Targeted Fiscal Policy *

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Abstract

How do supply chains affect the intensity with which an industry uses labor? I derive the *network-adjusted labor intensity* as the answer to this question. The network-adjusted labor intensity measures not just the direct labor intensity of a given industry, but also takes into account the labor intensity of all its inputs, its inputs' inputs, and so on. I show that this measure is the relevant sufficient statistic determining labor's share of income, the propagation of demand shocks, the relative rankings of government employment multipliers, and the composition of optimal fiscal policy. I use network-adjusted labor shares to decompose labor's share of income into disaggregated industrial components. Using a sample of 34 countries from 1995 to 2009, I find that labor's share of income has declined primarily due to a universal decrease in labor-use by all industries, rather than changes in households' consumption demands or firms' input demands. This is in contrast to the popular value-added decomposition, which gives a much larger role to industrial composition.

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1 Introduction

How labor intensive is a production process given the existence of supply chains? With constant returns to scale, the labor intensity of producing a good, if there are no intermediate inputs, is clear: we simply divide the wage bill by total revenue. This is the gross labor share of a firm. However, when there are intermediate inputs, it is insufficient to simply look at the gross labor share since some fraction of revenues is spent on intermediate inputs. A popular measure used in the literature to account for this is labor's share of value-added. This is a firm's total wage bill divided by its value-added (revenues minus intermediate input costs).

In a neoclassical model with independent industries, the value-added labor share is a key statistic that answers many important questions. For instance, an industry's value-added labor share converts demand for goods into demand for labor. As such, it determines how demand shocks or government spending shocks to different industries can move employment. Furthermore, a weighted-average of value-added labor shares determines labor's share of aggregate income. The aggregate labor share of income is a crucial object with implications for long-run growth, inequality, and macroeconomic dynamics.

This paper argues that in the presence of non-trivial firm-to-firm connections, the value-added labor share is the wrong measure of how much labor an industry uses. This is because the value-added labor share does not incorporate any information about the nature of an industry's supply chain. The key insight is that the labor intensity of an industry is *not* solely determined by how much of its revenues, or even its value-added, it spends on labor. To know how labor intensive an industry is, we also need to take into account the labor intensity of its entire supply chain. I derive an industry-level measure of labor intensity, the *network-adjusted labor intensity*, that takes these considerations into account. I show that the network-adjusted labor intensity is the key statistic determining labor's share of income, the employment multipliers from different kinds of government spending, and the propagation of demand shocks.

The network-adjusted labor intensity is conceptually distinct from the value-added labor share commonly seen in the literature, for example in Estrada and Valdeolivas (2012), Elsby et al. (2013), and Neiman and Karabarbounis (2014). While the network-adjusted labor intensity counts the contributions of labor to the production of a given good throughout its supply chain, the value-added labor share divides the wage bill by revenues net of intermediate inputs. This means that value-added measures do not take into account the labor/capital mix of the intermediate inputs of an industry.

Network-adjusted labor intensity and, the closely related, network-adjusted labor share are key statistics for understanding how industry-level changes affect aggregate outcomes. For example, the consumption-weighted average of network-adjusted labor shares is equal to labor's share of income. Writing the labor share of income as a consumption weighted average of network-adjusted labor shares allows us to decompose fluctuations in labor's share of income into more disaggregated parts. In particular, we can decompose changes in aggregate labor share into changes in consumption patterns, changes in supply chains (including trade), and changes in gross labor shares at the industry level for low, medium, and high-skill labor. This decomposition is related to the seminal work of Berman et al. (1994), but improves upon their work by explicitly accounting for intermediate inputs and trade in intermediate inputs. The resulting decomposition is more detailed, more closely tied to theory, and more stable than the value-added decomposition of labor share common in the literature. Furthermore, by explicitly accounting for imports, it allows us to distinguish between some competing theories of why the aggregate labor share is moving.

Using this decomposition and a sample of 34 countries over 15 years, I find that, on average over the sample, the overwhelming culprit behind the decline in aggregate labor's share of aggregate income is the decline in the gross labor share of all industries, rather than compositional effects across industries. In other words, it is not the case that labor intensive industries are getting smaller and capital intensive ones are getting larger – instead, all industries are using less labor. Furthermore, the decomposition casts doubt on theories of the decline of the aggregate labor share that rely directly on increased imports of intermediate and final consumption goods. These findings are in contrast to the conclusion one would reach if one relied on the popular (but misleading) value-added decomposition of the same data. The value-added decomposition attributes most of the changes to compositional effects between industries.

Another advantage of using network-adjusted labor shares is that we can analyze changes to an industry's labor share taking into account its entire supply chain. For example, we can analyze labor's share of manufacturing income, taking into account manufacturing's reliance on non-manufacturing labor. This contrasts with the manufacturing's value-added labor share, used for example in Oberfield and Raval (2012), which ignores the nature of manufacturing's supply chain. Using this approach, I show that for the United States, changes to the composition of industries is responsible for the decline in labor's share of manufacturing income. This is consistent with a story where increased imports are responsible for the drop in manufacturing's labor share. However, as stated previously, this is not a significant driver of the decline of the aggregate labor share.

Using network-adjusted labor shares by skill level, I also find that substitution of income across different types of labor, emphasized by Goldin and Katz (2009), dwarfs the substitution of income between labor and capital. To the extent that factor income shares are important determinants of inequality, this suggests that substitution across different labor types has been more important than substitution from labor to capital. I show that there is a near-universal trend of industries substituting from low and medium-skilled labor towards high-skilled labor in almost all countries in the sample. Once again, the culprits are the movements of the gross labor-shares of all industries, rather than changes in the supply chains or consumption patterns of households.

Not only are network-adjusted labor intensities important for studying long-run patterns in labor's share of income, but they are also important for analyzing short-run fluctuations. Network-adjusted labor intensities determine the relative ranking of employment multipliers from demand shocks. In particular, they show how fiscal policy should be targeted to maximize its impact on output and employment. I show that in a model with involuntary unemployment, the network-adjusted labor intensity is a key determinant of the composition of optimal countercyclical fiscal policy. Furthermore, the network adjustment allows us to compute the fraction of each dollar of government spending that is eventually paid out to different kinds of workers. I find that federal government consumption expenditures (defense and nondefense) are overwhelmingly tilted towards spending on high-skilled workers with at least 4 years of college education. On the other hand, state and local government investment and private investment spend much more on low-skilled workers without college degrees. These results speak to how government fiscal policy can be targeted to stimulate specific parts of the labor market.

The structure of the paper is as follows: in section 2, I develop a one-factor model where the network structure of the economy is irrelevant both in terms of labor's share of income, and the relative employment multipliers from government spending and demand shocks. This shows that in a very general sense, simply having a network structure is not enough to generate interesting answers to our questions; we need a second factor. In section 3, I introduce the benchmark model used throughout the rest of the paper that breaks the irrelevance of section 2 by adding capital, and I define the network-adjusted labor intensity. In section 4, I decompose labor's share of income into disaggregated components and show how each component has varied over time for a sample of 34 countries. In section 5, I characterize the relative employment multipliers from government expenditures in terms of network-adjusted labor intensities. I add a nominal rigidity and show that the network-adjusted labor intensity pins down the industrial composition of optimal fiscal policy when the zero lower bound constrains the central bank. I conclude in section 6.

2 An Irrelevance Result

In this section, I sketch a competitive constant returns to scale model where labor is the only non-constant returns to scale factor. I prove an irrelevance result in this environment showing that the network structure does not affect equilibrium employment in any meaningful way. Specifically, I show that the network does not affect labor's share of income nor the size of government multipliers. This drives home the point that models without a second factor, like Long and Plosser (1983) and Acemoglu et al. (2012), are uninformative about the determinants of labor's share of income or the composition of fiscal stimulus, despite having production networks. The intuition is that without profits or capital, all income is ultimately spent on labor. Therefore, the details of how industries are interconnected do not matter in terms of how shocks affect aggregate employment.

I use a dynamic framework so that the results can be directly compared to those in later sections. Let the representative household maximize

$$\max_{c_{it},l} \quad \sum_{t=0}^{\infty} \rho_t U(c_{1t}, \ldots, c_{nt}, l_t) \tag{1}$$

such that

$$\sum_{i=1}^{N} p_{it} c_{it} = w_t l_t + \Pi_t + (1 + i_{t-1}) B_{t-1} - B_t - \tau_t,$$

where ρ_t is the discount factor in period t, p_{it} is the price of good i and c_{it} is the quantity of good i consumed in period t, the wage is w_t, labor is l_t, lump sum taxes are τ_t , nominal government bonds are B_t with interest rate i_t, and Π_t denotes firm profits.

Assume that the representative firm in industry i maximizes profits

$$\max \quad p_{it} y_{it} - \sum_{j} p_{jt} x_{ijt} - w_t l_{it}$$

such that

$$y_{it} = F_i(l_{it}, x_{i1t}, \ldots, x_{int}),$$

where F_i is a constant returns to scale function and x_{ijt} are units of good j used by firm i in period t. The set of functions $\{F_i\}$ defines the network structure of this economy.

Let g_{it} be government consumption of good i, and assume that the government runs a balanced budget

$$\sum_{i} p_{it} g_{it} = \tau_t,$$

and sets the net supply of nominal bonds to be zero. Suppose that the distribution of government spending is given by the vector δ:

$$\delta_{it} = \frac{p_{it} g_{it}}{\sum_j p_{jt} g_{jt}}.$$

Note that in this basic setup, government consumption is socially wasteful, and is not consumed by the household.

Definition 2.1. A competitive equilibrium is a collection of prices $\{p_{it}\}_{i=1}^{N}$, sequence of wages w_t and interest rates i_t, and quantities $\{x_{ijt}, c_{it}\}_{ijt}$, and labor supplies l_t and labor demands $\{l_{it}\}_i$ such that for any given government policy $\{g_{it}\}_{i=1}^{N}$, and τ_t,

(i) Each firm maximizes its profits given prices,

(ii) the representative household chooses consumption and labor supply to maximize utility,

(iii) the government runs a balanced budget,

(iv) and markets for each good, labor, and bonds clear.

I focus on the steady-state equilibrium of this model, so time-subscripts are suppressed.

Definition 2.2. Labor's share of income is the wage bill wl divided by total expenditures on final goods $\sum_i p_i c_i + \sum_i p_i g_i$.

Trivially, labor's share of income, in this model, is always equal to one, regardless of the network structure. This follows from the fact that firms have constant returns to scale and make zero profits. More interestingly, we have the following result.

Theorem 1. *In the absence of profits or capital, the distribution of government expenditures has no effect on equilibrium employment. That is, equilibrium employment is not a function of δ.*

The intuition is that constant-returns-to-scale at the firm level mean that relative prices do not respond to δ. This allows us to use the Hicks-Leontief composite commodity theorem, see for example Woods (1979), to represent this economy as having only one aggregate consumption good. Therefore, the only way δ can change employment is through labor supply, or in other words, through the marginal utility of wealth. However, the marginal utility of wealth only depends on the amount the government taxes the household, not on how those taxes are spent because the household does not derive utility from government consumption. Therefore, it is only the total size of the government's budget, not its distribution, that matters.

Another way to see this is to note that constant returns to scale firms make zero profits in a competitive equilibrium. Therefore, all revenues are spent either on intermediate inputs or on labor. The portion of revenues spent on intermediaries is in turn either spent on other intermediate inputs or on labor. Ultimately, all firm revenues must be spent on labor, which means that changing the composition of government expenditures has no effect for a fixed amount of total spending. Of course, as stated above, this requires that the *composition* of government spending not affect labor supply directly.

Proof. See Appendix A. ■

3 Benchmark Model

To break the equivalence between GDP and compensation by labor, we need a second source of earnings. This could be profits or returns from land and capital. A second source of earnings is also necessary for the composition of government spending to affect equilibrium employment. Once a second "sink" for revenues is introduced in the model, the composition of government spending matters for labor demand, and labor's share of income is no longer equal to one. Then, in order to maximize employment, the government should concentrate its spending in a way that minimizes the amount of money being spent on the other factors as it travels through the supply chain. This is equivalent to the government varying the composition of its expenditures to boost labor's share of income.

In this section, the second "sink" is inelastically supplied capital rented out by households to firms in a spot market. In appendix D, I detail how profits, rather than inelastically supplied capital, can also play the role of a second sink. Ultimately, it does not matter for the results whether the second sink is returns to capital or profits. Either way, it is a notion of the network-adjusted labor intensity that acts as the relevant sufficient statistic.

The model in this section is neoclassical. Therefore, although fiscal policy can affect equilibrium employment, interventions are socially harmful. Nevertheless, in section 5, I show that the intuition from the neoclassical model carries over to models with involuntary unemployment and nominal rigidities.

3.1 Household's problem

The household chooses

$$\max_{c_{it},l_t,B_t} \quad \sum_{t=0}^{\infty} \rho^t \left(\log(C_t) - \frac{l_t^\theta}{\theta} \right),$$

where

$$C_t = u(c_{1t}, \ldots, c_{nt}),$$

and u has symmetric and constant elasticity of substitution across different consumption goods. The household's budget constraint is

$$\sum (1 + \tau_{it}) p_{it} c_{it} + q_t B_t = w_t l_t + r_t K + B_{t-1} + \Pi_t - \tau_t,$$

where p_{it} is the price of good i in period t, B_t is a nominal bond (in zero net supply), Π_t is firm profits, τ_t is lump sum taxes in period t. The new ingredients in this section are r_t, the rental rate of capital, and τ_{it}, an ad valorem consumption tax on good i. So that the problem is well-defined, suppose that there is a physical limit on the number of hours that can be worked

$$l_t \leq \bar{l},$$

although we assume that this is always non-binding. This simply allows for the inclusion of inelastic labor supply as a special case. To keep the exposition clear, for now, I assume a homogenous labor market. In section 3.9, I consider an extension with heterogenous labor markets. Assume that capital is inelastically supplied at K.

3.2 Firms' problem

Firms rent capital and labor on spot markets from the household, and reoptimize every period. Therefore, their problems are static, so I suppress time-subscripts. Since in a competitive equilibrium with constant returns to scale, firm size is indeterminate, I simply state the problem of the representative firm in industry i:

$$\max_{y_i, l_i, x_{ij}} \quad p_i y_i - \sum_j p_j x_{ij} - w l_i - r k_i$$

subject to the constant-returns to scale production function

$$y_i = F_i(l_i, k_i, G_i(x_{i1}, \ldots, x_{in})),$$

where l_i is labor, k_i is capital, and x_{ij} are inputs from industry j. Let F_i and G_i have constant and symmetric elasticities of substitution between their arguments σ_F and σ_G respectively. For simplicity, I assume that $\sigma_F = \sigma_G$, though this can be relaxed. Once again, the network structure of the economy is captured by the production functions. In particular, note that if all industries used no labor, the model in this section is a special case of the model in section 2.

3.3 Government behavior

The government runs balanced budgets every period

$$\sum p_i g_i = \tau_t + \sum_i \tau_{it} p_{it} c_{it}, \tag{2}$$

and the fraction of government expenditures on industry i is

$$\frac{\delta_i}{\sum_j \delta_j} = \frac{p_i g_i}{\sum_i p_i g_i}.$$

Before solving for the equilibrium, we need a a few key definitions that will serve us throughout the rest of the paper.

3.4 Network-adjusted labor intensity

Now we can define the network-adjusted labor intensity. It turns out that network-adjusted labor intensities play a key role in the determination of equilibrium employment. Recall that the representative firm in industry i has the following production function

$$y_i = F_i(l_i, k_i, G_i(x_{i1}, \ldots, x_{in})),$$

where F_i and G_i have symmetric elasticities of substitution between their arguments σ_F and σ_G respectively. Now define the following generalized elasticities of production with respect to inputs by

$$\hat{\omega}_{ij} := \frac{\mathrm{d}y_i}{\mathrm{d}x_{ij}} \left(\frac{x_{ij}}{y_i}\right)^{1/\sigma_G},$$

and

$$\alpha_i := \frac{\mathrm{d}y_i}{\mathrm{d}l_i} \left(\frac{l_i}{y_i}\right)^{1/\sigma_F}.$$

Let

$$\hat{\Omega} := \left[\hat{\omega}_{ij}\right]_{ij},$$

be the matrix of ω_{ij}'s and α be the column vector of α_i's. Define

$$\Psi := I + \hat{\Omega} + \hat{\Omega}^2 + \hat{\Omega}^3 + \cdots = (I - \hat{\Omega})^{-1},$$

to be the influence matrix. If we think of $\hat{\Omega}$ as defining a weighted directed graph, then the influence matrix is its inverse Laplacian. The ijth element of Ψ can be interpreted as the total intensity with which i uses inputs from j, taking into account both direct and indirect connections. Finally, the vector

$$\boxed{\tilde{\alpha} = \Psi\alpha}$$

is ***network-adjusted labor intensity.*** Intuitively, $\tilde{\alpha}_i$ captures both the direct and indirect uses of labor by industry i. Computationally, $\tilde{\alpha}_i$ is a weighted sum of the labor intensities of i, and i's suppliers, and i's suppliers' suppliers, and so on.[1]

A closely related object of interest is the network-adjusted labor share. To define this, let p_i denote the price of good i and w the wage. Then, let

$$\hat{w}_{ij} := \frac{p_j x_{ij}}{p_i y_i},$$

be industry i's expenditure share on input j, and

$$a_i := \frac{w l_i}{p_i y_i},$$

be its expenditure share on labor (gross labor share). Let

$$\hat{W} := \left[\hat{w}_{ij}\right]_{ij},$$

[1]We can also think of $\tilde{\alpha}$ and $1 - \tilde{\alpha}$ as the dominant eigenvectors of the matrix defined by $\begin{pmatrix} \hat{\Omega} & [\alpha \;\; \eta] \\ \mathbf{0} & I_2 \end{pmatrix}$, where I_2 is the 2×2 identity matrix.

be the matrix of w_{ij}'s and a be the column vector of a_i's. Define

$$\boxed{\tilde{a} = (I - \hat{W})^{-1} a}$$

to be the ***network-adjusted labor share.*** Intuitively, $\tilde{a}_i$ captures the total fraction of industry i's income that is eventually paid out to labor, whether directly by that industry itself, or through its supply chain. This fact is also noted by Valentinyi and Herrendorf (2008) who use it to measure factor income shares for the four major sectors of the US economy. Measures like $\tilde{\alpha}$ and $\tilde{a}$ behave like downstream versions of the measures derived by Antràs and Chor (2013) and Acemoglu et al. (2012), since they rely on an industry's *suppliers* rather than their *consumers*. The network-adjusted labor share also has an interpretation from the input-output literature pioneered by Leontief (1936). If we fix prices, and assume that production functions have a Leontief form, then the network-adjusted labor share is the total amount of labor required in order to produce a unit of a good.[2]

An important observation is that when F_i and G_i have Cobb-Douglas forms, the network-adjusted labor share and the network-adjusted labor intensity coincide. This makes Cobb-Douglas a very convenient modelling assumption, since it allows us to identify network-adjusted labor intensities from only expenditures data, and it makes the structural objects of interest $\tilde{\alpha}$ coincide with accounting objects of interest $\tilde{a}$. In Appendix C, I show how the network-adjusted labor intensities affect labor's share of income in a CES economy.

Intuitively, the network-adjusted labor intensity is always weakly greater than the gross labor intensity for every industry. This follows from the fact that taking into account the supply chain of a given industry can only increase the intensity with which labor is used.

Proposition 1. *For every industry i, we have*

$$\tilde{\alpha}_i \geq \alpha_i.$$

Proof. This follows from the non-negativity of Ω and α. ■

Just as with the value-added labor and capital shares, the network-adjusted labor and network-adjusted capital shares always sum to one.

Lemma 2. *Let a be the gross labor shares and c be the gross capital shares of industries. Then, the sum of the network-adjusted labor and capital share equals* 1 *for every industry.*

$$(I - W)^{-1} a + (I - W)^{-1} c = \mathbf{1}.$$

[2] As we shall see, when prices are being set flexibly, it is the network-adjusted labor intensity and not the network-adjusted labor share that determines equilibrium responses to shocks. So, with flexible prices and Leontief production functions, the usual input-output estimates of the impact of a demand shock are invalid.

Proof. See Appendix A. ■

This means that the network-adjusted labor share of industry i is *labor's share of income* from industry i. We can aggregate this observation up to get labor's share of total income.

Proposition 2. *Labor's share of aggregate income is equal to the final-consumption weighted average of network-adjusted labor shares. Specifically,*

$$\frac{wl}{GDP} = \frac{(H+G)'\tilde{\alpha}}{GDP},$$

where H is the vector of household spending net of consumption taxes, and G is the vector of government spending by industry.

Proof. See Appendix A. ■

It is crucial to note that proposition 2 is an *accounting identity*, and it will hold for all production and utility functions. Proposition 2 will serve as the foundation for a decomposition of labor's share of income into disaggregated components in section 4.

3.5 Response to Demand Shocks

In this subsection, I show that network-adjusted labor intensities allow us to trace out the aggregate effect of shocks to an industry's demand. For clarity, I assume that production and utility functions have Cobb-Douglas forms and leave the more general case to Appendix C. Competitive equilibrium is defined in the usual way. I focus on the steady-state equilibrium of this model and therefore, suppress time subscripts.

Definition 3.1. The employment multiplier of a taste shock to industry i is defined as $\mathrm{d}l/\mathrm{d}\beta_i$, where l is equilibrium employment and β_i is the Cobb-Douglas taste of the household for goods from industry i.

The following proposition shows how the network-adjusted labor intensities allow us to translate a change in final demand for goods into changes in equilibrium employment:

Proposition 3. *Employment multipliers of taste shocks satisfy*

$$\frac{\mathrm{d}l/\mathrm{d}\beta_i}{\mathrm{d}l/\mathrm{d}\beta_j} = \frac{\tilde{\alpha}_i - \beta'\tilde{\alpha}}{\tilde{\alpha}_j - \beta'\tilde{\alpha}}.$$

Proof. See Appendix A. ■

This shows that multipliers for industry i's demand are proportional to industry i's network-adjusted labor intensity minus the average labor intensity of consumption. This is quite intuitive since increasing the household's taste for good i will reorient household expenditures from all other industries towards industry i. To the extent that household preferences change at business cycle frequencies, this proposition shows the relative importance of preference shocks for aggregate employment. In section 5, I show that fiscal stimulus affects equilibrium employment in much the same way, and I derive the optimal industrial composition of fiscal policy when monetary policy is passive.

This result can be extended, without change, to the case where consumption and production functions have constant and symmetric elasticity of substitution, with β being the CES share parameters of consumption.

3.6 Comparison to labor share's share of value-added

The network-adjusted labor share is different from the value-added labor share commonly seen in the literature, for example in Estrada and Valdeolivas (2012), Elsby et al. (2013), Neiman and Karabarbounis (2014), or Oberfield and Raval (2012). Whereas the network-adjusted labor share counts the contributions of labor to the production of a given good up the supply chain, the value-added labor share divides the wage bill by revenues net of intermediate inputs.

(a) Single firm serving the household.

(b) Fragmented production process.

Figure 1: Fragmentation of the same production process. Each node represents a firm and the numbers under each node denote the gross labor and capital share of that firm. Edges denote the flow of goods and services. The labels on the edges denote the transaction's share of the downstream firm's total expenditures.

To see the difference, consider figure 1. There, we see how the same production process being broken from a single aggregate firm into two firms affects the network-adjusted and value-added labor shares. In the first panel, an aggregate firm provides a good to the household. In the second panel, the first firm supplies the capital part and the second firm the labor part of production. Intuitively, the activities of firm 1 in panel (b) are just as labor intensive as firm 1 in panel (a). However, the value-added labor share and the gross labor share of firm 1 in panel (b) are both equal to zero. On the other hand, the network-adjusted labor intensity of firm 1 is the same in both cases. So the network-adjusted factor shares and factor intensities are robust to changes in accounting rules or ownership structure, while the value-added measures are not.

Theoretically, there is no reason to expect a tight connection between the value-added labor share and the network-adjusted labor share. To see this, consider an industry whose inputs are made purely from labor but does not use labor directly. This industry's value-added labor share will be zero, but its network-adjusted labor share can be arbitrarily close to one. Reversing the roles of labor and capital produces the opposite result, with value-added labor share equal to one and a network-adjusted labor share that can be arbitrarily close to zero. In section 3.7, I compare observed network-adjusted, gross, and value-added measures of labor intensity for the U.S. economy.

In practice, the difference between these two measures becomes most apparent when considering primary industries with low-margins. The value-added approach will assign high labor shares of around 90% to primary industries like "Soybean and other oilseed processing," "Fiber, yarn, and thread mills," and "poultry processing" since their capital share (revenues minus labor and intermediate inputs) are close to zero. However, the network-adjusted labor share of these industries is quite low since their supply chains are not very labor-intensive.

The value-added labor share and the network-adjusted labor share will coincide when the supply chain of an industry uses the same capital/labor mix as the industry itself. The leading case of this is when an industry buys its intermediate inputs exclusively from itself (i.e. a degenerate input-output matrix with only diagonal elements). This intuition makes clear why the level of aggregation will be crucially important for whether or not the value-added labor share is a useful statistic. Once we aggregate the economy into a single sector, the input-output matrix is always diagonal (since it is a scalar), and so, the network-adjusted labor share and the value-added labor share coincide. However, at this level of aggregation, the value-added labor share is no longer informative about the industrial composition of the economy.

3.7 Calibration of $\tilde{\alpha}$ for the US

Now that we have defined and interpreted network-adjusted labor intensity $\tilde{\alpha}$, we turn to calibrating it for the United States. If we assume Cobb-Douglas functional forms, then we can measure $\tilde{\alpha}$ using national accounts data from the Bureau of Economic Analysis. There are two reasons to assume Cobb-Douglas. First, as discussed earlier, network adjustments are only interesting in cases where the underlying data is disaggregated. Assuming Cobb-Douglas allows us to use much more disaggregation since we only need expenditures data to calibrate the model. Calibrating a model with a non-unitary elasticity requires both price and quantity data. Second, for Cobb-Douglas, all network-adjusted labor intensities can also be interpreted as network-adjusted labor shares, which are accounting objects of independent interest. Once we deviate from Cobb-Douglas, the relevance of the computations will depend on how well we choose the elasticity of substitution, which is a controversial question outside the scope of this paper.

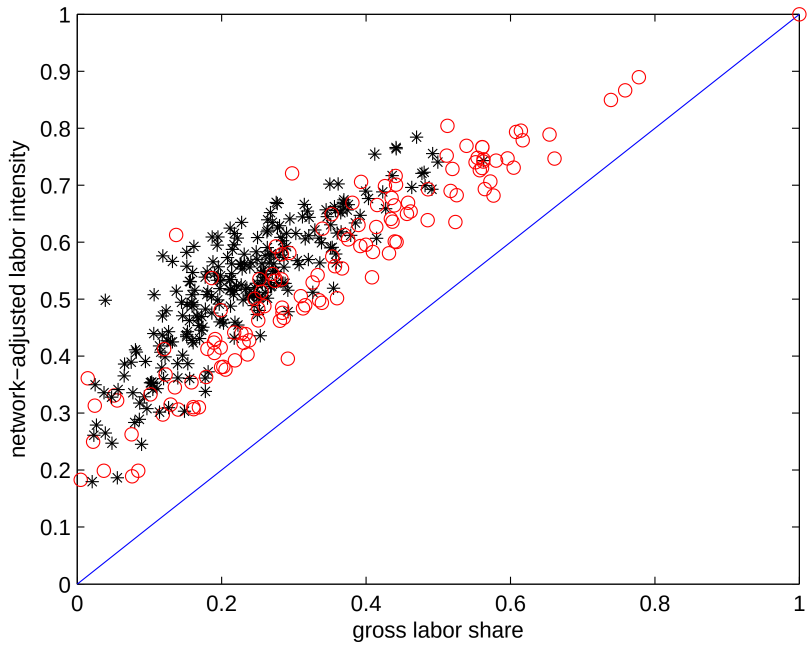


Figure 2: Labor intensities plotted against network-adjusted labor intensities using the detailed BEA input-output table. There are 381 industries in this plot. The black asterisks are manufacturing industries, while red circles represent non-manufacturing industries. Non-monotonicities represent cases where industries are ranked differently according to the different measures of labor use.

I use the detailed 2007 benchmark use-tables at purchaser values. This measures the dollar expenditures of a given industry on inputs. Since the Cobb-Douglas parameters are equal to the shares of expenditures, it is easy to calibrate the production functions of the various industries, as well as the utility function of the household using this data. I let the capital intensity of a given industry equal one minus the labor share and the intermediate input share. The calculations in this section abstract from world trade in production and assume all imports are final goods. The reason for this is that data on the breakdown of imports between intermediate and final use are not available from the Bureau of Economic Analysis' statistical data sources. In section 4, I explain how we can account for trade in this model by using other data sources. The results are plotted in figure 2. As implied by proposition 1, all points in figure 2 lie above the 45-degree line.

Industries with the largest and smallest $|\tilde{\alpha}_i - \alpha_i|$ are listed in table 1. Generally speaking, manufacturing industries are much more labor intensive than their gross labor shares would indicate, while service industries like "the postal service" or "office administrative services" are about as labor intensive as their gross labor share suggests. This is intuitive, since service industries have shorter, less labor-intensive, supply chains. The key exception to this general rule are some financial industries like "Funds, trusts, and other financial vehicles," which also have much higher

Table 1: Industries with the largest and smallest differences between their network-adjusted labor intensity and their labor share.

Industry	$\tilde{\alpha} - \alpha$	$\tilde{\alpha}$	α
Funds, trusts, and other financial vehicles	0.476	0.613	0.137
Light truck and utility vehicle manufacturing	0.459	0.498	0.039
Heavy duty truck manufacturing	0.457	0.576	0.119
Railroad rolling stock manufacturing	0.434	0.566	0.132
Motor vehicle seating and interior trim manufacturing	0.431	0.593	0.162
Automobile manufacturing	0.43	0.582	0.152
Other financial investment activities	0.423	0.721	0.298
Sawmills and wood preservation	0.422	0.609	0.187
News syndicates, libraries, archives and all other information services	0.104	0.396	0.292
Support activities for agriculture and forestry	0.105	0.682	0.577
Postal service	0.108	0.866	0.759
Civic, social, professional, and similar organizations	0.110	0.850	0.739
Office administrative services	0.111	0.889	0.778
Accounting, tax preparation, bookkeeping, and payroll services	0.111	0.635	0.524
Animal production, except cattle and poultry and eggs	0.113	0.189	0.076
Oil and gas extraction	0.114	0.199	0.085

labor intensities than one might infer from their gross labor share. The calculations here indicate that once supply chains are properly taken into account, the manufacturing sector is very labor intensive.

The alternative popular measure of labor intensity at the industry level is the value-added labor share. As discussed earlier, there is no reason to theoretically expect the network-adjusted labor share to be related to the value-added labor share. In the data, the correlation between the value-added and network-adjusted labor shares is 0.90, which is only slightly higher than the correlation between the gross and network-adjusted labor share at 0.87. The value-added and network-adjusted labor intensities are plotted in figure 3. Unlike figure 2, there are data points above and below the 45-degree line. Furthermore, unlike figure 2, where a gross labor share close to 1 implied that the network-adjusted labor intensity must also be close to 1, no such pattern need hold now.

The fact that the slope of the line of best fit in figure 3 is less than 1 implies that an industry's capital-labor mix is negatively correlated with its supply chain's capital-labor mix. This is because network-adjusted labor shares are higher (lower) when value-added labor shares are low (high), meaning that accounting for the supply chain properly increases (decreases) the labor share. In other words, the network-adjusted labor intensities have a smaller range than the value-added labor shares.

Another advantage of network-adjusted labor shares over value-added labor shares is that

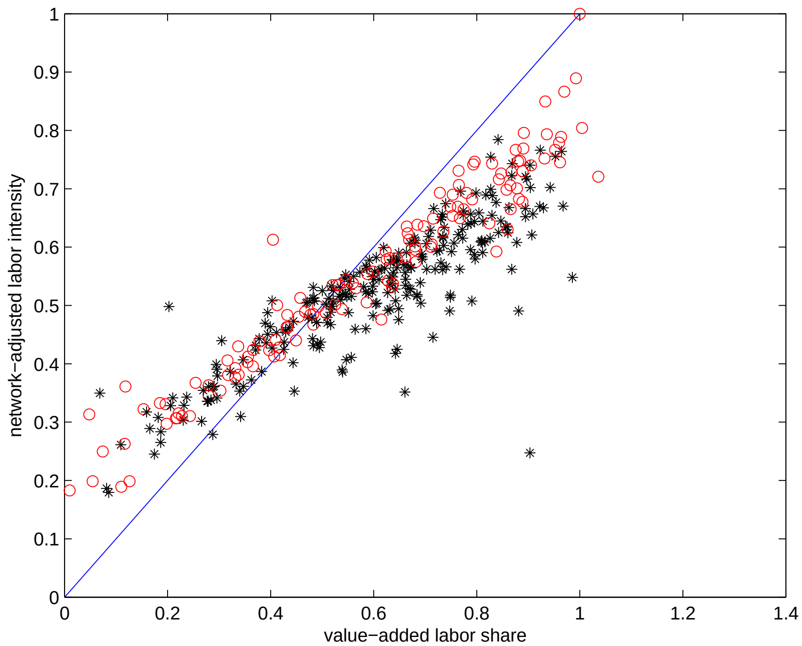


Figure 3: Value-added labor shares plotted against network-adjusted labor intensities using the detailed BEA input-output table. There are 381 industries in this plot. The black asterisks are manufacturing industries, while the red circles represent non-manufacturing industries. Non-monotonicities represent cases where industries are ranked differently according to the different measures of labor use.

they are more stable in time series. Value-added labor shares can move around violently at a high frequency if an industry's profits fluctuate. Network-adjusted labor shares, since they are weighted averages of many industries' labor and capital shares, are more stable over time. The increased time series stability suggests that secular changes in network-adjusted labor shares are more likely to reveal meaningful patterns. Furthermore, since they are averages over many industries, they are less badly affected by measurement error.

3.8 Upstream and Downstream Influence

Proposition 3 shows that $\tilde{\alpha}$ is the relevant influence measure of how industry-specific demand shocks move aggregate employment. Acemoglu et al. (2012) derive an alternative influence measure that they show maps supply (labor-augmenting productivity) shocks to aggregate output. To clarify $\tilde{\alpha}$'s network-theoretic properties, it helps to compare it with the alternative influence measure of Acemoglu et al. (2012). In this model, the Acemoglu et al. (2012) measure corresponds to $\tilde{\beta} \equiv \beta'\Psi$. This can be thought of as a network-adjusted consumption share. It takes into account both direct sales to households, as well as sales to industries who to sell to households, and sales to industries who sell to industries who sell to households, and so on. Acemoglu et al. (2012) show that $\tilde{\beta}$ is the key statistic determining how output responds to supply (labor-augmenting productivity) shocks. Since they are interested in the propagation of productivity shocks, Acemoglu et al. (2012) abstract away from capital and assume that labor is inelastically supplied. Since we are interested in the effect of demand shocks on employment, we need both of these ingredients, because as we saw in section 2, without them the model would give trivial answers to our questions.

To see the difference between the $\tilde{\alpha}$ and $\tilde{\beta}$, consider the example in figure 4.

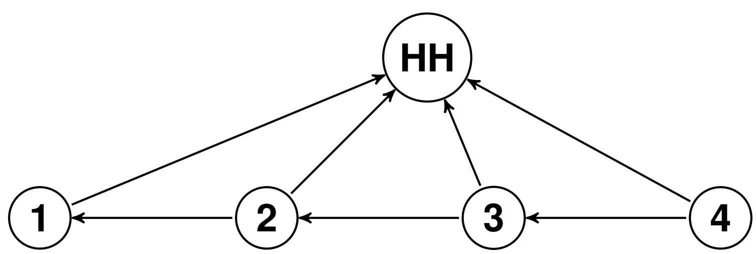


Figure 4: The arrows represent the flow of goods and services.

In figure 4, the network-adjusted labor intensity of firm (1) is

$$\tilde{\alpha}_1 = \alpha_1 + \omega_{12}\alpha_2 + \omega_{12}\omega_{23}\alpha_3 + \ldots,$$

while the network-adjusted labor intensity of the final firm (4) is simply equal to the regular labor intensity

$$\tilde{\alpha_4} = \alpha_4.$$

On the other hand, in figure 4, the network-adjusted consumption share of firm (1) is the same as its regular consumption share

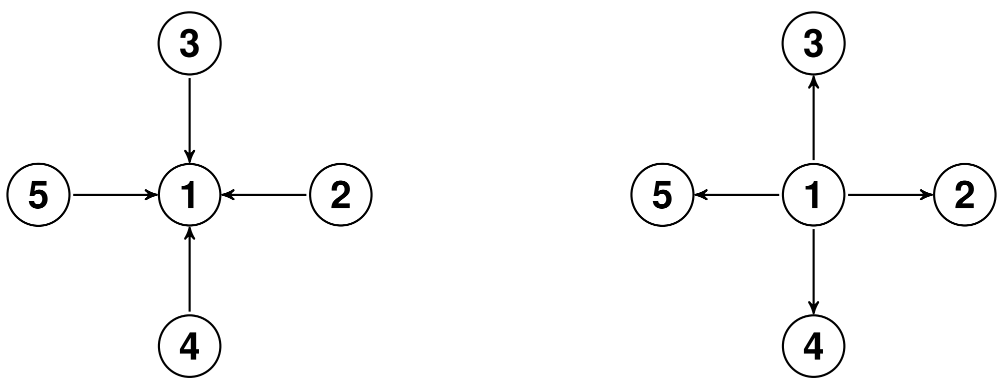
$$\tilde{\beta}_1 = \beta_1,$$

while the network-adjusted consumption share of firm (4) is

$$\tilde{\beta}_4 = \beta_4 + \beta_3\omega_{34} + \beta_2\omega_{23}\omega_{34} + \dots .$$

This simple example makes the difference clear: the network-adjusted labor share $\tilde{\alpha}$ is a downstream centrality measure, while the network-adjusted consumption share of Acemoglu et al. (2012) is an upstream centrality measure. This is because demand shocks travel upstream from consumers of inputs to producers of inputs, while supply shocks travel downstream from suppliers of inputs to consumers of inputs.[3]

The difference between the two measures can be seen most clearly by setting $(\beta_i, \alpha_i) = (\alpha, \beta)$ for all i and considering two different star economies in figure 5.



(a) Firm 1 is a star consumer

(b) Firm 1 is a star supplier

Figure 5: The arrows indicate the flow of goods and services.

Firm (1) in figure 5a will have the highest network-adjusted labor share and the lowest network-adjusted consumption share. The situation is exactly reversed in figure 5b. In the former case, firm (1) is an important conduit for the transmission of demand shocks and a poor conduit for the transmission of supply shocks, whereas in the latter, the opposite is true.

3.9 Heterogenous Labor Markets

We can easily extend the model to cover heterogenous labor markets. This allows us to analyze and craft policies to target specific parts of the labor market. To keep the notation clean, I suppress time subscripts. Suppose there are M different types of labor indexed by m and the production

[3]See Baqaee (2014b) for more details about the class of models where demand and supply shocks travel only in one direction.

function of industry i is given by

$$y_i = \left(\prod_{m}^{M} l_{im}^{\iota_{im}} \right)^{\alpha_i} k_i^{\eta_i} \prod_{j=1}^{N} x_{ij}^{\omega_{ij}}.$$

Now, labor market clearing for type m labor is given by

$$\begin{aligned} w_m l_m &= \sum_{i=1}^{M} p_i y_i \alpha_i \iota_{im}, \\ &= (H+G)' \Psi(\alpha \circ \iota_m), \end{aligned}$$

where $\circ$ denotes the element-wise product and ι_m is the column vector of ι_{im}'s for different industries i. Now define the *network-adjusted type-m labor intensity* by

$$\tilde{\alpha}_m = \Psi(\alpha \circ \iota_m).$$

The network-adjusted type-m labor intensity, for a Cobb-Douglas economy, is also that labor type's share of an industry's income. In the next section, this will allow us to break up labor income into labor income by skill level. Furthermore, we can now speak of demand-side interventions to specific labor markets. So for instance, if policy-makers wish to use fiscal policy to boost low-skill employment because the low-skill labor market is failing to clear, then they can tailor policy towards increasing demand for goods with high network-adjusted low-skill labor intensity.

Unfortunately, neither the BLS nor the BEA publish statistics about the intensity with which different types of labor are used by different industries. Therefore, I use the American Community Survey from 2007 to construct ι_m for each industry group in the detailed benchmark US input-output table of 2007. I divide labor into 11 types by educational attainment. In table 2, I report network-adjusted and gross labor shares for each type of labor for the industries that move the most when we take the input-output structure into account. For lower skill levels, manufacturing sectors gain the most from the network adjustment, while for higher skill levels, financial industries move the most.

In the next section, I build on these results to show how and why labor's share of income has changed over the past 15 years.

Education level	Industry	Network-adjusted labor share	Gross labor share
N/A or no schooling	Coffee and tea manufacturing	0.0048	0.0005
Nursery school to grade 4	Coffee and tea manufacturing	0.007	0.000
Grade 5, 6, 7, or 8	Coffee and tea manufacturing	0.023	0.001
Grade 9	Sawmills and wood preservation	0.014	0.004
Grade 10	Sawmills and wood preservation	0.015	0.004
Grade 11	Sawmills and wood preservation	0.015	0.005
Grade 12	Light truck and utility vehicle manufacturing	0.169	0.019
1 year of college	Light truck and utility vehicle manufacturing	0.067	0.006
2 years of college	Light truck and utility vehicle manufacturing	0.039	0.003
4 years of college	Funds, trusts, and other financial vehicles	0.219	0.057
5+ years of college	Funds, trusts, and other financial vehicles	0.156	0.030

Table 2: Industries with the largest difference between their network-adjusted labor share and gross labor share for each labor type. This table combines data from the 2007 American Community Survey from IPUMS-USA with the detailed Benchmark Input-Output table using purchaser prices for 2007 published by the BEA.

Education level	Industry	Network-adjusted labor share
N/A or no schooling	Support activities for agriculture and forestry	0.011953
Nursery school to grade 4,	Greenhouse, nursery, and floriculture production	0.018467
Grade 5, 6, 7, 8	Support activities for agriculture and forestry	0.057632
Grade 9	Civic, social, professional, and similar organizations	0.024194
Grade 10	Civic, social, professional, and similar organizations	0.026585
Grade 11	Electronic and precision equipment repair and maintenance	0.029375
Grade 12	Private households	0.407733
1 year of college	Private households	0.173831
2 years of college	Residential mental retardation, mental health, substance abuse and other facilities	0.115352
4 years of college	Management of companies and enterprises	0.360452
5+ years of college	Accounting, tax preparation, bookkeeping, and payroll services	0.385613

Table 3: Industries with the largest network-adjusted labor share for each labor type. This table combines data from the 2007 American Community Survey from IPUMS-USA with the detailed Benchmark Input-Output table using purchaser prices for 2007 published by the BEA.

4 Labor's share of income

As lemma 2 and proposition 2 show, network-adjusted labor shares are labor's share of an industry's income. Therefore, they can help us to decompose aggregate labor shares into disaggregated industrial components. Research on the evolution of labor's share of income has recently exploded. Piketty (2014) places labor's share of income at the heart of his theory of inequality. A variety of papers have been written on the causes and consequences of the decline in labor's share of income (see Neiman and Karabarbounis, 2014; Oberfield and Raval, 2012; Elsby et al., 2013). Popular theories for why labor's share of income has trended down include: increased globalization, increases in the capital stock, decreases in the price of investment goods, and increased automation in production.

This paper's contribution to this debate is to provide a coherent accounting framework for decomposing labor's share of income into disaggregated components. Using this framework, I find that the decline in labor's share of income is due primarily to a decrease in the gross labor share of all industries, and not changes to the composition of industries. In particular, I do not find strong evidence for the idea that labor's share of aggregate income has decreased due to substitution of imported inputs or imported consumption goods for domestic labor. I also find similar results for the income share of different labor types by education. Furthermore, consistent with the skill-biased technical change hypothesis of Goldin and Katz (2009), I find that changes in income share within labor types dwarfs changes between labor and capital's share of income, and that the source of these changes is within industries.

As already discussed, the network-adjusted labor share of an industry is precisely labor's share of that industry's income, and the GDP-weighted average of network-adjusted labor shares is equal to labor's share of aggregate income. This accounting identity must hold regardless of the underlying production and utility functions. This allows for a decomposition of labor's share of income into disaggregate industrial components.

For this section, I add international trade in intermediate and final goods to the model in section 3. This not only brings the model closer to the data I use, but it also allows us to account for the effects of globalization on labor's share of aggregate income. The key assumption of the model is that labor and capital are immobile, but other goods and services are traded with the rest of the world in consumption and production. The results in this section complement the work of Trefler and Zhu (2010) who account for intermediate inputs in computing the factor content of trade. They show that adjusting for the role of intermediate inputs significantly improves the Heckscher-Ohlin model's fit to the data.

The key result for this section, proposition 4, does not depend on structural assumptions, and relies only on accounting identities. Let W^* be the matrix whose ijth element is industry i's share of expenditures on the domestic industry j, and let b be the column vector whose ith element is the share of final-use expenditures on domestic industry i. Finally, let a be the column vector whose ith element is the gross labor share of industry i. Then, analogous to the closed-economy proposition 2, the following proposition holds with international trade.

Proposition 4. *Labor's share of income is*

$$\frac{wl}{GDP} = b'(I - W^*)^{-1}a = b'Pa = b'\tilde{a}, \tag{3}$$

where $P = (I - W^*)^{-1}$, and $\tilde{a}$ is the vector of network-adjusted domestic labor shares.

Proof. Let s_i denote the sales of industry i. Then, labor market clearing implies that

$$wl = s'a.$$

Furthermore, market clearing for good i gives

$$s' = (b)'\text{GDP} + s'W^*,$$

where W^* is the domestic input-output expenditure share matrix. Then,

$$s' = (b)'(I - W^*)^{-1}\text{GDP},$$

and so

$$wl = (b)'(I - W^*)^{-1}a\text{GDP}.$$

■

Note that this proposition holds for *any* structural model of the economy because it only makes use of accounting identities. A convenient way to give proposition 4 a structural interpretation is presented in Appendix B. There, I show that in an Armington model of trade, with unitary elasticity of substitution in consumption and production, the network-adjusted domestic labor share coincides with the network-adjusted domestic labor intensity. Adding trade to the model in this way does not change the model's qualitative properties. Analogues of all of the propositions in section 3 exist in the model with international trade. The presence of traded goods simply means that we must adjust the influence matrix for the fact that some fraction of expenditures on each good purchased was imported.

Using proposition 4, decompose labor's share of income into changes in its constituent parts

$$\Delta \frac{w_t l_t}{GDP_t} = (\Delta b)_t' P_t a_t + b_{t-1}'(\Delta P_t)\alpha_t + b_{t-1}' P_{t-1}(\Delta a_t), \tag{4}$$

where Δ is the time difference operator. Observe that if we assume Cobb-Douglas functional forms, b, P, and a will correspond to Cobb-Douglas parameters. Summing equation (4) over N

time periods gives

$$\frac{w^i_{t+N} l^i_{t+N}}{GDP_{t+N}} - \frac{w_t l_t}{GDP_t} = \underbrace{\sum_t^{t+N} \Delta b_t' P_t a_t}_{\text{consumption}} + \underbrace{\sum_t^{t+N} b_{t-1}' \Delta P_t a_t}_{\text{supply chain}} + \underbrace{\sum_t^{t+N} b_{t-1}' P_{t-1} \Delta a_t}_{\text{gross labor share}} . \tag{5}$$

Equation (5) decomposes changes to labor's share of income over time period t through $t + N$ into changes due to three different components: (1) changes to the composition of final goods consumption, (2) changes to the supply chain, including increased use of imported inputs, and (3) changes to the fraction of expenditures on labor by each industry.

We can see that the first summand is the effect of changes in consumption because Δb_t is the change in what final goods are demanded in the economy. To turn the effect of a change in final good demand into a change in aggregate labor share, we must multiply Δb_t by the network-adjusted labor shares $P_t a_t$ to capture the flow-on effects of changes in final good demand on intermediate industries.

We can see that the second summand is the effect of changes in supply chains because ΔP_t captures the changes in the input-output matrix. To turn the effect of a change in the domestic input-output matrix into a change in aggregate labor share, we must multiply it by final goods demand b_{t-1} and gross labor shares a_t.

Finally, we can see that the third summand is the effect of changes in gross labor shares because Δa_t captures changes in industry-level gross labor shares. To turn the effect of a change in gross labor shares into a change in aggregate labor share, we must multiply Δa_t by the total size of the industries $b_{t-1}' P_{t-1}$.

Of course, in practice, all of these terms will be moving together at the same time. However, changes are still interpretable. As an example, consider the case where industry i reduces its expenditures on labor, so a_{it} falls. This means that either that industry's gross capital share must be increasing or its intermediate input share must be increasing. If only the gross capital share increases, then we would observe a drop in the third component of the summand and no change in the second and first component, since $\Delta b = \Delta P_t = 0$. However, if the intermediate input share rises instead, then it depends on what intermediate input is being purchased. If that intermediate input uses a lot of labor, then we observe a drop in the third component and an increase in the second component. If that intermediate input uses a lot of capital or was imported, then we observe a drop in the third component and no change in the first and second components.

To summarize, the first component of (5) captures changes in how final goods consumption has changed across industries. This would capture changes in labor's share of income due to changes in household consumption patterns (either across different industries or between domestic/foreign production). The second component of (5) captures how changing supply chains are affecting labor's share of income. This would include either changes in the interconnections between industries, or increased use of imported intermediate inputs. The first two components capture changes in labor's share of income due to the changing composition of industries. This means that globalization-driven changes to the labor share, most recently emphasized by Elsby et al. (2013), should show up in the first two components. This is because if labor's share of income is falling due to households and firms buying more labor-intensive goods from overseas, this should show up in the first or second component of (5).

A further breakdown is possible if we assume that labor inputs consist of high skill, medium skill, and low skill labor. Then we can further decompose the changes in the high, medium, and low skill labor share as

$$\Delta \frac{w_t^i l_t^i}{GDP_t} = \underbrace{(\Delta b_t')P_t(a \circ \iota_t^i)}_{\text{consumption}} + \underbrace{b_{t-1}'(\Delta P_t)(a_t \circ \iota_t^i)}_{\text{supply chain}} + \underbrace{b_{t-1}'P_{t-1}\Delta(a_t \circ \iota_t^i)}_{\text{gross labor share}} + \underbrace{b_{t-1}'P_{t-1}(a_{t-1} \circ \Delta\iota_t^i)}_{\text{intralabor}}, \quad (6)$$

where ι_t^i is the vector of the shares of type i labor as a fraction of total labor used by the different industries in period t, and $\circ$ is the element-wise product. This formula allows us to decompose changes in labor type i's share of income into four components. The first three are the same as before, but now we have a fourth term capturing substitution within labor. The primary reason to suspect that the fourth term has changed is skill-biased technical change, emphasized by Goldin and Katz (2009).

For this section, I use data from the World Input-Ouput Database (WIOD). Using the WIOD, we can compute labor's share of income, and the decomposition of labor's share of income, implied by input-output tables of 34 different countries from 1995 to 2009. One of the great advantages of the WIOD over national input-output tables is that the WIOD includes data on trade in intermediate inputs. Whereas, many national data sources, like the BEA, do not provide this information. The downside to using the WIOD is that rather than having 381 industries, there are only 35 industries. For more information on the sources and construction of the WIOD see Timmer et al. (2012).

In figure 6, I plot the cross-country average (weighted by GDP) of equation (5) for the entire sample. This can be interpreted as a decomposition of labor's share of average income. We can see that the labor share at the industry level explains the majority of changes in labor's share of income. Changing consumption patterns also contribute, but their contribution is more than 3 times smaller. Changing supply chains, on average, are not causing any trends in labor's share of income.

The fact that the "consumption" and "supply chain" lines do not move very much in figure 6 is evidence against the idea that changes in the nature of supply chains or changes in imports of foreign goods for domestic goods have caused aggregate labor's share of income to drop in the sample (on average). However, by averaging over many countries, we are losing interesting variation within countries. If it turns out that in some countries globalization is increasing labor's

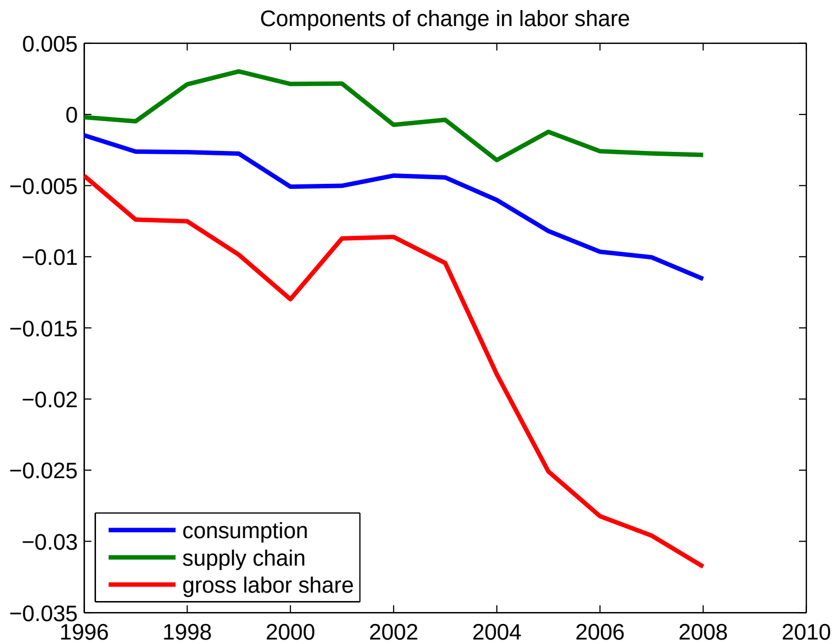


Figure 6: Cross-country average of cumulative changes in labor's share of income according to decomposition in equation (5). The data are from the World Input-Output Database. The data is in percentages.

share of income and in others it is decreasing labor's share, then we lose this by averaging. To get a sense of magnitudes, in figure 7, I show the total change, in absolute values, of each component of equation (5) over the sample for each country. The figure shows that by and large the largest movements in the labor's share of income are in the gross labor shares of all industries.

In figure 8, I break down the cross-country average into its effect for different labor types by equation (6). We see that the largest changes are attributable to the change in the composition of labor use from low-skill to high-skill. Adding the three lines in each plot gives the evolution of that labor-type's share of income. While low-skill and medium-skill labor shares have declined since 1995, high-skill labor's share of income has increased, primarily due to increased reliance on high-skilled labor relative to other types of labor. We can also see that changing supply chains have had differential effects on different types of labor. The changes in supply chains have contributed to increases in high-skilled labor's share of income but decreases in low and medium-skilled labor's shares of income.

In terms of substitution within labor's share of income, that is substitution between differently skilled labor, the global picture is much more homogenous. To show this, in table F.1, I report the total change owing to each of these components for all the countries in the sample for which all the data is available. The numbers in table F.1 show that, within labor-types, there are very strong and

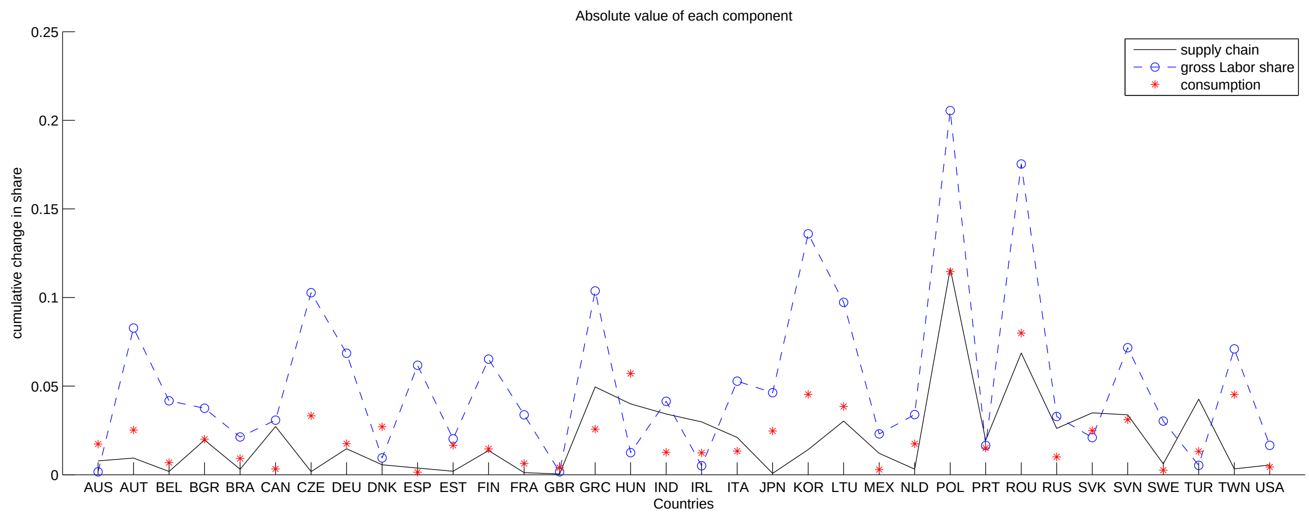


Figure 7: The absolute value of the total change in each component of equation (5) for each country from 1995 to 2009. The data are from the World Input-Output Database

near-universal compositional effects, with high-skill labor's share increasing and low-skill labor's share dropping. Column 5 shows that all countries in the sample, with the exception of Denmark and Estonia, feature declining low-skill labor share. Column 4 shows that large fractions of this decline are due to changes in the fraction of low-skill labor as a fraction of total labor. Similarly, all countries except Mexico feature increasing high-skill labor share of income. Therefore, the change between labor types is happening within industries and not across them. Adding the 5th, 10th, and 15th columns gives the overall change in the labor share.

These findings update and strengthen the results of Berman et al. (1994), who found that realloaction from unskilled to skilled labor in manufacturing over the 1980s in the US occurred within industries rather than between them. My work improves upon their decomposition by explicitly accounting for intermediate inputs and trade. These findings are also strongly supportive of the "skill-biased technical change" thesis of Goldin and Katz (2009) and Katz and Murphy (1992). The pattern of intra-labor substitution is particularly pronounced for the United States (see figure F.1), where the labor share has remained roughly constant from 1995 to 2009, but the relative shares of the different skill levels have changed drastically. This suggests that, at least for the United States, increases in income inequality are more likely linked to substitution from low-skill and medium-skill labor to high-skill labor, rather than increased use of imports or capital.

Figure F.1, in Appendix F, plots the decomposition of the labor share into its constituent high-skill, medium-skill, and low-skill components, as well as a decomposition of the changes in the each type of labor share for the US. Once again, the break-down shows that the largest component of the decline in low-skill labor intensity is the final component: low-skilled labor as a share of total labor. Although changing consumption patterns and supply chains contribute to fluctuations,

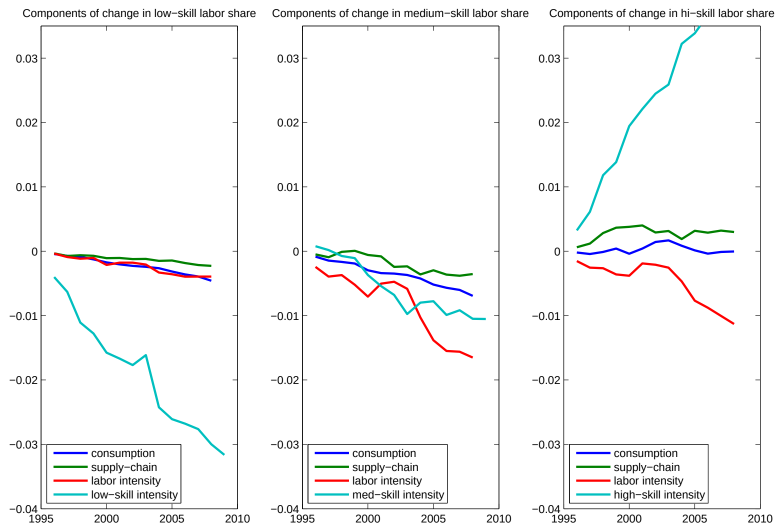


Figure 8: Cross-country average of changes in components of labor's share of income according to decomposition in equation (6). The data are from the World Input-Output Database.

there are no strong universal trends. This rules out theories of the decline in low and medium skilled labor's share of income that rely on substitution from labor towards imported intermediate inputs or changing composition of industries. It also rules out the possibility that changing supply lines, for example an increase of IT services in production, is driving the trend. If these factors were driving the trends, we should expect the "consumption" and "supply-chain" lines to be trending downwards. The trends we observe are consistent with skill-biased technical change.

4.1 Comparison to value-added decomposition

An alternative decomposition of labor's share of income common in the literature follows from the following accounting identity:

$$\frac{wl}{GDP} = \sum_i \frac{VA_i}{GDP} \hat{\alpha}_i,$$

where $VA_i = p_i y_i - \sum_j p_j x_{ij}$ is sales net of intermediate input costs, and $\hat{\alpha}_i$ is the value-added labor share defined as labor costs divided by value-added. This identity gives rise to

$$\Delta \frac{w_t l_t}{GDP_t} = \underbrace{\Delta \frac{VA}{GDP}_t^{\prime} \hat{\alpha}_t}_{\text{between-industries}} + \underbrace{\frac{VA}{GDP}_{t-1}^{\prime} \Delta \hat{\alpha}_{t-1}}_{\text{within-industries}}.$$

This decomposition, if the input-output matrix were diagonal, would have the following interpretation. The first summand captures changes to the composition of industries due to changing final-use expenditure patterns. For instance, households are spending a larger fraction on foreign-made goods or they are spending a larger fraction of their income on less-labor intensive sectors. The second summand, on the other hand, captures the labor/capital mix of each industry holding fixed sectoral compositions. These terms are commonly, but misleadingly, referred to as the between-industries and within-industries changes.

Since the input-output matrix is non-diagonal, these interpretations are not technically appropriate. In the empirically relevant case where the input-output matrix is non-diagonal, the value-added decomposition is difficult to interpret. This is because it is a non-linear transformation of the data that is not tightly connected with the theory. In figure 9, I plot the cross-country decomposition using value-added measures. Figure 9 is the value-added decomposition using the same data as figure 6. The two figures tell drastically different stories. The value-added decomposition gives the misleading impression that the composition term is much more important than within industry changes in explaining the decline in labor's share of aggregate income. This would incorrectly suggest that theories that affect industrial composition, could be the significant driver of the effect for the aggregate labor's share of income in this sample.

To see why the two decompositions may paint differing pictures, consider a simple example of an industry that uses labor, capital, and imported intermediate inputs to produce. Now suppose that this industry experiences capital-biased technological change, so that it changes its input mix, but it does not expand or shrink its sales. This industry substitutes away from intermediate inputs and labor towards using more capital. This would seem like a textbook case of within-industry change and under the network-adjusted decomposition, it would show up as purely a within-industry change. However, under the value-added decomposition, this would show up as both a change in the composition of industries (since the industry's value added would go up) and within-industries (since the industry's labor's share of value-added would go down).

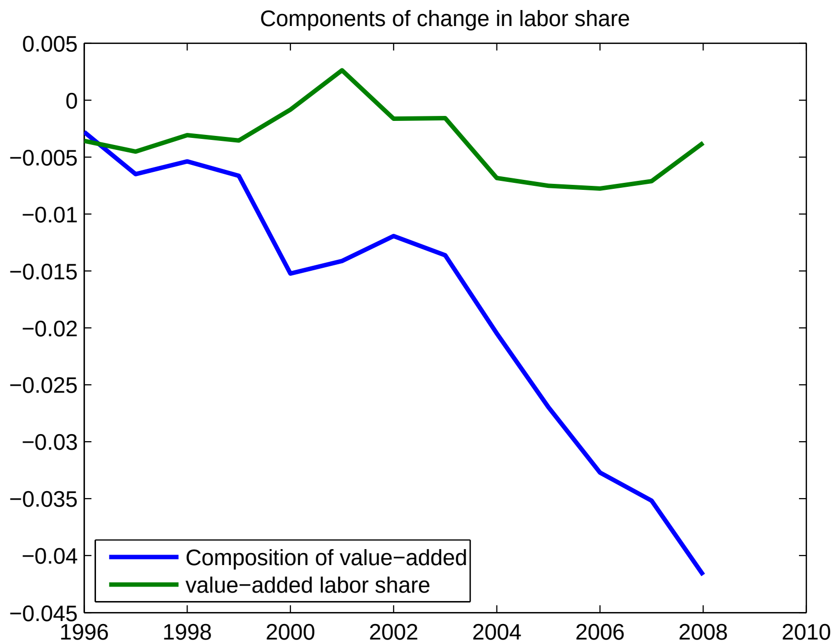


Figure 9: Cross-country average of changes in labor's share of income using the value-added decomposition. The data are from the World Input-Output Database.

4.2 Labor's Share of Manufacturing Income

A great advantage of this disaggregated approach is that we can zoom in on individual industries in a well-defined sense (without discarding the changes in their supply chains) and see which component is driving the change in their network-adjusted labor intensities for various labor types. In this subsection, manufacturing provides a good case-study.

The decline in manufacturing's labor share in the US has attracted much attention, for example, it forms part of the story behind the decline in labor's share of income in Elsby et al. (2013), and is the focus of Oberfield and Raval (2012). The papers in this literature focus on how manufacturing's value-added labor share has evolved over time. In figure 10, I plot the manufacturing sector's labor share, as measured by compensation of employees as a fraction of revenues ("gross labor share"), compensation of employees as a fraction of value added ("value-added labor share"), and the final consumption weighted network-adjusted labor share of the manufacturing sector for the US from 1995 to 2009. The network-adjusted labor share should be interpreted as the fraction of each dollar spent in manufacturing that is eventually paid to workers (even if they are non-manufacturing workers).

Formally, the value-added measure is defined as

$$\sum_{i \in M} \left(\frac{VA_i}{\sum_{j \in M} VA_j} \right) \left(\frac{\alpha_i}{\alpha_i + \eta_i} \right),$$

where M is the set of manufacturing industries, α_i is gross labor share, and η_i is gross capital share. Note that the value-added measure ignores how the supply chains of manufacturing are changing. The Network-adjusted labor share is given by

$$\sum_{i \in M} \left(\frac{\beta_i^*}{\sum_{j \in M} \beta_j^*} \right) \tilde{\alpha}_i,$$

where β^* is final-use expenditure shares. The supply chain of each industry is encapsulated in $\tilde{\alpha}_i$. Economically, the network-adjusted labor share of manufacturing captures the fraction of each dollar of expenditures in manufacturing eventually spent on labor (either directly or indirectly through intermediate inputs). This is precisely labor's share of manufacturing income.

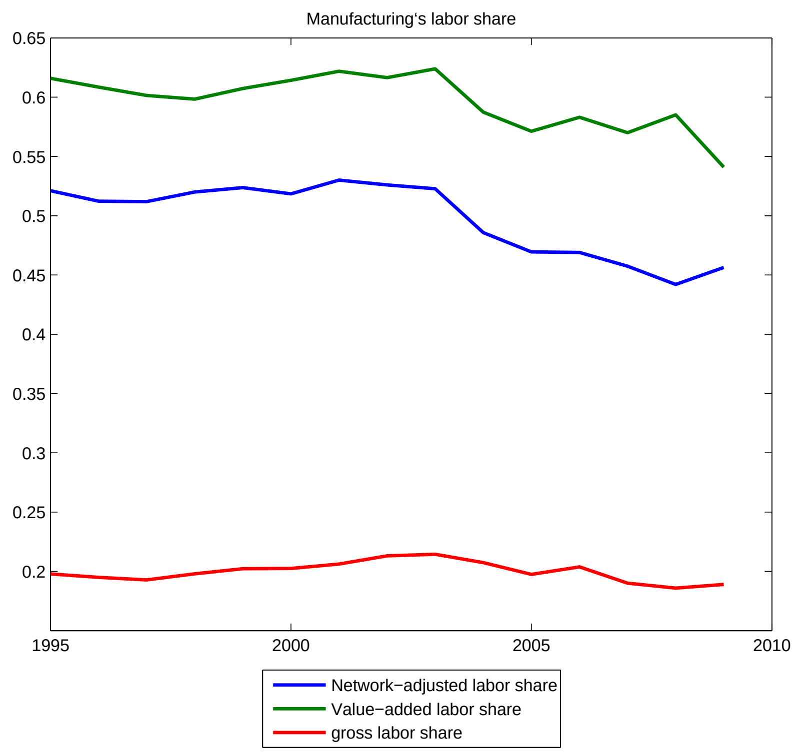


Figure 10: Evolution of labor use by the manufacturing industries of the US using the WIOD data from 1995-2009.

From 1995-2009, the manufacturing sector's value-added labor share fell by 12.2%. The network-adjusted labor intensity, however, dropped by 6.5%. The gross labor share fell by only 0.9%. The network-adjusted and value-added measures are highly correlated (correlation of 85%) but they're far from identical, either in levels or in changes. For instance, the drop in the network-adjusted measure is almost half as large as the one in the value-added share.

Crucially, we can go one step further and decompose the share in figure 10 according to equation (5). The results are plotted in figure 11. We see that in manufacturing, globalization and changing industrial composition have played a much larger role in labor's share of income than for the US economy as a whole. Unlike the the aggregate labor share, labor's share of US manufacturing income has been significantly affected by changing supply chains and consumption patterns. This finding is consistent with the idea that the changing composition of industries, which includes increased import competition, are responsible for the decline in labor's share of manufacturing income in the US, even if this does not aggregate up to be important for the economy as a whole. The idea that import competition has been important to workers involved in manufacturing is consistent with recent findings of Acemoglu et al. (2013).

Figure F.2 in Appendix F shows that the value-added decomposition is not misleading for US manfacturing. In this case, the composition effect *is* picking up the trend in aggregate labor shares in a way that's consistent with the results of figure 11. This suggests that once we aggregate over all manufacturing industries, assuming a block-diagonal input-output matrix, where manufacturing industries only use inputs from other manufacturing industries is not a bad assumption.

We can further decompose the network-adjusted labor intensity of manufacturing into network-adjusted high-skill, medium-skill, and low-skill intensity. These are plotted in figure 12. Here, we see the same pattern as in the rest of the data: high-skill use has trended upwards as medium and low-skill use has trended down. We see that the compositional changes, which include increased import competition, have had their biggest impact on medium-skill labor use of manufacturing.

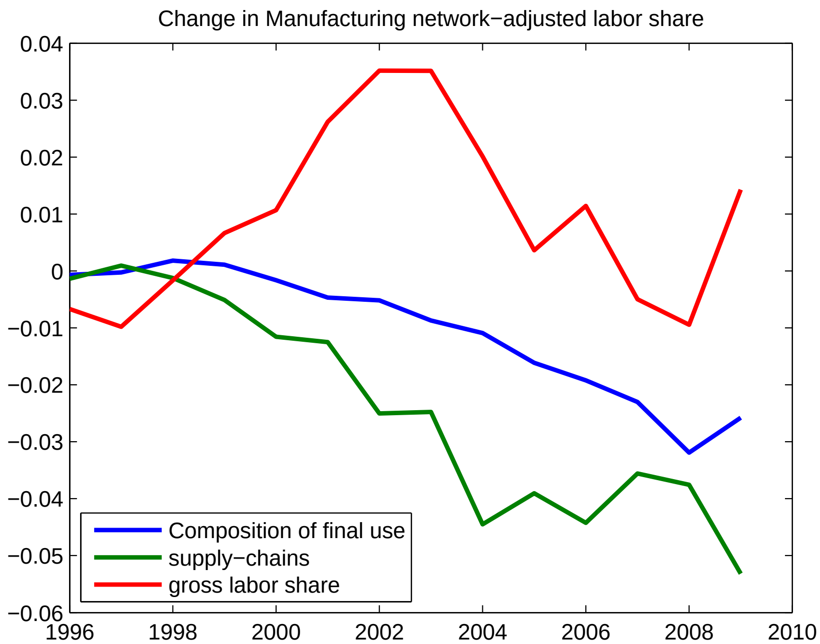


Figure 11: Decomposition of network-adjusted labor intensity of the manufacturing industries of the US using WIOD data from 1995-2009.

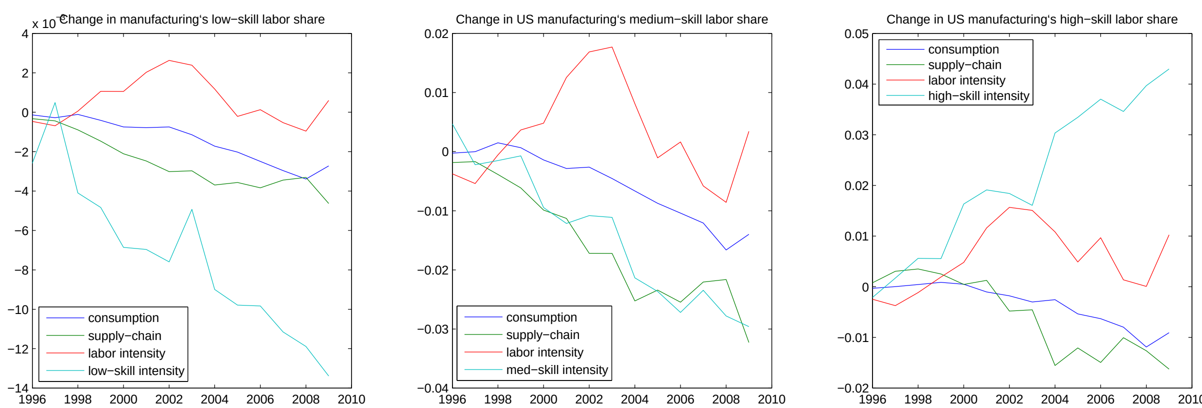


Figure 12: Network-adjusted labor intensity of the manufacturing industries of the US using WIOD data from 1995-2009.

5 Countercyclical Fiscal Policy

In this section, I move beyond the accounting applications of the network-adjusted labor intensities, and consider some of their policy implications. Received wisdom from Keynesian macroeconomics is that governments can use fiscal policy to stimulate employment at the zero-lower bound (for instance, see Christiano et al., 2011; Farhi and Werning, 2012). The question of exactly how they should do this is often left unexplored.

In section 3, I showed that the network-adjusted labor intensity tells us how more household demand for an industry's output eventually ends up as more demand for labor. In the neoclassical model of section 3, this information did not help us ask any normative policy questions because the equilibrium was efficient. However, we may think that in practice there are times when the government may want to pursue policies to raise employment. One such case, much studied in the literature, is in the context of a New Keynesian model at the zero-lower bound. In such a scenario, labor may be idle and government policy that expands employment can be welfare improving.

In this section, I show, in the context of a model with a production network, the network-adjusted labor intensities determine how much employment expands with increased government spending. When I introduce a nominal friction that causes involuntary unemployment, optimal government policy will then target those industries with the highest network-adjusted labor intensities. I will begin by deriving theoretical results linking the employment multiplier to the network-adjusted labor intensity and then calibrate how different types of government spending are expected to effect aggregate employment.

5.1 Neoclassical Benchmark

Before introducing any frictions, let us first see how the benchmark neoclassical model of section 3 responds to changes in fiscal policy. Since I focus on perturbations to the steady state of this model, changes in government policy are permanent changes to the steady state of the model. This implies that government spending has very strong crowding-out effects, since household's permanent income adjusts one-for-one with government expenditures.

Definition 5.1. The relative employment multiplier of government spending in industry i is defined as $\mathrm{d}l/\mathrm{d}\delta_i$, where l is equilibrium employment and δ_i is the share of government expenditures in industry i, holding fixed the total size of the governments' budget.

Proposition 5. *Government employment multipliers satisfy*

$$\frac{\mathrm{d}l/\mathrm{d}\delta_i}{\mathrm{d}l/\mathrm{d}\delta_j} = \frac{\tilde{\alpha}_i - \delta'\tilde{\alpha}}{\tilde{\alpha}_j - \delta'\tilde{\alpha}}. \tag{7}$$

Proof. See Appendix A. ∎

Proposition 5 shows that the relative multipliers from government spending are pinned down by the network-adjusted labor intensities. So, the government can boost employment by redirecting spending towards sectors with higher network-adjusted labor intensities. In other words, network-adjusted labor intensities also allow us to map how changes in final demand by the government translate into changes in equilibrium employment. Similar results hold for consumption taxes:

Proposition 6. *The employment multiplier from consumption taxes satisfy*

$$\frac{\mathrm{d}l/\mathrm{d}\tau_i}{\mathrm{d}l/\mathrm{d}\tau_j} = \frac{\beta_i\tilde{\alpha}_i}{\beta_j\tilde{\alpha}_j}\frac{(1+\tau_j)^2}{(1+\tau_i)^2}.$$

Proof. See Appendix A. ■

The employment response of a consumption tax is determined by how intensively an industry uses labor $\tilde{\alpha}$ and how intensively households consume that good β. This is because if households consume very little of a good, a consumption tax on that good will have small effects on employment even if that good uses labor intensively. This proposition demonstrates how targeted consumption taxes or subsidies, like "Cash for Clunkers," affect equilibrium employment. Proposition 6 shows that the efficacy of such government programs depends on the size of the subsidies, the household's tastes for the good being subsidized, and the network-adjusted labor intensity of the final industry producing the good.

Of course, in the benchmark model, there is no reason for the government to manipulate employment. The first welfare theorem holds and the optimal level of government taxation and expenditures is zero. However, once we allow for involuntary unemployment, these positive predictions become prescriptive. In the next section, I consider a second-best world where the zero-lower bound constrains the central bank, and neither the fiscal or monetary authority can commit to taking actions in the future. Furthermore, the only tools available to the fiscal authority are direct purchases by the government.

5.2 Keynesian Model Setup

Following the distinction made by Werning (2011), countercyclical fiscal policy can be optimal for opportunistic or stimulus reasons. Intuitively, opportunistic fiscal policy is when the government can provide useful goods and services to the household, and a recession is a particularly cheap time for the government to provide these goods and services. On the other hand, stimulus fiscal policy occurs when government expenditures are not directly valuable but still raise utility through a Keynesian multiplier effect. This is because government expenditures stimulate households to spend more and this raises private consumption. I add a few ingredients to the benchmark model in section 3 to study the model's normative properties for both types of fiscal policy.

To allow for opportunistic stimulus, I allow government purchases to enter the household's utility function directly. This gives fiscal policy a motive to increase expenditures during recessions. Second, I allow for heterogeneity in household types: specifically, there are some credit-constrained households that violate Ricardian equivalence and consume a constant fraction of their contemporaneous income. The presence of these households allows fiscal policy to have pure "stimulus" effects. Last, I make wages downwardly rigid so that the equilibrium after a shock is not necessarily efficient.

Households

As in Eggertsson and Krugman (2012), suppose there are two representative households with differing discount factors. The more patient household is called the saver and the less patient one the borrower. The fraction of savers in the population is $1-\chi$ and the fraction of borrowers is χ.

Let the saver maximize

$$\sum_t \rho^t \left[(1-\lambda)\log(c_t^s) + \lambda \log(G_t)\right], \quad \lambda \in (0,1),$$

where

$$c_t^s = \prod_k \left(c_{t,k}^s\right)^{\beta_k}$$

is private consumption by the saver, and

$$G_t = \prod_i g_{it}^{\phi_i}.$$

is government consumption services. We maintain the assumption that $\sum_k \beta_k = 1$. Since the government consumption good is additively separable from the household's private consumption, the government's consumption behavior does not directly distort the household's consumption choices through the utility function. The saver has budget constraint

$$\sum_k p_{t,k} c_{t,k}^s + B_t + D_t = (w_t l_t + r_t K_t)(1-\chi) + (1+i_{t-1})[D_{t-1} + B_{t-1}] - \tau_t^s,$$

where $p_{t,k}$ is the price of good k in time t. Nominal government bonds are B_t and debts of other households are D_t. The nominal net interest rate on debt is i_t. The household receives labor income $w_t l_t$ and capital income $r_t K_t$ in proportion to its share of the population. Households are endowed with an exogenous amount of labor and capital. Finally, savers face lump sum taxes τ_t^s.

The borrower, who has a smaller discount factor, faces the same problem as the saver but is subject to a borrowing limit on its debt:

$$D_t \leq \frac{D^h}{1+i_t} \frac{p_{t+1}}{p_t},$$

where p_t is the ideal price index for the households in period t.

Firms

The firms behave exactly as before. That is, the firms are competitive and rent capital and labor on spot markets from the household and reoptimize every period. Therefore, their problems are static.

$$\max_{y_{it}, l_{it}, x_{ijt}} \quad p_{it}y_{it} - \sum_j p_{jt}x_{ijt} - w_t l_{it} - r_t k_{it},$$

such that

$$y_{it} = (l_{it})^{\alpha_i} k_{it}^{\eta_i} \prod x_{ijt}^{(1-\alpha_i-\eta_i)\omega_{ij}}.$$

The government

The government faces the budget constraint

$$B_t = (1 + i_{t-1})B_{t-1} + \sum_k p_{t,k}G_{t,k} - \tau_t,$$

where τ_t is income from lump sum taxation. The government cannot target its tax base, so that taxes levied on borrowers and savers are proportional to their share of the population. Furthermore, the government cannot use consumption taxes, since, as shown by Correia et al. (2013), a government with access to a rich-enough set of taxes could replicate negative interest rates and achieve the first-best outcome. Unlike the household, the government is not subject to an exogenous borrowing limit (or at least, this limit does not bind for the purposes of our policy exercise).

Market clearing

Prices are flexible and the market for the goods and services clears:

$$p_{t,k}y_{t,k} = p_{t,k}(c^s_{t,k} + c^b_{t,k} + g_{t,k}) + \sum_j p_{t,k}x_{t,j,k}.$$

The rental rate of capital is also flexible and so capital is always fully employed. The bond market also clears; however, the price of bonds are set by the central bank according to a Taylor rule:

$$1 + i_t = \max\{1, (1 + R^n_t)\left(\frac{p_{t+1}}{p_t}\right)^{\phi}\},$$

where R^n_t is the net Wicksellian natural rate of interest and $\phi > 1$. The model can feature multiple equilibria, and we will discuss equilibrium selection later.

The labor market is subject to a nominal friction. Specifically, wages are downwardly rigid in the spirit of Patinkin (1965), Malinvaud (1977), and more recently Schmitt-Grohé and Uribe (2011). That is

$$l_t \leq \bar{l}, \quad w_{t-1} \leq w_t, \quad (w_t - w_{t-1})(l_t - \bar{l}) = 0.$$

Here, $\bar{l}$ is an exogenous endowment of labor, and w_t is the nominal wage in period t. This is a transparent and tractable way of adding nominal frictions into the model. The key assumption here is that in the event of a shortfall in nominal demand, it is the labor market that fails to clear, and not the capital market. Partial equilibrium in the labor market is shown in log-log terms in figure 13, where the sales of firms are held constant. There are two admissible regions: (1) $w_t \geq w_{t-1}$ and the labor market clears, and (2) $w_t = w_{t-1}$ and the labor market fails to clear with $l_t \in (0, \bar{l})$.

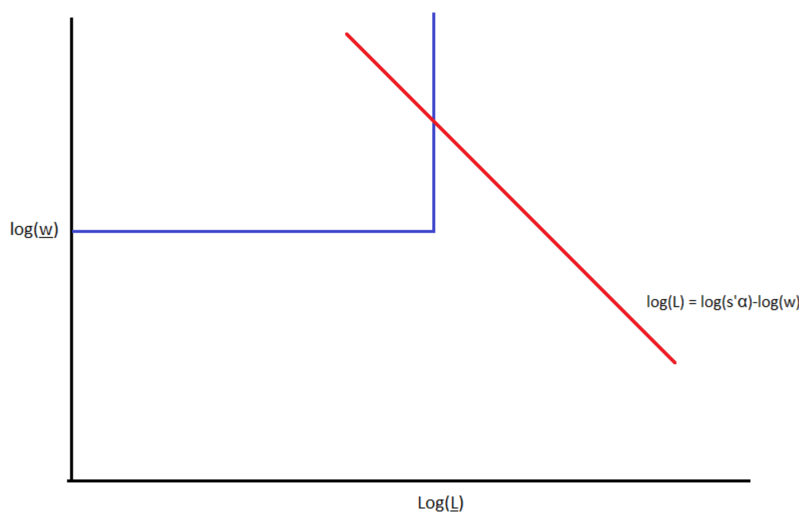


Figure 13: The labor market with downward wage rigidity. The blue line is labor supply and the red line is labor demand. The vector s is the sales of each industry, while the vector α is the gross labor share of each industry.

There is considerable empirical evidence for downward stickiness in wages, see for instance Barattieri et al. (2010), Baqaee (2014a), Dickens et al. (2007), and Bewley (1999). Downward wage rigidity is a particularly convenient modelling device in this paper since it makes the intuition for government intervention very transparent – there is idle labor and a unique efficient level of full employment. Such a stark assumption is not strictly necessary however, since similar forces operate as long as output is inefficiently low and labor (rather than capital) is the factor that adjusts. This can be accomplished with, for example, an elastic labor supply curve and sticky prices, and I sketch a version of this model in Appendix E.

5.3 Scenario I: Opportunistic Spending

In this section, I analyze optimal fiscal policy during a one-period liquidity trap with only the Ricardian households. In other words, I assume that the fraction of the population corresponding to impatient borrowers is zero. Under this assumption, there is no neoclassical multiplier effect of government spending since labor supply is inelastic (so there is no wealth effect of taxation). There is also no Keynesian multiplier effect for private consumption since we have a one-period shock. Therefore, the results of this subsection pertain to pure opportunistic fiscal policy.

The shock that pushes the economy into the zero lower bound, as is common in the representative agent zero-lower bound literature, following Krugman (1998), is a one-period unexpected discount factor shock. Suppose that there is an unexpected discount factor shock so that for the next period $\rho^* > 1$. I analyze the government's fiscal policy without commitment – that is, the government reoptimizes its expenditure plans each period.

Lemma 3. *Aggregate labor demand l_t when the zero lower bound binds is upward sloping in wage inflation, and is given by*

$$l_t = \frac{1}{\rho^*}\frac{w_{t+1}}{w_t}\bar{l} - \frac{1}{\rho^*}\delta'_{t+1}\tilde{\alpha}\tau_{t+1} - (\beta - \delta_t)'\tilde{\alpha}\tau_t. \tag{8}$$

Proof. See Appendix A. ■

Lemma 3 shows how government spending today τ_t, by deviating from private spending $(\beta - \delta_t)$, can increase employment.

Equilibrium employment is given by combining aggregate demand for labor with the aggregate supply curve for labor. Aggregate supply for labor is defined by $w_{t+1}/w_t \geq 1$ and $l_t \leq \bar{l}$. This situation is graphically depicted in figure 14. There are two candidates for equilibria. One is the neoclassical equilibrium where the wage rises by exactly enough tomorrow to ensure we maintain full employment. In this case, the government need not intervene to boost employment and government expenditure shares are equal to the Cobb-Douglas parameters. Such an equilibrium, however, is inconsistent with the inflation-targeting Taylor rule. The only equilibrium then features no wage inflation, $w_t = w_{t+1}$, and positive unemployment.

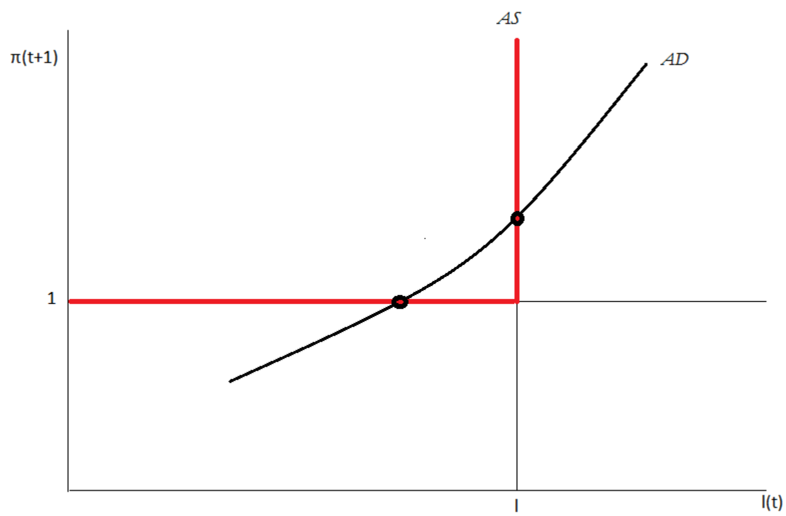
Proposition 7. *The optimal share of expenditures by the government in industry i relative to industry j satisfies*

$$\frac{\delta_i}{\delta_j} = \frac{\phi_i}{\phi_j}\left(\frac{const + (\mu_1 - \mu_2)\tilde{\alpha}_j}{const + (\mu_1 - \mu_2)\tilde{\alpha}_i}\right), \tag{9}$$

where μ_1 and μ_2 are lagrange multipliers corresponding to the labor and capital markets. When we are at the full employment steady state $\mu_1 = \mu_2$, so that spending shares are equal to the Cobb-Douglas parameters. When there is unemployment $\mu_1 < \mu_2$, so government tilts in favor of firms with higher network-adjusted labor intensities.

Proof. See Appendix A. ■

Figure 14: Zero-lower bound is binding and aggregate demand is upward sloping. The y-axis is wage inflation and the x-axis is employment.



The key intuition of this section is that when the zero lower bound binds, the government has an opportunity to provide goods and services to the household more cheaply than usual. Furthermore, the higher the network-adjusted labor intensity of an industry, the more cheaply the government can supply that good to the household. Therefore, the government tilts its expenditures in favor of industries with high network-adjusted labor intensity.

Equation (9) is intuitive to interpret. The production of each good uses a certain combination of labor and capital, directly and indirectly through inputs. When there is unemployment, there is idle labor that is essentially free to use for the government. However, capital is not free. Therefore, the government tilts its consumption of goods towards those that use labor more heavily than capital, since any capital used by the government crowds out the private sector, and bids up rents rather than expanding production. The existence of a wedge between private and public spending decisions follows from the logic set out by Farhi and Werning (2013).

The intuition for (9) can be illustrated by considering an extreme example with only two goods: one good only uses labor and the other only uses capital with no intermediaries. When there is unemployment, the government can use the labor-intensive good without reducing the household's consumption of labor. However any capital used by the government crowds out the household. Therefore, the government will use all the unemployed labor, but only use enough of the capital intensive good to equalize the marginal utility of government and household consumption.

5.4 Scenario II: Stimulus Spending

Now, let us consider the case where government expenditures have zero direct value, but since there are credit-constrained borrowers, multiplier effects of government spending give the government a motive to spend during a liquidity trap. Following Eggertsson and Krugman (2012), an exogenous debt limit on borrowers makes them non-Ricardian and forces them to behave as what Galí et al. (2007) call "hand-to-mouth consumers." The assumption that a fraction of households' consumption tracks their current income rather than permanent income accords with the empirical findings of Campbell and Mankiw (1990).

Set the fraction of borrowers to χ to be nonzero. To shut down the opportunistic channel, set the utility-value of government consumption $\lambda = 0$, so that government expenditures have no intrinsic value to the household. The only reason why government expenditures may be beneficial in this context is then the stimulus effect of spending due to the presence of non-Ricardian households.

Deleveraging Shock

Since the borrower has a smaller discount factor, the steady-state equilibrium of this model sees the borrower borrow up to his borrowing constraint from the household. We focus on the steady-state equilibrium with no inflation and no government spending or taxation.

Now suppose that a borrowing limit falls unexpectedly in period t so that

$$D_{t-1} = \frac{D^h}{1+i_{t-1}} \frac{p_t}{p_{t-1}}, \quad D_t = \frac{D^l}{1+i_t} \frac{p_{t+1}}{p_t},$$

where $D^h > D^l$. Assume that the borrower has to delever immediately to the new borrowing limit.

I analyze the equilibrium where the steady-state equilibrium features zero inflation, full employment, and no government spending and constant government taxes.

Lemma 4. *Aggregate demand for labor l_t when the zero lower bound binds is upward sloping in wage inflation and given by*

$$w_t l_t = \frac{\beta' \tilde{\alpha}}{\rho} \left(w_{t+1} \bar{l} + r_{t+1} \bar{k} \right) + \beta' \tilde{\alpha} \frac{D^l - \rho D^h}{\rho(1-\chi)} + \left(\beta' \tilde{\alpha} \left[\frac{1-(1-\rho)(1-\chi)}{\rho(1-\chi)} - 1 \right] + \delta' \tilde{\alpha} \right) p_t^g g_t. \quad (10)$$

Proof. See Appendix A. ■

Equation (10) shows that aggregate demand for labor can be stimulated in two ways by the government. The first channel is the same as the one in the Ricardian model: if the government buys labor intensive goods ($\delta_t' \tilde{\alpha} > 0$), then the government increases nominal demand for labor directly.[4] However, there is now a new, non-Ricardian channel. If $\chi > 0$, then there is a secondary effect because increased government spending increases the contemporaneous income of borrowers and therefore increases private expenditures.

The term

$$\left[\frac{1-(1-\rho)(1-\chi)}{\rho(1-\chi)} - 1\right],$$

is the government multiplier on nominal private GDP. As long as the fraction of borrowers χ is nonzero, this is greater than zero, and so there is a pure stimulus effect to government spending. If we set the fraction of borrowers to be zero, then the model is Ricardian and, as shown in the proof of proposition 8 in appendix A, the government multiplier on private nominal GDP is exactly zero.

Combining the aggregate demand equation (10) with the the aggregate supply relation gives the situation in figure 14. Once again, as we see from the graph, there are two equilibria. In the first, there is no inflation, positive unemployment, and increases in government spending increase output and weakly increase inflation. The other equilibrium is the neoclassical equilibrium where we have positive inflation equal to exactly the reciprocal of the gross natural interest rate, full employment, and increased government spending reduces inflation. Once again, the neoclassical equilibrium is inadmissible with the Taylor rule, so the relevant equilibrium is the one with positive unemployment.

By inspection of (10), we can see that the biggest bang for buck in terms of the government boost to employment comes from maximizing the size of the government's network-adjusted labor intensity $\delta'\tilde{\alpha}$. Maximizing employment is not the government's objective however. Optimal government policy seeks instead to maximize real GDP net of government consumption.

Proposition 8. *Optimal government spending in the period of the deleveraging shock has the government spend entirely on the industry with the highest network-adjusted labor intensity.*

The intuition here is that the industry with the highest network-adjusted labor intensity not only has the largest employment multiplier, since it employs the most amount of idle labor, but it is also the cheapest resource to waste (since all government spending is wasteful). Therefore, the industry with the highest network-adjusted labor intensity is targeted.

In this setup, borrowers and savers derive the same income from labor and capital. If we modify the model so that borrowers derive more income from labor than capital, as is empirically more relevant, these results would become even stronger. Then not only will high network-adjusted labor intensity imply that those goods are cheaper to waste, but it also means that their owners have higher marginal propensities to consume, and therefore, will have even higher multipliers.

[4] There is no crowding-out of the private sector because we are at the zero-lower bound, and current private nominal consumption is pinned down by the Euler equation.

Furthermore, labor is homogenous in this model. A simple extension of the model with different labor types would have the government target sectors that more intensively use low-skilled labor, both because it is cheaper to waste and because it gives larger multipliers. The relevant criteria for the target would be the network-adjusted labor intensity by type.

5.5 Practical Application

Both scenarios point to the government targeting its stimulus towards sectors with higher network-adjusted labor intensities. Table 4 reports the network-adjusted labor intensity for broad categories of final-use spending, including various types of government spending, assuming Cobb-Douglas functional forms. The network-adjusted labor intensities can be interpreted as determining the relative employment multipliers of an extra dollar of spending from the different final use sectors. Crucially, these numbers are also the fraction of each dollar of spending that is eventually spent on labor — or in other words, labor's share of income from that final-use sector.

If the government has the ability to finely tune stimulus spending, then the industry-level rankings discussed in section 3 are the relevant statistics for designing stimulus. If the government used value-added rankings instead, it would, to pick an extreme example, erroneously think that the multiplier for "Soybean and other oilseed processing" is 13 times larger than that of "other residential structures," since the former has a value-added labor share of 0.903, while the latter's value-added labor share is only 0.069. However, using the network-adjustment, we find that Soybean and other oilseed processing has a labor share of 0.247, while other residential structures has a labor share of 0.350 – a reversal in rankings.

Generally, for reasons outside of the model, we think that governments are unable to perfectly fine-tune stimulus spending at the industry level. Therefore, in table 4, we look at broad classes of government spending instead, since it may be easier to change spending patterns across these broader categories.

Final-Use	Network-adjusted	Value-Added
Personal consumption expenditures	0.503	0.537
Private fixed investment	0.572	0.594
Federal government defense: consumption expenditures	0.484	0.432
Federal government defense: gross investment	0.638	0.726
Federal government nondefense: consumption expenditures	0.747	0.796
Federal government nondefense: gross investment	0.685	0.800
State and local government consumption expenditures	0.747	0.880
State and local government gross investment	0.584	0.644

Table 4: Network-adjusted and gross labor intensities of various final goods consuming sectors using the BEA's detailed purchase price benchmark input-output table for 2007.

Since the data in table 4 are very heavily aggregated, we should expect the network structure to matter less for the rankings. Nonetheless, even with this level of aggregation, we see some interesting patterns. Namely, the network-adjusted labor intensities are much closer to one another than the value-added measures. We also see that nondefense investment is less labor intensive than nondefense consumption, despite the value-added measure being larger. The implied relative ranking of multipliers for defense and non-defense spending may help to explain why the literature estimating government multipliers tends to find smaller multipliers for defense spending than other types of government spending. For a summary of the contrasting estimates of defense and non-defense multipliers, see Yang et al. (2012). They find that non-defense multipliers are 1.5-2.0 times larger than defense multipliers.

5.6 Who Gets Paid?

The fact that unemployment rates vary significantly with education levels suggests that this exercise will be more informative if we focus on the multipliers associated with the lower skill types, since those labor markets are more likely to experience high cyclical unemployment. In table 5, I report each labor type's share of income by final-use normalized by that labor type's share of aggregate income. A number less than one implies that the final-use sector uses that labor type less intensively than average, whereas a number greater than one implies the opposite. Table 5 effectively answers the question of where the money for different kinds of government spending goes – a question that cannot be satisfactorily answered without the use of network-adjusted labor intensities.

Final-Use	Grades 0-9	Grade 10-12	1-2 years of college	4 years of college	5+ years of college
Personal consumption expenditures	0.902	0.908	0.938	0.911	0.935
Private fixed investment	1.338	1.243	1.016	1.016	0.715
Federal defense consumption	0.513	0.655	0.793	0.928	1.382
Federal defense investment	0.906	1.041	1.090	1.402	1.145
Federal nondefense consumption	0.569	0.855	1.185	1.523	2.347
Federal nondefense investment	0.851	1.002	1.164	1.608	1.304
State and local government consumption	1.090	1.227	1.304	1.271	1.405
State and local government investment	1.948	1.484	1.043	0.839	0.544

Table 5: Fraction of final use expenditures going to each type of labor normalized by that labor type's share of total GDP. A number greater than (less than) 1 indicates the final sector in that row spends more (less) heavily on that labor type compared to total GDP. This table combines data from the 2007 American Community Survey from IPUMS-USA with the detailed Benchmark Input-Output table using purchaser prices for 2007 published by the BEA.

Table 5 shows that private fixed investment and state and local government investment use low-skill labor much more and high-skill labor much less than average. On the other hand, federal consumption (defense and nondefense) are overwhelmingly tilted towards high-skill types. Table 5 implies that a uniform reallocation of funds from private consumption towards federal government spending would increase high-skilled labor's share of income at the expense of low-skilled labor. To the extent that low-skilled labor markets are experiencing greater slack, this table helps to explain why estimates of fiscal multipliers for state and local government expenditures, like those of Shoag (2010) and Chodorow-Reich et al. (2012) tend to find larger effects than estimates from federal expenditures.

6 Conclusion

This paper introduces the network-adjusted labor intensity as the relevant notion of labor intensity in an interconnected production economy. This captures how intensively a good or service uses labor in production by taking into account how heavily its entire supply chain relies on labor. Doing this adjusts for the artificial drop in the gross labor intensity resulting from fragmentation of the production process across industries.

The network-adjusted labor intensity plays a key role in determining how industry-level disturbances translate to aggregate employment, and this has both short-run and long-run implications. For instance, labor's share of income, a central object of interest in the study of growth and inequality, is a weighted average of network-adjusted labor intensities. This allows us to decompose labor's share of income into disaggregated components representing changing consumption patterns, changing supply chains (including trade), and changing capital/labor shares. In a sample of 34 countries over the past 15 years, this decomposition shows that the overwhelming driver of the secular decline in the labor share is a decline in the gross labor share of all industries. This contrasts with the usual value-added decomposition, which over the sample, attributes the drop to changing industrial composition.

The network adjustments also allow us to study individual industries, like manufacturing, without discarding information about changes in their supply chains. For the US manufacturing sector, increasing globalization and changing consumption patterns do explain a sizeable fraction of the decline in manufacturing's network-adjusted labor share, but these effects are not sizeable when aggregated up to the whole economy.

Over the short run, the network-adjusted labor intensities pin down the relative boost to employment from a marginal increase in spending in one industry versus another. This makes network-adjusted labor intensities important to policy makers interested in boosting employment through fiscal policy. I show that if labor is the factor that adjusts in a recession, then when the zero lower-bound binds, optimal fiscal stimulus should tilt in favor of stimulating demand in industries with higher network-adjusted labor intensities. The intuition is that, in a recession, the government should aim to expand production rather than to simply bid up rents. The way to expand production is to stimulate industries that are most reliant on unemployed resources. In my model, the unemployed resource is labor and the relevant notion of "reliant" is the network-adjusted labor intensity.

We can also compute measures of how intensively different types of government expenditures use different types of labor. I find that state and local government expenditures are much more reliant on low-skilled labor than average. On the other hand, federal government expenditures, defense and nondefense, are far more heavily reliant on very high-skilled workers with more than 4 years of college education than average. Furthermore, I find that on the whole defense expenditures are less reliant on labor than other types of government expenditures. These findings help explain the heterogeneity in estimates of the effect of government expenditures on employment found in the literature.

A Appendix: Proofs

Proof of theorem 1. The fact that labor's share of income is equal to 1 follows trivially from considering the aggregate budget constraint:

$$wl - \tau + \tau = \sum_i p_i(c_i + g_i).$$

To see that the distribution of government expenditures does not affect equilibrium employment, consider the cost minimization problem of firm i

$$c(x_i; \boldsymbol{p}, w) = \min_{x_{ij}, l_i} \left\{ \sum_j p_j x_{ij} + w l_i : F_i(x_{i1}, \ldots, x_{in}, l_i) = x_i \right\}.$$

Once again, note that since the problem of the firm is static, I have suppressed time subscripts. Note that for any $\alpha > 0$,

$$\begin{aligned}
c_i(\alpha x_i; \boldsymbol{p}, w) &= \min_{x_{ij}, l_i} \left\{ \sum_j p_j x_{ij} + w l_i : F_i(x_{i1}, \ldots, x_{in}, l_i) = \alpha x_i \right\}, \\
&= \min_{x_{ij}, l_i} \left\{ \sum_j p_j x_{ij} + w l_i : F_i(x_{i1}/\alpha, \ldots, x_{in}/\alpha, l_i/\alpha) = x_i \right\}, \\
&= \min_{x_{ij}, l_i} \left\{ \sum_j \alpha p_j \frac{x_{ij}}{\alpha} + w\alpha \frac{l_i}{\alpha} : F_i(x_{i1}/\alpha, \ldots, x_{in}/\alpha, l_i/\alpha) = x_i \right\}, \\
&= \alpha \min_{x_{ij}, l_i} \left\{ \sum_j p_j \frac{x_{ij}}{\alpha} + w \frac{l_i}{\alpha} : F_i(x_{i1}/\alpha, \ldots, x_{in}/\alpha, l_i/\alpha) = x_i \right\}, \\
&= \alpha c_i(x_i; \boldsymbol{p}, w).
\end{aligned}$$

So the marginal cost of firm i is

$$\frac{\partial c_i}{\partial x_i} = c_i(1; \boldsymbol{p}, w).$$

In other words, the marginal cost of firm i depends only on the wage and the prices of firm i's inputs.

In equilibrium, all firms must make zero profits, otherwise they would expand their size to infinity or shrink to zero. In particular, this means that price must equal marginal cost for each good

$$p_i = c_i(1; \boldsymbol{p}, w). \tag{11}$$

Furthermore, observe that p_i scale one-for-one with the wage w. That is, if $\boldsymbol{p}$ solves (11), then

$\tilde{\boldsymbol{p}} \equiv \alpha \boldsymbol{p}$ solves

$$\tilde{p}_i = c_i(1; \tilde{\boldsymbol{p}}, \alpha w).$$

So (11) implies that all prices are pinned down by technologies and the nominal wage. So let $\overline{\boldsymbol{p}}$ solve the following equations:

$$\overline{p}_i = c_i(1; \overline{\boldsymbol{p}}, 1),$$

and note that any equilibrium price vector must be $w\overline{\boldsymbol{p}}$.

At the steady-state equilibrium with zero inflation, $1 + i_{t+1} = \frac{\rho_t}{\rho_{t+1}}$. This implies that consumption of each good and labor are the same at every period. To see this, observe that the household's problem can be written as

$$\max \quad \sum_{\tau=t}^{\infty} \rho_\tau U(c_{1\tau}, \ldots, c_{n\tau}, l_\tau),$$

subject to

$$\sum_{s=1}^{\infty} \left(\sum_i p_{it+s} c_{it+s} + \tau_{t+s} \right) \left(\prod_{\tau=0}^{s-1} \frac{1}{1 + i_{t+\tau}} \right) = \sum_{s=1}^{\infty} w_{t+s} l_{t+s} \left(\prod_{\tau=0}^{s-1} \frac{1}{1 + i_{t+\tau}} \right),$$

where profits have been dropped since they are always equal to zero. The first order conditions, along with the assumption that $1 + i_{t+1} = \rho_t / \rho_{t+1}$ implies that

$$u_{it}(c_{1t} \ldots, c_{nt}, l_t) = u_{it+1}(c_{1t+1} \ldots, c_{nt+1}, l_t + 1),$$

for every t and i, where u_{it} is the marginal utility of good i in period t. Furthermore,

$$u_{lt}(c_{1t} \ldots, c_{nt}, l_t) = u_{lt+1}(c_{1t+1} \ldots, c_{nt+1}, l_t + 1).$$

These relations imply that consumption and labor supplied are the same in every period. This means that we can collapse the household problem into finding just the steady-state values of consumption and labor. In particular, we simply need to solve

$$\max \quad u(c_1, \ldots, c_n, l)$$

subject to

$$\sum_i p_i c_i = wl - \tau,$$

since perfect consumption-smoothing means that there is no variation across periods.

Define

$$V(C, l) = \max \left\{ u(c_1, \cdots, c_N, l) : \sum \overline{p}_i c_i = C \right\}.$$

Note that the indirect utility of the household, defined as the solution to the household's problem

(1) will coincide with the solution to

$$\max_{C,l} \quad V(C,l)$$

such that

$$wC = wl - \tau$$

The first order condition to this problem is

$$-\frac{V_l(l-\tau/w,l)}{V_c(l-\tau/w,l)} = \frac{w}{w}, \tag{12}$$

Now let labor be the numeraire so that $w = 1$. Then this equation pins down l. We see that the only way in which government policy appears in these two expressions is via the size of the government's budget τ. In particular, the distribution of spending is irrelevant. ■

An extension of the logic of theorem 1 to demand shocks is possible if we assume that the indirect utility function of the household $V(C,l)$ is quasi-linear in the composite consumption good.

Corollary A.1. *Suppose that*

$$V(C,l) = const\, C - v(l),$$

then equilibrium employment depends only on the disutility of labor.

Proof. The assumption of quasi-linearity allows us to write (12) as

$$v'(l) = \text{const}.$$

■

In particular, corollary A.1 implies that changes in the utility function of the household (demand-shocks) do not affect equilibrium employment so long as the indirect utility function remains quasi-linear in consumption. The leading example of this scenario is when the utility function is Cobb-Douglas or has the CES form in consumption. In that case, changes to the share parameters will not affect equilibrium employment.

Proof of Lemma 2.

$$\begin{aligned}\mathbf{1} &= I\Omega\mathbf{1},\\ &= (\text{diag}(a+c) + \text{diag}(1-a-c))W\mathbf{1},\\ &= \text{diag}(a+c)W\mathbf{1} + \text{diag}(1-a-c)W\mathbf{1},\\ &= a + c + \text{diag}(1-a-c)W\mathbf{1}.\end{aligned}$$

Rearrange this to get

$$(I - \text{diag}(1-a-c)W\mathbf{1} = a + c,$$

or

$$\mathbf{1} = (I - \text{diag}(1-a-c)W)^{-1}(a+c).$$

■

Proof of Proposition 2. Let s_i denote the sales of firm i. By definition,

$$p_j x_{ij} = w_{ij} s_i,$$

and

$$wl_i = a_i s_i.$$

Goods market clearing implies that

$$y_i = c_i + g_i + \sum_j x_{ji},$$

or equivalently

$$s_i = p_i c_i + p_i g_i + \sum s_j w_{ji}.$$

Solve this system of linear equations for s to get

$$s' = (H+G)'(I-W)^{-1},$$

where H is household expenditures net of taxes and G is government expenditures.

Finally, labor market clearing implies that

$$wl = \sum_i wl_i = s'a. \tag{13}$$

Hence,

$$wl = (H+G)'(I-W)^{-1}a = (H+G)'\tilde{a}. \tag{14}$$

■

Proof of Proposition 3. For simplicity, assume the government's budget is zero. Note that labor market clearing requires that

$$l^{\theta-1} = \frac{w}{PC}.$$

Rearrange this to get

$$l^{\theta} = \frac{wl}{PC} = \frac{s'\alpha}{PC} = \frac{\beta'\Psi\alpha PC}{PC} = \beta'\tilde{\alpha}.$$

Now simply differentiate implicitly with respect to the taste parameter β_i to get

$$\theta l^{\theta-1}\frac{\mathrm{d}l}{\mathrm{d}\beta_i} = \left(\frac{e_i}{\sum_j \beta_j} - \frac{\beta}{(\sum_j \beta_j)^2}\right)'\tilde{\alpha}.$$

Divide this by the same expression for β_j to get the desired result:

$$\frac{\mathrm{d}l/\mathrm{d}\beta_i}{\mathrm{d}l/\mathrm{d}\beta_j} = \frac{e_i'\tilde{\alpha} - \beta'\tilde{\alpha}}{e_j'\tilde{\alpha} - \beta'\tilde{\alpha}}.$$

■

Proof of Proposition 5. For simplicity, let the consumption taxes be equal to zero so that there is only lump-sum taxation. Firm cost minimization implies that

$$p_j x_{ij} = (1-\alpha_i-\eta_i)\omega_{ij}p_i x_i,$$

and

$$rk_i = \eta_i p_i y_i,$$

and

$$wl_i = \alpha p_i y_i.$$

Substitute firm i's demand for inputs into its production function to get that

$$p_i = \left(\frac{w}{\alpha_i}\right)^{\alpha_i}\left(\frac{r}{\eta_i}\right)^{\eta_i}\prod_j\left(\frac{(1-\alpha_i-\eta_i)\omega_{ij}}{p_j}\right)^{(1-\alpha_i-\eta_i)\omega_{ij}}. \tag{15}$$

Note that

$$\log(p_i) = \alpha_i(\log(w)-\log(\alpha_i)) + \eta_i(\log(r)-\log(\eta_i)) + \sum_j(1-\alpha_i-\eta_i)\omega_{ij}(\log(p_j)-\log(\omega_{ij})).$$

Rearrange this to get

$$\log(p) = (I - \hat{\Omega})^{-1}(\alpha \log(w) + \eta \log(r) - \Theta), \tag{16}$$

where

$$\hat{\Omega} = \text{diag}(1 - \alpha - \eta)\Omega,$$

and

$$\theta_i = \alpha_i \log(\alpha_i) + \eta_i \log(\eta_i) + \sum_j (1 - \alpha_i - \eta_i)\omega_{ij} \log((1 - \alpha_i - \eta_i)\omega_{ij}).$$

In the expressions above, logs of a vector or matrix are taken element by element. Equation (16) is informative, since it implies that the relative prices of consumption goods in the economy depend solely on the relative cost of the two factors and the technology.

Let $s_i = p_i y_i$. Then labor market clearing implies that

$$l = \sum_i l_i = \frac{s'\alpha}{w}. \tag{17}$$

Rearrange this to get

$$w = \frac{s'\alpha}{l}.$$

Plug this into the labor supply equation to get

$$l = \min\left\{ \left[\frac{s'\alpha}{lPC}\right]^{\frac{1}{\theta-1}}, \bar{l} \right\}.$$

Rearrange this to get

$$l = \min\left\{ \left[\prod \left(\frac{\beta_i}{p_i}\right)^{\beta_i} \frac{s'\alpha}{C}\right]^{\frac{1}{\theta}}, \bar{l} \right\}. \tag{18}$$

Similarly, capital market clearing implies that

$$K = \sum_i k_i = \frac{s'\eta}{r}. \tag{19}$$

Finally, goods market clearing implies that

$$y_i = c_i + g_i + \sum_j x_{ji}.$$

By market clearing,

$$p_i y_i = p_i(c_i + g_i) + \sum p_j x_j (1 - \alpha_j - \eta_j)\omega_{ji}$$

Denote the vector of s_i's by s. Then

$$s' = (H+G)'(I-\hat{\Omega})^{-1}, \tag{20}$$

where H is the vector of household expenditure and G is the vector of government expenditure.

Assume that $l \leq \bar{l}$ and substitute (20) into (18) to get

$$l = \left[(H+G)'(I-\hat{\Omega})^{-1}\alpha\right]^{\frac{1}{\theta}} \left(\frac{1}{PC}\right)^{\frac{1}{\theta}}. \tag{21}$$

So, the equilibrium labor is given by (21), with prices that satisfy (16), (19), and the normalization $w = 1$.

By changing the shares of government expenditures, the government affects equilibrium employment through three different channels. First, the government directly changes the demand for labor through its purchases. Second, the government changes demand for labor by affecting the price of labor relative to capital, and therefore, the relative prices of more and less labor intensive goods. Lastly, the government changes the income of households. Fortunately, all three of these forces can be expressed as multiples of the relative network-adjusted labor intensities of the various sectors. This makes the clean expression in (7) possible.

Let labor to be the numeraire $w = 1$ to form the following expression

$$l^\theta = \frac{\left(\beta'(l + rK - \tau) + \delta'\tau\right)\Psi\alpha}{PC}.$$

Note that equilibrium employment l, rental rate of capital r, and household expenditures PC all depend on δ. Implicitly differentiate this expression with respect to δ_i

$$\begin{aligned}\theta l^{\theta-1}\frac{\mathrm{d}l}{\mathrm{d}\delta_i} = &\left(\beta'\Psi\alpha\left(\frac{\mathrm{d}l}{\mathrm{d}\delta_i} + K\frac{\mathrm{d}r}{\mathrm{d}\delta_i}\right) + (e_i - \delta)'\Psi\alpha\tau\right)\frac{1}{PC} \\ &- \frac{1}{(PC)^2}\left(\frac{\mathrm{d}l}{\mathrm{d}\delta_i} + K\frac{\mathrm{d}r}{\mathrm{d}\delta_i}\right)\left(\beta' PC + \delta'\tau\right)\Psi\alpha,\end{aligned} \tag{22}$$

where e_i denotes the vector with zeros everywhere except the ith element. From (19), note that

$$K\frac{\mathrm{d}r}{\mathrm{d}\delta_i} = \beta'\Psi\eta\left(\frac{\mathrm{d}l}{\mathrm{d}\delta_i} + K\frac{\mathrm{d}r}{\mathrm{d}\delta_i}\right) + (e_i - \delta)'\Psi\eta.$$

Rearrange this to get $K\frac{\mathrm{d}r}{\mathrm{d}\delta_i}$ and substitute that into (22). After some rearranging, we get

$$PC\left[\theta l^{\theta-1} - \left(\beta'\Psi\alpha - \beta'\Psi\eta\right)\frac{1}{PC} + \frac{l}{(PC)^2}\frac{1}{\beta'\Psi\alpha}\right]\frac{\mathrm{d}l}{\mathrm{d}\delta_i} =$$
$$\left[(e_i-\delta)'\tau\Psi(\eta+\alpha) - \frac{1}{PC}l(e_i-\delta)'\Psi\eta\tau\frac{1}{\beta'\Psi\alpha}\right].$$

Divide the above expression for i by the same expression for k to get

$$\frac{\mathrm{d}l/\mathrm{d}\delta_i}{\mathrm{d}l/\mathrm{d}\delta_k} = \frac{(e_i-\delta)'\tau\left(\Psi\eta + \Psi\alpha - \frac{s'\alpha}{PC}\frac{1}{\beta'\Psi\alpha}\Psi\eta\right)}{(e_k-\delta)'\tau\left(\Psi\eta + \Psi\alpha - \frac{s'\alpha}{PC}\frac{1}{\beta'\Psi\alpha}\Psi\eta\right)}. \tag{23}$$

By lemma 2, note that $\Psi\eta = \mathbf{1} - \Psi\alpha$. Using this, we can simplify (23) to be

$$\begin{aligned}\frac{\mathrm{d}l/\mathrm{d}\delta_i}{\mathrm{d}l/\mathrm{d}\delta_k} &= \frac{(e_i-\delta)'(\Psi\eta + \Psi\alpha - \frac{s'\alpha}{PC}\frac{1}{\beta'\Psi\alpha}\Psi\eta)}{(e_k-\delta)'(\Psi\eta + \Psi\alpha - \frac{s'\alpha}{PC}\frac{1}{\beta'\Psi\alpha}\Psi\eta)} \\ &= \frac{(e_i-\delta)'(\mathbf{1} - \frac{s'\alpha}{PC}\frac{1}{\beta'\Psi\alpha}(\mathbf{1}-\Psi\alpha))}{(e_k-\delta)'(\mathbf{1} - \frac{s'\alpha}{PC}\frac{1}{\beta'\Psi\alpha}(\mathbf{1}-\Psi\alpha))}, \\ &= \frac{(e_i-\delta)'\Psi\alpha}{(e_k-\delta)'\Psi\alpha}.\end{aligned}$$

The final line follows from the fact that

$$e_i'\mathbf{1}\frac{s'\alpha}{PC}\frac{1}{\beta'\Psi\alpha} = \delta'\mathbf{1}\frac{s'\alpha}{PC}\frac{1}{\beta'\Psi\alpha}.$$

This completes the proof. ■

Proof of Proposition 6. For simplicity, assume that the government only uses consumption taxes and ensures a balanced budget with lump-sum taxes so there are no government purchases. Then, as before, the sales industry i are given by

$$p_i y_i = p_i c_i + \sum_j p_i x_{ji}.$$

Substituting household and firm input demands we get

$$p_i y_i = \frac{\beta_i PC}{1+\tau_i} + \sum_j \omega_{ji} p_j y_j.$$

Denote the vector of sales by s and household expenditure share on good i net of taxes by $\beta_i^* =$

$\beta_i/(1+\tau_i)$. Then

$$s' = (\beta^*)'\Psi PC.$$

Letting labor be the numeraire, labor demand is then given by

$$l = s'\alpha = (\beta^*)'\Psi\alpha PC = (\beta^*)'\tilde{\alpha}PC.$$

Substitute this into the labor supply equation to get equilibrium labor

$$l^\theta = (\beta^*)'\tilde{\alpha}.$$

Differentiate this expression with respect to τ_i to get

$$\theta l^{\theta-1}\frac{\mathrm{d}l}{\mathrm{d}\tau_i} = -\frac{\beta_i}{(1+\tau_i)^2}\tilde{\alpha}_i.$$

Rearrange this expression and divide through by the same expression for j to get

$$\frac{\mathrm{d}l/\mathrm{d}\tau_i}{\mathrm{d}l/\mathrm{d}\tau_j} = \frac{\beta_i\tilde{\alpha}_i}{\beta_j\tilde{\alpha}_j}\frac{(1+\tau_i)^2}{(1+\tau_j)^2}.$$

■

Proof of lemma 3. From the Euler equation, we have that

$$P_{t+1}C_{t+1} = \rho^*(1+i_t)P_tC_t.$$

Assume an equilibrium where $C_{t+1} = \overline{C}$, where $\overline{C}$ is the long-run efficient steady state value of consumption. Then the euler equation and the aggregate budget set imply that

$$1+i_t = \frac{P_{t+1}\overline{C}/\rho^*}{w_tl_t + r_tk - \tau_t}.$$

When the zero-lower bound is not binding, the central bank can ensure full employment and no inflation by setting the nominal rate equal to $1/\rho^*$. However, at the zero lower-bound, we have

$$\frac{P_{t+1}\overline{C}/\rho^*}{w_tl_t + r_tk - \tau_t} = 1. \tag{24}$$

Rearrange this for labor earnings to get

$$w_tl_t = \frac{P_{t+1}\overline{C}}{\rho^*} - r_tk + \tau_t.$$

Note that

$$r_t k = (1-\beta\tilde{\alpha})(w_t l_t + r_t k - \tau_t) + (1-\delta'\tilde{\alpha})\tau_t.$$

Rearrange this to get

$$r_t k = \frac{(1-\beta'\tilde{\alpha})w_t l_t + (\beta-\delta)'\tilde{\alpha}\tau_t}{\beta'\tilde{\alpha}}, \tag{25}$$

and substitute it into (24) to get

$$w_t l_t = \frac{P_{t+1}\overline{C}}{\rho^*}\beta'\tilde{\alpha} - (\beta-\delta)'\tilde{\alpha}\tau_t.$$

So labor earnings today depend on private nominal consumption tomorrow, and government policy today. Note that

$$\begin{aligned}
P_{t+1}\overline{C} =& w_{t+1}\bar{l} + r_{t+1}k - \tau_{t+1} \\
=& w_{t+1}\bar{l} + \frac{1-\beta'\tilde{\alpha}}{\beta'\tilde{\alpha}}w_{t+1}\bar{l} + \frac{(\beta-\delta_{t+1})'\tilde{\alpha}}{\beta'\tilde{\alpha}}\tau_{t+1} - \tau_{t+1}, \\
=& \frac{1}{\beta'\tilde{\alpha}}w_{t+1}\bar{l} - \frac{\delta'_{t+1}\tilde{\alpha}}{\beta'\tilde{\alpha}}\tau_{t+1}.
\end{aligned}$$

Substitute this into the previous expression to get

$$w_t l_t = \frac{1}{\rho^*}w_{t+1}\bar{l} - \frac{1}{\rho^*}\delta'_{t+1}\tilde{\alpha}\tau_{t+1} - (\beta-\delta_t)'\tilde{\alpha}\tau_t. \tag{26}$$

This gives the aggregate demand curve for labor. ■

Proof of Proposition 7. Since the government cannot commit to future policy τ_{t+1} and δ_{t+1}, we see that the only way the government can boost employment is via $(\delta_t - \beta)'\tilde{\alpha}\tau_t$.

The household Euler equation at the zero lower bound implies that

$$P_{t+1}C_{t+1} = \rho^*(1+i_t)P_t C_t = \rho^* P_t C_t.$$

The economy is back to its efficient full-employment steady state in period $t+1$. So $C_{t+1} = \overline{C}$. Note that as long as current government spending is not so high that it crowds out the private sector from the labor market, the equilibrium features $w_t = w_{t+1}$. Due to lack of commitment for fiscal policy, equation (25) pins down the rental rate of capital in period $t+1$ in terms of w_t:

$$r_{t+1}k = \frac{1-\beta'\tilde{\alpha}}{\beta'\tilde{\alpha}}w_t\bar{l} + \frac{(\beta-\phi)'\tilde{\alpha}}{\beta'\tilde{\alpha}}\overline{\tau} = \frac{1-\beta'\tilde{\alpha}}{\beta'\tilde{\alpha}}w_t\bar{l} + \frac{(\beta-\phi)'\tilde{\alpha}}{\beta'\tilde{\alpha}}\frac{\lambda}{1-\lambda}P_{t+1}\overline{C}.$$

Therefore, P_{t+1} is also pinned down at its long-run steady state value. Since both C_{t+1} and P_{t+1} do not respond to the shock, this means that in order for the Euler equation to hold, either P_t needs to fall or C_t needs to fall.

Since in period $t+1$, the economy returns to full employment, the government's problem can be separated in two. In period $t+1$, the government spends

$$p_{i,t+1}g_{i,t+1} = \frac{\lambda}{1-\lambda}\phi_i P_{t+1}C_{t+1}.$$

In period t, however, the government's problem is different because there is idle labor. The government's problem is

$$\max_{\delta,\tau_t}(1-\lambda)\log\left(\frac{P_tC_t}{P_t}\right) + \lambda\sum_i \phi_i \log(g_{it}).$$

The Euler equation pins down P_tC_t to be $\overline{PC}/\rho^*$, where $\overline{PC}$ is nominal GDP from period $t+1$ onwards. So we can rewrite the government's problem in period t as

$$\max_{\delta,\tau_t} -(1-\lambda)\log(P_t) + \lambda\sum_i \phi_i \log(g_{it}).$$

subject to the following constraints

$$w_tl_t = \beta'\Psi\alpha\frac{\overline{PC}}{\rho^*} + \delta'\Psi\alpha\tau_t, \tag{27}$$

$$r_tK = \beta'\Psi\eta\frac{\overline{PC}}{\rho^*} + \delta'\Psi\eta\tau_t, \tag{28}$$

$$w_{t-1} \le w_t, \tag{29}$$

$$(l_t-\bar{l})(w_t-w_{t-1}) = 0, \tag{30}$$

$$g_{it} = \frac{\delta_i\tau_t}{p_{it}}, \tag{31}$$

$$p_{it} = w_t^{\tilde{\alpha}_i}r_t^{\tilde{\eta}_i}\text{const}_i, \tag{32}$$

$$P_t = \prod_i^N\left(\frac{p_{it}}{\beta_i}\right)^{\beta_i}, \tag{33}$$

$$\sum_i \delta_i = 1. \tag{34}$$

The first order condition for δ_i is given by

$$\frac{\lambda\phi_i}{\delta_i} + \mu_1\tilde{\alpha}_k\tau_t + \mu_2\tilde{\eta}_k\tau_t + \mu_8 = 0,$$

where μ_1 is the lagrange multiplier on the labor market condition, μ_2 is the lagrange multiplier on the capital market condition, and μ_8 makes sure that the δ_i sum to 1. Rearrange this to get

$$\frac{\delta_i}{\delta_j} = \frac{\phi_i}{\mu_2 + (\mu_1 - \mu_2)\tilde{\alpha}_i + \mu_8} \frac{\mu_2 + (\mu_1 - \mu_2)\tilde{\alpha}_j + \mu_8}{\phi_j}. \tag{35}$$

When the labor market clears, μ_1 and μ_2 are equal, therefore, the government simply equates the marginal returns to various forms of government expenditures. However, during recession μ_2 exceeds μ_1, so that expenditures are tilted in favor of sectors with relatively high network-adjusted labor intensities.

To see this, note that the optimum features $w_t = w_{t-1}$ and $l_t = \bar{l}$. Substituting these into labor market clearing (27) for t and $t-1$ gives

$$\beta'\tilde{\alpha}\overline{PC}/\rho^* + \delta'\tilde{\alpha}\tau_t = \beta'\tilde{\alpha}\overline{PC} + \frac{\lambda}{1-\lambda}\overline{PC}\phi'\tilde{\alpha}.$$

Rearrange this to get

$$\delta'\tilde{\alpha} = \beta'\tilde{\alpha}(1 - 1/\rho^*)\overline{PC} + \frac{\lambda}{1-\lambda}\overline{PC}\phi'\tilde{\alpha}. \tag{36}$$

Substitute this into the first order condition for w_t to get

$$\begin{aligned}
\mu_1 + \frac{\mu_3}{l_t} &= \frac{(1-\lambda)\beta'\Psi\alpha + \lambda\phi'\Psi\alpha}{w_t l_t} - \mu_4\frac{(l_t - \bar{l})}{l_t}, \\
&= \frac{(1-\lambda)\beta'\Psi\alpha + \lambda\phi'\Psi\alpha}{w_t l_t}, \\
&= \frac{(1-\lambda)\beta'\Psi\alpha + \lambda\phi'\Psi\alpha}{\beta'\Psi\alpha\overline{PC}/\rho^* + \delta'\Psi\alpha\tau_t}, \\
&= \frac{(1-\lambda)\beta'\Psi\alpha + \lambda\phi'\Psi\alpha}{\beta'\Psi\alpha\overline{PC} + \frac{\lambda}{1-\lambda}\overline{PC}\phi'\Psi\alpha}, \\
&= \frac{1-\lambda}{\overline{PC}}.
\end{aligned}$$

Note that the first order condition for τ_t implies that

$$\mu_2\tau_t + (\mu_1 - \mu_2)\delta'\tilde{\alpha}\tau_t = \lambda.$$

Substitute (36) into this to get

$$\mu_2\tau_t\delta'\tilde{\eta} = \lambda\phi'\tilde{\eta} - (1-\lambda)\beta'\tilde{\alpha}(1 - 1/\rho^*). \tag{37}$$

Note that the first order equation for r_t gives

$$\begin{aligned}\mu_2 &= \frac{(1-\lambda)\beta'\tilde{\eta} + \lambda\phi'\tilde{\eta}}{r_t K}, \\ &= \frac{(1-\lambda)\beta'\tilde{\eta} + \lambda\phi'\tilde{\eta}}{\beta'\tilde{\eta}\overline{PC}/\rho^* + \delta\tilde{\eta}\tau_t}, \\ &= \frac{(1-\lambda)\rho^*}{\overline{PC}} + \frac{(1-\lambda)(\rho^*-1)}{\overline{PC}}\frac{\beta'\tilde{\alpha}}{\beta'\tilde{\eta}},\end{aligned}$$

where going from the second to the third line requires substituting in (37).

Now note that

$$\begin{aligned}\mu_1 - \mu_2 &= \frac{1-\lambda}{\overline{PC}} - \frac{(1-\lambda)\rho^*}{\overline{PC}} - \frac{(1-\lambda)(\rho^*-1)}{\overline{PC}}\frac{\beta'\tilde{\alpha}}{\beta'\tilde{\eta}} - \frac{\mu_3}{l_t}, \\ &= -\frac{(1-\lambda)(\rho^*-1)}{\overline{PC}} - \frac{(1-\lambda)(\rho^*-1)}{\overline{PC}}\frac{\beta'\tilde{\alpha}}{\beta'\tilde{\eta}} - \frac{\mu_3}{l_t}, \\ &< 0,\end{aligned}$$

as required. Note that $\mu_3 \geq 0$ by the KKT conditions.

We now see that the higher the labor-intensity of the household's consumption, the larger the required tilting by the government in its stimulus. It is also easy to verify that if the zero-lower bound does not bind ($\rho^* \leq 1$), then no intervention, and no tilting, is necessary. Indeed, when the zero-lower bound does not bind, we have that $\mu_1 = \mu_2$. ■

Proof of lemma 4. I analyze the equilibrium where the steady-state equilibrium features zero inflation, full employment, and no government spending after the first period and constant government taxes starting in period $t+1$. In period t, the intertemporal budget constraint of the saver and the Euler equation pin down his consumption in period t

$$p_t c_t^s = \frac{p_{t+1}c_{t+1}^s}{\rho(1+i_t)} = \frac{(w_{t+1}l_{t+1} + r_{t+1}K_{t+1})(1-\chi) + (1-\rho)\left[D^l \frac{p_{t+1}}{p_t} + B_t(1+i_t) - p_t^g g_t(1-\chi)\right]}{\rho(1+i_t)}.$$

The budget constraint for the borrower pins down his consumption in period t

$$p_t c_t^b = (w_t l_t + r_t K_t)\chi + \frac{D^l}{1+i_t}\frac{p_{t+1}}{p_t} - D^h\frac{p_t}{p_{t-1}}.$$

To find the real rate of interest R_t, we solve for the equilibrium interest rate that equates supply and demand. This requires that

$$c_t^s = \frac{w_t l_t + r_t k_t}{p_t} - c_t^b - \frac{p_t^g g_t}{p_t}.$$

Substituting the Euler equation into this and solving for the interest rate gives

$$1+R_t = \frac{\frac{(w_{t+1}\bar{l}+r_{t+1}\bar{k})(1-\chi)}{p_{t+1}} + \frac{D^l}{p_{t+1}} - \frac{p_t^g g_t(1-\chi)(1-\rho)}{p_{t+1}}}{\rho\left(\frac{(w_t l_t+r_t\bar{k})(1-\chi)}{p_t} + \frac{D^h}{p_t}\right) - \frac{p_t^g g_t}{p_t}}.$$

The natural rate of interest, on the other hand, is given by

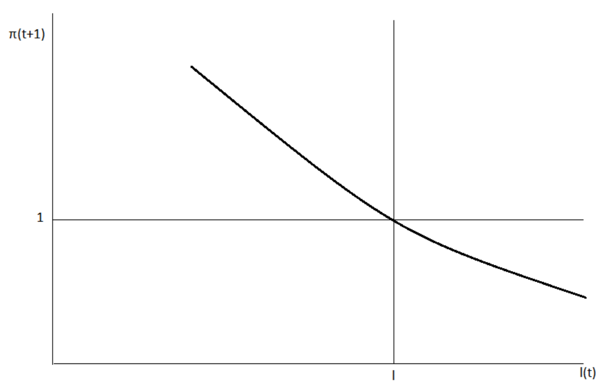
$$1+R_t^n = \frac{\frac{(w_{t+1}\bar{l}+r_{t+1}\bar{k})(1-\chi)}{p_{t+1}} + \frac{D^l}{p_{t+1}} - \frac{p_t^g g_t(1-\chi)(1-\rho)}{p_{t+1}}}{\rho\left(\frac{(w_t\bar{l}+r_t\bar{k})(1-\chi)}{p_t} + \frac{D^h}{p_t}\right) - \frac{p_t^g g_t}{p_t}}.$$

The no-arbitrage condition between nominal government bonds and household debt implies the Fisher equation

$$1+R_t = (1+i_t)\frac{p_t}{p_{t+1}}.$$

If $R_t^n > 0$, then the central bank can maintain full employment with zero inflation, and government spending is unambiguously bad since it wastes resources that would otherwise be going to the households. We can see that whether or not the zero lower bound binds depends on the inflation rate, the amount of government spending, and the size of the deleveraging shock. When the central bank is able to set $i_t = R_t^n$, we have full employment as in figure 15.

Figure 15: Zero-lower bound is not binding and aggregate demand is downward sloping



However, when the zero lower bound on the nominal interest rate binds, then the Fisher equation implies that

$$(1+R_t)\frac{p_{t+1}}{p_t} = 1.$$

Substituting the expression we have for R_t and solving for $w_t l_t$ we get

$$w_t l_t = \frac{1}{\rho}\left(w_{t+1}\bar{l} + r_{t+1}\bar{k}\right) + \frac{D^l - \rho D^h}{\rho(1-\chi)} + \frac{1-(1-\rho)(1-\chi)}{\rho(1-\chi)} p_t^g g_t - r_t\bar{k}.$$

Now substitute

$$r_t \bar{k} = \frac{(1-\beta'\tilde{\alpha})}{\beta'\tilde{\alpha}} w_t l_t + \frac{\beta'\tilde{\alpha} - \delta'\tilde{\alpha}}{\beta'\tilde{\alpha}} p_t^g g_t \tag{38}$$

into this expression and solve for $w_t l_t$ to get

$$w_t l_t = \frac{1}{\rho}\beta'\tilde{\alpha}\left(w_{t+1}\bar{l} + r_{t+1}\bar{k}\right) + \beta'\tilde{\alpha}\frac{D^l - \rho D^h}{\rho(1-\chi)} + \left(\beta'\tilde{\alpha}\left[\frac{1-(1-\rho)(1-\chi)}{\rho(1-\chi)} - 1\right] + \delta'\tilde{\alpha}\right) p_t^g g_t. \tag{39}$$

■

Proof of proposition 8. Societal welfare is given by real GDP net of government expenditures. That is,

$$\frac{w_t l_t + r_t \bar{k} - p_t^g g_t}{p_t} = \frac{1}{\rho}\frac{\left(w_{t+1}\bar{l} + r_{t+1}\bar{k}\right) + \frac{D^l - \rho D^h}{\rho(1-\chi)} + \left[\frac{1-(1-\rho)(1-\chi)}{\rho(1-\chi)} - 1\right] p_t^g g_t}{c_1 w_t \left(\frac{1-\beta'\tilde{\alpha}}{\beta'\tilde{\alpha}}\frac{l_t}{\bar{k}} + \frac{\beta'\tilde{\alpha} - \delta'\tilde{\alpha}}{\beta'\tilde{\alpha}}\frac{p_t^g g_t}{\bar{k} w_t}\right)^{1-\beta'\tilde{\alpha}}}, \tag{40}$$

where the numerator comes from combining (38) with (10) and c_1 is a constant. The denominator comes from combining (38) with the household's price index

$$p_t \propto \prod \left(w_t^{\tilde{\alpha}_i} r_t^{1-\tilde{\alpha}_i}\right)^{\beta_i}.$$

Equation (40) gives real GDP in period t net of government consumption, so it warrants close inspection. The term

$$\left[\frac{1-(1-\rho)(1-\chi)}{\rho(1-\chi)} - 1\right] > 0$$

in the numerator is the government multiplier on nominal private GDP. This term does not depend on the composition of government spending since income from either factor is distributed between the two households uniformly. The denominator, which gives the price level, does however the depend on the composition of spending, since government purchases of capital-intensive goods directly crowds out the household.

The numerator of (40) does not depend on the composition of spending, so we can focus on minimizing the denominator. Note that w_t is fixed as long as the zero lower bound is binding, so we can substitute l_t using (10) into the denominator of (40) and treat w_t as a constant, to get

$$\frac{1}{p_t} \propto \left[\frac{1-\beta'\tilde{\alpha}}{\beta'\tilde{\alpha}}\left(\frac{c_2}{w_t\bar{k}} + \delta'\tilde{\alpha}\frac{p_t^g g_t}{w_t \bar{k}}\right) + p_t^g g_t - \frac{\delta'\tilde{\alpha}}{\beta'\tilde{\alpha}}\frac{p_t^g g_t}{\bar{k} w_t}\right]^{\beta'\tilde{\alpha}-1} = \left(c_3 - \beta'\tilde{\alpha}\delta'\tilde{\alpha} p_t^g g_t\right)^{\beta'\tilde{\alpha}-1},$$

where c_3 and c_2 are constants not affected by $p_t^g g_t$ or δ. This is maximized when the inner term is minimized because the exponent is less than one. The inner term is minimized when $\delta'\tilde{\alpha}$ is maximized, as required. ■

B Appendix: World Trade

In this section, I augment the model in section 3 with trade in goods and services but immobile labor and capital. Assume capital is inelastically supplied at quantity K. The household chooses

$$\max \quad \sum_{t=0}^{\infty} \rho^t \left(\log(C_t) - \frac{l_t^\theta}{\theta} \right),$$

where

$$C_t = \prod_{i=1}^{N} c_{it}^{\beta_i},$$

where

$$c_{it} = \left(c_{it}^h\right)^{\kappa_i} \left(c_{it}^f\right)^{1-\kappa_i},$$

subject to budget constraint

$$\sum \left(p_{it}^h c_{it}^h + p_{it}^f c_{it}^f \right) + q_t B_t = w_t l_t + r_t K + B_{t-1} + \Pi_t - \tau_t,$$

where B_t is a nominal bond (in zero net supply), Π_t is firm profits, τ_t is lump sum taxes, r_t is the rental rate of capital, and c_{it}^h is quantity of domestically produced good i and c_{it}^f is quantity of foreign produced good i consumed. Suppose that there is a physical limit to the number of hours that can be worked

$$l_t \leq \bar{l}.$$

B.1 Firms' problem

Firms rent capital and labor on spot markets from the household, and reoptimize every period. Therefore, their problems are static, so I suppress time-subscripts. Since, in a competitive equilibrium with constant returns to scale, firm size is indeterminate, I simply state the problem of the representative firm in industry i:

$$\max_{y_i^h, l_i, x_{ij}} \quad p_i^h y_i^h - \sum_j p_j x_{ij}^h - \sum_j p_j^f x_{ij}^f - w l_i - r k_i$$

subject to the production function

$$y_i^h = l_i^{\alpha_i} k_i^{\eta_i} \prod x_{ij}^{(1-\alpha_i-\eta_i)\omega_{ij}},$$

with

$$x_{ij} = \left(x_{ij}^h\right)^{\kappa_{ij}} \left(x_{ij}^f\right)^{1-\kappa_{ij}},$$

where superscript h denotes domestic and f foreign use of input j by firm i. The network structure of the economy is captured by the parameters of the Cobb-Douglas production function.

B.2 Government behavior

Let the government run balanced budgets every period

$$\sum p_i g_i = \tau. \tag{41}$$

Also, define the fraction of government expenditures on industry i to be

$$\delta_i = \frac{p_i^h g_i}{\sum_i p_i^h g_i}.$$

To keep the notation simple, the government is assumed to only make domestic purchases.

B.3 The Rest of the World

The rest of the world's behavior is treated as being exogenous. The world simply spends its earnings from trading with home on buying goods and services from home, so that

$$\sum_i p_{it}^h e_{it} = \sum_j \left(\sum_i p_{jt}^f x_{jt}^f + p_{jt}^f c_{jt}^f \right),$$

where $e_i t$ is exports of good i to foreign.

B.4 Market Clearing

The market for good or service i clears so that

$$p_{it}^h e_{it} + p_{it}^h c_{it}^h + p_{it}^h g_{it} + \sum_j p_{it}^h x_{jit}^h = p_{it}^h y_{it}^h.$$

The variable d_{it} is foreign demand for good i in domestic currency. The expenditures of foreigners on each good and service is treated as being exogenous. This would follow from a Cobb-Douglas utility function for the rest of the world.

B.5 Equilibrium

I will focus on the steady state of this model, and will therefore suppress time-subscripts.

Definition B.1. The steady-state competitive equilibrium of this economy is a collection of prices $\{p_i^h\}_{i=1}^N$, wage w, quantities $\{x_{ij}, x_{ij}^h, x_{ij}^f, c_i, c_i^h, c_i^f\}_{ij}$, and labor supply l and labor demands $\{l_i\}_i$

such that for a given δ and τ,

(i) Each firm maximizes its profits given prices,

(ii) the representative household chooses consumption basket $\{c_i\}$ and labor supply l every period to maximizes utility,

(iii) the government runs a balanced budget,

(iv) and markets for each good, labor, and capital clear.

Definition B.2. The employment multiplier of government spending in industry i is defined as $\mathrm{d}l/\mathrm{d}G_i$, where l is equilibrium employment and G_i is government expenditures in industry i.

Since I am focusing on perturbations to the steady state of this model, the changes in government policy are permanent changes to the steady state of the model. This implies that government spending has very strong crowding-out effects.

Definition B.3. The relative employment multiplier of government spending in industry i is defined as $\mathrm{d}l/\mathrm{d}\delta_i$, where l is equilibrium employment and δ_i is the share of government expenditures in industry i, holding fixed the total size of the governments' budget.

The presence of trade with the rest of the world means that we must adjust the influence matrix for trade. To that end, let

$$\Psi^* \equiv (I - \mathrm{diag}(1-\alpha-\eta)(\kappa \cdot \Omega))^{-1}$$

represent the influence matrix with trade and ψ^*_{ij} represent the jth element of the ith row of this matrix. Similarly, let

$$\beta^*_i \equiv \beta_i \kappa_i$$

for each i and β^* denote the column vector β^*_i.

Proposition 9. *The relative government multiplier for shares of expenditure satisfies*

$$\frac{\mathrm{d}l/\mathrm{d}\delta_i}{\mathrm{d}l/\mathrm{d}\delta_k} = \frac{e_i'\Psi^*\alpha - \delta'\Psi^*\alpha}{e_k'\Psi^*\alpha - \delta'\Psi^*\alpha}. \tag{42}$$

Furthermore, labor's share of income is equal to

$$\frac{wl}{GDP} = \frac{(PC\beta^* + \tau\delta + E)'\Psi^*\alpha}{GDP},$$

where E is the vector of expenditures on foreign exports.

The proof for this proposition is very similar to that of proposition 2 and 5.

Proof. Let s_i denote sales of domestic industry i and note that

$$l = \sum_i l_i = \frac{s'\alpha}{w}.$$

Market clearing implies that

$$p_i^h y_i^h = p_i^h c_i + p_i^h g_i + p_i^h e_i + \sum_j p_j^h y_j^h (1 - \alpha_j - \eta_j)\kappa_{ji}\omega_{ji}.$$

Rearrange this to get

$$s' = (PC\beta^* + \tau\delta + E)'\Psi^*.$$

Equating Labor supply and labor demand gives

$$l = \left((PC\beta^* + \tau\delta + E)'\Psi^*\alpha\right)^{\frac{1}{\theta}} \left(\frac{1}{PC}\right)^{\frac{1}{\theta}}.$$

Take derivatives, rearrange, and use lemma 2 to get the desired result. ■

C Appendix: Non-unit Elasticity of Substitution

C.1 Response to Demand Shocks

Recent work by Atalay (2013) suggests that, at business cycle frequencies, the elasticity of substitution may be significantly less than one. In this section, I sketch how the optimal policy results can be generalized to cases with non-unitary elasticities.

Suppose that household utility is given by

$$\max_{c_i} \quad \left(\sum_{i=1}^{N} \beta_i^{\frac{1}{\varepsilon}} c_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is the elasticity of substitution, and β_i is the share parameter for good i. The household faces the same budget set as before

$$\sum p_i c_i = wl + rk - \tau.$$

The government runs a balanced budget

$$\sum_i^N p_i g_i = \tau.$$

Government purchases are given by

$$g_i = \left(\frac{p_i}{P_G}\right)^{-\varepsilon} \delta_i G, \tag{43}$$

where

$$G = \left(\sum_i \delta_i^{\frac{1}{\varepsilon}} g_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{44}$$

and

$$P_G = \left(\sum_i^N \delta_i p_i^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{45}$$

The instrument of government policy are the share parameters δ_i. Observe that as $\varepsilon \to 1$ we recover the Cobb-Douglas production function as before.

The representative firm in each industry is competitive. It chooses inputs and prices to maximize profits:

$$\max_{p_i, l_i, k_i, x_{ij}} \quad p_i y_i - \sum_j p_j x_{ij} - w l_i - r k_i$$

using the production technology

$$y_i \leq \left(\alpha_i^{\frac{1}{\varepsilon}} l_i^{\frac{\varepsilon-1}{\varepsilon}} + \eta_i^{\frac{1}{\varepsilon}} k^{\frac{\varepsilon-1}{\varepsilon}} + \sum_i^N \omega_{ij}^{\frac{1}{\varepsilon}} x_{ij}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

We have the following proposition:

Proposition 10. *In the presence of downward sticky wages, when the labor market fails to clear, we have*

$$\frac{\mathrm{d}l/\mathrm{d}\delta_i}{\mathrm{d}l/\mathrm{d}\delta_i} = \frac{\tilde{\alpha}_i - \delta'\tilde{\alpha}}{\tilde{\alpha}_j - \delta'\tilde{\alpha}},$$

This proposition shows that the qualitative logic of the previous results carry-over without change to the case with non-unitary elasticities. Of course, quantitatively, the share parameters Ω and α no longer correspond to expenditure shares. For simplicity, I have assumed that the household's elasticity of substitution across goods is the same as the firms' elasticity of substitution across inputs. This assumption can be relaxed without losing analytical tractability.

Proof. Note that the price vector P raised element-wise by $1-\varepsilon$ satisfies

$$P^{1-\varepsilon} = \Psi(\alpha w^{1-\varepsilon} + \eta r^{1-\varepsilon}).$$

Let $s_i = p_i^{\varepsilon} y_i$. Then

$$s' = (P_c^{\varepsilon} C\beta + P_G^{\varepsilon} G\delta)'\Psi.$$

Without loss of generality, suppose that the wage is stuck at $w = 1$ and that the labor market is not clearing. Then, labor is determined by demand, so we have

$$l = s'\alpha.$$

Therefore,

$$\frac{\mathrm{d}l}{\mathrm{d}\delta_i} = \left(\beta \frac{\mathrm{d}P_c^{\varepsilon} C}{\mathrm{d}\delta_i} + \delta \frac{\mathrm{d}P_G^{\varepsilon} G}{\mathrm{d}\delta_i} + e_i P_G^{\varepsilon} G\right)' \Psi\alpha. \tag{46}$$

Note that

$$\begin{aligned} P_c^{\varepsilon} C &= \frac{l + rK - \tau}{P_c^{1-\varepsilon}}, \\ &= \frac{\mathrm{d}l/\mathrm{d}\delta_i + \mathrm{d}rK/\mathrm{d}\delta_i}{P_c^{1-\varepsilon}} + (\varepsilon - 1) P_c C P_c^{\varepsilon} \left(\beta' \Psi \eta \mathrm{d}r^{1-\varepsilon}/\mathrm{d}\delta_i\right), \end{aligned}$$

where the last line uses the fact that

$$P_c^{1-\varepsilon} = \beta\Psi(\alpha + \eta r^{1-\varepsilon}).$$

Market clearing for capital implies that

$$rK = (1 - s'\alpha) r^{1-\varepsilon}.$$

Therefore,

$$r^{1-\varepsilon} = \left(\frac{1 - s'\alpha}{K}\right)^{\frac{1-\varepsilon}{\varepsilon}}.$$

So,

$$\frac{\mathrm{d}r^{1-\varepsilon}}{\mathrm{d}\delta_i} = \left(\frac{\varepsilon - 1}{\varepsilon}\right)\left(\frac{s'\eta}{K}\right)^{\frac{1-\varepsilon}{\varepsilon}-1} \frac{\mathrm{d}s'\alpha}{\mathrm{d}\delta_i}\frac{1}{K}.$$

Furthermore,

$$\frac{\mathrm{d}rK}{\mathrm{d}\delta_i} = \frac{\mathrm{d}r^{1-\varepsilon}}{\mathrm{d}\delta_i} - r^{1-\varepsilon}\frac{\mathrm{d}s'\alpha}{\mathrm{d}\delta_i}.$$

Combine these two equations with (46) to get

$$\left[\left(1-\beta'\tilde{\alpha}P_c^{\varepsilon-1}(1-r^{1-\varepsilon})\right)-\beta'\tilde{\alpha}P_c^{\varepsilon-1}\left(1+(\varepsilon-1)P_c^2(1-s'\alpha)\right)\frac{\varepsilon-1}{\varepsilon}\left(\frac{s'\eta}{K}\right)^{\frac{1-\varepsilon}{\varepsilon}-1}\frac{1}{K}\right]\frac{\mathrm{d}s'\alpha}{\mathrm{d}\delta_i}$$
$$=\tilde{\alpha}_i P_G^{\varepsilon}G+\delta'\tilde{\alpha}\frac{\mathrm{d}P_G^{\varepsilon}G}{\mathrm{d}\delta_i},$$
$$=\tilde{\alpha}_i P_G^{\varepsilon}G+\delta'\tilde{\alpha}\frac{\mathrm{d}\tau/P_G^{1-\varepsilon}}{\mathrm{d}\delta_i},$$
$$=\tilde{\alpha}_i P_G^{\varepsilon}G+\delta'\tilde{\alpha}\frac{P_G^{\varepsilon}G}{P_G^{\varepsilon-1}}\left[\tilde{\alpha}_i+\tilde{\eta}_i r^{1-\varepsilon}+\delta'\tilde{\eta}\frac{\mathrm{d}r^{1-\varepsilon}}{\mathrm{d}\delta_i}\right].$$

Rearrange this to get

$$\Theta\frac{\mathrm{d}s'\alpha}{\mathrm{d}\delta_i}=\tilde{\alpha}_i-\frac{\delta\tilde{\alpha}}{P_G^{1-\varepsilon}}\left(\tilde{\alpha}_i+\tilde{\eta}_i r^{1-\varepsilon}\right),$$

where Θ does not depend on δ_i. Recall that $\mathrm{d}s'\alpha/\mathrm{d}\delta_i=\mathrm{d}l/\mathrm{d}\delta_i$. Divide the derivative of labor with respect to δ_i by the derivative with respect to δ_j to get

$$\begin{aligned}\frac{\mathrm{d}l/\mathrm{d}\delta_i}{\mathrm{d}l/\mathrm{d}\delta_j}&=\frac{\tilde{\alpha}_i-\frac{\delta\tilde{\alpha}}{P_G^{1-\varepsilon}}\left(\tilde{\alpha}_i+\tilde{\eta}_i r^{1-\varepsilon}\right)}{\tilde{\alpha}_j-\frac{\delta\tilde{\alpha}}{P_G^{1-\varepsilon}}\left(\tilde{\alpha}_j+\tilde{\eta}_j r^{1-\varepsilon}\right)}\\&=\frac{\tilde{\alpha}_i-\frac{\delta\tilde{\alpha}}{P_G^{1-\varepsilon}}\left(\tilde{\alpha}_i+\tilde{\eta}_i r^{1-\varepsilon}\right)}{\tilde{\alpha}_j-\frac{\delta\tilde{\alpha}}{P_G^{1-\varepsilon}}\left(\tilde{\alpha}_j+\tilde{\eta}_j r^{1-\varepsilon}\right)}\\&=\frac{\tilde{\alpha}_i-\frac{\delta\tilde{\alpha}/P_G^{1-\varepsilon}r^{1-\varepsilon}}{1-\delta\tilde{\alpha}/P_G^{1-\varepsilon}(1-r^{1-\varepsilon})}}{\tilde{\alpha}_j-\frac{\delta\tilde{\alpha}/P_G^{1-\varepsilon}r^{1-\varepsilon}}{1-\delta\tilde{\alpha}/P_G^{1-\varepsilon}(1-r^{1-\varepsilon})}}.\end{aligned}\tag{47}$$

Note that

$$\begin{aligned}\frac{\delta\tilde{\alpha}/P_G^{1-\varepsilon}r^{1-\varepsilon}}{1-\delta\tilde{\alpha}/P_G^{1-\varepsilon}(1-r^{1-\varepsilon})}&=\frac{\delta'\tilde{\alpha}r^{1-\varepsilon}}{P_G^{\varepsilon-1}-\delta'\tilde{\alpha}(1-r^{1-\varepsilon})}\\&=\frac{\delta'\tilde{\alpha}r^{1-\varepsilon}}{\delta'(\tilde{\alpha}+\tilde{\eta}r^{1-\varepsilon})-\delta'\tilde{\alpha}(1-r^{1-\varepsilon})}\\&=\frac{\delta'\tilde{\alpha}r^{1-\varepsilon}}{r^{1-\varepsilon}}\\&=\delta'\tilde{\alpha}.\end{aligned}$$

Substitute this fact into expression (47) to get the desired result.

■

C.2 Labor Share of Income with CES

The results of section 4 take advantage of the Cobb-Douglas form of the production functions, but they can be viewed in a more general reduced-form manner. Whatever the underlying production functions of the different industries, we can define Ω, α, η, and β to be the observed expenditure shares. Market clearing will then imply that equation (3) must hold. This in turn allows us to carry out the decomposition in (5). Therefore, since these depend only on accounting identities, the resulting calculations still tell us how changing expenditure shares are changing labor's share of income regardless of the underlying production functions. Of course, without a structural model, it is impossible to know the causes of these changes.

On the structural front, the benchmark model can be extended to allow for non-unit symmetric elasticity of substitution. Let the composite consumption of household be given by

$$C = \left(\sum_k \beta_k^{\frac{1}{\varepsilon_h}} c_k^{\frac{\varepsilon_h - 1}{\varepsilon_h}} \right)^{\frac{\varepsilon_h}{\varepsilon_h - 1}} .$$

Note that as $\varepsilon_h \to 1$, we recover the utility function of the benchmark model. Let the production function of the representative firm in industry i be given by

$$y_i \leq \left(\alpha_i^{\frac{1}{\varepsilon}} l_i^{\frac{\varepsilon-1}{\varepsilon}} + \eta_i^{\frac{1}{\varepsilon}} k^{\frac{\varepsilon-1}{\varepsilon}} + \sum_i^N \omega_{ij}^{\frac{1}{\varepsilon}} x_{ij}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

Once again, note that letting $\varepsilon \to 1$ recovers the production functions of the benchmark model.

Now, labor's share of private GDP can be written as

$$\frac{wl}{GDP} = \left(\beta \circ P^{\varepsilon - \varepsilon_h} \right)' \tilde{\alpha} w^{1-\varepsilon} P_c^{\varepsilon_h - 1}, \tag{48}$$

where P is a column vector of the price of each good, w is the nominal wage, and P_c is the price level of aggregate consumption. The network-adjusted labor intensities are still defined as $\tilde{\alpha} = (I - \hat{\Omega})^{-1}\alpha$, but now these numbers pertain to the share parameters rather than the observed expenditure shares. Equation (48) allows us to carry out a decomposition similar to (5), although now we need both price and quantity data to identify the relevant parameters. A further restriction of $\varepsilon_h = \varepsilon$ gives us the even simpler expression

$$\frac{wl}{GDP} = \beta' \tilde{\alpha} \left(\frac{w}{P_c} \right)^{1-\varepsilon} .$$

Note that these equations will hold as long as consumption and production have the CES form, and do not depend on assumptions about other aspects of the model like labor supply, intertemporal decision-making, and capital accumulation.

D Appendix: Profits

In this section, I show that firm profits, rather than inelastically supplied capital, can play the role of a non-labor sink and break the irrelevance result in section 2. A tractable way of showing this is to use Dixit-Stiglitz monopolistic competition.

Let the representative household maximize

$$\max_{c_{it}, l_t, B_t} \quad \sum_{t=0}^{\infty} \rho^t \left(\log(C_t) - \frac{l_t^\theta}{\theta} \right),$$

where

$$C_t = \prod_{i=1}^{N} c_{it}^{\frac{\beta_i}{\sum_j \beta_j}},$$

subject to budget constraint

$$\sum (1 + \tau_{it}) p_{it} c_{it} + q_t B_t = w_t l_t + B_{t-1} + \Pi_t - \tau_t,$$

where p_{it} is the price of good i in period t, B_t is a nominal bond (in zero net supply), Π_t is firm profits, τ_t is lump sum taxes in period t. Note that we no longer have capital income.

The government runs balanced budgets every period

$$\sum p_i g_i = \tau + \sum_i \tau_{it} p_{it} c_{it}, \tag{49}$$

and the fraction of government expenditures on industry i is

$$\frac{\delta_i}{\sum_j \delta_j} = \frac{p_i g_i}{\sum_i p_i g_i}.$$

Without loss of generality, suppose that there is a unit mass of firms in each industry. Assume that these firms are monopolistically competitive so that they make positive profits in equilibrium, and the elasticity of substitution across firms producing different varieties in industry i is given by $\varepsilon_i > 1$. The representative firm in industry i maximizes profits

$$\max_{y_i, l_i, x_{ij}} \quad p_i y_i - \sum_j p_j x_{ij} - w l_i$$

subject to the production function

$$y_i = (l_i)^{\alpha_i} \prod x_{ij}^{(1-\alpha_i)\omega_{ij}},$$

where ω_{ij} is the intensity with which the representative firm in industry i uses inputs from industry j. Assume that $\sum_j \omega_{ij} = 1$ for all i to maintain constant returns to scale. In equilibrium, firm i sets its price equal to

$$p_i = \frac{\varepsilon_i}{\varepsilon_i - 1}\lambda_i,$$

where λ_i is its marginal cost. Let μ_i denote the reciprocal of the markup of industry i.

Cost minimization by the firm implies that

$$wl_i = \alpha_i \lambda_i y_i = \alpha_i \mu_i p_i y_i,$$

and

$$p_j x_{ij} = \omega_{ij} \lambda_i y_i = \omega_{ij} \mu_i p_i y_i.$$

Denote the sales of industry i by s_i. Then market clearing for industry i's goods implies

$$s' = H + G + s'\mu \mathrm{diag}(1-\alpha)\Omega = \beta'(H+G) + s'\mu\hat{\Omega},$$

where μ is a diagonal matrix whose ith diagonal element is μ_i. This implies that

$$s' = (H+G)'(I - \mu\hat{\Omega})^{-1}\alpha.$$

Market clearing for labor implies that

$$wl = s'\alpha = (H+G)'(I - \mu\hat{\Omega})^{-1}\alpha.$$

The network-adjusted labor intensities are now given by

$$\tilde{\alpha} = (I - \mu\hat{\Omega})^{-1}\alpha.$$

Labor supply is the same as before

$$l = \left(\frac{wl}{PC}\right)^{\frac{1}{\theta}}.$$

Combining these two equations we get that equilibrium employment must equal

$$l = \left((H+G)'\tilde{\alpha}P\right)^{\frac{1}{\theta}}.$$

It is easy to verify that this model behaves in the same way as the benchmark model in section 3.

E Appendix: Sticky Prices

In this section, I sketch how the basic intuition of the case with sticky wages can be extended to sticky prices, as long as labor (and not capital) is the factor that falls during the recession. Let household utility be given by

$$\sum_t \rho^t \left[(1-\lambda)\log(c_t) - \frac{l_t^\theta}{\theta} + \lambda \log(G_t) \right], \quad \lambda \in (0,1),$$

where

$$c_t = \sum_k (c_{t,k})^{\beta_k}$$

is private consumption, and

$$G_t = \prod_i g_{it}^{\phi_i},$$

is government consumption services. We maintain the assumption that $\sum_k \beta_k = 1$. The household's budget is

$$\sum_k p_{t,k} c_{t,k} + B_t = (w_t l_t + r_t K_t) + (1 + i_{t-1}) B_{t-1} + \Pi_t - \tau_t,$$

where $p_{t,k}$ is the price of good k in time t. Nominal government bonds are B_t and Π_t is firm profits in period t. The nominal net interest rate on debt is i_t. The household receives labor income $w_t l_t$ and capital income $r_t K_t$. Households are endowed with an exogenous amount of capital, and both the wage and the rental rate of capital are flexible. Finally, savers face lump sum taxes τ_t.

Suppose that each industry consists of a fraction ξ of firms who set their prices ever period and $1-\xi$ whose prices are pre-determined. The production function of firms in industry i are given by

$$y_{it} = (l_{it})^{\alpha_i} k_{it}^{\eta_i} \prod x_{ijt}^{(1-\alpha_i-\eta_i)\omega_{ij}}.$$

I assume that demand for goods from industry i are a CES bundle of goods from the firms with pre-determined prices and firms with flexible prices.

E.1 Discount Factor Shock

Suppose that there is an unexpected discount factor shock so that for the next period, $\rho^* > 1$. I analyze the government's fiscal policy without commitment when interest rates are at the zero lower bound.

Proposition 11. *The relative employment multiplier of government spending satisfies*

$$\frac{\mathrm{d}l/\mathrm{d}\delta_i}{\mathrm{d}l/\mathrm{d}\delta_i} = \frac{\tilde{\alpha}_i - \delta'\tilde{\alpha}}{\tilde{\alpha}_j - \delta'\tilde{\alpha}}. \tag{50}$$

Proof. As before, the Euler equation pins down current household expenditures to be

$$P_t C_t = \frac{\overline{PC}}{\rho^*}.$$

Labor supply then implies that

$$l_t^{\theta-1} = \frac{w_t \rho^*}{\overline{PC}}.$$

Furthermore, combining labor supply and labor demand implies that the equilibrium wage is given by

$$w_t = \left(\left(\beta' \frac{\overline{PC}}{\rho^*} + \delta'\tau\right)\tilde{\alpha}\right)^{\frac{\theta-1}{\theta}} (\overline{PC})^{\theta}.$$

Define government expenditure shares on industry i to be

$$\frac{\delta_i}{\sum_k \delta_k},$$

where we assume that $\sum_k \delta_k = 1$. Now Implicitly differentiate equilibrium employment with respect to δ_i evaluated at $\sum_k \delta_k = 1$ to get

$$(\theta-1)\frac{\overline{PC}}{\rho^*} l^{\theta-2} \frac{\mathrm{d}l}{\mathrm{d}\delta_i} = \frac{\theta-1}{\theta}(w_t l_t)^{-\frac{1}{\theta}} \left(\overline{PC}\right)^{\theta} \tau(\tilde{\alpha}_i - \delta'\tilde{\alpha}).$$

Divide this expression through by its counterpart for δ_j to get the desired result. ∎

Proposition 11 shows that the government multipliers behave much in the same way with sticky prices and elastic labor supply as with sticky wages and inelastic labor supply. Welfare analysis in this context is considerably less clean however, since unlike the case with downward sticky wages, in this case, the efficient level of employment depends on the marginal utility of consumption, so there is no "employment-targeting".

Proposition 12. *The optimal share of expenditures by the government in industry i relative to industry j satisfies*

$$\frac{\delta_i}{\delta_j} = \frac{\phi_i}{\phi_j} \frac{const - \tilde{\alpha}_j}{const - \tilde{\alpha}_i}, \tag{51}$$

where const > 1 *. So the government tilts spending according to network-adjusted labor intensities.*

Proof. As before, the Euler equation pins down current household expenditures to be

$$P_t C_t = \frac{\overline{PC}}{\rho^*}.$$

Therefore current consumption is

$$C_t = \frac{\overline{PC}}{\rho^*} \frac{1}{P_t}.$$

Using the above expression for household consumption, the government's optimization problem in period t can be written as

$$\max \quad \frac{1}{\varepsilon - 1} \sum_i (\beta_i + 1) \log(\xi p^*_{it} + (1-\xi) p_{it}) + \lambda \sum_i \phi \log(\delta_i) + \lambda \log(\tau_t) - \frac{l_t^\theta}{\theta},$$

subject to

$$\begin{aligned}
p^*_{it} &= c_i w_t^{\tilde{\alpha}_i} r_t^{\tilde{\eta}_i}, \quad \text{for each } i \\
l_t^{\theta-1} &= \rho^* \frac{w_t}{\overline{\overline{PC}}}, \\
w_t &= \left(\left(\beta' \frac{\overline{PC}}{\rho^*} + \delta' \tau \right) \tilde{\alpha} \right)^{\frac{\theta-1}{\theta}} (\overline{PC})^\theta, \\
r_t &= \beta' \tilde{\eta} + \delta' \tilde{\eta} \frac{\tau}{\overline{k}}, \\
1 &= \sum_i \delta_i.
\end{aligned}$$

The first order conditions for this can be written as

$$\frac{\delta_j}{\delta_i} = \frac{\phi_j}{\phi_i} \frac{\text{const} - \tilde{\alpha}_i}{\text{const} - \tilde{\alpha}_j},$$

where

$$\text{const} = -\frac{\mu_4 \tau_t / \overline{k} + \mu_5}{\mu_3 \frac{\theta-1}{\theta} (wl)^{-1/\theta} - \mu_4 \tau_t / \overline{k}},$$

where μ_k is the lagrange multiplier corresponding to the kth constraint. The term $\mu_4 \tau_t / \overline{k}$ captures the scarcity of capital, while the term $\mu_3 \frac{\theta-1}{\theta} (wl)^{-1/\theta}$ captures the scarcity of labor.

∎

F Appendix: Tables and Graphs

Table F.1: Cumulative changes to each component of labor share from 1996-2009. Rows represent the different countries in the sample. Columns 1-4 show the fraction of change in the labor share for low-skilled labor attributable to each component. The fifth column gives the change in the labor share for low-skilled labor. The columns 6-9 show the fraction of change in the labor share for medium-skilled labor attributable to each component. The tenth column gives the change in the labor share for medium-skilled labor. Columns 11-14 show the fraction of change in the labor share for high-skilled labor attributable to each component. The final column gives the change in the labor share for high-skilled labor.

	cons	supply	labor	lowskill	total change lowskill	cons	supply	labor	medskill	total change medskill	cons	supply	labor	highskill	total change highskill
AUS	0.142	0.128	-0.014	0.745	-0.055	4.232	1.019	0.524	-4.774	-0.002	0.074	0.046	-0.043	0.924	0.036
AUT	0.100	-0.038	0.207	0.731	-0.038	0.235	-0.226	0.734	0.257	-0.050	-0.056	0.248	-0.234	1.042	0.039
BEL	0.040	-0.024	0.107	0.877	-0.078	0.109	0.072	-0.296	1.115	0.033	0.208	0.106	-0.036	0.721	0.043
BGR	0.339	0.145	0.054	0.462	-0.069	0.201	0.383	-0.210	0.625	0.020	0.213	0.294	-0.124	0.618	0.031
BRA	0.030	-0.039	0.032	0.977	-0.052	-0.079	0.056	0.105	0.918	0.019	-0.069	-0.044	-0.033	1.146	0.029
CAN	0.013	-0.022	0.091	0.918	-0.011	-0.126	-0.647	0.786	0.987	-0.028	0.068	0.186	-0.192	0.938	0.040
CZE	0.423	0.566	-2.379	2.391	-0.003	-0.405	-0.200	1.949	-0.343	0.047	-0.058	0.107	0.542	0.409	0.056
DEU	0.116	0.132	0.243	0.509	-0.016	0.138	0.102	0.319	0.440	-0.066	-0.111	0.098	-0.225	1.239	0.030
DNK	-0.425	0.098	0.366	0.960	0.013	0.248	-0.065	-0.150	0.967	-0.055	0.105	0.184	0.023	0.688	0.058
ESP	0.025	-0.012	0.172	0.815	-0.113	0.097	0.037	-0.584	1.449	0.024	0.223	0.228	-0.860	1.409	0.040
EST	-0.567	-0.001	0.756	0.812	0.009	-1.033	0.013	1.481	0.540	0.017	0.453	0.564	0.770	-0.787	0.020
FIN	0.054	-0.100	0.146	0.900	-0.060	-0.914	2.916	-3.186	2.184	0.004	0.060	0.278	-0.199	0.860	0.052
FRA	0.007	0.023	0.080	0.890	-0.060	2.209	0.886	5.022	-7.118	-0.001	-0.060	0.131	0.012	0.916	0.048
GBR	0.047	0.021	-0.086	1.018	-0.069	-0.704	-0.352	1.707	0.349	0.007	0.099	0.100	0.029	0.771	0.088
GRC	0.322	0.512	-0.860	1.026	-0.059	-0.146	-0.242	0.661	0.728	0.052	0.229	-0.080	0.501	0.351	0.066
HUN	0.228	0.188	0.083	0.500	-0.036	0.326	0.272	0.144	0.258	-0.079	-0.346	-0.086	0.210	1.223	0.031
IND	0.264	0.199	0.181	0.355	-0.074	0.086	0.345	0.264	0.305	-0.026	0.400	-0.104	-0.523	1.228	0.028
IRL	0.228	0.163	-0.111	0.720	-0.079	0.446	0.372	-0.129	0.310	-0.040	0.162	-0.103	0.033	0.908	0.076
ITA	0.086	-0.013	0.070	0.857	-0.114	-0.096	0.201	-0.184	1.079	0.059	0.099	0.192	-0.093	0.802	0.042
JPN	0.122	0.033	0.062	0.784	-0.044	0.331	-0.020	0.508	0.181	-0.045	-0.117	0.173	-0.375	1.319	0.032
KOR	0.146	-0.080	0.159	0.776	-0.075	0.218	-0.127	0.566	0.343	-0.087	-0.210	0.482	-2.882	3.609	0.024
LTU	2.320	2.251	-3.948	0.376	-0.003	-5.861	-5.846	14.188	-1.480	0.004	0.138	0.003	0.726	0.134	0.054
MEX	-0.009	0.106	0.166	0.737	-0.028	0.197	-0.131	-0.641	1.575	0.023	-0.492	0.100	0.319	1.073	-0.014
NLD	0.175	-0.011	0.063	0.772	-0.040	0.047	-0.082	0.212	0.823	-0.039	0.132	0.058	-0.066	0.877	0.072
POL	0.273	0.290	0.250	0.187	-0.123	0.224	0.257	0.422	0.097	-0.330	-0.258	-0.113	-1.358	2.730	0.020
PRT	0.521	0.105	-0.600	0.974	-0.045	0.159	0.133	0.003	0.706	0.026	0.261	0.094	0.057	0.588	0.045
ROU	2.388	1.658	-3.950	0.904	-0.039	0.096	-0.179	0.272	0.811	0.020	0.307	0.025	0.134	0.534	0.035
RUS	0.107	-0.008	0.147	0.754	-0.014	-0.099	-1.336	2.057	0.379	-0.018	0.261	0.417	-0.031	0.352	0.049
SVK	0.106	0.199	-0.074	0.769	-0.010	1.317	1.905	-2.862	0.640	-0.011	-0.052	-0.019	0.424	0.646	0.023
SVN	0.272	0.171	0.218	0.339	-0.052	0.196	0.134	0.347	0.323	-0.072	0.099	0.016	0.054	0.832	0.049
SWE	0.028	0.061	0.094	0.816	-0.050	-0.013	0.141	0.498	0.374	-0.035	-0.035	0.041	-0.175	1.168	0.046
TUR	0.939	-0.851	-0.337	1.249	-0.033	0.101	0.497	-0.174	0.577	0.026	0.467	0.244	-0.304	0.593	0.043
TWN	0.165	0.043	0.235	0.557	-0.098	0.355	-0.108	0.717	0.036	-0.028	-0.175	0.141	-0.532	1.567	0.036
USA	0.175	0.159	-0.055	0.721	-0.013	0.137	0.221	0.001	0.641	-0.055	0.156	0.009	-0.042	0.878	0.051

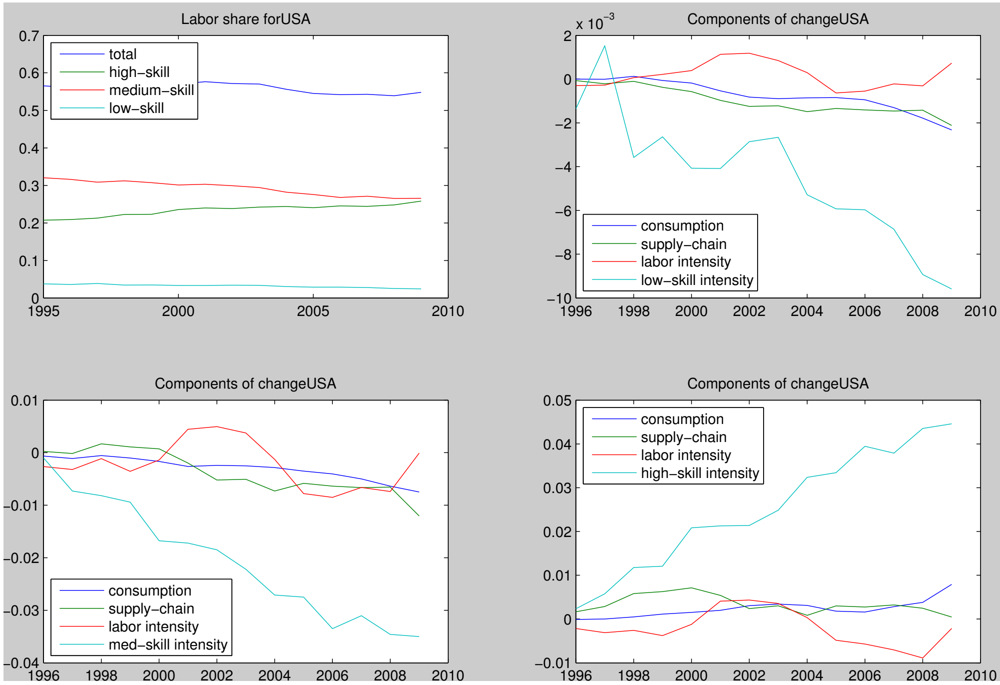


Figure F.1: Evolution of the labor share over time for the US using the WIOD.

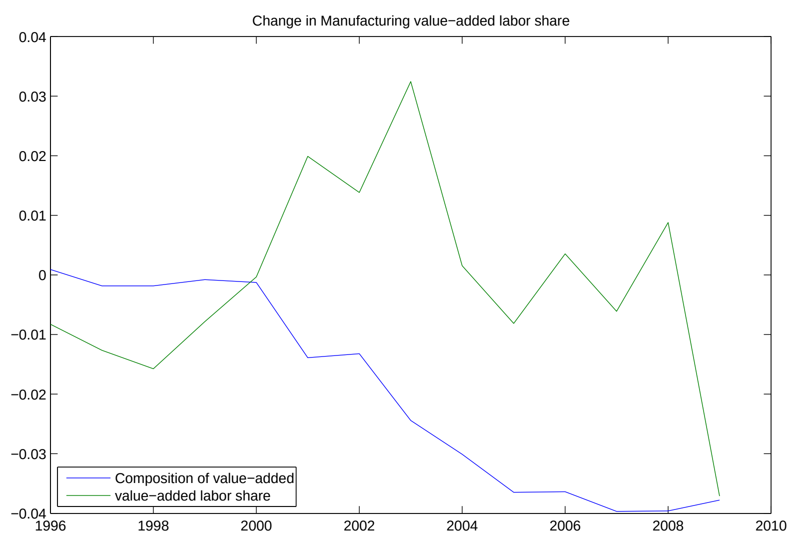


Figure F.2: Evolution of labor use by the manufacturing industries of the US using the WIOD data from 1995-2009.

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