Uncertainty Quantification on Spent Nuclear Fuel with Multifidelity Monte Carlo

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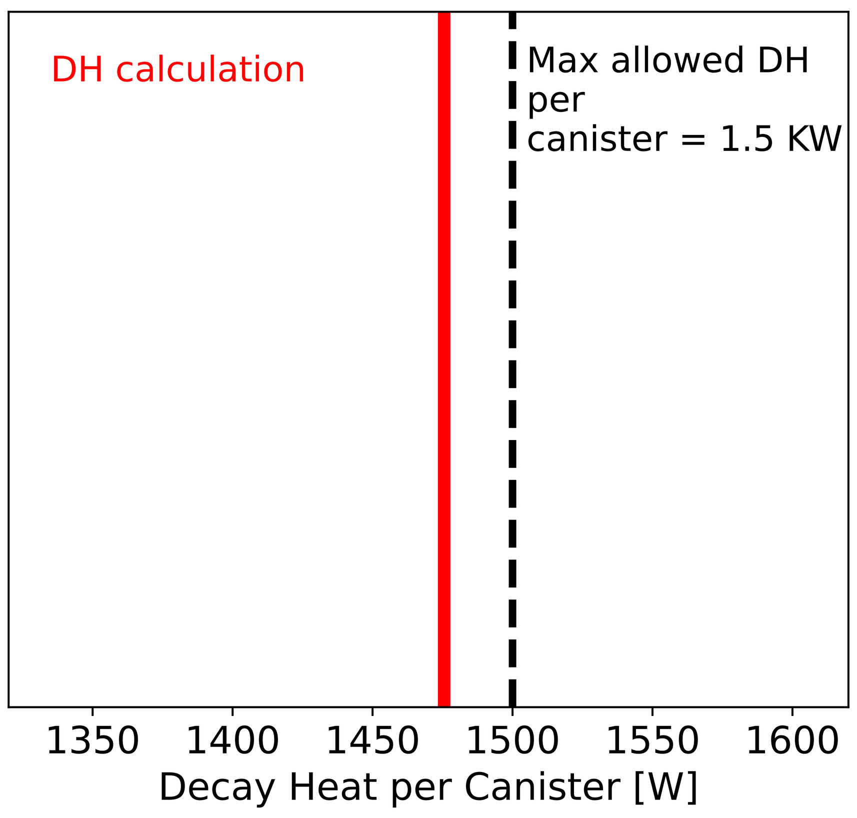
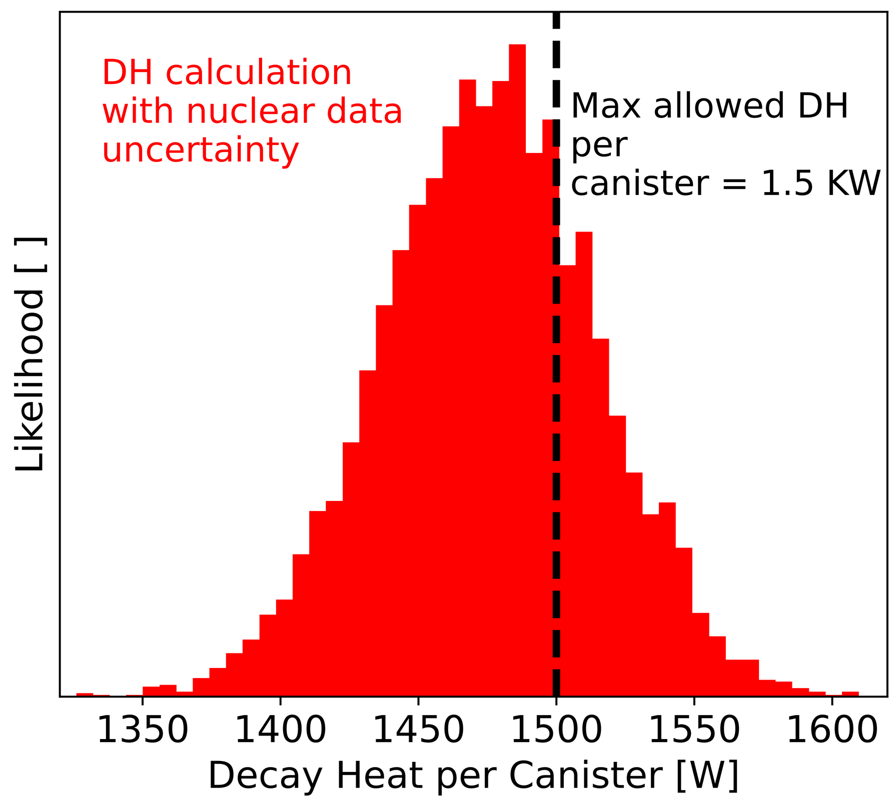
Goal: Reducing computational cost of uncertainty quantification in spent nuclear fuel

The accurate characterisation of spent nuclear fuel (SNF) is crucial to ensure its safe storage and disposal. This is achieved by means of nuclear computations that simulate and predict SNF characteristics such as decay heat, nuclide content, or k-effective. It is important that these predictions include uncertainty estimates such that both risks and costs may be reduced. Traditional uncertainty quantification (UQ) methods such as Monte Carlo or surrogate modelling are computationally expensive, and inaccurate when the number of uncertain inputs is large. In this work Multifidelity Monte Carlo (MFMC), a modern method unexplored in the field of nuclear computations as of yet, is applied to the UQ of nuclide content and decay heat for SNF [1].





Old Method: Simple Monte Carlo [2]

$$f : \underbrace{\begin{pmatrix} \texttt{Fresh fuel parameters} \\ \texttt{Design parameters} \\ \texttt{Operation Parameters} \\ \texttt{Reaction cross sections} \\ \texttt{Neutron multiplicity} \\ \texttt{Fission yields} \\ \texttt{Fission spectrum} \end{pmatrix}}_{\text{Uncertain nuclear data} \in \mathbb{R}^{15\,000}} \rightarrow \underbrace{\begin{pmatrix} \texttt{Decay Heat} \\ \texttt{Nuclide Content} \\ \texttt{k-eff} \end{pmatrix}}_{\text{Uncertain characteristics of SNF}}$$

For every SNF assembly:

1. Sample M inputs from covariance matrix
2. **Perform** M **expensive simulations** with different inputs
3. Compute sample estimators μ_M and σ_M.

- Slow error convergence $SE(\sigma_M) \simeq \sqrt{\frac{\mathrm{Var}[f]}{2M}}$
- $M \sim 1000$ simulations required per assembly!
- Up to **9 hours per simulation** (CASMO5, OpenMC, ...)
- Expected $> 12\,000$ SNF assemblies in Switzerland

New Method: 2LMC = Multifidelity MC [3] + Lasso [4]

For one SNF assembly:

1. Compute μ_M and σ_M with Simple MC and $M > 500$
2. Train a Lasso machine learning model $\tilde{f}$ with the M samples

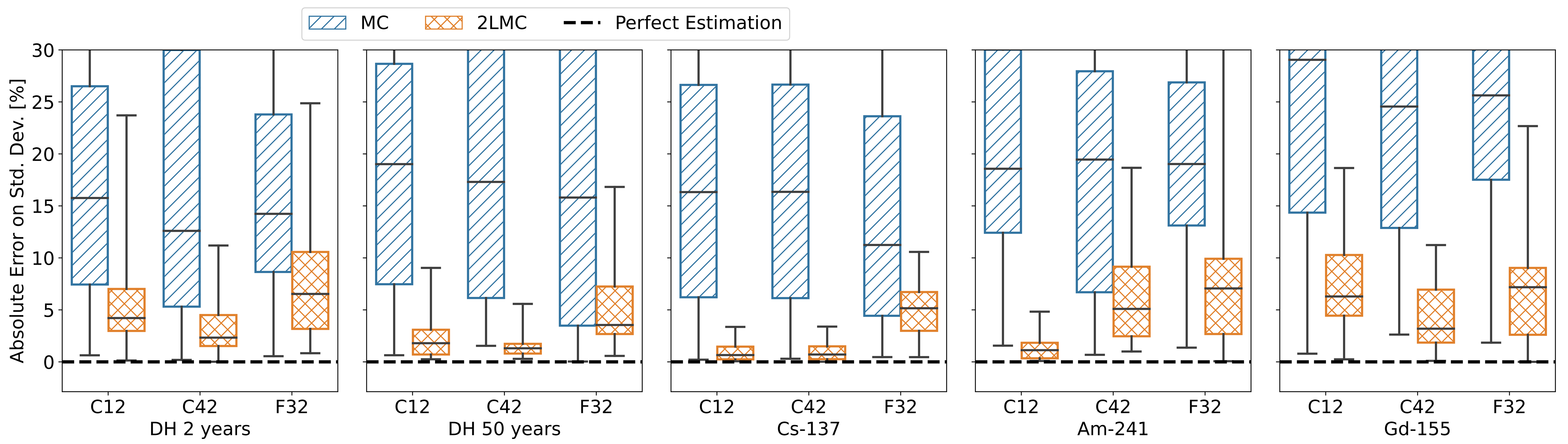
For all other (similar) SNF assemblies:

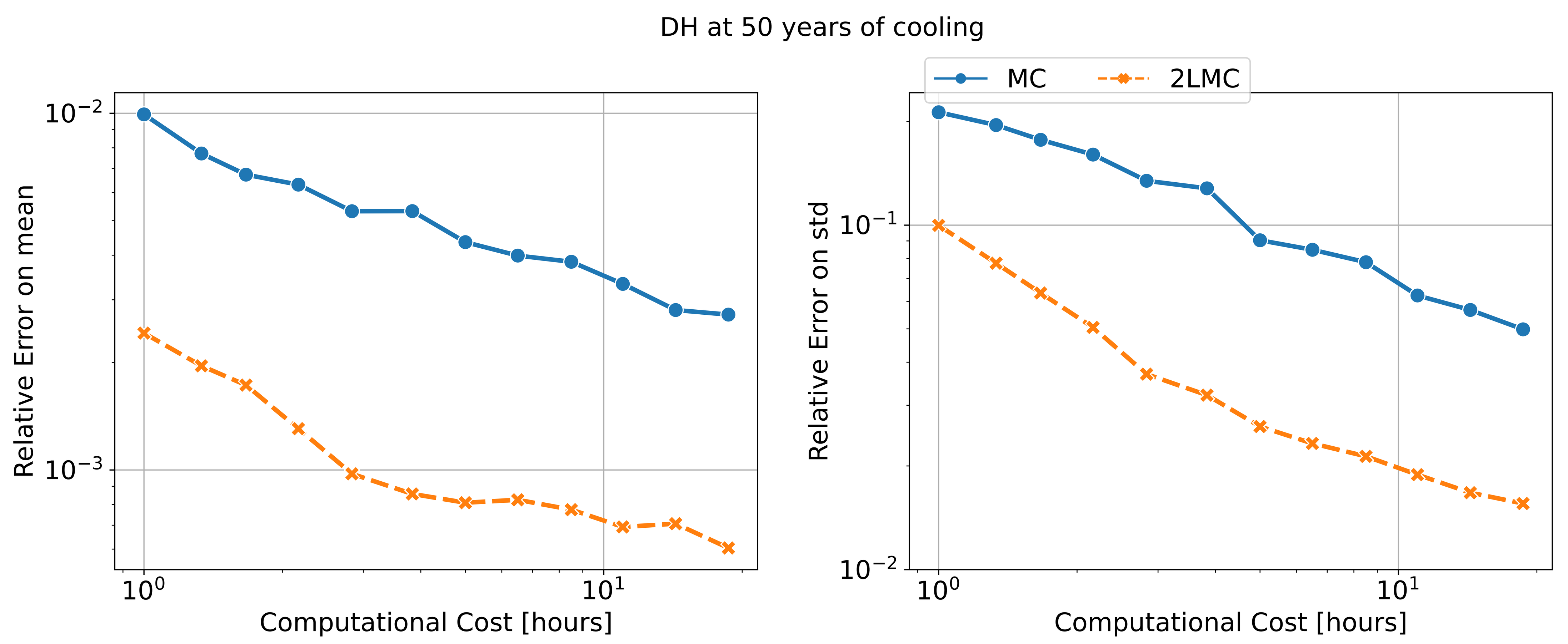
1. Perform $N \sim 10$ expensive simulations
2. Compute the MFMC estimators

$$\mu_{N,M} = \frac{1}{M}\sum_{i=1}^{M} \alpha\tilde{f}(z_i) + \frac{1}{N}\sum_{i=1}^{N} f(x_i) - \alpha\tilde{f}(x_i)$$
$$\sigma^2_{N,M} = \sigma^2_M(\alpha\tilde{f}) + \sigma^2_N(f) - \sigma^2_N(\alpha\tilde{f})$$

with $\alpha \in \mathbb{R}$ adjusted to minimise error.

- Smaller error than MC
  $SE(\sigma_{N,M}) \simeq \sqrt{\frac{\mathrm{Var}[f]}{2N}}\sqrt{1 + \frac{\mathrm{Var}^2[\tilde{f}]}{\mathrm{Var}^2[f]} - \frac{\mathrm{Cov}[f^2, \tilde{f}^2]}{\mathrm{Var}^2[f]}}$
- **Fewer simulations required**
- **Computational cost reduced up to a factor of 10**





[1] Manuscript in preparation.

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[3] S. Krumscheid, F. Nobile, and M. Pisaroni. Quantifying uncertain system outputs via the multilevel Monte Carlo method — Part I: Central moment estimation. *Journal of Computational Physics*, 414:109466, August 2020.

[4] R. Tibshirani. Regression Shrinkage and Selection via the Lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, 58(1):267–288, 1996.

Acknowledgement

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