Endogenous Input-Output Linkages and Structural Changes

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Abstract

This paper presents a model of economic growth with endogenous input-output linkages to study the interplay between structural changes and the evolution of production networks. Endogenous linkages translate the static difference among industries – the fixed cost of forming firm relationships being lower in some industries than in others – into different productivity growth rates, bringing about structural changes. The expanding industries also become more prominent intermediate input suppliers, a prediction consistent with the empirical pattern in the United States. A simple calibration of the model to the U.S. economy suggests that, comparing to a model with a fixed production network, the endogenous adjustment of linkages and the resulting structural changes double the welfare gains from a technology shock that lowers the linkage fixed cost universally.

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1 INTRODUCTION

Economic growth is often accompanied by structural changes, defined as the shifts of economic resources and activities across sectors or industries. Common measures of structural changes – sectoral shares in consumption expenditure, value-added, and employment – highlight the decline of agriculture, the hump-shaped evolution of manufacturing, and the rise of services over time.[1] However, another prominent feature of the modern economy is the extensive input-output linkages weaving firms of various industries into a production network. It is then natural to doubt the premise that production networks are fixed structures when industries experience such different patterns of growth. One may even ask: could the endogenous adjustment of linkages be driving structural changes in the first place?

Figure 1 plots the sectoral shares in the domestic intermediate input expenditure of the United States, providing a first glimpse of the changing input-output structure. Over the past two decades, the U.S. economy has decreased its use of manufacturing intermediates while relying more on the service sector for intermediate inputs. Since firms constitute not only the units of economic activities but also the "nodes" in production networks, sectoral firm dynamics are relevant to both structural changes and the evolution of the input-output structure. Figure 2 contrasts the trend in the number of manufacturing establishments with that of the service sector, suggesting a steady shift of establishment mass from manufacturing to services. Moreover, Figure 3 reveals that this reallocation of establishment mass is driven mostly by the difference in establishment entry rate between the two sectors. In summary, evolution of the production networks occurs along side structural changes: the trend in sectoral relative sizes echos the evolution of their relative importance as intermediate input suppliers.

In this paper, I propose a growth model with many industries and endogenous input-output linkages to study the interplay between structural changes and the evolution of production networks. In the model, industries have only one intrinsic difference: they vary in the efficiency of adopting upstream linkages.[2] Each firm produces a differentiated variety for two purposes: to meet final consumption demand and to satisfy the intermediate input demand of other firms. The differentiated varieties are produced from labor and bundles of intermediate inputs (i.e., other varieties) so that labor is the only factor of production in this economy. Both the consumption basket and the intermediate input bundles aggregate varieties in a nested CES fashion with the elasticity of substitution higher within than across industries. While households can access all varieties in the economy, which varieties a firm is able to source as intermediate inputs depends on the production network structure – a variety is accessible only if the linkage exists. In reality, establishing firm-to-firm relationships requires economic

[1] These stylized facts are established and strengthened by a long strand of literature dating back to Kuznets (1957). Recent empirical works by Buera and Kaboski (2012) as well as Adler, Boppart, and Müller (2018) confirm these patterns for multiple countries over a long period of time.

[2] To focus on reallocation across industries, I abstract away from asymmetry within industries.

resources and I therefore introduce a per-linkage fixed cost that is decreasing in the linkage efficiency of the downstream firm.[3] Firms operate as long as they are not hit by the exogenous exit shock and new firms can be created subject to the sunk costs of entry. Thus, growth in this model is achieved through expanding product variety à la Romer (1990).

I solve the model by looking at the problem of a social planner.[4] In each period, the planner decides how to allocate labor among firms and intermediates along input-output linkages, how to distribute firm mass across industries, and how to form linkages among firms. The dynamic problem facing the planner is then to allocate output minus the linkage fixed costs between consumption and firm creation. To focus on the role of endogenous linkages, there is no exogenous productivity growth in the model so that the economy converges to a steady state. During the transition, linkages are redistributed towards industries with relatively high linkage efficiency. The non-stationary distribution of linkages is the result of the following trade off. On one hand, the planner prefers to connect suppliers to firms with relatively high linkage efficiency because the associated fixed costs are lower. On the other hand, the planner would like to maintain certain product diversity in all industries, even the ones very inefficient in adopting linkages, due to the lower elasticity of substitution across industries than within. Consequently, the planner allows the most efficient firms to access all intermediate input varieties while connecting the less efficient ones only partially to the production network. As the economy grows through the accumulation of firm mass, the number of possible bilateral firm relationships expands exponentially, forcing the planner to be more selective in which firms to grant complete upstream linkages.

Due to the love for variety embedded in the CES production function, the redistribution of supplier linkages alters firm productivity through the changes in input diversity. Therefore, endogenous linkages translate the static difference among industries (linkage efficiency) into different productivity growth rates. The direction of structural changes then depends on the cross-industry elasticity of substitution. If varieties of different industries are substitutes, resources are reallocated towards better-connected industries; if there is complementarity among varieties across industries, then it is the less-connected industries that expand in relative sizes. In either case, industries with intermediate levels of linkage efficiency undergo a non-monotonic growth pattern, as they experience the transition

[3] There are many reasons for why linkage fixed costs could differ across industries. For example, introducing an additional intermediate input may require significant changes to the existing production lines in some industries but little modification in others. Additionally, products differ in the extent to which their attributes can be communicated in a systematic way, that is, their codifiability. As a result, product information may be specified via the phone or the internet for some industries, while in-person product inspection may be necessary for others. Empirical works by Fort (2017) and Juhász and Steinwender (2018) suggest that product codifiability affects the degree to which firms or countries engage in domestic or international intermediate input trade.

[4] I then show that the planner's solution can be decentralized in a market equilibrium with monopolistically competitive firms, once the production of varieties is optimally subsidized through lump-sum taxation.

from enjoying full access to all suppliers in the economy to having only an incomplete set of upstream linkages.

This paper belongs to the literature trying to understand the economic forces behind structural changes. Broadly speaking, this literature takes two approaches. One approach focuses on the demand side, emphasizing the role of non-homothetic preferences.[5] The other approach highlights a supply-side explanation: productivity growth rates differ across industries.[6] By adopting CES preferences, this paper falls into the second approach. However, structural changes in my model are not contingent on exogenous differences in industry productivity growth rates. In fact, industry productivity in my model grows endogenously at different speed, because the rearrangement of input-output linkages alters the production technology of firms in a biased manner, favoring those most efficient in adopting linkages.

This paper also speaks to the literature on the macroeconomic relevance of production networks, from the earlier work by Long and Plossor (1983) to recent contributions such as Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Baqaee (2018). In particular, this paper joins the budding literature that introduces the endogenous formation of input-output linkages to macroeconomic models, such as Lim (2018) and Oberfield (2018). While these two papers keep the mass of firms fixed, my model accommodates firm entry which is essential in driving both the evolution of the production network and structural changes.

The rest of the paper is organized as follows. Section 2 sets up the model and solves the planner's static problem. Section 3 turns to the dynamic problem of the planner and characterizes the condition under which structural changes can occur. Section 4 presents a simple calibration of the model to the U.S. economy and studies the welfare impact of a technology shock that leads to a universal reduction in the linkage fixed cost. Section 5 concludes. Proofs and derivation details are relegated to the appendix.

[5]Examples from this strand of literature include Matsuyama (1992, 2002), Echevarria (1997), Laitner (2000), Caselli and Coleman (2001), and Buera and Kaboski (2006) among others.

[6]This technology-based explanation can be traced back to Baumol (1967) and is recently explored by Ngai and Pissarides (2007) as well as Acemoglu and Guerrieri (2008). In addition, Boppart (2014) combines non-homothetic preferences with differential productivity growth in a single framework and shows that the income effect emphasized by the demand-side approach and the relative price effect emphasized by the supply-side approach are of similar quantitative importance in accounting for the observed structural changes in the United States.

2 The model

This section sets up a growth model with many industries and endogenous input-output linkages. I derive the equilibrium as the solution to a social planner problem. In the appendix, I show that the planner's allocation can be sustained in a market equilibrium with monopolistically competitive firms, once the appropriate policy intervention is in place.

2.1 Preferences and technology

The economy hosts a constant mass L of identical households, which supply labor inelastically and have preferences

$$\int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}-1}{1-\gamma} dt$$

where c_t is consumption per capita at time t.

There is a unit-mass continuum of industries indexed by $i \in [0,1]$. Each industry hosts an endogenous mass of firms, each producing a differentiated variety indexed by $j \in [0, N_i]$.[7] The differentiated varieties can be used to produce the final good and be adopted as intermediate inputs by other firms. The distribution of input-output linkages determines whether a variety is accessible to intermediate customers (i.e., other firms).

Production The unique final good is assembled from the differentiated varieties via a nested CES aggregator:

$$Y = \left\{ \int_0^1 \left[\int_0^{N_i} X_i^\beta (j)\, dj \right]^{\frac{\alpha}{\beta}} di \right\}^{\frac{1}{\alpha}}$$

where $\alpha < \beta < 1$ so that the elasticity of substitution is higher within than across industries.

The differentiated varieties are produced from labor and a bundle of intermediate inputs:

$$q_i(j) = l_i(j)^\sigma m_i(j)^{1-\sigma}$$

where $0 < \sigma < 1$ and $m_i(j)$ is the firm's intermediate input bundle which aggregates the differentiated varieties accessible via input-output linkages:

$$m_i(j) = \left\{ \int_0^1 \left[\int_0^{N_{i'}} x_{i,i'}(j,j')^\beta \, \mathbb{I}_{i,i'}(j,j')\, dj' \right]^{\frac{\alpha}{\beta}} di' \right\}^{\frac{1}{\alpha}}$$

[7] Throughout this paper, "firms" refer specifically to the producers of the differentiated varieties, and therefore I use "firms" interchangeably with "varieties" .

where $\mathbb{I}_{i,i'}(j,j')$ is an indicator variable taking on value 1 if firm j' from industry i' is an intermediate input supplier and 0 otherwise.

To focus on cross-industry differences, I assume that all varieties in any given industry are symmetric. Under this assumption of within-industry symmetry, the aggregate production function can be written as

$$Y = \left(\int_0^1 X_i^{\alpha} N_i^{\frac{\alpha}{\beta}} di\right)^{\frac{1}{\alpha}}$$

Accordingly, the intermediate input bundle used by a firm reduces to

$$m_i = \left[\int_0^1 x_{i,i'}^{\alpha} \left(\mu_{i,i'} N_{i'}\right)^{\frac{\alpha}{\beta}} di'\right]^{\frac{1}{\alpha}}$$

where $\mu_{i,i'} \equiv \int_0^{N_{i'}} \mathbb{I}_{i,i'}(j,j')\, dj'/N_{i'}$ gives the fraction of industry-i' firms supplying to a firm industry-i.[8]

Linkage and firm creation Firms operate as long as they are not hit by the exogenous exit shock, which occurs with probability δ every period. Given the total stock of firms $N \equiv \int_0^1 N_i di$ in the economy, the social planner is free to redistribute firm mass across industries by choosing $\{N_i\}_{i\in[0,1]}$ period by period. The social planner also chooses input-output linkages $\left\{\mu_{i,i'}\right\}_{i,i'\in[0,1]}$, subject to a per-linkage fixed cost in units of the final good that has to be paid every period. Specifically, the fixed cost of establishing a firm-to-firm relationship between an industry-i buyer and an industry-i' seller is κ/ϕ_i, where ϕ_i is the efficiency of an industry-i firm in incorporating an additional supplier into its intermediate input bundle and is the only source of cross-industry asymmetry in this model. Consequently, the economy admits a single state variable, the aggregate firm mass N, with law of motion:

$$\dot{N}_t = N_t^e - \delta N_t$$

where N_t^e is the mass of new firms. The social planner can create firms at a cost of ν units of the final good per new firm. Therefore, the aggregate resource constraint of the economy is

$$C + \int_0^1 \int_0^1 \frac{\kappa}{\phi_i} \mu_{i,i'} N_i N_{i'} di di' + \nu N^e = Y$$

where $C \equiv cL$ is aggregate consumption and $\int_0^1 \int_0^1 (\kappa/\phi_i)\, \mu_{i,i'} N_i N_{i'} di di'$ is the total fixed costs for creating all the firm-to-firm linkages in the economy.

[8]Throughout the paper, whenever a variable has two subscripts, the former index always refers to the industry of the customer firm and the latter index always refers to the industry of the supplier firm.

2.2 THE STATIC PROBLEM

The static problem of the social planner is to maximize aggregate output Y net of all the linkage fixed costs by choosing: (1) the allocation of labor across industries $\{l_i\}_{i\in[0,1]}$; (2) the allocation of variety output among intermediate users $\{x_{i,i'}\}_{i,i'\in[0,1]}$; (3) the distribution of firm mass across industries $\{N_i\}_{i\in[0,1]}$; and (4) the distribution of input-output linkages $\{\mu_{i,i'}\}_{i,i'\in[0,1]}$ subject to the constraint of aggregate labor supply:

$$\int_0^1 l_i N_i di = L$$

the identity of total firm mass:

$$\int_0^1 N_i di = N$$

and the constraints of variety quantities:

$$q_i = X_i + \int_0^1 x_{i',i}\mu_{i',i}N_{i'}di' \qquad \text{for all } i \in [0,1]$$

which states that variety output q_i must meet both the final demand X_i and all the intermediate demand $\int_0^1 x_{i',i}\mu_{i',i}N_{i'}di'$.

The solution to the static problem is characterized by four sets of optimality conditions. First, the efficient allocation of labor requires the equalization of marginal product of labor across industries:

$$\frac{\partial Y}{\partial X_i}\frac{\partial X_i}{\partial l_i}\frac{1}{N_i} = \frac{\partial Y}{\partial X_{i'}}\frac{\partial X_{i'}}{\partial l_{i'}}\frac{1}{N_{i'}} \qquad \text{for all } i, i'$$

Second, when choosing the intermediate input quantity $x_{i,i'}$ that a firm in industry i sources from a supplier in industry i', the planner faces the following trade-off: increasing $x_{i,i'}$ leads to more output of industry-i variety while diverting industry-i' output from consumption to fulfilling intermediate input demand. Therefore, the efficient allocation of intermediate input varieties requires balancing this trade-off:

$$\frac{\partial Y}{\partial X_i}\frac{\partial X_i}{\partial x_{i,i'}} = -\frac{\partial Y}{\partial X_{i'}}\frac{\partial X_{i'}}{\partial x_{i,i'}} \qquad \text{for all } i, i' \tag{1}$$

Third, the planner distributes total firm mass across industries so as to equalize the marginal returns to varieties:

$$\frac{\partial Y}{\partial N_i} - \int_0^1 \frac{\kappa}{\phi_{i''}}\mu_{i'',i}N_{i''}di'' = \frac{\partial Y}{\partial N_{i'}} - \int_0^1 \frac{\kappa}{\phi_{i''}}\mu_{i'',i'}N_{i''}di'' \qquad \text{for all } i, i' \tag{2}$$

Finally, the planner sets up input-output linkages according to the following first order condition:

$$\frac{\partial Y}{\partial \mu_{i,i'}} - \frac{\kappa}{\phi_i} N_i N_{i'} \begin{cases} \leqslant 0 & \text{if } \mu_{i,i'} = 0 \\ = 0 & \text{if } \mu_{i,i'} \in [0,1] \\ \geqslant 0 & \text{if } \mu_{i,i'} = 1 \end{cases} \tag{3}$$

where the left hand side corresponds to the return to an additional firm-to-firm relationship between an industry-i buyer and an industry-i' seller. Using these four sets of optimality conditions, I characterize the solution to the planner's static problem in the proposition below.

Proposition 1 *Given the state variable, total firm mass $N \equiv \int_0^1 N_i di$, the planner's static problem yields the following solution: labor is allocated according to $l_i = L/N$ for all i; input-output linkages are formed according to*

$$\mu_{i,i'} \equiv \tilde{\mu}_i = \begin{cases} \frac{\phi_i}{\underline{\phi}} & \text{if } \phi_i < \underline{\phi} \\ 1 & \text{if } \phi_i \geqslant \underline{\phi} \end{cases} \tag{4}$$

where the efficiency cutoff is

$$\underline{\phi} = \kappa \left(\frac{\sigma}{1-\sigma} \right) \left(\frac{\beta}{1-\beta} \right) \frac{N^2}{Y}; \tag{5}$$

the distribution of firm mass across industries is given by

$$N_i = \tilde{\mu}_i^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} \frac{N}{A} \quad \text{where } A \equiv \int_0^1 \tilde{\mu}_i^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} di \tag{6}$$

and the allocation of intermediate inputs is given by

$$x_{i,i'} = \left(\frac{1-\sigma}{\sigma} \right) N^{-\left(\frac{1+\alpha}{\alpha}\right)} Y \tilde{\mu}_i^{-1} N_{i'}^{\frac{\beta-\alpha}{\alpha\beta}}. \tag{7}$$

Consequently, aggregate output in the social optimum is

$$Y = \sigma (1-\sigma)^{\frac{1-\sigma}{\sigma}} L A^{\frac{\beta-\alpha}{\sigma\alpha\beta}} N^{\frac{1-\beta}{\sigma\beta}} \tag{8}$$

Since all firms share the same labor productivity, the planner equalizes employment per firm across the economy. In contrast, the returns to input-output linkages depend on the efficiency ϕ_i of downstream firms in adopting additional intermediate input varieties. This leads to an efficiency cutoff $\underline{\phi}$ partitioning all firms into two groups: for firms with linkage efficiency $\phi_i \geqslant \underline{\phi}$, the planner allows them to source intermediate inputs from all other firms in the economy by setting $\tilde{\mu}_i = 1$; for firms with $\phi_i < \underline{\phi}$, the planner connects them only partially to the production networks by setting $\tilde{\mu}_i \in (0,1)$

where $\tilde{\mu}_i$ is increasing in ϕ_i. Therefore, the number of suppliers that a firm has weakly increases in its linkage efficiency ϕ_i. The distribution of firm mass depends crucially on the cross-industry elasticity of substitution $1/(1-\alpha)$. If varieties of different industries are substitutes ($\alpha > 0$), N_i increases in $\tilde{\mu}_i$, suggesting that the planner allocates more firms to better-connected industries. If there is complementarity among varieties across industries ($\alpha < 0$), N_i is then decreasing in $\tilde{\mu}_i$, implying that larger industries have fewer upstream linkages. By Proposition 1, total fixed costs of establishing all the input-output linkages in the economy is

$$\int_0^1 \int_0^1 \frac{\kappa}{\phi_i} \mu_{i,i'} N_i N_{i'} di di' = \kappa \frac{Z}{A} N^2$$

where

$$Z \equiv \int_0^1 \phi_i^{-1} \tilde{\mu}_i^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)+1} di$$

2.3 The dynamic problem

The dynamic problem of the planner is to allocate output minus linkage costs between consumption and firm creation:

$$\max_{\{C_t, N_t\}_{t \geqslant 0}} \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma} - 1}{1-\gamma} dt$$

subject to

$$\dot{N}_t = \frac{1}{\nu} \left(Y_t - \kappa \frac{Z_t}{A_t} N_t^2 - C_t \right) - \delta N_t \tag{9}$$

and the initial condition $N_0 > 0$. Taking relevant partial derivatives of the Hamiltonian yields the Euler equation for consumption growth:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\gamma \nu} \left[\frac{\partial}{\partial N_t} \left(Y_t - \kappa \frac{Z_t}{A_t} N_t^2 \right) - \nu (\delta + \rho) \right] \tag{10}$$

with transversality condition:

$$\lim_{t \to \infty} e^{-\rho t} C_t^{-\gamma} N_t = 0$$

3 Structural changes and aggregate growth

This section characterizes the dynamic equilibrium of the economy and examines under what conditions structural changes can take place. To facilitate the discussion about structural changes, I define the firm mass shares of industries:

$$n_i \equiv \frac{N_i}{N} = \begin{cases} \left(\frac{\phi_i}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} A^{-1} & \text{if } \phi_i < \underline{\phi} \\ A^{-1} & \text{if } \phi_i \geqslant \underline{\phi} \end{cases} \tag{11}$$

where the linkage efficiency cutoff $\underline{\phi}$ and the term A are defined as in (5) and (6). Since all firms in the economy have the same employment size by Proposition 1, $\{n_i\}_{i\in[0,1]}$ coincide with the employment shares of industries and therefore can represent industry relative sizes. The next proposition relates the growth rate of relative industry sizes n_i to the dynamics of the linkage efficiency cutoff $\underline{\phi}$, an aggregate variable.

Proposition 2 *Suppose that linkage efficiency ϕ_i follows a continuous distribution $\mathscr{F}$ with support $[\phi_{min}, \phi_{max}]$. The dynamics of relative industry sizes satisfy*

$$\frac{\dot{n}_i}{n_i} = \begin{cases} -\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)\,\Phi\frac{\dot{\underline{\phi}}}{\underline{\phi}} & \text{if } \phi_i < \underline{\phi} \\ \alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)\,(1-\Phi)\,\frac{\dot{\underline{\phi}}}{\underline{\phi}} & \text{if } \phi_i \geqslant \underline{\phi} \end{cases} \tag{12}$$

where

$$\Phi \equiv \frac{\int_{\underline{\phi}}^{\phi_{max}} d\mathscr{F}(\phi)}{\int_{\phi_{min}}^{\underline{\phi}} \left(\frac{\phi}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi) + \int_{\underline{\phi}}^{\phi_{max}} d\mathscr{F}(\phi)}$$

Furthermore, for two industries i and i' with $\phi_i > \phi_{i'}$, the growth rate of their relative sizes satisfies

$$\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_{i'}}{n_{i'}} = \begin{cases} 0 & \text{if } \phi_{i'} < \phi_i < \underline{\phi} \text{ or } \underline{\phi} < \phi_{i'} < \phi_i \\ \alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)\,\frac{\dot{\underline{\phi}}}{\underline{\phi}} & \text{if } \phi_{i'} < \underline{\phi} < \phi_i \end{cases} \tag{13}$$

Proposition 2 suggests that the distribution of firm mass across industries is non-stationary as long as $\dot{\underline{\phi}} \neq 0$ and $\phi_{min} < \underline{\phi} < \phi_{max}$. Furthermore, if we divide industries into two groups according to whether the linkage efficiency is above or below the cutoff $\underline{\phi}$, then firm mass shares evolve in the same way for all industries within a group. If $\alpha > 0$, firm mass (hence employment) distribution shifts towards better-connected industries (those with $\phi_i \geqslant \underline{\phi}$) when the linkage efficiency cutoff rises over time ($\dot{\underline{\phi}} > 0$) and in the opposite direction when the cutoff falls ($\dot{\underline{\phi}} < 0$). If $\alpha < 0$, firm mass is redistributed towards industries with fewer upstream linkages (those with $\phi_i < \underline{\phi}$) when $\dot{\underline{\phi}} > 0$ and in the opposite direction when $\dot{\underline{\phi}} < 0$.

3.1 GROWTH WITHOUT STRUCTURAL CHANGES

I define the absence of structural changes as the state where $\dot{n}_i/n_i = \dot{n}_{i'}/n_{i'}$ for all i and i'. One immediately observes that structural changes cannot take place if the distribution of input-output linkages $\{\tilde{\mu}_i\}_{i\in[0,1]}$ is held fixed, because $\dot{\underline{\phi}} = 0$ in this case of exogenous production networks. In the case of endogenous linkages, structural changes can still be absent if all industries lie above or below the efficiency cutoff $\underline{\phi}$ (i.e., $\underline{\phi} \leqslant \phi_{min}$ or $\underline{\phi} \geqslant \phi_{max}$). Since the parameter κ regulates the fixed cost of establishing linkages, the planner's allocation features a solution for the linkage efficiency cutoff $\underline{\phi}$ outside the support $[\phi_{min}, \phi_{max}]$ when κ is sufficiently low or high. The next proposition identifies the threshold levels of κ below or above which the economy grows without structural changes and characterizes the aggregate dynamics in each case.

Proposition 3 *If $\kappa \geqslant \overline{\kappa}$, all industries lie below the linkage efficiency cutoff ($\underline{\phi} \geqslant \phi_{max}$). In this case, the dynamic equilibrium is given by the following system of differential equations:*

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\gamma}\left\{\frac{1}{\nu}\left[1-2(1-\sigma)\right]\left(\frac{1-\beta}{\sigma\beta}\right)\Lambda N_t^{\frac{[1-2(1-\sigma)]\frac{1-\beta}{\sigma\beta}}{1-\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)}-1} - \delta - \rho\right\} \tag{14}$$

$$\frac{\dot{N}_t}{N_t} = \frac{1}{\nu}\left\{\left[1-\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)\right]\Lambda N_t^{\frac{[1-2(1-\sigma)]\frac{1-\beta}{\sigma\beta}}{1-\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)}-1} - \frac{C_t}{N_t}\right\} - \delta \tag{15}$$

where Λ is a constant term. If $\kappa \leqslant \underline{\kappa}$, all industries lie above the linkage efficiency cutoff ($\underline{\phi} \leqslant \phi_{min}$). In this case, the dynamic equilibrium is given by the following system of differential equations:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\gamma}\left\{\frac{1}{\nu}\left[\left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1-\sigma}{\sigma}} L N_t^{\frac{1-\beta}{\sigma\beta}-1} - 2\kappa\int_{\phi_{min}}^{\phi_{max}} \frac{1}{\phi} d\mathscr{F}(\phi) N_t\right] - \delta - \rho\right\} \tag{16}$$

$$\frac{\dot{N}_t}{N_t} = \frac{1}{\nu}\left[\sigma(1-\sigma)^{\frac{1-\sigma}{\sigma}} L N_t^{\frac{1-\beta}{\sigma\beta}-1} - \kappa\int_{\phi_{min}}^{\phi_{max}} \frac{1}{\phi} d\mathscr{F}(\phi) N_t - \frac{C_t}{N_t}\right] - \delta \tag{17}$$

The full expressions of the threshold levels $\overline{\kappa}$ and $\underline{\kappa}$ as well as the constant Λ are given in the appendix. In both cases, there exists a unique steady state that is locally saddle-path stable, provided that $2(1-\sigma) < 1$ and $(1-\beta)/\beta < \sigma$. Furthermore, the steady-state aggregate firm mass N^{SS} satisfies $dN^{SS}/d\kappa < 0$ and $dN^{SS}/dL > 0$. Finally, $\underline{\kappa} < \overline{\kappa}$ provided that labor endowment L is sufficiently large.

For sufficiently small linkage fixed costs ($\kappa \leqslant \underline{\kappa}$), all firms in the economy are fully connected with each other via input-output linkages. Contrastingly, when linkage fixed costs are too high ($\kappa \geqslant \overline{\kappa}$), all firms in the economy adopt intermediate inputs from only a subset of their peers. In both cases, a small technology shock that reduces κ or a market size shock that raises L to the economy already at the steady state leads it to a new steady state with a higher number of firms. During the transition to the new steady state, all industries expand at the same rate $\dot{N}_t/N_t$ and the distribution of linkages $\{\tilde{\mu}_i\}_{i\in[0,1]}$ remain unchanged.

3.2 GROWTH WITH STRUCTURAL CHANGES

I define structural changes as the state where $\dot{n}_i/n_i \neq \dot{n}_{i'}/n_{i'}$ for at least some i and i'. Proposition 2 establishes that structural changes can occur only if the planner's choice of the linkage efficiency cutoff $\underline{\phi}$ partitions firms into two groups by whether they have complete or incomplete upstream linkages. This case requires intermediate levels of the linkage fixed cost parameter κ. The next proposition characterizes the aggregate dynamics when growth is accompanied by structural changes.

Proposition 4 *The economy undergoes structural changes as long as $\phi_{min} < \underline{\phi} < \phi_{max}$ and $\dot{\underline{\phi}} \neq 0$. In this case, the dynamic equilibrium is given by the following system of differential equations:*

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\gamma}\left\{\frac{1}{\nu}\left[\left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1-\sigma}{\sigma}} LA_t^{\frac{\beta-\alpha}{\sigma\alpha\beta}} N_t^{\frac{1-\beta}{\sigma\beta}-1} - 2\frac{Z_t}{A_t}\kappa N_t\right] - (\delta+\rho)\right\} \tag{18}$$

$$\frac{\dot{N}_t}{N_t} = \frac{1}{\nu}\left[\sigma(1-\sigma)^{\frac{1-\sigma}{\sigma}} LA_t^{\frac{\beta-\alpha}{\sigma\alpha\beta}} N_t^{\frac{1-\beta}{\sigma\beta}-1} - \frac{Z_t}{A_t}\kappa N_t - \frac{C_t}{N_t}\right] - \delta \tag{19}$$

$$\frac{\dot{\underline{\phi}}_t}{\underline{\phi}_t} = \left(2-\frac{1-\beta}{\sigma\beta}\right)A_t\left[A_t - \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)\int_{\phi_{min}}^{\underline{\phi}_t}\left(\frac{\phi}{\underline{\phi}_t}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi)\right]^{-1}\frac{\dot{N}_t}{N_t} \tag{20}$$

where A_t and Z_t are functions of $\underline{\phi}_t$ whose full expressions are given in the appendix. A sufficient condition for this system to admit a unique locally stable steady state is $\phi_{min}/\phi_{max} > \alpha(1-\beta)(1-\sigma) / [(1-\alpha)\beta - \alpha(1-\beta)\sigma]$. Furthermore, the steady-state aggregate firm mass N^{SS} and linkage efficiency cutoff $\underline{\phi}^{SS}$ satisfies $dN^{SS}/d\kappa < 0$, $dN^{SS}/dL > 0$, $d\underline{\phi}^{SS}/d\kappa > 0$, and $d\underline{\phi}^{SS}/dL > 0$. Given initial condition $N_0 < N^{SS}$, the economy experiences structural changes throughout the entire transition to the steady state if the linkage fixed cost parameter κ satisfies $\underline{\kappa}' \leqslant \kappa \leqslant \overline{\kappa}'$, where the threshold levels $\underline{\kappa}'$ and $\overline{\kappa}'$ are given in the appendix. If $\underline{\kappa} < \kappa < \underline{\kappa}'$ or/and $\overline{\kappa}' < \kappa < \overline{\kappa}$, the economy experiences structural changes only during part of the transition to the steady state. Finally, $\underline{\kappa} < \underline{\kappa}'$ and $\overline{\kappa}' < \overline{\kappa}$ provided that labor endowment L is sufficiently large.

In this model, structural changes are driven by the cross-industry differences in linkage efficiency ϕ_i, as well as the assumption that varieties are less substitutable across industries than within industries. Since the fixed costs of creating linkages decrease in the linkage efficiency of the downstream industries (i.e., the intermediate input customers), the planner prefers to concentrate linkages in industries with high ϕ_i, which is why the optimal linkage density $\tilde{\mu}_i$ weakly increases in industry linkage efficiency ϕ_i. However, the planner also faces the trade-off between allocating resources to industries most efficient in adopting linkages and maintaining sufficient product diversity in all industries, because households care about not only the total number of varieties but also how the varieties are distributed across industries (a direct consequence of the elasticity of substitution being lower across than within industries). Due to this trade-off, the constraint $\tilde{\mu}_i \leqslant 1$ becomes binding for industries with $\phi_i \geqslant \underline{\phi}$. As the economy grows, the linkage efficiency cutoff $\underline{\phi}$ rises, because the number of total possible firm relationships N^2 increases twice as fast as that of total firm mass N, prompting the planner to be more "selective" in which industries to concentrate linkages in. Consequently, $\tilde{\mu}_i$ falls in industries with $\phi_i < \underline{\phi}$ by Proposition 1, which implies that upstream linkages become sparser for these industries below the cutoff. Furthermore, the endogenous adjustment of linkages leads to differential productivity growth on the industry level:

$$\frac{\dot{(q_i/l_i)}}{q_i/l_i} = (1-\sigma)\frac{\dot{(Y/L)}}{Y/L} + (1-\sigma)\left(\frac{1-\beta}{\beta}\right)\frac{\dot{\tilde{\mu}}_i}{\tilde{\mu}_i}$$

The above equation decomposes the growth rate of industry productivity into a common component driven by aggregate productivity growth and an idiosyncratic component driven by the rearrangement of input-output linkages.[9] Specifically, industries low in linkage efficiency (those with $\phi_i < \underline{\phi}$) have relatively low productivity growth rate because linkages are being reallocated away from them ($\dot{\tilde{\mu}}_i/\tilde{\mu}_i < 0$) as the economy grows. Since the CES production function entails a "love of variety", the relative loss of input diversity due to linkage rearrangement ultimately results in productivity disadvantage. To summarize, endogenous linkages translate the static difference among industries (linkage efficiency) into uneven productivity growth.

To an economy already at the steady state (and satisfies $\underline{\kappa}' \leqslant \kappa \leqslant \overline{\kappa}'$), a small technology shock that reduces κ or a market size shock that raises L leads the economy to a new steady state with a larger aggregate firm mass ($N_1^{SS} > N_0^{SS}$) and a lower linkage efficiency cutoff ($\underline{\phi}_1^{SS} < \underline{\phi}_0^{SS}$). Since the linkage efficiency cutoff $\underline{\phi}$ rises as the economy accumulates firm mass during the transition, it must be that on impact $\underline{\phi}$ "overshoots" the new steady-state cutoff $\underline{\phi}_1^{SS}$, falling to $\underline{\phi}_1 < \underline{\phi}_1^{SS}$. During the transition to the new steady state, industry dynamics follow different patterns depending on the linkage efficiency ϕ_i relative to the cutoff. If $\alpha > 0$, industries with $\phi_i > \underline{\phi}_1^{SS}$ ($\phi_i < \underline{\phi}_1$) see their firm mass shares n_i grow (shrink) monotonically throughout the entire transition, whereas those with $\underline{\phi}_1 < \phi_i < \underline{\phi}_1^{SS}$ first expand then shrink in their relative sizes. If $\alpha < 0$, industries with $\phi_i < \underline{\phi}_1$ ($\phi_i > \underline{\phi}_1^{SS}$) see their

[9] To derive this decomposition, we start from (28), then substitute in $l_i = L/N$, (6), and (8), and finally time-differentiate both side of the equation.

firm mass shares n_i grow (shrink) monotonically throughout the entire transition, whereas those with $\underline{\phi}_1 < \phi_i < \underline{\phi}_1^{SS}$ first shrink then expand in their relative sizes.

Finally, I study the relationship between structural changes and the evolution of intermediate input expenditure shares. In a competitive equilibrium that decentralizes the planner's allocation, the price of an industry-i variety is given by $(\partial Y/\partial X_i)/N_i$. Therefore, the expenditure share of intermediate inputs supplied by industry i in the economy's total spending on intermediates is

$$\Lambda_i \equiv \frac{\int_0^1 x_{i',i}\mu_{i',i}N_iN_{i'}di'\,(\partial Y/\partial X_i)/N_i}{\int_0^1\int_0^1 x_{i',i}\mu_{i',i}N_iN_{i'}di'\,(\partial Y/\partial X_i)/N_i di}$$

The next proposition relates the relative changes in intermediate expenditure shares to the relative changes in firm mass shares:

Proposition 5 *For any two industries i and i', their relative importance as intermediate input suppliers is related to their relative sizes as given by the following equation:*

$$\frac{\dot{\Lambda}_i}{\Lambda_i} - \frac{\dot{\Lambda}_{i'}}{\Lambda_{i'}} = \frac{\dot{n}_i}{n_i} - \frac{\dot{n}_{i'}}{n_{i'}} \tag{21}$$

Proposition 5 implies that structural changes ($\dot{n}_i/n_i \neq \dot{n}_{i'}/n_{i'}$) are necessary for there to be redistribution of intermediate input expenditure shares across industries. Specifically, industries that gain more firm mass also become more prominent intermediate input suppliers, a prediction consistent with the patterns presented in Figure 1 and Figure 3.

4 Quantitative Example

In this section, I perform a simple calibration of the model. I then use the calibrated model to study the impacts of a technology shock that lowers the linkage fixed cost parameter κ universally. The motivation for such a shock is the information and communication technologies (ICT) revolution during the 1990s, when the surging usage of internet and mobile phones (as illustrated in Figure 4) arguably made it easier for businesses to establish supplier-customer relationships with each other.

4.1 Calibration

To calibrate the model, I first impose a parametric assumption on the distribution of linkage efficiency ϕ across industries: ϕ follows a Pareto distribution with shape parameter ζ and support $[\phi_{min}, \infty)$.[10]

[10] In order for output to be finite, the Pareto shape parameter must satisfy $\zeta > (1-\sigma)\alpha(1-\beta)/(\beta-\alpha)$. This condition also guarantees the local stability of the steady state when $\phi_{max} = \infty$.

Accordingly, this model is characterized by 11 parameters: α, β, σ, κ, ν, δ, ρ, γ, ζ, ϕ_{min}, L and one initial condition N_0. I choose values for these parameters as follows. I consider two cases of α: $\alpha = -1/3$ and $\alpha = 1/5$. These values correspond respectively to the cross-industry elasticity of substitution $1/(1-\alpha)$ being 0.75 and 1.25, consistent with the empirical finding that this high-level elasticity of substitution is close to 1 (Atalay 2017, Oberfield and Raval 2014). For the elasticity of substitution within an industry $1/(1-\beta)$, I set it at 4 (implying $\beta = 3/4$), which lies within the range estimated by the empirical literature (e.g., Broda and Weinstein 2006) and adopted by the quantitative trade literature (e.g. Costinot and Rodríguez-Clare 2014). For the inter-temporal preference parameters, I choose $\rho = 0.02$ and $\gamma = 1$ as benchmarks. Since it is the ratio $\underline{\phi}/\phi_{min}$ that matters for the aggregate variables, I thus normalize ϕ_{min} to 1.

I calibrate the rest of the parameters to relevant statistics of the U.S. economy. The exogenous firm exit rate $\delta = 0.096$ corresponds to the average of U.S. establishment exit rate during 1997-2015.[11] Labor endowment $L = 105.58$ (in millions) is set to the U.S. employment size averaged over the same time period. In the model, σ gives the common proportion of variety output that goes into final consumption: $\sigma = X_i/q_i$. I show in the appendix that σ is also the ratio of GDP to gross output and set $\sigma = 0.56$ corresponding to the period average of this ratio. Finally, I assume that the U.S. economy was at the steady state in 1997 and calibrate the remaining three parameters (κ, ν, and ζ) to jointly match aggregate output $Y = 8.61$ (in trillion USD), aggregate consumption $C = 5.56$ (in trillion USD), and total firm mass $N = 5.37$ (in millions) as observed in 1997. Additional calibration details are given in the appendix.

4.2 The impacts of a technology shock

Using the calibrated model, I now study the impacts of a 10% permanent reduction in κ, which governs the level of linkage fixed costs. Figure 5 shows the aggregate dynamics in response to this technology shock for two cases: varieties of different industries are substitutes ($\alpha = 1/5$) or complements ($\alpha = -1/3$). The left panel illustrates the overshooting behavior of the linkage efficiency cutoff $\underline{\phi}$. On impact, this cutoff drops since linkages have become cheaper to form universally. During the transition to the new steady state, the cutoff rises as the economy gains firm mass (the right panel) because the exponentially-growing linkage possibilities force the planner to be more selective in which firm pairs to connect. Quantitatively, the cumulative welfare gains from this technology shock amount to 74.0% and 86.5% of the initial welfare level for the cases of $\alpha = 1/5$ and $\alpha = -1/3$ respectively. In an alternative model where the production network is fixed (i.e., neither the firm mass distribution $\{N_i\}_{i\in[0,1]}$ nor the linkage distribution $\{\tilde{\mu}_i\}_{i\in[0,1]}$ is responsive to shocks), the cumulative

[11] I choose 1997 as the initial point because expenditure data on domestic intermediates is not available for the earlier years.

welfare gains, which is equivalent to the discounted sum of a series of static gains, are 35.9% and 40.6% for the two cases of α respectively. Therefore, more than half of the gains from this technology shock can be attributed to the endogenous adjustment of input-output linkages and the resulting structural changes.

Figure 6 shows the dynamic responses on the industry level. To highlight the patterns of structural changes, I plot the evolution of relative industry size n_i for three levels of the linkage efficiency ϕ corresponding to 0.5, 0.95, and 2 times the new steady-state linkage efficiency cutoff $\underline{\phi}$. As discussed in Section 3.2, the elasticity of substitution across industries determines the pattern of structural changes. If $\alpha > 0$, firm mass shifts towards industries with high linkage efficiency and therefore more linkages. If $\alpha < 0$, firm mass is redistributed in the opposite direction. In both cases, industries with intermediate levels of linkage efficiency that are surpassed by the rising cutoff during the transition experience non-monotonic changes in relative size.

5 Conclusion

This paper presented a growth model with endogenous input-output linkages to study the interplay between structural changes and the evolution of the production networks. The model is consistent with the stylized empirical fact that the expanding sector also becomes more important intermediate input supplier, as measured by the sectoral shares in aggregate intermediate input expenditure. The model is also able to generate non-monotonic industry growth patterns, another empirical regularity. Nevertheless, the model relies on several strong assumptions which I hope to relax in future research. First, one may introduce exogenous aggregate productivity growth to explore the existence of a constant growth path, as in Acemoglu and Guerrieri (2008), and also to bring the model closer to the actual growth experience of the U.S. both in a qualitative and a quantitative sense. Second, one may also relax the assumption of a fixed firm boundary to study the interaction among structural changes, endogenous linkages, and the organization of the firm. Third, a significant portion of input-output linkages in the real world are cross-country relationships. It would be interesting to see how the expansion of global value chains would affect structural changes in different countries.

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FIGURES

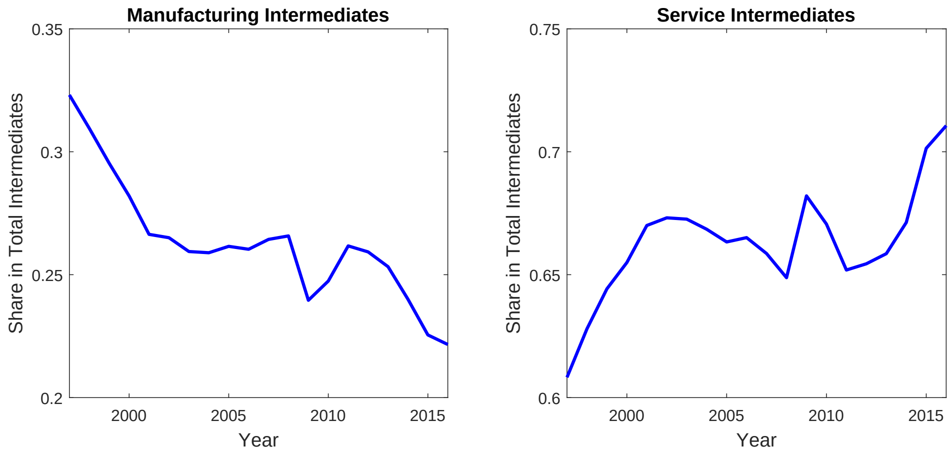

Figure 1: Shares of manufacturing and service intermediates in total U.S. domestic intermediate input expenditure. The data source is the Input-Output Accounts Data of the Bureau of Economic Analysis. Domestic intermediate input expenditure is calculated from the after-redefinition Use Tables (producer value) and Import Matrices. In this figure, "Services" include all the NAICS sectors other than "Agriculture, Forestry, Fishing, and Hunting", "Mining", "Construction", "Manufacturing", and "Public Administration".

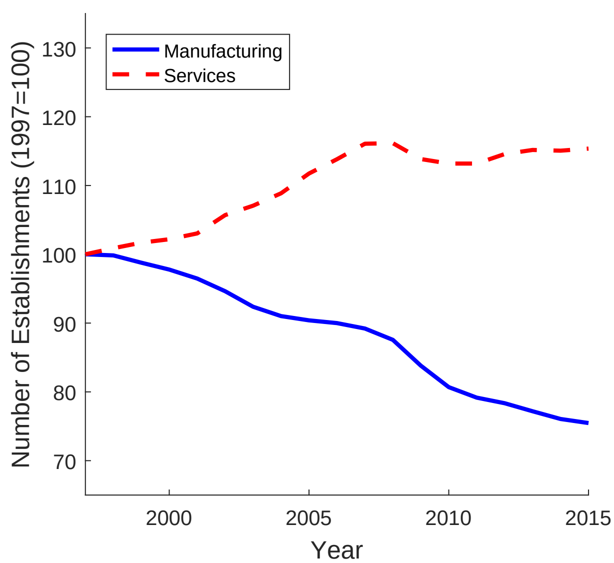

Figure 2: Number of establishments in the U.S. manufacturing and services industries (1997=100). The data source is the Longitudinal Business Database of the U.S. Census Bureau. In this figure, "Services" include the following SIC 87 major divisions: "Transportation and Public Utilities", "Wholesale Trade", "Retail Trade", "Finance, Insurance, and Real Estate", and "Services" .

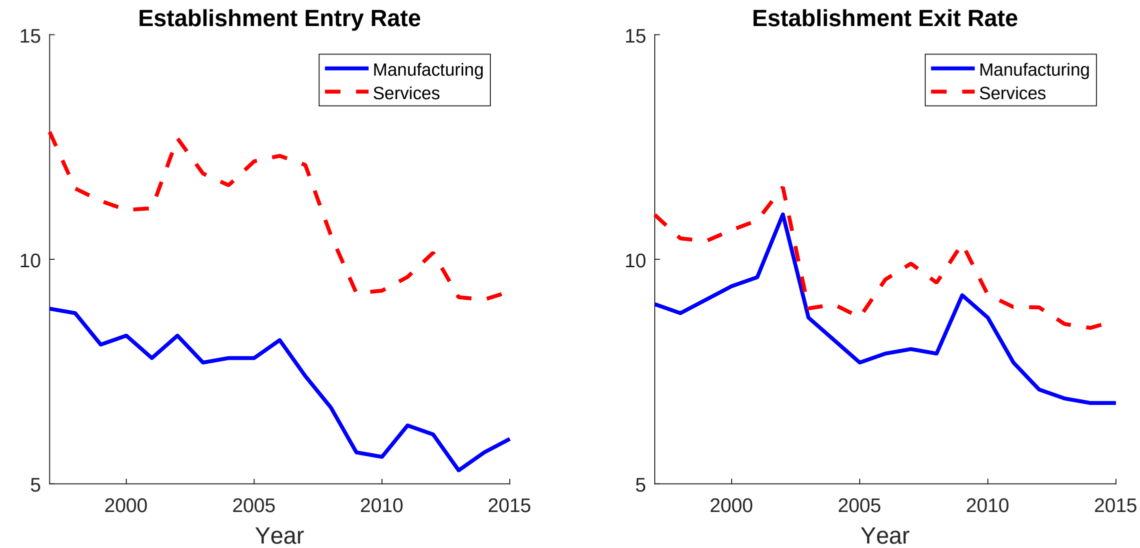

Figure 3: Establishment dynamics in the U.S. manufacturing and services industries. The data source is the Longitudinal Business Database of the U.S. Census Bureau. In this figure, "Services" include the following SIC 87 major divisions: "Transportation and Public Utilities", "Wholesale Trade", "Retail Trade", "Finance, Insurance, and Real Estate", and "Services" .

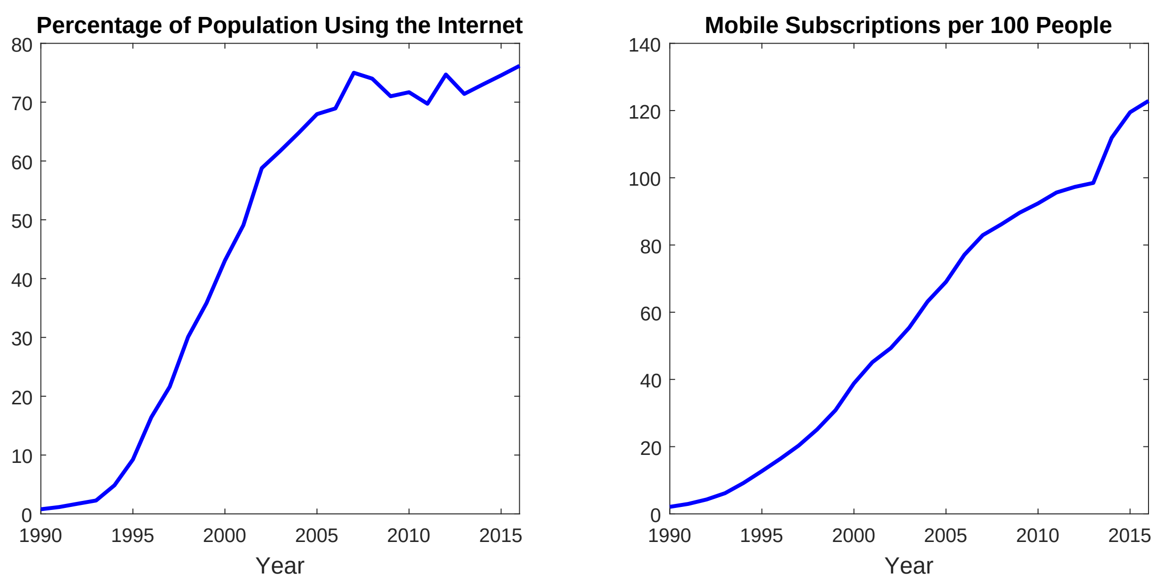

Figure 4: Time trend of the usage of information and communication technologies (ICT) in the United States. The data source is the World Bank World Development Indicators.

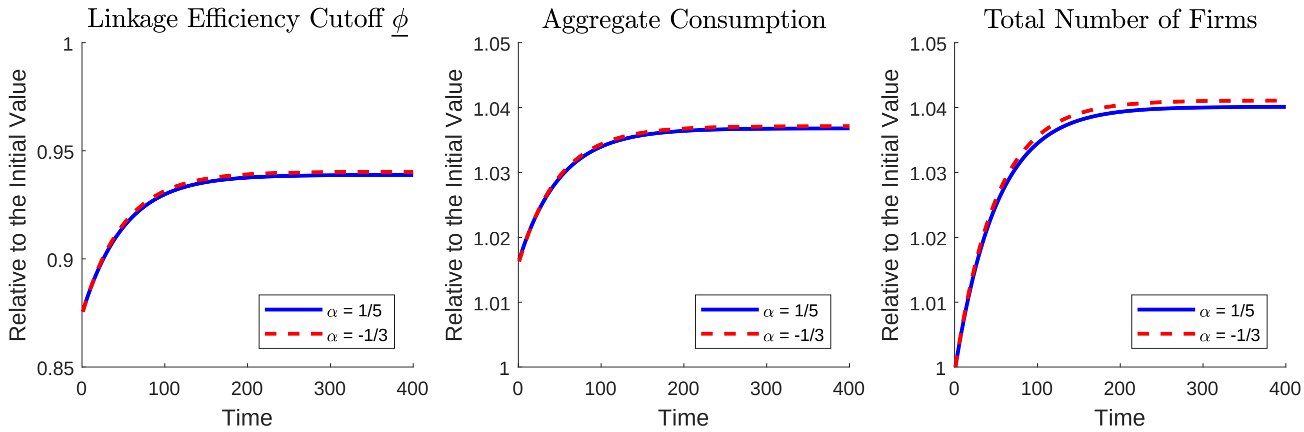

Figure 5: Aggregate dynamics in response to a 10% permanent decrease in κ, which implies a 10% universal reduction in the fixed cost per linkage.

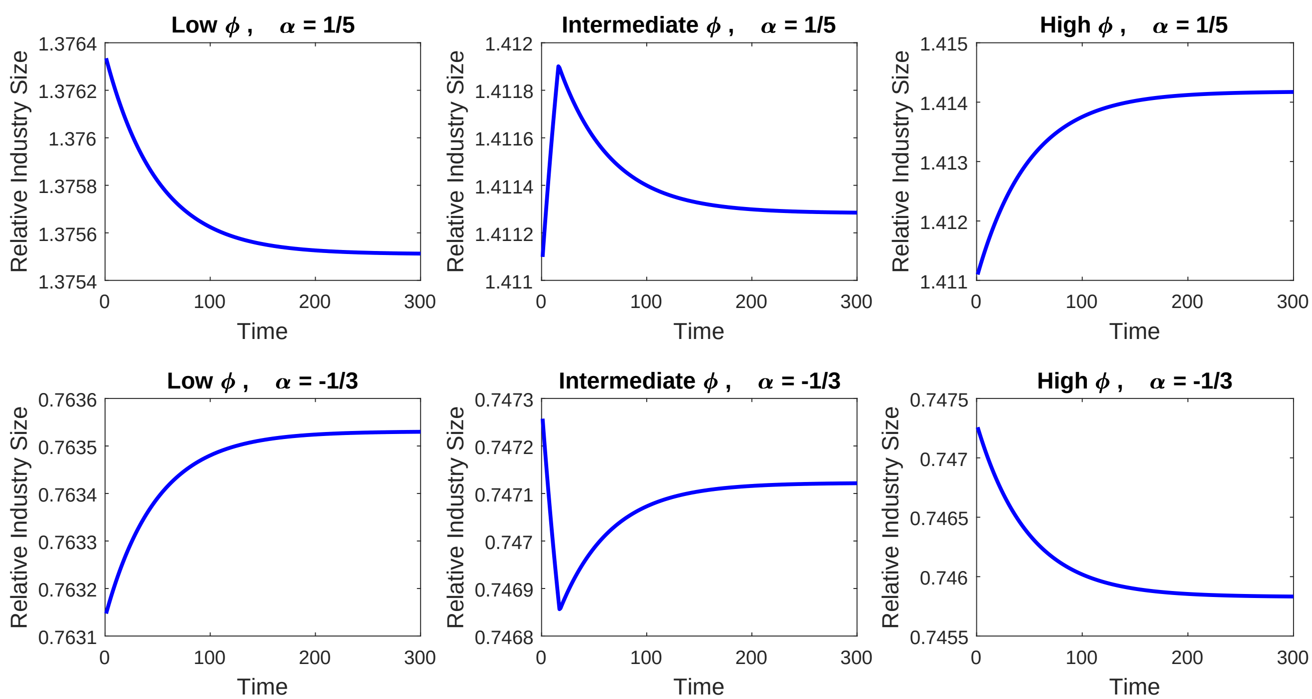

Figure 6: Patterns of structural changes in response to a 10% permanent reduction in linkage fixed cost κ. The low, intermediate, and high levels of the linkage efficiency ϕ correspond respectively to 0.5, 0.95, and 2 times the new steady-state linkage efficiency cutoff $\underline{\phi}$.

Appendix A: Decentralization

This section replicates the planner's allocation in a decentralized economy with perfectly competitive markets for labor and the final good, as well as monopolistically competitive markets with free entry for the differentiated varieties. Since monopolistic market power is the only source of distortion, I show that a production subsidy for the differentiated varieties to correct the markup is sufficient for aligning the market equilibrium with the social optimum.

Households

Households supply labor inelastically and earn competitive wage w. Static utility maximization by households implies the following final demand for the differentiated varieties:

$$X_i = YP^{\frac{1}{1-\alpha}} p_i^{\frac{1}{\alpha-1}} N_i^{\frac{\alpha-\beta}{\beta(1-\alpha)}}$$

where $P \equiv \left[\int_0^1 p_i^{\frac{\alpha}{\alpha-1}} N_i^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)} di\right]^{\frac{\alpha}{\alpha-1}}$ is the consumer price index.

Firms

The markets for varieties are characterized by monopolistic competition. In each period, firms are free to choose which industry to operate in and moving across industries incurs no cost. Firms take their upstream linkages (i.e., their suppliers) as given, but can choose downstream linkages (i.e., customers) in every period as long as they pay a fixed cost for every linkage they form. Specifically, if a firm in industry i wishes to sell its variety as intermediate inputs to a firm in industry i', it first needs to pay a fixed cost of $\kappa/\phi_{i'}$ units of the final good to establish a supply-customer relationship, where $\phi_{i'}$ is the linkage efficiency of the buyer firm. Profit maximization by firms implies the following input demand:

$$\begin{aligned}
x_{i,i'} &= m_i \left(P_i^S\right)^{\frac{1}{1-\alpha}} p_{i'}^{\frac{1}{\alpha-1}} \left(\mu_{i,i'} N_{i'}\right)^{\frac{\alpha-\beta}{\beta(1-\alpha)}} \\
m_i &= q_i \left(\frac{w}{\sigma}\right)^{\sigma} \left(\frac{P_i^S}{1-\sigma}\right)^{-\sigma} \\
l_i &= q_i \left(\frac{w}{\sigma}\right)^{\sigma-1} \left(\frac{P_i^S}{1-\sigma}\right)^{1-\sigma}
\end{aligned}$$

where $P_i^S \equiv \left[\int_0^1 p_{i'}^{\frac{\alpha}{\alpha-1}} \left(\mu_{i,i'} N_{i'}\right)^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)} di'\right]^{\frac{\alpha-1}{\alpha}}$ is the producer price index facing an industry-i firm. The firm sets monopolistic price to maximize operating profit:

$$\max_{p_i} (1+\psi)\, q_i p_i - q_i \left(\frac{w}{\sigma}\right)^{\sigma} \left(\frac{P_i^S}{1-\sigma}\right)^{1-\sigma}$$

where $\psi \geqslant 0$ is a subsidy financed by lump-sum taxation. The case of $\psi = 0$ corresponds to the laissez-faire market equilibrium. Optimal pricing implies

$$p_i = \frac{1}{(1+\psi)\,\beta} \left(\frac{w}{\sigma}\right)^{\sigma} \left(\frac{P_i^S}{1-\sigma}\right)^{1-\sigma} \tag{22}$$

The firm chooses downstream linkages to maximize flow profit, subject to the demand function for its variety:

$$\begin{aligned} \max_{\{\mu_{i',i}\}_{i'\in[0,1]}} \quad & \pi_i = (1+\psi)(1-\beta)\, q_i p_i - \int_0^1 \frac{\kappa}{\phi_{i'}} \mu_{i',i} N_{i'} di' \\ \text{s.t.} \quad & q_i = X_i + \int_0^1 x_{i',i} \mu_{i',i} N_{i'} di' \end{aligned} \tag{23}$$

The first order condition for linkage formation is

$$(1+\psi)(1-\beta)\, x_{i',i} p_i - \frac{\kappa}{\phi_{i'}} \begin{cases} \geqslant 0 & \text{if } \mu_{i',i} = 1 \\ = 0 & \text{if } \mu_{i',i} \in [0,1] \\ \leqslant 0 & \text{if } \mu_{i',i} = 0 \end{cases}$$

Characterization of the Static Equilibrium

Using the final good as the numeraire, I normalize $P = 1$. Substituting the final demand and the intermediate demand functions into the market clearing condition for a variety, we have

$$\begin{aligned} & q_i = X_i + \int_0^1 x_{i',i} \mu_{i',i} N_{i'} di' \\ \Leftrightarrow \quad & q_i p_i^{\frac{1}{1-\alpha}} N_i^{\frac{\beta-\alpha}{\beta(1-\alpha)}} = Y + \int_0^1 (1+\psi)\,\beta\,(1-\sigma)\, q_{i'} p_{i'} \left(P_{i'}^S\right)^{\frac{\alpha}{1-\alpha}} \mu_{i',i}^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)} N_{i'} di' \end{aligned} \tag{24}$$

In order for the last equality to hold for all i, we must have that $\mu_{i',i}$ does not depend on the supplier industry i (so that $\mu_{i',i} = \tilde{\mu}_i$ which is to be determined) and $q_i p_i^{\frac{1}{1-\alpha}} N_i^{\frac{\beta-\alpha}{\beta(1-\alpha)}} \equiv D$ is a constant to be determined. Furthermore, the free mobility of firms across industries implies that flow profit (23) must be equalized throughout the economy, which in turn implies that gross output of a firm $q_i p_i$ is constant. Substituting the labor demand $l_i = (1+\psi)\beta\sigma q_i p_i/w$ into the labor market clearing condition $\int_0^1 l_i N_i di = L$, we have $q_i p_i = Lw/[(1+\psi)\sigma\beta N]$, which implies that $p_i^{\frac{\alpha}{\alpha-1}} N_i^{\frac{\alpha-\beta}{\beta(1-\alpha)}} = q_i p_i/D = Lw/[(1+\psi)\sigma\beta ND]$. Substituting the last result into the price normalization $\int_0^1 p_i^{\frac{\alpha}{\alpha-1}} N_i^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)} di = 1$, we have $D = Lw/[(1+\psi)\sigma\beta]$. Using the above results to rewrite the optimal pricing condition (22), we have

$$
\begin{aligned}
& p_i = \frac{1}{(1+\psi)\beta}\left(\frac{w}{\sigma}\right)^{\sigma}\left\{\frac{1}{1-\sigma}\left[\int_0^1 p_{i'}^{\frac{\alpha}{\alpha-1}} (\tilde{\mu}_i N_{i'})^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)} di'\right]^{\frac{\alpha-1}{\alpha}}\right\}^{1-\sigma} \\
\Leftrightarrow \quad & p_i = \frac{1}{(1+\psi)\beta}\left(\frac{w}{\sigma}\right)^{\sigma}\left(\frac{1}{1-\sigma}\tilde{\mu}_i^{\frac{\beta-1}{\beta}}\right)^{1-\sigma}
\end{aligned}
$$

Substituting the above results into the first order condition for linkage formation, we have the following equilibrium distribution of linkages:

$$
\tilde{\mu}_i = \begin{cases} \frac{\phi_i}{\underline{\phi}} & \text{if } \phi_i < \underline{\phi} \\ 1 & \text{if } \phi_i \geqslant \underline{\phi} \end{cases}
$$

where the linkage efficiency cutoff is

$$
\underline{\phi} = \frac{\kappa[1-(1+\psi)\beta(1-\sigma)]N^2}{(1+\psi)^2\beta(1-\beta)(1-\sigma)Y}
$$

Accordingly, the equilibrium distribution of firm mass across industries is given by $N_i/N = \tilde{\mu}_i^{(1-\sigma)\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)}/A$ where $A \equiv \int_0^1 \tilde{\mu}_i^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} di$. Finally, substituting all of the above results into (24) and the identity $\int_0^1 N_i di = N$, we obtain the expressions of the two aggregate variables in the static equilibrium:

$$
\begin{aligned}
w &= \left[\frac{(1+\psi)\sigma\beta}{1-(1+\psi)\beta(1-\sigma)}\right]\frac{Y}{L} \\
Y &= [1-(1+\psi)\beta(1-\sigma)]\,[(1+\psi)\beta(1-\sigma)]^{\frac{1-\sigma}{\sigma}} LA^{\frac{\beta-\alpha}{\sigma\alpha\beta}} N^{\frac{1-\beta}{\sigma\beta}}
\end{aligned}
$$

The Dynamic Problem of the Households

Households allocate disposable income (wage and interest earnings minus the lump-sum tax) between consumption and the holding of assets M, implying the dynamic budget constraint as follows:

$$
\dot{M}_t = w_t L + r_t M_t - T_t - C_t
$$

Assets in this economy take the form of a mutual fund aggregating the value of all firms:

$$M_t = \int_0^1 V_{i,t} N_{i,t} di \quad \text{where} V_{i,t} = \int_t^\infty e^{-\int_t^\tau (r_s+\delta)ds} \pi_{i,\tau} d\tau$$

Entrepreneurs borrow from the households at interest rate r_t to create firms, upon paying the entry sunk cost ν in units of the final good. The free entry condition thus implies

$$\int_0^1 V_{i,t} di = \nu$$

The lump-sum tax for financing the variety production subsidy amounts to

$$\begin{aligned} T &\equiv \int_0^1 \psi q_i p_i N_i di \\ &= \left[\frac{\psi}{1-(1+\psi)\beta(1-\sigma)} \right] Y \end{aligned}$$

Since flow profits π are equalized across industries due to the free mobility of firms, firm value V is also the same in all industries. Substituting the Bellman equation of firm value $(r_t+\delta)V_t = \pi_t + \dot{V}_t$, the free entry condition $V_t = \nu$ and the expression of T_t into the dynamic budget constraint of the households, we have

$$\begin{aligned} \dot{M}_t &= w_t L + r_t M_t - T_t - C_t \\ \Leftrightarrow \quad \nu N_t^e &= \left[\frac{(1+\psi)\sigma\beta - \psi}{1-(1+\psi)\beta(1-\sigma)} \right] Y_t + \pi_t N_t - C_t \\ \Leftrightarrow \quad C_t &= Y_t - \kappa \frac{Z_t}{A_t} N_t^2 - \nu N_t^e \end{aligned} \tag{25}$$

where the last equality follows from the expression of the flow profit:

$$\begin{aligned} \pi_i &= (1+\psi)(1-\beta) q_i p_i - \int_0^1 \frac{\kappa}{\phi_{i'}} \mu_{i',i} N_{i'} di' \\ &= \left[\frac{(1+\psi)(1-\beta)}{1-(1+\psi)\beta(1-\sigma)} \right] \frac{Y_t}{N_t} - \kappa \frac{Z_t}{A_t} N_t \end{aligned}$$

where $Z \equiv \int_0^1 \phi_i^{-1} \tilde{\mu}_i^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)+1} di$.

Implement the Social Optimum

To correct the monopolistic markup, the production subsidy ψ should be set such that $(1+\psi)\beta = 1$, which implies the optimal subsidy

$$\psi = \frac{1-\beta}{\beta}$$

It is straightforward to check that, once the optimal subsidy ψ is in place, the linkage efficiency cutoff $\underline{\phi}$ in the decentralized economy is aligned with the planner's solution, so are the distribution of linkages $\{\tilde{\mu}_i\}_{i\in[0,1]}$ and the derived expressions A and Z. Conditional on the same firm stock N, aggregate output Y in the market equilibrium under the optimal subsidy is the same as in the planner's allocation. Furthermore, once the static inefficiency is corrected, the household dynamic budget constraint (25) agrees with the aggregate resource constraint in the planner's problem, and therefore the market equilibrium replicates the social optimum.

Appendix B: Proofs and derivation

Proof of Proposition 1

First we observe that, in order for the optimality condition (2) to hold, we must have $\partial Y/\partial N_i$ be constant across industries and $\mu_{i,i'}$ be the same across supplier industries indexed by i', which allows us to write

$$\mu_{i,i'} \equiv \tilde{\mu}_i \quad \text{for all } i, i'$$

The first order condition with respect to l_i states

$$\sigma Y^{1-\alpha} X_i^{\alpha-1} N_i^{\frac{\alpha-\beta}{\beta}} q_i l_i^{-1} = \lambda_L \quad \text{for all } i \tag{26}$$

where λ_L is the Lagrange multiplier corresponding to the labor supply constraint $\int_0^1 l_i N_i di = L$. The optimality condition (1) implies

$$x_{i,i'} = \left[(1-\sigma) X_i^{\alpha-1} N_i^{\frac{\alpha-\beta}{\beta}} l_i^{\frac{\alpha\sigma}{1-\sigma}} q_i^{\frac{1-\sigma-\alpha}{1-\sigma}} \tilde{\mu}_i^{\frac{\alpha-\beta}{\beta}} X_{i'}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} \quad \text{for all } i, i' \tag{27}$$

which can be substituted into the production function of differentiated varieties to yield

$$q_i = l_i \left[(1-\sigma) X_i^{\alpha-1} N_i^{\frac{\alpha-\beta}{\beta}} \tilde{\mu}_i^{\frac{1-\beta}{\beta}} Y^{1-\alpha}\right]^{\frac{1-\sigma}{\sigma}} \quad \text{for all } i \tag{28}$$

Substituting (26), (27), and (28) into the identity of variety quantity, we have

$$
\begin{aligned}
X_i &= q_i - \int_0^1 x_{i',i}\mu_{i',i}N_{i'}di' \\
X_i &= l_i\left[(1-\sigma)X_i^{\alpha-1}N_i^{\frac{\alpha-\beta}{\beta}}\tilde{\mu}_i^{\frac{1-\beta}{\beta}}Y^{1-\alpha}\right]^{\frac{1-\sigma}{\sigma}} \\
&\quad -\int_0^1 Y^{\frac{1-\sigma-\alpha}{\sigma}}\left[(1-\sigma)X_{i'}^{\alpha-1}N_{i'}^{\frac{\alpha-\beta}{\beta}}\right]^{\frac{1}{\sigma}} l_{i'}\tilde{\mu}_{i'}^{\frac{\alpha-\beta}{\beta(1-\alpha)}+\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)\frac{1-\sigma-\alpha}{\alpha\sigma}} X_i\tilde{\mu}_{i'}N_{i'}di' \\
X_i &= l_i\lambda_L\left(\sigma Y^{1-\alpha}X_i^{\alpha-1}N_i^{\frac{\alpha-\beta}{\beta}}\right)^{-1} \\
&\quad -\int_0^1 Y^{\frac{1-\sigma-\alpha}{\sigma}}\left[(1-\sigma)X_{i'}^{\alpha-1}N_{i'}^{\frac{\alpha-\beta}{\beta}}\right]^{\frac{1}{\sigma}} l_{i'}\tilde{\mu}_{i'}^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-\sigma}{\sigma}\right)} X_iN_{i'}di'
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad \int_0^1 X_i^{\alpha}N_i^{\frac{\alpha}{\beta}}di &= \lambda_L\left(\sigma Y^{1-\alpha}\right)^{-1}\int_0^1 l_iN_idi \\
&\quad -\int_0^1\int_0^1 X_i^{\alpha-1}N_i^{\frac{\alpha}{\beta}}Y^{\frac{1-\sigma-\alpha}{\sigma}}\left[(1-\sigma)X_{i'}^{\alpha-1}N_{i'}^{\frac{\alpha-\beta}{\beta}}\right]^{\frac{1}{\sigma}} l_{i'}\tilde{\mu}_{i'}^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-\sigma}{\sigma}\right)} X_iN_{i'}di'di \\
\int_0^1 X_i^{\alpha}N_i^{\frac{\alpha}{\beta}}di &= \frac{1}{\sigma}\lambda_LY^{\alpha-1}\int_0^1 l_iN_idi - \left(\frac{1-\sigma}{\sigma}\right)\lambda_LY^{-1}\int_0^1 l_{i'}N_{i'}di'\int_0^1 X_i^{\alpha}N_i^{\frac{\alpha}{\beta}}di \\
Y^{\alpha} &= \lambda_LY^{\alpha-1}L \\
\lambda_L &= \frac{Y}{L}
\end{aligned}
$$

From (26), (28) and $\lambda_L = Y/L$, we have

$$X_i = \left[\sigma(1-\sigma)^{\frac{1-\sigma}{\sigma}}L\right]^{\frac{\sigma}{1-\alpha}}Y^{1-\frac{\sigma}{1-\alpha}}N_i^{\frac{\alpha-\beta}{\beta(1-\alpha)}}\tilde{\mu}_i^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-\sigma}{1-\alpha}\right)} \tag{29}$$

which we then substitute into the aggregate production function, yielding

$$Y = \sigma(1-\sigma)^{\frac{1-\sigma}{\sigma}}L\left[\int_0^1 \tilde{\mu}_i^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)(1-\sigma)}N_i^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)}di\right]^{\frac{1-\alpha}{\sigma\alpha}} \tag{30}$$

The observation that $\partial Y/\partial N_i$ must be constant across industries then implies (6). By the first order condition (3), the interior solution of $\mu_{i,i'} \equiv \tilde{\mu}_i$ is given by

$$
\begin{aligned}
\frac{\partial Y}{\partial\tilde{\mu}_i} &= \frac{\kappa}{\phi_i}N_iN \\
\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-\sigma}{\sigma}\right)Y\frac{\tilde{\mu}_i^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)(1-\sigma)-1}N_i^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)}}{\int_0^1\tilde{\mu}_i^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)(1-\sigma)}N_i^{\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)}di} &= \frac{\kappa}{\phi_i}N_iN \\
\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-\sigma}{\sigma}\right)Y &= \frac{\kappa}{\phi_i}\tilde{\mu}_iN^2
\end{aligned}
$$

where the last line follows from (6). Therefore, the solution of $\mu_{i,i'} \equiv \tilde{\mu}_i$ is given by

$$\tilde{\mu}_i = \begin{cases} \frac{\phi_i}{\kappa}\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)\frac{Y}{N^2} & \text{if } \phi_i < \kappa\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\beta}{1-\beta}\right)\frac{N^2}{Y} \\ 1 & \text{if } \phi_i \geqslant \kappa\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\beta}{1-\beta}\right)\frac{N^2}{Y} \end{cases}$$

From (27), (28), and the identity of variety quantity, we also have

$$\begin{aligned} q_i &= X_i + \sigma(1-\sigma)^{\frac{1}{\sigma}} Y^{-\alpha+\frac{1-\sigma-\alpha}{\sigma}} L X_i \int_0^1 q_{i'} \left(X_{i'}^{\alpha-1} N_{i'}^{\frac{\alpha-\beta}{\beta}}\right)^{\frac{1+\sigma}{\sigma}} \mu_{i'}^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-\sigma}{\sigma}\right)} N_{i'} di' \\ q_i &= X_i + \frac{\sigma(1-\sigma)^{\frac{1}{\sigma}} Y^{-\alpha+\frac{1-\sigma-\alpha}{\sigma}} L X_i \int_0^1 X_i \left(X_i^{\alpha-1} N_i^{\frac{\alpha-\beta}{\beta}}\right)^{\frac{1+\sigma}{\sigma}} \mu_i^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-\sigma}{\sigma}\right)} N_i di}{1-\sigma(1-\sigma)^{\frac{1}{\sigma}} Y^{-\alpha+\frac{1-\sigma-\alpha}{\sigma}} L \int_0^1 X_i \left(X_i^{\alpha-1} N_i^{\frac{\alpha-\beta}{\beta}}\right)^{\frac{1+\sigma}{\sigma}} \mu_i^{\left(\frac{1-\beta}{\beta}\right)\left(\frac{1-\sigma}{\sigma}\right)} N_i di} \\ \frac{q_i}{X_i} &= \frac{1}{\sigma} \end{aligned}$$

Finally, substituting $q_i = X_i/\sigma$ and (6) back into (27) and (28) leads to $l_i = L/N$ and (7). Substituting (6) back into (8) yields the expression of aggregate output (8).

Proof of Proposition 2

Using the definition of A and (4), we have

$$\begin{aligned} A &\equiv \int_0^1 \mu_i^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} di \\ &= \int_{\phi_{min}}^{\underline{\phi}} \mu(\phi)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi) + \int_{\underline{\phi}}^{\phi_{max}} \mu(\phi)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi) \\ &= \int_{\phi_{min}}^{\underline{\phi}} \left(\frac{\phi}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi) + \int_{\underline{\phi}}^{\phi_{max}} d\mathscr{F}(\phi) \end{aligned}$$

Time-differentiating A yields

$$\begin{aligned} \dot{A} &= \frac{dA}{d\underline{\phi}}\dot{\underline{\phi}} \\ &= \left[-\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)\underline{\phi}^{-1}\int_{\phi_{min}}^{\underline{\phi}}\left(\frac{\phi}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi) + \mathscr{F}'\left(\underline{\phi}\right) - \mathscr{F}'\left(\underline{\phi}\right)\right]\dot{\underline{\phi}} \\ &= -\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)\int_{\phi_{min}}^{\underline{\phi}}\left(\frac{\phi}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi)\frac{\dot{\underline{\phi}}}{\underline{\phi}} \end{aligned}$$

Time-differentiating both sides of (11) and using the expression of $\dot{A}$ derived above, we have (12), and (13) follows directly from (12).

Proof of Proposition 3

If $\underline{\phi} \geqslant \phi_{max}$, from Proposition 2 we have $\tilde{\mu}_i = \phi_i/\underline{\phi}$ for all i, which then implies

$$A = \int_{\phi_{min}}^{\phi_{max}} \left(\frac{\phi}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi) \quad \text{and} \quad Z = \frac{1}{\underline{\phi}} \int_{\phi_{min}}^{\phi_{max}} \left(\frac{\phi}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi)$$

In this case, aggregate output and total linkage fixed costs are respectively given by

$$Y_t = \Lambda N_t^{\frac{[1-2(1-\sigma)]\frac{1-\beta}{\sigma\beta}}{1-\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)}} \quad \text{and} \quad \kappa \frac{Z_t}{A_t} N_t^2 = \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right) Y_t$$

where Λ is a constant:

$$\Lambda \equiv \left\{ \sigma(1-\sigma)^{\frac{1-\sigma}{\sigma}} L \left[\frac{1}{\kappa}\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)\right]^{\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)} \left[\int_{\phi_{min}}^{\phi_{max}} \phi^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi)\right]^{\frac{\beta-\alpha}{\sigma\alpha\beta}} \right\}^{\frac{1}{1-\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)}}$$

Substituting the above results into (9) and (10) yields the dynamic system (14) and (15). In the steady state, total firm mass is:

$$N^{SS} = \left\{ [1-2(1-\sigma)] \left(\frac{1-\beta}{\sigma\beta}\right) \frac{\Lambda}{\nu(\delta+\rho)} \right\}^{\frac{1-\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)}{1-\frac{1-\beta}{\sigma\beta}+\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)}}$$

which implies $dN/d\kappa < 0$ and $dN/dL > 0$. The steady state is locally saddle-path stable provided that

$$\frac{\partial \dot{C}_t}{\partial N_t}\Big|_{N^{SS}} < 0 \quad \Leftrightarrow \quad \frac{[1-2(1-\sigma)]\frac{1-\beta}{\sigma\beta}}{1-\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)} - 1 < 0$$

which holds under the parameter restrictions $2(1-\sigma) < 1$ and $(1-\beta)/\beta < \sigma$. Provided that the initial condition $N_0 < N^{SS}$, the linkage efficiency cutoff $\underline{\phi}$ rises over time ($\dot{\underline{\phi}} > 0$) since $d\underline{\phi}/dN > 0$. Therefore, the condition on κ for this case to prevail is

$$\begin{aligned} & \underline{\phi}_0 \geqslant \phi_{max} \\ \Leftrightarrow \quad & \kappa \geqslant \overline{\kappa} \equiv \phi_{max}\left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1}{\sigma}} L \left[\int_{\phi_{min}}^{\phi_{max}} \left(\frac{\phi}{\phi_{max}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi)\right]^{\frac{\beta-\alpha}{\sigma\alpha\beta}} N_0^{\frac{1-\beta}{\sigma\beta}-2} \end{aligned}$$

In anticipation for the proof of Proposition 4, we also derive another threshold of κ defined by

$$\begin{aligned} \underline{\phi}^{SS} \quad & \geqslant \quad \phi_{max} \\ \Leftrightarrow \quad \kappa \quad & \geqslant \quad \overline{\kappa}' \equiv \phi_{max}(1-\sigma)\left[\left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1-\sigma}{\sigma}} L\right]^{\frac{1}{\frac{1-\beta}{\sigma\beta}-1}} \left[\frac{\nu(\delta+\rho)}{1-2(1-\sigma)}\right]^{\frac{2-\frac{1-\beta}{\sigma\beta}}{1-\frac{1-\beta}{\sigma\beta}}} \times \\ & \left[\int_{\phi_{min}}^{\phi_{max}} \left(\frac{\phi}{\phi_{max}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi)\right]^{\frac{\frac{\beta-\alpha}{\sigma\alpha\beta}}{\frac{1-\beta}{\sigma\beta}-1}} \end{aligned} \tag{31}$$

If $\underline{\phi} \leqslant \phi_{min}$, from Proposition 2 we have$\tilde{\mu}_i = 1$ for all i, which then implies

$$A = 1 \quad \text{and} \quad Z = \int_{\phi_{min}}^{\phi_{max}} \frac{1}{\phi} d\mathscr{F}(\phi)$$

In this case, aggregate output and total linkage fixed costs are respectively given by

$$Y_t = \sigma (1-\sigma)^{\frac{1-\sigma}{\sigma}} L N_t^{\frac{1-\beta}{\sigma\beta}} \quad \text{and} \quad \kappa \frac{Z_t}{A_t} N_t^2 = \int_{\phi_{min}}^{\phi_{max}} \frac{\kappa}{\phi} d\mathscr{F}(\phi) N_t^2$$

Substituting the above results into (9) and (10) yields the dynamic system (16) and (17). In the steady state, total firm mass N^{SS} is given implicitly by the following equation:

$$\frac{1}{\nu}\left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1-\sigma}{\sigma}} L \left(N^{SS}\right)^{\frac{1-\beta}{\sigma\beta}-1} - \frac{2}{\nu}\int_{\phi_{min}}^{\phi_{max}} \frac{\kappa}{\phi} d\mathscr{F}(\phi) N^{SS} - \delta - \rho = 0$$

which implies $dN^{SS}/d\kappa < 0$ and $dN^{SS}/dL > 0$ by the Implicit Function Theorem. The steady state is locally saddle-path stable provided that

$$\frac{\partial \dot{C}_t}{\partial N_t}|_{N^{SS}} < 0$$

which holds under the parameter restrictions $(1-\beta)/\beta < \sigma$. Since $\dot{\underline{\phi}} > 0$ as argued above, the condition on κ for this case to prevail is

$$\begin{aligned} & \underline{\phi}^{SS} \leqslant \phi_{min} \\ \Leftrightarrow \quad & \kappa \leqslant \underline{\kappa} \equiv \phi_{min}(1-\sigma)\left[\left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1-\sigma}{\sigma}} L\right]^{\frac{1}{\frac{1-\beta}{\sigma\beta}-1}} \left[\frac{\nu(\delta+\rho)}{1-2(1-\sigma)\int_{\phi_{min}}^{\phi_{max}} \frac{\phi_{min}}{\phi} d\mathscr{F}(\phi)}\right]^{\frac{2-\frac{1-\beta}{\sigma\beta}}{1-\frac{1-\beta}{\sigma\beta}}} \end{aligned}$$

In anticipation for the proof of Proposition 4, we also derive another threshold of κ defined by

$$\begin{aligned} \underline{\phi}_0 \quad & \leqslant \quad \phi_{min} \\ \Leftrightarrow \quad \kappa \quad & \leqslant \quad \underline{\kappa}' \equiv \phi_{min} L \left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1}{\sigma}} N_0^{\frac{1-\beta}{\sigma\beta}-2} \end{aligned} \tag{32}$$

Finally, since $d\underline{\kappa}/dL < 0$ while $d\overline{\kappa}/dL > 0$, $\underline{\kappa} < \overline{\kappa}$ is satisfied when L is sufficiently large.

Proof of Proposition 4

If $\phi_{min} < \underline{\phi} < \phi_{max}$, Proposition 2 implies

$$A = \int_{\phi_{min}}^{\underline{\phi}} \left(\frac{\phi}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi) + \int_{\underline{\phi}}^{\phi_{max}} d\mathscr{F}(\phi)$$

$$Z = \frac{1}{\underline{\phi}} \int_{\phi_{min}}^{\underline{\phi}} \left(\frac{\phi}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi) + \int_{\underline{\phi}}^{\phi_{max}} \frac{1}{\phi} d\mathscr{F}(\phi)$$

Substituting the above results into (9) and (10) yields the differential equations (18) and (19). Both of these two differential equations depend on $\underline{\phi}_t$. To derive the law of motion of $\underline{\phi}_t$, we first substitute the expression of aggregate output (8) into that of $\underline{\phi}_t$ (5), obtaining an equation relating total firm mass N_t to $\underline{\phi}_t$:

$$N_t = \left[\left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1}{\sigma}} \frac{L}{\kappa}\underline{\phi}_t A_t^{\frac{\beta-\alpha}{\sigma\alpha\beta}}\right]^{\frac{1}{2-\frac{1-\beta}{\sigma\beta}}} \tag{33}$$

Time-differentiating both sides of the above equation yields the third differential equation (20). To study the local stability of this system, we substituting (20) into (19) to obtain

$$\frac{\dot{\underline{\phi}}_t}{\underline{\phi}_t} = \frac{\left(2-\frac{1-\beta}{\sigma\beta}\right)A_t\left\{\frac{1}{\nu}\left[\sigma(1-\sigma)^{\frac{1-\sigma}{\sigma}} LA_t^{\frac{\beta-\alpha}{\sigma\alpha\beta}} N_t^{\frac{1-\beta}{\sigma\beta}-1} - \kappa\frac{Z_t}{A_t}N_t - \frac{C_t}{N_t}\right]-\delta\right\}}{A_t - \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\beta}{\beta}\right)\int_{\phi_{min}}^{\underline{\phi}_t}\left(\frac{\phi}{\underline{\phi}_t}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi)}$$

Using (33), the above differential equation and (18) constitute a dynamic system of C_t and $\underline{\phi}_t$ only, where the partial derivatives evaluated at the steady state satisfy: $\partial\dot{C}_t/\partial C_t|_{SS}=0$, $\partial\dot{\underline{\phi}}_t/\partial C_t|_{SS}<0$, and $\partial\dot{\underline{\phi}}_t/\partial\underline{\phi}_t|_{SS}>0$. Thus, the system is locally saddle-path stable provided that $\partial\dot{C}_t/\partial\underline{\phi}_t|_{SS}<0$. A sufficient parameter restriction for $\partial\dot{C}_t/\partial\underline{\phi}_t|_{SS}<0$ is

$$\frac{\underline{\phi}_{min}}{\phi_{max}} > \frac{\alpha(1-\beta)(1-\sigma)}{(1-\alpha)\beta-\alpha(1-\beta)\sigma}$$

which guarantees that

$$\begin{aligned} & \frac{1-\alpha}{\sigma\alpha}\left[1-\frac{\alpha}{\beta}\left(\frac{1-\beta}{1-\alpha}\right)\right]\mathscr{E}_A^{\underline{\phi}}|_{SS} - \mathscr{E}_{Z/A}^{\underline{\phi}}|_{SS} - \left[2-\frac{1}{\sigma}\left(\frac{1-\beta}{\beta}\right)\right]\mathscr{E}_N^{\underline{\phi}}|_{SS} < 0 \\ \Leftrightarrow \quad & \left[1+\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)\right]\left[1-\frac{\int_{\underline{\phi}}^{\phi_{max}}\frac{\phi}{\underline{\phi}}d\mathscr{F}(\phi)}{\int_{\underline{\phi}}^{\phi_{max}}d\mathscr{F}(\phi)}\right] - \frac{\int_{\underline{\phi}}^{\phi_{max}}\frac{\phi}{\underline{\phi}}d\mathscr{F}(\phi)}{\int_{\phi_{min}}^{\underline{\phi}}\left(\frac{\phi}{\underline{\phi}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi)} < 1 \end{aligned} \tag{34}$$

where $\mathscr{E}_A^{\underline{\phi}}|_{SS}$, $\mathscr{E}_{Z/A}^{\underline{\phi}}|_{SS}$, and $\mathscr{E}_N^{\underline{\phi}}|_{SS}$ are respectively the elasticity of A, Z/A, and N with respect to $\underline{\phi}$ evaluated at the steady state. The steady-state values N^{SS} and $\underline{\phi}^{SS}$ are jointly defined implicitly by the following two equations:

$$\nu(\delta+\rho) = \left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1-\sigma}{\sigma}} L\left(A^{SS}\right)^{\frac{\beta-\alpha}{\sigma\alpha\beta}}\left(N^{SS}\right)^{\frac{1-\beta}{\sigma\beta}-1} - 2\kappa\frac{Z^{SS}}{A^{SS}}N^{SS} \tag{35}$$

$$N^{SS} = \left[\left(\frac{1-\beta}{\beta}\right)(1-\sigma)^{\frac{1}{\sigma}}\frac{L}{\kappa}\underline{\phi}^{SS}\left(A^{SS}\right)^{\frac{\beta-\alpha}{\sigma\alpha\beta}}\right]^{\frac{1}{2-\frac{1-\beta}{\sigma\beta}}} \tag{36}$$

where A^{SS} and Z^{SS} are A and Z (both are functions of $\underline{\phi}$) evaluated at the steady state. Substituting (36) into (36), applying the Implicit Function Theorem and using $\partial\dot{C}_t/\partial\underline{\phi}_t|_{SS}<0$, we have $d\underline{\phi}^{SS}/d\kappa>0$ and $d\underline{\phi}^{SS}/dL>0$. We can also show that the product $\underline{\phi}^{SS}\left(A^{SS}\right)^{\frac{\beta-\alpha}{\sigma\alpha\beta}}$ is increasing in $\underline{\phi}^{SS}$, which implies that $dN^{SS}/dL>0$. Furthermore, under the aforementioned parameter restriction that guarantee $\partial\dot{C}_t/\partial\underline{\phi}_t|_{SS}<0$, the elasticity of the product $\underline{\phi}^{SS}\left(A^{SS}\right)^{\frac{\beta-\alpha}{\sigma\alpha\beta}}$ with respect to κ is between 0 and 1, and therefore $dN^{SS}/d\kappa<0$. Finally, given initial condition $N_0<N^{SS}$, the dynamic system given by (18), (19), and (20) is applicable to the entire transition to the

steady state if the linkage fixed cost parameter κ satisfies $\underline{\kappa}' \leqslant \kappa \leqslant \overline{\kappa}'$, where the threshold levels $\underline{\kappa}'$ and $\overline{\kappa}'$ are given by (31) and (32) in the proof of Proposition 3. The condition for $\underline{\kappa}' > \underline{\kappa}$ and $\overline{\kappa}' < \overline{\kappa}$ to both be satisfied is

$$L > \left(\frac{\beta}{1-\beta}\right)(1-\sigma)^{\frac{\sigma-1}{\sigma}}\left[\frac{\nu(\delta+\rho)}{1-2(1-\sigma)}\right] N_0^{1-\frac{1-\beta}{\sigma\beta}}\left[\int_{\phi_{min}}^{\phi_{max}}\left(\frac{\phi}{\phi_{max}}\right)^{\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)} d\mathscr{F}(\phi)\right]^{\frac{\alpha-\beta}{\sigma\alpha\beta}}$$

which also ensures that $\underline{\kappa} < \overline{\kappa}$.

Proof of Proposition 5

In the proof of Proposition 1, we established that $q_i = X_i/\sigma$, which can be substituted into (28) to yield

$$X_i = N^{-\frac{1}{\alpha}} Y N_i^{\frac{\beta-\alpha}{\alpha\beta}}$$

Substituting the above equation and (7) into the definition of intermediate expenditure shares Λ_i, we have

$$\begin{aligned}
\Lambda_i &= \frac{\int_0^1 \left(\frac{1-\sigma}{\sigma}\right)\frac{1}{N} X_i \mu_{i'}^{-1}\mu_{i'} N_i N_{i'} di' Y^{1-\alpha} X_i^{\alpha-1} N_i^{\frac{\alpha-\beta}{\beta}}}{\int_0^1\int_0^1 \left(\frac{1-\sigma}{\sigma}\right)\frac{1}{N} X_i \mu_{i'}^{-1}\mu_{i'} N_i N_{i'} di' Y^{1-\alpha} X_i^{\alpha-1} N_i^{\frac{\alpha-\beta}{\beta}} di} \\
&= \frac{N_i}{\int_0^1 N_i di} \\
&= n_i
\end{aligned}$$

which implies

$$\frac{\Lambda_i}{\Lambda_{i'}} = \frac{n_i}{n_{i'}} \qquad \text{for all } i, i'$$

Time-differentiating the last equation yields (21).

Derivation details of Section 4.1

In a competitive equilibrium that decentralizes the planner's allocation, the price of an industry-i variety is given by $(\partial Y/\partial X_i)/N_i$. Therefore, gross output of the economy is

$$\begin{aligned}
Q &\equiv \int_0^1 q_i \left(\frac{\partial Y}{\partial X_i} N_i^{-1}\right) N_i di \\
&= \int_0^1 \frac{1}{\sigma} X_i Y^{1-\alpha} X_i^{\alpha-1} N_i^{\frac{\alpha}{\beta}} di \\
&= \frac{1}{\sigma} Y^{1-\alpha} \int_0^1 X_i^{\alpha} N_i^{\frac{\alpha}{\beta}} di \\
&= \frac{1}{\sigma} Y
\end{aligned}$$

Therefore, $\sigma = Y/Q$ corresponds to the ratio of GDP to gross output. To calibrate κ, ν, and ζ, I assume that the U.S. economy was at the steady state in 1997 with the initial linkage efficiency cutoff $\underline{\phi} > \phi_{min}$. Specifically, the steady state is characterized jointly by the following system of four equations:

$$\left(\frac{1-\beta}{\beta\sigma}\right)Y - 2\frac{Z}{A}\kappa N^2 = \nu\left(\delta+\rho\right)N$$

$$Y - \frac{Z}{A}\kappa N^2 - C = \nu\delta N$$

$$\left[\frac{L}{\kappa}\left(\frac{1-\beta}{\beta}\right)\left(1-\sigma\right)^{\frac{1}{\sigma}}\underline{\phi}A^{\frac{\beta-\alpha}{\sigma\alpha\beta}}\right]^{\frac{1}{2-\frac{1-\beta}{\sigma\beta}}} = N$$

$$\sigma\left(1-\sigma\right)^{\frac{1-\sigma}{\sigma}}LA^{\frac{\beta-\alpha}{\sigma\alpha\beta}}N^{\frac{1-\beta}{\sigma\beta}} = Y$$

where

$$A = \zeta\underline{\phi}^{-\zeta}\left[\frac{\underline{\phi}^{\zeta-\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)}-1}{\zeta-\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)\left(1-\sigma\right)}+\frac{1}{\zeta}\right]; \quad Z = \zeta\underline{\phi}^{-\zeta-1}\left[\frac{\underline{\phi}^{\zeta-\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)(1-\sigma)}-1}{\zeta-\alpha\left(\frac{1-\beta}{\beta-\alpha}\right)\left(1-\sigma\right)}+\frac{1}{\zeta+1}\right]$$

Setting aggregate output Y, aggregate consumption C, and the total number of firms N at their observed values in 1997 (respectively 8.61 trillion USD, 5.56 trillion USD, and 5.37 millions) the above system gives four equations with four unknowns (κ, ν, ζ, and $\underline{\phi}$), which allows us to back out the values of κ, ν, and ζ.