Bottom-up Markup Fluctuations *

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Abstract

We study markup cyclicality in a macroeconomic model with oligopolistic competition and firm-level shocks. Given changes in demand or productivity of individual firms, we characterize the co-movement of firm, sectoral, and economy-wide markups with sectoral and aggregate output. We then quantify the model's ability to reproduce salient features of the cyclical properties of markups in French firm level data, from the bottom (firm) level to the aggregate level. Our model with firm-level shocks only goes a long way in rationalizing various measures of markup cyclicality in the French data.

Keywords: Markup Cyclicality; Firm Dynamics; Aggregate Fluctuations

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Introduction

Ascertaining the cyclical properties of macro variables has played a crucial role in the development of modern business cycle models. Yet, while there is a broad consensus on moments concerning, for example, the behavior of consumption, investment or unemployment over the business cycle, there exists a lingering disagreement on the behavior of markups over the business cycle and the implied role of markups for cyclical fluctuations in the labor wedge. Indeed, the upshot of a large body of empirical work on this topic is a wide range of estimates of the correlation between markups and economic activity that vary in sign and magnitude. This basic impasse in the literature can be ascribed to the inherent difficulty of measuring markups at various levels of aggregation (product, firm, sector, and aggregate levels) and the differing empirical strategies pursued.

In this paper we study the cyclical properties of markups from the bottom (firm) level to the aggregate level, both theoretically and empirically based on French administrative data. We use a macroeconomic model with oligopolistic competition, flexible prices and firm-level granular shocks only as in Gabaix (2011). Shocks to individual firms result in changes in the size and market power of individual firms and, aggregating these firm-level outcomes, in sectoral and economy-wide output and markups. We abstract from exogenous aggregate shocks which, in our model, do not affect firm-level markups and would only impact aggregate markups via between-sector reallocation (see Nekarda and Ramey (2013) for an empirical analysis of how the cyclicality of aggregate markups in the U.S varies across types of shocks). We assess the model's ability with firm-level shocks only to reproduce salient features of the cyclical properties of markups in French firm level data at various levels of disaggregations.

To model in a tractable way the determination of markups in an economy featuring a large but finite number of sectors with a discrete number of single-product firms that internalize the impact of their choices on sectoral outcomes, we use the nested CES demand structure studied in Atkeson and Burstein (2008). A fixed number of firms per sector differ in a productivity/demand composite, which follows a discrete Markovian process as in Carvalho and Grassi (2019). We characterize, up to a first-order approximation, how markups, productivity, and output respond to firm-level shocks from the bottom (firm) level to the aggregate level. We provide simple expressions that show how the sign of markup cyclicality depends on the level of aggregation, market structure within and across industries, and the set of shocked firms. At the level of individual firms, markups are increasing in within-sector expenditure shares. Aggregating outcomes across firms, we show that sectoral output and markups co-move positively in response to shocks to large firms in the sector, while output and markups co-move negatively in response to shocks to small firms. The effect of such shocks on aggregate markup depends on the distribution of sector-level markup and sectoral expenditure shares. Given i.i.d. shocks across firms (such that large firms drive the cycle in each sector), we provide sufficient conditions for a positive correlation (over long samples) between sectoral markups and sectoral output, sectoral markups and aggregate output, as well as aggregate markup and aggregate output.

We compare theoretically the implications of our model with an alternative specification in which firm-level markups are heterogeneous but constant in response to shocks (so that sectoral and aggregate markups only change due to between-firm reallocation and not within-firm markup changes). We show that, while within-firm markup changes account for exactly half of sectoral markup fluctuations in the variable markup mode, changes in sectoral and aggregate markups can be larger or smaller than in the constant markup model depending on parameter values. We also compare the volatility of sectoral and aggregate output under the two model specifications. Given that the pass-through rates of firm-level shocks to prices are decreasing in firm size, we show (assuming a high elasticity of labor supply) that the volatility of sectoral and aggregate output is smaller in the model with variable markups than in the model with constant markups.

Our theoretical results reveal that the sign of markup cyclicality depends on the level of aggregation, the origins of the shock, and market structure within and across all industries. Moreover, the sign and magnitude of the analytic covariances may differ from those in finite samples. We thus conduct a quantitative exploration, calibrating our model based on French administrative data covering approximately 500,000 firms in 504 sectors in 2014. We measure markups in the data for the period 1994-2016 using the methodology introduced by De Loecker and Eeckhout (2017) and De Loecker et al. (2018).

We perform, on French data and on data generated by our calibrated model, a variety of reduced-form regressions on markup cyclicality that have been considered in the literature. By using a single dataset and a measure of markups that is consistent across different levels of aggregation, we can rule out differences in methodologies or datasets as the source of conflicting measures of markup cyclicality.

The French data supports the positive correlation between firm-level markup and firm-level market share in its sector, both in the cross-section and in the time series, that is built-in in our model. Moreover, the data corroborates a positive relation between sectoral markups and sectoral concentration, both across sectors and over the business cycle.

Our calibrated model can account for various measures of markup cyclicality in the French data on the extent to which markups are pro-cyclical, acyclical, or countercyclical. We first explore a notion of firm-level markup cyclicality recently proposed by Hong (2017). In particular we ask whether firm markups covary systematically with respect to sector-level output. As in Hong (2017) for major European countries, we find that this reduced form relation is “counter-cyclical” for the average firm in the French data, but can switch sign for large firms. We then proceed to evaluate notions of sector-level markup cyclicality. Following Nekarda and Ramey (2013), we ask whether sector markups comove with sector output over the business cycle. Like Nekarda and Ramey (2013) for the U.S., we find evidence for a positive systematic comovement between the two measures, or “pro-cyclicality”, in the French data. Finally, we follow the recent work of Bils et al. (2018) who investigate yet another notion of cyclicality: the extent to which sector level markups comove with aggregate output. According to this measure, we find pro-cyclical point estimates which, however, are not statistically different from zero (Bils et al. (2018) document a negative correlation for the U.S).

Our calibrated model can go a long way in accounting for (and reconciling) these wide range of observed correlations within a single dataset and measure of firm-level markups. Our theoretical results help us understand why our reduced form estimates of markup cyclicality should not be invariant across specification and levels of aggregation. We show, however, that the magnitude and sign of these measures of markup cyclicality can vary substantially across 20-year sequences of random firm-level shocks (and hence may differ from our asymptotic results).

We then turn our attention to quantifying the extent of aggregate markup and output fluctuations in response to idiosyncratic firm-level shocks. Much of the work on the granular origin of business cycles, such as Gabaix (2011) and Carvalho and Grassi (2019), abstracts from movements in markups that can partly offset the impact of own firm-level shocks or magnify the impact from shocks to competitors. We quantify to what extent aggregate fluctuations are reduced, for a given distribution of firm-level shocks, in the presence of oligopolistic competition. We also quantify, and compare with data, the size of movements in aggregate markups relative to movements in aggregate output induced by firm-level shocks. Our calibrated model with firm-level shocks only account for roughly 35% of the volatility of aggregate output in our data. The ratio of the markup volatility relative to output volatility is roughly 0.5 in the data and model (on average across 22-year samples of random shocks). Finally, our model implies that point estimate of the correlation between aggregate output and markups is too high relative to the data, but there is large variation in point estimate across different 20-year samples of random shocks. Moreover, adding aggregate shocks to account for the overall volatility of aggregate output reduces this correlation.

Related Papers

Our paper relates to various strands in the literature. TO BE COMPLETED.

- Variable Markups among Heterogeneous firms: Kimball (1995), Atkeson and Burstein (2008), Bilbiie et al. (2012), Klenow and Willis (2016), Amiti et al. (2019)

- Market Power in Macro. Cycles: Rotemberg and Woodford (1992), Grassi (2018), Mongey (2017), Jaimovich and Floetotto (2008)

  Trends: De Loecker and Eeckhout (2017), Edmond et al. (2018), Baqaee and Farhi (2017), Eggertsson et al. (2018), Berger et al. (2019)

- Markup Cyclicality in the Data: Bils (1987), Hall (1988), Hong (2017), Nekarda and Ramey (2013), Bils et al. (2018), Anderson et al. (2018), Stroebel and Vavra (2019)

- Micro-Origins of Aggregate Outcomes: Carvalho (2010), Foerster et al. (2011), Gabaix (2011), Acemoglu et al. (2012), Carvalho and Grassi (2019), Grassi (2018), Gaubert and Itskhoki (2018)

  Baqaee and Farhi (2017) provide a very general characterization of the impact of microeconomic shocks on aggregate productivity and output in a large class of models in which productivities and wedges (such as markups) are exogenous primitives. They use a simple chain to consider the response of wedges and productivities to other primitive fundamentals.[1]

  Our paper also relates to Grassi (2018), which studies the role of input-output linkages and endogenous markups à la Atkeson and Burstein (2008) in shaping comovement of sector-level variables. Grassi (2018) provides an analytical characterization of the impact of microeconomics shocks on aggregate output under an approximation of the deep parameters of the model. Our analytic results make use of a different approximation with respect to firm-level idiosyncratic shocks, similar to the one used in e.g. Gopinath et al. (2010), Burstein and Gopinath (2014), and Amiti et al. (2019) in the context of exchange rate shocks. As in Grassi (2018), we calibrate our model by matching each sector's Herfindahl index in the data with their model counterpart. Our quantitative results solve for the full non-linear equilibrium numerically.

1 Model

In this section we describe the model and characterize the equilibrium. We first describe the preferences, technology and market structure. Second, we aggregate our model from the firm-level to the sector and aggregate level.

[1] Baqaee and Farhi (2017) consider the case of variable markups in the model of Atkeson and Burstein (2008), as we do, restricted with one large firm and many infinitesimally small firms.

1.1 Preferences and technologies

Households have preferences at time t over consumption of a final good at time, Y_t, and labor, L_t, given by

$$U\left(Y_t, L_t\right) = \frac{1}{1-\eta} Y_t^{1-\eta} - \frac{\theta}{1+f^{-1}} L_t^{1+f^{-1}},$$

where $\eta \leq 1$ and $f \geq 0$ are respectively the constant relative risk aversion and the Frisch elasticity of labor supply. The final good aggregates output of N sectors according to a Constant-Elasticity-of-Substitution (CES) aggregator

$$Y_t = \left[\sum_{k=1}^{N} A_k^{\frac{1}{\sigma}} Y_{kt}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where Y_{kt} denotes sector k output, A_k is a demand shifter for sector k (which we assume is constant over time), and $\sigma \geq 1$ is the elasticity of substitution across sectors.

Each sector k is itself a CES aggregator of the output of N_k individual firms given by

$$Y_{kt} = \left[\sum_{i=1}^{N_k} A_{kit}^{\frac{1}{\varepsilon}} Y_{kit}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where Y_{kit} denotes output of firm i in sector k, A_{kit} is a firm-demand shock (independently drawn across firms), and ε is the elasticity of substitution between the output of firms in sector k (where $\sigma \leq \varepsilon$).

Firm i in sector k produces output according to the constant returns to scale technology

$$Y_{kit} = Z_{kit} L_{kit}, \tag{1}$$

where Z_{kit} denotes productivity of firm i in sector k (independently drawn across firms) and L_{kit} denotes its employment level. In Appendix A.4, we provide analytic results if we allow for decreasing returns to scale at the firm level.

Finally, we assume that workers are perfectly mobile across firms. For the labor market to clear, the sum of employment across all firms must equal aggregate labor, L_t.

1.2 Market structure and equilibrium

In this section, we describe the market structure, that is, how firms compete and interact. We then derive the equations that characterize the equilibrium.

Firm i in sector k, given prices of individual goods $\{P_{kit}\}$, faces demand

$$Y_{kit} = A_k A_{kit} \left(P_{kit}\right)^{-\varepsilon} \left(P_{kt}\right)^{\varepsilon-\sigma} P_t^{\sigma} Y_t,$$

where the sectoral price, P_{kt}, is

$$P_{kt} = \left[\sum_{i=1}^{N_k} A_{kit} P_{kit}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{2}$$

and the aggregate price, P_t, is

$$P_t = \left[\sum_{k=1}^{N} A_k P_{kt}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

The markup for firm i in sector k, μ_{kit}, is defined as the ratio of price to marginal cost:

$$\mu_{kit} \equiv \frac{Z_{kit} P_{kit}}{W_t}, . \tag{3}$$

This markup determines how firm's revenues are split into labor payments and profit, such that

$$L_{kit} W_t = \mu_{kit}^{-1} P_{kit} Y_{kit}, \qquad \text{and} \qquad \Pi_{kit} = \left(1 - \mu_{kit}^{-1}\right) P_{kit} Y_{kit}.$$

The market share of firm i in sector k, that is, the share of revenue of this firm relative to the revenue of its sector, is defined by

$$s_{kit} \equiv \frac{P_{kit} Y_{kit}}{P_{kt} Y_{kt}},$$

can be re-expressed in terms of markups and a composite of demand and productivity shifters, $V_{kit} \equiv A_{kit} Z_{kit}^{\varepsilon-1}$, as

$$s_{kit} = \frac{A_{kit} P_{kit}^{1-\varepsilon}}{\sum_{i'=1}^{N_k} A_{ki't} P_{ki't}^{1-\varepsilon}} = \frac{V_{kit} \mu_{kit}^{1-\varepsilon}}{\sum_{i'=1}^{N_k} V_{ki't} \mu_{ki't}^{1-\varepsilon}}. \tag{4}$$

Below we show that the split of the firm-level demand/productivity composite V_{kit} into demand and productivity shifters is required to solve for firm-level quantity and prices, but not for any other model outcome.

Firms choose price to maximize current profits, taking into account that they are large enough in their sector to impact sectoral output and prices. [2] As in Atkeson and Burstein

[2] We assume, as in Atkeson and Burstein (2008), that when setting prices or quantities firms do not internalize their impact on aggregate prices P_t, output Y_t, and the wage rate W_t. This effect would be absent with a continuum of sectors but not with a finite number of sectors.

(2008), we consider two alternative market structures: firms take other firms' prices as given (Bertrand competition), or firms take other firms' quantities as given (Cournot competition). Some of our analytic results and our quantitative analysis focuses on the case of Cournot competition.

Equilibrium markups and expenditure shares in each sector k, $\{\mu_{kit}\}_{k=1}^{N_k}$ and $\{s_{kit}\}_{k=1}^{N_k}$, solve the non-linear system of equations given by (4) and

$$\mu_{kit} = \begin{cases} \frac{\varepsilon}{\varepsilon-1}\left[1-\left(\frac{\varepsilon/\sigma-1}{\varepsilon-1}\right)s_{kit}\right]^{-1} & \text{under Cournot,} \\ \frac{\varepsilon}{\varepsilon-1}\left[\frac{1-\left(\frac{\varepsilon-\sigma}{\varepsilon}\right)s_{kit}}{1-\left(\frac{\varepsilon-\sigma}{\varepsilon-1}\right)s_{kit}}\right] & \text{under Bertrand.} \end{cases} \tag{5}$$

Under both formulations, markups are increasing in expenditure shares s_{kit}, a property which is satisfied in a variety of models with variable elasticity of demand (see e.g. the reviews in Burstein and Gopinath (2014) and Arkolakis and Morlacco (2017)).

Moreover, by equations (4) and (5) we can observe that firm-level expenditure shares and markups in sector k depend only on relative demand/productivity shifters across firms within this sector. This implies that firm-level expenditure shares and firm-level markups in sector k do not vary in response to proportional shifts in the demand/productivity composite to all firms in sector k, changes in the wage or sectoral demand shifter A_k, or shocks in other sectors.

1.3 Sectoral outcomes

We now describe how the model aggregates outcomes from the firm-level to the sector-level. Specifically, we show how the joint distribution of market share and markup across firm determines sectoral markup. Importantly, we show how in this model sectoral markup relates to concentration. Furthermore, we show that the joint distribution of markup and firm-level shocks entirely characterized the sectoral productivity.

Consistently with the firm-level markup, we define the sectoral markup as the ratio of sectoral revenues to labor payments,

$$\mu_{kt} \equiv \frac{P_{kt}Y_{kt}}{W_t L_{kt}}, \tag{6}$$

where sectoral employment is $L_{kt} = \sum_{i=1}^{N_k} L_{kit}$. Sectoral markups can be expressed as a function of firm-level markups and expenditure shares,

$$\mu_{kt}^{-1} = \sum_{i=1}^{N_k} \mu_{kit}^{-1} s_{kit}. \tag{7}$$

Substituting equation (5) under Cournot competition, we can express the sectoral markup, μ_{kt}, as a simple function of the Herfindahl index, $HHI_{kt} = \sum_{i=1}^{N_k} s_{kit}^2$:

$$\mu_{kt} = \frac{\varepsilon}{\varepsilon - 1} \left[1 - \left(\frac{\varepsilon/\sigma - 1}{\varepsilon - 1} \right) HHI_{kt} \right]^{-1}. \tag{8}$$

The Herfindahl index is the sum of the market share squared, it is also equal to the average of market share weigthed by market share and thus varies from zero to one. The above equation implies a positive relationship between sectoral markup and Herfindahl which takes the exact same form under Cournot as the firm-level relationship between markup and market share of equation (5). In the same way that a firm with a large market share charges a higher markup, a sector with a large average market share, that is a high Herfindahl, have a high sectoral markup.[3]

Sectoral productivity is defined as the ratio between sectoral output and employment,

$$Z_{kt} \equiv \frac{Y_{kt}}{L_{kt}}, \tag{9}$$

which can be expressed in terms of firm level markups and demand/productivity shifters as

$$Z_{kt} = \frac{\left(\sum_{i=1}^{N_k} V_{kit} \left(\mu_{kit}\right)^{1-\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}}}{\sum_{i=1}^{N_k} V_{kit} \left(\mu_{kit}\right)^{-\varepsilon}}. \tag{10}$$

In this equation, the sectoral productivity is entirely determined by the distribution of markup and the composite of demand and productivity shifters.

The sectoral price index can be expressed as the product of the sectoral markup and sectoral marginal cost defined as the ratio of wage to sectoral productivity,

$$P_{kt} = \frac{\mu_{kt} W_t}{Z_{kt}},. \tag{11}$$

Sectoral output is recovered from the CES demand for the sector-level good, that is, it is determined by the aggregate output, the sector-specific demand shifter A_k and the relative sectoral price,

$$Y_{kt} = A_k P_{kt}^{-\sigma} P_t^{\sigma} Y_t. \tag{12}$$

1.4 Aggregate outcomes

We describe now the model aggregates outcomes from the sector level to the aggregate level.

[3] A similar mapping between sectoral markups and concentration indices can be obtained under Bertrand competition (see Grassi, 2018).

Consistently with the firm and sector-level markup, we define the aggregate markup as the ratio of aggregate revenues and labor payments,

$$\mu_t \equiv \frac{P_t Y_t}{W_t L_t} = \left[\sum_{k=1}^{N} s_{kt}\mu_{kt}^{-1}\right]^{-1}, \tag{13}$$

which can be expressed as a harmonic weighted average sectoral markups. Alternatively, for the Cournot case, we can express the aggregate markup in terms of a weighted average of sectoral Herfindahl indices,

$$\mu_t = \frac{\varepsilon}{\varepsilon-1}\left[1-\left(\frac{\varepsilon/\sigma-1}{\varepsilon-1}\right)\sum_{k=1}^{N} s_{kt}HHI_{kt}\right]^{-1}.$$

Note that the weighted average of sectoral Herfindahl indices $\sum_{k=1}^{N} s_{kt}HHI_{kt}$ is also equal to the average market share, s_{kit} across firms weighted by firms' revenue share in the whole economy, $P_{kit}Y_{kit}/(P_tY_t)$. Under Cournot, when the (weighted) average market share in the economy is high the aggregate markup is high.

Sectoral expenditure shares, s_{kt}, are determined by sectoral markups and sectoral demand/productivity shifters $V_{kt} \equiv A_k Z_{kt}^{\sigma-1}$ as

$$s_{kt} \equiv \frac{P_{kt}Y_{kt}}{P_tY_t} = \frac{V_{kt}\left(\mu_{kt}\right)^{1-\sigma}}{\sum_{k'} V_{kt'}\left(\mu_{kt'}\right)^{1-\sigma}}. \tag{14}$$

Aggregate productivity is defined as the ratio between aggregate output and aggregate labor, $Z_t \equiv \frac{Y_t}{L_t}$, and can be expressed in terms of sectoral markups and demand/productivity shifters as

$$Z_t = \frac{\left(\sum_{k=1}^{N} V_{kt}\mu_{kt}^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{k=1}^{N} V_{kt}\mu_{kt}^{-\sigma}\right)}. \tag{15}$$

Similarly to the sectoral case, the aggregate price index can be expressed as the product of aggregate markup with wage divided by aggregate productivity,

$$P_t = \frac{\mu_t W_t}{Z_t}. \tag{16}$$

The above equation in combination with the Euler equation,

$$Y_t^{-\eta}\frac{W_t}{P_t} = \theta L_t^{\frac{1}{f}}, \tag{17}$$

yields the following expression for aggregate output

$$Y_t^{\eta+\frac{1}{f}} = \frac{Z_t^{1+\frac{1}{f}}}{\theta \mu_t}, \tag{18}$$

while aggregate labor can be computed using

$$L_t = \frac{Y_t}{Z_t}. \tag{19}$$

Combining equations (16) and (17), the labor wedge is equal to the aggregate markup, μ_t.

The following proposition describes an algorithm to solve equilibrium variables at the firm, sector, and aggregate level.

Proposition 1 ***(Summary of equilibrium solution)*** *Consider a given realization at time t of firm-level demand/productivity shifters, $\{V_{kit}\}$, and sectoral demand shifters, $\{A_k\}$. Equilibrium firm-level markups and expenditure shares, μ_{kit} and s_{kit}, are the solution to the non-linear system of equations (4) and (5). Sectoral markups and productivities, μ_{kt} and Z_{kt}, are solved for from equations (7) and (10), respectively, and sectoral expenditure shares, s_{kt}, from equation (14). Aggregate markup, productivity, output, and employment, μ_t, Z_t, Y_t, and L_t, are solved for from equations (13), (15), (18) and (19), respectively. Setting $W_t = \bar{W}$ as the numeraire, sectoral and aggregate price levels, P_{kt} and P_t, are solved for from equations (11) and (16), respectively. Sectoral output and employment are solved for from equations (12) and (9), respectively. Firm-level expenditures and employment, $P_{kit}Y_{kit}$ and L_{kit} are solved from from $P_{kit}Y_{kit} = s_{kit}P_{kt}Y_{kt}$ and equation (6), respectively. Finally, given realization of firm-level productivities $\{Z_{kit}\}$, firm-level output and price are solved from equations (1) and (3), respectively.*

From Proposition 1, we note that the split of the firm-level demand/productivity composite V_{kit} into demand and productivity shifters is required to solve for firm-level quantity and prices, but not for any other model outcomes.

We also note from Proposition 1 that aggregate productivity and aggregate markup (and hence the real wage) are determined only as functions of sectoral markups, sectoral prices, and exogenous sectoral demand shifters, for a given elasticity of substitution between sectors σ. Given the utility parameters η and f, we can then solve for sectoral and aggregate output and employment. Note that with linear disutility of labor ($f \to \infty$), $Y_t = \left(\theta P_t / \bar{W}\right)^{-1/\eta}$, so changes in aggregate output are pinned down by changes in the aggregate price (which are determined by changes in sectoral prices).

In the next section we use a first-order approximation to characterize the response of sectoral markups and sectoral prices to changes in firm-level demand/productivity shifters. We then map changes in sectoral outcomes to aggregate outcomes.

2 Analytic results

In this section we characterize the response of sectoral markups, prices and productivity to firm-level shocks, up to a first order approximation, and the impact of these sectoral outcomes on aggregate output, productivity, and markup.[4] We derive expressions for the variance of sectoral prices (which shape the volatility of aggregate output) and the covariance between sectoral prices and sectoral output. We analyze the role of the heterogeneous response of markups across firms in shaping these responses. In the quantitative analysis that follows we fully solve the non-linear equilibrium of the model.

We first introduce a definition. Recall from equation (5) that equilibrium markups are increasing in expenditure shares s_{kit}. We denote by Γ_{ki} the markup elasticity with respect to s_{kit}, that is $\Gamma_{ki} \equiv \frac{\partial \ln \mu_{ki}}{\partial \ln s_{ki}}$, which is equal to

$$\Gamma_{ki} = \begin{cases} \mu_{ki}\left(\frac{\varepsilon-1}{\varepsilon}\right) - 1 & \text{under Cournot,} \\ \left[\varepsilon\left(\frac{\mu_{ki}-1}{\mu_{ki}}\right) - 1\right](\mu_{ki}-1) & \text{under Bertrand,} \end{cases}$$

If $\varepsilon = \sigma$, then $\mu_{ki} = \varepsilon/(\varepsilon-1)$ and markup elasticities Γ_{ki} are zero. If $\varepsilon > \sigma$, then $\mu_{ki} \geq \varepsilon/(\varepsilon-1)$ and $\Gamma_{ki} \geq 0$ (with strict inequality if $s_{ki} > 0$). Moreover, within sectors, markup elasticities Γ_{ki} are increasing in markups and, since larger firms set higher markups, in expenditure shares s_{ki}. This property that the elasticity of markups to expenditure shares is increasing in size is satisfied by a variety of demand models with variable elasticity, as discussed in e.g. Burstein and Gopinath (2014) and Arkolakis and Morlacco (2017).

We consider an initial equilibrium in sector k and feed in shocks to demand/productivity shifters, V_{kit}. Up to a first-order approximation around the initial equilibrium (i.e. for infinitesimally small shocks), changes in equilibrium expenditures shares and markups are, by equation (4) and the definition of Γ_{ki}, the solution to the system of equations

$$\widehat{s}_{kit} = \widehat{V}_{kit} + (1-\varepsilon)\,\hat{\mu}_{kit} - \sum_{i'=1}^{N_k} s_{ki'}\left(\widehat{V}_{ki't} + (1-\varepsilon)\,\Gamma_{ki'}\widehat{s}_{ki't}\right) \tag{20}$$

$$\widehat{\mu}_{kit} = \Gamma_{ki}\widehat{s}_{kit}. \tag{21}$$

For any variable x_t, $\widehat{x}_t$ denotes its log difference relative to the initial equilibrium. In order to solve this linear system of equations, we require expenditure shares s_{ki} and markup elasticities Γ_{ki} in the initial equilibrium, demand/productivity shifters $\widehat{V}_{kit}$, and the demand elasticity parameter ε.

[4] We thank Dmitry Mukhin for his valuable input in deriving these analytic results.

In what follows we use this approximation to characterize sectoral and aggregate outcomes in response to idiosyncratic firm-level shocks. We provide simple expressions for markup cyclicality at different layers of aggregation that we then use to interpret our reduced form empirical results.

Sectoral prices We first consider changes in sectoral prices which, by equation (12), shape changes in sectoral output for given changes in aggregates, as well as changes in a sector's expenditure share in total expenditures. Taking a first-order approximation of (2) and using the expression for changes in firm-level prices, $\widehat{P}_{kit} = -\widehat{Z}_{kit} + \widehat{\mu}_{kit}$, log-changes in sectoral prices are

$$\widehat{P}_{kt} = \frac{1}{1-\varepsilon} \sum_{i=1}^{N_k} s_{ki} \left[\widehat{V}_{kit} + (1-\varepsilon)\, \widehat{\mu}_{kit} \right]. \tag{22}$$

By equations (20), (21) and (22), we can express log-changes in expenditure shares as

$$\widehat{s}_{kit} = \frac{\widehat{V}_{kit}}{1+(\varepsilon-1)\,\Gamma_{ki}} + \frac{(\varepsilon-1)\,\widehat{P}_{kt}}{1+(\varepsilon-1)\,\Gamma_{ki}}. \tag{23}$$

Substituting (21) and (23) into (22), changes in sector k price can be expressed as a weighted average of demand/productivity shifters, $\{\widehat{V}_{kit}\}_{i=1}^{N_k}$,[5]

$$\widehat{P}_{kt} = -\frac{1}{\varepsilon-1} \frac{\sum_{i=1}^{N_k} s_{ki}\alpha_{ki}\widehat{V}_{kit}}{\sum_{i=1}^{N_k} s_{ki}\alpha_{ki}}, \tag{24}$$

where the weights are given by the product of expenditure shares, s_{ki}, and pass-through rates, α_{ki}, with these defined as

$$\alpha_{ki} = \frac{1}{1+(\varepsilon-1)\,\Gamma_{ki}}.$$

We refer to α_{ki} as pass-through rate because α_{ki} governs how firm-level prices respond to idiosyncratic shocks (for given changes in sectoral prices), according to

$$\widehat{P}_{kit} = \alpha_{ki}\left(-\widehat{Z}_{kit} + \Gamma_{ki}\widehat{A}_{kit}\right) + (1-\alpha_{ki})\,\widehat{P}_{kt}. \tag{25}$$

Conversely, $1-\alpha_{ki}$ governs how prices respond to changes in sectoral price (due to variable markups). Since markup elasticities are increasing in expenditure shares (if $\varepsilon > \sigma$), α_{ki} is de-

[5] In the Appendix we return to equation (23) and provide expressions for the elasticity of firm-level expenditure shares with respect to own shocks and for the variance of expenditure shares.

creasing in expenditure shares. If $\varepsilon = \sigma$, or imposing that markups are constant in response to shocks, $\alpha_{ki} = 1$.[6]

Since $\varepsilon \geq 1$, sectoral prices fall in response to an increase in demand/productivity shifter. To understand how sectoral price changes are shaped by pass-through rates, note that if $\alpha_{ki} = \alpha_k$, then $\widehat{P}_{kt}$ is independent of α_k for given expenditure shares s_{ki} in the initial equilibrium. That is, the response in sectoral price is identical to that if markups were fixed at their initial level ($\alpha_{ki} = 1$). Intuitively, as pass-through α_k falls, the larger markup increase of a firm to an own positive shock is exactly offset by the larger markup decrease of its competitors.

With heterogeneity in pass-through rates, since α_{ki} is weakly decreasing in s_{ki}, there exists a single value $\bar{s}_k^p$ such that a positive shock to firm i with $s_{ki} > \bar{s}_k^p$ results in a smaller reduction in sectoral prices than if markups were fixed at their initial level. Intuitively, the markup increase of by firm i more than offsets the markup decrease of its competitors. Conversely, a positive shock to firm i with $s_{ki} < \bar{s}_k^p$ results in a larger reduction in sectoral prices than if markups were fixed at their initial level.[7]

From equation (24) we can calculate the asymptotic variance of sectoral price changes. Specifically, with shocks to the demand/productivity composite V_{kit} that are independently distributed across firms with variance $\sigma_v^2 = \mathbb{V}ar\left[\widehat{V}_{kit}\right]$, the asymptotic variance of price changes in sector k (around the initial equilibrium) is

$$\mathbb{V}ar\left[\widehat{P}_{kt}\right] = \left(\frac{\sigma_v}{\varepsilon - 1}\right)^2 \sum_{i=1}^{N_k} \left(\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}\right)^2. \tag{26}$$

If markups are fixed at their initial level (or more generally if $\alpha_{ki} = \alpha_k$), then this variance is proportional to the sectoral Herfindahl index, as in Gabaix (2011): $\mathbb{V}ar\left[\widehat{P}_{kt}\right] = \left(\frac{\sigma_v}{\varepsilon-1}\right)^2 \sum_{i=1}^{N_k} s_{ki}^2$. Comparing this expression with (26), $\mathbb{V}ar\left[\widehat{P}_{kt}\right]$ is lower under variable markups than under constant markups if and only if the variance of $\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}$ is lower than the variance of s_{ki}. Since α_{ki} is decreasing in s_{ki}, this condition is satisfied if $s_{ki}\alpha_{ki}$ is increasing in s_{ki} (see condition 2 below).

[6] We can further solve for $\widehat{P}_{kit}$ using $\widehat{P}_{kt} = s_{ki}\widehat{P}_{kit} + (1 - s_{ki})\widehat{P}_{k-it}$, where $\widehat{P}_{k-it}$ is the competitors' price index defined in Amiti et al. (2019). We can re-write (25) as $\widehat{P}_{kit} = \tilde{\alpha}_{kit}\left(-\widehat{Z}_{ki} + \Gamma_{ki}\widehat{A}_{kit}\right) + (1 - \tilde{\alpha}_{ki})\widehat{P}_{k-it}$, where $\tilde{\alpha}_{ki} = \frac{\alpha_{ki}}{1-(1-\alpha_{ki})s_{ki}}$, which is a U-shape function of expenditure shares s_{ki}.

[7] The threshold $\bar{s}^p$ is defined implicitly by $\alpha_k(\bar{s}^p) = \sum_{i=1}^{N_k} s_{ki}\alpha_{ki}$.

Sectoral markups Given equilibrium changes in firm-level shares and markups, changes in sectoral markups are, up to a first order by equations (7) and (21), given by

$$\widehat{\mu}_{kt} = \sum_{i=1}^{N_k} s_{ki} \frac{\mu_k}{\mu_{ki}} \left(\widehat{\mu}_{kit} - \widehat{s}_{kit} \right). \tag{27}$$

Equation (27) decomposes changes in sectoral markups into changes in markups *within* firms and reallocation of expenditures *between* firms with heterogeneous markups.

The following Proposition states that ex-ante firm heterogeneity is a necessary condition for sectoral markups to change in response to firm-level shocks:

Proposition 2 *If firms in sector k are symmetric in the initial equilibrium then, up to a first-order, sectoral markups are unchanged in response to firm-level shocks.*

Proof *In a symmetric initial equilibrium, s_{ki} and μ_{ki} are equal across all firms in sector k. By equation (21), (27), and $\sum_{i=1}^{N_k} s_{ki}\widehat{s}_{ki} = 0$, it follows that $\widehat{\mu}_{kt} = 0$.*

We now characterize changes in sectoral markups allowing for ex-ante firm heterogeneity. Substituting equations (21), (23), and (24) into (27) yields,

$$\widehat{\mu}_{kt} = \mu_k \sum_{i=1}^{N_k} s_{ki}\alpha_{ki} \left[\left(\frac{\Gamma_{ki} - 1}{\mu_{ki}} \right) - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'} \left(\frac{\Gamma_{ki'} - 1}{\mu_{ki'}} \right)}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \right] \widehat{V}_{kit}. \tag{28}$$

A positive shock to firm i results in an increase in sectoral markup if and only if $(\Gamma_{ki} - 1)/\mu_{ki}$ is higher than its average (weighted by $s_{kj} \times \alpha_{kj}$). Under our assumptions on market structure, the ratio $(\Gamma_{ki} - 1)/\mu_{ki}$ is increasing in markup μ_{ki} (and hence also increasing in expenditure share s_{ik}). Specifically,

$$\frac{\Gamma_{ki} - 1}{\mu_{ki}} = \begin{cases} \frac{\varepsilon - 1}{\varepsilon} - \frac{2}{\mu_{ki}} & \text{under Cournot,} \\ \varepsilon \left(\frac{\mu_{ki} - 1}{\mu_{ki}} \right)^2 & \text{under Bertrand.} \end{cases}$$

Therefore, positive shocks to relatively large (small) firms in the sector increase (decrease) sectoral markups.

Under Cournot competition, (28) can be re-expressed using (5) and the above equation as

$$\widehat{\mu}_{kt} = 2 \left(\frac{1}{\sigma} - \frac{1}{\varepsilon} \right) \mu_k \sum_{i=1}^{N_k} s_{ki}\alpha_{ki} \left[s_{ki} - \frac{\sum_{i'} s_{ki'}^2 \alpha_{ki'}}{\sum_{i'} s_{ki'}\alpha_{ki'}} \right] \widehat{V}_{kit}. \tag{29}$$

We therefore have the following proposition.

Proposition 3 *Consider a positive demand/productivity shock to firm i in sector k. Then, under Cournot competition, sectoral markup rises (and comoves negatively with sectoral price) if and only if $s_{ki} > \sum_{i'} s_{ki'}^2 \alpha_{ki'} / \sum_{i'} s_{ki'} \alpha_{ki'}$.*

The “2" in front of (29) reflects the fact that, in terms of the decomposition of sectoral markups introduced in equation (27), the magnitude of the within term is equal to the magnitude of the between term (and hence each accounts for 50% of changes in sectoral markups). In the Appendix we show that this 50-50 within/between decomposition of changes in sectoral markups under Cournot competition holds globally (not only up to a first order). With Bertrand competition, the within/between decomposition is not pinned down at 50-50.[8]

How do changes in sectoral markups compare under fixed markups and variable markups? If firm-level markups are fixed at their initial level after the realization of firm-level shocks, changes in sectoral markups (obtained by setting $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$ in equation 28) are:

$$\widehat{\mu}_{kt} = \sum_{i=1}^{N_k} s_{ki} \left(1 - \frac{\mu_k}{\mu_{ki}}\right) \widehat{V}_{kit}.$$

In this case, sectoral markups move only due to between-firm reallocation. In response to positive shock to firm i, sectoral markups rise if and only if $\mu_{ki} > \mu_k$.

In the general, we do not obtain a simple characterization comparing the above equation with (29). To make analytic progress, we restrict the extent of *ex-ante firm heterogeneity*. Specifically, we assume that in sector k there are N_k^A type A firms and $N_k^B = N_k - N_k^A$ type B firms, and in the initial equilibrium firms within each type have equal demand/productivity composite, V_{kit}. In the initial equilibrium, each firm of type $g = A, B$ has expenditure share s_k^g, markup μ_k^g, and markup elasticity Γ_k^g. Firms of type A are indexed by $i = 1, ..., N_k^A$ and firms of type B are indexed by $N_k^A + 1, ..., N_k$. In this case, equation (28) under Cournot competition can be written as

$$\widehat{\mu}_{kt} = \frac{2}{1 + (\varepsilon - 1)\widetilde{\Gamma}_k} \left[s_k^A \left(1 - \frac{\mu_k}{\mu_k^A}\right) \sum_{i=1}^{N_k^A} \widehat{V}_{kit} + s_k^B \left(1 - \frac{\mu_k}{\mu_k^B}\right) \sum_{i=N_k^A+1}^{N_k} \widehat{V}_{kit} \right], \tag{30}$$

[8] Following similar steps, under Bertrand competition we obtain

$$\widehat{\mu}_{kt} = \mu_k \varepsilon \sum_{i=1}^{N_k} s_{ki} \alpha_{ki} \left[\left(\varepsilon - s_{ki} \left(\varepsilon - \sigma\right)\right)^{-2} - \frac{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'} \left(\varepsilon - s_{ki'} \left(\varepsilon - \sigma\right)\right)^{-2}}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}} \right] \widehat{V}_{kit}.$$

where

$$\widetilde{\Gamma}_k = N_k^B s_k^B \Gamma_k^A + N_k^A s_k^A \Gamma_k^B.$$

The term in square brackets in equation (30) corresponds to the change in the sectoral markup under fixed markups as express above. Therefore, given the same firm-level shocks, sectoral markups change by more (and the variance is higher) under variable markups than under constant markups if and only if the term in front of the square brackets in equation (30) is higher than one, which is the case if $(\varepsilon - 1)\widetilde{\Gamma}_k < 1$. This condition is violated if σ is sufficiently low and/or ε sufficiently high. Intuitively, changes in sectoral markups can be smaller under variable markups than under constant markups because the larger response of sectoral markups due to changes in firm-level markups is more than offset by a smaller extent of between-firm reallocation due to incomplete pass-through.

To summarize, even though sectoral markups in the model with variable markups are twice as large as the between-firm reallocation term, variable markups do not necessarily magnify changes in sectoral markups relative to the model specification with constant markups (in which sectoral markups change only due to between-firm reallocation).

Covariance between sectoral prices and sectoral markups We calculate the asymptotic covariance between price and markup in sector k, up to a first-order, given a long realization of shocks to V_{kit} that are independently distributed across firms with variance σ_v^2.

In the case of constant (but heterogeneous) markups,

$$\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] = -\frac{1}{\varepsilon - 1}\sum_{i=1}^{N_k} s_{ki}^2 \left[1 - \frac{\mu_k}{\mu_{ki}}\right] \times \sigma_v^2.$$

Thus, sectoral markups and prices are negatively correlated as long as large firms within sector charge higher markups. Intuitively, shocks to small firms induce a positive comovement while shocks to large firms induce a negative comovement. Overall, comovement is negative because shocks to large firms induce larger changes in sectoral price compared to shocks to small firms.

Under Cournot competition, by equations (24) and (29),

$$\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] = -\left(\frac{2\mu_k}{\varepsilon - 1}\right)\left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right)\sum_{i=1}^{N_k} s_{ki}\alpha_{ki}\left[s_{ki} - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}s_{ki'}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]\frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \times \sigma_v^2.$$

The square bracket in the above equation indicates, consistent with Proposition 3, that shocks to large firms induce a negative covariance, while positive shocks to small firms induce a positive covariance. This equation can be re-expressed as

$$\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] = -\left(\frac{2\mu_k}{\varepsilon-1}\right)\left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)\sum_{i'=1}^{N_k} s_{ki'}^2\alpha_{ki'}\sum_{i=1}^{N_k}\left[\frac{s_{ki}^2\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}^2\alpha_{ki'}} - \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]\frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\times\sigma_v^2.$$

Therefore, when $\varepsilon > \sigma$, sectoral prices and markups are negatively correlated in long samples if and only if

$$\sum_{i=1}^{N_k}\left[\frac{s_{ki}^2\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}^2\alpha_{ki'}} - \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]\frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} > 0, \tag{31}$$

where the product of expenditure shares and pass-through rates, $s_{ki}\alpha_{ki}$, is given under Cournot by

$$s_{ki}\alpha_{ki} = \frac{\left(1-\frac{1}{\varepsilon}\right)s_{ki} - \left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)s_{ki}^2}{1-\frac{1}{\varepsilon}+(\varepsilon-2)\left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)s_{ki}}.$$

If firms are ex-ante homogeneous, then equation (31) holds with equality and sectoral markups are constant over time, consistent with Proposition 2. If firms are heterogeneous in the initial equilibrium, inequality (31) may or may not hold. The following proposition states that if pass-through rates do not fall too strongly with expenditure shares, then inequality (31) holds, so sectoral prices and markups are negatively correlated.

Proposition 4 *With Cournot competition, if firms are ex-ante heterogeneous and $s_{ki}\alpha_{ki}$ is increasing in s_{ki}, then $\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] < 0$.*

Proof *Define $f(.)$ and $g(.)$ as probability density functions given by $f(s) = \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}$ and $g(s_{ki}) = s_{ki}f(s_{ki})a$ with $a = \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}{\sum_{i'=1}^{N_k} s_{ki'}^2\alpha_{ki'}} > 1$. Since the likelihood ratio $g(s)/f(s) = sa$ is increasing in s, $g(.)$ first-order stochastically dominates $f(.)$. If $s_{ki}\alpha_{ki}$ is increasing in s_{ki}, then $f(s)$ is increasing in s. It then follows that $\sum_{i=1}^{N_k}\left[g(s_{ki}) - f(s_{ki})\right]f(s_{ki}) > 0$, which corresponds to inequality (31). Note that if $s_{ki}\alpha_{ki}$ is decreasing in s_{ki}, then inequality (31) is reversed.* $\square$

The condition that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} implies, by equation (24), that sectoral prices are more responsive to large firm shocks than to small firm shocks, contributing to a negative covariance between sectoral price and markup as discussed in the case of constant markups. Note that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} if and only if

$$2\left(\frac{\varepsilon-1}{\varepsilon}\right)s_{ki} + \left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)(\varepsilon-2)\,s_{ki}^2 < \frac{\sigma\,(\varepsilon-1)^2}{\varepsilon\,(\varepsilon-\sigma)}.$$

Since the left hand side of this equation is increasing in s_{ik} (for $s_{ik} \leq 1$), this inequality holds for $s_{ki} \leq \tilde{s}_k$, where $\tilde{s}_k$ is a function of σ and ε. Note that the condition that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} is sufficient but not necessary for inequality (31) to hold. In particular, inequality (31) may hold even if $s_{ki}\alpha_{ki}$ is increasing in some regions of the distribution of market shares in a sector but decreasing at the upper tail of the distribution.

The covariance expression (31) was derived for a long (asymptotic) realization of shocks. With finite samples, the sign and magnitude of the covariance depends on the realization of shocks.

Sectoral output and productivity Changes in sectoral output are, by equation (12), determined not only by changes in sectoral prices but also by changes in aggregate output and price,

$$\widehat{Y}_{kt} = -\sigma \widehat{P}_{kt} + \sigma \widehat{P}_t + \widehat{Y}_t. \tag{32}$$

If sector k is small in the aggregate ($s_k \to 0$), then sectoral output rises when sector price falls. More generally, in the Appendix we derive the following expression for the change in sector k output in response to sector k shocks taking into account changes in aggregate output and price:

$$\widehat{Y}_{kt} = -\left[\sigma\left(1-s_k\right) + \left(\frac{f+1}{f\eta+1} + \left(\frac{\sigma-1}{f\eta+1}\right)\left(1-\frac{\mu}{\mu_k}\right)\right)s_k\right]\widehat{P}_{kt} + \frac{s_k\mu}{\mu_k}\frac{\widehat{\mu}_{kt}}{f\eta+1}. \tag{33}$$

With linear disutility of labor ($f \to \infty$), this expression simplifies to

$$\widehat{Y}_{kt} = -\left[\sigma\left(1-s_k\right) + \eta^{-1}s_k\right]\widehat{P}_{kt}, \tag{34}$$

so sectoral output necessarily moves in the opposite direction as the sectoral price. With finite f and if sector k is sufficiently large in the aggregate, sectoral output can potentially fall when sectoral price falls if sectoral markup μ_k is very low relative to the aggregate markup and/or if sector k markup falls substantially when the sectoral price falls.

The asymptotic covariance between sectoral output and sectoral markups is

$$\mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kt}\right] = -\left[\sigma\left(1-s_k\right) + \frac{f+1+(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)}{f\eta+1}s_k\right]\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] + \frac{s_k\mu}{\mu_k}\frac{1}{f\eta+1}\mathbb{V}ar\left[\widehat{\mu}_{kt}\right].$$

From this equation we obtain the following sufficient conditions for $\mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kt}\right]$ to have the opposite sign than $\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right]$, so that sectoral markups are pro-cyclical with respect to sectoral output.

Proposition 5 *Under the conditions of Proposition 4,* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right]>0$ *if at least one of these three conditions holds: (i)* $s_k\to 0$*, (ii)* $f\to\infty$*, (iii)* $\sigma\to 1$*. If all three conditions (i)-(iii) are violated,* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right]>0$ *as long as sectoral markup* μ_k *is not too low relative to aggregate markup.*

We also calculate the covariance between firm i markup and sector k output. We consider the case of $f\to\infty$. By equations (21), (23), (24), and (34), we have

$$\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kit}\right]=\left(\sigma\left(1-s_k\right)+\eta^{-1}s_k\right)\frac{\alpha_{ki}\Gamma_{ki}}{\left(\epsilon-1\right)\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\left[s_{ki}\alpha_{ki}-\frac{\sum_{i'=1}^{N_k}\left(s_{ki'}\alpha_{ki'}\right)^2}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\right]\times\sigma_v^2. \quad (35)$$

From this expression, we obtain the following Proposition:

Proposition 6 *If* $s_{ki}\alpha_{ki}$ *is increasing in* s_{ki} *and* $f\to\infty$*, then* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kit}\right]>0$ *if and only if* $s_{ki}>\bar{s}_k^{\mu}$ *and* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kit}\right]<0$ *if and only if* $s_{ki}<\bar{s}_k^{\mu}$*, where* $\bar{s}_k^{\mu}$ *is defined by the condition that the square bracket in (35) is equal to zero.*

The cutoff $\bar{s}_k^{\mu}$ differs from the cutoff defined in Proposition 3. This is because the condition in Proposition 3 is based on a shock to one firm only whereas the asymptotic covariance in Proposition 6 takes into account shocks to all firms in the sector. Intuitively, firm-level markups are positively correlated with sectoral output in response to own-shocks and negatively correlated in response to competitors' shocks. Since large firms have a disproportionate impact on sectoral output (as long as $s_{ki}\alpha_{ki}$ is increasing in s_{ki}), it follows that firm-level markups are pro-cyclical for large firms and counter-cyclical for small firms.

Finally, by equations (10) and (21), changes in sectoral productivity are, up to a first order, given by

$$\widehat{Z}_{kt}=\sum_{i=1}^{N_k}s_{ki}\left[\left(\frac{\varepsilon}{\varepsilon-1}-\frac{\mu_k}{\mu_{ki}}\right)\widehat{V}_{kit}-\varepsilon\left(1-\frac{\mu_k}{\mu_{ki}}\right)\Gamma_{ki}\widehat{s}_{kit}\right],$$

where changes in expenditure shares are given by (23).

The term $s_{ki}\times\frac{\varepsilon}{\varepsilon-1}$ corresponds to the elasticity of sectoral productivity under monopolistic competition. The remaining terms reflect changes in efficiency due to reallocation across firms with heterogeneous markups, as discussed in detail in (Baqaee and Farhi (2017)).

Aggregate markup and aggregate output We now discuss how sectoral markup and price outcomes discussed above map into changes in aggregate price (i.e. the negative of the real wage), markup, productivity, and output.

Up to a first order, changes in the aggregate price are

$$\widehat{P}_t = \sum_k s_k \widehat{P}_{kt}. \tag{36}$$

Based on our results above, any positive firm-level shock in sector k reduces the corresponding sectoral price and therefore reduces the aggregate price (or increases the real wage) proportionately to the expenditure share of sector k. Whether the real wage rises more or less under variable markups relative to constant markups depends, as discussed above (see equation 24) on the shocked firm's relative size in its sector.

The change in aggregate markup can be decomposed into a reallocation term and a within-sector markup change (analogous to the decomposition of sectoral markups in equation (27):

$$\widehat{\mu}_t = \sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} + \sum_k s_k \left(1 - \frac{\mu}{\mu_k}\right) \widehat{s}_{kt}. \tag{37}$$

Using $\widehat{s}_{kt} = (1-\sigma)\left(\widehat{P}_{kt} - \widehat{P}_t\right)$, we can re-express (37) as

$$\widehat{\mu}_t = \sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} + (1-\sigma) \sum_k s_k \left(1 - \frac{\mu}{\mu_k}\right) \widehat{P}_{kt}. \tag{38}$$

Consider an increase in demand or productivity for a firm in sector k. The first (within) term in (38) is positive if the shocked firm is relatively large (and sets a higher markup) in sector k. The second (between) term in (38) is positive, when $\sigma > 1$, if sector k has a relatively high markup relative to the aggregate markup.

Changes in aggregate productivity, using $\widehat{Z}_t = \widehat{\mu}_t - \widehat{P}_t$, can be expressed in terms of changes in sectoral markups and prices as

$$\widehat{Z}_t = \sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} - \sum_k s_k \left[1 + (\sigma - 1)\left(1 - \frac{\mu}{\mu_k}\right)\right] \widehat{P}_{kt}. \tag{39}$$

Recall that the sectoral price falls ($\widehat{P}_{kt} < 0$) in response to positive firm-level shocks. Aggregate productivity typically rises, but can fall if shocked firms are relatively small in their sector (such that the sectoral markup falls) or belong to low markup sectors and $\sigma > 1$.

Finally, by equation (18), changes in aggregate output are

$$\widehat{Y}_t = (f^{-1} + \eta)^{-1} \left[f^{-1} \widehat{Z}_t - \widehat{P}_t \right], \tag{40}$$

which can be written only in terms of changes in sectoral markups and prices as

$$\widehat{Y}_t = (1 + f\eta)^{-1} \left[\sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} - \sum_k s_k \left[1 + f + (\sigma - 1) \left(1 - \frac{\mu}{\mu_k} \right) \right] \widehat{P}_{kt} \right]. \tag{41}$$

With inelastic labor supply ($f \to 0$), $\widehat{Y}_t = \widehat{Z}_t$. With linear disutility of labor ($f \to \infty$), the aggregate productivity term drops, so $\widehat{Y}_t = -\widehat{P}_t / \eta$, or

$$\widehat{Y}_t = -\eta^{-1} \sum_k s_k \widehat{P}_{kt}. \tag{42}$$

By equation (42), a positive firm-level shock in sector k reduces the corresponding sectoral price and increases aggregate output. The resulting increase in aggregate output is smaller (larger) under variable markups compared to constant markups if shocked firms are relatively large (small) in their sectors.

We now calculate the covariance between aggregate output and sector k markup. Using the fact that sector k markups are affected only by shocks to sector k firms, we can express this covariance as

$$\begin{aligned} \mathbb{C}ov \left[\widehat{Y}_t, \widehat{\mu}_{kt} \right] &= \mathbb{C}ov \left[\widehat{Y}_{kt}, \widehat{\mu}_{kt} \right] + \sigma (1 - s_k) \mathbb{C}ov \left[\widehat{P}_{kt}, \widehat{\mu}_{kt} \right] \\ &= -s_k \left[\frac{f + 1 + (\sigma - 1) \left(1 - \frac{\mu}{\mu_k} \right)}{f\eta + 1} \right] \mathbb{C}ov \left[\widehat{P}_{kt}, \widehat{\mu}_{kt} \right] + \frac{s_k \mu}{\mu_k} \frac{1}{f\eta + 1} \mathbb{V}ar \left[\widehat{\mu}_{kt} \right]. \end{aligned}$$

The following Proposition states that the covariance between aggregate output and sector k markups is positive and lower than the covariance between sector k output and sector k markup.

Proposition 7 *Under the conditions of Proposition 5,*

$$0 < \mathbb{C}ov \left[\widehat{Y}_t, \widehat{\mu}_{kt} \right] \leq \mathbb{C}ov \left[\widehat{Y}_{kt}, \widehat{\mu}_{kt} \right] \tag{43}$$

where the second inequality holds strictly if $s_k < 1$.

The variance of aggregate output (when $f \to \infty$) is

$$\mathbb{V}ar \left[\widehat{Y}_t \right] = \eta^{-2} \sum_k s_k^2 \mathbb{V}ar \left[\widehat{P}_{kt} \right]. \tag{44}$$

Based on the discussion that follows equation (26), $\mathbb{V}ar\left[\widehat{Y}_t\right]$ is lower under variable markups compared to constant (and heterogeneous markups) under the condition of Proposition 4.

Finally, the covariance between aggregate output and aggregate markups (when $f \to \infty$) is

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_t\right] = -\frac{\mu}{\eta}\sum_k \frac{s_k^2}{\mu_k}\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] + \frac{\sigma-1}{\eta}\sum_k s_k^2\left(1-\frac{\mu}{\mu_k}\right)\mathbb{V}ar\left[\widehat{P}_{kt}\right]. \tag{45}$$

The first term in (45) is positive if sectoral markups and sectoral prices comove negatively (which we discussed above). The second term in (45) is non-negative unless larger sectors have relatively lower markups.

From these theoretical results, it is clear that the notion of the sign of markup cyclicality depends on the level of aggregation, market structure within and across all industries, and the set of shocked firms. Moreover, the sign and magnitude of covariances in finite samples may differ from those of the asymptotic covariances we derived.

In what follows we calibrate the model to match salient features of the French firm-level data. We evaluate quantitatively its implications for the cyclicality of markups, as well as its ability to generate aggregate fluctuations in output and markups in response to idiosyncratic firm-level shocks.

3 Data, Estimation and Calibration

In this section, we describe how we calibrate our model using French administrative firm-level data. First, we introduce the data. Second, we describe how we estimate firm-level markups. Third, we present our calibration strategy.

3.1 Data

Our empirical results rely on firm-level data containing the universe of French firms between 1994 and 2016. These information are sourced from two datasets: the FICUS data covering the period from 1994 to 2007, and, the FARE dataset covering the period 2008 to 2016. Both of these datasets are produced from firms' tax statement by the Institut National de la Statistique et des Etudes Economiques (INSEE), the French national statistical institute, and the French tax administration. The FICUS-FARE dataset contains, for each firm and each year, income statement, balance sheet, and, demographic information. To the best of our knowledge, this is the first time, that the full panel covering the period 1994 to 2016 have been used.[9,10]

In our analysis, we use a subset of the variables available in the FICUS-FARE dataset. Specifically, we use the total revenue of the firm, the wage bill (sum of wages and social security payments), we compute capital from the permanent inventory method. Importantly, we extract information on inputs expenditures. We sum the expenditure and the variation of stock of materials and merchandises (ACHAMPR and ACHAMAR respectively), this is our baseline measure of variable input that we call materials, with abuse of language.The variable ACHAMPR is defined as "everything that the firm purchase to be transformed", while ACHAMAR is defined as "everything that the firm purchase to be sold as is". We are also using the expenditure on service input. The variables AUTACH is defined as to "correspond to expenditure on services" as opposed to "ACHAMAR and ACHAMPR that correspond to expenditures on goods". It includes, among many things, study and research, outsourcing cost, and, external personnel cost (temporary workers). We drop firms in the Finance and Insurance sector and firms with zero or negative revenue, wage bill, and capital. We deflate firm's income using 2-digit sector price index provided by EU-KLEMS, capital and wage bill are deflated using the GDP deflator. To comply with confidentiality rule, we drop all the firms in the sector that have less than 12 firms in a given year. Sectors are classified according to the Nomenclature d'Activités Française (NAF2008) classification which is the French version of the NACE Rev 2 at the 4-digits level. After this treatment, we end up with $10,928,469$ firm-year across 504 sectors.

3.2 Markup Estimation

To estimate firm-level markups we follow the literature and assume a production function function that is more flexible than the one we use in our theoretical model (in which labor is the only factor of production). Specifically, we allow for intermediate inputs and adjustment frictions for a subset of factors of production.

We estimate firm-level markup following the method proposed by Hall (1988) using the FICUS-FARE data described above. Cost minimization by firm i in sector k with respect to

[9] During this period, an important change in the NACE and NAF classification happen in 2008. For each firm, we construct a consistent industry code. For firms that are observed with the old and the new codes, the new code is applied to all earlier years. For firms that are observed with only one of either, the combination of industry codes that is observed most frequently is assigned.

[10] We thanks Isabelle Mejean for sharing code to help merging the FICUS and FARE datasets.

a variable input v (that is not subject to adjustment costs or to any intertemporal decision) assuming price taking in factor markets,[11] implies

$$\mu_{kit} = \theta^v_{kit} \frac{P_{kit} Y_{kit}}{P^v_{kit} V_{kit}}$$

where $P^v_{kit} V_{kit}$ is the expenditure on input v by firm i in sector k, $P_{kit} Y_{kit}$ is the revenue of this firm, and, θ^v_{kit} is the output elasticity with respect to input v. Given measures of expenditures on a variable input as a share of revenues and estimates of output elasticity with respect to this variable input, we can compute firm-level markups.

As a baseline, we use for expenditures on variable input the sum of material and merchandises expenditures (ACHAMPR and ACHAMAR), taking into account variation in stocks. Arguably this input is the least subject to adjustment costs among the various input measures in the data. For robustness, we consider the sum of all inputs expenditures including the wage bill, which is similar to Cost Of Good Sold (COGS) in Compustat.[12] This second measure of input expenditures is more prone to adjustment cost and frictions given that a significant share of the COGS is due to expenditure on labor and services.

Our results on the cyclical pattern of markup presented below are robust across these variable input choices. To understand this, note that since the wage bill and services expenditures are more likely to be subject to frictions, they are fairly constant across years. Year-to-year variation in input expenditures is dominated by year-to-year variation in material and merchandises expenditures.

Given a choice of variable input, the last step is to estimate firm-level production functions and elasticities θ^v_{kit}. To do so we follow the iterative GMM procedure proposed by De Loecker and Warzynski (2012) and De Loecker and Eeckhout (2017), who extend the methodology developed by Olley and Pakes (1996), Levinsohn and Petrin (2003) and Ackerberg et al. (2007). We make three main assumptions regarding the production function. First, we assume that firms have four factors of production: labor, capital, materials, and services. Second, we assume that the production function is identical across firms within 2-digit sectors. This choice is based on the availability of price deflator at the 2-digit level only. In the data appendix, we provide additional estimation details. Finally, we assume that firms have a translog production function. Under such parameterization, elasticities θ^v_{kit} are heterogeneous across firms and increasing in the level of input.

[11] Morlacco (2019) relaxes this assumption and shows that with one more variable input, one can estimate the markdown on inputs.

[12] Unlike COGS in Compustat, this measure include all labor cost and is therefore consistent across sectors.

3.3 Calibration

In this section, we describe how we parameterize the model presented in section 1 to match salient features of the sectoral French data in 2014. Before describing the calibration procedure, we specify a process for firms' productivity.

Firm-Level Productivity Process

We assume that firm-level demand shocks, A_{kit}, are fixed over time so that the composite V_{kit} is driven only by productivity shocks.[13] Following Carvalho and Grassi (2019), we assume that firm-level productivity, Z_{ikt}, follows a discretized random growth process introduced by Córdoba (2008). Precisely, firm productivity in sector k evolves on an evenly spaced log grid, $\Phi_k = \{1, \varphi_k, \varphi_k^2, \ldots, \varphi_k^S\}$ where φ_k is greater than one and where S is an integer. Note that $\varphi_k^n = \varphi_k \varphi_k^{n-1}$. This process is a Markov chain where the associate matrix of transition probabilities is equal to:

$$P = \begin{pmatrix} a_k + b_k & c_k & 0 & \cdots & \cdots & 0 & 0 \\ a_k & b_k & c_k & \cdots & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a_k & b_k & c_k \\ 0 & 0 & 0 & \cdots & 0 & a_k & b_k + c_k \end{pmatrix}$$

where $1 > a_k, b_k, c_k > 0$ and $a_k + b_k + c_k = 1$. As shown in Córdoba (2008) and summarized by Carvalho and Grassi (2019) such process implies that firm-level productivity conditional growth is independant of the current level, and that, the stationary distribution is Pareto with a tail index equal to $\delta_k = \ln(\frac{a_k}{c_k})/\ln(\varphi_k)$.

With this assumption in place, and with the oligopolistic structure assumed in this model, Grassi (2018) shows that the growth rate of firms' market share is decreasing in firm size, even if firm-level productivity growth is not (see the discussion in Appendix A.2.Furthermore, Carvalho and Grassi (2019) show that with this productivity process, moments of the productivity distribution (up to some power) follow a stochastic process that takes an autoregressive form. This flexible process yields realistic firm dynamics while having tractable aggregation properties.

The Pareto stationary distribution of productivity implied by this firm-level process guides our calibration strategy. Given the parsimonious parametrization of the Pareto distribution,

[13] While our analytic results do not take a stand on the prevalence of productivity versus demand firm-level shocks, in the data we construct sectoral output by deflating nominal value added by industry price indices that typically do not take into account high frequency changes in demand or quality shifter. Therefore, for consistency we abstract from shocks to demand shifters.

the tail index δ_k pins down the moments of the productivity distribution. We use this property to match the Herfindahl-Hirschman Index (HHI) in sector k by calibrating the tail index δ_k. While δ_k is the tail index of the stationary productivity distribution in sector k, the HHI is the second moments of the market share distribution. However, as shown above, the market share distribution is entirely determined by the firm-level productivity distribution in each sector which justifies our calibration strategy.

Given a productivity grid, the firm-level productivity process described above has three parameters a_k, b_k and c_k. These parameters sum to one and thus we are only left with two parameters to calibrate. The calibration of the tail index δ_k identifies the ratio between a_k and c_k. The last parameter is calibrated to match volatility of small firms market-share of 10%, a value at the lower hand of the empirical estimate in the literature.

Calibration Strategy

We assume Cournot competition in all sectors. We choose a value for the elasticity of substitution across firms within sectors, $\varepsilon = 6$, which are in the range of estimates in the literature.

Given ε, we calibrate the elasticity of substitution across sector, σ, to match the regression coefficient of the inverse sector-level markup on HHI with sector and time fixed effects. According to equation (8), this coefficient is equal to $\frac{\frac{\varepsilon}{\sigma}-1}{\varepsilon}$. Thus, given an estimate of this coefficient and a value of ε, we obtain the implied σ. For our baseline specification the estimated coefficient $\frac{\frac{\varepsilon}{\sigma}-1}{\varepsilon}$ is equal to 0.444 (s.e 0.109) which implies a value of $\sigma = 1.638$.

We use the 2014 vintage of the data described above to compute HHI for each sectors of the Nomenclature d'Activités Française (NAF2008).[14] They are $513,457$ firms across $N = 504$ sectors. We compute market share as the ratio between the revenue of a firm and the sum of the revenue of all the firms in its sector. The median HHI across sectors in 2014 is equal to 0.049, which implied that in the median sector firms have on average 4.9% market share. The median number of firms in each a sector is 317. The top and bottom quartile of the distribution of firm number across sector are equal to 92 and to $1,037$, the top 5% of sectors have more than $5,485$ firms. The number of firms can be compared to "effective" number of firms defined as the inverse of the HHI. If in a sector all the firms have the same market share then the inverse of the HHI is equal to the number of firms. The median number of firms is 317, while the median of the inverse of the HHI is equal to 21. This indicates that even the median sector is quite concentrated. The second and third quartile of the distribution

[14] Classified by the Institut National de la Statistique et des Etudes Economiques (INSEE), the Nomenclature d'Activités Française (NAF2008) is imbricated in the NACE 2 classification used by the European Commission.

Table 1: Baseline Calibration

Parameters	Cournot	Description	Target/Source
ε	6	substitution across firms	see main text
σ	1.6376	substitution across sectors	slope of μ_{kt}^{-1} on HHI_{kt}
f	1	Frisch Elasticity of Labor Supply	
η	1	relative risk aversion	log utility
N	504	# of sectors	NAF with >11 firms
N_k	325	median # firms in a sector	FARE 2014
$\{N_k\}_k$	95, 1021, 4852	Q1,Q3 and top 5% of # firms	FARE 2014
$\sum_k N_k$	513 457	total # of firms	FARE 2014
HHI_k	4.80	median Herfindahl (pp)	FARE 2014
$\{HHI_k\}_k$	1.6, 12.7, 41.2	Q1,Q3 and top 5% of Herfindahl (pp)	FARE 2014
$1/HHI_k$	20.83	median of inverse of Herfindahl	FARE 2014
a_k, c_k	0.47, 0.42	median Firm-level pdty process	Vol. 10% and HHI_k
A_k	0.078	median HH preference shifts (pp)	Income share

of HHI across sector are equal to 1.6% and 12.9% respectively. The top 5% of sectors have an HHI greater than 41.3%.

As described above, we calibrate the firm-level productivity process in each of the 504 sectors to match the HHI and the volatility of small firms market share of 10%. Precisely, for sector k and given a productivity grid φ_k, we calibrate the tail index of the stationary productivity distribution, δ_k, to match the measure HHI in this sector. Given this tail parameter, we are left with one parameter that we calibrate to match the volatility of small firms market share of 10%. Note that we fit for each of the 504 sectors, a Pareto distribution that matches the second moment of the market share distribution, the HHI. To do so, we are solving 504 non-linear systems of N_k equations without relying on any approximation. This sector-by-sector strategy is a contribution to the existing standard in the literature on macroeconomics model of oligopolistic competition.

The final step of our calibration strategy is to calibrate the constant sector-level demand shifter, A_k. To do so we use the first-order-condition of the household to match the income share across the 504 sectors. We set the relative risk aversion to one (log utility) and the Frisch labor supply elasticity to 0.5 that are standard values in the business cycle literature. Table 1 summarizes our baseline calibration, the associated values (or median across sectors), and the targets.

The calibrated model is used as a data generating process to simulate aggregate-level time series, sector-level and firm-level panels. These simulated data are used in our quantitative results described below. We use the simulated sector-level and firm-level panels to run similar regressions that we run on actual data. We also compute business cycle statistics using the simulated aggregate time-series that we compare to their counterpart in the data.

4 Model meets Data

4.1 Inspecting the Mechanism

4.1.1 Markups, Market Shares and Concentration: Firm and Sector-level Evidence

Hardwired into our model are two key relations between markups and measures of concentration. At the firm-level, and following the discussion in Section 1.2, markups increase with a firm's market share. In turn, this immediately gives rise to a notion of markup procyclicality at the micro-level: a firm's markup increases whenever its market share increases. At the sector-level, as discussed in Section 1.3, equilibrium aggregation of firm-level outcomes yields that sectoral markups increase in the Herfindahl-Hirschman Index (HHI) of that sector. By the same token, this yields a simple notion of markup cyclicality at the sector level: a sector's markup increases whenever its level of concentration increases. In this section, we evaluate whether these two relations, at the firm and sector levels, hold empirically in the data.

Starting at the micro-level, note that taking the inverse of equation (5) for Cournot yields a simple relation between the firm's market share and its markup:

$$\mu_{kit}^{-1} = \frac{\varepsilon - 1}{\varepsilon} - \frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon} s_{kit} \tag{46}$$

where μ_{kit}^{-1} is the inverse of the (gross) markup of firm i in sector k at time t and s_{kit} gives its market share. In turn, this suggests the following simple empirical specification:

$$\mu_{kit}^{-1} = \gamma_i + \alpha_t + \beta s_{kit} + \epsilon_{kit}$$

where β is the coefficient of interest, predicted by the model to be negative. To guard against the possibility that our results are driven by unobserved heterogeneity in firm-level markup, we further control for γ_i, a firm fixed-effect, and α_t, a year fixed effect controlling for unobserved shifters of markup, common across all firms.

We start by inspecting these firm-level relations in the French census data. Recall from our discussion in the previous section that we have estimated firm-level markups for the population of French firms over the period 1994-2016. This yields the empirical counterpart of

Table 2: Firm Inverse Markup and Market Share: Level

	Dependent Variable: μ_{kit}^{-1}			
	(1)	(2)	(3)	(4)
s_{kit}	-4.853	-4.852	-.663	-.589
	(.273)	(.273)	(.103)	(.101)
Year FE	N	Y	N	Y
Firm FE	N	N	Y	Y
Number of Firms	1 284 905	1 284 905	1 154 181	1 154 181
Observations	10 928 469	10 928 469	10 727 745	10 727 745

NOTE: μ_{kit}^{-1} is the inverse of firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k. Column (1-4) reports empirical estimates for the FICUS-FARE (1994-2016) data. Standard errors (in parenthesis) are clustered at firm level. Markups are winsorized at the 2% level.

μ_{kit} above. Firm-level market shares are immediate to calculate in data by dividing firm-level turnover by the corresponding 5-digit NAF sector turnover. This yields time series for s_{kit} for each firm in data.

Table 2 summarizes the results. Columns (1) and (2) evaluate the firm-level relation between inverse markup and market share in the data. Pooling all firm-level data (across sectors and years) for a total of over 10 million observations of markups and market shares gives a negative and statistically significant coefficient, validating the prediction of the model. Further, including year fixed effects does not alter this relation.

In columns (3) and (4), by additionally imposing firm-fixed effects (or firm and year fixed effects), we allow for unobserved heterogeneity across firm markups. In agreement with the model, we again find a negative and significant coefficient albeit the point estimate is now smaller, indicating the importance of controlling for unobserved heterogeneity in the data.

Turning to our sector-level predictions, by the same logic as above, note that taking the inverse of equation (8) yields:

$$\mu_{kt}^{-1} = \frac{\varepsilon - 1}{\varepsilon} - \frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon} HHI_{kt}$$

where μ_{kt} is sector k's markup at time t, and HHI_{kt} is the Herfindahl-Hirschman Index. Following the same strategy, we test this relationship in data with the following empirical specification:

$$\mu_{kt}^{-1} = \gamma_k + \alpha_t + \beta HHI_{kt} + \epsilon_{kt} \tag{47}$$

Table 3: Sector Inverse Markup and Sector Herfindahl: Level

	Dependent Variable: μ_{kt}^{-1}			
	(1)	(2)	(3)	(4)
HHI_{kt}	-.7267	-.7281	-.4335	-.4436
	(.2310)	(.2315)	(.1102)	(.1094)
Year FE	N	Y	N	Y
Sector FE	N	N	Y	Y
Number of Sectors	504	504	504	504
Observations	11 592	11 592	11 592	11 592

NOTE: μ_{kt} is sector k gross markup in year t, HHI_{kt} gives Herfindahl-Hirschman index of concentration k. Column (1-4) reports empirical estimates for the FICUS-FARE (1994-2016) data, aggregated to sector level. Standard errors (in parenthesis) are clustered at sector level.

where γ_k is now a sector fixed-effect controlling for unobserved sector-specific heterogeneity, α_t a time fixed-effect, and the coefficient of interest is again β, predicted to be negative by the model. Notice that we can again readily construct the dependent and independent variables in this regression by aggregating from the firm-level French census data. In particular, we aggregate firm-level markups to their sector-level counterparts by taking an harmonic weighted average of firm-level markups - as instructed by the model equation (7) - within narrowly defined 5-digit NAF sectors, for a total of 504 sectors. Taking the sum of squared firm-level market shares readily produces series for sectoral HHI indexes (at the same level of disaggregation). Table 3 summarizes the results.

As before, columns (1) and (2) of Table 3 show, respectively, the results of a pooled regression across all sectors and years and pooled regression across sector but controlling for time fixed effects. In agreement with model predictions, both indicate a negative and significant relation between the level of concentration and the inverse of markups in a sector. Column (3) further controls for unobservable heterogeneity by controlling for sector-fixed effects while Column (4) includes sector and time fixed effects. The implied estimates again assert a negative, stable and statistically significant relation between concentration and inverse sectoral markups.

Taken together this set of estimates confirms the basic qualitative predictions of our model in the French data, both at the firm and sector level. Note, however, that our model additionally imposes cross-equation restrictions. To see this, compare the two expressions above and note that, according to the model, the slope coefficients of these two relations - i.e. the slope of the inverse of firm markup on market share and the slope of the inverse sector markup on HHI - should coincide. We can formally test for the equality of two slope coefficients in the data by forming a simple Z-score. We focus on the specifications controlling for cross-sectional and time fixed effects (i.e. the estimate in Table 2, Column 4 versus that in Table 3, Column 4) and find that, despite the differences in the point estimates, we cannot reject the null that the two coefficients are the same (Z-score of 1.20).

Finally, as an additional robustness check, in Appendix B.4 we report the results of an alternative first-difference (rather than level) specification, where we investigate the cross-sectional and within-firm relations between firm-level (respectively, sector) markups and market shares (resp., HHI), for which our model carries the same predictions. Overall, the qualitative results of this alternative econometric specification again confirm the conclusions set out above.

4.2 Reduced Form Varieties of Markup Cyclicality

Our theoretical framework yields a simple relation between markups and competition: the level of a firm's markup is determined by its market share within a sector. Further, as shown above, general equilibrium aggregation of this relation yields a relation between a sector's markup and its level of concentration, both across sectors and dynamically, over the business cycle. As we've seen, the data broadly support all these relations.

In contrast, there is a large applied literature investigating different definitions of "markup cyclicality". This literature yields a variety of results, with some contributions arguing for pro-cyclicality while others conclude in favor of counter-cyclicality.

In this section, we argue that these conflicting empirical results can be largely ascribed to the alternative reduced-form exercises pursued and, in particular, to the reduced-form definitions of markup cyclicality being deployed in the literature. Importantly, as we will show, our model with firm-level shocks only can go a long away in accounting for these seemingly conflicting reduced-form relations in data.

To do this, we start by exploring a notion of firm-level markup cyclicality recently proposed in the literature Hong (2017). In particular we ask whether firm markups covary systematically with respect to sector-level output. We find that this reduced form relation is weakly counter-cyclical in the data. We then proceed to evaluate notions of sector-level markup cyclicality. Following Nekarda and Ramey (2013), we ask whether sector markups comove with sector output over the business cycle. Like Nekarda and Ramey (2013), we find evidence for a positive systematic comovement between the two measures, or 'pro-cyclicality'. Finally, we follow the recent contribution of Bils et al. (2018) who investigate yet another notion of cyclicality: do sector level markups comove systematically with aggregate output (i.e. GDP)?

Like the authors we find no strong evidence for pro-cyclicality according to this definition. In the French data this correlation is positive but statistically insignificantly different from zero.

By reproducing these distinct reduced-form investigations in our data, we thus arrive at the same apparently conflicting conclusions as the literature: for the same data, and using the same consistent measure of markups throughout, we can conclude that markups are countercyclical, procyclical or acyclical, depending on the notion of cyclicality we espouse. Importantly, we show that our model can reconcile these apparently contradictory results.

4.2.1 Firm-Level Evidence

We start by analyzing a firm-level notion of reduced-form markup cyclicality and ask whether, in the data, firm markups covary with the respective sector-level output.

Before going to the data, it is useful to first recall that our setting is a granular one, in which extensive within sector-heterogeneity in the firm size distribution renders possible that large firm dynamics lead the sector business cycle. In particular, in our setting with idiosyncratic firm-level shocks only, sector output fluctuations are necessarily led by shocks to very large firms. To make matters concrete consider a positive idiosyncratic (demand or technology) shock hitting a large market share firm. Given the granular nature of the economy, the corresponding sector output will increase. Following our analysis in Sections 3 and 5.1 above, we additionally know that - within the model - this large firm will increase its market share and its markup. This implies that large firm markups should comove positively with sector output.

By the same token, and following our discussion in Proposition 6, the average (small) firm in a given sector is losing market share to the very largest firms: if sector output expansions are led by large firms, the latter will be increasing their market share while the average firm, by virtue of our independent shock assumption, loses competitiveness - as evaluated by its market share within the sector. Again, due to the markup-market share relation in our setting, this implies that the average firm-markup is expected to comove negatively with sector output, as summarized by Proposition 6.

To evaluate this prediction we implement the following reduced-form regression, both in the data and in our model-simulated data:

$$\ln(\mu_{kit}) = \alpha_i + \alpha_t + \beta_1 s_{kit} + \beta_2 \widehat{Y}_{k,t} + \beta_3 \widehat{Y}_{k,t} * s_{kit} + \epsilon_{it} \tag{48}$$

where, consistently with our notation throughout μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t and $\widehat{Y}_{k,t}$ is the deviation of sector

Table 4: Firm Markup and Sector Output

Coefficient	Estimate (data)	Cournot ($\varepsilon = 6, \sigma = 1.6376$) (model)
$\widehat{Y}_{k,t}$	-.068	-.023
	(.035)	
$\widehat{Y}_{k,t} * s_{kit}$	.192	.124
	(.043)	
Firm Controls	Y	Y
Firm FE	Y	Y
Year FE	Y	Y
Number of Firms	1 267 705	506 576

NOTE: μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t and $\widehat{Y}_{k,t}$ is the deviation of sector k (log) real value added in year t from its HP trend. Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data. Column (2) reports estimates based on model simulated data. We control for the time-varying firm market share, s_{kit}, in all regressions. Standard errors clustered at sector-level.

k (log) real value added in year t from its HP trend.[15]. Finally, α_i is a firm fixed effect, controlling for time-invariant firm-level unobservables determining the average level of a firm's markup while α_t is a year fixed effect. In this specification, β_2 captures the average correlation between firm markups and their respective sector output while coefficient β_3, in the interaction term, captures heterogeneity in this relation as a function of a firm's market share.

Before proceeding, note that Hong (2017) runs a version of this regression, where (i) Y is aggregate (rather than sector) value added; (ii) markups are estimated following a similar strategy to that used in our paper and (iii) data correspond to firms in manufacturing sectors for four large European countries from the well known BVD-Amadeus database. For this data, Hong finds a negative β_2 estimate, concluding that (i) in the data 'markups are countercyclical' and (ii) that there is 'substantial heterogeneity in markup cyclicality across firms, with small firms having significantly more countercyclical markups than large firms.'

The second column in Table 4 summarizes the estimates obtained when implementing the above reduced-form regression on our French census data. We see that, for the average firm, markups are weakly 'countercyclical' with respect to own sector output. Further, we additionally confirm that there is substantial heterogeneity in this relation. In particular, the estimates on the interaction term imply that large firms - roughly with market shares above 0.35 - are procyclical with respect to the dynamics of sectoral output.[16]

[15] To obtain sector real value added, we sum firm-level nominal value added to the NAF 5 digit level and deflate using EUKLEMS sectoral price deflators

[16] We also arrive to similar results when we do not include the market share as a separate control.

The third column in Table 4 implements the same reduced form regressions on model simulated data. Specifically, given our model calibration, we simulate 100 period histories for 514 207 firms distributed across 504 sectors.[17] We then record the simulated data of firms' output, market share and equilibrium sectoral output. Finally, we implement the same reduced form regression on this simulated dataset.

Overall, the model is able to qualitatively reproduce the patterns observed in the data. Consistent with Proposition 6, markups for the average firm are weakly countercyclical with respect to own sector output. Additionally, the model predicts that - as in the data - large firms' markups are procyclical with respect to own sectoral output.

4.2.2 Sector Level Evidence

We now explore sector-level, reduced-form, notions of markup cyclicality. In particular, we first follow Nekarda and Ramey (2013) and ask whether sector markups covary with own-sector output?

It is useful to first recall what the bottom-up mechanics of our environment imply for such correlation. In our setting firms are granular which implies that large firm dynamics drive the business cycle at the sector level. At the firm-level, a positive idiosyncratic demand or technology shock to, say, the sector's largest firm implies that the firm's output and markup both increase (along with its market share). By virtue of granularity, this shock will not "average out" at the sector-level, implying that the corresponding sector's output and markup also increase. This in turn implies that, as encoded in Propositions 3 and 5, we should expect a positive covariance between sector markup and sector output.

To test this intuition in the data, we implement the following sector panel regression:

$$\widehat{\mu}_{kt} = \alpha_k + \alpha_t + \beta \widehat{Y}_{k,t} + \epsilon_{kt}, \tag{49}$$

where $\widehat{\mu}_{kt}$ ($\widehat{Y}_{kt}$) denotes sector k's log change in markup (output) between $t-1$ and t. Sector-level markups are aggregated from firm-level estimates, according to a harmonic weighted average as instructed by the model. We measure output in the data by real value added. Finally, we control for possible sector level and year level unobservable correlates by allowing for α_k and α_k, respectively sector and year fixed effects.

Note that Nekarda and Ramey (2013) run exactly this regression on U.S. sectoral data by sourcing data from the NBER CES manufacturing database and estimating markups based

[17] The next draft will include longer simulations from which we can then study the small sample behavior of the model as we currently do below in Sections 4.2.3 and 4.3.

Table 5: Sector Markup and Sector Output

Coefficient	Estimate (data)	$\varepsilon = 6, \sigma = 1.64$ (model)
$\Delta Y_{k,t}$	.102	.155
	(.028)	[.040; .176]
Sector FE	Y	Y
Year FE	Y	Y
Number of Sectors	504	504

NOTE: Regression of sector-level markup growth ($\widehat{\mu}_{kt}$) on sector real value added growth ($\widehat{Y}_{kt}$). Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data and standard errors (in parenthesis) are clustered at the sector level. Columns (2) and (3) report estimates based on model simulated data. Point estimates for these columns give the median coefficient obtained from running the reduced form regression over 5000 independent simulated samples, each of the same length (22 years) as the French data. Terms in square brackets give, respectively, the 0.025 and 0.975 quantiles of coefficient estimates from simulated data.

on the dynamics of the labor share (at the sector level) and generalizations thereof. They find that β is robustly positive and significant and therefore that "markups are generally procyclical (...) hitting troughs during recessions and reaching peaks in the middle of expansions." Table 5 reports the coefficient estimates obtained with our data. Despite the differences regarding the country of analysis, sample period and the methods deployed to estimate markups, we confirm the qualitative findings of Nekarda and Ramey (2013): sector markups comove positively and significantly with sector output.

The final column in Table Table 5 additionally reports the coefficient obtained by performing the same reduced form regression on our model simulated data. In particular, this coefficient is obtained by simulating 5000 independent samples, each of the same length (22 years) as the French data. For each simulated sample, we implement the sector-level regression above and store the implied estimate. Table 5 reports the resulting median coefficient. Figure 1 gives the full histogram of estimated coefficients for the 5000 simulated samples. As in the data, the model implies a positive correlation between sector markups and sector output.

The recent work by Bils et al. (2018) explores yet another reduced-form notion of markup cyclicality: do sector-level markups comove systematically with aggregate GDP fluctuations? Note that, unlike Nekarda and Ramey (2013), markup cyclicality is evaluated with respect to its comovement with aggregate GDP rather than sector-level output.

To understand this form of comovement in the context of our model, it is useful, to first note that sector markups only react to within-sector firm shocks. Thus, as summarized in Proposition 7, positive comovement of a sector's markup with aggregate GDP is to be expected if, in a given cyclical episode, the fluctuation in aggregate economic activity is due to large firm

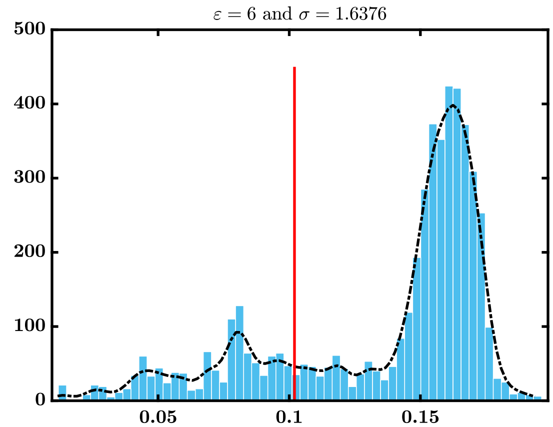

Figure 1: Histogram of Sector Markup on Sector Output Slopes in Model Simulations

NOTE: Kernel density of estimated regression coefficient on model simulated data from equation (49) based on 5000 repetitions of independent samples.

dynamics in the same sector. However, whenever a sector comoves negatively with aggregate output, negative correlation of that sector's markup with aggregate output will obtain. More generally, with idiosyncratic shocks and no input-output linkages, if aggregate output dynamics movement reflects shocks hitting other sectors in the economy we should expect a weak correlation between the average sector's markup and aggregate GDP fluctuations.

To explore this intuition, we follow Bils et al. (2018) and implement the following regression:

$$\widehat{\mu}_{kt} = \alpha_k + \beta \widehat{Y}_t + \epsilon_{kt} \tag{50}$$

where $\widehat{\mu}_{kt}$ is the deviation of sector k's markup in year t from its HP trend, $\widehat{Y}_t$ gives the HP-trend deviation of (log) aggregate real value added in year t and α_k is a sector fixed effect.

Bils et al. (2018) implement this specification based on US KLEMS industries from 1987-2012. They conclude that, for this data, "the price markup is estimated to be highly countercyclical" with the possible exception of service industries for which they find evidence favoring procyclicality.

Table 6 summarizes the estimates obtained from our French census data. Unlike, Bils et al. (2018) we do not find a statistical significant relation between sectoral markups and aggregate GDP. Rather, for the average French sector, the data suggests that this relation is acyclical.

The final columns in Table 6 repeat the exercise but this time based on model-simulated data, again under two distinct parametrizations for the parameter governing elasticities of

Table 6: Sector Markup and Aggregate Output

Coefficient	Estimate (data)	$(\varepsilon = 6, \sigma = 1.64)$ (model)
$\widehat{Y}_t$	.191	.471
	(.161)	[-.077, 1.103]
Sector FE	Y	Y
Number of Sectors	504	504

NOTE: Regression of sector k's markup in year t in deviation from its HP trend $\widehat{\mu}_{kt}$ on $\widehat{Y}_t$, the HP-trend deviation of (log) aggregate real value added in year t. Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data. Standard errors (in parenthesis) are clustered at the sector level. Columns (2) and (3) reports estimates based on model simulated data. Point estimates for these columns give the median coefficient obtained from running the reduced form regression over 5000 independent simulated samples, each of the same length (22 years) as the French data. Terms in square brackets give, respectively, the 0.025 and 0.975 quantiles of coefficient estimates from simulated data.

subsitution across varieties. As in our previous exercise, the numbers on Table 6 give the median estimate of the slope of simulated sectoral markups on equilibrium aggregate GDP across 5000 independent samples of 22 years. Figure 2 below gives the full histogram of the implied slope coefficients for each simulated sample.

Our model implies a positive median slope between the average sector markup and aggregate GDP. However, as in the data, the histogram points to considerable uncertainty over the magnitude and sign of this slope parameter. This is consistent with the intuition above: over small samples we may expect either positive or negative correlation to arise depending on whether the sectors driving within-sample aggregate dynamics have, respectively, higher or lower levels of sectoral markups with respect to the aggregate. However, over long enough samples (or many independent small samples as we have here) we expect the markup of the average sector to be weakly positively correlated with aggregate GDP. Thus, both the model and the data imply that markups are acyclical when evaluated through this particular reduced form statistic.

4.3 Granular Markup Cyclicality

In this final section we turn our attention to aggregate markup fluctuations. Recall that, in our environment there are no aggregate shocks and all markup movements - at the firm, sector and aggregate levels - have their origins in idiosyncratic firm-level shocks and general equilibrium adjustments both within and across sectors.

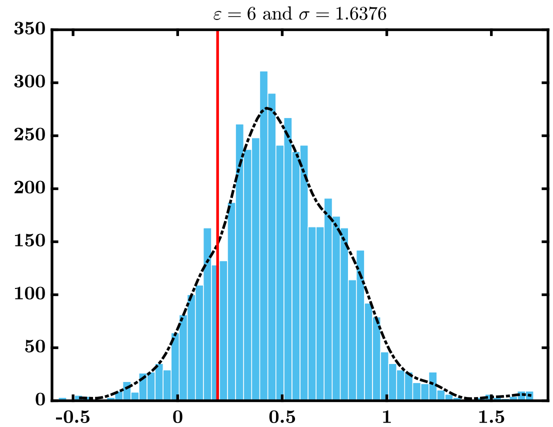

Figure 2: Histogram of Sector Markup on Aggregate Output Slopes in Model Simulations

NOTE: Kernel density of estimated regression coefficient on model simulated data from equation (50). 5000 repetitions of independent samples. The median regression coefficient in Panel A (respectively, Panel B) is 0.208 (resp 0.072).

Thus, in our setting, any aggregate fluctuation needs to have a granular origin in the dynamics of large firms (see Gabaix (2011) and Carvalho and Grassi (2019)) so that idiosyncratic shocks do not average out. However, while relying on these very large firms - which arguably hold significant market power - the extant literature (see Grassi (2018) for an exception) has simultaneously maintained that these firms are price takers and do not internalize the effects of their decisions on sector and economy-wide aggregates. The quantitative exercise below thus extends previous insights by allowing for the strategic behavior of large firms and asking whether aggregate markup fluctuations can have granular origins.

Note also that, in a granular world, allowing for oligopolistic competition is expected to weaken the aggregate output response to given firm-level shocks. To see this note that, if following a positive productivity shock to a large firm, the latter raises markups, its equilibrium output response (taking other aggregates as given) will necessarily be smaller than if pass-through was complete. This, in turn, raises the possibility that a granular setup with variable markups - as we have here - may yield quantitatively weak aggregate output responses relative to a constant markups (or perfect competition) case, as discussed in Section 2, equation (42). Therefore, a second question of interest is whether sizable aggregate effects (as documented in Carvalho and Grassi (2019)) survive in this more general oligopolistic environment.

Finally, we are interested in understanding markup cyclicality in the aggregate. Recalling Proposition (7) in Section 2, our setting implies a positive comovement between aggregate

	(1) Data			(2) Cournot ($\varepsilon = 6, \sigma = 1.64$)			(3) Monop ($\varepsilon = 6, \sigma = 1.64$)		
	σ_x	σ_x/σ_Y	$\rho(x, Y)$	σ_x	σ_x/σ_Y	$\rho(x, Y)$	σ_x	σ_x/σ_Y	$\rho(x, Y)$
Y_t	1.71	1	1	0.64	1	1	0.78	1	1
μ_t	0.96	0.57	0.13	0.18	0.46	0.84	0	0	na

Table 7: Aggregate Markup and Aggregate Output

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time series correlations, $\rho(x, Y)$, for aggregate output Y_t and aggregate markup μ_t, both in deviations from their HP trend. Column (1) reports empirical estimates for the FICUS-FARE (1994-2016) data. Column (2) and (3) report estimates based on model simulated data under Cournot and monopolistic competition respectively.

output and aggregate markups unless a particular expansionary episode is driven by sufficiently low sectoral markup sector, in which case negative comovement may obtain. This in turn implies that, while over sufficiently long samples we should observe positive comovement, in any given short sample, comovement may be absent or negative depending on sectors driving the dynamics over the short sample dynamics.

To assess this we again rely on our model calibration and the simulated histories of roughly five hundred thousand firms distributed across 504 sectors (which we've used in the previous section). After recording the simulated data for firms' markups outputs and market shares, we construct aggregate markup series, μ_t, by taking a weighted harmonic mean of firm-level markups and aggregate GDP, Y_t, as the sum of firm level value-added. We then implement the exact same procedure in the FICUS-FARE census data and its firm-level markup estimates and HP-detrend all series. Table 7 compares the data- and model-based estimates we obtain for aggregate output and markup fluctuations.

A first conclusion from Table 7 is that, qualitatively, the moments calculated from the aggregated firm-level data and from our simulated firm-level panel agree: the volatility of aggregate output is larger than the volatility of aggregate markups and aggregate markups are procyclical with respect to aggregate output.

Second, quantitatively, in our calibrated model with variable markups, the volatility of aggregate output is 37% that observed in the data. This ratio is roughly 45% in a monopolistic competitive specification of the model given the same volatility of shocks. Thus, while incomplete pass-through reduces the aggregate volatility implied by firm-level shocks, the magnitude of aggregate fluctuations remains quite sizable.

Third, the volatility of the aggregate markup relative to the volatility of output (on average across shock realizations) is 57% in the data and 46% in our calibrated model. Thus, while

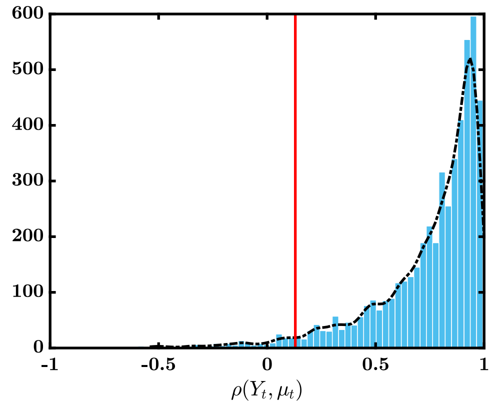

Figure 3: Histogram of $\rho(\mu_t, Y_t)$ in Model Simulated Data

NOTE: Kernel density of $\rho(\mu_t, Y_t)$, the correlation coefficient between aggregate markups and aggregate output on model simulated data based on 5000 repetitions of independent 22 periods samples.

the model only generates 37% of aggregate output volatility in the data, the level of volatility of markups relative to output is quite close to that observed in data.

Finally, both in the data and in the model, aggregate markup is procyclical with respect to aggregate output. Our model predicts much higher aggregate procyclicality than that observed in data: the correlation between the aggregate markup and aggregate output is 13% in the data and 84% (on average across shock realizations) in the model.

However, as discussed above, our model also predicts variation in this correlation coefficient across small samples, depending on which sectors are driving aggregate dynamics and their relative levels of sectoral markups. To see this variation at play, Figure 3 plots the histogram of the correlation coefficient $\rho(\mu_t, Y_t)$ from 5000 independent simulated samples, each of the same length (22 years) as the French data. As is clear from the histogram, there is a substantial amount of variation in model-implied aggregate markup cyclicality over small samples. Indeed, we find a non-negligible number of samples where model simulations display only weak procyclicality (as in the data) or even countercyclicality. To understand this, note that over small samples aggregate output dynamics may be driven by positive shocks to large firms in large sectors that nevertheless have relatively lower markups. If this is the case, as shown by equation (45) in Section 2, then the positive covariance of aggregate output and aggregate markups weakens and may turn negative.

Further, note that if we were to superimpose aggregate TFP shocks in the model, the implied correlation between aggregate output and aggregate markups would further decline. This is a direct implication of the fact that firm-level markups do not respond to proportional shifts in productivity affecting all firms, as discussed in Section 1.2. Therefore the correlation between aggregate markup and aggregate output would decline, as aggregate markups would remain unchanged but aggregate output volatility would now reflect this additional (common) stochastic shifter in productivity. If we include aggregate TFP shocks to match the aggregate output volatility in the data, the correlation between aggregate markups and aggregate output falls to 0.05.

5 Conclusion

[TBA]

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A Analytic results

A.1 Global between / within decomposition of changes in sectoral markups

The change in the inverse of the sectoral markup between two time periods is, by equation (7),

$$\frac{1}{\mu_{kt'}} - \frac{1}{\mu_{kt}} = \sum_{i=1}^{N_k} \left(\frac{s_{kit'}}{\mu_{kit'}} - \frac{s_{kit}}{\mu_{kit}} \right) \tag{51}$$

This change in sectoral markups can be decomposed into a within term (i.e. changes in firm-level markups evaluated at firms' expenditure share averaged over both time periods) and a between term (i.e. changes in expenditure shares evaluated at firm-level markups averaged over both time periods) as follows:

$$\frac{1}{\mu_{kt'}} - \frac{1}{\mu_{kt}} = \sum_{i=1}^{N_k} \frac{1}{2} \left[(s_{kit'} + s_{kit}) \left(\frac{1}{\mu_{kit'}} - \frac{1}{\mu_{kit}} \right) + \left(\frac{1}{\mu_{kit'}} + \frac{1}{\mu_{kit}} \right) (s_{kit'} - s_{kit}) \right] \tag{52}$$

Note that if markups are equal across firms (as is the case with $\sigma = \varepsilon$), then all terms in (52) are equal to zero.

It is straightforward to show that, by equation (5) under Cournot competition, the within and the between terms in (52) are equal to

$$\frac{1}{2} \sum_{i=1}^{N_k} (s_{kit'} - s_{kit}) (s_{kit'} + s_{kit}) \left(\frac{1}{\sigma} + \frac{1}{\varepsilon} \right)$$

Therefore, under Cournot competition the contribution in changes in sectoral markups of the between and the within terms is 50% each, irrespective of the values of σ and ε (as long as $\sigma \neq \varepsilon$). If σ is close to ε, then firm-level markups are less responsive to shocks (reducing the within term), but firm-level markups are also less heterogeneous across firms (reducing the between term).

A.2 Firm-level expenditure shares

We now return to changes in expenditure shares introduced in equation (23). Combining (23) and (24),

$$\widehat{s}_{kit} = \alpha_{ki} \left[\widehat{V}_{kit} - \frac{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'} \widehat{V}_{ki't}}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}} \right]. \tag{53}$$

The response of firm i's expenditure share to a firm i shock is

$$\widehat{s}_{kit} = \alpha_{ki} \left[1 - \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \right] \widehat{V}_{kit}. \tag{54}$$

Finally, we can express the variance of expenditure shares as

$$\mathbb{V}ar\left[\widehat{s}_{kit}\right] = \left(\frac{\alpha_{ki}\sigma_v}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \right)^2 \left[\left(\sum_{i'\neq i}^{N_k} s_{ki'}\alpha_{ki'} \right)^2 + \sum_{i'\neq i}^{N_k} \left(s_{ki'}\alpha_{ki'}\right)^2 \right]. \tag{55}$$

A.3 Additional derivations for equations (33)

In response to sector k shocks only, the change in aggregate output is (by equation 41)

$$\widehat{Y}_t = (1+f\eta)^{-1} s_k \left[-\left(f + 1 + (\sigma-1)\left(1 - \frac{\mu}{\mu_k}\right)\right) \widehat{P}_{kt} + \frac{s_k\mu}{\mu_k}\widehat{\mu}_{kt} \right] \tag{56}$$

and the change in aggregate price is $\widehat{P}_t = s_k\widehat{P}_{kt}$. Substituting these expressions into equation (32), we obtain equation (33).

A.4 Decreasing returns to scale

The production function is now given by

$$Y_{kit} = Z_{kit}L_{kit}^{\beta}. \tag{57}$$

where $\beta \leq 1$. Marginal cost is

$$MC_{kit} = \beta^{-1}W_t\left(Y_{kit}\right)^{(1-\beta)/\beta}\left(Z_{kit}\right)^{-1/\beta}. \tag{58}$$

or using $Y_{kit} = s_{kit}P_{kt}Y_{kt}$,

$$MC_{kit} = \beta^{-1}W_t\mu_{kit}^{\beta-1}\left(P_{kt}Y_{kt}s_{kit}\right)^{(1-\beta)}\left(Z_{kit}\right)^{-1}. \tag{59}$$

The firm-level markup, μ_{kit}, is defined as the ratio of price to marginal cost, and is related to expenditure shares by equation (5) which does not depend on β.

Labor payments of firm i in sector k are

$$L_{kit}W_t = \beta\mu_{kit}^{-1}P_{kit}Y_{kit}$$

and profits (revenues minus labor payments) are

$$\Pi_{kit} = \left(1 - \beta\mu_{kit}^{-1}\right) P_{kit} Y_{kit}.$$

We define the sectoral markup as the ratio of sectoral revenues to labor payments,

$$\mu_{kt} \equiv \frac{P_{kt} Y_{kt}}{W_t L_{kt}}, \tag{60}$$

which can be expressed as a function of firm-level markups and expenditure shares,

$$\mu_{kt}^{-1} = \beta \sum_{i=1}^{N_k} \mu_{kit}^{-1} s_{kit}. \tag{61}$$

The 50-50 between/within decomposition of changes in sectoral markups under Cournot competition derived in Appendix 1 holds irrespectively of the value of β.

The expenditure share of firm i in sector k, using $P_{kit} = \mu_{kit} MC_{kit}$, satisfies

$$s_{kit} = \frac{V_{kit}\left(\mu_{kit}^{\beta} s_{kit}^{1-\beta}\right)^{1-\varepsilon}}{\sum_{i'=1}^{N_k} V_{ki't}\left(\mu_{ki't}^{\beta} s_{ki't}^{1-\beta}\right)^{1-\varepsilon}}. \tag{62}$$

Equilibrium firm-level expenditure shares and markups are the solution to equations (5) and (62).

Log-linearizing this equation, and using $\widehat{\mu}_{kit} = \Gamma_{ki}\widehat{s}_{kit}$, we obtain the analog to equations (20) and (21):

$$\widehat{s}_{kit} = \widehat{V}_{kit} + (1-\varepsilon)\,\Lambda_{ki}\widehat{s}_{kit} - \sum_{i'=1}^{N_k} s_{ki'}\left(\widehat{V}_{ki't} + (1-\varepsilon)\,\Lambda_{ki'}\widehat{s}_{ki't}\right), \tag{63}$$

where $\Lambda_{ki} = \beta\Gamma_{ki} + 1 - \beta$. Note that $\Gamma_{ki} < \Lambda_{ki}$ if and only if $\Gamma_{ki} < 1$.

We can follow similar steps to obtain expressions for changes in sectoral markups and prices to firm-level shocks, as well as the implied variances and covariances.

B Empirics

B.1 Data

Data description

B.2 Aggregate Markup

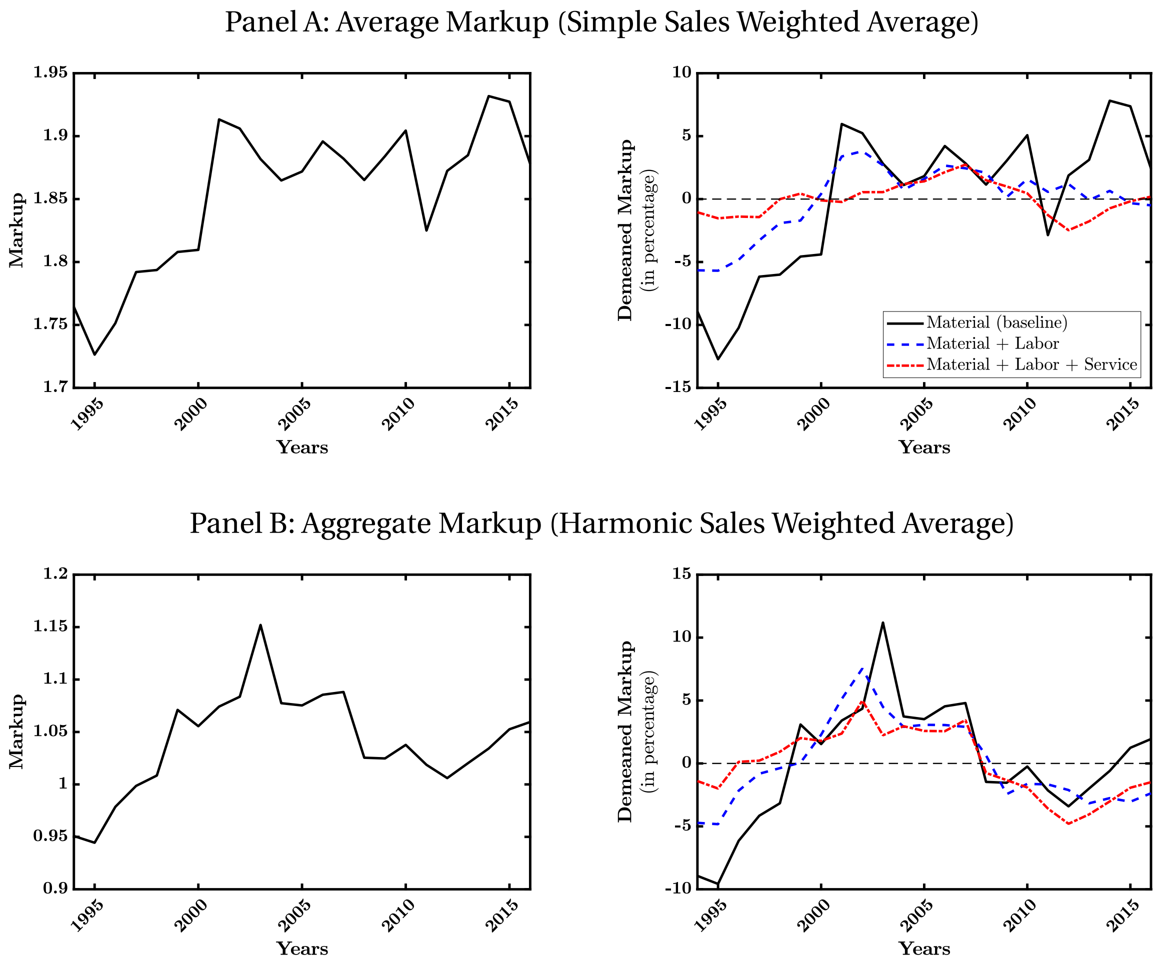

Figure 4: Trend in Average and Aggregate Markup in France

NOTE: Panel A reports sales-weighted simple average of firm level markup. Panel B reports sales-weighted harmonic average of firm level markups. The panels on the rights report average markup substract by their mean over the time periods for various measure firm-level markup. The black line is the average of our baseline markup measure. The blue dashed line reports the average of firm-level markup based on the sum of labor and materials as a variable input. The red dotted line reports the average of firm-level markup based on the sum of labor, materials and service as a variable input.

B.3 Markups, Market Shares and Concentration: Robustness

This appendix derives additional cross-sectional and within-firm predictions for the empirical relation between (a) firm markup and firm market share and (b) sector markup and sector concentration.

Table 8: Firm Inverse Markup and Market Share: First Difference

	Dependent Variable: $\Delta\mu_{kit}^{-1}$			
	(1)	(2)	(3)	(4)
Δs_{kit}	-.511	-.508	-.157	-.129
	(.156)	(.156)	(.140)	(.140)
Year FE	N	Y	N	Y
Firm FE	N	N	Y	Y
Number of Firms	1 135 547	1 135 547	976 612	976 612
Observations	9 328 004	9 328 004	9 169 029	9 169 029

NOTE: $\Delta\mu_{ikt}^{-1}$ is the first difference of the inverse of firm i sector k gross markup in year t, Δs_{kit} gives the first difference of market share of firm i in sector k. Column (1-4) reports empirical estimates for the FICUS-FARE (1994-2016) data. Standard errors (in parenthesis) are clustered at firm level. Markup are winsorized at the 2% level.

Proceeding in parallel to the results in the main text, taking first difference of equation (46) yields

$$\Delta\mu_{kit}^{-1} = \frac{\varepsilon - 1}{\varepsilon} - \frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon}\Delta s_{kit}$$

where $\Delta\mu_{kit}^{-1}$ and Δs_{kit} are the first difference across time of the inverse firm-level markup and market share respectively. Testing this relationship empirically yields the results in Table 8 that shows a negative and significant coefficient even after controlling for time and firm-fixed effect.

Turning to our sector-level predictions, we follow the same methodology as at the firm-level by taking first difference of equation 47 which yields

$$\Delta\mu_{kt}^{-1} = \gamma_k + \alpha_t + \beta\Delta HHI_{kt} + \epsilon_{kt}.$$

This last equation can be brough to the data by estimating

$$\Delta\mu_{kt}^{-1} = \gamma_k + \alpha_t + \beta\Delta HHI_{kt} + \epsilon_{kt}.$$

Table 9 reports the results with and without firm and time fixed effects. This table shows as predicted by the model a negative and significant estimated coefficient.

Table 9: Sector Inverse Markup and Sector Herfindahl: First Difference

	Dependent Variable: $\Delta\mu_{kt}^{-1}$			
	(1)	(2)	(3)	(4)
ΔHHI_{kt}	-.3395	-.3498	-.3447	-.3553
	(.1465)	(.1466)	(.1491)	(.1527)
Year FE	N	Y	N	Y
Sector FE	N	N	Y	Y
Number of Sectors	504	504	504	504
Observations	11 088	11 088	11 088	11 088

NOTE: μ_{kt} is sector k gross markup in year t, HHI_{kt} gives Herfindahl-Hirschman index of concentration k. Column (1-4) reports empirical estimates for the FICUS-FARE (1994-2016) data, aggregated to sector level. Standard errors (in parenthesis) are clustered at sector level.