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NOTES AND COMMENTS

CASCADING FAILURES IN PRODUCTION NETWORKS

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This paper analyzes a general equilibrium economy featuring input-output connections, imperfect competition, and external economies of scale owing to entry and exit. The interaction of input-output networks with industry-level market structure affects the amplification of shocks and the pattern of diffusion in the model, generating cascades of firm entry and exit across the economy. In this model, sales provide a poor measure of the systemic importance of industries. Unlike the relevant notions of centrality in competitive constant-returns-to-scale models, systemic importance depends on the industry's role as both a supplier and a consumer of inputs, as well as the market structure of industries. A basic calibration of the model suggests that aggregate output is three times more volatile in response to labor productivity shocks when compared to a perfectly competitive model.

KEYWORDS: Production networks, entry and exit, external economies of scale.

1. INTRODUCTION

THIS PAPER STUDIES the role product creation and destruction play in amplifying and propagating shocks in production networks. I show that entry and exit provide a new mechanism for propagating shocks, and that the industrial organization of industries interacts with network interconnections to have important implications for the volatility of GDP. Entry and exit also change the pattern in which shocks travel from one industry to another as compared to a perfectly competitive model, creating additional feedback loops.

The idea that key consumers or key suppliers can have outsized effects on an economy, depending on the forward and backward linkages between industries, dates back at least to Hirschman (1958). However, results like those of Hulten (1978) suggest that, as long as the economy is efficient, the systemic importance of firms and industries can be approximated by their sales, even in the presence of linkages and complementarities. Therefore, size is a sufficient statistic for systemic importance. This paper breaks the equivalence between sales and systemic importance by introducing external economies of scale and imperfect competition,[1] showing that what may look like a diversified economy can in fact be very fragile. In such an economy, shocks to small industries can have large effects on output.

A memorable example of the interaction between entry-exit and input-output relationships was discussed by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) in the

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[1]By external economies of scale, I refer to the idea that an industry becomes more productive as it experiences more entry; in this paper, external economies arise due to either reductions in markups or increases in product variety.

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case of the U.S. automobile industry. In the fall of 2008, the president of Ford Motor Company lobbied for a government bailout of General Motors and Chrysler, but not of Ford itself. His reasoning was that the failures of General Motors and Chrysler would result in the failure of many of their suppliers. Since Ford relied on many of those same suppliers, he feared that the knock-on effect on its suppliers would put Ford itself out of business. This paper introduces a model that allows for just this type of knock-on effect, and highlights the importance of industrial organization in transmitting the shock. Exits in one industry make that industry less productive, due to lower product variety or higher markups, and these reductions in productivity can cause cascades of exits in upstream and downstream industries.

This paper connects the literature on the macroeconomic importance of networks, as typified by Acemoglu et al. (2012), to the literature on the macroeconomic importance of product entry and exit, for example, Bilbiie, Ghironi, and Melitz (2012). It also contributes to the literature seeking to derive aggregate fluctuations by combining microeconomic shocks with local interactions. Empirical work on this front includes Foerster, Sarte, and Watson (2011), Di Giovanni, Levchenko, and Méjean (2014), Atalay (2017), and Acemoglu, Akcigit, and Kerr (2016). Theoretical work, building on the canonical model of Long and Plosser (1983), can be divided into two categories: those with efficient equilibria and those without. The first category includes Carvalho (2010), Gabaix (2011), Acemoglu et al. (2012), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017). This paper belongs to the second category, which introduces frictions, like Jones (2011), Bigio and La'O (2016), and subsequent work by Liu (2017), and Grassi (2017).[2]

Separately, Broda and Weinstein (2010) and Dekle, Kawakami,Kiyotaki, and Miyagawa (2015) have shown, using U.S. and Japanese data, that net product creation comoves strongly with the business cycle, and is quantitatively large enough to significantly alter the costs of business cycles, even without input-output linkages. Work by Hopenhayn (1992), Bilbiie, Ghironi, and Melitz (2012), Clementi and Palazzo (2016), and Carvalho and Grassi (2015) has emphasized that extensive margin adjustment within industries can have macroeconomic consequences. This paper shows how these forces can be further amplified by the presence of interdependencies between industries. This paper predicts effects consistent with the empirical findings of Carvalho, Nirei, and Saito (2014), who showed that firm entry and exit have important spill-overs on other firms through supply and demand chains.

The structure of the paper is as follows. In Section 2, I set up the model and define its equilibrium. In Section 3, I characterize comparative statics of output with respect to value-added productivity shocks, and demonstrate the intuition by way of some examples. I also relate these results to the benchmark perfectly competitive model. Section 4 is a basic quantitative example of the model showing that the new forces in the model can have sizable effects on output volatility. I conclude with speculations for future work in Section 5.

2. MODEL

This section spells out the environment and defines the equilibrium. The model is static, and there are two types of agents: households and firms. Each firm belongs to an industry,

[2]A parallel literature in operations research, for example, Bimpikis, Fearing, and Tahbaz-Salehi (2014) and Bimpikis, Candogan, and Ehsani (2015), has also emphasized how disruptions in supply-chains can result in inefficient network formation and amplify aggregate risk.

and produces a single differentiated product variety. The number of industries is exogenous and equal to N. Since each firm produces only one product, as in Bilbiie, Ghironi, and Melitz (2012), one can think of firms as production-lines for specific products.

Household's Problem

The households in the model are homogeneous with a unit mass. The representative household maximizes consumption

$$U(c_1,\dots,c_N)=\frac{C}{\overline{C}}=\left(\sum_{k=1}^{N}\beta_k^{\frac{1}{\sigma}}\left(\frac{c_k}{\overline{y}_k}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where c_k represents composite consumption of varieties from industry k, $\sigma>0$ is the elasticity of substitution across industries, and the vector of weights $\beta\geq 0$ determines household tastes for goods and services from the different industries. Throughout the paper, a variable with an overline like $\overline{y}_k$ is a normalizing constant denoted in the same units as the variable it is dividing (in this case c_k). Here, $\overline{y}_k$ is the total physical production of industry i and $\overline{C}$ is real output at steady state.[3] The composite consumption good produced by industry k is given by

$$c_k=\left(M_k^{-\varphi_k}\int_0^{M_k}c(k,i)^{\frac{\varepsilon_k-1}{\varepsilon_k}}\,\mathrm{d}i\right)^{\frac{\varepsilon_k}{\varepsilon_k-1}},$$

where $c(k,i)$ is household consumption from firm i in industry k and $\varepsilon_k>1$ is the elasticity of substitution across firms within industry k. The within-industry elasticity of substitution ε_k is greater than the cross-industry elasticity of substitution σ for every industry k. When $\varepsilon_k\to\infty$, varieties are perfect substitutes and there is no product differentiation. The mass of product varieties offered by industry k is M_k. Finally, the term $M_k^{-\varphi_k}$ controls the returns to product variety or the love-of-variety effect. If we let $\varphi_k=0$, we recover the Dixit and Stiglitz (1977) demand system, whereas $\varphi_k=1/\varepsilon_k$ removes love of variety.

The household's budget constraint is

$$\sum_{k=1}^{N}\int_0^{M_k}p(k,i)c(k,i)\,\mathrm{d}i=wl+\sum_{k=1}^{N}\int_0^{M_k}\pi(k,i)\,\mathrm{d}i,$$

where $p(k,i)$ is the price and $\pi(k,i)$ is the profit of firm i in industry k. The wage is w and labor is inelastically supplied at $l=\overline{l}$. For the rest of the paper, labor is the numeraire and the supply of labor is fixed at $\overline{l}=1$.

Firms' Problem

Firm i in industry k produces a variety $y(k,i)$ using inputs from other firms in other industries, as well as labor. Each firm must hire labor for two purposes: production labor $l(k,i)$ and non-production labor f_k. The profits are given by

$$\pi(k,i)=p(k,i)y(k,i)-\sum_{l=1}^{N}\int_0^{M_l}p(l,j)x(k,i,h,j)\,\mathrm{d}j-wl(k,i)-w\frac{f_k}{z_k^m},$$

[3]Steady state here means the model evaluated at the point where the productivity shocks, introduced later, are set equal to 1.

where inputs from firm j in industry h used by firm i in industry k are $x(k, i, h, j)$.

In order for each firm to operate and regardless of how many units it produces, each firm must hire f_k/z_k^m effective units of labor. We can think of this as representing non-production management workers whose productivity may be subject to industry-level shocks z_k^m. Since equilibrium features free entry and zero profits, we may also think of the management workers as entrepreneurs, since they are the residual claimants of the firm's operating surplus. Free entry ensures that workers are indifferent between being entrepreneurs or production workers (in other words, both labor types are paid the same wage).

The firm's production function (once non-production overhead labor has been hired) is constant-returns-to-scale

$$\frac{y(k,i)}{\overline{y}_k} = \left(\alpha_k^{\frac{1}{\sigma}}\left(z_k^w \frac{l(k,i)}{\overline{l}}\right)^{\frac{\sigma-1}{\sigma}} + \sum_{h=1}^{N} \omega_{k,h}^{\frac{1}{\sigma}}\left(\frac{x(k,i,h)}{\overline{y}_k}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

Here, $\sigma > 0$ is again the elasticity of substitution among inputs, and $\omega_{k,h}$ is the share parameter for how intensively firms in industry k use composite inputs from industry h. The parameters $\omega_{k,h}$ determine the network-structure of this economy. The parameter $\alpha_k > 0$ gives the intensity with which firms in industry k use labor (net of overhead labor costs). Finally, z_k^w are productivity shocks for production workers. Value-added for each firm is given by the sum of payments to its production and management workers $w(l(k, i) + f_k)$. The productivity parameters z_k^w and z_k^m are key, since they are the source of shocks in the model.

The composite intermediate input from industry h used by firm i in industry k is

$$x(k,i,h) = \left(M_h^{-\varphi_h} \int_0^{M_h} x(k,i,h,j)^{\frac{\varepsilon_h-1}{\varepsilon_h}}\,\mathrm{d}i\right)^{\frac{\varepsilon_h}{\varepsilon_h-1}},$$

where ε_h is the elasticity of substitution across different varieties within industry h. Note that the elasticities of substitution are the same for all users of an industry's output.

Equilibrium

To define the equilibrium, first define a reduced-form markup function $\mu_k : \mathbb{R} \to \mathbb{R}$ to be a mapping of the mass of entrants M_k to markups for industry k. With some abuse of notation, define μ to be the diagonal matrix whose kth diagonal element is equal to μ_k. I assume that firms set their prices to equal the markup function times their marginal cost. Now we can define general equilibrium as follows.

DEFINITION 1: A general equilibrium is a collection of prices $p(i, k)$, wage w, and inputs $x(i, k, l, j)$, outputs $y(i, k)$, consumptions $c(i, k)$, production labor $l(i, k)$, and masses M_i such that, for vectors of productivity shocks and markup function $\mu(M)$:

(i) each firm minimizes its costs subject to demand for its goods given the markup function,

(ii) the representative household chooses consumption to maximize utility subject to its budget set,

(iii) markets for each good and labor clear,[4]

[4]Labor market clearing requires total production and non-production labor demand to equal the endowment: $\sum_{k=1}^{N}\int_0^{M_k}(l(i,k)+f_k)\,\mathrm{d}i = \overline{l}$.

(iv) profits are zero.

Conditions (ii)–(iv) are standard, but condition (i) merits further discussion. Condition (i) requires that the firms minimize their costs given the demand they face, and characterizes their choice over prices with a reduced-form markup function. The specification of the markup function μ, which depends on market structure, will play an important role in our analysis. However, for now, we can leave the market structure relatively general.[5]

Our goal in this paper is to derive comparative statics of output with respect to the productivity parameters z_k^w and z_k^m. To this end, we begin by characterizing firm profits which, through the zero profit condition, will determine how net entry responds to shocks.

Characterizing Firm Profits

We start by recalling some key statistics from the literature on monopolistic competition.

DEFINITION 2—Price Indices: The price index p_k and the total output y_k of industry k are given by

$$p_k = \left(M_k^{-\varphi_k \varepsilon_k} \int_0^{M_k} p(k,i)^{1-\varepsilon_k}\,\mathrm{d}i\right)^{\frac{1}{1-\varepsilon_k}},$$

$$y_k = \left(M_k^{-\varphi_k} \int_0^{M_k} y(k,i)^{\frac{\varepsilon_k-1}{\varepsilon_k}}\,\mathrm{d}i\right)^{\frac{\varepsilon_k}{\varepsilon_k-1}}.$$

The consumer price index P_c, which represents the price level for the household, is given by

$$P_c\overline{C} = \left(\sum_k^N \beta_k^\sigma (p_k \overline{y}_k)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{1}$$

These are the "ideal" price and quantity indices. The reason we do not simply average prices to get a price index or add outputs to get total output is because, even within each industry, each firm is producing a slightly different product.

Since marginal costs are constant, we can write firm i's profits as

$$\pi(k,i) = \frac{1}{M_k}\left(1 - \frac{1}{\mu_k}\right) p_k y_k - w\frac{f_k}{z_k^m}. \tag{2}$$

Combining this with the zero profit condition implies

$$M_k = z_k^m \left(1 - \frac{1}{\mu_k}\right)\frac{p_k y_k}{w f_k}. \tag{3}$$

Hence, ceteris paribus, the equilibrium mass of entrants is increasing in the sales share, productivity of managers, and markups in industry k, but decreasing in overhead labor

[5]Dixit–Stiglitz, Cournot, and oligopolistic competition are special cases, as discussed in Appendix C of the Supplemental Material (Baqaee (2018)).

costs of operation wf_k. Collectively, $(1-1/\mu_k)p_k y_k$ is industry k's gross operating surplus, which is divided amongst the firms in industry k and eventually paid out to the managers (by free entry).

In order to characterize firm profits, it helps to establish the following notation. Let Ω be the $N \times N$ matrix whose ijth element is equal to ω_{ij}. Let M be the $N \times 1$ vector whose ith element is M_i, and let $\tilde{M}$ be the $N \times N$ diagonal matrix whose ith diagonal element is equal to $M_i^{\frac{1-\varphi_i\varepsilon_i}{\varepsilon_i-1}}$. Note that when $\varphi_k = 1/\varepsilon_k$, the case with no return to product variety, this term is equal to 1. Let $\circ$ denote the element-wise or Hadamard product. Let e_i denote the ith standard basis vector, and $\operatorname{diag} : \mathbb{R}^N \to \mathbb{R}^{N^2}$ be the operator that maps a vector to a diagonal matrix, formally, $\operatorname{diag}(x) = \sum_i x_i e_i e_i'$. To simplify notation, when a variable appears without a subscript, this represents the matrix of all such variables. The identity matrix is I.

The following two centrality measures will help us characterize the distribution of sales, and hence profits. Lemma 1, below, motivates why we are interested in these centrality measures.

DEFINITION 3—Centrality Measures: The supplier centrality $\tilde{\beta}$ and the supply-side influence matrix Ψ_s are

$$\tilde{\beta}' = \beta'\Psi_s, \qquad \Psi_s = \left(I - \tilde{M}^{\sigma-1}\mu^{-\sigma}\Omega\right)^{-1}. \tag{4}$$

The consumer centrality $\tilde{\alpha}$ and the demand-side influence matrix Ψ_d are

$$\tilde{\alpha} = \Psi_d\left(\alpha \circ \left(z^w\right)^{\sigma-1}\right), \qquad \Psi_d = \left(I - \mu^{1-\sigma}\tilde{M}^{\sigma-1}\Omega\right)^{-1}\mu^{1-\sigma}\tilde{M}^{\sigma-1}. \tag{5}$$

We can think of $\tilde{\beta}$ as the *network-adjusted* consumption share of the industries. The kth element of $\tilde{\beta}$ captures demand from the household that reaches industry k, whether directly or indirectly through other industries who use k's products. The supplier centrality only depends on the industry's role as a supplier, hence we can think of it as capturing the importance of an industry as a supplier to the household.[6] It is the natural generalization of the *influence vector* defined by Acemoglu et al. (2012), which allows for imperfect competition, product differentiation, and a non-unitary elasticity of substitution.

On the other hand, the consumer centrality $\tilde{\alpha}$ is a *network-adjusted* measure of factor use. It captures how each industry uses the underlying factor (in this case, production workers) both directly and indirectly through its use of inputs from other industries. The consumer centrality only depends on the industry's demand for inputs, hence we can think of it as capturing the importance of an industry as a consumer of inputs. It is also the natural generalization of the *network-adjusted labor intensity* defined by Baqaee (2015).

The following lemma, which will help us characterize profits, establishes why $\tilde{\beta}$ and $\tilde{\alpha}$ are interesting objects in equilibrium.

LEMMA 1: *In equilibrium*,

$$\tilde{\beta}_i = \left(\frac{p_i^\sigma y_i/\overline{y}_i^{1-\sigma}}{P_c^\sigma C/\overline{C}^{1-\sigma}}\right) \quad \textit{and} \quad \tilde{\alpha}_i = \left(\frac{p_i\overline{y}_i}{w\overline{l}}\right)^{1-\sigma}.$$

The normalizing constants ensure that the centrality measures are unitless.

[6]Propositions 4 and 5 in the Supplemental Material (Baqaee (2018)) formalize this intuition.

An immediate consequence of Lemma 1 is that $\tilde{\beta}_k\tilde{\alpha}_k$ is proportional to sales. When the elasticity of substitution σ is not 1, the sales share of a given industry depends both on its supply-chain (captured by $\tilde{\alpha}$) and on its demand-chain (captured by $\tilde{\beta}$). Note that in the Cobb–Douglas limit, where $\sigma = 1$, only the supplier centrality $\tilde{\beta}_i$ is needed, and consumer centrality is equal to unity. This is because in the Cobb–Douglas case, relative sales shares do not depend on relative prices.

Putting Lemma 1 and equation (2) together allows us to give the following characterization of active firms' profit functions in terms of supplier and consumer centrality measures.

LEMMA 2—Characterization of Profits: *The profit of firm i in industry k is equal to*

$$\pi(k,i) = \underbrace{\left(1-\frac{1}{\mu_k}\right)\frac{1}{M_k}}_{\text{industry competition}} \times \underbrace{P_c C}_{\text{aggregate demand}} \times \underbrace{\left(\frac{w}{P_c}\right)^{1-\sigma}}_{\text{aggregate supply}} \times \underbrace{\tilde{\beta}_k}_{\text{supplier centrality}} \times \underbrace{\tilde{\alpha}_k}_{\text{consumer centrality}} - \underbrace{wf_k/z_k^m}_{\text{overhead cost}}.$$

The product of $\tilde{\beta}_k$ and $\tilde{\alpha}_k$ is proportional to industry k's share of sales, and $(1 - 1/\mu_k)\tilde{\beta}_k\tilde{\alpha}_k$ is proportional to industry k's gross operating surplus. The term $(1 - 1/\mu_k)$ is called the Lerner index, and is one of the standard measures of market power in industrial organization. The aggregate general equilibrium terms, P_cC and w/P_c, are economy-wide shifters of all industries' profits. Dividing through by M_k converts an industry's gross profits into firm-level gross profit. Finally, we arrive at a firm's net profits by subtracting the overhead costs of operation from gross profit. Lemma 2 implies that the profits of a firm are determined by a few intuitive key statistics. Crucially, the supplier and consumer centrality are nonlinear functions of markups μ and the mass of entrants M, and this means that the mass of entrants in different industries will generally respond to changes in markups or entrants in other industries.

Lemma 2 also helps us think through how shocks diffuse in this model. Normalize $w = 1$, and observe that $C = 1/P_c$. This follows from the fact that, with free entry, profits are zero in equilibrium and real output is equal to the real wage. Then we can write the sales of industry k as

$$\log(p_k y_k) = \log(\tilde{\beta}_k) + \log(\tilde{\alpha}_k) + (1-\sigma)\log(C). \tag{6}$$

Hence the sales of k can only be affected by a shock to industry i through either changes in $\tilde{\beta}$, $\tilde{\alpha}$, or through changes in output. The first two channels are *network* propagation, while the final general equilibrium channel is standard to all multisector models. In the next section, we investigate how shocks to productivity perturb $\tilde{\beta}$ and $\tilde{\alpha}$, and through these, how output responds to these shocks. As a consequence of equation (6), this also helps us understand how shocks propagate from industry to industry through the network.

3. THE IMPACT OF PRODUCTIVITY SHOCKS

In this section, we study how output and entry respond to productivity shocks z_k^w and z_k^m. We also touch on how shocks propagate from industry to industry, and contrast this with the perfectly competitive benchmark. This is where we take advantage of the characterization of firm profits in Lemma 2. For simplicity of notation and without loss of generality, I set all normalizing constants (variables with overlines) equal to 1, with the understanding that this is equivalent to absorbing them into the CES share parameters.

3.1. *Overhead-Labor Productivity Shocks*

We start by analyzing the impact of manager productivity shocks on output. There are several reasons to study such shocks. First, as equation (3) makes clear, this comparative static can be thought of as a change in the exogenous costs of starting a new business: ceteris paribus, a one percent increase in z_k^m causes a one percent increase in M_k. However, in equilibrium, this change in the number of entrants will set off changes to relative prices and markups, which change the distribution of sales and profits across industries, which in turn affect the mass of entrants, and so on. Thus, entry decisions will cascade through the network. Therefore, comparative statics in z_k^m are a clean way to see how the extensive margin of entry operates in this model, since z_k^m have no direct effect on relative prices.

Second, we may think that entry costs for starting new businesses can change over time or be affected by policy, so these comparative statics are of independent interest, since an increase in the productivity of managers is analogous to a reduction in entry barriers for an industry. Furthermore, empirical work by Broda and Weinstein (2010) showed that there is substantial variation in net product creation and firm entry over the course of the business cycle and net entry is highly procyclical. It is plausible that at least some of this turnover may be due to time-series variation in the cost of creating new products or firms, for instance, due to cyclical variation in the cost of credit required to finance new entry.

The third and final reason to start with these comparative statics is because understanding the comparative statics with respect to z_k^m is necessary for understanding the impact of shocks to worker productivity z_k^w as well. This is because shocks to production labor can trigger extensive margin changes, and these extensive margin changes have a similar impact on output as when they are caused by a shock to z_k^m.

PROPOSITION 1—Overhead-Labor Shocks: *Let*

$$\tilde{D}_{kk} = \frac{\partial \log(1 - 1/\mu_k)}{\partial \log(M_k)}, \qquad \Psi_1 = \frac{\partial \tilde{\beta}}{\partial \log(M)}, \qquad \Psi_2 = \frac{\partial \tilde{\alpha}}{\partial \log(M)}, \tag{7}$$

then denote

$$\begin{aligned} \Lambda &= \frac{\partial \log(1 - 1/\mu_k)}{\partial \log(M)} + \frac{\partial \log(\tilde{\beta})}{\partial \log(M)} + \frac{\partial \log(\tilde{\alpha})}{\partial \log(M)} \\ &= \tilde{D} + \operatorname{diag}(\tilde{\beta})^{-1}\Psi_1 + \operatorname{diag}(\tilde{\alpha})^{-1}\Psi_2. \end{aligned} \tag{8}$$

The elasticity of output with respect to z_k^m, denoted by $\tilde{v}_k$, is given by

$$\mathrm{d}\log C/\mathrm{d}\log z_k^m = \frac{\mathrm{d}\log C}{\mathrm{d}\log p}\frac{\mathrm{d}\log p}{\mathrm{d}\log M}\frac{\mathrm{d}\log M}{\mathrm{d}\log z^m} = \tilde{v}_k = \beta'\Psi_2(I - \Lambda)^{-1}\,\mathrm{const}, \tag{9}$$

where const is a constant equal to $\frac{1}{\sigma - 1}\frac{1}{\beta'\tilde{\alpha} + \beta'\Psi_2(I-\Lambda)^{-1}\mathbf{1}}$.[7]

[7]The partial derivatives here treat μ, $\tilde{\beta}$, and $\tilde{\alpha}$ as functions of M defined in Definition 3. Throughout the paper, partial derivatives should be interpreted as perturbations holding fixed the mass of entrants. Total derivatives take into account the fact that the mass of entrants can change in equilibrium with entry.

The intuition for Proposition 1 is the following. A shock to overhead labor in industry k affects the mass of products offered in industry k. This has a direct effect on the supplier and consumer centralities of all other industries since $\tilde{\beta}$ and $\tilde{\alpha}$ generically depend on the mass of firms through markups μ and product variety $\tilde{M}$. Since industry-level markups may depend on the number of entrants, there may also be a direct effect on the profit margins of the industry. These partial equilibrium effects are captured by the matrices $\tilde{D}$, Ψ_1, and Ψ_2 in (7). For the moment, take $\tilde{D}$, Ψ_1, and Ψ_2 as given; we characterize these matrices later.

According to equation (3), the mass of firms M in each industry is proportional to that industry's gross operating surplus $(1 - 1/\mu_k) \times \tilde{\beta}_k \times \tilde{\alpha}_k$. If Λ in equation (8) is nonzero, this means the gross operating surplus shares will respond to the shock, and hence, the equilibrium mass of entrants itself must respond to the initial entry into industry k. Of course, this is merely a partial equilibrium effect.[8] To deduce the full general equilibrium impact of the shock, we must take into account the fact that entry or exit will take place in response to the first round of entrants (and yet more rounds of entry can occur in response to these responses). In general, the total impact of the shock to the equilibrium mass of entrants can be written as

$$\frac{\mathrm{d}\log M}{\mathrm{d}\log z_k^m} = (I - \Lambda)^{-1}\left(e_k - \mathbf{1}\frac{\mathrm{d}\log C^{1-\sigma}}{\mathrm{d}\log z_k^m}\right), \tag{10}$$

where e_k is the kth standard basis vector, and $\mathbf{1}$ is a column vector of all ones, so that $(I - \Lambda)^{-1}e_k$ is the kth column of $(I - \Lambda)^{-1}$.[9] This formula is reminiscent of the impact of a shock in input-output models (see Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015)), due to the presence of $(I - \Lambda)^{-1}$, which has the form of a Leontief inverse. However, Λ is not an input-output matrix, but the elasticity of the profit shares (gross of entry costs) with respect to a change in the mass of entrants (in matrix form). As such, although it is shaped by the underlying input-output structure of the economy, it also depends strongly on other factors like the market structure of different industries and the elasticities of substitution.

Once we are in possession of the general equilibrium response of masses to the shock $\mathrm{d}\log M/\mathrm{d}\log z_k^m$ from equation (10), it is a simple matter to translate this into the impact on output using the chain rule. In this model, output is the reciprocal of the consumer price index in equation (1), meaning we can write

$$\frac{\mathrm{d}\log C}{\mathrm{d}\log z_k^m} = \frac{1}{\sigma - 1}\frac{1}{C^{1-\sigma}}\sum_i \beta_i \frac{\mathrm{d}p_i^{1-\sigma}}{\mathrm{d}\log z_k^m}. \tag{11}$$

Hence, the change in output from the shock z_k^m can be written as the change in the price index in terms of industry-level prices. However, by Lemma 1, we know that, in equilibrium, consumer centralities $\tilde{\alpha}_k$ are equal to $p_k^{1-\sigma}$. Hence, the change in industry-level

[8] From Lemma 2, the kth column of Λ is proportional to the change in the gross operating surplus of industries with respect to M_k, holding all other masses constant.

[9] Proposition 5 in the Supplemental Material (Baqaee (2018)) formally shows this relationship. As long as the largest eigenvalue of Λ is less than unity, $(I - \Lambda)^{-1}$ is equal to $\sum_{i=0}^{\infty} \Lambda^i$, where we can interpret each term in this series as capturing an additional round of entry in response to the previous round.

prices can be decomposed into

$$\frac{\mathrm{d}p_j^{1-\sigma}}{\mathrm{d}\log z_k^m} = \sum_i \frac{\mathrm{d}p_j^{1-\sigma}}{\mathrm{d}\log M_i}\frac{\mathrm{d}\log M_i}{\mathrm{d}\log z_k^m} = \sum_i \frac{\mathrm{d}\tilde{\alpha}_j}{\mathrm{d}\log M_i}\frac{\mathrm{d}\log M_i}{\mathrm{d}\log z_k^m}. \tag{12}$$

The first part of this summand is given by Ψ_2 and the second part of the summand is proportional to $(I - \Lambda)^{-1}e_k$. Putting all this together gives us Proposition 1, where $\mathrm{d}\log C/\mathrm{d}\log p$, $\mathrm{d}\log p/\mathrm{d}\log M$, and $\mathrm{d}\log M/\mathrm{d}\log z$ are proportional to β, Ψ_2, and $(I - \Lambda)^{-1}$, respectively.

3.2. *Production-Labor Productivity Shocks*

Now, we turn our attention to comparative statics in how shocks to the productivity of production workers z_k^w affect output.

PROPOSITION 2—Production-Labor Shocks: *In equilibrium*,

$$\begin{aligned}\frac{\mathrm{d}\log(C)}{\mathrm{d}\log(z_k^w)} &= \left(\tilde{v}' + \frac{1}{\sigma-1}\frac{\beta'\operatorname{diag}(\tilde{\alpha})}{\beta'\tilde{\alpha} + \beta'\Psi_2(I-\Lambda)^{-1}\mathbf{1}}\right)\frac{\partial\log(\tilde{\alpha})}{\partial\log(z_k^w)} \\ &= \left((\sigma-1)\tilde{v}' + \frac{\beta'\operatorname{diag}(\tilde{\alpha})}{\beta'\tilde{\alpha} + \beta'\Psi_2(I-\Lambda)^{-1}\mathbf{1}}\right)\operatorname{diag}(\tilde{\alpha})^{-1}\Psi_d e_k \alpha_k \left(z_k^w\right)^{\sigma-1}.\end{aligned} \tag{13}$$

The intuition we built for Proposition 1 proves useful in interpreting this formula. There are two reasons why output moves in response to a shock to the productivity of production labor. The first is that the productivity shock changes prices along the intensive margin, holding fixed the mass of entrants. The second reason is that the change in relative prices may cause a change in gross operating surplus, and hence change the equilibrium mass of entrants. More formally, at steady state, we can write this as

$$\begin{aligned}\frac{\mathrm{d}\log C}{\mathrm{d}\log z_k^w} &= \frac{1}{\sigma-1}\sum_i \beta_i \frac{\mathrm{d}p_i^{1-\sigma}}{\mathrm{d}\log z_k^w} \\ &= \frac{1}{\sigma-1}\sum_i \beta_i\left(\frac{\partial p_i^{1-\sigma}}{\partial\log z_k^w} + \frac{\partial p_i^{1-\sigma}}{\partial\log M}\frac{\mathrm{d}\log M}{\mathrm{d}\log z_k^w}\right) \\ &= \frac{1}{\sigma-1}\sum_i \beta_i\left(\frac{\partial\tilde{\alpha}_i}{\partial\log z_k^w} + \frac{\partial\tilde{\alpha}_i}{\partial\log M}\frac{\mathrm{d}\log M}{\mathrm{d}\log z_k^w}\right).\end{aligned} \tag{14}$$

The final equality follows from Lemma 1. The first set of summands, on the right-hand side, is the intensive margin effect, holding fixed masses, and the second set of summands is the extensive margin response to the intensive margin shock. I describe the two effects in turn.

First, a change in z_k^w changes industry-level prices, measured by $\tilde{\alpha}$, holding fixed the mass of entrants. This channel is qualitatively the same as the traditional input-output channel, although its magnitude is different due to the presence of markups. This change is given by

$$\frac{\partial p^{1-\sigma}}{\partial\log z_k^w} = \frac{\partial\tilde{\alpha}}{\partial\log z_k^w} = (\sigma-1)\Psi_d e_k \alpha_k\left(z_k^w\right)^{\sigma-1}. \tag{15}$$

The change in the price of industry i, holding fixed the mass of entrants, is given by how intensively industry k uses labor α_k and how intensively industry i uses inputs from k, measured by the ith element of $\Psi_d e_k$. Qualitatively, this effect is the same as the one in traditional input-output models. If the economy were perfectly competitive, then the ith element of $\Psi_d e_k$ would be the "total requirements" of k by i, or the ikth element of the Leontief inverse. Although qualitatively this is the traditional input-output channel, quantitatively, it has a different magnitude due to the presence of markups and product variety effects. We call this the *intensive margin* effect, since it operates via changing prices holding fixed the mass of entrants.

Now, let us turn to the second set of summands in equation (14). Conditional on the change in prices, if the elasticity of substitution $\sigma \neq 1$, then the gross operating surplus of industries will change in response to the change in relative prices. According to equation (3), this induces changes in the mass of varieties in different industries. These changes in the mass of entrants then behave very similarly to Proposition 1, which explains the appearance of $\tilde{v}$ in equation (13). Here $\tilde{v}$ is simply the expression on the right-hand side of (9), and not literally the output elasticity with respect to z_k^m, since those are being held constant. Hence, the intensive margin productivity shocks can trigger entry and exits across the network if they alter the profit share of industries, and the impact of these changes on output is similar to Proposition 1. We call this second effect the *extensive margin* effect, since it operates via an endogenous change in the mass of products.[10]

3.3. *Characterizing the Diffusion of Entry and Exit*

Both Propositions 1 and 2 require knowledge of the matrix Λ, which is the partial equilibrium elasticity of industry-level profit share with respect to the mass of entrants. Equation (7) shows that Λ can be written in terms of the markup elasticity $\tilde{D}$, and Ψ_1 and Ψ_2 which are the (partial equilibrium) elasticity of supplier and consumer centrality with respect to the mass of entrants. In this section, we characterize Ψ_1 and Ψ_2. For simplicity, I impose the Dixit–Stiglitz demand system: this implies that the markup μ_k is constant and equal to $\varepsilon_k/(\varepsilon_k - 1)$ so that $\tilde{D} = \mathbf{0}$. Furthermore, the love-of-variety parameter is $\varphi_k = 0$ for every k, so there are positive returns to product variety. Although I use the Dixit–Stiglitz demand system for simplicity, the proof and qualitative insights are very similar for alternative specifications of the markup function and product-variety effect.[11]

With Dixit–Stiglitz, the only source of external economies is returns to product variety (since markups are constant). In the symmetric equilibrium where $p(k, i)$ is the same for all i in industry k, we have the following relationship between the industry price-level and the individual prices:

$$p_k = M^{\frac{1}{1-\varepsilon_k}} p(k, i), \quad \text{hence} \quad \frac{\partial \log p_k}{\partial \log M_k} = \frac{1}{1 - \varepsilon_k}.$$

[10]Cobb–Douglas is a knife-edge case because the profit share does not respond to changes in relative prices. Therefore, in the Cobb–Douglas case, shocks to production labor do not change the equilibrium mass of entrants and so $\tilde{v}_k$ drops out of the expression in Proposition 2.

[11]The proofs in Supplemental Material Appendix B (Baqaee (2018)) are for the more general case without assuming Dixit–Stiglitz. See also Supplemental Material Appendix C (Baqaee (2018)) for how markups could be variable in this model.

In other words, the returns to entry are given by $1/(\varepsilon_k - 1)$, holding fixed firm-level prices. As an industry experiences more entry, that industry becomes more productive due to this effect.[12] The following lemma completes our characterization of the comparative statics.

LEMMA 3—Response of Consumer/Producer Centrality to Entry: *For the Dixit–Stiglitz demand system*,

$$\frac{\partial \tilde{\beta}}{\partial \log(M)} = \Psi_1 = (\sigma - 1)\big(\Psi_s' - I\big) \operatorname{diag}(\varepsilon - 1)^{-1} \operatorname{diag}(\tilde{\beta}), \tag{16}$$

$$\frac{\partial \tilde{\alpha}}{\partial \log(M)} = \Psi_2 = (\sigma - 1)\Psi_d \operatorname{diag}(\varepsilon - 1)^{-1} \mu^{\sigma-1} \tilde{M}^{1-\sigma} \operatorname{diag}(\tilde{\alpha}). \tag{17}$$

We begin by unpacking (16). It helps to write it element-wise:

$$\frac{\partial \tilde{\beta}_i}{\partial \log(M_k)} = \frac{\sigma - 1}{\varepsilon_k - 1}\big([\Psi_s]_{ki} - \mathbf{1}(i = k)\big)\tilde{\beta}_k.$$

Since supplier centrality depends only on the role of an industry as a supplier, we can think of $\tilde{\beta}_i$ as capturing total "demand" that reaches industry i, ultimately emanating from the household, taking into account double-marginalization and product-variety effects. The change in the demand reaching i, resulting from a change to the mass M_k of industry k, depends on how much demand $\tilde{\beta}_k$ reaches industry k, how intensively k buys inputs from i, measured by the kith element of Ψ_s, the percent change in the price of k from increased entry $1/(\varepsilon_k - 1)$, and the sensitivity of demand to relative price changes $(\sigma - 1)$. In terms of network interactions, it is worth noting that in order for (16) to be large, the shocked industry k must be a larger supplier (as measured by $\tilde{\beta}_k$) and it must be a large consumer (as measured by the kth row of Ψ_s'). In terms of elasticities, the magnitude of (16) is increasing in how sensitive cross-industry demand is to changes in relative prices $|\sigma - 1|$ and in external economies of scale from entry. In the Dixit–Stiglitz special case, the returns from entry are due purely to product-variety effects and measured by $1/(\varepsilon_k - 1)$, but in general, they can also arise from a reduction in markups.

The logic of (17) is analogous but the direction of the network effects is reversed. We can think of $\tilde{\alpha}_k$ as capturing the total "supply" of inputs, ultimately emanating from labor, that reaches industry i, as before, taking into account double-marginalization and product-variety effects. The change in consumer centrality of industry i with respect to a change in the mass of entrants in industry k then depends on the consumer centrality of industry k, and the importance of i as a consumer of inputs from industry k. Once again, these effects are mediated by the elasticity of substitution across industries $\sigma - 1$ and the returns to entry $1/(\varepsilon_k - 1)$. As with equation (16), equation (17) shows that both the connections going into industry k (measured by $\tilde{\alpha}_k$) and the connections going out of industry k (measured by the kth column of Ψ_d) matter. The fact that the expressions in (16) and (17) are multiplicative is what gives the model its complex diffusion patterns. We analyze diffusion patterns in the next section and contrast them with the competitive model.

[12]Intuitively, this effect tends to zero as products in industry k become perfect substitutes. The returns to product variety can be thought of as a reduced-form representation of a more complex model where increasing product variety improves the match between products and consumers. Anderson, De Palma, and Thisse (1992) provided a microfoundation for this via a discrete choice model where each consumer consumes only a single variety, but there exists a CES representative consumer for the population of consumers.

3.4. *Comparison With Competitive Model*

In this section, we compare the comparative statics results of Propositions 1 and 2, and the model's diffusion patterns, to the perfectly competitive benchmark (which is nested as a limiting case where products in every industry are perfect substitutes $\varepsilon_k \to \infty$). This limit is a generalization of the canonical Cobb–Douglas input-output model of Acemoglu et al. (2012), allowing for CES demand. Since the competitive model satisfies the conditions of Hulten (1978), we know that the elasticity of output with respect to z_k^w can be measured via the industry's wage bill relative to GDP:

$$\lim_{\varepsilon\to\infty} \frac{\mathrm{d}\log(C)}{\mathrm{d}\log(z_k^w)} = \frac{wl_k}{P_c C}. \tag{18}$$

Therefore, the observed wage bill as a share of GDP is a sufficient statistic regardless of the underlying network structure and the elasticity of substitution σ.[13]

Finally, we can also revisit the pattern of diffusion of shocks. In light of equation (6) and Lemma 3, we can deduce the following result about the diffusion of z_k^w for both the perfectly competitive and monopolistically competitive model with entry.

PROPOSITION 3—Diffusion of Shocks: *For the perfectly competitive limit*, *at the steady state where* $z^w = \mathbf{1}$,

$$\frac{\mathrm{d}\log(p_i y_i)}{\mathrm{d}\log(z_k^w)} = (\sigma - 1)\frac{[\Psi_d]_{ik}\alpha_k}{\tilde{\alpha}_i} - (1-\sigma)\frac{\mathrm{d}\log(P_c)}{\mathrm{d}\log(z_k^w)}. \tag{19}$$

For the Dixit–Stiglitz model with entry, *at the steady state where* $z^w = \mathbf{1}$,

$$\frac{\mathrm{d}\log(p_i y_i)}{\mathrm{d}\log(z_k^w)} = \sum_j^N (I-\Lambda)^{-1}_{ij}\left[(\sigma-1)\frac{[\Psi_d]_{jk}\alpha_k}{\tilde{\alpha}_j} - (1-\sigma)\frac{\mathrm{d}\log(P_c)}{\mathrm{d}\log(z_k^w)}\right]. \tag{20}$$

Equation (19) has a simple intuition: in the perfectly competitive model, the change in i's sales, when k is shocked, depends on the change in the price of i relative to the change in CPI and on the elasticity of substitution. The first term captures how the price of i changes in response to the shock, and the second term captures how the price index changes. The first term is the ratio of the intensity with which i uses the labor of industry k as a share of its total inputs. If industry i does not buy from industry k (either directly or indirectly), then it is not affected by the shock to k through the network. The second term is a general equilibrium term that hits all industries in the same way regardless of their

[13]In fact, we can go further, and write

$$C = \left(\sum_k \tilde{\beta}_k \alpha_k z_k^{\sigma-1}\right)^{\frac{1}{\sigma-1}},$$

where $\tilde{\beta}_k \alpha_k = wl_k/P_c C$, so that distribution of value-addeds by industry are global sufficient statistics. On the other hand, shocks to the productivity of overhead labor have no effect on output: $\lim_{\varepsilon\to\infty} \frac{\mathrm{d}\log(C)}{\mathrm{d}\log(z_k^m)} = 0$. In the perfectly competitive limit, the mass of firms in each industry approaches zero due to overhead costs, but since the mass of firms is irrelevant in the limit, this does not matter. Shocks to overhead labor also do not diffuse to other industries in the perfectly competitive limit. Proposition 6, in the Supplemental Material (Baqaee (2018)), extends this to the case when only some industries are perfectly competitive.

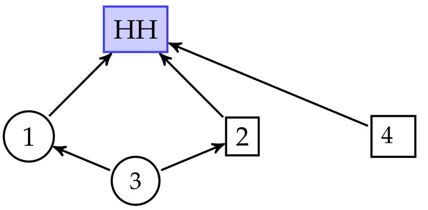


FIGURE 1.—Rectangular industries are competitive and circular ones are monopolistically competitive. The direction of arrows gives the flow of goods. Industries 1 and 2 share a common supplier with each other.

position in the network. In other words, in the competitive benchmark, the shocks only diffuse from suppliers to consumers and not the other way around. In fact, this result holds more generally: as long as M does not respond to shocks in equilibrium, the expression in equation (19) holds.

Propagation in the model with entry looks similar to the competitive model, but there is a qualitatively new channel for propagation from the presence of $(I - \Lambda)^{-1}$. When there is entry, both $\tilde{\beta}$ and $\tilde{\alpha}$ can change in response to net entry, and these changes bring about additional propagation. Without entry, $\Lambda = \mathbf{0}$, equation (20) collapses to equation (19). To bring out this intuition, we turn to a simple example, inspired by the anecdote about the automakers, showing how shocks diffuse in the model with entry.

The Role of $\sigma \neq 0$

Consider the economy depicted in Figure 1, which is inspired by the example about the U.S. automakers in the Introduction. To simplify the algebra, let industries 4 and 2 be perfectly competitive, but not 1 and 3. Let $\alpha_1 = \alpha_2 = 0$ and $\alpha_3 = \alpha_4 = 1$, so that industries 1 and 2 do not use production labor directly, and industries 3 and 4 only use labor. Let $\beta = (1/3, 1/3, 0, 1/3)$, so that the household derives the same utility from consuming goods from either industry 1, 2, or 4, but does not consume 3 directly. Apply Proposition 3 to get the following:

EXAMPLE 1: If $\sigma > 1$, then there exists ε_3^* such that $\mathrm{d}p_2y_2/\mathrm{d}z_1^m > 0$ if, and only if, $\varepsilon_3 < \varepsilon_3^*$.

In other words, the sales of 2 increase if entry barriers in 1 are lowered, as long as ε_3 is below some cutoff. The intuition for this can be seen via equation (6), and the changes in $\tilde{\beta}$ and $\tilde{\alpha}$ as more firms enter industry 1. As firms enter 1, the supplier centrality of the common supplier $\tilde{\beta}_3$ increases, which causes more entry into industry 3. More entry into 3 increases the consumer centrality $\tilde{\alpha}_2$, which in turn boosts the sales of 2. This positive effect on 2 must be weighed against increased competition in the product market, which reallocates demand away from 2 and towards 1. Which of these two effects dominates depends on the strength of the external economies of scale in the common supplier, here captured by ε_3. In the absence of adjustments in the mass of entrants, only the general equilibrium term in equation (6) would change.[14]

[14]When $\sigma < 1$, then reduced entry barriers for 1 would always positively affect 2. When $\sigma = 1$, the Cobb–Douglas limit, there would be no effect on 2, 3, and 4 in response to the shock to 1, because there would be no substitution effects.

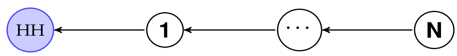


FIGURE 2.—Vertical economy, where the solid arrows represent the flow of goods. Only industry N directly employs production labor and only industry 1 sells to the household. The flow of profits and wages from firms to households has been suppressed in the diagram.

This example shows the importance of market structure for how shocks spread across the network. To go back to our motivating example, if we think of 1, 2, and 3 as GM, Ford, and Toyota, external economies of scale mean that Ford cannot expand to simply take up the market share of GM in the event of GM's failure, since some demand is stolen by Toyota. This shrinks the demand that reaches their common supplier, and if the upstream industry is very uncompetitive (proxied by a low ε_3), then the lost suppliers are more costly to Ford than the gain from fewer competitors. This example also shows how, via changing $\tilde{\beta}$ and $\tilde{\alpha}$, shocks can reverse direction as they travel the network. A consequence of Proposition 3 is that it is impossible to generate the diffusion pattern in Example 1 with productivity shocks in the competitive model.

Example With a Vertical Economy

Before moving on to a quantitative exercise, we can demonstrate the wide disparity between the output elasticities with respect to shocks and the sales shares using a simple example of a vertical economy, shown in Figure 2.[15] Here, only the final industry N uses production workers, and every other industry only uses overhead labor.

EXAMPLE 2: Applying Propositions 1 and 2 to the vertical economy, at the steady state,

$$\frac{d\log C}{d\log z_k^m} = \frac{1}{\varepsilon_k - 1}, \qquad \frac{d\log C}{d\log z_k^w} = \alpha_k = \begin{cases} 0, & k < N, \\ 1, & k = N, \end{cases}$$

while sales and wage bill as a share of GDP are

$$\frac{p_k y_k}{P_c C} = \prod_{i=1}^{k-1} \mu_i^{-1}, \qquad \frac{w l_k}{P_c C} = \prod_{i=1}^{k-1} \mu_i^{-1} \alpha_k. \tag{21}$$

In this simple example, $d\log C/d\log z_k^m$ captures purely the extensive margin effect, while $d\log C/d\log z_k^w$ captures purely the intensive margin effect, since in this example, without loss of generality, we can set $\sigma = 1$ (and intensive margin shocks do not set off extensive margin effects). We can compare the response of this vertical economy to shocks with its perfectly competitive model counterpart using equation (21), since, by Hulten (1978), the expenditure shares give the output elasticity with respect to productivity shocks in the competitive model. For this example, the output response to a change in entry costs z_N^m for industry N is $1/(\varepsilon_N - 1)$, which is just the return to product variety in that industry, and can be unboundedly large as $\varepsilon_N \to 1$. Intuitively, the less substitutable goods within an industry, the larger the impact of a change in the number of products available in that

[15] I borrow the term vertical economy, representing a production chain with only one input at each step, from Bigio and La'O (2016).

industry.[16] In particular, the output elasticity with respect to shocks does not depend on the length of the chain N. On the other hand, the sales of industry N can be arbitrarily close to zero, either if a downstream industry charges high markups or if the chain is sufficiently long. This simple example, in which shocks to an arbitrarily small industry can have an arbitrarily large effect on output, serves as an important qualification to Gabaix (2016), who wrote that "networks are a particular case of granularity rather than an alternative to it: if all firms had small sales, the central limit theorem would hold and idiosyncratic shocks would all wash out."[17]

4. QUANTITATIVE EXAMPLE

In this section, I perform a back-of-the-envelope calibration of the model, and investigate its implications for the volatility of output. I use the 2007 detailed input-output table from the BEA. The detailed input-output accounts are the finest level of disaggregation available for the United States, with each industry roughly corresponding to four-digit SIC definitions. A rough summary of the calibration strategy is as follows: for a given value of σ, I assume that all industry-level goods in the economy are measured in dollars. Therefore, the model is calibrated so that, without shocks, all industry-level prices are equal to 1.[18] The share parameters Ω and α are set so that the model matches the input-output expenditure shares and gross labor share, respectively, in steady state ($z^m = z^w = \mathbf{1}$). The gross profit of each industry is set equal to gross operating surplus net of a depreciation rate of 10% and taxes. Gross profits are the payments to the overhead factors of production. The share of gross profits is used to pin down the within-industry elasticity of substitution (assuming a Dixit–Stiglitz markup). I winsorize the distribution of industry-level elasticities of substitution on their left tail at the first percentile, so that four industries with $\varepsilon_k \in (1, 2)$ have their ε_k set to 2. This implies an average markup of 13%, and a maximum markup of 100%. To calibrate the household taste parameters β, I use final-use expenditure shares. The details of the calibration are in Supplemental Material Appendix D (Baqaee (2018)).

Using our comparative statics, we can then write

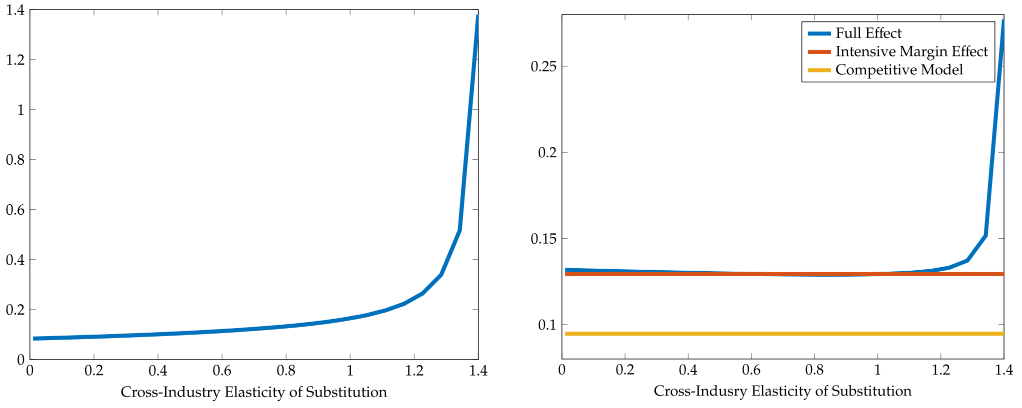
$$\log C \approx \log \overline{C} + \sum_{i=1}^{N} \frac{\mathrm{d}\log C}{\mathrm{d}\log z_k^w} \mathrm{d}\log z_k^w + \sum_{i=1}^{N} \frac{\mathrm{d}\log C}{\mathrm{d}\log z_k^m} \mathrm{d}\log z_k^m, \tag{22}$$

which we can compute using Propositions 1 and 2. Supposing that $\mathrm{d}\log z_k^w$ and $\mathrm{d}\log z_k^m$ are independent with mean zero and variance s_w^2 and s_m^2, then we can approximate the

[16]These results are consistent with the empirical findings of Barrot and Sauvagnat (2016), who found that negative shocks to a firm have a greater impact on its partners if the affected firm produces less substitutable goods.

[17]See also Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017) and Baqaee and Farhi (2017) for other reasons why the central limit theorem could be misleading, even when Hulten's theorem applies.

[18]Similarly, steady-state M is equal to 1 (since we are interested in elasticities, the units in which the mass of products is expressed is irrelevant).



(a) (b)

FIGURE 3.—(a) Standard deviation of log output as a fraction of the standard deviation of shocks to overhead labor as a function of σ. (b) Standard deviation of log output as a fraction of the standard deviation of shocks to production labor as a function of σ.

variance of output, in logs, as

$$\begin{aligned}\operatorname{Var}\bigl(\log(C/\overline{C})\bigr) &\approx s_w^2 \sum_{i=1}^{N}\left(\frac{\mathrm{d}\log C}{\mathrm{d}\log z_k^w}\right)^2 + s_m^2 \sum_{i=1}^{N}\left(\frac{\mathrm{d}\log C}{\mathrm{d}\log z_k^m}\right)^2 \\ &= s_w^2 \left\|\frac{\mathrm{d}\log C}{\mathrm{d}\log z^w}\right\|^2 + s_m^2 \left\|\frac{\mathrm{d}\log C}{\mathrm{d}\log z^m}\right\|^2.\end{aligned} \tag{23}$$

Hence, we can think of $\|\frac{\mathrm{d}\log C}{\mathrm{d}\log z^w}\|$ and $\|\frac{\mathrm{d}\log C}{\mathrm{d}\log z^w}\|$ as measures of the strength of diversification in the model. When there are only production-labor shocks ($s_m = 0$), then $\|\frac{\mathrm{d}\log C}{\mathrm{d}\log z^w}\|$ is the ratio of the standard deviation of log output to labor productivity shocks. On the other hand, when there are only overhead-labor shocks ($s_w = 0$), then $\|\frac{\mathrm{d}\log C}{\mathrm{d}\log z^m}\|$ is the ratio of the standard deviation of log output to overhead-labor shocks.

Figure 3(a) plots the diversification factor as a function of σ for overhead-labor shocks. We see that as σ increases, the amplification channels become much stronger. The intuition here is that, as the degree of substitutability increases, the market can reallocate more forcefully to take advantage of variation in the productivity of different industries, amplifying the spill-over effects from shocks. When an industry becomes more productive, its expenditure share rises, so it experiences further entry, which further increases its expenditure share and so on.

Figure 3(b) plots the diversification factor as a function of σ for production-labor shocks. I also plot the equivalent diversification factor for the perfectly competitive model as implied by Hulten (1978). As we might expect, Figure 3(b) is non-monotonic and achieves its lowest value in the region of $\sigma = 1$, since at this point the extensive margin response to productivity shocks is reduced.[19] The orange line is constant as a function of σ since wage bills as a share of GDP, at steady state, are equal to the observed wage

[19]When $\sigma = 1$, the profit shares do not respond to changes in z_k^w because markups are constant. Hence, the equilibrium mass of entrants remains constant in this case. This knife-edge depends on overhead costs being in units of labor rather than output.

bill ratios and do not depend on σ. The intensive margin effect comes from setting the second set of summands in (14) to zero and only taking the intensive margin effect, given by equation (15), into account.

One way to discipline the results further is to calibrate σ to match the observed volatility in net product creation given the standard deviation of industrial productivity shocks. Since our focus is on fluctuations rather than growth, we match on the standard deviation of net product entry rather than the mean of net product entry. Broda and Weinstein (2010) showed that the standard deviation in the aggregate net product creation rate, measured as the share of sales of new products minus the share of sales of destroyed products, is roughly 1% at annual frequency in the United States.[20] For some shocks $\mathrm{d}\log z$, the equivalent object in this model is the expenditure-share weighted average of the change in entrants

$$\sum_i \frac{p_i y_i}{\mathrm{GDP}} \mathrm{d}\log M = \sum_i \frac{p_i y_i}{\mathrm{GDP}} \frac{\mathrm{d}\log M}{\mathrm{d}\log z} \mathrm{d}\log z. \tag{24}$$

For a given standard deviation of the shocks $\mathrm{d}\log z$, the standard deviation of equation (24) gives us a number that we can compare to the 1% reported by Broda and Weinstein (2010). Acemoglu et al. (2012) calibrated the standard deviation of their industry-level value-added productivity shocks to be around 5% for this level of aggregation. In this model, value-added is split between overhead labor and production labor. To make the comparison with the competitive benchmark easier, I simply assume that $s_m = 0$ and all volatility is due to production labor $s_w = 5\%$. In this case, the volatility of net entry is equal to 1% when $\sigma = 1.4$. For this value of σ, the diversification factor $\| \frac{\mathrm{d}\log C}{\mathrm{d}\log z^w} \|$ is 0.28, relative to 0.095 for the perfectly competitive model, an almost tripling of volatility. Hence, the volatility of log GDP, for this benchmark calibration, is $0.28 \times 0.05 = 0.0135$, rather than the $0.095 \times 0.05 \approx 0.005$ that would be implied by a perfectly competitive model. To put these numbers into perspective, the standard deviation of the growth rate of aggregate TFP over the sample for which the productivity shocks were estimated was 0.0147.[21]

Overall, the numerical results suggest that the extensive margin becomes a more potent force when $\sigma > 1$. The model is obviously quite stylized for the sake of analytical tractability. In practice, cross-industry elasticities of substitution are likely to be heterogeneous, and depending on the level of aggregation and time horizon, they may be greater than or less than 1. At single- or two-digit levels of aggregation, empirical work by Atalay (2017) and Boehm, Flaaen, and Pandalai-Nayar (2015) suggests the elasticities of substitution are less than 1. The data set I use has 384 industries, and while some of these industries are certainly gross complements, others are likely gross substitutes.[22] In practice, cross-industry elasticities of substitution that are greater than and less than 1 are both likely to be empirically relevant depending on the context.

[20]Since the model is static, and varieties within each industry are symmetric, entry and exit matter only in how they affect the total mass of products in each industry. The model is silent on whether a given change in the total mass comes about as a result of a large or modest amount of both entry and exit.

[21]With only z_k^w shocks, the model-implied volatility of net product creation decreases to zero as we approach $\sigma = 1$ from either side, but there is no value of $\sigma < 1$ where the volatility of net creation is close to 1%. Even for $\sigma = 0.001$, the volatility of net product creation is only 0.3%. See Supplemental Material Appendix A (Baqaee (2018)) for more details. This finding depends on not allowing shocks to entry costs, or variable markups, both of which would result in more entry volatility with lower values of σ.

[22]As an example, "Newspaper publishers," "Periodical Publishers," and "Directory, mailing list, and other publishers," separate industries in this data set, are probably gross substitutes.

5. CONCLUSION

This paper highlights the importance of net entry in propagating and amplifying shocks in a production network. In an input-output model with external economies of scale, notions of systemic influence are decoupled from firm and industry size. The systemic importance of an industry depends on its role as a supplier and as a consumer of inputs, as well as on the market structure of the different industries. A simple calibration suggests that these forces significantly affect the volatility of output compared to a perfectly competitive model. These forces likely affect other moments of output, like its mean and skewness, too. Investigating these seems to be a valuable area for future work. Finally, for analytical tractability, I impose a uniform cross-industry elasticity of substitution; it would also be interesting to relax this assumption and allow for heterogeneous elasticities.

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