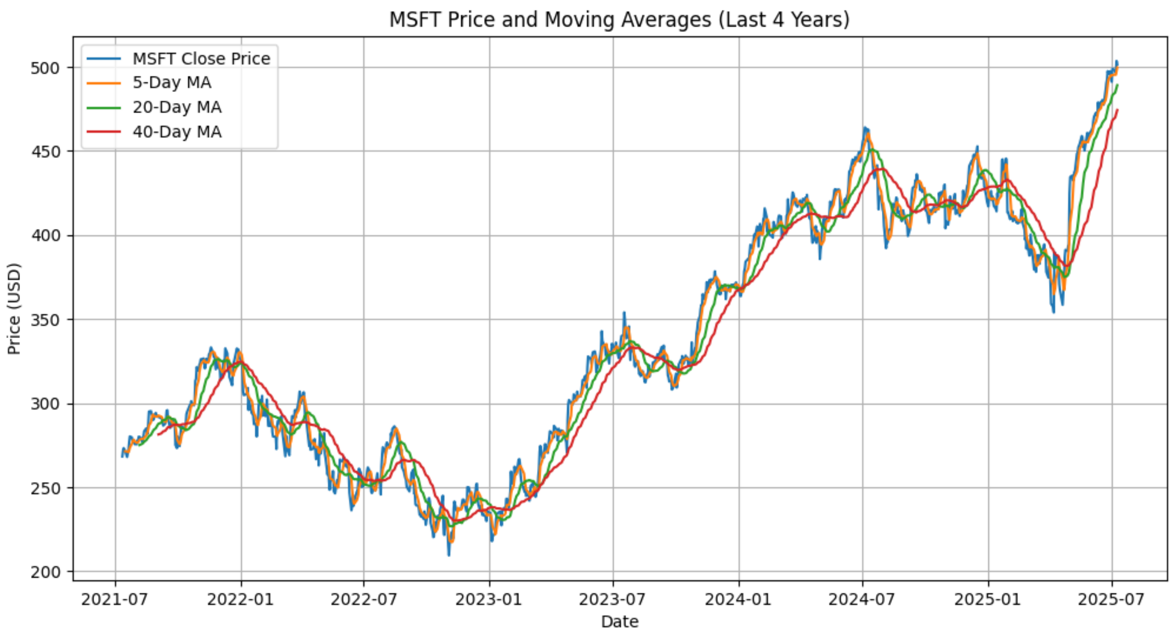
b)

A **moving average (MA)** smooths out price data by averaging past values. The longer the window (e.g., 40-day vs 20-day), the **greater the smoothing**, but also the **greater the lag**.

Crossing Mas is a classic technical analysis signal:

When the **20-day MA crosses above the 40-day MA**, it indicates **short-term momentum is improving**. Often a buy signal

When the **20-day MA crosses below the 40-day MA**, it suggests **weakening momentum**. Often a sell signal




```
import yfinance as yf

import matplotlib.pyplot as plt


# Download MSFT data for 4 years

data = yf.download("MSFT", period="4y", progress=False)

close = data["Close"]


# Compute moving averages

ma5 = close.rolling(window=5).mean()

ma20 = close.rolling(window=20).mean()

ma40 = close.rolling(window=40).mean()


# Plot

plt.figure(figsize=(12,6))

plt.plot(close, label='MSFT Close Price')

plt.plot(ma5, label='5-Day MA')

plt.plot(ma20, label='20-Day MA')

plt.plot(ma40, label='40-Day MA')

plt.title("MSFT Price and Moving Averages (Last 4 Years)")

plt.xlabel("Date")

plt.ylabel("Price (USD)")

plt.legend()

plt.grid(True)

plt.show()
```

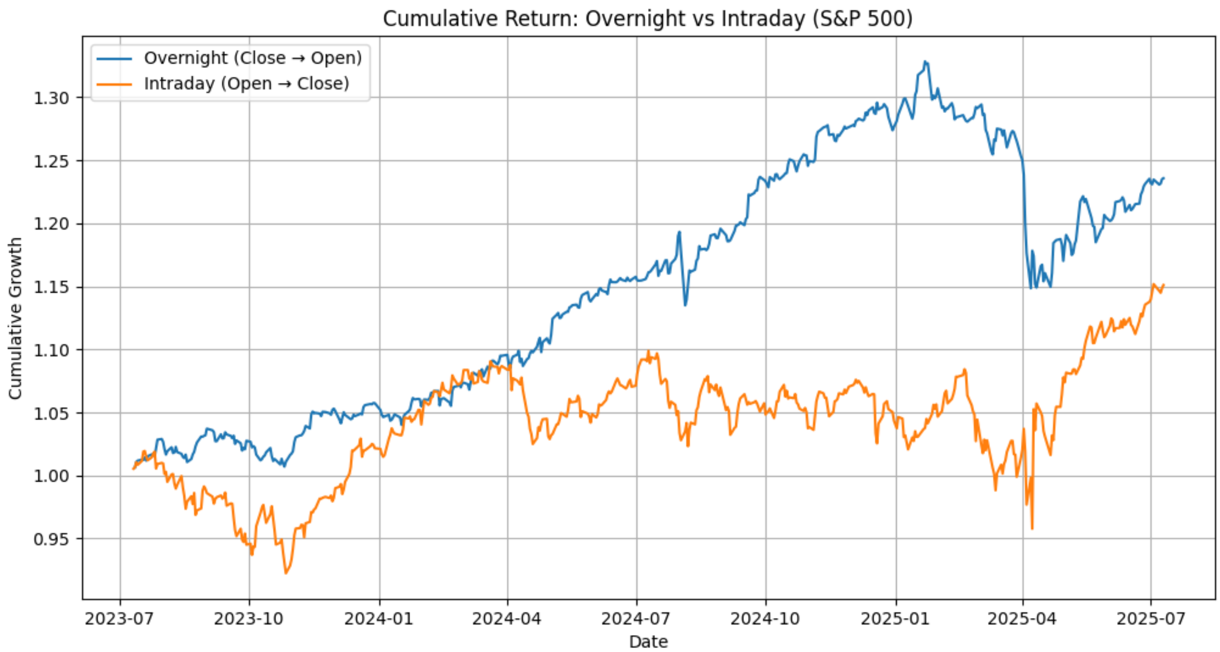
c)

In many equities and indices (especially large-cap and ETFs like the S&P 500):

Overnight returns (close → open) are often positive on average.

Intraday returns (open → close) are sometimes flat or even negative over long periods.

This contradicts intuition! You would expect most of the gains to happen when markets are open, but often it is the overnight drift that dominates!




```
import yfinance as yf
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# Fetch data
data = yf.download("^GSPC", period="2y", progress=False)
data = data.dropna()

# Calculate returns
overnight_returns = data["Open"] / data["Close"].shift(1) - 1  # Close → Open
intraday_returns = data["Close"] / data["Open"] - 1          # Open → Close

# Cumulative returns
cum_overnight = (1 + overnight_returns).cumprod()
cum_intraday = (1 + intraday_returns).cumprod()

# Plot
plt.figure(figsize=(12,6))
plt.plot(cum_overnight, label="Overnight (Close → Open)")
plt.plot(cum_intraday, label="Intraday (Open → Close)")
plt.title("Cumulative Return: Overnight vs Intraday (S&P 500)")
plt.xlabel("Date")
plt.ylabel("Cumulative Growth")
plt.legend()
plt.grid(True)
plt.show()
```