Oxford Review of Economic Policy, Volume 35, Number 4, 2019, pp. 614–637

The role of networks in antitrust investigations

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Abstract Antitrust investigations typically focus on the competitive pressures coming from within the defined markets of interest. However, competitive pressures can also come from other markets. Even when individually these markets place only weak constraints on one another, collectively they may matter. A networks approach to modelling competition permits a systemic view of competition that can sometimes paint a more accurate picture. We demonstrate this through some simple examples, and show more generally how tools from the networks literature can be applied to capture competition across a system of interrelated markets. As a leading example, we consider antitrust investigations into supermarkets where local geographic markets have been used as the basis of investigation.

Key words: antitrust, competition, networks, networked market

JEL classification: L13, L14, L40

I. Introduction

Despite a surge of research over the last decade or so related to networks and markets, the ideas from this research programme have not yet influenced antitrust investigations in a meaningful way. In this article we investigate the possible reasons why not, and consider whether the academic literature on networks has practical insights, methodologies, and approaches that could be incorporated into antitrust investigations to help them better serve the interests of consumers.

We argue that antitrust investigations could benefit from a more holistic view that the lens of network theory can provide, but also that there are substantial impediments to this being achieved. The substantial impediment is not data—the required data are already largely used to define markets—it is more fundamental than that. Getting the benefits a networks approach can provide requires rethinking the basic framework in which antitrust investigations take place.

A central tenet of both merger and abuse of a dominant position investigations is market definition. We argue that this largely excludes the competitive forces that a systemic network-based approach could internalize. Considering, rather than abstracting from, the cycles and

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We thank Simon Cowan for very helpful comments. Financial support from the European Research Council under the grants 'EMBED' #757229 (Elliott) and #283454 (Galeotti) is gratefully acknowledged.

doi:10.1093/oxrep/grz022

chains of interactions between market participants can be important for anticipating the effects of various antitrust interventions on consumer surplus. The networks literature provides the tools to undertake such a systemic approach and, as already mentioned, often this approach will only require data that are already collected to define markets.

We limit the scope of the article to investigating how a systemic approach might aid antitrust investigations, and focus only on the UK antitrust authorities and UK antitrust cases. Our analysis is mainly confined to some revealing examples that we would like to be taken as a proof of concept. While our focus is on horizontal merger analysis, we note that the points we make apply also to other issues including, but not limited to: vertical merger analysis, exclusive dealing contracts, and price discrimination.

The structure of the article is as follows. First, we briefly review the related networks literature in section II. In section III we review some key components of anti-trust investigations and pose the question of why the related networks literature has had little impact to date. To provide more context, in section IV, we discuss the 2008 Competition Commission Supermarkets Inquiry. This leads us to develop a simple theoretical model in section V capable of capturing competition between markets, as defined within the Competition Commission Inquiry, as well as within markets. In section V we present our main results. Section VI investigates the robustness of our findings to the way in which we model competition. Section VII concludes.

II. Brief overview of the academic literature on networks and markets

Traditionally, economics sees markets as populated by agents interacting anonymously through the price system. The classical paradigm of general equilibrium presumes large economies in which goods are homogeneous, perfectly divisible, information is complete, and market participants take prices as given. Under this paradigm, the law of one price emerges, and, through the market system, resources are allocated efficiently.

But these premises are in contrast to a body of evidence accumulated over time that shows that individual relationships and bonds of trust affect economic outcomes in many relevant cases. The importance of business relationships in markets finds a strong foundation in the sociological literature; see, for example, Granovetter (1973, 1985). As specific supplier-to-supplier, supplier-to-manufacturer, and manufacturer-to-consumer relationships are formed, market participants develop competitive advantages from doing business with specific others. Relevant information is held within a specific relationship, firms differentiate and position themselves in niche markets, and, often, market power and chains of intermediation emerge.

Economists have deepened their understanding of markets by relaxing the assumed absence of market power, leading to the development of a comprehensive theory of oligopoly markets (see Tirole (1988)), a major tool used by competition authorities.[1]

[1] There is also an important body of theoretical work that provides foundations based on game theory for the law of one price, see Rubinstein and Wolinsky (1985, 1990), Gale (1986*a*,*b*, 1987).

But a systematic study of how connections across market participants affect market outcomes, and how this could inform authorities in their methodology to investigate and intervene in markets, has only started to be developed in the last two decades or so; see part VII of Bramoulle *et al.* (2016) for a survey and perspective of this literature. We can divide the study of markets using networks in two related categories: the study of buyer–seller networks and the study of intermediation in networks.[2]

Buyer and seller networks

Manufacturer–supplier relationships take different forms depending on the specific industry. At one extreme, in industries where inputs and final products are standardized, these relationships tend to be determined by anonymous markets. At the other extreme, when a manufacturer needs very specific inputs from the supplier, there is a pressure towards vertically integrated manufacturer–supplier relationships. In the middle, we have networks: manufacturers tend to form closed relationships with a small subset of suppliers. These suppliers acquire firm-specific information and firm-specific know-how to supply made-to-measure inputs to the manufacturer. Yet, manufacturer–supplier relationships are not exclusive and this generates inefficiencies, for example, in investments. The level of these inefficiencies is affected by the competition faced by manufacturers and the structure of the network between suppliers and manufacturers.

Kranton and Minehart (2000, 2001) are probably the first to formalize an economic model of buyer–seller networks and study the implications of these networks for economic performance. In turn, this has led to an active research agenda on buyer–seller networks, see Corominas-Bosch (2004), Polanski (2007), Manea (2011), Elliott (2015), Talamas (2016), and Elliott and Nava (2019). Recently, buyer–seller network models have also been adopted and extended to study the impact of exclusive contracts in markets, see Ramezzana (2019). For a survey on buyer–seller networks see Manea (2016).

Intermediation and resale

In a wide range of markets trade involves a long list of middlemen, connecting producers to buyers. Production and distribution create supply chains, a major example of chains of intermediation. Chains of intermediation are typical in the market for agricultural goods, as well as in financial markets for the trade of assets sold over-the-counter. Resale of event tickets, art, and collectables are another example of intermediated markets often discussed by policy-makers. As products flow downstream along the chain of intermediaries, the terms of trade agreed in each step depend both on horizontal competition among the middlemen as well as on vertical complementarities often present between intermediaries operating at different phases of the chain. The implication of, say, a merger between two intermediaries on the market outcome depends on how it affects horizontal competition viz. the vertical complementarities.

These ideas have recently been formalized in the following articles. Manea (2018) and Condorelli *et al.* (2017) study bargaining models of resale in networks. Gale and Kariv

[2] There is also a body of work that incorporates networks into matching models of markets. The focus of this literature is to derive conditions under which competitive outcomes can be achieved despite trade being decentralized, and possibly organized through trading networks, e.g. Ostrovsky (2008), Hatfield *et al.* (2013) and Fleiner *et al.* (2019).

(2009), Blume *et al.* (2009), and Choi *et al.* (2017) study posted price in networks, Nava (2015) studies quantity competition in networks. Bimpikis *et al.* (2019) study multi-market Cournot competition in a networked market. Malamud and Rostek (2017) and Babus and Condor (2018) study asset trading in networks. For a survey on intermediation and resale in networks, see Condorelli and Galeotti (2016).

At a general level, both in the case of buyer–seller networks and in the case of intermediation networks, the structure of the network matters because it determines outside options and opportunity costs of each relationship. Hence, the effect of a change on a particular connection will not be confined to that relationship. It will spread to close-by connections, and from there to the entire network. To fix ideas:

- In a buyer–seller network, as supplier A develops a new connection with manufacturer B, the terms of trade of supplier A with her other connections, say manufacturer C, changes. In turn, this affects the competitiveness of manufacturer C and B with the manufacturers in the markets that use other suppliers, and these effects will keep spreading in the network. The understanding of the effect for consumer surplus of a vertical merger or a new exclusive contract between a supplier and a manufacturer needs to take into account these feedback effects.
- In networks of intermediaries, a horizontal merger between two middlemen affects the terms of trade between those intermediaries and their upstream trading partners and their downstream partners. In turn, these effects will propagate upstream to initial producers and downstream to final customers.

III. Antitrust investigations

Despite the very active academic research in the last two decades, we are not aware of investigations by competition authorities that take into account these possibly intricate network feedbacks. There are several possible explanations. First, a network approach might require data that are not typically available. The network approach is systemic. All interconnections are considered at once, and it might be expected that the data required for this, rather than breaking the system into parts, would be prohibitively difficult to collect and analyse. Second, it might be expected that, in practice, the network effects are relatively small, and can be safely ignored. Considering within-market competitive constraints might provide a good approximation of overall competition. As long as the market is well defined, the effective competitive pressure could come from within the market. Evaluating these possibilities requires understanding how antitrust investigations are conducted.

The Competition and Markets Authority (CMA) is largely responsible for the enforcement of antitrust policy in the UK, having taken over from the Office of Fair Trading (OFT) and Competition Commission following the Enterprise and Regulatory Reform Act 2013. The first step the CMA takes, as was the case with the OFT and Competition Commission before it, is to define the markets that may be affected. This is true for both a merger and anticompetitive behaviour investigation. Although the focus of this article is on UK antitrust policy, it is worth noting that similar approaches are taken by the European Commission (under Articles 101–109 of the Treaty on the Functioning of the European Union), and in US investigations (under the Sherman Act, Clayton Act, and Federal Trade Commission Act, among others). Only after markets have been defined are anti-competitive effects considered, and this is done through the lens of the defined markets.

Markets are defined using a hypothetical monopolist test. If the market were monopolized, could the monopoly effect a small but significant non-transitory increase in price (SSNIP)? This is referred to as the SSNIP test and in practice is taken to mean about a 5 per cent increase in prices. So, the SSNIP test is used to identify the smallest relevant market within which a hypothetical monopolist would impose a large enough increase in price. In practice, the SSNIP test checks whether an increase in price of about 5 per cent for a basket of goods and services (corresponding to the goods and services offered in the market of reference) would lead consumers to still buy those goods (in which case the SSNIP test is passed), or would induce consumers to switch to substitute products (in which case the definition of market of reference will be expanded). In effect, the SSNIP test calculates the residual elasticity of demand of the proposed basket of goods, and expands the basket until demand is sufficiently inelastic.

The data required for this exercise are not trivial. Indeed, they are closely related to the data requirements of a systemic, network-based approach. In order to exclude a good from a market it must be shown that it exerts a sufficiently weak competitive impact on the market. Once the competitive impact is being estimated for this purpose, the very same estimation can be used to carry out the systemic analysis that we propose. In the main, the data required for a network approach to be taken are already collected in order to define markets. Often data requirements should not be a major impediment to a network approach being used.

Market definition is an important part of antitrust investigations. The submissions of the interested parties and the final reports published by the antitrust authorities often devote considerable space to the issue of market definition. It can be the main focus of investigations. Defining markets first helps to limit the scope of antitrust investigations and simplify them. It may thus create a more certain business environment and help to dissuade some anti-competitive behaviour by allowing firms to better anticipate the outcome of potential investigations.

However, this framework, by largely abstracting from the indirect effects between the defined markets, may also inhibit the ability of regulatory authorities from making decisions that achieve their legislative goal of maximizing consumer surplus or other objectives. The SSNIP test necessarily excludes non-local effects. It misses the cumulative impact of many individually weak interactions between firms that are considered to operate in different markets. A theme from the networked markets literature is that these small local effects can interact in ways that are collectively significant across markets. Further, these interactions can spread anticompetitive effects, as well as constrain them. The way in which the markets are defined necessarily abstracts from these interactions even though their inclusion could affect the qualitative and quantitative conclusions reached by an investigation.

It is precisely the ability to account for all the indirect competitive effects in a complex setting that the networked markets literature brings to the table. This is what it adds on top of simple examples of related phenomena, such as hold-up, the impact of outside options on bargaining, and so on, that are already well known in the industrial organization literature. As the standard approach taken in antitrust investigations is predisposed to ignore these indirect effects, it is perhaps unsurprising that the networked markets literature has had relatively little impact in antitrust investigations to date.[3]

As there is a trade-off involved in taking a more systemic approach to competitive forces— despite the potential to reach better decisions, it also makes the analysis more complicated and potentially less transparent—it is important to know whether a systemic approach might bring substantial benefits or whether the standard approach does a good job approximating competitive forces. Can the weak competitive effects captured by taking a systemic approach to antitrust investigations be safely excluded?

IV. Competition Commission's Supermarkets Inquiry 2008

To evaluate the potential impact of taking a more systemic approach to antitrust it is helpful to consider specific cases rather than argue in the abstract. Here we focus on the Competition Commission's Supermarkets Inquiry (Competition Commission, 2008). Given the recently concluded and somewhat controversial investigation that blocked the merger between Asda and Sainsbury's, a re-examination of this inquiry is timely. The foundations and framework for the Asda and Sainsbury investigation was laid down in this market inquiry. We also focus on this inquiry because market definition was crucial as well as contentious (although as claimed above this is not uncommon), and because the analysis of supermarkets has received considerable attention and benefitted from related academic work. Important and influential academic work has helped to identify competitive forces within markets (see, for example, Smith (2004)) and led to sophisticated econometrics being used in the Competition Commission inquiry, albeit within the context of the current framework.

Further, this investigation built on previous related investigations within the same sector (e.g. investigations into the Cooperative Group's acquisition of Somerfield, and the acquisition of Safeway by Morrisons) and so represents a mature understanding of these markets that has attracted substantial resources and, in our view, been done to a high standard. It provides a high-water mark to improve upon.[4]

In the 2008 Competition Commission inquiry into UK supermarkets the Competition Commission considered defining markets along various dimensions, including store size, product offering, and geography. We pay particular attention here to the geographical dimension, but, as we argue later, considering the other dimensions only adds weight to our conclusions. To inform their choice of how to define markets geographically, the Competition Commission undertook an empirical analysis of profit margins by local

[3] In terms of comparison, we note that authorities regulating financial systems use systemic analysis to shape their policies and regulations. The idea of systemic effects (often referred to as systemic risk) has been central to the policy debate in finance since the financial crisis, and subsequent regulations and market interventions have explicitly tried to address these concerns (see Gai and Kapadia (2019), this issue).

[4] At the time we were revising this article the Competition and Markets Authority (CMA) ruled to block the merger between Asda and Sainsbury (25 April 2019). The CMA report indicates that the methodology used to define the market of reference is based on similar considerations to those that were used in the 2008 inquiry.

competition. They found that on average as the number of competing facias (firms, as opposed to stores which may be run by the same firm) within 10 minutes' drive-time of a store increased, that store's profit margin decreased (paragraph 4.110, Appendix 4.4).

A related finding was that the influence of competitor stores declined with drive-time (4.112, Appendix 4.4). Based on this, a qualitative analysis and the submissions of different interest groups and their comments to the Competition Commission's own analysis, the Competition Commission based the geographic dimension of its market definition on a 10-minute drive-time (and supported its results with several robustness checks).

Once markets are defined locally, competitive pressures from outside the market, and the impacts of increasing profit margins in one market, by raising price or reducing quality, to other markets are not considered. Through feedback loops that are naturally present, we contend that these effects might be substantial. The anti-competitive effects of increasing price or reducing quality can lead others outside the market to make similar choices, that can lead others to make similar choices, and so on, feeding back into the market of reference. These systemic network effects can lead not only to an underestimation of the harm a price increase in a local market can have on consumer surplus overall, but also on consumer surplus within the market that the authority had defined. Moreover, when several such markets are affected at once, for example through a merger of facias (firms), the cumulative impact of these effects can have a substantial global impact on the market that cannot be fully appreciated in a market-by-market analysis.

The market definition framework may have impeded the Competition Commission in the supermarket inquiry from considering such effects. Affected markets had to be defined and a national market definition would have ignored local market heterogeneities that are important, whereas limiting the geographic scope of the market ignores the feedback effects discussed above.

V. Analysis

The Competition Commission inquiry used relatively sophisticated econometric techniques to estimate competitive effects within local markets and to address potential identification problems. And given the market definitions framework it operated within, it seems to have done a good job. However, this framework leads to a focus on local rather than global effects. We introduce a simple and intentionally superficial model of supermarket competition to illustrate how a network approach might be important for determining the impact of a merger. This is intended as a proof of concept, in contrast to the sophisticated econometric model of supermarket competition developed by Smith (2004).

(i) A model of supermarket competition

There are k markets, and in each market there are n independently operated supermarkets. Each supermarket i has a mass α_i of locked/loyal customers. This provides a way of incorporating switching costs into the model (see Klemperer (1987)). Supermarkets also compete for customers who are price sensitive. This is formalized using the Hotelling model to model differentiation in a network context as we explain now.

There are two types of links: strong and weak links. *Strong links* connect supermarkets operating in the same market. We model markets as cliques: supermarkets within a market are fully connected. In each strong link there is a mass 1 of consumers distributed uniformly on the unit interval. *Weak links* connect firms across markets. If market l and l' are connected, this means that there are weak links between every pair of supermarket i in market l and supermarket j in market l'. In each weak link there is a mass $w < 1$ of consumers, also distributed uniformly on the unit interval.

Consumers have unit demand, valuation v, and face a linear transportation cost t meaning that they prefer to purchase from supermarkets they are located closer to. Those consumers located on the edge connecting supermarket i and j choose to buy either from i or j given the prices p_i and p_j, their location on the unit interval between these firms, and the transportation cost. Firm i produces at a constant marginal cost c_i.

Figure 1 illustrates competition both within and across markets. Two structures are considered for the links between markets. A circle network (a one-dimensional torus illustrated in panel (a) of Figure 1), and a torus network (a two-dimensional torus illustrated in panel (b) of Figure 1). In both examples, each market contains four supermarkets (see panel (c) of Figure 1).

For each of the networks we perform the following exercise. First, we derive the equilibrium prices across all markets at once, allowing these prices to be set optimally. We contrast this with the equilibrium prices obtained under the assumption that prices outside the market are fixed. In this latter case, the strategic responses of firms ignore out-of-market competition. In both cases, we then simulate a merger of facias—the merger of two supermarket chains that each operate one store in each market—and compare the two predictions.

We consider two cases.

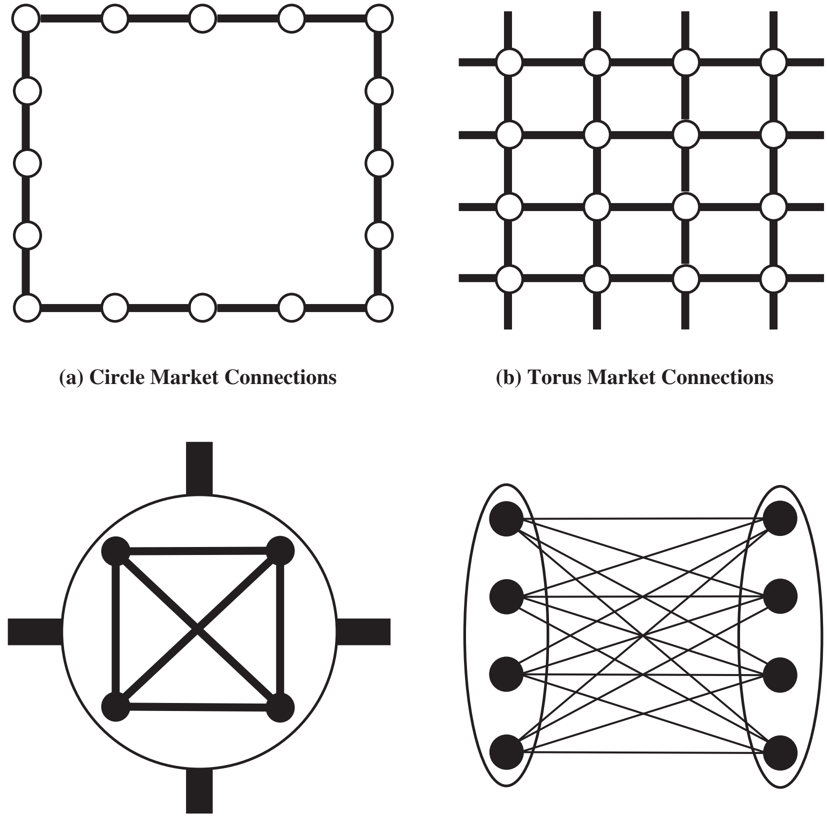
(i) We first suppose that two stores competing across markets compete for 1/10th the mass of customers that two stores competing within a market compete for, i.e. $w = 0.1$.
(ii) We then suppose that two stores competing across markets compete for 1/5th the mass of customers, i.e. $w = 0.2$.

We remark that, in both cases, our choice of w is sufficiently small for each market we define to pass the SSNIP test—a hypothetical monopoly of one of our markets would be able to raise prices by more than 5 per cent.[5] Moreover, the selected values of w are in line with the estimates produced by the Competition Commission report. In comparison, considering Figure 4.9 from the Competition Commission report, the revenue impact of a medium- or large-sized store located within 5 minutes' drive time has 3–4 times the impact on revenues of a store located 10–15 minutes' drive time away (which is on the boundary of being excluded from the market).

We find that ignoring the out-of-market competition leads pre-merger prices to be estimated correctly. However, when we consider a merger of facias and study the resulting

[5] We do not report the calculation here, but they are available upon request from the authors.

Figure 1: Competition within and across markets



(a) Circle Market Connections **(b) Torus Market Connections**

(c) Within Market Connections **(d) Across Market Connections**

Notes: Panels (a) and (b) illustrate connections between markets. Each node represents a market and a link in these panels represents that every pair of firms in these different but connected markets competes for a small mass of customers located on a Hotelling line between them. Panel (c) illustrates competition within a market. Nodes are firms in the same market and links are strong—there is a unit mass of customers located on a Hotelling line between the two firms which they compete for. Panel (d) shows in more detail the structure of weak links between firms in adjacent markets. Each node is a firm and the links are weak.

price increases, we conclude that ignoring the out-of-market competition leads the price increases to be *underestimated*. The magnitude of the underestimation is substantial despite a given across-market link being weak.

(ii) Short discussion of modelling choice

Given the market structures described, firms simultaneously choose what prices to set. An alternative approach would be to let firms choose how much to produce in each market and model Cournot competition. We consider this alternative in section VI, and show our qualitative results are robust to the change.[6]

We assume price discrimination is not possible. As customers come to supermarkets they face the same prices and so the supermarket is unable to set different prices to different customers. Thus each store chooses a single price. Although we have in mind that the same facias own stores in the various markets, we assume prices are set locally to maximize store profits.

In representations to the Competition Commission it was argued by supermarket chains that prices are set at a national rather than local level.[7] However, the Competition Commission concluded that competition was local. For example, part of their reasoning involved estimating the impact of a new store entering the market on a similarly sized store. They found the impact of the entering firm on the incumbent's revenues dropped off with distance and was fairly small for journey times of more than 10–15 minutes (see Figure 4.9 of the Competition Commission report). For robustness, two geographic market definitions were used in this analysis. The first included only stores within 10 minutes' drive-time, the second included only stores within 15 minutes' drive-time. They also argued that in practice competition is multi-dimensional and occurs in terms of the quality of the shopping experience as well as pricing. Multidimensional competition is beyond the scope of our simple model. We take competition in prices as a proxy for more general competition.

An implication of permitting prices to be set locally is that stores operated by the same facia compete with each other, albeit to a limited extent because by our construction they are located in adjacent markets. A quarter of a firm's across-market links are to stores operated by the same facia. It may be realistic to think about local managers optimizing the performance of their store without internalizing the impact on other stores under the same ownership. We also do not think our results would change substantially if we modelled prices as being set at the facia level.

(iii) Equilibrium analysis

To ease exposition, in what follows we set the mass of loyal consumers and the marginal cost of each firm to 0. The Appendix provides the general derivation. Consider a link between supermarket i and supermarket j; without loss of generality, we suppose i is located at 0 and j is located at 1. For a given p_i and p_j, the indifferent consumer is located at l such that:

[6] A possible advantage of working with differentiated price competition is that, in a standard Cournot model with identical firms, a merger between two firms in a market of size $n > 2$ is unprofitable (Salant *et al.*, 1983). This and related results are known as the Cournot paradox. Of course, if there are synergies, then such mergers can be profitable. Indeed, a literature evolving from Farrell and Shapiro (1990) takes a Cournot approach to merger analysis by using the implied profitability of mergers that are proposed. This provides a lower bound on the size of synergies that is incorporated into the analysis.

[7] When making this argument the chains were claiming that a single geographic market should be defined rather than many local markets.

$$l = \frac{t + p_j - p_i}{2t}.$$

Under the assumption that customers are uniformly located over the unit interval and the market is covered (v is sufficiently high), i's demand from the edge ij is

$$x_{ij}(p_i, p_j) = \frac{1}{2} + \frac{p_j - p_i}{2t}.$$

Denote the network describing connections between supermarkets in the same market as $\boldsymbol{g}^{LM}$ with $g_{ij}^{LM} = 1$ if i and j are in the same market; $g_{ij}^{LM} = 0$ otherwise. The network describing connections between supermarkets in different markets is denoted by $\boldsymbol{g}^{AM}$: $g_{ij}^{AM} = w$ if, and only if, i and j are in connected markets. The network $\boldsymbol{g} = \boldsymbol{g}^{LM} + \boldsymbol{g}^{AM}$ describes both connections within and across markets and $d_i = \sum_j g_{ij}$.[8] Then, the demand of supermarket i is:

$$D_i(\boldsymbol{p}) = \sum_j g_{ij} x_{ij}.$$

Thus, the problem of a manager operating supermarket i is

$$\max_{p_i \geq 0} \Pi_i(\boldsymbol{p}) = (p_i - c_i) D_i(\boldsymbol{p}).$$

The equilibrium price profile $\boldsymbol{p}^*$ will then solve, for all i,

$$D_i(\boldsymbol{p}^*) + (p_i^* - c_i) \frac{\partial D_i(\boldsymbol{p}^*)}{\partial p_i} = 0.$$

Define $\hat{\boldsymbol{g}}^{LM}$ and $\hat{\boldsymbol{g}}^{AM}$ such that $\hat{g}_{ij}^{LM} = g_{ij}^{LM}/d_i$ and $\hat{g}_{ij}^{AM} = g_{ij}^{AM}/d_i$; furthermore, let $b_i = t + c_i/2$. The first order condition for i can then be written as

$$p_i^* - \frac{1}{2} \sum_j \left(\hat{g}_{ij}^{LM} + \hat{g}_{ij}^{AM} \right) p_j^* = b_i.$$

Thus, letting $\boldsymbol{I}$ be the identify matrix, in matrix notation the conditions for equilibrium prices $\boldsymbol{p}^*$ can be written as follows:

$$\left[\boldsymbol{I} - \frac{1}{2} \left(\hat{\boldsymbol{g}}^{LM} + \hat{\boldsymbol{g}}^{AM} \right) \right] \boldsymbol{p}^* = \boldsymbol{b}.$$

And so, the unique Nash equilibrium prices are given by[9]

[8] As a convention we set $g_{ii} = 0$ for all i.

[9] This requires that the matrix $I - \frac{1}{2}(\hat{g}^{LM} + \hat{g}^{AM})$ is invertible; a sufficient condition is that 1/2 times the largest (in absolute value) eigenvalue of $\hat{g}^{LM} + \hat{g}^{AM}$ is less than 1; this holds when w is sufficiently small.

$$\boldsymbol{p}^* = \left[\boldsymbol{I} - \frac{1}{2}\left(\hat{\boldsymbol{g}}^{LM} + \hat{\boldsymbol{g}}^{AM}\right)\right]^{-1}\boldsymbol{b}. \tag{1}$$

This formula is familiar from many other contexts in the networks literature, including the analysis of network games (e.g. Ballester *et al.* (2006), Bramoulle *et al.* (2014), Galeotti *et al.* (2017)), financial networks literature (e.g. Elliott *et al.* (2014), Acemoglu *et al.* (2015), and Cabrales *et al.* (2017)), the analysis of the input–output structure of economies (e.g. Leontief (1936), Acemoglu *et al.* (2012)) and the analysis of power and centrality in networks in the quantitative social network literature (e.g. Bonacich (1987) and Wasserman and Faust (1994)). By using the Neumann series

$$\left[\boldsymbol{I} - \frac{1}{2}\left(\hat{\boldsymbol{g}}^{LM} + \hat{\boldsymbol{g}}^{AM}\right)\right]^{-1} = \sum_{z=0}^{\infty}\left(\frac{1}{2}\right)^{z}\left[\hat{\boldsymbol{g}}^{LM} + \hat{\boldsymbol{g}}^{AM}\right]^{z}. \tag{2}$$

This captures the infinite sum of indirect effects that occur in equilibrium. The price that a supermarket in market k ends up charging does not depend only on the other three supermarkets in the same market; that is, does not depend only on $\hat{\boldsymbol{g}}^{LM}$. It also depends on the supermarkets that operate in the other markets—the network $\hat{\boldsymbol{g}}^{AM}$ matters. Each of these indirect effects is small by construction (as we think of w being small). But, in combination, these indirect effects can be important. How important they are will depend on how many cycles there are in the network and the strength of these cycles. To be more precise, note that, for each z, the entry (ij) of the matrix

$$\left(\frac{1}{2}\right)^{z}\left[\hat{\boldsymbol{g}}^{LM} + \hat{\boldsymbol{g}}^{AM}\right]^{z}$$

sums up the paths from supermarket i to supermarket j of length z, where each of these paths is weighted depending on the strength of each link composing the path and a decay factor that decreases with the length of the path, $(1/2)^z$. In turn, row i of the matrix

$$\sum_{z=0}^{\infty}\left(\frac{1}{2}\right)^{z}\left[\hat{\boldsymbol{g}}^{LM} + \hat{\boldsymbol{g}}^{AM}\right]^{z}$$

measures how much supermarket i is globally connected to each of the other supermarkets, not just directly. Maybe supermarket i has only a weak link with supermarket j as they operate in different markets; but the way they are globally connected to each other depends on the possibly intricate structure of the networks. And, in our example, an important determinant of this will be how many markets there are that are interacting with one another. The locally defined markets for the supermarkets inquiry is an instance in which there are many such markets that may be interacting with one another.

(iv) Equilibrium analysis without out-of-market connections

As a comparison, suppose we turn off the strategic interaction across markets. Since markets are symmetric, we focus on the price charged by firms in one of the markets. We assume that these firms choose their price optimally, taking the prices charged by the firms outside their market as fixed. Without loss of generality, we fix the price of each firm outside the market to be $\bar{p}$. The first order condition for a given firm i in the market is now

$$\tilde{p}_i^* - \frac{1}{2}\sum_j \hat{g}^{LM}\tilde{p}_j^* = b_i + \frac{1}{2}\sum_j \hat{g}_{ij}^{AM}\bar{p}.$$

Notice that there are now just n unknown prices: the price of each firm in the market we are analysing. Let $\tilde{\boldsymbol{p}}^*$ denote the equilibrium prices of these firms. Furthermore, let $\tilde{b}_i = b_i + \frac{1}{2}\sum_j \hat{g}_{ij}^{AM}\bar{p}$ and let $\tilde{\boldsymbol{g}}^{LM}$ denote the n-by-n adjacency matrix of the local network describing the connections among the n firms in the market. We have

$$\tilde{\boldsymbol{p}}^* = \left[\boldsymbol{I} - \frac{1}{2}\left(\tilde{\boldsymbol{g}}^{LM}\right)\right]^{-1}\tilde{\boldsymbol{b}}. \tag{3}$$

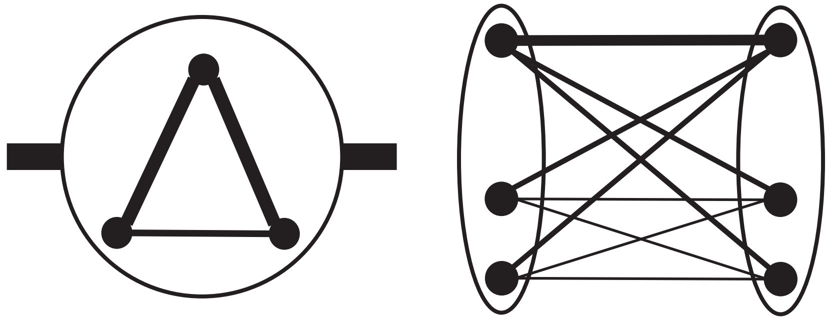
(v) Illustration of mergers: numerical example

In this section we compare equilibrium prices obtained by ignoring out-of-market interactions ($\tilde{\boldsymbol{p}}^*$ given by equation 3) to equilibrium prices obtained taking into account the feedback effects ($\boldsymbol{p}^*$ given by equation 1). We compare these prices both prior to and after a merger between facias that operate one store in each market. We denote the post-merger prices by $\tilde{\boldsymbol{p}}^*_{\text{post}}$ and $\boldsymbol{p}^*_{\text{post}}$ respectively. For these numerical examples we set $t = 1$, c_i=0 for all firms i and consider $k = 16$ markets each containing $n = 4$ stores.

We start by considering the case in which the markets are connected in a circle as shown in panel (a) of Figure 1. Using expression (1) for equilibrium prices, we find that all firms charge a price of $\boldsymbol{p}^* = 1$. If we had ignored out-of-market effects by considering the pricing problem in one market while fixing the prices set in other markets to $\bar{p} = 1$, we find that local prices are still $\tilde{\boldsymbol{p}}^* = 1$. This makes sense. When the firms outside of the market choose to set a price of 1, firms within the market best respond to these prices and each other by also setting a price of 1. If the prices outside the market are instead fixed at 1, then the firms in the market face the same trade-off and therefore their best responses do not change.

Consider now a merger between two facias each operating a store in one market. If supermarket i and supermarket j merge to form supermarket m, then the weight of the network connection from the merged supermarket m to a store k is $g_{mk} = g_{ik} + g_{jk}$ for all $k \neq i,j$. We assume the mass of consumers that i and j previously competed over,

Figure 2: Link structure within and across markets post-merger



(a) Within Market Connections **(b) Across Market Connections**

Notes: Panels (a) and (b) show the post-merger links between firms. Within each panel, the relative size of the nodes and links represents the size of the firms and the strength of connections respectively. Panel (a) illustrates the within-market links and panel (b) the across-market links. The connections between markets is as shown in panels (a) and (b) of Figure 1.

a mass of 1 given that i and j are in the same market, now become captive.[10] The link structure within markets and across markets post-merger is shown in Figure 2.

The price set by the stores that have merged increases by $\Delta\, p_i^* = \frac{\left(p_{i,POST}^* - p_i^*\right)}{p_i^*} = 23.4$ per cent after the merger, while the other stores increase their prices by 9.1 per cent. However, had out-of-market competition been ignored, the price increases would have been underestimated. For the merged stores the price increase would be estimated at $\Delta\, \tilde{p}_i^* = \frac{\left(\tilde{p}_{i,POST}^* - \tilde{p}_i^*\right)}{\tilde{p}_i^*} = 20.0$ per cent, while for the other stores the price increase would be estimated at 6.1 per cent. We are interested in by how much the market-by-market approach underestimates the price increase. We thus let U_i denote the percentage of the underestimation:

$$U_i := \frac{\Delta p_i^* - \Delta \tilde{p}_i^*}{\Delta \tilde{p}_i^*}.$$

So, for connections between markets that take the form of a circle and for $w = 0.1$, $U_i = 16.9$ per cent for stores that merge, while for the other stores in the market $U_i = 49.4$ per cent. These underestimates are fairly substantial. Moreover, the network structure and parameters that have been chosen are conservative. In practice, even just looking at the geographical dimension, heterogeneity is two dimensional. Out-of-market competition is likely to come from the north, south, east, and west. The two-dimensional torus network structure captures this better (panel (b) of Figure 1). Each market is then constrained by four other markets. We thus repeat the same exercise but assuming the network structure connecting markets is a two-dimensional torus.

[10] The appendix shows how this is incorporated into our calculations. In the context of these calculations, we still have $g_{mm} = 0$, but $\alpha_m = \alpha_i + \alpha_j + 1$. As we have set $\alpha_i = 0$ for all i so far, this implies that $\alpha_m = 1$.

The second adjustment we make is to increase the intensity of out-of-market competition. So far we have assumed that firms in neighbouring markets compete for a mass of customers one-tenth the mass of customers two firms in the same market compete for. Closer to alignment with the way markets were defined by the Competition Commission in their investigation, and in particular which stores were excluded, we now let the mass of consumers that the two stores within a market compete over be five times larger than the mass of consumers that stores in neighbouring markets compete over, instead of ten times larger. We report our results in Tables 1 and 2.

As shown in Tables 1 and 2, markets are more competitive when there are more connections between markets (torus versus circle) and when these connections are stronger ($w = 0.2$ instead of $w = 0.1$). In these cases the price increases resulting from a merger of facias is smaller. However, the approximation given by the market-by-market approach also becomes worse. The relative amount by which the price increases are underestimated gets substantially larger. When the torus network is considered with $w = 0.2$, the price increase is underestimated by almost 35 per cent for those stores merging and by over 180 per cent for those stores not merging.

Some intuition for why the underestimate gets worse can be obtained by considering the Neumann series representation of prices in equation (2). This equation shows how cycles of feedback effects increase prices. Suppose a store i in a neighbouring market to store j increases its price. Then store j will increase its price (as, in the terminology of Bulow *et al.* (1985), prices are strategic complements), which incentivizes store i to increase its price further. This is true for all stores in markets neighbouring the market of store i. However, the effects do not stop there. Suppose store l is in a neighbouring market to store j but not store i. Then store l also increases its price. If store m is in a neighbouring market to store l and i, but not j, then the increase in price by store l causes store m to increase its price, and this further incentivizes store i to increase its price. Thus all cycles of these competitive effects matter, and this is what equation (2) is

Table 1: Merging firms' price increases following a facia merger

		Market-by-market price		Actual price		Underestimate (U_i)
Network	***w***	**Pre-merger**	**Post-merger**	**Pre-merger**	**Post-merger**	**%**
Circle	0.1	1.00	1.20	1.00	1.23	16.9
Circle	0.2	1.00	1.15	1.00	1.19	25.7
Torus	0.1	1.00	1.15	1.00	1.19	25.7
Torus	0.2	1.00	1.10	1.00	1.13	34.4

Table 2: Non-merging firms' price increases following a facia merger

		Market-by-market price		Actual price		Underestimate (U_i)
Network	***w***	**Pre-merger**	**Post-merger**	**Pre-merger**	**Post-merger**	**%**
Circle	0.1	1.00	1.06	1.00	1.09	49.4
Circle	0.2	1.00	1.04	1.00	1.07	95.0
Torus	0.1	1.00	1.04	1.00	1.07	95.0
Torus	0.2	1.00	1.02	1.00	1.05	181.4

capturing. Although individually these cycles might not have much impact, when there are many of them, collectively they can. The impact of an individual cycle depends on w, while the number of these cycles depends on the network structure connecting markets. Thus the underestimate is worse for the torus than the circle network and for $w = 0.2$ than $w = 0.1$.

In practice, there are likely to be further competitive constraints that we continue to abstract from even with the torus market structure. In the supermarket investigation markets were also defined based on the size of stores, with smaller stores excluded from the market. However, smaller stores may provide (another) weak competitive constraint, further increasing the dimensionality of the competitive space in which we embed firms. The indirect effects would then be stronger still.

It is important that firms do not need to be hyper-rational and able to accurately calculate all the intricate network effects we highlight. We do need firms to be optimizing the prices they set based on the prices set by those they directly compete for customers with (i.e. in our model, those in the same or adjacent markets), but that is all. If all firms are pricing in this way, then the equilibrium outcomes we describe will obtain. In the case of supermarkets, there is evidence that stores carefully monitor the prices being charged by stores with which they compete for customers.[11]

VI. Robustness

Section V develops a model of price competition and emphasizes the importance of taking a network approach when considering competitive forces. It shows that a local approach to markets can lead to underestimation of a merger's effects on market prices. However, under price competition, the strategic actions of the oligopoly firms are strategic complements. When one firm prices higher, the other firms best respond to these changes by also increasing their prices. On the other hand, if, for example, there is quantity competition, the strategic actions of firms are strategic substitutes—when one firm produces more, the other firms best respond by producing less. It is therefore important to test whether estimates of the competitive effects of a merger that confine attention to local effects remain biased under strategic complements and, if so, whether the direction of the bias is the same.

With this aim, we develop a model of supermarkets competition, in which firms compete in quantities. We show that the results are qualitatively similar to the results under price competition.

(i) A simple model of quantity competition

We maintain the assumption that there are k markets, and in each market there are n independently operated supermarkets. Supermarkets simultaneously choose a quantity

[11] See paragraphs 4.66–4.68 of Competition Commission (2008).

to produce. If supermarket i chooses quantity q_i then $\gamma_{im}q_i$ is sold in market m, with $\gamma_{im} \in [0, 1]$ and $\sum_{m=1}^{k} \gamma_{im} = 1$, for all i. We think of γ_{im} as the magnitude of the presence of supermarket i in market m. The $(n \times k)$ matrix Γ collects γ_{im} for each supermarket i and market m.

Given quantity profile $\mathbf{q}$, the price of the product in market m is given by the standard linear inverse demand:

$$p_m(\mathbf{q}) = \alpha - bQ_m \quad \text{where} \quad Q_m = \sum_i \gamma_{im} q_i.$$

Thus, the problem of a manager operating supermarket i is:

$$\max_{q_i} \Pi_i(\boldsymbol{q}) = \sum_m q_i \gamma_{im}(p_m - c_i).$$

The equilibrium quantity profile $\mathbf{q}^*$ will then solve, for all i, the following condition:

$$\sum_m \gamma_{im}(p_m - c_i) - \sum_m q_i \gamma_{im}^2 b = 0.$$

After some manipulation, the system of equilibrium conditions is:

$[\mathbf{I} + \mathbf{g}]\mathbf{q}^* = \mathbf{a}$ which is equivalent to $\mathbf{q}^* = [\mathbf{I} + \mathbf{g}]^{-1}\mathbf{a}$,

where the $n \times n$ matrix $\mathbf{g}$ is such that $g_{ij} = \dfrac{\sum_m \gamma_{im}\gamma_{jm}}{\sum_m \gamma_{im}^2}$ and $\mathbf{a}$ is a vector with $a_i = \dfrac{\alpha - c_i}{b \sum_m \gamma_{im}^2}$.

A few observations follow. First, the network $\mathbf{g}$ summarizes competition effects across supermarkets. The element $g_{ij} \in [0, 1]$ measures how similar supermarkets i and j are in their presence across different markets. When supermarket i and supermarket j serve distinctive subsets of markets, i.e. $\gamma_{im}\gamma_{jm} = 0$ for all m, we have that $g_{ij} = 0$. Whenever supermarket i and supermarket j serve the same markets with the same intensity, i.e. $\gamma_{im} = \gamma_{jm}$ for all m, then $g_{ij} = 1$.

The second observation is that, similarly to the case of price competition, we can express equilibrium quantities as an infinite sum of indirect effects that occur in equilibrium, i.e.

$$[\mathbf{I} + \mathbf{g}]^{-1} = \sum_{z=0}^{\infty} (-1)^z \mathbf{g}^z.$$

Differently from price competition, the indirect strategic effects oscillate between being positive (pushing production up), and being negative (pushing production down). The value of g_{ij} measures the direct effect on q_i of supermarket j's choice and, because of strategic substitutes, the larger is q_j the lower is q_i. The value of $\{\boldsymbol{g}^2\}_{ij}$ measures the (indirect) effect on q_i of other supermarkets that have a connection with j. This effect is positive. Consider the chain $\{g_{ij}, g_{jl}\}$. As supermarket l produces more, this impacts negatively on the quantity supermarket j chooses to produce, and this creates a positive effect on the quantity supermarket i chooses to produce. More generally, strategic substitutes create negative effects for odd length cycles and positive effects for even length cycles.

To compare the result with the model developed in the previous session, we consider the same market connections: market connections organized in a circle and market connections organized in a torus, as illustrated in Figure 1. Each node is a market, and so there are 16 markets. In each market there are four supermarkets located there, and we let each supermarket sell a proportion γ of its output in its home market. The supermarkets also make sales in adjacent markets. In these adjacent markets they sell the remaining quantity produced. More precisely, when the markets are connected in a circle, each supermarket makes sales in three markets: a proportion γ to their primary market and a proportion $(1 - \gamma)/2$ to each of the two adjacent markets. When the markets are connected in a torus, each supermarket makes sales in five markets: a proportion γ to their primary market and a proportion $(1 - \gamma)/4$ to each of the four adjacent markets.

We think of γ as being relatively large so that each supermarket sells mostly to customers in its primary market. A large value of γ also implies that most of the competition is within local markets. Recall that g_{ij} measures the intensity of competition between supermarkets i and j. Note that:

- In the circle and in the torus, for each supermarket i and supermarket j whose primary market is the same, $g_{ij} = 1$;
- In the circle, for each supermarket i whose primary market is m and for each supermarket j whose primary market is m' and the two markets are adjacent, then

$$g_{ij} = \frac{2\gamma\left(\frac{1-\gamma}{2}\right)}{\gamma^2 + 2\left(\frac{1-\gamma}{2}\right)^2}.$$

For example, if $\gamma = 0.6$, then $g_{ij} = \frac{0.24}{0.44} \approx 0.55$ for adjacent supermarkets, and if two markets just share a neighbour in common, then

$$g_{ij} = \frac{\left(\frac{1-\gamma}{2}\right)^2}{\gamma^2 + 2\left(\frac{1-\gamma}{2}\right)^2} = \frac{0.04}{0.44} \approx 0.09.$$

- In the torus, for each supermarket i whose primary market is m and for each supermarket j whose primary market is m' and the two markets are adjacent, then

$$g_{ij} = \frac{2\gamma\left(\frac{1-\gamma}{4}\right)}{\gamma^2 + 4\left(\frac{1-\gamma}{4}\right)^2}.$$

For example, if $\gamma = 0.6$, then $g_{ij} = 0.12/0.4 = 0.3$ for adjacent supermarkets, and if two markets share a neighbour in common, then $g_{ij} = 0.02/0.4 = 0.05$.[12]

For each of the networks we perform the following exercise, which mirrors the approach we took when considering price competition. First, we derive the equilibrium output choices of supermarkets, and calculate the resulting equilibrium prices. For all markets we then (simultaneously) merge two supermarkets located in that market, and derive the percentage change in price in that market. Our interpretation of this exercise, as before, is of a facia merger but with independently run stores.[13] This is the price effect of the merger the competition authority would calculate were it to take a network approach to analysing competition. We contrast this with the pre- and post-merger equilibrium quantities and resulting change in the price obtained under the assumption that supermarkets operate only in their primary market. This is the merger price effect that a competition authority would estimate under the standard within-market approach.

We set $\gamma = 0.6$. For both the circle and the torus this choice of γ is sufficiently high for each market we define to pass the SSNIP test. We also set $\alpha = b = 1$ and $c = 0$, but our results do not depend on this parametrization. We have described above the pre-merger network $\mathbf{g}$ for both the circle and torus. We also have $a_i = 1/0.44 \approx 2.27$ for the circle and $a_i = 1/0.4 = 2{:}5$ for the torus. Using the formula $\mathbf{q}^* = [\mathbf{I} + \mathbf{g}]^{-1}\mathbf{a}$, equilibrium outputs are about $q_i = 0.225$ for the circle and $q_i = 0.227$ for the torus, giving market prices of about $p = 0.10$ for the circle and $p = 0.09$ for the torus.

After the merger of facias, there are three firms operating in each market. However, the value of g_{ij} does not change for any two firms i and j that remain after the merger, including the firm created as a result of the merger. This is because, after the merger, we have the same situation as before, but with three identical firms in each market instead of four. Specifically, there is the same overlap in market presence for two firms in the same location, the same overlap in market presence between firms located in adjacent markets, and so on. Moreover, we still have $a_i = 1/0.44 \approx 2.27$ for the circle and $a_i = 1/0.4 = 2.5$ for the torus. Thus the post-merger outputs are $q_i = 0.291$ and $q_i = 0.294$ for the circle and torus, respectively, while the post-merger prices are 0.128 and 0.118 for the circle and torus, respectively. Thus the merger results in a 29.1 per cent price increase for the circle and a 29.4 per cent price increase for the torus.

As a comparison, suppose that instead the effect of the merger was estimated assuming that pre-merger there are four firms operating in each market, and these firms make all their sales within the market. Then, in equilibrium, pre-merger each firm would choose $q_i = 1/5$ and the market price would be $p = 0.2$. Post-merger each firm would choose $q_i = 1/4$ and the market price would be $p = 0.25$. Thus prices would be expected to increase by 25 per cent post-merger instead of between 29.1 and 29.4 per cent. This means that the estimated impact on prices is between 80 and 85 per cent of the actual price increase.

These differences are far from trivial, but less striking than we found under price competition. This is because under price competition all effects are reinforcing, while

[12] Note that, because of the torus structure, if i shares a neighbour in common with j, then i must share two neighbours in common with j.

[13] As we discuss at the end of this section, we expect the difference between the network and non-network approach to be more pronounced when there is a single merger, but consider this exercise more useful as a pedagogical device, given our earlier treatment of price competition.

under quantity competition the effects go in different directions. In this case, considering a facia merger rather than an individual store merger, worked against the overall network effects. This is because the direct effects of mergers in adjacent markets have a countervailing impact. Indeed, had we considered a store merger instead of a facia merger, all cycles in both the torus and cycle networks would have been of even length, and thus all effects would have been reinforcing. While this is in part due to the specific structure of circle and torus networks, cycles of even length will in general be more common than cycles of odd length. The first-order non-local effects of a store merger occur in adjacent markets, and incentivize the stores located here to increase their outputs, and this further reinforces the incentives of the firm created by the store merger to restrict its output.

An interesting implication of our analysis is that there should be systematic misestimation of merger effects when there are many weak out-of-market constraints that interact with each other. In principle, this is something that might be tested for. While there is some limited academic work on the *ex post* analysis of merger decisions (and also some work in this regard undertaken on behalf of regulators), this work is typically done on a case-study basis (e.g. Aguzzoni *et al.*, 2016). Unfortunately, there is no work we are aware of that compares *ex ante* predictions about the competitive effects of many mergers with the outcomes of those mergers (when they are permitted).

VII. Conclusions: other networked markets

The supermarkets investigation provides one instance in which markets are networked and a systemic approach might have been appropriate. There are many others. In the case of supermarkets the network naturally arose from the geographical locations of different stores and the preferences of consumers. In other investigations the network structure will arise for different reasons. Other types of horizontal differentiation and vertical differentiation can generate networks in which certain firms compete weakly with each other for consumers, and not all firms compete directly with one another. For example, in investigations that consider public transportation (see the Competition Commission's *Local Bus Services Market Investigation* (Competition Commission, 2011)) a key question is the extent to which different modes of transportation provide a competitive constraint. When do train and bus routes compete? How about driving, walking, or cycling? The same issues regarding market definition and taking a market-by-market approach instead of a systemic approach arise here, too. In the case of healthcare provision, publicly subsidized services and privately financed treatments have been considered separate markets, but may provide additional competitive constraints that interact with the geographical locations of hospitals and other healthcare facilities (see the CMA's market investigation into private healthcare markets (Competition and Markets Authority, 2014), and the investigation into the proposed merger between Royal Bournemouth and Christchurch Hospitals NHS Foundation Trust and Poole Hospital NHS Foundation Trust (Competition Commission, 2013)).

We are witnessing a proliferation of platforms in the digital economy, from classical search engines and online-marketplaces to platforms associated with the sharing economy. Some of these platforms have expanded to a level that antitrust competition authorities around the world are considering or conducting investigations into possible abuses of dominant positions.[14] Platform-mediated networks connect multiple sides of markets, each side with possibly different objectives. Those on one side of the market typically care about access to those on other sides. When markets are intermediated by multiple platforms in this way, networks can be used to represent who has access to whom. In such settings, a change in regulation that affects the service that the platforms offer to a particular side will also impact on the other sides. Even though there is a well-developed literature on two-sided markets,[15] this is another area in which we believe network theory could be used by antitrust authorities to inform their assessments.

We have emphasized the value a systemic network-based approach can have for incorporating and aggregating effects that would otherwise be out-of-market. However, within defined markets there can also be constraints on competition between the firms that a networks approach can capture. For example, switching costs can create customers who are locked in to purchasing from a given firm within a market, limiting competition among firms in such markets (Klemperer, 1987). The presence of switching costs thereby generates a particular networked structure. Each firm will have some locked-in consumers over whom they have monopoly power, and some other consumers for whom they compete. These can be important in several markets. For example, in the market for banking services, it might be the case that the largest banks do not exert the strongest competitive pressures if they also have a larger proportion of locked-in customers (see, for example, the Competition Commission report on the proposed merger between Lloyds-TSB and Abbey National (Competition Commission, 2001), which was blocked). In markets for advice (lawyers, accountants, etc.), conflicts of interest can prevent all firms from competing for all consumers. In others, not all consumers may be aware of all products. Often consumers do not have access to all retailers and, in this case, exclusive dealing contracts might prevent all consumers from having access to all products (Ramezzana, 2019). More generally, when effort is required to sustain effective supply relationships between producers and retailers, such networks can also emerge (e.g. Kranton and Minehart, 2001; Elliott, 2015).

While the importance of these intricacies of competition between different products is starting to be recognized (Armstrong and Vickers, 2019), networks provide a useful way to visualize these interactions,[16] and a set of tools that have been developed in different contexts that can be readily applied.

[14] The European Commission has already taken actions with respect to abusive practice in online advertising by Google. Similar concerns and discussions are now salient in the USA.

[15] Two of the early and seminal contributions to this literature are Rochet and Tirole (2003) and Armstrong (2006).

[16] Technically, any analysis that is possible with a networks approach is also possible without it. A network can be mathematically represented by an adjacency matrix, and properties of networks have analogous properties in this matrix. However, it is much easier to see a path or cycle on a graph than it is by staring at an adjacency matrix, and it is often such things that are important for the economics.

Appendix: Mathematics supporting claims in section V(iii)

In this section we solve a more general problem that supports the analysis and claims in section V(iii). Specifically we permit $c_i \geq 0$ and $\alpha_i \geq 0$. The manager of firm i then solves the following problem:

$$\max_{p_i \geq 0} (p_i - c_i) \left[\alpha_i + \sum_j g_{ij} \left(\frac{1}{2} + \frac{p_j - p_i}{2t} \right) \right].$$

The FOC is: $$0 = \alpha_i + \sum_{j \neq i} g_{ij} \left(\frac{1}{2} + \frac{p_j - p_i}{2t} \right) - \frac{1}{2t}(p_i - c_i) \sum_{j \neq i} g_{ij},$$
$$= \alpha_i + \sum_{j \neq i} g_{ij} \left(\frac{1}{2} + \frac{p_j - 2p_i}{2t} + \frac{c_i}{2t} \right),$$
$$= \alpha_i + \sum_{j \neq i} g_{ij} \left(\frac{1}{2} + \frac{c_i}{2t} \right) + \sum_{j \neq i} g_{ij} \left(\frac{p_j}{2t} \right) - \sum_{j \neq i} g_{ij} \left(\frac{p_i}{t} \right).$$

We therefore have:

$$p_i = \frac{t\alpha_i}{d_i} + t \left(\frac{1}{2} + \frac{c_i}{2t} \right) + \frac{1}{2} \sum_{j \neq i} \left(\frac{g_{ij}}{d_i} \right) p_j.$$

Define $\hat{\boldsymbol{g}}$ such that $\hat{g}_{ij} = g_{ij}/d_i$, where $d_i = \Sigma_j g_{ij}$ is the degree of i and let

$$b_i = \frac{t\alpha_i}{d_i} + \frac{t}{2} + \frac{c_i}{2}.$$

Then, as long as $[\boldsymbol{I} - \frac{1}{2}\hat{\boldsymbol{g}}]$ is invertible, the unique Nash equilibrium prices are given by

$$\boldsymbol{p} = [\boldsymbol{I} - \frac{1}{2}\hat{\boldsymbol{g}}]^{-1}\boldsymbol{b}.$$

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