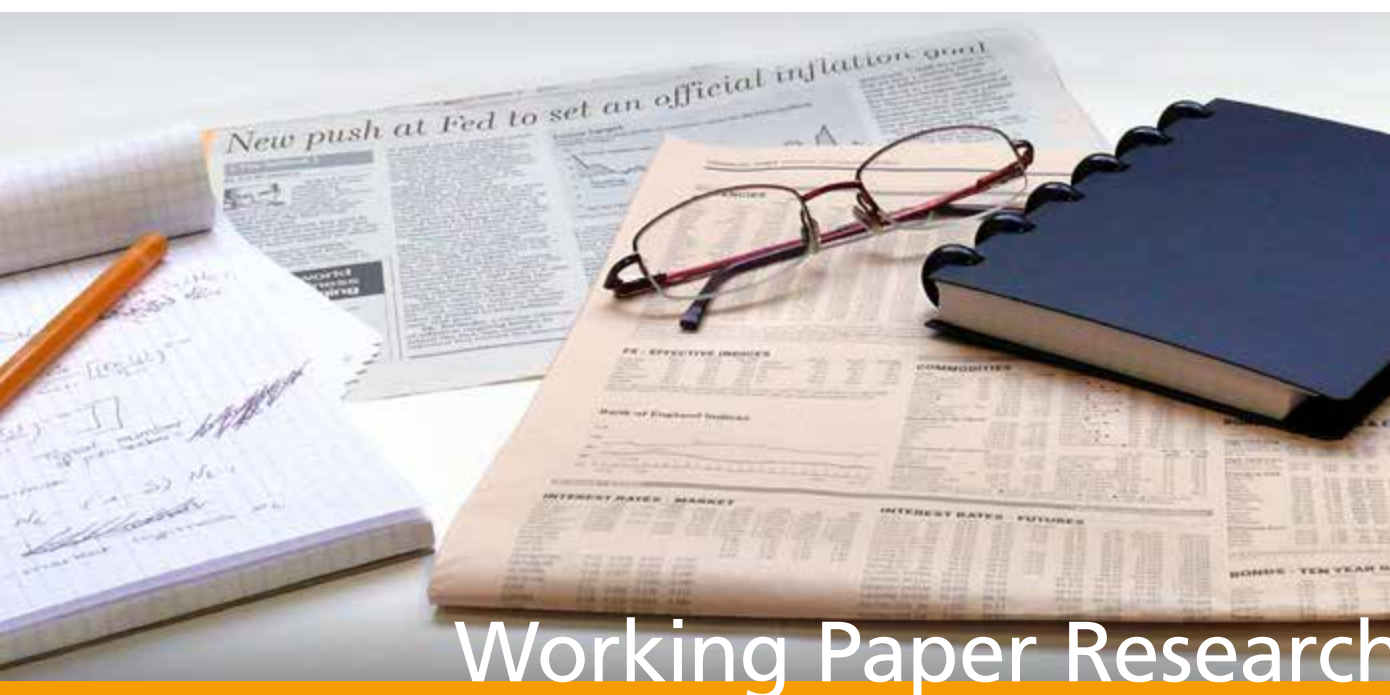


Heterogeneous firms and the micro origins of aggregate fluctuations



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and Jan Van Hove

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Abstract

This paper evaluates the impact of idiosyncratic productivity shocks to individual firms on aggregate output. Two sources of firm-level heterogeneity contribute to aggregate fluctuations: (i) asymmetries in supplier-buyer relationships and (ii) the skewed distribution of sales to final demand. We first develop a model with monopolistic competitive firms and derive a generalized centrality measure that takes these two sources of heterogeneity into account.

The model is subsequently estimated using unique data on firm-to-firm transactions across all economic activities in Belgium. The model generates aggregate volatility from micro origins in the same order of magnitude as observed volatility in GDP. The top 100 firms contribute to 90% of the volatility generated by the model, underlining a strong granularity of the economy. Counterfactual analysis further shows that both sources of micro heterogeneity contribute substantially to aggregate fluctuations, while the relative contribution of each channel crucially depends on the labor share in the economy.

Keywords: Heterogeneous firms, networks, input-output linkages, aggregate volatility.

JEL Classification: E3, L1.

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The results presented fully respect confidentiality restrictions associated with the data sources used in our analysis.

The views expressed are those of the authors and do not necessarily reflect the views of the National Bank of Belgium. All remaining errors are of course idiosyncratic.

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1 Introduction

Traditionally, real business cycle models and New Keynesian models (e.g. [Kydland and Prescott \(1982\)](#), [Prescott \(1986\)](#), [Christiano et al. \(2005\)](#)) focus on large and common shocks to the economy (technology shocks, monetary and fiscal policies, government expenditures, aggregate demand shocks) as potential mechanisms to explain movements in aggregate variables such as GDP. However, while macroeconomic shocks are without doubt important, a large fraction of aggregate volatility remains unexplained by models that postulate this source of volatility ([Cochrane \(1994b\)](#)). Moreover, the 2008-2009 global crisis has made painfully clear that aggregate volatility is not always the result of these large and common shocks: instead, firm-level idiosyncrasies can propagate through relationships with other economic agents, generating sizable aggregate fluctuations.

In this paper, we argue that idiosyncratic productivity shocks to influential firms in the economy can contribute substantially to these aggregate movements. The distribution of firms' influences is governed by two sources of underlying micro-economic heterogeneity: the distribution of sales to final demand and that of input-output linkages at the firm level. There is ample real-world evidence to support this view. First, shocks to firms with large sales to final demand can have a direct impact on aggregate output. For instance, Walmart's US sales represented 2% of US GDP in 2010, equally large as the sales of the next five largest US retailers combined. Nokia contributed 25% to Finland's GDP growth over the period 1998-2007, with a peak of 40% in 2000 ([Seppala and Ali-Yrkko \(2011\)](#)). Second, disruptions in firms' supplier-buyer linkages can have a large impact on aggregate output through propagation effects across the production network. For example, Volkswagen Group (which accounts for 1.3% of German GDP in 2014), experienced the recent "Dieselgate" with a yet unmeasured impact on its own output, that of downstream distribution systems and on sales to the final consumer ([Commission \(2015\)](#)).

To formalize these concepts, we first develop a static firm-level framework with monopolistically competitive firms. Firms are heterogeneous in terms of input requirements and productivity, the two underlying channels for aggregate fluctuations from micro origins in this paper. The model builds on the sectoral input-output presentation of [Acemoglu et al. \(2012, 2016\)](#), where the interaction is now at the firm level and we allow for monopolistic competition with heterogeneous firms as in [Melitz \(2003\)](#) and [Melitz and Redding \(2014\)](#). The model provides micro-economic foundations for the existence of the skewed firm size distribution in reality: more productive firms charge lower prices, sell more to downstream firms and to final demand and earn higher profits than do less productive firms. If there are no intermediate goods, the model collapses back to [Melitz \(2003\)](#). In our more general setting, firms' revenues depend on productivities and the whole input-output structure of production. If the distribution of firms' influences is sufficiently skewed, idiosyncratic productivity shocks can then contribute to aggregate fluctuations, as in [Gabaix \(2011\)](#) and [Acemoglu et al. \(2012\)](#).

The key relationship in this paper is the contribution of these two micro channels in the prediction of aggregate volatility through the combined arrival of temporary and independent shocks to individual firms. We build on the influence vector of [Acemoglu et al. \(2012\)](#) and [Acemoglu et al. \(2016\)](#) to include a structural measure of heterogeneity in final demand. Aggregate volatility is then ultimately a weighted average of productivity shocks to individual firms, with

weights given by the elements of this influence vector, which is in turn governed by the two micro-economic sources of heterogeneity.

The main contribution of the paper lies in the empirical validation of the model’s prediction of aggregate volatility. Using novel data on the universe of firm-to-firm transactions within Belgium, we present three key results. First, idiosyncratic firm-level volatility - measured as the standard deviation of TFP growth - contributes substantially to observed aggregate volatility. The model predicts 1.11% aggregate volatility from idiosyncratic micro shocks, compared to observed volatility of GDP of 1.99% over the same period. This mechanism is complementary to existing macro shocks and other propagation mechanisms such as R&D synchronization (e.g. [Gourio and Kashyap \(2007\)](#)). Importantly, the bulk of aggregate fluctuations predicted by the model remains after controlling for comovement at the level of the economy and highly disaggregated (4-digit NACE) sectors. If sectoral idiosyncrasies would be the underlying source of aggregate volatility rather than firm-specific volatility, this remaining variance should be zero. Second, both micro sources of heterogeneity contribute to aggregate fluctuations. Counterfactually shutting down either channel generates substantially lower predictions of aggregate volatility. Third, the influence of individual firms is severely skewed: the 100 most influential firms explain 90% of the variance in aggregate movement generated by the model. A variance decomposition across industries furthermore shows that firms in Services contribute most to aggregate volatility (40%), followed by Utilities (36%), Manufacturing (24%) and with only a marginal role for the Primary industry (0%). Additional sensitivity analysis, accounting for exports, imports and alternative measures for firm-level productivity growth in the data confirm our main results. Moreover, extensions to the baseline model to account for heterogenous labor shares and investment goods also confirm our main findings.

Finally, we present two additional results. First, as we cannot additively decompose the relative contribution of each channel on aggregate volatility, we instead perform simple counterfactuals across a sequence of economies. We show that the relative contribution of each channel crucially depends on the labor share in production: in the limit, labor is the only factor of production, all sales are directly to final demand and any network propagation mechanisms are mute. As the labor share decreases, relatively more intermediate inputs are used in production and the fragmentation of the production process generates network multipliers ([Carvalho \(2014\)](#)) as in [Acemoglu et al. \(2012, 2016\)](#). Second, [Gabaix \(2011\)](#) and [Acemoglu et al. \(2012\)](#) provide necessary conditions for the influence vector under which idiosyncrasies can surface in the economy. We empirically confirm that these necessary conditions are satisfied in our setting.

This paper contributes to three strands of literature that posit micro origins for aggregate fluctuations. First, a large literature focuses on the aggregate effects of sectoral shocks via the input-output structure of the economy, using the standard deviation of aggregate output growth as a sufficient statistic for aggregate volatility ([Long and Plosser \(1983\)](#), [Horvath \(1998, 2000\)](#), [Dupor \(1999\)](#), [Shea \(2002\)](#), [Foerster et al. \(2011\)](#), [Carvalho and Gabaix \(2013\)](#), [Acemoglu et al. \(2015\)](#) and [Atalay \(2015\)](#)). Shocks to sectors that are important suppliers to other sectors can propagate through the network of input-output linkages and show up in aggregate output. [Acemoglu et al. \(2012\)](#) present a tractable theoretical framework and derive conditions for when shocks to key sectors can show up in the aggregate. We build on this literature by bringing

the network structure to the firm level, while we allow for structural firm heterogeneity, a key component in this paper. Importantly, the mechanisms in this paper rely on observable quantities, allowing us to estimate the model in the data.

Even within narrowly defined industries, there is a large amount of heterogeneity in productivities (see [Syverson \(2011\)](#) for an excellent survey). Hence shocks to firms, rather than to sectors, can directly show up in the aggregate. A second strand of literature thus focuses on the impact of individual firms on aggregate output (e.g. [Durlauf \(1993\)](#), [Carvalho and Grassi \(2016\)](#)). Some firms are large relative to the size of the economy, and [Gabaix \(2011\)](#) shows that the skewness of firm sizes, measured by the sales Herfindahl index, can generate aggregate volatility from firm-level shocks. We contribute to this literature by developing a model that accounts for heterogeneity in productivity, while explicitly allowing for intermediate input consumption.¹ While we abstract from changes in extensive margins, a few other papers evaluate the effects of non-linearities and cascades in production networks, such as [Elliot et al. \(2014\)](#); [Baqae \(2015\)](#).

Third, we add to a growing literature on the empirical validation of networks in a production economy. Applications at the sector level have identified the network structure of production to be the major channel of contribution to aggregate fluctuations ([Atalay \(2015\)](#)). Some first evidence at the firm-level however, shows that the firm-specific component dominates sector level and aggregate sources of aggregate fluctuations ([di Giovanni and Levchenko \(2012\)](#)). Empirical evidence from exogenous shocks such as the Tohoku earthquake in 2011 also point to sizable supply chain distortions at the firm level ([Carvalho et al. \(2015\)](#), [Boehm et al. \(2015\)](#)). Few other papers have substantial information on domestic firm-to-firm linkages, such as [Bernard et al. \(2015a\)](#) for Japan. However, these contain only information on the existence of a link, not its value. We add to this literature by providing a structural firm-to-firm network analysis at this level of detail.

Closest to us is [di Giovanni et al. \(2014\)](#), who empirically evaluate the impact of micro shocks on aggregate volatility in the presence of production networks. However, we differ in three key aspects. First, while these authors start from a decomposition of [Melitz \(2003\)](#) into variance and covariance components, we develop a structural model of production networks. Second, while the authors assume comovement in growth rates is correlated with unobserved firm-level input output linkages, we do observe these linkages at the firm level. Finally, we evaluate the propagation of idiosyncratic productivity shocks, rather than movements in the outcome variable of sales.

¹ Identical to [Gabaix \(2011\)](#) and [Acemoglu et al. \(2012, 2016\)](#), a sufficient statistic for the aggregate impact of firm-level idiosyncratic shocks in our model is given by [Hulten \(1978\)](#). However, we are specifically interested in the relative contribution of the two underlying micro channels to aggregate fluctuations in the data.

2 Model

In this section, we develop a static model of a production economy with intermediate goods. The model largely follows Long and Plosser (1983); Acemoglu et al. (2012, 2016), where we now allow for monopolistic competitive firms with heterogeneous productivities as in Melitz (2003); Melitz and Redding (2014).

2.1 Environment

The representative household in the economy has constant elasticity of substitution (CES) preferences over n goods, and maximizes utility

$$U = \left(\sum_{i=1}^n q_i^\rho \right)^{1/\rho}$$

where q_i is the quantity consumed of good i and $\eta = \frac{1}{1-\rho} > 1$ is the elasticity of substitution, common across goods. Each household is endowed with one unit of labor, supplied inelastically and paid in wages w . The size of the economy is normalized to 1 and labor is the only source of value added in this economy, so that $w = Y$, where Y represents total spending on final consumption. Residual demand for consumption of good i is thus given by $q_i = \frac{p_i^{-\eta}}{P^{1-\eta}} Y$, where p_i is the price of good i and the aggregate price index is given by $P^{1-\eta} = \sum_{i=1}^n p_i^{1-\eta}$ (see Appendix A for the model derivation).

The sequence of events is as follows. *Ex ante*, there is a large pool of potential entering firms. After payment of an investment cost $f_e > 0$, each firm i observes its own productivity ϕ_i (drawn from a Pareto distribution) and it also receives a blueprint for production, stipulated by particular input requirements ω_{ji} . If expected profits are lower than the investment cost, which is sunk after payment, firms exit before producing. The existence of fixed costs of production dictates that there is a cutoff productivity ϕ_i^* , below which firm i cannot make positive profits. Productivity thresholds for entry are firm-specific due to the particular input requirements of i . Sufficiently productive firms then search for input suppliers. The blueprint is contingent, so that the firm learns how to produce the input in house if it is not available on the market (e.g. due to exiting firms with below-threshold productivities). We also assume that in-house production is more expensive than sourcing these inputs (e.g. due to diseconomies of scope). Next, the remaining set of firms enter the market and start producing.²

Ex post, the economy consists of n heterogeneous firms, competing in monopolistic competition. Upon entry, each firm produces a differentiated good, which can be used for consumption and as an input by other firms. Output of firm i is given by $x_i = \sum_j^n x_{ij} + q_i$, where x_{ij} is output produced by i and used as an input for production by firm j . Production entails fixed costs f , common across all firms, so that x_i output requires a total cost of $\Gamma_i = \frac{x_i}{\phi_i} + f$ of inputs in terms of labor and intermediate goods.³ After payment of fixed costs f , output x_i follows a

² In this setting, aggregate profits are absorbed by the entry costs as in Melitz (2003). Alternatively, one can imagine a setting as in Chaney (2008). Then the set of potential entrants is exogenous and profits are redistributed across consumers through a fund in which all workers have equal shares, so that $Y = w + \sum_i \pi_i$.

³ This formulation of the total cost function is the same as in Bernard et al. (2007). Both f_e and f are paid in terms of labor and inputs. This ensures that the production function remains homothetic and no reallocation

Cobb-Douglas production technology with constant returns to scale

$$x_i = (z_i l_i)^\alpha \prod_{j=1}^n x_{ji}^{(1-\alpha)\omega_{ji}} \quad (1)$$

where z_i is total factor productivity (TFP) of firm i (independent across firms), l_i is the amount of labor needed to produce x_i , $\alpha \in (0, 1]$ is the share of labor (common across firms), x_{ji} is the amount of inputs j used in production of i and $\omega_{ji} \in [0, 1]$ is the share of input j in total intermediate input use of firm i .⁴ If firm i does not use input j in its production, $\omega_{ji} = 0$. Let $\sum_j^n \omega_{ji} = 1$ so that constant returns to scale between labor and inputs hold. Let $\varepsilon_i \equiv \ln z_i$ and denote the distribution of ε_i by $F(\varepsilon)$. Furthermore assume $E(\varepsilon_i) = 0$ and $\text{var}(\varepsilon_i) = \sigma_i^2 \in (\underline{\sigma}^2, \bar{\sigma}^2)$ with $0 < \underline{\sigma}^2 < \bar{\sigma}^2 < \infty$. Next, the unit cost function associated with (1) is given by

$$c_i = B_i \left(\frac{w}{z_i} \right)^\alpha \prod_{j=1}^n p_{ji}^{(1-\alpha)\omega_{ji}} \quad (2)$$

where B_i is a constant that maps x_i to c_i , and p_{ji} is the price of input j to firm i .⁵ Assuming no arbitrage (or equivalently no price discrimination), we can set $p_{ji} = p_j$. Marginal costs are given by $\frac{c_i}{\phi_i}$ and from monopolistic competition and CES preferences, prices are then set as a constant markup over marginal cost, so that $p_i = \frac{c_i}{\rho \phi_i}$.⁶

2.2 Competitive equilibrium

Firms maximize profits given optimal amounts of labor and inputs, l_i and x_{ji} , and obtain output prices p_i , taking as given the prices of wages and inputs, w and p_j , and the total cost function Γ_i . The firm's problem can then be written as

$$\pi_i = p_i x_i - w l_i - \sum_{j=1}^n x_{ji} p_j - w f - f \sum_{j \in \mathcal{S}_i} p_j$$

where \mathcal{S}_i represents the set of suppliers of firm i . Denoting revenues from total sales by $r_i \equiv p_i x_i$, we can write equilibrium firm revenues as

$$r_i(\phi) = \underbrace{(1-\alpha) \sum_{j=1}^n \omega_{ij} r_j(\phi)}_{\text{downstream revenue}} + \underbrace{p_i q_i(\phi_i)}_{\text{final demand revenue}} \quad (3)$$

of inputs or labor is possible through these fixed costs in ways that change the marginal rate of technical substitution.

⁴ In [Appendix B](#), we derive an extension of the model with heterogeneous labor shares. We then re-estimate the model predictions for aggregate volatility with this extra source of heterogeneity, generating almost identical results. In [Appendix C](#), we also derive an extension with capital as a second factor of production and re-estimate the model. Again, results are very similar to our baseline model.

⁵ In particular, $B_i = \alpha^{-\alpha} \prod_j^n ((1-\alpha)\omega_{ji})^{-(1-\alpha)\omega_{ji}}$.

⁶ Firms that are large relative to the size of the economy could potentially internalize the impact of their pricing strategy on the aggregate price index. This can lead to variable markups, even with CES preferences ([Melitz and Redding \(2014\)](#)). However, this is a general critique to heterogeneous firms models with CES preferences ([Dixit and Stiglitz \(1993\)](#), [di Giovanni and Levchenko \(2012\)](#)) and not specific to our setup. With non-CES preferences, pass-through can be incomplete (see for instance [Amiti and Konings \(2007\)](#)).

Relative revenues can be written as

$$\frac{r_1(\phi)}{r_2(\phi)} = \frac{(1 - \alpha) \sum_{j=1}^n \omega_{1j} r_j(\phi) + Y \left(\frac{\rho P \phi_1}{c_1} \right)^{\eta-1}}{(1 - \alpha) \sum_{j=1}^n \omega_{2j} r_j(\phi) + Y \left(\frac{\rho P \phi_2}{c_2} \right)^{\eta-1}} \quad (4)$$

Firm profits are given by

$$\pi_i = \frac{r_i}{\eta} - c_i f = \underbrace{\frac{1 - \alpha}{\eta} \sum_{j=1}^n \omega_{ij} r_j(\phi)}_{\text{downstream}} + \underbrace{\left(\frac{\rho \phi_i P}{c_i} \right)^{\eta-1} \frac{Y}{\eta}}_{\text{final demand}} - c_i f \quad (5)$$

Some things are worth mentioning at this point. First, (4) and (5) provide micro foundations for the skewed distribution of firm sizes observed in reality: all other things equal, more productive firms charge a lower price through $p_i = \frac{c_i}{\rho \phi_i}$, generate higher revenues from (4) and earn higher profits than do less productive firms from (5). When there are no intermediate inputs ($\omega_{ij} = 0$ for all j and $c_i = w$), (4) collapses to Melitz (2003), with $\frac{r_1(\phi)}{r_2(\phi)} = \left(\frac{\phi_1}{\phi_2} \right)^{\eta-1}$. Then, relative revenues only depend on relative productivities and the elasticity of substitution: the closer substitutes goods are, the more small differences in productivity result in higher revenues. In our more general setting, firms' cost functions (2) are also heterogeneous in terms of input requirements, and relative revenues subsequently depend on the whole input-output structure of production.

Second, (3) shows the relationship between revenues, productivities, and the network structure of production. Revenues have a recursive structure: sales of i depend on sales of j , which in turn depend on sales of buyers of j etc. Similarly, sales to final demand depend on a firm's productivity ϕ_i , while sales to intermediate demand depend on the productivities of all of i 's buyers, ϕ_j , which in turn depend on productivities of *their* buyers etc. All other things equal, if the intermediate input share $1 - \alpha$ is higher, if downstream firms j are larger or if they simply have higher input requirements from i through ω_{ji} , firm i sells more to any given j . Conditional on final demand, from (4) we see that larger firms sell to more intermediate suppliers directly (summing over all j) and/or indirectly (through the value chain of r_j 's). These are the first-order and higher-order effects respectively as defined at the sectoral level in Acemoglu et al. (2012). Compared to the latter, our model has the additional channel of heterogeneous sales to final demand. In Acemoglu et al. (2016), the authors derive a similar influence vector as in this paper, stemming from perfectly competitive firms and consumer heterogeneous preference shares instead.

Third, (3) provides a mechanism for idiosyncratic shocks to propagate over the network of production. A positive shock z_i to a firm leads to a decrease of its output price through c_i . This leads to lower input prices for downstream buyers j , resulting in lower output for their goods, affecting *their* buyers, and so on up to final demand.⁷

⁷ Acemoglu et al. (2015) show that with Cobb-Douglas technologies, supply shocks only propagate downwards through the price channel of the model, while final demand shocks propagate upwards through the quantity channel. With more general production technologies, productivity shocks can affect both upstream and downstream production through a price and quantity effect. As shown by Shea (2002) and Acemoglu et al. (2012) however, these upstream effect exactly cancel out in a Cobb-Douglas production setting: a negative shock to a firm for instance, leads to an increase of its output price, raising the demand for inputs. At the same time,

2.3 The network structure of production

We can also view the production economy as a weighted directed graph, in which vertices represent firms and directed edges from i to j are given by the non-negative elements of the adjacency matrix $\omega_{ij} \in \Omega$. We can write (3) in matrix form:

$$\mathbf{r} = [\mathbf{I} - (1 - \alpha)\Omega]^{-1} \mathbf{b} \quad (6)$$

where $p_i q_i \equiv b_i \in \mathbf{b}$. \mathbf{b} represents the vector of equilibrium revenues from final demand and is of dimension $n \times 1$. \mathbf{I} is the identity matrix and Ω is a square matrix with elements $\omega_{ij} \in [0, 1]$, both of dimension $n \times n$. Hence, aggregate output depends on the network structure of production through a Leontief inverse $[\mathbf{I} - (1 - \alpha)\Omega]^{-1}$, on the share of intermediate inputs $(1 - \alpha)$, on input requirements ω_{ij} , on the productivities of firms through $r_i(\phi)$, and on final demand revenues b_i . Additionally, since Ω is row stochastic, $[\mathbf{I} - (1 - \alpha)\Omega]$ is invertible for values $\alpha \in (0, 1]$ establishing existence and uniqueness of equilibrium.

We define the *influence vector* \mathbf{v} , similar to Acemoglu et al. (2012, 2016), as

$$\mathbf{v} \equiv \frac{\alpha}{Y} \mathbf{r} = \frac{\alpha}{Y} [\mathbf{I} - (1 - \alpha)\Omega]^{-1} \mathbf{b} \quad (7)$$

The influence vector captures how important each firm is in contributing to aggregate value added Y . \mathbf{v} is also known as a generalized Bonacich (1987) centrality, where \mathbf{b} contains firm-specific elements outside the network (Newman (2010)).

Note that α is the share of value added in output, so we can write $v_i = \frac{\alpha r_i}{\alpha R}$, where v_i is the i^{th} element of the influence vector \mathbf{v} and $R = \sum_{i=1}^n r_i$ represents gross output of the economy. The influence vector thus equivalently reflects the vector of equilibrium market shares in the economy. As shown by Acemoglu et al. (2012) and reproduced in Appendix A, the relationship between the revenue vector and the influence vector is derived from a variant of Hulten (1978) in the presence of Harrod-neutral productivity shocks. In our setup however, we derive equilibrium sales shares for heterogeneous firms in a production network, where firms can be large or influential because of a combination of (i) sales to final demand \mathbf{b} , and (ii) sales to other firms in the network Ω . Importantly, and different from previous settings, we can empirically evaluate the relative contribution of both channels to aggregate fluctuations at the firm level.

The framework in this paper nests several existing contributions on the micro origins of aggregate fluctuations. First, if firms source equally from all other firms in the economy and also sell in equal proportions to final demand, (7) collapses to $\mathbf{v} = \frac{1}{n} \mathbf{b}$. This is a restatement of the classical diversification argument by Lucas (1977): aggregate volatility is then proportional to $1/\sqrt{n}$ and shocks to individual firms wash out in the aggregate from a law of large numbers argument, as shown below.

Second, with homogeneous input shares and heterogeneity in final demand, (7) collapses to $\mathbf{v} = \frac{\alpha}{Y} \left[\mathbf{I} - \frac{(1-\alpha)}{n} \right]^{-1} \mathbf{b}$. For large n , this can be approximated as $\mathbf{v} \simeq \frac{\alpha}{Y} \mathbf{b}$. Hence the influence of individual firms is proportional to their sales to final demand. If additionally $\alpha = 1$, (7) becomes $\mathbf{v} = \frac{1}{Y} \mathbf{b} = \frac{1}{R} \mathbf{r}$. This is a direct statement of the granular hypothesis presented by

the firm's output quantity decreases, reducing the demand for inputs, exactly offsetting the increased demand for inputs. See for instance Carvalho et al. (2015) for upstream effects of productivity shocks in a CES setting.

Gabaix (2011): in the presence of a sufficiently skewed sales Herfindahl index that is consistent with a power law with fat tails, shocks to large firms can contribute significantly to aggregate fluctuations.

Third, with heterogeneous input shares and homogeneous sales to final demand, (7) collapses to $\mathbf{v} = \frac{\alpha}{n} [\mathbf{I} - (1 - \alpha)\mathbf{\Omega}]^{-1} \mathbf{1}$, where $\mathbf{1}$ is a vector of ones. This specification is the influence vector in Acemoglu et al. (2012): if the distribution of \mathbf{v} is consistent with a power law distribution with fat tails, idiosyncratic shocks to important suppliers can propagate through the network of production and show up in the aggregate of the economy.

Finally, our setup allows for heterogeneity in both $\mathbf{\Omega}$ and \mathbf{b} . Skewness in sales to final demand or the input-output structure of the economy can then drive aggregate fluctuations.

2.4 Aggregate volatility

Log-linearizing (1) and subsequently summing over all firms in the economy allows us to write (7) as

$$\ln Y \equiv y = \mu + \mathbf{v}'\varepsilon \quad (8)$$

where μ is a constant, independent of the vector of idiosyncratic productivity shocks ε , and it represents the mean output of the economy. The log of aggregate value added is thus a random variable, and given $E(\varepsilon_i) = 0$, it follows that $E(y) = \mu$; i.e. any deviation from steady state output is only the result of idiosyncratic shocks ε , weighted by their impact through the influence vector \mathbf{v} . (8) is the link between the production network of the economy, idiosyncratic shocks and the resulting aggregate volatility. We can then write the standard deviation of aggregate output, σ_y , as

$$\sigma_y = \sqrt{\sum_{i=1}^n v_i^2 \sigma_i^2} \quad (9)$$

Consider for simplicity of exposition $\sigma_i = \bar{\sigma}$ for all i , so that all firms experience the same volatility $\bar{\sigma}$. Also note that we can write the Euclidean norm of the influence vector as $\|\mathbf{v}\|_2 = \sqrt{\sum_{i=1}^n v_i^2}$. In the extreme case that all firms are completely homogeneous, $v_i = 1/n$ for all i , and so $\sigma_y = \bar{\sigma}/\sqrt{n}$. Then, as $n \rightarrow \infty$, σ_y decays rapidly to zero. This is the classical diversification argument: in an economy with potentially millions of firms, idiosyncratic shocks to individual firms have a negligible impact on the aggregate and aggregate output y rapidly converges to its mean, μ . With asymmetric firm sizes however, micro shocks can show up in the aggregate if the distribution of the influence vector is sufficiently skewed, the crux of the argument in Gabaix (2011). With the additional existence of heterogeneous input-output linkages, v_i represents the influence of firm i through the whole network structure of production, generating large firms from a combination of heterogeneity in firm productivities and input requirements. These two sources of heterogeneity then drive the norm: σ_y clearly increases in $\|\mathbf{v}\|_2$, and with a power law distributed \mathbf{v} with infinite variance, idiosyncratic productivity shocks can contribute to aggregate volatility.⁸

⁸ Gabaix (2011) and Acemoglu et al. (2012) elegantly prove that economies with fat-tailed distributions of v_i , consistent with a power law distribution with tail exponents of $\beta \in [1, 2)$, can generate sizable fluctuations in the

2.5 Extensive margins of the network

Idiosyncratic shocks could have substantial effects on the extensive margins of the economy, i.e. the adding or dropping of transactions and the entry and exit of firms. Firms would then respond to shocks by switching suppliers instead of taking the price changes as given. In addition, when shocks are sufficiently large, this leads to firm entry and exit.

Both theoretical reasons and empirical evidence support our choice of a fixed network in this paper. First, from our model, and consistent with Hopenhayn (1992) and Melitz (2003), entering and exiting firms are at the margin in terms of productivity, employment and turnover. Hence, their direct impact on the aggregate in terms of sales to downstream firms and final demand is negligible. Additionally, less productive firms are also marginal in terms of the number of buyers and suppliers they have (Bernard et al. (2015b)), so that their impact through the propagation mechanism is also peripheral. See Elliot et al. (2014) and Baqaee (2015) for models with discontinuous drops in valuation, forcing large firms to exit in equilibrium and triggering cascades of failures over the network. Second, a large empirical literature suggests that the volatility of aggregate growth rates is generally dominated by the intensive margin (e.g. Bernard et al. (2009) for the US; Bricongne et al. (2012), Osotimehin (2013), di Giovanni et al. (2014) for France; Behrens et al. (2013) and Magerman et al. (2016) for Belgium).

There is also evidence for a marginal role for changes in the extensive margins of domestic transactions as a response to shocks (Bernard et al. (2016)). First, in terms of the number of firms, there is a churning of around 3% entering and 3% exiting firms per year. The median turnover and input consumption of entering/exiting firms is one third of turnover and inputs of incumbents. The median number of buyers entering/exiting firms have is 2, and they have a median of 6 suppliers, compared to 10 and 17 respectively for incumbents. Hence, entering/exiting firms are much smaller in terms of turnover, input consumption and the network they possess. Second, in terms of values, the median value of new transactions is half that of continuing transactions. This is even less for new transactions between incumbents, compared to transactions that involve an entering or exiting firm. Supplier switching between incumbents would suggest that transactions of similar sizes are added and dropped in the data as a response to shocks, which we do not observe in the data.

Finally, new transactions account for around 3% in total turnover for sellers and around 2% in total inputs of buyers. Any remaining changes in the extensive margin are thus peripheral in terms of turnover and inputs.

2.6 Identification strategy

The model is a static representation of an economy's steady state output with disturbances. While it exhibits a stationary equilibrium, GDP is not stationary in reality (which would imply that the economy is not growing over time). In most of the empirical analysis that follows, we exploit the unique panel dimensions of our data and focus on the stationary series of aggregate value added *growth* to predict aggregate volatility.

aggregate. Volatility is then proportional to $1/n^{1-1/\beta}$, much larger than the prediction from the diversification argument. We confirm this requirement in Section 5.

From (8), it is straightforward to generate a simple dynamic interpretation of the model. The growth rate of the economy can then be written as

$$\Delta y = \ln Y_t - \ln Y_{t-1} = \mathbf{v}'(\varepsilon_t - \varepsilon_{t-1}) \quad (10)$$

Assuming independence of shocks over time and using the sample variance as an estimator for the population variance, this results in the estimation equation to be used in Section 4:

$$\hat{\sigma}_{\Delta y} = \sqrt{\sum_{i=1}^n \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_{it})} \quad (11)$$

where $\hat{\sigma}_{\Delta y}$ is the predicted standard deviation of GDP growth, \hat{v}_i denotes an empirical estimate of v_i and $d\hat{\varepsilon}_{it} = (\hat{\varepsilon}_{it} - \hat{\varepsilon}_{it-1})$ denotes estimated TFP growth of firm i at time t .

A few remarks. First, in the model, the network structure \mathbf{v} is exogenous. Hence, shares v_i are constant over time, and there is no meaningful entry or exit of enterprises. Entry is governed by the *ex ante* draws of productivity and input requirements, which are fixed thereafter. Given fixed costs of production and $E(\varepsilon_i) = 0$, enterprises do not exit in the presence of idiosyncratic shocks: firms stay in the market as long as they cover variable costs of production. Second, idiosyncratic productivity shocks are estimated as firm-level TFP growth. Our structural estimation of TFP follows Wooldridge (2009), implying the assumption of a first-order Markov process of TFP growth. Hence, idiosyncratic shocks are independent across firms as well as over time and thus mean reverting. Third, as argued by Cochrane (1994a), supply shocks are mostly transitory, after which the system returns to its steady state, as in our setup. Demand shocks in contrast, are generally more permanent in nature and are related to a shift in the mean output of the economy. This additionally underlines our choice of modeling mean-reverting output shocks as productivity shocks. Finally, given that we observe firm-level transactions in the data instead of implied linkages such as those from sectoral input-output tables, the data we use in this paper arguably provides better identification from firm-level idiosyncratic shocks.

In Section 4, we fix \mathbf{v} for the year 2012 and calculate the volatility over the years 2002-2012 for enterprises that are active in 2012. We use an unbalanced panel structure and calculate $\text{Var}(d\hat{\varepsilon}_{it})$ over the observed time series per firm. We then compare this to the volatility of GDP growth over the same period.⁹

3 The Belgian network of production

3.1 The Belgian VAT system and filings

The Belgian VAT system requires that virtually all enterprises (excluding the financial sector) charge VAT on top of the delivery of their goods and services.¹⁰ This tax is levied in successive stages of the production and distribution process: with each transaction, enterprises charge VAT

⁹ This procedure is similar to di Giovanni et al. (2014), who fix the weights of individual growth rates over time when aggregating to country-level growth rates.

¹⁰ These also include foreign companies with a branch in Belgium or whose securities are officially listed in Belgium. Enterprises that only perform financial transactions or medical or socio-cultural activities are exempt.

on top of their sales and pay VAT on inputs sourced from their suppliers, in effect only paying taxes on the value added at that stage. The tax is neutral to the enterprise and the full burden of the tax ultimately lies with the final consumer. The standard VAT rate in Belgium is 21%, but for some goods a reduced rate of 12%, 6% or 0% applies.¹¹ Since we capture much more than only manufacturing in the VAT data we use, we prefer to talk about enterprises instead of firms in this Section.

VAT liable enterprises have to file periodic VAT declarations and VAT listings to the tax administration. The VAT declaration contains the total sales value, the VAT amount charged on those sales (both to enterprises and to final consumers), the total amounts paid on inputs sourced and the VAT paid on those inputs.¹² This declaration is due monthly or quarterly, depending on some size criteria, and it is the basis for the balance of VAT due to the tax authorities every period. Additionally, at the end of every calendar year, VAT liable enterprises have to file a complete list of their Belgian VAT liable customers over that year.¹³ An observation in this list refers to the value of a sales transaction from enterprise i to enterprise j and the VAT amount due. All yearly transactions larger or equal to 250 euro have to be reported and sanctions for incomplete and mis-reporting guarantee a very high quality of the data.

3.2 Data sources and construction

The empirical analysis mainly draws from five enterprise-level data sources for the years 2002-2012, administered by the National Bank of Belgium (NBB): (i) the novel and confidential NBB B2B Transactions Dataset based on the VAT listings, (ii) annual accounts from the Central Balance Sheet Office at the NBB, (iii) VAT declarations of enterprises as reported to the NBB, (iv) the main economic activity of the enterprise by NACE classification from the Crossroads Bank of Belgium, and (v) enterprise-level import and export data from the Foreign Trade Statistics at the NBB. Enterprises are identified by their VAT number, which is unique and common across these databases. This implies that observations are at the level of the enterprise, rather than at the plant or establishment level.

Enterprise-to-enterprise transactions The NBB B2B Transactions Dataset contains the values of yearly domestic transactions between all VAT liable Belgian enterprises (hence excluding the financial sector) for the years 2002 to 2012. We use the sales values excl. VAT for the analysis. A detailed description of the collection and cleaning of the NBB B2B Transactions Dataset is given in [Dhyne et al. \(2015\)](#).

Enterprise-level characteristics Virtually all enterprises have to file annual accounts at the end of their fiscal year.¹⁴ We extract enterprise-level information from the annual accounts at the NBB and annualize all variables from fiscal years to calendar years. This transformation

¹¹See ec.europa.eu/taxation_customs for a complete list of tariffs. These rates did not change over our sample period.

¹²Sample forms of the declarations can be found [here](#) (French) and [here](#) (Dutch).

¹³Sample forms of the listings can be found [here](#) (French) and [here](#) (Dutch).

¹⁴See [here](#) for filing requirements and exceptions.

ensures that all enterprise-level information in our database is consistent with observations in the VAT listings data.¹⁵ We mainly extract information on turnover, intermediate inputs (both in euro) and employment (in average full-time equivalents (FTE)). Turnover and intermediate inputs do not have to be reported by small firms in the annual accounts, hence we use the VAT declarations to extract these values for small enterprises.¹⁶ We additionally extract information on labor costs, debts and assets (all in euro) for enterprise-level TFP estimation and labor shares. Finally, for some of the sensitivity analyses in [Section 4](#), we extract exports and imports values of enterprise i in year t from the Foreign Trade Statistics at the NBB.

Turnover is defined as total sales of the enterprise in a given calendar year. Intermediate inputs are defined as the sum of material and service inputs to the enterprise. Employment is recorded as the average number of FTE's in that year. We calculate value added of the enterprise as turnover minus intermediate inputs. A detailed description of the TFP estimation procedure is provided in [Appendix D](#). For the subsequent analysis, we drop enterprises that generate negative value added to ensure well-defined influence vectors and weighted growth rates (see [subsection 3.4](#)). We also drop enterprises that have less than one FTE or that do not report employment to account for very small enterprises (including management enterprises).

Sector membership We obtain information on the main economic activity of the enterprise at the NACE 4-digit level from the Crossroads Bank of Belgium. The NACE Rev. 2 classification is the current official EU classification system of economic activities, active since Jan 1, 2008. The 2-digit levels coincide with the international ISIC Rev 4. classification, while the 4-digit levels are particular to the EU. We concord NACE codes over time to the NACE Rev. 2 version to cope with changes in the NACE classification over our panel from Rev. 1.1 to Rev. 2. As we have insufficient information on the financial sector, we drop remaining enterprises in NACE 2-digit sectors 64-66, which have activities related to financial services and insurance.¹⁷ We group sectors into industries across the following NACE 2-digit codes: Primary (NACE 01-09), Manufacturing (NACE 10-33), Utilities (NACE 35-43) and Services (NACE 45-63, 68-82 and 94-96). See [Table 1](#) for an overview of the sectors covered in our dataset.

3.3 Descriptive statistics

The final dataset contains information on enterprises that are VAT liable and additionally report annual accounts in 525 sectors at the NACE 4-digit level. [Table 2](#) reports the share of the aggregate industries in terms of total gross output of the economy and in terms of GDP. **These industries cover 81% of gross output of the Belgian economy and 71% of GDP in 2012 as reported by the National Accounts.** These shares are very stable over our sample period.

[Table 3](#) presents summary statistics for the main variables in 2012. By far the most novel information is that on business transactions. In our final sample, there are 3,505,207 domestic

¹⁵This annualization procedure has a relatively modest impact on our variables of interest: in the data, 78% of enterprises have annual accounts that coincide with calendar years, while 98% of enterprises have fiscal years of 12 months.

¹⁶See [here](#) for the size criteria and filing requirements for either full-format or abridged annual accounts.

¹⁷This choice is purely methodological and bears no impact on our results. These sectors are marginal both in numbers of firms and values.

ISIC 4 Code	NACE Code	Description	Type of input
A	01-03	Agriculture, forestry and fishing	Primary
B	05-09	Mining and quarrying	Primary
C	10-33	Manufacturing	Manufacturing
D	35	Electricity, gas, steam and air conditioning supply	Utilities
E	36-39	Water supply; sewerage, waste management and remediation activities	Utilities
F	41-43	Construction	Utilities
G	45-47	Wholesale and retail trade; repair of motor vehicles and motorcycles	Services
H	49-53	Transportation and storage	Services
I	55-56	Accommodation and food service activities	Services
J	58-63	Information and communication	Services
K	64-66	Financial and insurance activities	–
L	68	Real estate activities	Services
M	69-75	Professional, scientific and technical activities	Services
N	77-82	Administrative and support service activities	Services
O	83	Public administration and defense; compulsory social security	–
P	85	Education	–
Q	86-88	Human health and social work activities	–
R	90-93	Arts, entertainment and recreation	–
S	94-96	Other service activities	Services
T	97-98	Activities of households as employers	–
U	99	Activities of extraterritorial organizations and bodies	–

Table 1: Sectoral Classification.

Coverage	Gross output	GDP
Primary (NACE 01-09)	1%	1%
Manufacturing (NACE 10-33)	28%	14%
Utilities (NACE 35-41)	11%	9%
Services (NACE 45-63, 68-82 and 94-96)	41%	47%
Total Economy	81%	71%

Notes: Industry contribution to gross output and GDP for the Belgian economy in 2012. Industries are aggregated over 2-digit NACE classes. Source: National Accounts (NBB).

Table 2: Industry coverage (2012).

transactions between 79,788 enterprises in 2012. The average transaction value between any two Belgian enterprises is 46,403 euro, with a standard deviation of 1.2 million euro. The table also reports values by typical quantiles and the top 100th and top 10th observations. The median transaction value is 2,000 euro, but the distribution is clearly skewed with extreme outliers over 300 million euro. There are 67,466 enterprises that have at least one business customer in 2012. The median enterprise has 11 business customers, but the distribution is very skewed. On the input side, there are 79,689 enterprises with at least one supplier in the network and the median enterprise sources from 28 suppliers.

The median turnover of enterprises is 0.8 million euro, with a standard deviation of 150 million euro. Input consumption follows very similar patterns with a mean input of 0.5 million euro and a standard deviation of 141 million euro. The median enterprise employs 4 FTE, with a standard deviation of 244 FTE. Each of these enterprise size distributions is very skewed, as is well-documented in the literature (e.g. Axtell (2001) for employment and Gabaix (2011) for turnover of US firms). Results for other years in our sample are very similar (not reported).

Variable	n	mean	st. dev.	Percentiles						
				10th	25th	50th	75th	90th	Top 100	Top 10
Transaction value r_{ij} (euro)	3,505,207	46,403	1.2 mio	382	700	2,000	7,983	34,437	81 mio	327 mio
Number of Customers	67,466	52	236	1	3	11	39	109	2,260	8,020
Number of Suppliers	79,689	44	66	8	16	28	48	89	720	1,574
Turnover r_i (mio euro)	79,788	8.1	150	0.2	0.3	0.8	2.3	8.6	671	4,433
Inputs (mio euro)	79,788	6.6	141	0.1	0.2	0.5	1.6	6.5	573	3,848
Employment (FTE)	79,788	20	244	1	2	4	9	27	1,224	8,815

Table 3: Summary statistics (2012).

3.4 Construction of the influence vector

In order to estimate our theoretical object $\mathbf{v} = \frac{\alpha}{Y} [\mathbf{I} - (1 - \alpha)\mathbf{\Omega}]^{-1} \mathbf{b}$, we need information on input shares $\omega_{ij} \in \mathbf{\Omega}$ and final demand $b_i \in \mathbf{b}$. We obtain intermediate input shares to j , $\hat{\omega}_{ij} \in \hat{\mathbf{\Omega}}$, as the value of transactions r_{ij} in total input consumption of j .¹⁸ We construct sales to final demand of enterprise i , \hat{b}_i as the residual of turnover minus the total of its business transactions observed in the dataset: $\hat{b}_i = r_i - \sum_{j=1}^n r_{ij}$. Final demand thus contains final consumption, government expenditure, exports and domestic sales to other enterprises not in the observed network.¹⁹

Table 4 reports the summary statistics of $\mathbf{\Omega}$ and \mathbf{b} . The median input share across all suppliers in the economy is 0.3% with a standard deviation of 8%. This distribution is clearly skewed: most suppliers represent only a small fraction of input requirements, but some linkages represent key input supplier-buyer relationships. This distribution is very similar when we compare the list of input shares enterprise by enterprise (not reported): invariant to the number of suppliers an enterprise has, most suppliers account for small input shares, while there are a few key suppliers. The median enterprise sells for 0.2 million euro to final demand with a standard deviation of 133 million euro. It is clear that the distributions of both input shares and final demand are heavily skewed, underlining the rationale for our two sources of heterogeneity in this paper.

Variable	n	mean	st. dev.	Percentiles					Top 100	Top 10
				10th	25th	50th	75th	90th		
input shares $\hat{\omega}_{ij}$ (%)	3,505,207	2	8	.01	.05	.3	1	5	100	100
Final demand \hat{b}_i (mio euro)	79,788	6.1	133	0.1	0.2	0.5	1.5	5.6	529	3,730

Table 4: Summary statistics, model variables (2012).

To get a simple estimate for the labor share in production, α , we consider three measurements and take the simple average as our baseline estimate for α . First, from a macro perspective, we calculate the labor share year-by-year from the National Accounts as the labor cost aggregated over all 2-digit sectors in our analysis, divided by total gross output of these sectors. These shares are very stable over our sample period, ranging between 0.18 and 0.20. We take the

¹⁸Since $\sum_{i=1}^n \omega_{ij} = 1$ in the model, we renormalize each entry in the transaction data so that input shares sum to one per buyer, or $\hat{\omega}_{ij} = \frac{r_{ij}}{\sum_{i=1}^n r_{ij}}$. This ensures that our dataset lines up with the model: all value added is generated from transactions in the network and sales to final demand. In Section 4, we additionally allow for imported inputs m_j , such that $\hat{\omega}_{ij} = \frac{r_{ij}}{\sum_{i=1}^n r_{ij} + m_j}$.

¹⁹In Appendix C, we develop a simple extension of the model that includes capital, so that final demand additionally contains investments in line with the typical National Accounting Identity. Note also that while changes in inventories are part of aggregate final demand in the National Accounts, we do not observe these inventories in the VAT data. However, these changes contribute typically less than 1% to GDP.

simple average across years, resulting in $\hat{\alpha} = 0.19$. Second, from a micro perspective, we regress enterprise turnover on labor costs and inputs (all in logs) for the years 2002-2012, using OLS and controlling for year fixed effects, resulting in $\hat{\alpha} = 0.17$.²⁰ Third, we calculate the labor share in production in 2012 from a combination of micro and macro data as

$$\hat{\alpha} = \frac{\text{value added}}{\text{total output}} \times \frac{\text{total labor cost}}{\text{value added}} = \left[\frac{1}{n} \sum_{i=1}^n \left(1 - \frac{\text{inputs}_i}{\text{turnover}_i} \right) \right] \times \frac{\text{total labor cost}}{\text{GDP}} = 0.31 \times 0.69 = 0.21$$

For the first factor, we use annual accounts information; for the second, we use the average labor share in GDP for Belgium over the period 2002-2010 of 0.69, as reported by the OECD, resulting in $\hat{\alpha} = 0.21$. The arithmetic average of these estimates then results in our baseline $\hat{\alpha} = 0.19$.

Finally, as we only observe a subset of enterprises in the economy contributing to observed GDP (we do not observe the financial sector and some small firms in every other sector), we calibrate Y by normalizing \mathbf{v} so that $\sum_i v_i = 1$.

3.5 Estimation of enterprise-level volatilities

We use enterprise-level TFP growth as our measure of productivity shocks g_{it} , so that²¹

$$g_{it} = \Delta \ln TFP_{it} = \ln TFP_{it} - \ln TFP_{it-1}$$

Following the recent literature (e.g. Gabaix (2011), di Giovanni et al. (2014)), we winsorize growth rates at $g_{it} = \pm 1$ (i.e. a doubling or halving of productivity from $t-1$ to t) to account for extreme outliers such as mergers and acquisitions and possible measurement error in the micro data.²² Our measure of enterprise-level volatility is then obtained as the standard deviation of individual growth rates: $\hat{\sigma}_i = \sqrt{\frac{1}{T} \sum_{t=1}^T \left(g_{it} - \frac{1}{T} \sum_{t=1}^T g_{it} \right)^2}$. If the number of observations in the time dimension T is less than or equal to 4, we do not calculate the firm's volatility. Alternative constraints on the time dimension bear no impact on our results.

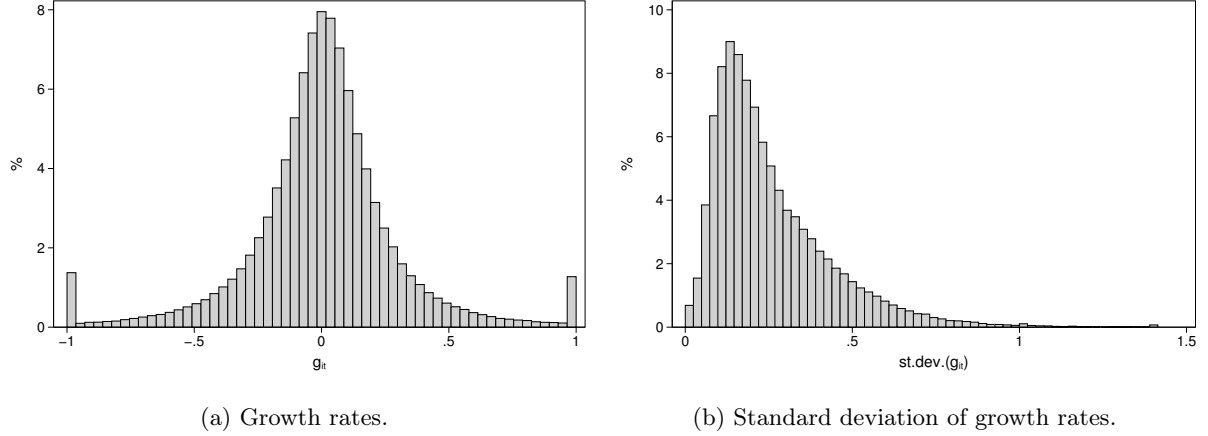
Figure 1 shows the histograms of enterprise-level growth rates (g_{it}) in panel (a) (pooled over the period 2002-2012) and enterprise-level volatilities (st. dev. (g_{it})) in panel (b). The distribution of growth rates is symmetric with mean 0.5%, close to zero. Hence by the law of large numbers, micro shocks could cancel out in the aggregate with homogeneous enterprises as proposed by Lucas (1977). This distribution is very similar when comparing year-by-year, across industries and using alternative measures of growth rates as in subsection 4.3 (not reported). When we turn to panel (b), we see that most enterprises experience a relatively low volatility. The mean volatility is 25.7% with a standard deviation of 17.7%; the median is 20.7%. Again, results are presented across the economy and pooled over our sample period, but these moments

²⁰The estimated coefficients are 0.17 and 0.81 respectively, good support for our constant returns to scale assumption in the model.

²¹Remark that we use yearly growth rates as in similar studies (e.g. Gabaix (2011); Acemoglu et al. (2012); di Giovanni et al. (2014)), as our data dimensions are mainly driven by availability of the different enterprise-level variables. This is slightly different from the bulk of the previous real business cycles literature, which mostly uses quarterly or monthly aggregate output data.

²²The cutoff of 1 we use is extremely liberal. Gabaix (2011) winsorizes growth rates for the top 1,000 firms in his Compustat sample at 20%, while di Giovanni et al. (2014) trim (and thus drop) observations with double and half growth rates. In our data, this procedure leads to 2% of observations to be winsorized for TFP growth.

are very similar across aggregated industries and using alternative growth rate measures.



Notes: Growth rates expressed as log-differences, winsorized at ± 1 .

Figure 1: Productivity growth rates and volatility.

As a simple way to account for aggregate shocks, we follow the literature (e.g. [Stockman \(1988\)](#), [Gabaix \(2011\)](#), [Carvalho and Gabaix \(2013\)](#)), and extract common comovement at aggregate levels by demeaning individual growth rates. For instance, we extract economy-wide comovement as $d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$, where \bar{g}_t is the growth rate of the economy at time t . To construct \bar{g}_t , we use the weighted arithmetic mean of individual growth rates of all enterprises in our panel. In particular, $\bar{g}_t = \sum_i \theta_{it} g_{it}$, where θ_{it} is the share of value added of enterprise i in total value added in the sample at time t . Similarly, we account for sector-level comovement by generating sector-demeaned growth rates by subtracting the 2-digit sector growth rate from the individual growth rates: $d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_{2t}}$, where $\bar{g}_{I_{2t}}$ is the mean growth rate of the 2-digit sector I at time t , to which enterprise i belongs. Similarly for comovement at the 4-digit sector level, as $d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_{4t}}$, where $\bar{g}_{I_{4t}}$ is the mean growth rate of 4-digit sector I at time t . We demean individual growth rates with sector aggregates only if there are at least ten enterprises in the same sector.²³

4 Empirical validation

This Section describes the empirical implementation of our model. We then turn to the underlying micro sources of aggregate fluctuations. Finally, we present several sensitivity analyses.

4.1 Aggregate volatility

With our estimates for v_i and $d\varepsilon_i$, we obtain the model's prediction of aggregate volatility from

²³From our definition of g_{it} , and using the same methodology as in [di Giovanni et al. \(2014\)](#), growth rates are conditional on enterprises surviving from $t-1$ to t , but enterprises are allowed to drop in and/or out the sample over time. We then calculate the volatility over the observed growth rates in the sample. See [Section 2](#) for a discussion on entry/exit, its impact, and our choice for analyzing the evolution of the intensive margin.

$$\hat{\sigma}_{\Delta y} = \sqrt{\sum_{i=1}^n \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_{it})}$$

We estimate the model and three benchmark specifications of \mathbf{v} to evaluate the relative contribution of each source of heterogeneity to aggregate volatility and its relation to the literature.

In particular:

1. **Model:** both Ω and \mathbf{b} are heterogeneous, so that $\mathbf{v} = \frac{\alpha}{Y} [I - (1 - \alpha)\Omega]^{-1} \mathbf{b}$.
2. **Network benchmark** (Acemoglu et al. (2012)): Ω is heterogeneous and \mathbf{b} is homogeneous, so that $\mathbf{v} = \frac{\alpha}{n} [I - (1 - \alpha)\Omega]^{-1} \mathbf{1}$.
3. **Final Demand benchmark:** Ω is homogeneous and \mathbf{b} is heterogeneous, so that $\mathbf{v} = \frac{\alpha}{Y} \left[I - \frac{(1-\alpha)}{n} \right]^{-1} \mathbf{b}$.
4. **Diversification benchmark** (Lucas (1977)): both Ω and \mathbf{b} are homogeneous, so that $\mathbf{v} = \frac{1}{n}$.

Table 5 presents the main results of this paper. The top left cell shows the model’s predicted volatility of 1.11% over the period 2002-2012, while the observed volatility of GDP ($\sigma_{\Delta y}$) was 1.99% over the same period. The second to fourth rows show predictions of aggregate volatility, accounting for aggregate comovement at different levels of aggregation, up to the 4-digit sector level. Predicted volatility decreases monotonically as we demean growth rates at more disaggregated levels, but most of aggregate volatility remains after accounting for aggregate comovement. The second column reports predictions of the Network benchmark. Predicted volatility is then 0.69%. Again predictions naturally decrease when accounting for aggregate comovement. The results from the Final Demand benchmark are given in column three, with a predicted volatility of 0.32%. The last column reports predicted volatility under the assumption of the Diversification benchmark. Predicted volatility is then 0.09%, an order of magnitude smaller than the model prediction.

A few remarks. First, these results suggest that the model predicts a plausible aggregate volatility from idiosyncratic micro origins. As in Gabaix (2011) and Acemoglu et al. (2012), aggregate fluctuations are of the same order of magnitude as those observed in reality. Moreover, our model generates a higher point estimate than any of our benchmarks, suggesting a distinct contribution of both sources of micro heterogeneity to aggregate volatility.

Second, this source of aggregate volatility is complementary to existing macro shocks and other amplification mechanisms such as R&D synchronization (Gourio and Kashyap (2007)). Moreover, while there are other potential sources of micro heterogeneity as natural candidates for contributing to aggregate fluctuations in the light of our model, we find that our results are very robust to these alternative specifications. For instance, we allow for heterogeneous labor shares at the enterprise level in Appendix B and capital goods Appendix C, without any impact on our main findings.

Third, the network structure of production has an important role in explaining aggregate fluctuations. This empirically confirms the findings in Acemoglu et al. (2012), but now at the

enterprise level. At the same time, there is also a significant role for heterogeneity in sales to final demand. This relates to findings of the granular economy as in [Gabaix \(2011\)](#). Moreover, confirming earlier findings, the classical diversification argument grossly underestimates observed aggregate volatility from micro origins. Under this homogeneity restriction, the contribution of micro shocks to aggregate volatility declines rapidly, consistent with [Lucas \(1977\)](#).

$\hat{\sigma}_{\Delta y}$ (%)	Model	Network	Final Demand	Diversification
$d\hat{\varepsilon}_{it} = g_{it}$	1.11	0.69	0.32	0.09
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	1.11	0.69	0.32	0.09
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_{2t}}$	0.96	0.66	0.24	0.09
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_{4t}}$	0.89	0.60	0.22	0.09
$\sigma_{\Delta y}$ (%)	1.99			

Notes: Volatility predictions $\hat{\sigma}_{\Delta y}$ estimated over the period 2002-2012. \bar{g}_t , $\bar{g}_{I_{2t}}$ and $\bar{g}_{I_{4t}}$ represent the mean weighted growth rates of the economy, 2-digit and 4-digit industries respectively. $\sigma_{\Delta y}$ is the observed standard deviation of GDP growth over the same period.

Table 5: Aggregate volatility (2002-2012).

4.2 The micro origins of aggregate fluctuations

The above results suggest that there is a significant role for individual enterprises in explaining aggregate volatility. We now consider these micro origins more carefully. First, from (11), it is straightforward to additively decompose the contribution of the top k enterprises to total volatility of the economy as

$$\hat{\sigma}_{\Delta y}^2 = \sum_{i=1}^k \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_i) + \sum_{i=k+1}^n \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_i) \quad (12)$$

where we set $k = 1,000$ and 100 to capture the variance of the model explained by shocks to the top 1,000 and top 100 enterprises respectively. Here, we use the variance of aggregate movement ($\hat{\sigma}_{\Delta y}^2$) instead of the standard deviation ($\hat{\sigma}_{\Delta y}$) to ensure that volatility shares sum to one. In particular, we obtain the contribution of the top k enterprises from

$$\hat{\sigma}_{\Delta y|i=\{1,\dots,k\}}^2 = \frac{\sum_{i=1}^k \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_i)}{\sum_{i=1}^k \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_i) + \sum_{i=k+1}^n \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_i)}$$

[Table 6](#) presents the share of variance explained by the k most influential enterprises, expressed as $\frac{\hat{\sigma}_{\Delta y|i=\{1,\dots,k\}}^2}{\hat{\sigma}_{\Delta y}^2}$. In our baseline setting, 98.9% of volatility predicted by the model is generated by only the top 1,000 enterprises out of around 80,000 enterprises used in estimation. When we consider the top 100 most influential enterprises, we still find a contribution of 91.3%. Again, demeaning has a slightly decreasing impact on prediction, but the vast majority of variance remains explained by the top k enterprises.

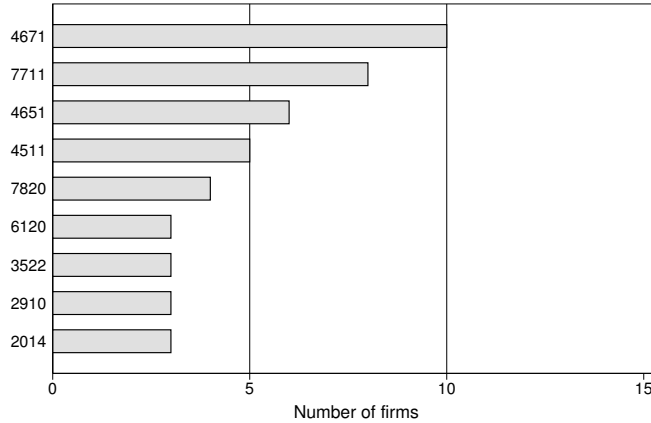
These results indicate that there is a key role for only a very small subset of enterprises. For instance, the top 100 enterprises represent just 0.15% of observations. The classical diversification argument would then allocate a role of $0.09\% \times 0.15\% = 0.0135\%$ to these top 100 enterprises, compared to around 90% in the data. Again, this underlines the rationale for a

framework in which heterogeneous enterprises contribute to aggregate volatility.

Share of variance (%)	Top 1,000	Top 100
$d\hat{\varepsilon}_{it} = g_{it}$	98.9	91.3
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	98.9	91.2
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	98.5	88.4
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	98.3	86.8

Table 6: Share of variance explained by the model, by top k influential enterprises.

Second, Figure 2 shows the distribution of the top 100 most influential enterprises across 4-digit sectors predicted by (12).²⁴ Enterprises in Solid, Liquid and Gaseous Fuels (4671) and in Renting and Leasing of Cars and Light Motor Vehicles (7711) are among the most influential enterprises in the economy predicted by our model. Other inputs such as Wholesale of Computers, Computer Peripheral Equipment and Software (4651), Sale of Cars and Light Motor Vehicles (4511) and Temporary Employment Agencies (7820) are also prominent. Interestingly, there are only two Manufacturing sectors present in this ranking: Manufacture of Other Organic Basic Chemicals (2014) and Manufacture of Motor Vehicles (2910).



NACE Codes: Manufacture of Other Organic Basic Chemicals (2014); Manufacture of Motor Vehicles (2910); Distribution of Gaseous Fuels Through Mains (3522); Sale of Cars and Light Motor Vehicles (4511); Wholesale of Computers, Computer Peripheral Equipment and Software (4651); Solid, Liquid and Gaseous Fuels (4671); Wireless Telecommunication Activities (6120); Renting and Leasing of Cars and Light Motor Vehicles (7711) and Temporary employment agency activities (7820).

Figure 2: Sector membership top 100 enterprises.

Finally, we evaluate the distribution of aggregate volatility across aggregated industries. We allocate each i to industry $I = \{\text{Primary, Manufacturing, Utilities, Services}\}$ and obtain volatility shares similar to (12), but now across I 's:

$$\hat{\sigma}_{\Delta y|i \in I}^2 = \frac{\sum_{i \in I} \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_i)}{\sum_{i \in I} \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_i) + \sum_{i \notin I} \hat{v}_i^2 \text{Var}(d\hat{\varepsilon}_i)}$$

Table 7 reports the shares of variance explained by enterprises in these industries. Each entry represents the share of aggregate volatility explained by every aggregated industry. Clearly the

²⁴Due to confidentiality reasons, we only report sectors with at least 3 enterprises in a 4-digit sector.

Primary industry has a negligible impact on aggregate volatility. Conversely, Manufacturing accounts for 23.8% of aggregate fluctuations in the baseline setting, while Utilities and Services account for 35.6% and 40.5% respectively. This underlines the need for a comprehensive view on the economy, spanning all economic activities: studies with access to Manufacturing data only might incorrectly extrapolate findings for Manufacturing to the rest of the economy.

Share of variance (%)	Primary	Manufacturing	Utilities	Services
$d\hat{\varepsilon}_{it} = g_{it}$	0.0	23.8	35.6	40.5
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	0.0	23.9	35.9	40.3
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	0.0	11.9	34.1	54.0
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	0.0	12.3	36.3	51.4

Table 7: Share of variance explained by the model, by industries (2002-2012).

4.3 Sensitivity analysis

We present a series of sensitivity analyses in [Appendix F](#), and briefly discuss the additional insights here.

First, we acknowledge that the data generates an upward bias towards final demand in the model. Final demand is obtained as the residual of turnover minus sales to other enterprises in the Belgian economy. The residual thus contains final consumption, government spending, exports and sales to other enterprises not observed in the network. However, enterprise-level exports contain sales to foreign final demand as well as exports of intermediate goods. This leads to an overestimation of the impact of consumer final demand in the baseline model. To evaluate the impact of this bias, we perform two extra estimations. First, we estimate the model with only domestic turnover (including that of exporters), ignoring exports and assuming that all final demand is domestic. Second, we alternatively split up exports into business sales and final demand sales, applying the ratio of both channels for enterprises their domestic sales. This implies assuming that enterprise-level exports contain the same ratio of sales to final demand and sales to other businesses as their domestic sales.²⁵ [Table 11](#) presents additional results for these specifications. Predictions for aggregate volatility are very robust to these alternatives.

Second, the baseline model normalizes input shares to sum to one. We additionally allow for imports to be used as inputs and re-estimate the model with these recalculated input shares. [Table 12](#) presents these extra results. The model now generates a slightly higher aggregate volatility. Two remarks are in order here. First, we do not observe enterprise-level counterparts for foreign suppliers, only product-country-level information. Given the different units of observation compared to the domestic data, it is impossible to capture import transactions in the same way as we do with inputs from domestic suppliers. Second, it is also difficult to consider changes in imports as true idiosyncrasies at the enterprise level (see e.g. [Kramarz et al. \(2015\)](#) for correlated shocks across exporters and importers). Given these data limitations and the structure of the model, we cannot capture the impact of international trade shocks on domestic

²⁵In particular, domestic sales are obtained as turnover minus exports, and domestic sales to final demand as the residual of domestic sales minus sales to other enterprises in the network. We then construct the share of domestic sales to final demand out of domestic sales. We apply this share to exports and adjust final demand downwards from this correction.

growth rates. See [Dhyne et al. \(2016\)](#) for further analysis of imported shocks in a related setting.

Third, we alternatively use unweighted averages ($\bar{g}_{It} = \frac{1}{n} \sum_{i \in I} g_{it}$) in the calculation of aggregate growth rates. When using weighted averages, enterprises with high value added shares drive the growth rate of their sector. This can lead to an under- or overestimation of the model predictions of aggregate volatility, guided by the growth rates of the highest value added firms. [Table 13](#) presents these results and shows that the unweighted version actually generates slightly increasing volatility when demeaning at more disaggregated levels.

Finally, we use several other measures for productivity growth, as it is difficult to measure enterprise-level movement due to the measurement in micro data, identifying assumptions on the link between productivity and observables etc. ([Gabaix \(2011\)](#)). We derive enterprise growth using labor productivity (expressed as value added per employee), sales per worker, sales, value added and FTE as alternative measures to calculate g_{it} . [Table 14](#) presents these additional results. The main findings are robust to these alternative measures. Note that labor productivity generates higher predictions for aggregate volatility. This is consistent with the observation in [Acemoglu et al. \(2012\)](#), their footnote 28: “To the extent that total factor productivity is measured correctly, it approximates the variability of idiosyncratic sectoral shocks. In contrast, the variability of sectoral value added is determined by idiosyncratic shocks as well as the sectoral linkages, as we emphasized throughout the paper.”

5 Additional evidence

In this Section, we first evaluate the non-linear effect of the labor share α on the model predictions. Then, we empirically confirm that our influence vector satisfies the necessary condition for micro shocks to surface in the aggregate.

5.1 Counterfactual analysis of a change in the labor share

We investigate how the relative contributions of the network structure and sales to final demand change, as the labor share α varies across a sequence of counterfactual economies. In particular, when $\alpha = 1$, the intermediate input share is zero, output only requires labor, all sales are to final demand and aggregate value added equals gross output. In this case, (7) converges to $\frac{b_i}{Y} = \frac{r_i}{R}$. The propagation mechanism through the network of production is then mute, and all aggregate variance in the model directly comes from large enterprises selling to final demand.

As the labor share α decreases, relatively more intermediate inputs are used in production. This has a non-linear impact on the influence vector from the interaction between an increase in the Leontief inverse $[\mathbf{I} - (1 - \alpha)\mathbf{\Omega}]^{-1}$ and a decrease in the constant term $\frac{\alpha}{Y}$. When $\alpha \rightarrow 0$, the propagation mechanism of the network dominates the contribution of micro shocks to aggregate volatility. Since we cannot additively decompose the effect of a shift in α , we simulate \mathbf{v} for different values of α and re-estimate aggregate volatility for the four specifications in [Section 4](#). We use the baseline specification of $d\hat{\varepsilon}_{it} = g_{it}$ for exposition below, but results are quasi identical for the demeaned growth rates.

[Figure 3](#) shows the comparative static results of changes in α . The X-axis shows the counterfactual α , ranging from 0.01 to 0.99; the Y-axis shows the predicted aggregate volatility under

this restriction. We start with the analysis of the benchmarks. The dotted line represents the prediction of the classical diversification argument as $\hat{\sigma}_{\Delta y} \propto 1/\sqrt{n}$, or 0.09% in our data. By construction, this is independent of the labor share in the economy.

The dashed line shows predicted volatility using the Network benchmark. Volatility decreases monotonically as α increases: with a low α , the network structure is important in production, and multiplier effects generate sizable aggregate fluctuations. However, as α increases, multiplier effects die out, \mathbf{v} converges to $1/n$ in the limit as $\alpha \rightarrow 1$, and aggregate volatility declines exponentially. This observation is consistent with [Acemoglu et al. \(2012\)](#), who provide conditions for aggregate volatility to remain bounded away from zero, even as the number of production units in the economy tends to infinity.²⁶

The dash-dotted line depicts aggregate volatility for the Final Demand benchmark. The production structure thus represents a fully balanced and complete network, in which the intensity of sourcing depends on α . For large n , we can approximate the influence vector as $\mathbf{v} \simeq \frac{\alpha}{Y} \mathbf{b}$. Predicted volatility as a function of α is then a straight line, with slope given by $\sqrt{\sum_{i=1}^n \left(\frac{b_i}{Y}\right)^2}$: the more skewed the distribution of sales to final demand, the steeper the slope.

We now turn to the predictions of our model, depicted by the solid line. Predicted volatility follows a U-shaped curve as a function of α : if α is close to zero, almost all aggregate volatility is generated from the propagation of shocks in the network. Increasing α however, leads to a drop in volatility until around $\alpha = 0.2$, after which volatility increases again. The dominant mechanism in aggregate volatility from micro origins then becomes the channel of sales to final demand. As in the limit $\alpha \rightarrow 1$, all volatility is generated from sales to final demand, and this coincides with the sales shares vector of the economy.

Our model captures both sources of heterogeneity in the presence of a production network. The simulations show that the level of α matters in explaining which channel is dominant in generating aggregate volatility. This is not surprising in view of \mathbf{v} as a centrality measure: \mathbf{v} collapses to a [Bonacich \(1987\)](#) centrality in the case of [Acemoglu et al. \(2012\)](#), where all the adjustments are made inside the network. The formulation of our model however is consistent with a generalized Bonacich centrality where differences in final demand matter, outside the network of intermediate goods. The interaction of α in the model then results in non-linearities in the prediction of aggregate volatility through the influence vector.

These mechanisms are intuitive when we think of different types of economies. A post-industrial economy heavily relies on labor or human capital as input. The majority of idiosyncratic shocks has to surface directly in the aggregate, as few transmission mechanisms are active. Conversely, a highly industrialized economy heavily depends on its production network and shocks then dominantly propagate throughout this network. Our model could serve as a framework for real economies with any share of labor and inputs.

5.2 Empirical distribution of the influence vector

[Gabaix \(2011\)](#) and [Acemoglu et al. \(2012\)](#) derive a necessary condition under which the skewed distributions of the influence vector generate aggregate fluctuations from micro origins. We

²⁶Note that [Acemoglu et al. \(2012\)](#) perform a counterfactual analysis for a sequence of economies as $n \rightarrow \infty$. We keep n fixed and vary the importance of the network structure in aggregate output through α .

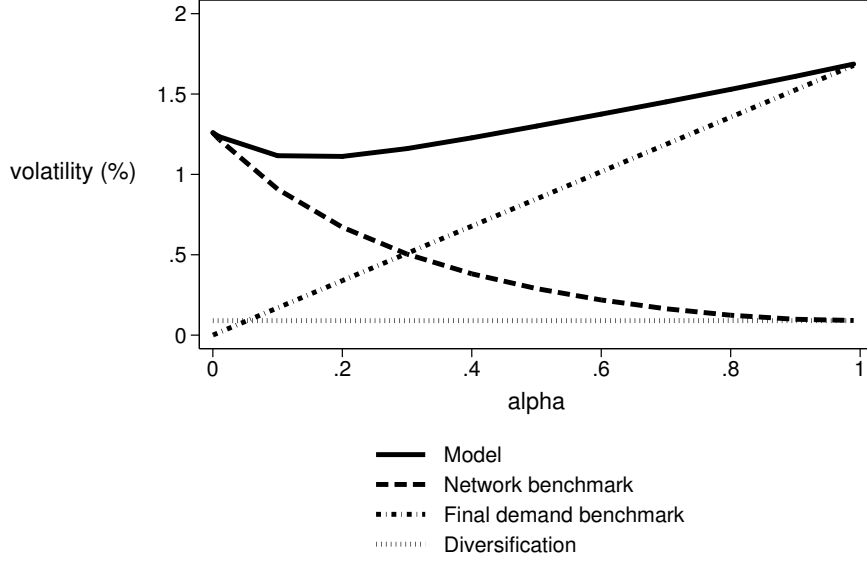


Figure 3: Counterfactual analysis of change in labor share α .

now show that the distribution of the influence vector from our generalized model satisfies this condition.

Figure 4 depicts the empirical counter-cumulative distribution function (CCDF) of the influence vector predicted by our model. The X-axis labels the values of the elements of the influence vector $v_i \in (0, 1)$, while the Y-axis represents the probability that i 's influence is larger than the observed value. Both axes are in log scales. Most enterprises have a negligible total impact on the economy, in the order of a share of 10^{-9} to 10^{-7} . However, some enterprises are very central in the economy, with a total influence of more than 1%, as indicated by the outliers in the plot.

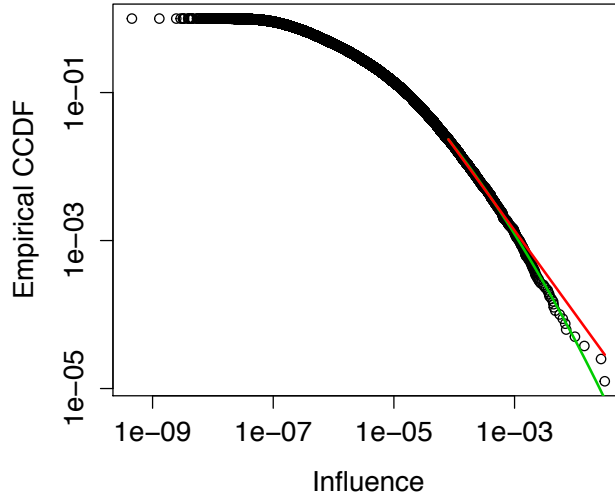
Idiosyncratic shocks to enterprises can thus contribute to aggregate fluctuations if some enterprises are very influential relative to the rest of the economy. In particular, if the tail of the distribution of the influence vector can be approximated by a power law distribution with exponent $\beta \in [1, 2)$, aggregate fluctuations are predicted to have a size of $\sigma/n^{(1-1/\beta)}$. This is much larger than σ/\sqrt{n} , the size suggested by the diversification argument as in Lucas (1977). We define a *power law distribution* as follows:

DEFINITION: The distribution of a variable x is consistent with a power law distribution if, there exists a $\beta > 1$ and a slowly varying function $L(x)$ so that for all $a > 0$, $\lim_{x \rightarrow \infty} L(ax)/L(x) = 1$, the counter-cumulative distribution function can be written as²⁷

$$\Pr(v > x) = x^{-\beta} L(x)$$

The shape parameter β captures the scaling behavior of the tail of the distribution. As β

²⁷Typical representations are $L(x) = \kappa$, or $L(x) = \kappa \ln x$, where κ is some non-zero constant (Gabaix (2011)). In its most familiar form, the log of the probability density function can then be written as $\ln p(x) = \ln L(x) - \beta \ln x$ for sufficiently large values of x .



Notes: The empirical counter-cumulative distribution function (CCDF) is plotted against the values of the influence vector, both in log scales. The red line indicates a power law fit for the tail of the distribution, using a maximum likelihood estimation with endogenous cutoff. The green line indicates a log-normal fit.

Figure 4: Influence vector distribution.

decreases, the distribution becomes more skewed, generating more observations with extreme values. For the range $\beta \in [2, \infty)$, the first two moments of the distribution are well-defined, and Gabaix (2011) and Acemoglu et al. (2012) show that micro fluctuations average out in the aggregate at a rate consistent with the diversification argument. However for $\beta \in [1, 2)$, the second moment diverges, i.e. the distribution is fat-tailed as the variance goes to infinity (see e.g. Gabaix (2009)). In this parameter range, the law of large numbers does not hold and aggregate volatility decays more slowly than presented by the standard diversification argument. For $\beta = 1$ (i.e. Zipf's law), volatility decays at a rate proportional to $1/\ln(n)$. For values of $\beta < 1$, none of the moments are defined.

We fit a power law to the tail of the distribution of the influence vector generated by our data. We present our results in Table 8; the red line in Figure 4 show the graphical representation of the fit. Our baseline estimation method is the numerical maximum likelihood Hill estimator with endogenous cutoff for the tail (Clauset et al. (2009)). The estimated coefficient for the influence vector from our model is $\hat{\beta} = 1.12$, with a standard error of 0.03, confirming that micro shocks can surface in the aggregate.²⁸

The majority of the literature imposes exogenous cutoffs that represent some fraction of the observations (e.g. top 5% observations) or a visual cutoff to define the minimum value of x , x_{min} , above which the fit is performed. However, estimated β 's can be very sensitive to changes in the cutoff, since there is much less mass in the tail. This generates biased results for the

²⁸For the maximum likelihood estimation, approximate standard errors are calculated as $\frac{\hat{\beta}-1}{\sqrt{N}}$ (Clauset et al. (2009)).

scaling behavior of the distribution and we are particularly interested in the behavior of these outliers. The endogenous cutoff method we use ensures the best fit given the data by minimizing the Kolmogorov-Smirnov distance between the proposed fit and the data. Additionally, many papers resort to an OLS estimation on the log-density function $\ln p(x) = \ln L(x) - \beta \ln x$, where $L(x)$ is constant for large enough values of x . Using the x_{min} endogenously dictated by the MLE method, we find an estimate of $\hat{\beta} = 1.20$. In any case, these results confirm that the distribution of the influence vector is sufficiently skewed and consistent with a power law with infinite variance, satisfying the condition stipulated in [Gabaix \(2011\)](#) and [Acemoglu et al. \(2012\)](#).

Furthermore, it is possible to fit a power law to any distribution to get an estimate for β and other distributions might actually provide better fits. As a natural alternative, we fit a log-normal distribution on the influence vector using MLE (see [Saichev et al. \(2010\)](#) for a lengthy discussion on power law versus log-normal in firm sizes and other examples). We impose the same cutoffs for these estimations as in our earlier estimations. The log-normal fits are represented by the green line in [Figure 4](#). We then perform a [Vuong \(1989\)](#) likelihood ratio test to compare the fits of both models. In particular, the test statistic is given by $R = \ln \frac{L(\theta_1|x)}{L(\theta_2|x)}$, where L is the likelihood function and θ_1 is a vector of parameters for model 1 (power law). Similarly for model 2 (log-normal). The sign of R indicates which model is closer to the true (unobserved) model: if $R > 0$, the test statistic presents evidence in favor of model 1. Results are presented in the last two rows of [Table 8](#). R presents evidence that the log-normal distribution is closer to the true model, with a p -value of 9%. While our model is data driven and does not crucially depend on the power law assumption, this does suggest however that in general, there is room for alternative models with log-normal distributions of enterprise size and influence.

Power law estimation			
$\hat{\beta}_{MLE}$	1.12	$\hat{\beta}_{OLS}$	1.20
	(.003)		(.002)
x_{min}	7.99×10^{-5}		7.99×10^{-5}
N	1,861		1,861
R value	-1.70		
p -value	0.09		

Notes: For maximum likelihood estimation, approximate standard errors are calculated as $\frac{\hat{\beta}-1}{\sqrt{N}}$. x_{min} is the endogenous cutoff for the estimated fit, N denotes the number of observations in the tail fit. R value denotes the likelihood ratio test statistic and p -value denotes the significance of the R value being statistically different from zero.

Table 8: Power law fit influence vector.

6 Conclusion

This paper shows that firm-level idiosyncrasies are important drivers for aggregate fluctuations. We have developed a model in which both firm productivity and the network structure of production interact, providing channels for random growth to surface in the aggregate output of an economy. This view generalizes existing contributions on the microeconomic sources of aggregate volatility in which either of both channels matter.

More importantly, we are the first to evaluate the impact of these two sources of heterogeneity on aggregate fluctuations at this level of detail. We have calibrated and estimated the model using firm-to-firm transaction data for Belgian firms and found that a large part of aggregate volatility originates at the firm level, even after accounting for aggregate movement and highly disaggregated sector comovement.

This paper presents a framework in which heterogeneous firms depend on each other in the network structure of production. We believe this has interesting applications in several domains. Using this type of micro data, the model proposes an important policy tool for the evaluation and simulation of different types of micro shocks on the economy at large. It provides a yardstick for the impact of idiosyncrasies to the largest firms, the most connected firms, geographically clustered firms etc. Importantly, the model reveals patterns that remain hidden when using a standard firm-level framework: for instance, the amount of fragmentation of the value chain dictates the importance of additional network multipliers to proliferate.

All European countries have the same VAT system and tax authorities in every Member State have to collect similar data for VAT transactions as does Belgium. If this data is made available for research, it would be very useful to test the model across different economies with different levels of interaction between the network structure and sales to final demand.

This paper embeds the [Melitz \(2003\)](#) model, where inputs now contain the whole network of production rather than only wages. Future work might explore how the network structure changes when firms open up to international trade and reshuffling appears as firms drop below the productivity thresholds for production from international competition. A first step in this direction is [di Giovanni and Levchenko \(2012\)](#), who show that opening up to trade leads to higher volatility from the reallocation of factors to larger and more productive firms. The effect on the network structure of production however, is not explored yet.

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A Model derivation

Demand Each household is endowed with one unit of labor, supplied inelastically (i.e. there is no leisure) and the size of the economy is normalized to 1 so that labor market clearing implies $\sum_i l_i = 1$, where l_i is the amount of labor needed to produce good i . Each household then maximizes utility over CES preferences:

$$U = \left(\sum_{i=1}^n q_i^\rho \right)^{1/\rho} \quad (13)$$

subject to its budget constraint $\sum_{i=1}^n p_i q_i = Y$, where q_i is the quantity consumed of good i and $\eta = \frac{1}{1-\rho} > 1$ is the elasticity of substitution, common across goods. Labor is paid in wages w and it is the only source of value added in this economy, so that $Y = w$, where Y represents total spending on final goods.²⁹ Residual demand then follows $q_i = \frac{p_i^{-\eta}}{P^{1-\eta}} Y$. As each firm faces a downward sloping demand, in equilibrium no two firms will produce the same good.

Firm environment After payment of an investment cost $f_e > 0$, each firm i observes its own productivity ϕ_i (drawn from a Pareto distribution) and also receives a contingent blueprint of production, stipulated by particular input requirements ω_{ji} . After payment of fixed costs, output x_i follows a Cobb-Douglas production technology with constant returns to scale:

$$x_i = (z_i l_i)^\alpha \prod_{j=1}^n x_{ji}^{(1-\alpha)\omega_{ji}} \quad (14)$$

with associated unit cost function:

$$c_i = B_i \left(\frac{w}{z_i} \right)^\alpha \prod_{j=1}^n p_{ji}^{(1-\alpha)\omega_{ji}} \quad (15)$$

Cost minimization can be about cheaper, better or more novel inputs, all isomorphic to the model. Marginal costs are given by $\frac{c_i}{\phi_i}$ and from monopolistic competition and CES preferences, prices are set as a constant markup over marginal cost $p_i = \frac{c_i}{\rho \phi_i}$. Hence, prices are also stochastic through c_i .

Competitive equilibrium A static competitive equilibrium is given by the following equilibrium quantities: prices (p_1, \dots, p_n) , final demands (q_1, \dots, q_n) , quantities (l_i, x_{ij}, x_i) , profits (π_1, \dots, π_n) and cutoff productivities $(\phi_1^*, \dots, \phi_n^*)$.

Total cost is given by $\Gamma_i = \left[f + \frac{x_i}{\phi_i} \right] c_i$.³⁰ The firm's problem is to optimize the amount of inputs l_i and x_{ji} , delivering its output price p_i , taking as given the prices of inputs w and p_{ji} ,

²⁹Note that the last equality is not an accounting identity, but comes from the absorption of profits from free entry under monopolistic competition. This allows us to model the effects of shocks and their propagation only. See for instance [Chaney \(2008\)](#) where positive profits are shared among households from participation in global funds.

³⁰This specification is similar to [Bernard et al. \(2007\)](#) when there are no intermediate goods, in which case Γ_i collapses to $\left[f + \frac{q_i}{\phi_i} \right] c_i$. It is also similar to [Melitz \(2003\)](#) when there are no factors of production, only labor, in which case Γ_i collapses to $f + \frac{q_i}{\phi_i}$, where wages are normalized to one.

and its total cost function Γ_i . The firm's problem can then be written as:

$$\pi_i = p_i x_i - w l_i - \sum_{j=1}^n x_{ji} p_j - w f - f \sum_{j \in \mathcal{S}_i} p_j \quad (16)$$

subject to $x_i = (z_i l_i)^\alpha \prod_{j=1}^n x_{ji}^{(1-\alpha)\omega_{ji}}$, and where we have used the fact that l_i is the amount of variable labor needed to produce x_i/ϕ_i output and x_{ji} is the amount of variable input j needed to produce x_i/ϕ_i . \mathcal{S}_i is the set of input suppliers j to i . Just plugging in x_i and taking first-order conditions with respect to l_i : $\frac{\partial \pi_i}{\partial l_i} = \alpha p_i z_i^\alpha l_i^{\alpha-1} \prod_{j=1}^n x_{ji}^{(1-\alpha)\omega_{ji}} = w$ and with respect to x_{ji} : $\frac{\partial \pi_i}{\partial x_{ji}} = (1-\alpha)\omega_{ji} p_i (z_i l_i)^\alpha x_{ji}^{(1-\alpha)\omega_{ji}-1} \prod_{k \neq j} x_{ki}^{(1-\alpha)\omega_{ki}} = p_j$, leads to optimal factor and inputs demands:

$$\begin{cases} w l_i = \alpha p_i x_i \\ x_{ji} p_j = (1-\alpha)\omega_{ji} p_i x_i \end{cases}$$

Plugging x_{ij} back into goods market clearing $x_i = \sum_j x_{ij} + q_i$ leads to equilibrium revenues per firm:

$$\begin{aligned} x_i &= \sum_{j=1}^n x_{ij} + q_i \\ \Leftrightarrow r_i &\equiv \underbrace{p_i x_i}_{\text{total revenue}} = \underbrace{\sum_{j=1}^n p_i x_{ij}}_{\text{downstream intermediate revenue}} + \underbrace{p_i q_i}_{\text{final demand revenue}} \\ \Leftrightarrow r_i &= \sum_{j=1}^n p_i \frac{(1-\alpha)\omega_{ij} p_j x_j}{p_i} + p_i q_i \\ \Leftrightarrow r_i &= (1-\alpha) \sum_{j=1}^n \omega_{ij} r_j + \left(\frac{p_i}{P}\right)^{1-\eta} Y \end{aligned} \quad (17)$$

where the last equation follows from plugging in optimal demands from the utility maximization. From constant markups, this allows us to write profits as:

$$\pi_i = \frac{r_i}{\eta} - c_i f = \underbrace{\frac{1-\alpha}{\eta} \sum_{j=1}^n \omega_{ij} r_j}_{\text{downstream}} + \underbrace{\left(\frac{\rho \phi_i P}{c_i}\right)^{\eta-1} \frac{Y}{\eta}}_{\text{final demand}} - c_i f$$

The network structure of production We can write (17) in matrix form as

$$\mathbf{r} = (1-\alpha)\mathbf{\Omega}\mathbf{r} + \mathbf{b}$$

where directed edges from i to j are given by the non-negative elements of the adjacency matrix $\omega_{ij} \in \mathbf{\Omega}$. This leads to

$$\mathbf{r} = [\mathbf{I} - (1-\alpha)\mathbf{\Omega}]^{-1} \mathbf{b} \quad (18)$$

where \mathbf{I} is the $n \times n$ identity matrix and $\mathbf{\Omega}$ is the input-output matrix, with input shares ω_{ij} as elements.

Finally, we derive the relationship between the revenue vector and the influence vector. A variant of [Hulten \(1978\)](#) theorem as in [Acemoglu et al. \(2012\)](#), states that, when individual firms are hit with Harrod-neutral productivity shocks ε_i , aggregate output Y changes as $dY = \frac{p_i x_i}{\sum_j p_j x_j} d\varepsilon_i$. Whenever production follows a Cobb-Douglas specification, [Hulten \(1978\)](#) holds, and accounting for heterogeneous firms does not influence the outcome. The result is thus the same as in [Acemoglu et al. \(2012\)](#). Let production be given by $x_i = e^{\alpha \varepsilon_i} f(x_{i1}, \dots, x_{in}, l_i)$. Goods market clearing in the economy is given by $x_i = \sum_j x_{ij} + q_i$. The social optimum is given by

$$\max_{q_i, l_i, x_{ij}} U(q_1, \dots, q_n) \quad s.t. \quad \begin{cases} \sum_{j=1}^n x_{ij} + q_i = e^{\alpha \varepsilon_i} f(\cdot) \\ \sum_{i=1}^m l_i = 1 \end{cases}$$

The Lagrangian of the economy can be written as

$$\mathcal{L} = U(\cdot) + \sum_{i=1}^n p_i \left[e^{\alpha \varepsilon_i} f(\cdot) - q_i - \sum_{j=1}^n x_{ij} \right] + Y \left[1 - \sum_{i=1}^m l_i \right]$$

Aggregate output is given by, $Y = w = \sum_{i=1}^n p_i q_i = \alpha \sum_{i=1}^n p_i x_i$. If a Harrod-neutral shock hits individual firms, welfare changes as

$$dY = \sum_{i=1}^n p_i [e^{\alpha \varepsilon_i} f(\cdot) d\alpha \varepsilon_i] = \sum_{i=1}^n p_i x_i d\alpha \varepsilon_i$$

Finally, since $Y = \alpha \sum_{i=1}^n p_i x_i$, $dY = \frac{p_i x_i}{\alpha \sum_{j=1}^n p_j x_j} d\alpha \varepsilon_i$ and so $v_i \equiv \frac{dY}{d\varepsilon_i} = \frac{p_i x_i}{\sum_{j=1}^n p_j x_j}$. \square

Aggregate output and the influence vector for the economy From the firm's production function, plug in optimal inputs l_i and x_{ji} :

$$x_i = \left(\frac{z_i \alpha p_i x_i}{w} \right)^\alpha \prod_{j=1}^n \left(\frac{(1-\alpha) \omega_{ji} p_i x_i}{p_j} \right)^{(1-\alpha) \omega_{ji}}$$

Taking logarithms and using $\sum_j \omega_{ji} = 1$:

$$\begin{aligned} \ln x_i &= \underbrace{\alpha \ln z_i}_{\equiv \varepsilon_i} + \underbrace{\alpha \ln \alpha + (1-\alpha) \ln(1-\alpha)}_{\equiv C} + \alpha \ln p_i + \alpha \ln x_i - \alpha \ln w \\ &\quad + (1-\alpha) \sum_{j=1}^n \omega_{ji} \ln \omega_{ji} + (1-\alpha) \ln p_i + (1-\alpha) \ln x_i - (1-\alpha) \sum_{j=1}^n \omega_{ji} \ln p_j \end{aligned}$$

where $H = -\sum_j \omega_{ji} \ln \omega_{ji}$ is the [Shannon \(1948\)](#) entropy of the system.³¹ Collecting terms and dividing by α :

$$\ln w = \varepsilon_i + C/\alpha + \ln p_i + \frac{(1-\alpha)}{\alpha} \sum_{j=1}^n \omega_{ji} \ln \omega_{ji} - \frac{(1-\alpha)}{\alpha} \sum_{j=1}^n \omega_{ji} \ln p_j$$

Premultiply with the i -th element of the influence vector $\mathbf{v} = \frac{\alpha}{Y} [\mathbf{I} - (1-\alpha)\mathbf{\Omega}]^{-1} \mathbf{b}$ and sum over all firms i :

$$\ln w \equiv y = \mu + \mathbf{v}'\varepsilon$$

where we have used $\sum_{i=1}^n v_i = 1$ and μ is a mean shifter for the output of the economy, independent of the vector of shocks ε : $\mu = C/\alpha + \sum_{i=1}^n v_i \ln p_i + \frac{(1-\alpha)}{\alpha} \sum_{i=1}^n \sum_{j=1}^n v_i \omega_{ji} \ln \omega_{ji} - \frac{(1-\alpha)}{\alpha} \sum_{i=1}^n \sum_{j=1}^n v_i \omega_{ji} \ln p_j$.

Firm entry, exit and cutoff productivities Firms pay a fixed investment cost $f_e > 0$, before entering the market, paid in terms of labor and all inputs. After payment, this cost is sunk and it entitles the firm to a productivity draw ϕ_i and a blueprint for production of a particular product (i.e. its set of ω_{ji}). ϕ_i is drawn from a Pareto distributed cumulative distribution function $G(\phi) \equiv 1 - \left(\frac{\phi_{min}}{\phi}\right)^\theta$ with support $[\phi_{min}, \infty)$, where $\phi \geq \phi_{min} > 0$ and we require $\theta > 1$ for the distribution to have a finite mean. θ is the shape parameter of the distribution, common across all firms and is inversely related to the variance of the distribution. The blueprint is contingent, in that it stipulates how to produce a product in-house if it is not available on the market as an input. After observing its productivity and blueprint for production, firms decide whether to start producing at their technologies Γ_i or exit immediately without producing. The existence of fixed costs of production dictates that there is a cutoff productivity ϕ^* , below which firms cannot make positive profits and it is endogenously determined by the zero profit condition for the marginal firm:

$$\phi_i^{*\eta-1} = \frac{\eta \left(c_i f - \frac{1-\alpha}{\eta} \sum_{j=1}^n \omega_{ij} r_j \right) \left(\frac{c_i}{\rho} \right)^{\eta-1}}{Y P^{\eta-1}}$$

Note that cutoff productivities are i -specific in our setup, due to the input requirements of downstream firms and the obtained blueprint for production by i . Also note that we take as given the productivities of other firms, similar to taking as given prices of inputs.

B Model with heterogeneous labor shares

The baseline model assumes a common labor share across all firms in the economy, in line with previous input-output models at the sector level as in [Long and Plosser \(1983\)](#) and [Acemoglu](#)

³¹ Shannon entropy represents the average amount of information contained in an event and ranges between 0 and $\ln n$. A higher entropy indicates more potential states of the system. If $\omega_{ji} = 1$ for all i , then all firms only use 1 firm as input and the economy is represented by a circle graph. As $\omega_{ji} \rightarrow 0$ for all i and j , the number of possible states increases. Interestingly in our model, H shows that the realized production recipes determine the possible outcome space of the economy, dictated by the phase space of the system.

et al. (2012). Here, we derive a simple extension with heterogeneous labor shares and re-estimate the model, allowing for this extra source of heterogeneity. Output of firm i is now given by

$$x_i = (z_i l_i)^{\alpha_i} \prod_{j=1}^n x_{ji}^{(1-\alpha_i)\omega_{ji}} \quad (19)$$

where α_i denotes the labor share for firm i . Revenues can then be written as

$$r_i = \sum_{j=1}^n (1 - \alpha_j) \omega_{ij} r_j + p_i q_i \quad (20)$$

where downstream revenues naturally depend on the intermediate input share of downstream buyers j . The impact of this heterogeneity on aggregate fluctuations is *a priori* not clear, as the diffusion of shocks depends on the net impact of all downstream buyers j .

We then estimate the model under these relaxed constraints. We first obtain firm-level labor shares from the annual accounts in 2012 as $\alpha_i = \frac{\text{labor cost}_i}{\text{inputs}_i + \text{labor cost}_i}$. Figure 5 shows the distribution of labor shares for firms used in the estimation procedure, confirming our earlier estimate of a common labor share around $\alpha = 0.2$.

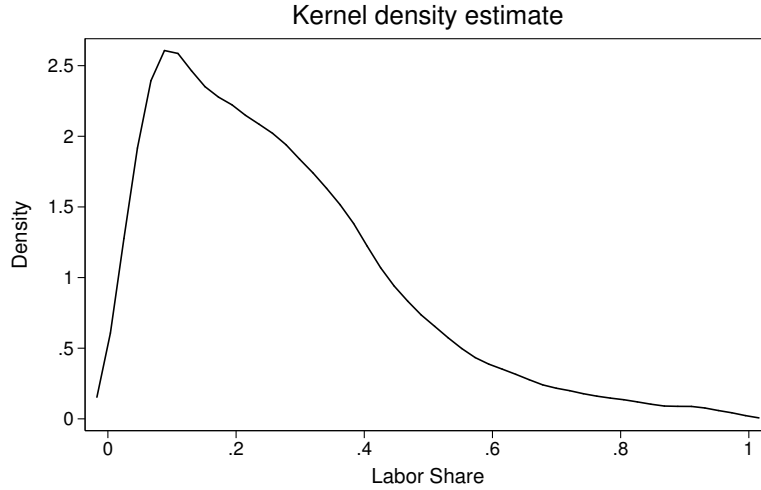


Figure 5: Labor share distribution across firms (2012).

We re-estimate the model using (20). Results for aggregate volatility are given in Table 9. The baseline model generates an aggregate volatility of 1.11%. Accounting for heterogeneous labor shares turns out to have a negligible impact on the model's prediction of aggregate volatility, with prediction now being 1.12% for the baseline specification.

$\hat{\sigma}_{\Delta y}$ (%)	Heterogeneous labor
$d\hat{\varepsilon}_{it} = g_{it}$	1.12
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	1.12
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	1.11
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	1.10

Notes: Volatility predictions $\hat{\sigma}_{\Delta y}$ estimated over the period 2002-2012. \bar{g}_t , \bar{g}_{I_2t} and \bar{g}_{I_4t} represent the mean weighted growth rates of the economy, 2-digit and 4-digit industries respectively. $\sigma_{\Delta y}$ is the observed standard deviation of GDP growth (private sector excluding the financial sector) over the same period.

Table 9: Aggregate volatility (heterogeneous labor shares).

C Model with capital goods

One drawback of the NBB B2B dataset is that transactions can be either intermediate inputs or capital goods (e.g. machinery, construction). We cannot directly disentangle these goods, as we only observe sales values between enterprises across all economic activities. This generates a bias in the estimation of the Leontief inverse in our model as capital goods potentially skew the input shares matrix of the economy towards this type of goods.

Here, we develop a simple extension to the model, allowing for the factor capital in production. Output of firm i is now given by

$$x_i = (z_i l_i)^\alpha k_i^\beta \prod_{j=1}^n x_{ji}^{(1-\alpha-\beta)\omega_{ji}} \quad (21)$$

where k_i are capital inputs for firm i with capital share β . Note that capital goods can be sold as output and used as input, but it is not part of the input-output matrix $\mathbf{\Omega}$. Capital goods become part of the final demand residual as “investment”, consistent with the typical National Accounting Identity. This implies that the baseline model over-estimates the network effect in our data.

We calculate the intermediate inputs ratio for sector I in total transactions from the sectoral Input-Output matrix for Belgium at the NACE 2-digit level in 2012 as $\frac{\text{total inputs}_I}{\text{total inputs}_I + \text{investment}_I}$. [Figure 6](#) shows the distribution of sectors and the corrected intermediate inputs ratio: 41 out of 70 sectors are unaffected by this correction factor. The distribution at the firm level is very similar (not reported). After correction, 14% of the total value of previous business transaction are redirected towards final demand. [Table 10](#) reports results for these corrected flows. The model prediction is now 1.14%, and results are very similar to the baseline specification in the paper.

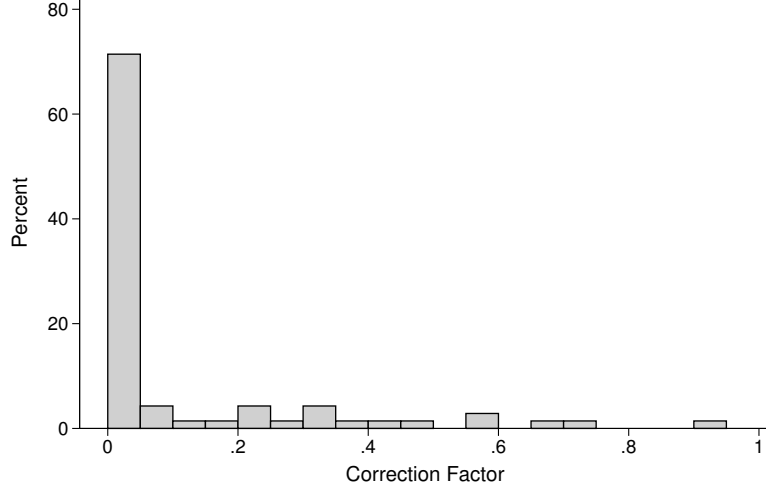


Figure 6: Corrected intermediate inputs ratio (2012).

$\hat{\sigma}_{\Delta y}$ (%)	Capital goods correction
$d\hat{\varepsilon}_{it} = g_{it}$	1.14
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	1.14
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	0.97
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	0.91

Notes: Volatility predictions $\hat{\sigma}_{\Delta y}$ estimated over the period 2002-2012. \bar{g}_t , \bar{g}_{I_2t} and \bar{g}_{I_4t} represent the mean weighted growth rates of the economy, 2-digit and 4-digit industries respectively. $\sigma_{\Delta y}$ is the observed standard deviation of GDP growth (private sector excluding the financial sector) over the same period.

Table 10: Aggregate volatility (corrected for capital goods).

D TFP estimation

TFP estimation follows the CompNet procedure (Lopez-Garcia et al. (2014)). In particular, real value added output Y_{it} of firm i in year t is assumed to follow a Cobb-Douglas production function, so that:

$$Y_{it} = A_{it} K_{it}^{\alpha_K} L_{it}^{\alpha_L} M_{it}^{\alpha_M}$$

where A_{it} is productivity of i at time t , K_{it} , L_{it} and M_{it} are capital, labor and intermediate inputs respectively. In logs:

$$y_{it} = \alpha_0 + \alpha_{kt}k_{it} + \alpha_{lt}l_{it} + \alpha_{mt}m_{it} + \varpi_{it} + u_{it} \quad (22)$$

where $\ln A_{it} = \alpha_0 + \varpi_{it} + u_{it}$, and α_0 represents mean efficiency across firms and time, while $\varpi_{it} + u_{it}$ denotes the firm-specific deviation from the mean; ϖ_{it} is the unobserved productivity component (known by the firm) and u_{it} is the i.i.d. error term.

As the firm optimizes its inputs given its own observed productivity, variable inputs are arguably correlated with ϖ_{it} . Estimating (22) using OLS then leads to biased estimated parameters of interest. Following Wooldridge (2009), we structurally estimate (22) using a control function approach. In particular, consider the following assumptions:

ASSUMPTION 1 (Monotonicity): $m_{it} = f(k_{it}, \varpi_{it})$, where $f(\cdot)$ is strictly monotonically increasing in ϖ_{it} . Then, unobserved productivity can be inverted out, so that: $\varpi_{it} = f^{-1}(k_{it}, m_{it})$.

ASSUMPTION 2 (Capital): $K_{it} = I_{it-1} + (1 - \delta)K_{it-1}$ where I_{it} is the investment of firm i at t . K_{it-1} is independent of current shocks.

ASSUMPTION 3 (Markov Process): $\varpi_{it} = E(\varpi_{it}|\varpi_{it-1}) + \nu_{it}$, so that productivity follows a first-order Markov process.

Then, we can write:

$$y_{it} = \alpha_L l_{it} + \alpha_M m_{it} + \psi(m_{it}, k_{it}) + u_{it} \quad (23)$$

where $\psi(m_{it}, k_{it}) = \alpha_0 + \alpha_K k_{it} + f^{-1}(k_{it}, m_{it})$ is a non-parametric function, proxied by a third-order polynomial in capital and intermediate inputs. We can then estimate (23) using GMM or an instrumental variables (IV) approach, leading to efficient estimation of the parameters of interest, with robust standard errors (and without relying on bootstrap methods). We follow Petrin and Levinsohn (2012), and use a pooled IV approach where we instrument current values of l_{it} with lagged values l_{it-1} . (23) is estimated within the 2-digit NACE industry level, i.e. firms within that industry share the same technologies and deviations from mean output given the same inputs are firm-specific. For investment/inputs, we use value added deflators at the NACE 2 digit level. Capital stock is deflated using gross fixed capital formation deflators. We include year fixed effects to purge general yearly trends and cluster standard errors at the i level.

E Identification of shocks

First, we derive the economy and individual growth rates from Section 3. The dynamic interpretation of the model is given by

$$\Delta \ln Y = \ln Y_t - \ln Y_{t-1} = \mu_t + \mathbf{v}'_t \varepsilon_t - (\mu_{t-1} + \mathbf{v}'_{t-1} \varepsilon_{t-1})$$

We assume $\mu_t = \mu$ (steady state) and $\mathbf{v}'_t = \mathbf{v}'$ (fixed network from *ex ante* draws). Then $\Delta \ln Y = \mathbf{v}'(\varepsilon_t - \varepsilon_{t-1})$, and so the variance of GDP growth is

$$Var(\Delta \ln Y) = Var(\mathbf{v}'(\varepsilon_t - \varepsilon_{t-1}))$$

$$\iff Var(\Delta \ln Y) = \mathbf{v}'^2 [Var(\varepsilon_t) + Var(\varepsilon_{t-1}) - 2Cov(\varepsilon_t, \varepsilon_{t-1})]$$

We assume that ε_t is independent over time, so that $\sigma_{\Delta y} = \sqrt{2 \sum_{i=1}^n v_i^2 \sigma_i^2}$ (this independence assumption is also reflected in the structural estimation of TFP). Then, $g_{it} = \varepsilon_{it} - \varepsilon_{it-1}$ results

in $Var(g_{it}) = 2\sigma_i^2$, plugging back in leads to $\sigma_{\Delta y} = \sqrt{\sum_{i=1}^n v_i^2 Var(g_{it})}$.

F Sensitivity analysis

$\hat{\sigma}_{\Delta y}$ (%)	Domestic Sales	Export Split
$d\hat{\varepsilon}_{it} = g_{it}$	1.10	1.13
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	1.10	1.13
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	0.94	0.96
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	0.87	0.88

Table 11: Final demand corrections (2012).

$\hat{\sigma}_{\Delta y}$ (%)	Imports
$d\hat{\varepsilon}_{it} = g_{it}$	1.20
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	1.20
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	0.95
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	0.89

Table 12: Accounting for imports.

$\hat{\sigma}_{\Delta y}$ (%)	Unweighted
$d\hat{\varepsilon}_{it} = g_{it}$	1.11
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	1.11
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	1.16
$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	1.20

Table 13: Unweighted demeaning procedure.

	$\hat{\sigma}_{\Delta y}$ (%)	Model	Network	F.D.	Divers.
$g_{it} \cong \Delta \ln(\text{value added}_{it}/\text{FTE}_{it})$	$d\hat{\varepsilon}_{it} = g_{it}$	1.43	0.86	0.47	0.17
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	1.40	0.84	0.46	0.17
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	1.18	0.78	0.35	0.17
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	1.10	0.74	0.32	0.17
$g_{it} \cong \Delta \ln(\text{value added}_{it})$	$d\hat{\varepsilon}_{it} = g_{it}$	1.46	0.86	0.48	0.16
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	1.44	0.85	0.47	0.16
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	1.19	0.78	0.36	0.16
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	1.11	0.74	0.32	0.16
$g_{it} \cong \Delta \ln(\text{turnover}_{it}/\text{FTE}_{it})$	$d\hat{\varepsilon}_{it} = g_{it}$	1.08	0.69	0.33	0.09
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	0.97	0.62	0.30	0.09
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	0.71	0.52	0.17	0.09
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	0.58	0.36	0.17	0.09
$g_{it} \cong \Delta \ln(\text{turnover}_{it})$	$d\hat{\varepsilon}_{it} = g_{it}$	1.05	0.67	0.34	0.07
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	0.85	0.53	0.28	0.07
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	0.64	0.46	0.17	0.07
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	0.53	0.34	0.17	0.07
$g_{it} \cong \Delta \ln(\text{FTE}_{it})$	$d\hat{\varepsilon}_{it} = g_{it}$	0.42	0.32	0.10	0.08
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_t$	0.43	0.32	0.10	0.08
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_2t}$	0.42	0.31	0.09	0.08
	$d\hat{\varepsilon}_{it} = g_{it} - \bar{g}_{I_4t}$	0.42	0.32	0.09	0.08

Notes: F.D. represents the Final demand benchmark, Divers. represents the Diversification benchmark.

Table 14: Alternative growth rate measures.

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