Verkfræðilegar Bestunaraðferðir

Day 12 - Group 2

T-423-ENOP

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Exercise 1

Response Correction Surrogate Models

Let $f(x) = \frac{\exp(x/3)\cdot 0.1\cdot x^2}{1+0.1\cdot x^2}$ and $c(x) = \exp(x/3)$. Perform the following tasks:

- Set up a surrogate model using a beta-correlation method at $x(1) = 1$ and then at $x(1) = 3$.
- Set up a surrogate model using a general response correction at $x(1) = 1$ and then at $x(1) = 3$.
- Visualize each of the above cases in a similar way as shown during the lecture (plots of f, c, s as well as linear approximation of f at the respective points).

Beta-correlation (left) and general response correction (right) models at $x(1) = 1$ ($f(x)$ – blue line, $c(x)$ – black line, surrogate model (red line), and tangent (dashed red line)):

Functionality

For the β-correlation method, we simply apply the formula given in lecture slides at points $x^{(k)} \in \{1,3\}$. The derivatives are approximated numerically.

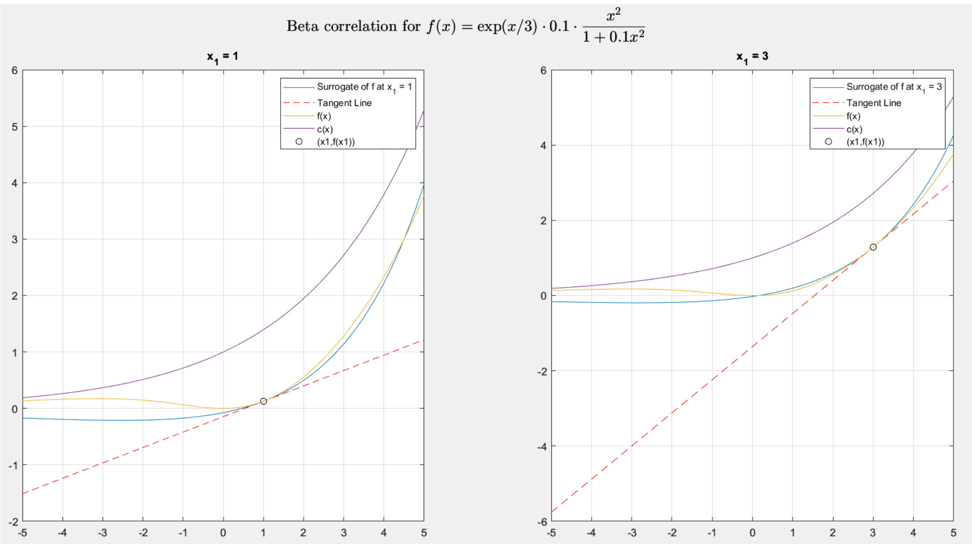
$$s_k(x) = \left(\frac{f(x^{(k)})}{c(x^{(k)})} + \frac{f'(x^{(k)})c(x^{(k)}) - c'(x^{(k)})f(x^{(k)})}{c(x^{(k)})^2}\right)^T (x - x^{(k)}) \cdot c(x)$$

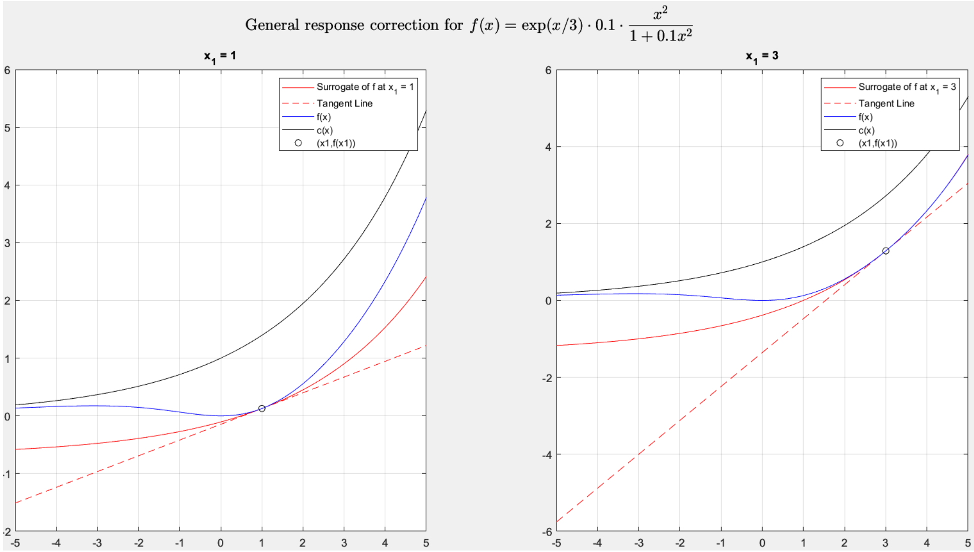
For the General response correction we also apply the formula given in lecture slides at the same points,

$$s_k(x) = f(x^{(k)}) + \frac{f'(x^{(k)})}{c'(x^{(k)})}[c(x) - c(x^{(k)})]$$

Solution

We plot the tangent line, $x(x)$, $f(X)$ and $s(x)$ using each method for each point.




General response correction for $f(x) = \exp(x/3) \cdot 0.1 \cdot \dfrac{x^2}{1+0.1x^2}$
$x_1 = 1$
Surrogate of f at $x_1 = 1$
Tangent Line
f(x)
c(x)
(x1,f(x1))
$x_1 = 3$
Surrogate of f at $x_1 = 3$
Tangent Line
f(x)
c(x)
(x1,f(x1))

Exercise 2

Description

Consider the following data pairs:

0.0000	1.0253	0.8000	-0.4323
0.1000	0.8702	0.9000	-0.0802
0.2000	0.5632	1.0000	0.2176
0.3000	0.1260	1.1000	0.4952
0.4000	-0.2467	1.2000	0.6125
0.5000	-0.5407	1.3000	0.5570
0.6000	-0.6864	1.4000	0.4531
0.7000	-0.5969	1.5000	0.2293

Set up radial basis function surrogate model for this data using Gaussian basis functions.

Use cross validation method to obtain the value of control parameter c of the basis function for which the generalization error of the model is minimized.

Practical hints:

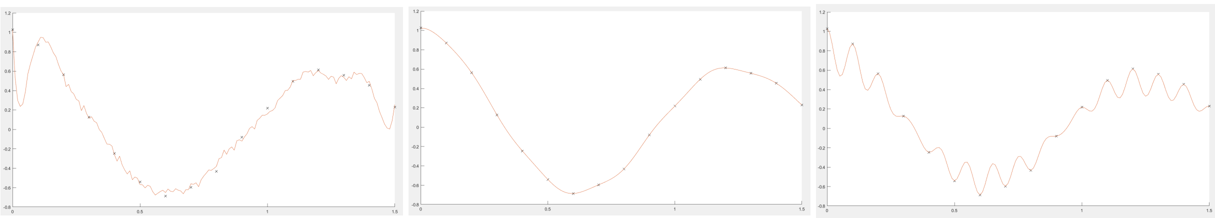
- Split data into eight groups for the sake of cross validation procedure.
- Consider c in the interval $c \in [2, 1000]$; optimize c in logarithmic scale.

Functionality

The radial basis function model is constructed in the exact same way as it was in previous exercises. The data points are then split into groups, where two points are used for testing and the other 14 are used for training, we do this 8 times. We then iterate from 2 to 1000 and compute the root mean square of the errors. Then we take the average between all groupings to find an accurate value for c.

Result

RBF Model for c = 2,32(which was found to be the best value as seen in the next figure) and 500, respectively



And here we can see the error plotted as a function of c on a loglog graph