



# Lasso Monte Carlo (LMC): A Novel Method for High-Dimensional Uncertainty Quantification in Nuclear Burn-up Calculations

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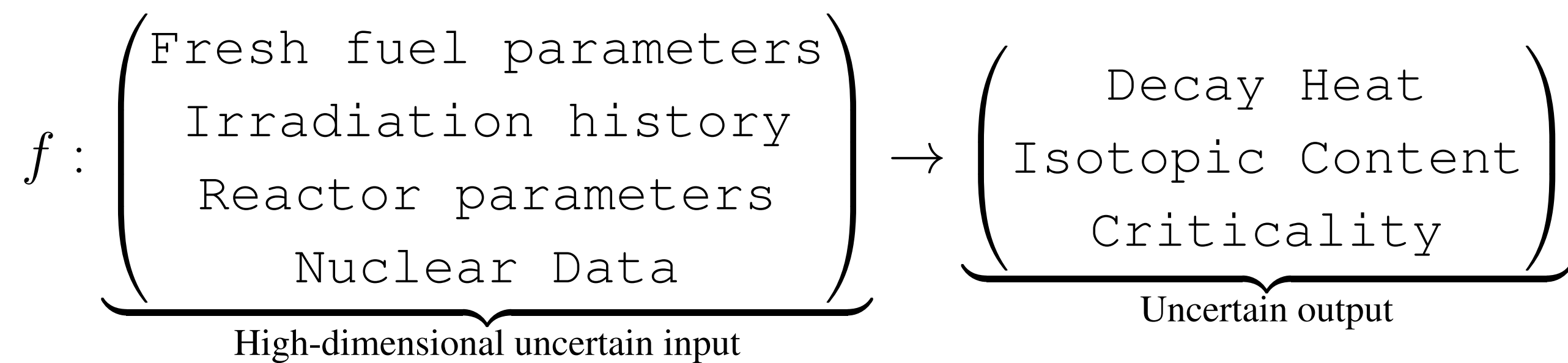


PSI FoKo Poster Event  
7<sup>th</sup> December 2022

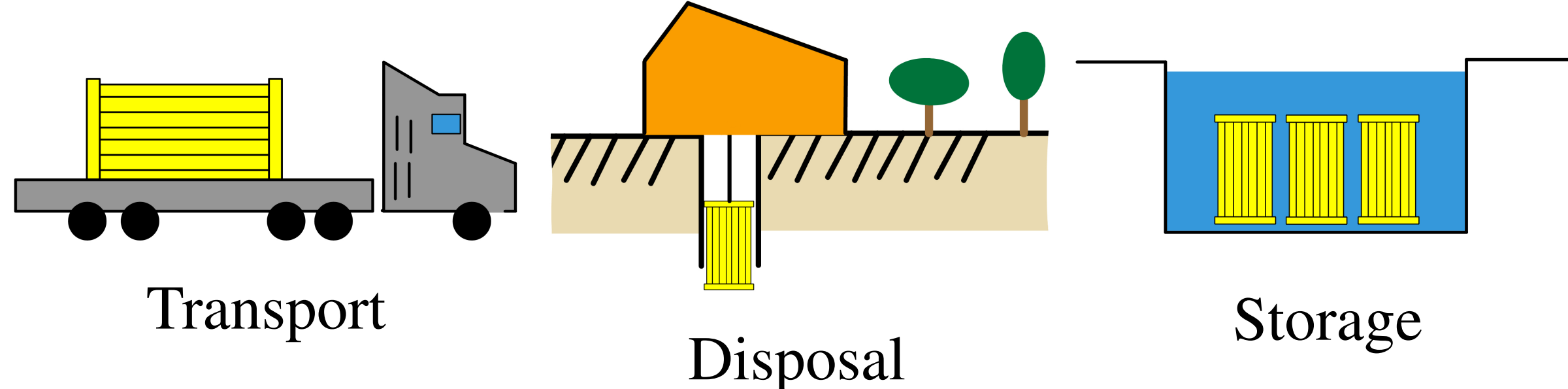
## Background and Motivation

**Goal: Efficient uncertainty quantification (UQ) for spent nuclear fuel (SNF)**

Nuclear Burn-up calculation:



Knowledge of SNF parameters **and uncertainty** is necessary to **reduce risks and costs** of

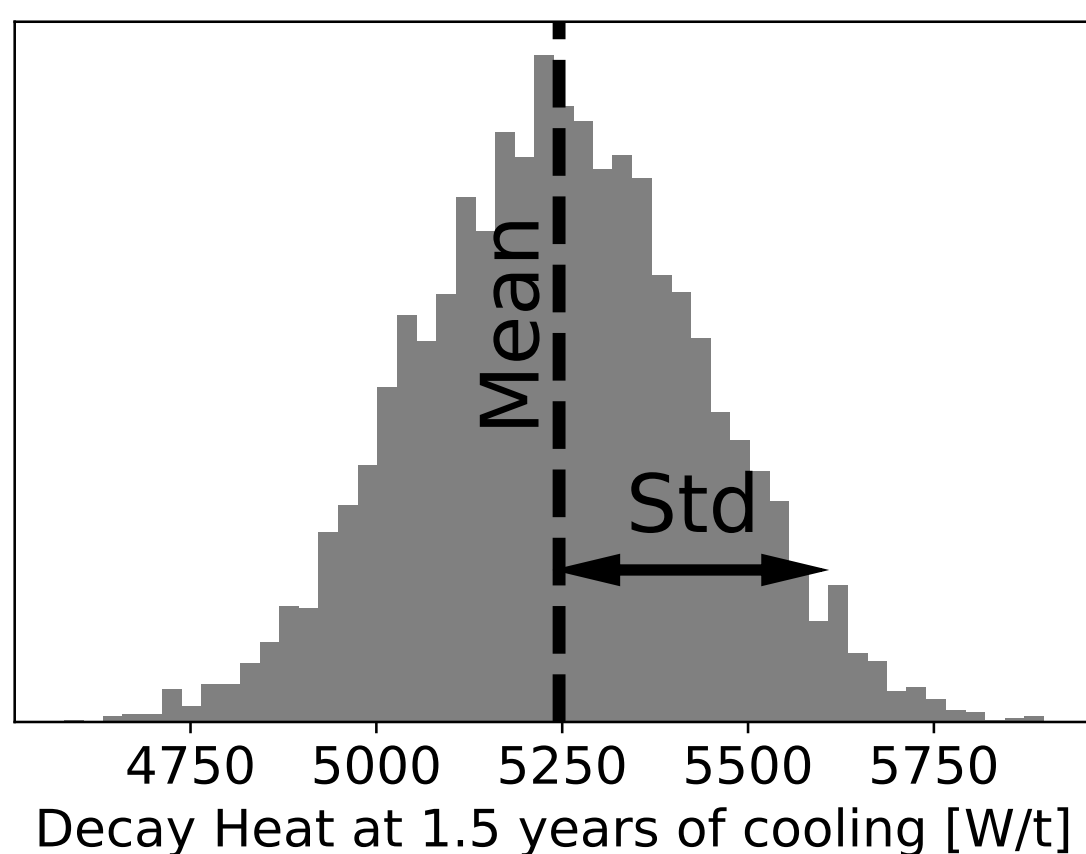


## Current Common Methods for UQ

### Monte Carlo (MC) [1]

- Requires  $N$  expensive simulations
- Slow error convergence  
 $\text{error} \sim \frac{1}{\sqrt{N}}$ ,  
**many simulations required**

Output uncertainty:



### Surrogate models

- Biased
- Require large training set, i.e. **many simulations (curse of dimensionality)**

**Current methods are not suited for high-dimensional UQ!**

**Goal of new method:** Estimate output mean  $\mu$  and variance  $\sigma^2$  with **small number of simulations  $N$** .

In Switzerland  $> 12000$  SNF are expected, each requiring UQ.

## New Method: LMC

**Lasso Monte Carlo (LMC) [2]** is a new UQ method, that combines **Lasso Regression [3]**, **Multi Level MC [4]**.

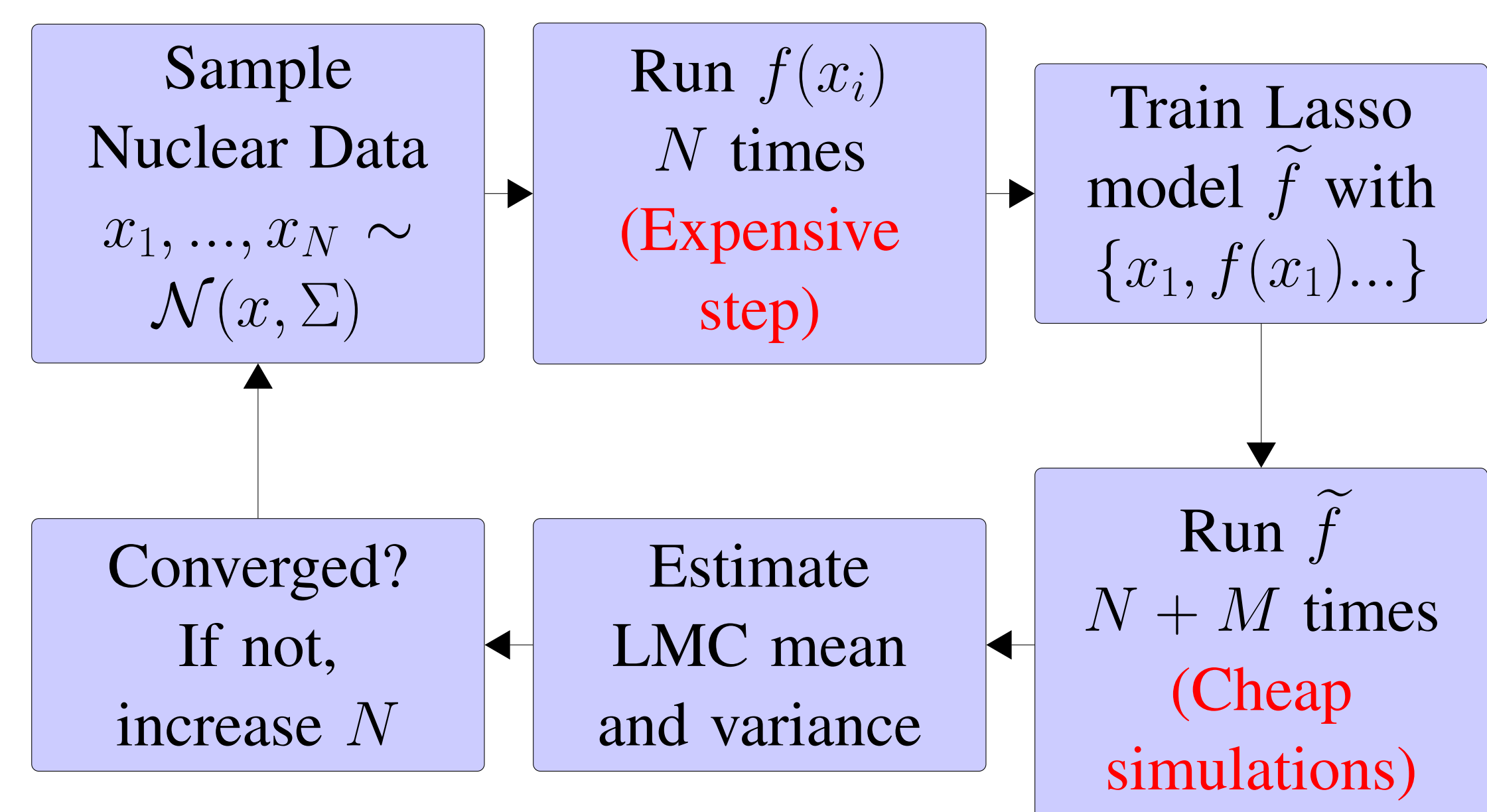
LMC ingredients:

- A **computationally expensive** model (e.g. CASMO, SCALE, few hours per simulation)

$$f: \mathbb{R}^{10^4} \rightarrow \mathbb{R}$$

Nuclear Data  $\rightarrow$  Decay Heat

- A surrogate model  $\tilde{f} \sim f$  with **strong regularisation**, such that it can be trained with few samples, e.g. **Lasso**.
- Combine  $f$  and  $\tilde{f}$  with **MLMC** to estimate mean and variance.



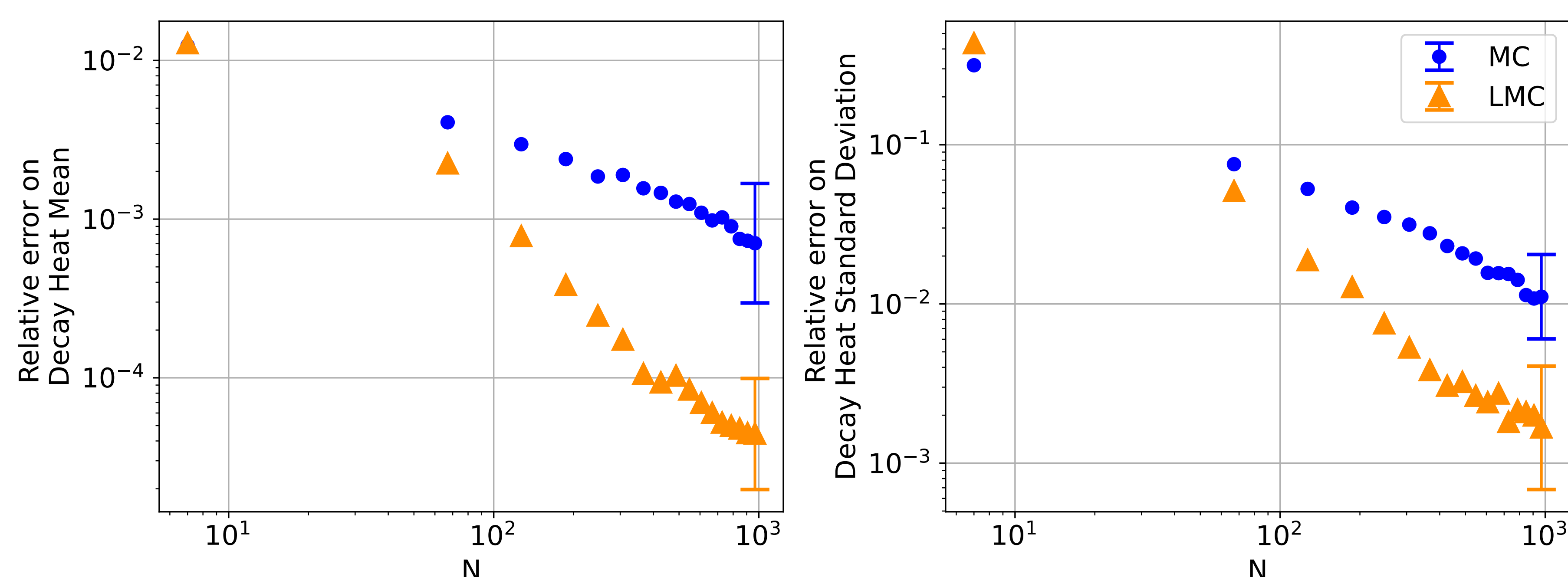
With  $N \ll M$

$$\text{Mean}_{LMC} = \underbrace{\frac{1}{M} \sum_{i=1}^M \tilde{f}(z_i)}_{\text{leading term}} + \underbrace{\frac{1}{N} \sum_{i=1}^N f(x_i) - \tilde{f}(x_i)}_{\text{correction term}}$$

$$\text{Variance}_{LMC} = \underbrace{\tilde{\sigma}_M^2}_{\text{leading term}} + \underbrace{\sigma_N^2 - \tilde{\sigma}_N^2}_{\text{correction term}}$$

**LMC is unbiased, and more accurate than simple MC for a given  $N$ .**

## Method Comparison: predicting decay heat of SNF

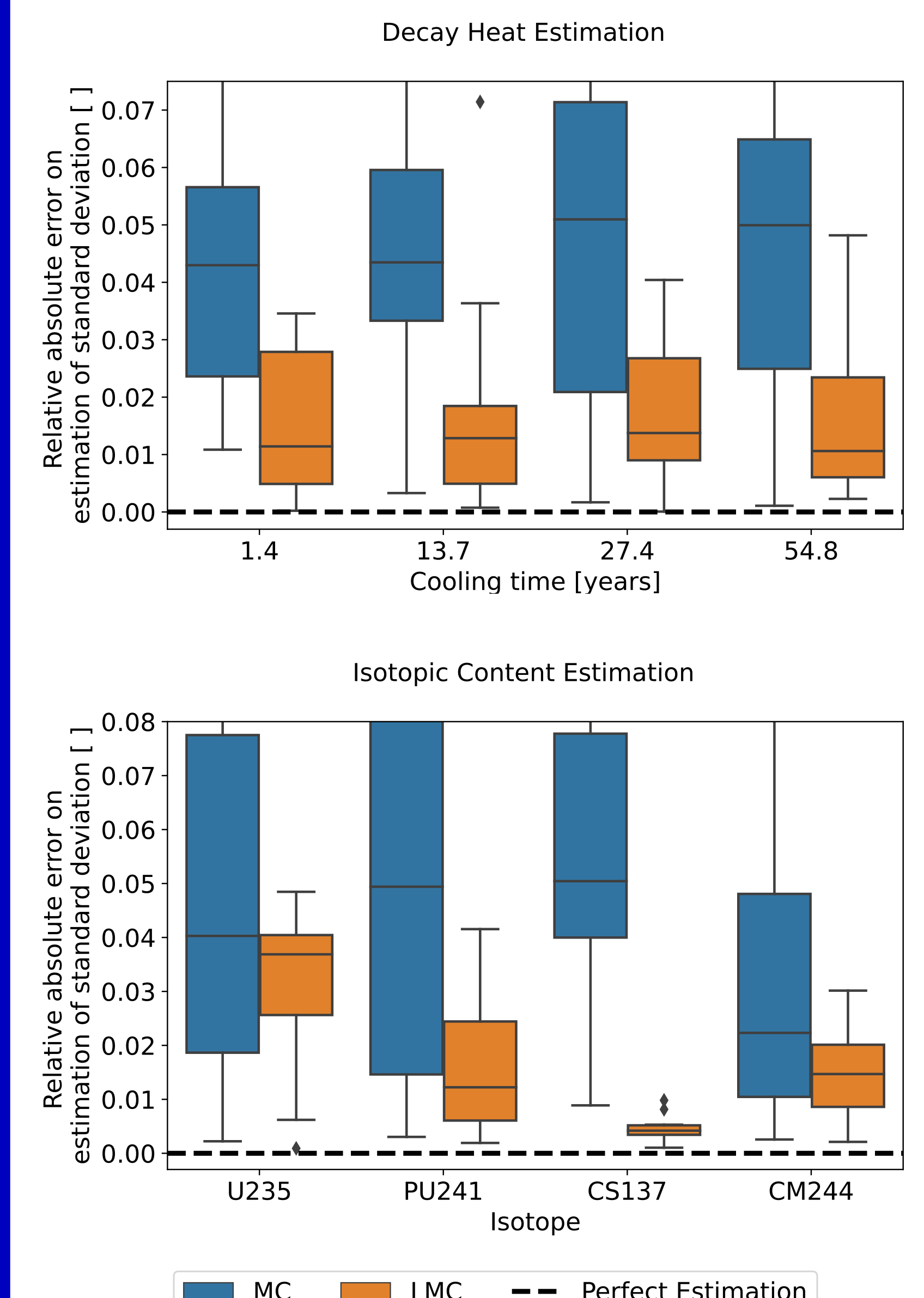


Prediction of decay heat at 500 days of cooling, for the Ringhals-2 PWR [5].

## Ongoing and Future Work

- Applying method to other types of fuel and different sources of uncertainty
- Preprint for applied mathematics journal available [2], publication in nuclear physics journal being prepared
- Try other surrogate models with strong regularisation (sparse PCE, NNs)

## Estimating Other Quantities



Estimations with  $N = 150$

## Acknowledgement

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[1] O. Leray, H. Ferroukhi, M. Hursin, A. Vasiliev, and D. Rochman. Methodology for core analyses with nuclear data uncertainty quantification and application to Swiss PWR operated cycles. *Annals of Nuclear Energy*, 110:547–559, December 2017.

[2] A. Albà, R. Boiger, D. Rochman, and A. Adelmann. Lasso Monte Carlo, a novel method for high dimensional uncertainty quantification, 2022. 10.48550/ARXIV.2210.03634.

[3] R. Tibshirani. Regression Shrinkage and Selection via the Lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, 58(1):267–288, 1996.

[4] S. Krumscheid, F. Nobile, and M. Pisaroni. Quantifying uncertain system outputs via the multilevel Monte Carlo method — Part I: Central moment estimation. *Journal of Computational Physics*, 414:109466, August 2020.

[5] F. Sturek, L. Agrenius, and O. Osifo. Measurements of decay heat in spent nuclear fuel at the Swedish interim storage facility, Clab. Technical Report R-05-62, Svensk Kärnbränslehantering AB, December 2006.