Class Test 4 - Solution

T-503-AFLE: Class Test 4 - Solution

- **Duration:** 1.5 hours
- **Total Marks:** 20 marks

1: Example 2.2.1 (Calculation of $F_{0,T}^{P}$ with Discrete Dividends) (2 marks)

A certain stock costs 40 today and will pay an annual dividend of 6 for the next 4 years. An investor wishes to purchase a 4-year prepaid forward contract for this stock. The first dividend will be paid one year from today and the last dividend will be paid just prior to delivery of the stock. Assume an annual effective interest rate of 5%.

Calculate the price of the prepaid forward contract.

Solution:
By (2.2.1), the price of the 4 -year prepaid forward contract is

$$\begin{aligned} F_{0,4}^{P} &= S(0) - \mathrm{PV}_{0,4}(\text{ Div }) \\ &= 40 - 6\left(1/1.05 + 1/1.05^{2} + 1/1.05^{3} + 1/1.05^{4}\right) \\ &= 18.72. \quad \text{(Answer: (D))} \end{aligned}$$

2: Example 3.1.1 (Hedging an implicit long position) (2 marks)

Determine which of the following risk management techniques can hedge the financial risk of an oil producer arising from the price of the oil that it sells:
I. Short forward position on the price of oil
II. Long put option on the price of oil
III. Long call option on the price of oil

This is a multiple choice question. Please select one of the answers below.
(A) I only
(B) II only
(C) III only
(D) I, II, and III
(E) The correct answer is not given by (A), (B), (C), or (D)

Solution:
Because the oil producer is to sell oil in the future, he/she will benefit from increases in the price of the oil. More precisely, his/her payoff of selling each unit of oil is the future (random) oil price, $S(T)$. As a result, he/she is long with respect to oil. The oil producer is therefore in need of positions which can help him hedge against the downside risk he/she faces arising from oil price. Here I and II can serve this purpose.

For I, entering into a short forward position means that the oil producer agrees to sell its oil for a predetermined price in contrast to a random price at a fixed time in the future, which protects the producer from decreases in oil price.

For II, buying a put option allows the producer to sell oil for a minimum price, the strike price, which protects the producer from drops in oil price below the strike price. This sets up a floor. **(Answer: (E))**

Remark. (i) Watch out! The producer is not short with respect to oil, although he/she is to sell oil.
(ii) Note that III protects the buyer of oil, who is vulnerable to increases in oil price, not the seller.

3: Problem 4.6.1. (Valuing a strangle) (2 marks)

Consider a 50-65 1-year strangle strategy. You are given:
(i) The stock currently sells for $\$55$.

(ii) In one year, the stock will either sell for $\$70$ or $\$45$.
(iii) The effective annual risk-free interest rate is 10%.

Calculate the current price of the strangle.

Solution:
It turns out that the strangle pays $(50-70)_+ + (70-65)_+ = 5$ in the up state and $(50-45)_+ + (45-65)_+ = 5$ in the down state as well. It is the same as a risk-free bond with a face value of 5 . The current price of the strangle is therefore the present value of 5 , or

$$V_0 = \frac{5}{1.1} = 4.5455.$$

Remark. There is no need to determine the tree parameters and the risk-neutral probability of an up move. For your information, they are $u = 70/55 = 14/11$, $d = 45/55 = 9/11$ and

$$p^* = \frac{(1+0.1) - 9/11}{14/11 - 9/11} = 0.62$$

4: Problem 6.4.15 (Pricing a Futures Option - II) (3 marks)

Assume the Black-Scholes framework.
You are given:
(i) The current price of the P&K 777 index is 500 .
(ii) The P&K 777 index pays dividends continuously at a rate proportional to its price. The dividend yield is 2%.
(iii) The continuously compounded risk-free interest rate is 6%.
(iv) The current prices and volatility of futures contracts on P&K 777 of various maturities:

Maturity (in Years)	1	2	3	4
Current Price	520.41	541.64	563.75	586.76
Volatility	30%			

Calculate the price of a 2-year 550-strike European put option on a 1-year futures contract (i.e., the futures matures at the end of 3 years).

Solution:
Note that the underlying futures contract matures three years from now. From the table in (iv), it has a current price of 563.75 and a volatility of 30%. With

$$\begin{aligned}
d_1 &= \frac{\ln(563.75/550) + \left(0.3^2/2\right)(\overbrace{\quad 2 \quad}^{\text{lifespan of option}})}{0.3\sqrt{2}} = 0.27033, \\
d_2 &= d_1 - 0.3\sqrt{2} = -0.15393, \\
N\left(-d_1\right) &= 0.39345, \\
N\left(-d_2\right) &= 0.56117,
\end{aligned}$$

the current price of the required put option is

$$P = 550\mathrm{e}^{-0.06(2)}(0.5596) - 563.75\mathrm{e}^{-0.06(2)}(0.3936) = 77.02.$$

BLACK-SCHOLES FRAMEWORK			
Equity		**FX**	**Futures**
S(0)=	563.75	X(0)	F_0,T^F
r=	0.06	r_d	
delta=	0.06	r_f	r
sigma=	0.3		
T=	2		
K=	550		
OPTION PRICES			
		Either:	
F^P_0,T=	500.0014	S(0)*exp(-delta*T)	
		S(0) - PV(Dividends)	
	True Val	Z-Table	
d_1=	0.270333	0.27	
d_2=	-0.15393	-0.15	
Phi(d_1)=	0.606548	0.6064	
Phi(d_2)=	0.438832	0.4404	
Phi(-d_1)=	0.393452	0.3936	
Phi(-d_2)=	0.561168	0.5596	
Call Optio	89.20982	88.37098	
Put Optio	77.01466	76.17582	

5: Problem 6.4.28. (Given the constituent prices, deltas, and elasticities, find the portfolio elasticity) (3 marks)

Assume the Black-Scholes framework. Consider a portfolio consisting of three European options, X, Y, and Z, on the same stock. You are given:

	X	Y	Z
Option price	6.8268	?	1.9299
Option delta	?	-0.4269	0.3537
Option elasticity	5.6496	-6.8755	9.1627

Calculate the elasticity of the portfolio.

Solution:
Considering option Z, we have

$$\Omega_Z = \frac{S\Delta_Z}{V_Z} = \frac{S(0.3537)}{1.9299} = 9.1627$$

which gives $S = 50$.
Turning to options X and Y , we get

$$\begin{cases} \Omega_X = \frac{S\Delta_X}{V_X} = \frac{50\Delta_X}{6.8268} = 5.6496 \\ \Omega_Y = \frac{S\Delta_Y}{V_Y} = \frac{50(-0.4269)}{V_Y} = -6.8755 \end{cases}$$

resulting in $\Delta_X = 0.7714$ and $V_Y = 3.1045$.
Finally, the elasticity of the portfolio is

$$\Omega_{\text{portfolio}} = \frac{S\Delta_{\text{portfolio}}}{V_{\text{portfolio}}} = \frac{50(0.7714 - 0.4269 + 0.3537)}{6.8268 + 3.1045 + 1.9299} = 2.9432.$$

6: Problem 7.4.4. (Calculation of holding profit for short calls with dividends - v2) (6 marks)

Assume the Black-Scholes framework. You are given:
(i) The current stock price is 50 .
(ii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%.
(iii) The volatility of the stock is 16%.
(iv) The prices of 1-year at-the-money European call and put options are 4.348 and 1.981, respectively.

Timothy has just written 200 units of the call in (iv), and he delta-hedged his position immediately. After 3 months, the stock price rises to 55 and the call price increases to 7.316 .

a) Calculate the cost of Timothy's initial investment. (3 marks)
b) Calculate the Timothy's payoff if he closes out his position at 3-months. (2 marks)
c) Calculate Timothy's 3-month holding profit. (1 mark)

(Hint: Remember to take care of dividends.)

Solution:
a)
r: 0.08
Z-table delta: -126.488
Z-table investment: 5, 454.79

By put-call parity, we have

$$4.348-1.981=50\mathrm{e}^{-0.03(1)}-50\mathrm{e}^{-r(1)},$$

which gives $r=0.08$. The delta of each 50 -strike 1-year call is $\mathrm{e}^{-0.03} N\left(d_1\right)$, where

$$d_1=\frac{\ln(50/50)+\left(0.08-0.03+0.16^2/2\right)(1)}{0.16\sqrt{1}}=0.3925,$$

so Timothy has incurred a delta of $-200\mathrm{e}^{-0.03} N(0.3925)=-200\mathrm{e}^{-0.03}(0.65266)=$ -126.6742. This means that he needs 126.6742 shares of the stock for delta-hedging. His net initial investment is

$$126.6742(50)-200(4.348)=5,464.1098$$

Mark allocation: 1 for r, 1 for delta, 1 for final answer.

b) **Z-table payoff:** 5546.005

$$[126.6742 \underbrace{e^{0.03(0.25)}}_{\text{div, reinvested}}(55)-200(7.316)]=5556.283$$

Mark allocation: 1 for correctly accounting for dividends, 1 for final answer

c) **Z-table profit:** -18.9824

$$[126.6742 \underbrace{e^{0.03(0.25)}}_{\text{div. reinvested}}(55)-200(7.316)]-5,464.1098\mathrm{e}^{0.08(0.25)}=-18.16.$$

PROBLEM 7.4.4						
BLACK-SCHOLES GREEKS						
Equity		FX				
S(0)=	50	X(0)		C	4.348	
r=	0.08	r_d		P	1.981	
delta=	0.03	r_f		r	0.080012	
sigma=	0.16					
T=	1					
K=	50				Time-0 Cashflow	
					True Val	Z-Table
FAIR VALUE					-5464.067	-5454.79
		Either:				
F^P_0,T=	48.52228	S(0)*exp(-delta*T)			Time-t Cashflow	
		S(0) - PV(Dividends)			t=	0.25
					S(t)=	55
	True Val	Z-Table			C(t)=	7.316
d_1=	0.3925	0.39				
d_2=	0.2325	0.23			5556.283	5546.005
Phi(d_1)=	0.652656	0.6517				
Phi(d_2)=	0.591925	0.591			Time-t Profit	
Phi(-d_1)=	0.347344	0.3483			-18.16558	-18.9824
Phi(-d_2)=	0.408075	0.409				
phi(d1)=	0.369366	0.3697				
phi(d2)=	0.388304	0.3885				
Put Optio	1.981087	1.97742				
CALL OPTION						
	True Val	Z-Table				
Fair Value	4.347546	4.34388				
Delta	0.633367	0.632439		Delta_V	-126.6733	-126.488
Vega	17.92249	17.93869				
Theta	-2.66941	-2.66868				
Rho	27.32079	27.27809				
Epsilon	-31.6683	-31.622				
Gamma	0.044806	0.044847				
Omega	7.284186	7.27966				

7: Problem 7.4.6. (Testing your conceptual understanding of Figure 7.1.1 - v2) (2 marks)

Assume the Black-Scholes framework. Yesterday, you sold a European call option on a nondividendpaying stock. The parameters were:

$$S(0) = K = 50, \quad r = 0.08, \quad \delta = 0, \quad \sigma = 0.25, \quad T = 1.$$

You immediately delta-hedged the commitment with shares of the stock. Today, you decide to close out all positions.

Which of the following statements about your delta-hedged portfolio today is incorrect?
(A) Your delta when today's stock price is $\$50$ is (approximately) zero.
(B) Stock price risk is not completely eliminated.
(C) You lose from large stock price moves in either direction.
(D) The larger the stock price today, the smaller your delta.
(E) Your gamma is a negative constant.

Solution:
Only Statement (E) is incorrect. Your gamma, although negative, is not a constant. It varies with today's stock price.