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Thesis Report

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Dynamic Portfolio Allocation Using Markov Regime Switching Model: Momentum & Low-Volatility Strategies

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Abstract

This research explores the application of dynamic portfolio allocation using the Markov Regime Switching Model (MRSM), focusing on transitions between Minimum Volatility and Momentum strategies. The goal of this dynamic strategy is to combine the complementarity stability of the Minimum Volatility strategy with the growth potential of the Momentum strategy. Thereby, the strategy balances risk and return by adapting to market conditions. The study employs advanced data preprocessing techniques to identify the variables subset. It includes feature selection through Elastic Net regression and Akaike Information Criterion (AIC) optimization. In addition, the Expanding Window method has been used to reduce potential forward bias and improve regime detection over time. The results underscore the efficacy of MRSM application to dynamic portfolio allocation strategy. The strategy achieves an annualized return of 8.7%, comparable to the MSCI World Index, and maintains a low volatility of 8.2%. With a Sharpe ratio of 1.06, the strategy outperforms other analysed strategies. This performance can be attributed to the model's precision in regime allocation, its ability to capture long-term dependencies, and its performance gains relative to the switch decision. Additionally, the dynamic nature of the strategy mitigates drawdowns during periods of uncertainty and takes advantage of recovery and growth opportunities. Also, the low transaction frequency improves the practicality of the investment strategy by minimizing transactions and operational costs. Finally, this study contributes to the Markov Regime Switching Model applications and offers direction for future research in adaptive portfolio management.

Contents

Introduction

This research explores the application of dynamic portfolio allocation using the Markov Regime Switching Model (MRSM), focusing on transitions between Minimum Volatility and Momentum strategies. The motivation for this research comes from a collaboration with portfolio manager Jean-Mark Lueder, who has implemented a Minimum Volatility strategy to address client demand for low-risk equity investments. While the strategy demonstrated strong performance, it experienced notable underperformance during the 2023–2024 period, represented by an upward-trending market condition. This highlighted a gap in the strategy's ability to capture returns during growth phases. To address this limitation, the Momentum strategy was identified as a complementary solution capable of enhancing portfolio performance during favourable market trends. The portfolio manager wanted to maintain exposure to the minimum volatility strategy while sporadically exposing the portfolio to a slightly riskier approach that would deliver a higher return on investment. This is why the dynamic allocation between these strategies comes naturally and would be performed by the Markov Regime Switching Model. To ensure transparency and neutrality, this study employs the MSCI World Momentum Index and MSCI World Low Volatility Index to avoid reliance on proprietary methods.

The thesis is organized into key sections and starts with the Literature Review. In this section, the theorical foundation of Markov Regime Switching Models has been assessed, parkouring its financial application. It started from the foundation written by Goldfeld and Quandt (1973) and its pioneering application to financial econometrics by Hamilton (1989). Then, the review highlights the growing use of the regime-switching model in market regime identification with, for instance, Guidolin and Timmermann (2004), demonstrating the increasing importance of this field. Additionally, the benefits and limitations of Momentum and Minimum Volatility strategies have been assessed to demonstrate their complementary nature. Finally, this chapter defines the critical research gap: The lack of an MRSM-based dynamic allocation strategy that integrates minimum volatility and momentum strategies.

The second section is the Data Preparation chapter. It provides an overview of the dataset construction process, which is composed of quality data extraction and preprocessing for the model implementation. It starts with extracting financial and economic variables, comprising the benchmark, two portfolio strategies, and a selected set of predictors. Then, the data relevancy is improved through the following preprocessing steps: data cleaning, feature extraction, handling outliers, and finally, the standardization of the dataset. The last subsection assesses the multicollinearity among variables, employing a correlation matrix and VIF. These meticulous data preparations create a qualitative dataset for MRSM analysis.

Then followed the Model Implementation chapter. It elaborates on the MRSM implementation in association with dynamic portfolio allocation. The first step is an advanced feature selection method, regrouping Elastic Net regression and AIC optimization. The combination of those algorithms selects the final dataset of the model. Then, the theoretical basis of MRSM is presented, including the model assumptions. Building on this outcome, the calculation for the dynamic portfolio allocation strategy is derived using the principles of the Markov process. Finally, it enhances the strategy with methods avoiding forward bias and low signal regime transitions. This chapter establishes the model's robustness and effectiveness in dynamic portfolio allocation, giving valuable insights for analysis.

The Results & Discussion section presents the principal outcomes of the research, which are analysed to improve understanding. It starts with model quality evaluation. The model has significant variables and good model evaluation metrics, assessing the firm foundation of the model implementation. It follows with the regime allocation. The model identifies the correct regime by capturing long-term tendencies but sometimes misses opportunities by assessing the wrong regime. However, those misidentifications do not affect the model performance significantly, as shown in the switch efficiency part. During this period, the allocation decision produced an interesting relative outperformance, partly explaining the satisfying result of the strategy. After that, the performance metrics of the dynamic portfolio allocation strategy are discussed. The strategy achieves an annualized return of 8.7%, comparable to the MSCI World Index, and maintains a low volatility of 8.2%. With a Sharpe ratio of 1.06, the strategy outperforms other analysed strategies and attests to the strong quality of the Markov Regime Switching Model. To further test the model, a shorter-term investment horizon and worst drawdown scenarios are simulated to evaluate the consistency and resilience of the model. Lastly, its practical design is discussed in a dedicated subsection.

Finally, the Conclusion synthesizes the key findings of the thesis, emphasizing the MRSM's ability to deliver adaptive portfolio allocation with superior risk-adjusted returns. Then, the chapter nuances the statement by addressing the model's limitation, regrouping its sensitivity to unconventional market conditions, interpretability challenges, and the size of the dataset. Future research is then discussed to address these limitations, highlighting how the MRSM framework can be further improved to refine the strategy. By addressing the gap in dynamic allocation strategies with the MRSM, this research offers contributions to financial research. It extends the academic understanding of regime-switching models with a new application and provides a dynamic strategy implementable to optimize portfolio allocation according to market conditions.

Literature Review

The literature review gives the academic foundation for the Markov Regime-Switching Models (MRSM) study and its application to the financial industry. An MRSM is a regression model that identifies structural changes in time-series data by allowing transitions between distinct regimes. A current regime Markov Process assesses the probability of moving from one regime to another. The model is then particularly effective for capturing dynamic changes in financial markets. Regarding dynamic allocation, this investment strategy refers to a portfolio management approach where changing market conditions adjust asset weights over time. The primary objective of this research paper is to optimize returns and manage risks more effectively by adapting an investment strategy to market conditions. This section examines the development of MRSM and portfolio strategies to highlight the following research gap: Integrating a dynamic portfolio allocation composed of Minimum Volatility and Momentum strategies within an MRSM framework.

Markov Regime-Switching Models are introduced by Goldfeld and Quandt in 1973 (Stephen M. Goldfeld, 1973). The study develops a system with multiple regression equations corresponding to distinct regimes, and a Markov process gives transitions between states. This innovation allowed the analysis of complex dynamics in time-series data. Their early work laid a solid base for future research by providing a statistical model applicable to various fields. The MRSM application to financial sector starts in 1989 with the work of James D. Hamilton (Hamilton, 1989). The American economist advanced the field by adopting the Markov regime-switching model for nonstationary time series analysis. This approach allows the identification of different economic regimes, such as periods of economic expansion and recession. With this new approach to the business cycle, Hamilton pioneered the Regime Switching Application to the financial industry.

Then, progressively, some studies coupled the Markov Regime Switching with investment strategies. Massimo Guidolin and Allan Timmermann published a study in 2004 that attempted to identify strategic asset allocation and consumption choices within an MRSM (Timmermann, 2004). The authors identify four distinct regimes: crash, slow growth, bull, and recovery, and analyze how optimal asset allocation varies across these states. The study demonstrates that regime switching has important implications for strategic asset allocation, and ignoring these dynamics can lead to suboptimal performances. This study highlights the possibility of a link between the regime identified by the model and the economic condition. Progressively, the Markov Regime Switching model has been adapted for portfolio optimization. As an example from 2012, the research paper of Agostino Capponi and José E. Figueroa-Lopez addresses a portfolio optimization problem in a market with regime-switching dynamics and default risk (Figueroa-Lopez, 2012). The authors develop an easy portfolio allocation model where an investor allocates wealth among a defaultable bond, a stock, and a money market account. The market coefficients are related to the economic regime. Researchers found that accounting for sudden changes in the market can improve investment outcomes, demonstrating the significance of considering regime shifts in portfolio optimization and asset allocation.

The two previously discussed research papers underscore the efficacy of combining the Markov Regime Switching model with an investment strategy and their associated market conditions. This research takes a deeper dive into the specific advantages of Minimum Volatility and Momentum strategies by exploring the individual benefits of each static strategy. Minimum Volatility strategy focuses on constructing portfolios that minimize risk while remaining diversified. This strategy avoids uncertainties associated with forecasting the expected returns because it relies uniquely on the covariance of asset returns. Research by Clarke, de Silva, and Thorley (2006) demonstrates that the strong risk-adjusted returns of Minimum Volatility strategies are particularly effective during periods of market instability. Historical analyses highlight the minimum volatility strategy's outperformance of traditional market-cap-weighted portfolios. The portfolio achieves comparable or superior returns with less risk over extended periods. These characteristics position Minimum Volatility as an excellent investment approach for risk-averse investors. (Roger G Clarke, 2006). On the other hand, the Momentum strategy leverages the benefits of return trends. A key study by Foltice and Langer (2015) observed that the Momentum strategy generates excess returns by buying past winning stocks. The strategy takes advantage of bullish market trends, and past winners allow for substantial gains (Langer, 2015). However, the strategy is sensitive to market downturns. Moreover, it tends to underperform during periods of high volatility or bearish conditions, as highlighted by Butt, Kolari, and Sadaqat (Hilal Anwar Butt, 2024). Despite these limitations, Momentum's strategy's ability to enhance returns in a favourable market is invaluable.

Recent research explores the combination of MRSM with momentum portfolio strategies for a dynamic asset allocation based on market conditions. For instance, Uhl (2021) applies MRSM to implied volatility in the S&P 500 index (Uhl, 2021). The model generates a binary signal for switching between past winners and past losers portfolios based on the regime outcome. Shorting past losers and longing for past winners enhanced the regular momentum strategy with an MRSM. Similarly, Gu and Mulvey examined the application of MRSM to a risk-parity strategy with an overlay of a factor momentum strategy. Their model optimized overall performance by dynamically adjusting the momentum layer (Mulvey, 2021). These studies highlight the practical utility of momentum strategies in MRSM. Despite the model's identification of market regimes, the adoption of minimum volatility strategies in these settings remains largely unexplored.

In summary, while the integration of momentum strategies into MRSM has been explored, the application of minimum volatility strategies within the same model remains neglected. Adding this portfolio strategy to the model makes it a new contribution to financial research. Furthermore, beyond the gap in the single strategy usage, no existing framework alternates between those two portfolio strategies based on regimes, representing the central gap identified in this literature review. Therefore, in this research, a Markov Regime Switching model will seek to allocate between a minimum volatility strategy during uncertainty periods and a momentum strategy during favourable market conditions. It leverages the complementary strengths of these strategies and the adaptability of MRSM. By doing so, this thesis contributes to the literature on dynamic portfolio allocation and provides practical information for institutional investors.

Data Preparation

The full implementation of the MRSM model is documented in a dedicated GitHub repository, see the Appendix B: Code Accessibility.

Data Extraction & Cleaning

Data Extraction

The dataset is extracted from the Bloomberg platform because of the quality and the large amount of data provided (Bloomberg, 2024). Twenty years of data have been collected in a daily format, covering a period from January 1, 2005, to November 24, 2024. The extended period of the dataset contains various market conditions, including periods of growth, volatility, and economic downturns. This diversity comprehensively represents different regimes and will improve the model's ability to fit and generalize across market cycles. The extracted dataset is represented in Table 1.

	BENCH MARK	PORTFOLIO STRATEGIES		INDEPENDENT VARIABLES							
DATE	**MSCI**	**MIN VOL**	**MOM**	**VIX**	**CS**	**GDP**	**IR**	**CPI**	**Liq**	**P/C**	**Crude Oil**
01.01.2005	2531	1311	634	13.3	188.2	4.47	4.22	3.26	0.02	0.45	43.5
02.01.2005	2531	1311	634	13.3	188.2	4.47	4.22	3.26	0.02	0.45	43.5
…	…	…	…	…	…	…	…	…	…	…	…
21.11.2024	11864	5245	4740	16.9	143.4	2.72	4.42	2.60	-0.05	0.53	70.1
22.11.2024	11864	5245	4740	16.2	143.4	2.72	4.42	2.60	-0.05	0.53	70.7

Table 1: Raw Dataset

The data extraction includes a benchmark, the MSCI World Index (MSCI, 2024). This way, the implemented strategy performance could be compared to the global equity performance. The extraction follows with two portfolio strategies employed for dynamic portfolio allocation: The MSCI World Minimum Volatility Index (MSCI, 2024) and the MSCI World Momentum Index (MSCI, 2024). These two MSCI investment strategies allow the replication of the strategies described in the Literature Review. Then, independent variables are extracted based on their relevance to market behaviour and economic cycles, as well as recommendations from previous studies (Mandalà, 2020). The independent variables include measures of market volatility, such as the VIX Index; macroeconomic indicators, such as the World GDP, US Interest Rate, and CPI, as well as additional variables like the Credit Spread, PUT/CALL Ratio, Liquidity Measures, and Crude Oil Prices. The variety of the data selection ensures that the MRSM has access to the necessary inputs to identify distinct market regimes and effectively allocate between Minimum Volatility and Momentum strategies[1].

Data Cleaning

Once the data is extracted, it needs to be cleaned, and it starts with transforming the daily format into a weekly one. In a portfolio management context, rebalancing more frequently than weekly is unmanageable. In addition, the potential daily trading activity would significantly increase transaction fees, which would impact the strategy's overall performance. Therefore, weekly rebalancing balances between reducing trading frequency and capturing market trends. The weekly transformation also helps prevent overfitting because daily data fluctuates significantly, inducing noise and potential short-term reactions in the model. The weekly format allows then to focus on longer-term dynamics, improving the model’s generalization ability across different market conditions. After the transformation, the quality and completeness of the dataset need to be verified. A detailed examination confirmed that the dataset is complete, with no missing values requiring additional forward-filling or interpolation. This is mainly due to the consistency furnished by the data provider. The data regularity allows a solid foundation for the analysis and reduces the number of data-cleaning measures.

Preprocessing Techniques

Feature Extraction

Firstly, feature extraction is applied to add meaningful information to the Markov Regime Switching Model. Initially, the dataset was composed uniquely of the extracted value format, and with feature extraction, two formats would be added: returns and log returns. These additional formats capture relative changes over time, providing more profound information on the time series dynamics. In this way, the model could better catch underlying trends and relationships. All three variable formats are retained for the following research because they maximize the information available for future feature selection.

[1] All the ticker of the extracted variables are defined in Table 15 of the Appendix section.

Handling Outliers

Handling outliers is a crucial step in data preprocessing because extreme can distort statistical results and lead to incorrect results. Outliers can arise from natural variability, measurement errors, or data entry mistakes, impacting the model implementation (Bonthu, 2024). One practical approach to managing outliers is the capping method. This technique modifies extreme values by capping them at a certain threshold based on percentile. In the research, the cap is set at percentile 5% and 95%; in this way, the impact of extreme is reduced. Table 2 and Table 3 resumed the effect of handling outliers on the raw dataset.

	VIX	CS	GDP	IR	CPI	LIQUIDITY	PUT/CALL
MEAN	19.06	247.58	2.95	2.90	2.59	0.28	0.64
STD	8.76	79.18	1.95	1.12	1.94	0.48	0.13
MIN	9.14	124.85	-2.93	0.51	-2.10	-0.61	0.32
25%	13.23	188.87	2.71	2.00	1.47	-0.03	0.56
50%	16.59	229.91	3.13	2.75	2.23	0.17	0.62
75%	22.14	288.84	4.37	3.83	3.46	0.47	0.70
MAX	82.69	622.42	6.26	5.29	9.06	3.84	2.40

Table 2: Summary Statistics Raw Dataset

	VIX	CS	GDP	IR	CPI	LIQUIDITY	PUT/CALL
MEAN	18.49	241.30	2.95	2.90	2.57	0.24	0.64
STD	6.39	58.28	1.61	1.06	1.72	0.34	0.09
MIN	11.07	160.61	-1.36	1.25	-0.04	-0.22	0.49
25%	13.36	187.97	2.71	2.00	1.47	-0.02	0.58
50%	16.66	228.80	3.13	2.74	2.21	0.17	0.63
75%	22.21	288.99	4.26	3.82	3.42	0.48	0.69
MAX	33.80	352.03	4.61	4.71	6.91	0.99	0.82

Table 3: Summary Statistics Capped Raw Dataset: Percentile 5%.

The impact of capping outliers varies significantly depending on the variable analysed. For instance, the adjustment has a relatively minor effect on Interest Rate (IR) and Gross Domestic Product (GDP), with reductions of 10% and 27% in their maximum values, respectively. In contrast, the Liquidity and PUT/CALL ratio variables experience more changes. The maximum value of Liquidity is reduced by four, while the PUT/CALL ratio is reduced by three, highlighting the importance of capping outliers in this case. The capping method has also been applied to return and log return formats to ensure consistency in outlier handling across all formats.

Standardization

Standardization is fundamental in data preprocessing; it allows features to be on the same scale and prevents those with larger magnitudes or units from dominating the analysis. This is particularly important when the data features are measured in different units, such as VIX levels (ranging from 10 to 80) and PUT/CALL ratios (not exceeding 2.40). Standardizing variables helps prevent MRSM, which is sensitive to scale, from producing biased results. It also enhances model interpretability and performance by rescaling the data to a uniform range. (Choi, 2024) Table 4 is the summary of statistics after standardization and capped outliers:

	VIX	CS	GDP	IR	CPI	LIQUIDITY	PUT/CALL
MEAN	0.00	0.00	0.00	0.00	0.00	0.00	0.00
STD	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MIN	-1.16	-1.39	-2.68	-1.56	-1.52	-1.39	-1.72
25%	-0.80	-0.92	-0.15	-0.85	-0.64	-0.79	-0.68
50%	-0.29	-0.21	0.11	-0.15	-0.21	-0.20	-0.13
75%	0.58	0.82	0.82	0.86	0.50	0.70	0.63
MAX	2.40	1.90	1.04	1.70	2.53	2.20	2.13

Table 4: Summary Statistics: Standardisation & Outliers

The table shows that the mean of all variables has been standardized to 0, with a standard deviation of 1. The data now exhibit a more regular and uniform range, and each feature contributes equally to the model without any variable disproportionately affecting the results.

Multicollinearity Assessment

The final step in data preprocessing involves examining the correlation and multicollinearity of variables using two primary tools: The Correlation Matrix and the Variance Inflation Factor.

Correlation Matrix

The Correlation Matrix quantifies the correlation between independent variables across the entire dataset. The matrix excludes potential combinations of features that show excessive correlation.

This multicollinearity could be found in their format's variation, such as raw values, log returns, and returns, or between different economic variables. The correlation matrix is calculated with the following formula (Damodar N. Gujarati, 2009):

$$r_{ij} = \frac{\sum_{t=1}^{n}(x_{it} - \bar{x}_i)(x_{jt} - \bar{x}_j)}{\sqrt{\sum_{t=1}^{n}(x_{it} - \bar{x}_i)^2 \sum_{t=1}^{n}(x_{jt} - \bar{x}_j)^2}}$$

Equation 1: Correlation Matrix

Where:

- r_{ij} is the Pearson correlation coefficient between variables x_i and x_j,
- x_{it} *and* x_{jt} *are the observed values of variables* i *and* j *at time* t
- $\bar{x}_i$ *and* $\bar{x}_j$ *are their respective means*
- n *is the total number of observations*

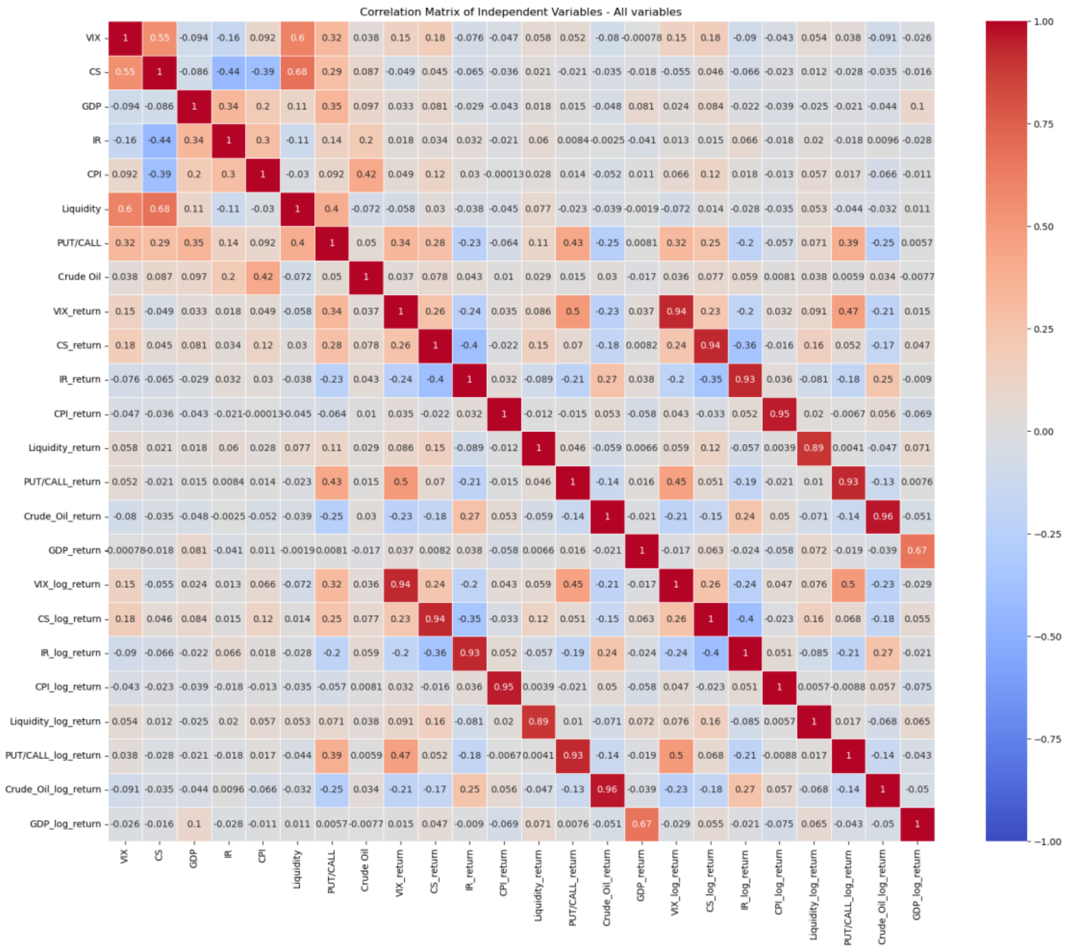

Figure 1: Correlation Matrix of Independent Variables - All Variables

According to Gujarati's "Basic Econometrics" (2009, p. 359), a correlation coefficient exceeding 0.8 is considered highly correlated (Damodar N. Gujarati, 2009). Variables exceeding this threshold need further investigation and may be excluded to avoid multicollinearity. On the other hand, values closer to zero signify weak correlations and are preferable. As it is possible to see in Figure 1, combinations of variables that should be excluded primarily involve variables in their return and log-return formats. For instance, the VIX Return and VIX Log return exhibit a high Pearson correlation coefficient of 0.94. Aside from these cases, some variables, such as VIX, Credit Spread, and Liquidity, show moderate correlations of approximately 0.6, which is higher than others but still manageable. Regarding other correlations, most are close to 0, which could be attributed to earlier steps in the data preprocessing process, such as standardization and outlier capping, designed to reduce variability and align the variables on a standard scale, thereby minimizing excessive correlations.

Variance Inflation Factor (VIF)

As the correlation matrix, the Variance Inflation Factor (VIF) measures the multicollinearity among independent variables. This analysis calculates the VIF for the data formats: raw values, returns, and log returns. This separation guarantees that the VIF measures the correlation between distinct variables rather than simply detecting relationships among derived features of the same variable. The formula used for VIF calculation is as follows:

$$VIF_i = \frac{1}{(1 - {R_i}^2)}$$

$${R_i}^2 = {r'}_{ij} R^{-1} r_{ij}$$

Equation 2: Variance Inflation Factor

Where:

- $R^{-1}: Inverse\ of\ correlation\ matrix.$
- $r_{ij}: Pearson\ correlation\ coefficient$

The thresholds for Variance Inflation Factor levels can be categorized into three main groups. A VIF value below 5 is generally considered acceptable for most models, indicating low multicollinearity. Values between 5 and 10 suggest moderate multicollinearity, and the feature should be carefully examined. Finally, VIF values exceeding 10 indicate high multicollinearity, the problematic variable should probably be removed (F.Hair, 2009).

As illustrated in Table 5, the VIF analysis shows that only the raw Credit Spread (CS) exceeds the low-multicollinearity threshold, with a value of 5.6. This suggests moderate collinearity between Credit Spread and other variables in the dataset. All other variables have a low VIF, and the correlation between economic variables in the dataset is then particularly low. In addition, return and log-return formats exhibit even lower VIF values than the extracted format. Overall, the two multicollinearity analyses on the dataset show weak dependencies, highlighting its strong explanatory power.

	EXTRACTED VALUE	**RETURN**	**LOG RETURN**
FEATURE		**VIF**	
VIX	2.0	1.5	1.5
CS	5.6	1.27	1.3
GDP	1.4	1.0	1.0
IR	1.8	1.3	1.3
CPI	2.6	1.0	1.0
LIQUIDITY	3.2	1.0	1.0
PUT/CALL	1.4	1.4	1.4
CRUDE OIL	2.0	1.1	1.1

Table 5: VIF Data after Data Preprocessing

Model Implementation

Once the data is prepared, it is organized into two matrices. The first matrix consists of 24 independent variables, which are the predictors of the model. The 24 variables are the eight economic variables in the three-format described in the Preprocessing Techniques part. The second matrix contains the dependent variables representing the two portfolio strategies, Minimum Volatility and Momentum, in the log return format. The dataset is a time series with weekly observations for 20 years.

Feature Selection & Optimization

The Momentum strategy in log return format is the dependent variable because model results are higher than in combination with other dependent variable possibilities. The AIC, BIC, and Sharpe ratio are better, meaning that the model identifies more easily market regimes with this entry. The Markov Regime Switching Model is fitted on the first matrix of independent variables to identify market regimes. Selecting the best combination from the 24 available variables is essential to give meaningful results. Traditional methods like forward or backward stepwise regression could have been used; however, they are computationally intensive for large datasets. Therefore, this research uses modern regularization techniques such as Lasso and Elastic Net regression. Both algorithms can handle large variable sets while reducing risks of overfitting. Lasso simplifies the model by selecting a minimal subset of predictors, improving interpretability and computational efficiency while reducing collinearity among selected variables (Tibshirani, 1996). On the other hand, Elastic Net provides a more balanced approach; the model allows correlated variables but keeps the model stable and generalizable (Hastie, 2005). Together, Lasso and Elastic Net are complementary practical tools for the feature selection in this study.

Lasso & Elastic Net

The Lasso (Least Absolute Shrinkage and Selection Operator) adds an L_1-norm penalty to the loss function, which is the sum of the absolute values of the regression coefficients.

The Lasso's objective function to minimize is (Tibshirani, 1996):

$$Minimize: \frac{1}{2N}\sum_{i=1}^{N}(y_i - X_i\,\beta)^2 + \lambda\sum_{j=1}^{p}|\beta_j|$$

Equation 3: Lasso

Where:

- N *is the number of observations.*
- P *is the number of predictors.*
- X_i *is the matrix of independent variables.*
- y_i *is the dependent variable.*
- β *is the vector of coefficients.*
- λ *is the regularization parameter controlling the penalty's strength.*

The Lasso algorithm was applied with the MSCI World Momentum (log return) as the dependent variable, the 24 independent variables for X_i, 1037 weekly observations for N, and a regularization parameter 0.05. Find in Table 6 the seven variables selected by the algorithm.

LASSO SELECTED VARIABLES

VARIABLES	VIX	CPI	PUT/CALL	IR Return	VIX Log Return	CS Log Return	Crude Oil Log Return

Table 6: Intermediate Results: Lasso Selected Variables

Elastic Net has a regularization parameter with two penalties, the L_1-norm (Lasso) and L_2-norm (Ridge) penalties. The Elastic Net's objective function to minimize is (Hastie, 2005):

$$Minimize: \frac{1}{2N}\sum_{i=1}^{N}(y_i - X_i\,\beta)^2 + \lambda\left(\alpha\sum_{j=1}^{p}|\beta_j| + \frac{1-\alpha}{2}\sum_{j=1}^{p}{\beta_j}^2\right)$$

Where:

- α *controls the balance between* L_1 *and* L_2 *penalties.*
- *The other terms are defined as above.*

The Elastic Net algorithm was applied with the same parameters as the Lasso. Table 7 shows the 11 variables selected by the algorithm.

ELASTIC NET SELECTED VARIABLES

VARIABLES	VIX	GDP	IR	CPI	Liquidity	PUT /CALL	IR Return	VIX Return	VIX Log Return	CS Log Return	Crude Oil Log Return

Table 7: Intermediate Results: Elastic Net Selected Variables

AIC and BIC Algorithms

The Lasso and Elastic Net algorithms selected a subset of variables by addressing overfitting and reducing the dimension of the dataset size. However, the Lasso and Elastic Net do not guarantee the best performant model. This is why the feature selection is further optimized using a forward stepwise AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) algorithm. This two-stage process ensures the MRSM captures the most relevant predictors while ensuring simplicity and goodness of fit. The algorithm works as follows:

1. Start with an empty model.
2. Each variable is evaluated, and the one that improves the selection criterion (AIC or BIC) the most is added.
3. The model is updated.
4. This process repeats until no further improvements to the selection criterion are possible.

The following algorithm is applied to the two subsets given by Lasso and Elastic Net, and each is optimized on AIC and BIC. Therefore, four sets of variables have been calculated, and their statistics can be compared in Table 8.

	AIC	BIC	SHARPE RATIO[2]	# SWITCH
LASSO | BEST AIC	1930.1	2029.9	1.0	28
LASSO | BEST BIC	1940.7	2019.8	0.99	28
ELASTIC | BEST AIC	1907.2	2035.7	1.04	28
ELASTIC | BEST BIC	1934.5	2013.6	0.99	30

Table 8: Summary of Variable Selection Algorithm

Before selecting the final set of variables, it is essential to understand the purpose of the different presented metrics. AIC and BIC assess model quality by balancing goodness of fit and complexity, but they have different approaches. AIC focuses on minimizing prediction error, while BIC imposes a stricter penalty for additional parameters, especially in larger datasets (Damodar N. Gujarati, 2009). Since the dataset in this study (24 variables and 1,037 observations) is relatively small, AIC is a stronger criterion for the model selection. However, BIC remains relevant for avoiding unnecessary variables. The Sharpe Ratio is a measure of the dynamic

[2] The Sharpe Ratio has been calculated with a risk-free rate equal to 0, the same assumption has been used for all the research.

portfolio allocation risk-adjusted performance. The Sharpe ratio is calculated on the strategy implemented in the Dynamic Portfolio Allocation Strategy chapter. A higher Sharpe Ratio indicates better overall performance. Finally, the number of transitions reflects how often the model switches between regimes during the analysis period. Fewer switches are preferable for practical implementation and minimizing transaction costs.

For the MRSM implementation, variables from Elastic Net AIC have superior statistical performance. The model implemented with those variables has the lowest AIC value, indicating an optimal balance between model fit and complexity. It also achieves the highest Sharpe Ratio, suggesting better risk-adjusted returns than the alternatives. The BIC value is higher than that of some models; however, BIC is not our priority because of the dataset length. Finally, the number of switches is mainly similar for all the selection of variables and is adequate for an actual case implementation. In conclusion, the Elastic Net AIC is selected for the MRSM due to its superior statistical performance among the four options. This set of variables balances the goodness of fit and simplicity, delivers the Sharpe ratio, and maintains a good number of switches for the strategy implementation.

Final Selected Variables

The final selected variables are derived from the Elastic Net and Best AIC method, resulting in a set of 10 variables summarized in Table 9:

FINAL SELECTED VARIABLES

VARIABLES	VIX	GDP	IR	CPI	Liquidity	PUT/CALL	IR Return	VIX Log Return	CS Log Return	Crude Oil Log Return

Table 9: Final Selected Variables from Elastic Net and Best AIC Method

These diversified variables capture various aspects of economic and financial systems, including macroeconomic performance, market sentiment, and liquidity conditions. Two selected variables are represented in multiple formats, such as VIX and IR. As discussed in the chapter Multicollinearity Assessment, this could potentially lead to higher levels of correlation. To address this concern, the correlation matrix of the final variables is presented in Figure 2 to confirm their suitability for the model:

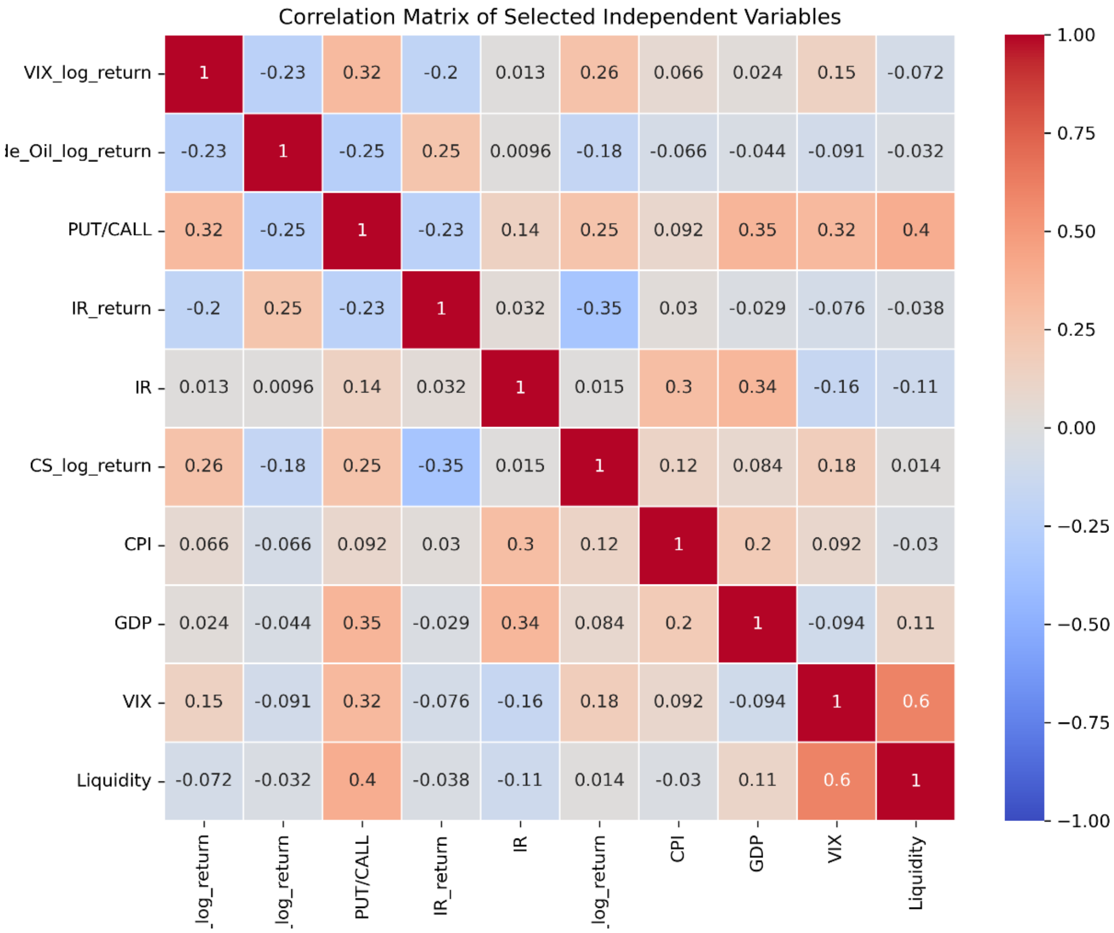

Figure 2: Correlation Matrix of Selected Independent Variables

As shown in Figure 2, no variable combination exceeds the critical value of 0.8. The highest value is the combination of VIX and Liquidity, with a Pearson correlation of 0.6. Therefore, those variables need monitoring for the following research. Regarding the correlation between VIX and VIX log return, the correlation is low, at 0.15. They can be added to the model without redundancy.

Markov Regime Switching Model

This thesis employs the Markov Regime Switching Model to analyse the dynamic relationships between portfolio strategies and economic variables. It uses time-series data to identify regimes and their associated parameters. Each regime has its own means, variances, and regression coefficients, allowing it to identify different market conditions. The transitions between regimes are governed by a Markov process, relying solely on the current regime. The MRSM uses a likelihood-based approach to estimate the model parameters and transition. The Hamilton filter calculates forward probabilities (Hamilton, 1989), while the Kim smoother computes backward probabilities to refine regime identification (Kim, 1991). The theoretical formulation of the Markov Regime Switching Model represents a time series as follows (Hamilton, 2005):

$$y_t \ = \ X_t \beta_{S_t} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma^2{}_{S_t}),$$

Equation 4: Markov Regime Switching Model

Where:

- $y_t is\ the\ dependent\ variable\ at\ time\ t$
- $X_t\ is\ the\ independent\ variable\ at\ time\ t$
- $\beta_{S_t}\ is\ the\ regime\ specific\ regression\ coefficients$
- $S_t\ is\ the\ latent\ state\ variable\ at\ time\ t, indicating\ the\ current\ regime$
- $\varepsilon_t\ is\ the\ normally\ distributed\ error\ termwith\ regim\ specific\ variance.$

The latent state variable S_t follows a Markov process with transition probabilities P, defined as:

$$P_{ij} \ = \ P(S_t = j \mid S_{t-1} \ = \ i), \sum_{j=1}^{m} P_{ij} \ = \ 1\ \forall i$$

Where:

- $Pij: Probabilities\ of\ switching\ from\ i\ to\ j, with\ i, j\ \in\ \{0,1\} the\ two\ regimes$

The model estimates parameters $\left(\beta_{S_t}, \sigma^2{}_{S_t}\right)$ and transition probabilities by maximizing the likelihood:

$$L \ = \ \prod_{t=1}^{T} \sum_{S_t} P(S_t \mid S_{t-1}) \, f(y_t \mid S_t)$$

Based on these theoretical foundations, the practical implementation of the MRSM was done in Python using the following assumptions. The regime-switching model is fitted once on the whole dataset because it is easier to calibrate and select variables. Other methods would be approached in the Chapter Model Enhancement to avoid the potential forward bias of this approach. The MRSM model for this research is implemented with a Python library and takes four main entries (statsmodels, 2024):

- The endogenous variable: The MSCI World Momentum in the log-return format (1 variable x 1037 times)
- The exogenous variable: The Selected Variables dataset (11 variables x 1037 times)
- The number of Regime: Two Regimes, one for the Momentum Portfolio and one for the Minimum Volatility Portfolio
- The switching variance: The *"True"* selection of the Regime switching variance allows the model error term's variance to change over time, reflecting better the different regimes.

Dynamic Portfolio Allocation Strategy

Once the model is fitted, the regime-switching Model provides a marginal probabilities matrix with regime allocation probabilities output for each dataset time. The matrix is composed in Table 10.

	REGIME 0 (MOM)	**REGIME 1 (MIN-VOL)**
2005-01-16	0.13	0.87
2005-01-23	0.11	0.89
…	…	…
2024-11-17	0.00	1.00
2024-11-24	0.01	0.99

Table 10: Regime Allocation Probabilities

	REGIME 0 (MOM)	**REGIME 1 (MIN-VOL)**
2005-01-16	0	1
2005-01-23	0	1
…	…	…
2024-11-17	0	1
2024-11-24	0	1

Table 11: Regime Allocation Binary

The matrix provides the dynamic allocation of portfolio strategies, where Regime 0 corresponds to the momentum strategy and Regime 1 represents the minimum volatility strategy. Continuous probabilities have been converted into binary signals to avoid excessive transaction costs and facilitate weight turnover. Each time, the strategy with the higher regime probability is fully allocated. This binary approach keeps the strategy dynamic and respects the practical constraints of portfolio implementation. Figure 3 represents the regime allocation over time, and Table 11 describes the new matrix format.

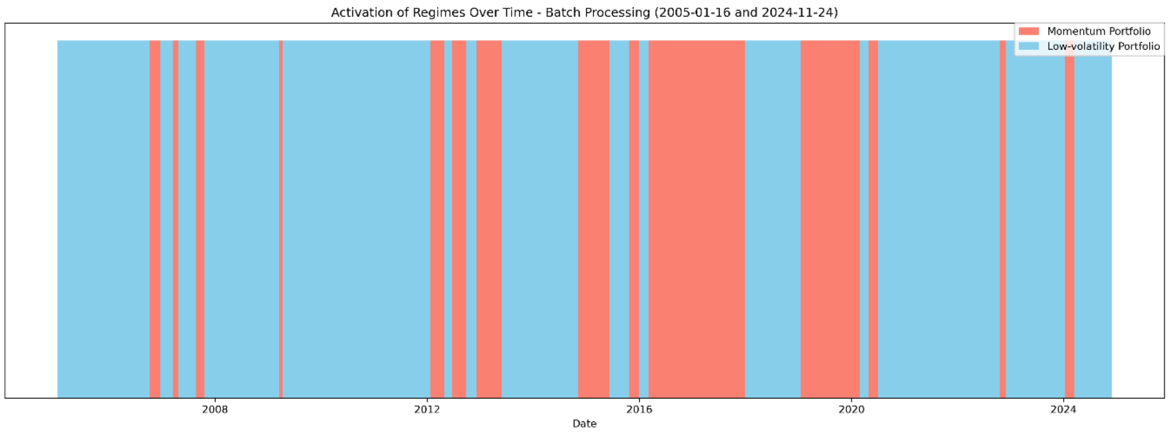

Figure 3: Activation of Regime Over Time – Batch Processing

Once the regime allocation is refined, the dynamic portfolio allocation strategy must be implemented with the binary allocation matrix and the lag return of the associated strategy. The following formula represents the dynamic strategy implementation:

$$r_{t+1,dynamic} = x_{t,Regime0}\, r_{t+1,Momentum} + x_{t,Regime1}\, r_{t+1,Minimum\,Volatilty}$$

$$x_{t,Regime0} \in \{0,1\}, where\ x_{t,Regime0} = 1\ (active)\ and\ x_{t,Regime0} = 0\ (not\ active)$$
$$x_{t,Regime1} \in \{0,1\}, where\ x_{t,Regime1} = 1\ (active)\ and\ x_{t,Regime1} = 0\ (not\ active)$$
$$x_{t,Regime0} + x_{t,Regime1} = 1$$

Equation 5: Dynamic Portfolio Allocation Strategy

Where:

- $r_{t+1,dynamic}$: $Return\ of\ the\ regime\ switching\ strategy$
- $r_{t+1,Minimum\,Volatility}$: $Return\ of\ the\ MSCI\ World\ Minimum\ Volatility$
- $r_{t+1,Momentum}$: $Return\ of\ the\ MSCI\ World\ Momentum$

A one-period lag is introduced in the return calculation since the regime-switching decision made at time t affects the return from period t to t plus 1 rather than from period $t-1$ to t. To evaluate the effectiveness of this approach, the dynamic portfolio allocation strategy has been fitted on the entire dataset, giving the following annualized performance results:

- Annualized Mean (%): 11.11
- Annualized Std Dev (%): 10.70
- Sharpe Ratio: 1.04

Model Enhancement

This chapter has implemented a few methods to enhance the model. Two main potential issues have been tested to improve the model: first, addressing the potential forward bias associated with fitting the model to the entire dataset simultaneously, and second, evaluating the potential impact of low signal switches caused by the binary rule.

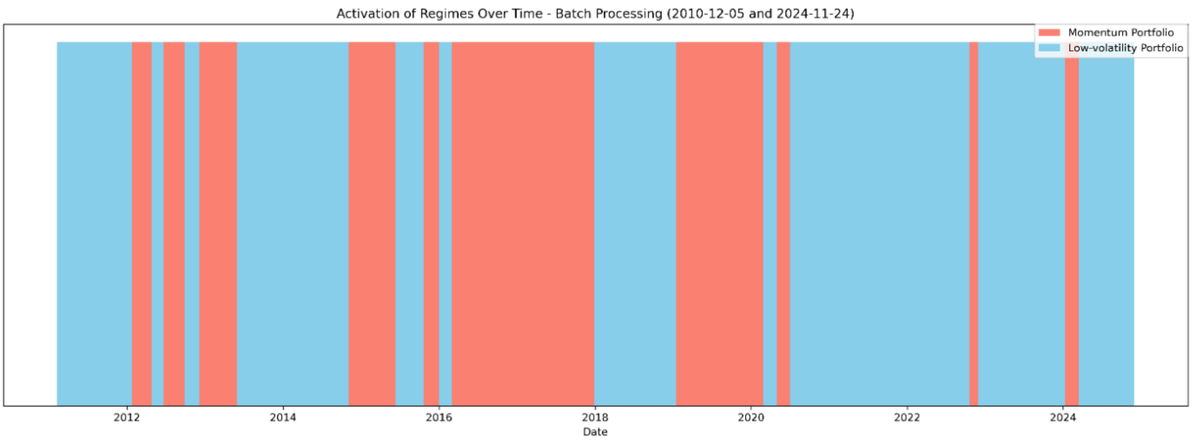

Figure 4: Activation of Regime Over Time – Batch Processing (2010-12-05 and 2024-11-24)

Forward Bias

Fitting the MRSM to the entire dataset may introduce forward bias. Although the regime determination depends only on the current time, initializing the model parameters using the whole dataset could inadvertently affect the model's outcomes, with parameter initialization being an example. Two alternative approaches have been tested to address this issue and reduce the risk of forward bias: the Rolling Window and the Expanding Window methods.

Rolling Window

The Rolling Window method trains the model on a fixed-length window that updates over time. However, this approach introduces a small-sample bias because of the limited training data available at each iteration, resulting in excessive regime switches and imprecise regime identification. The previous arguments can be verified in Figure 5 below. Additionally, the Sharpe ratio drops significantly to 0.78, reducing the strategy's risk-adjusted performance. The Rolling Window method is not suitable for this research and is discarded.

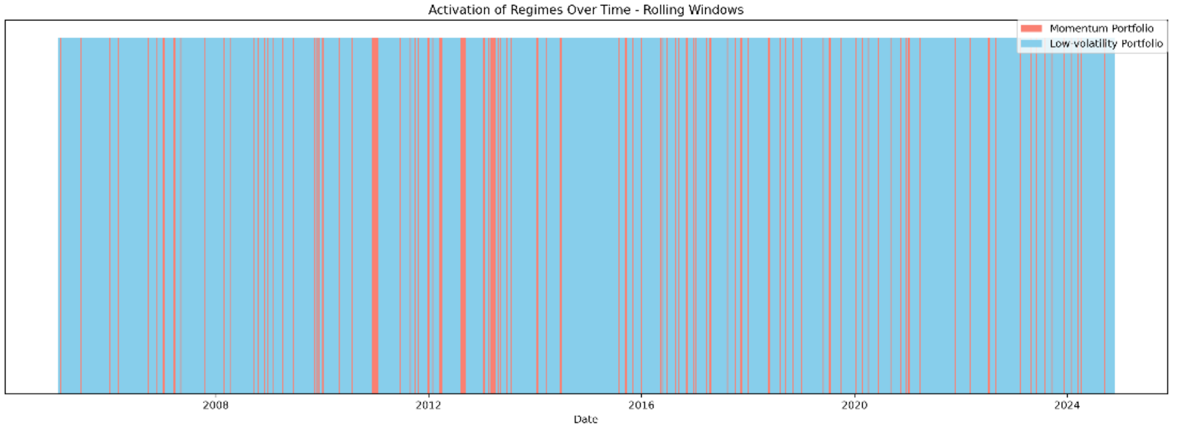

Figure 5: Activation of Regimes Over Time - Rolling Windows

Expanding Window

The Expanding Window method reduced the impact of the small-sample bias because it trains the model on a growing dataset. The algorithm starts with an initial dataset size of 317 weeks and then incorporates new data 10 weeks at a time. The larger initial dataset size provides an advantage by reducing the impact of limited training data available during the early iterations of the algorithm. Furthermore, using a data addition window of 10 reduced the calculation time of the algorithm.

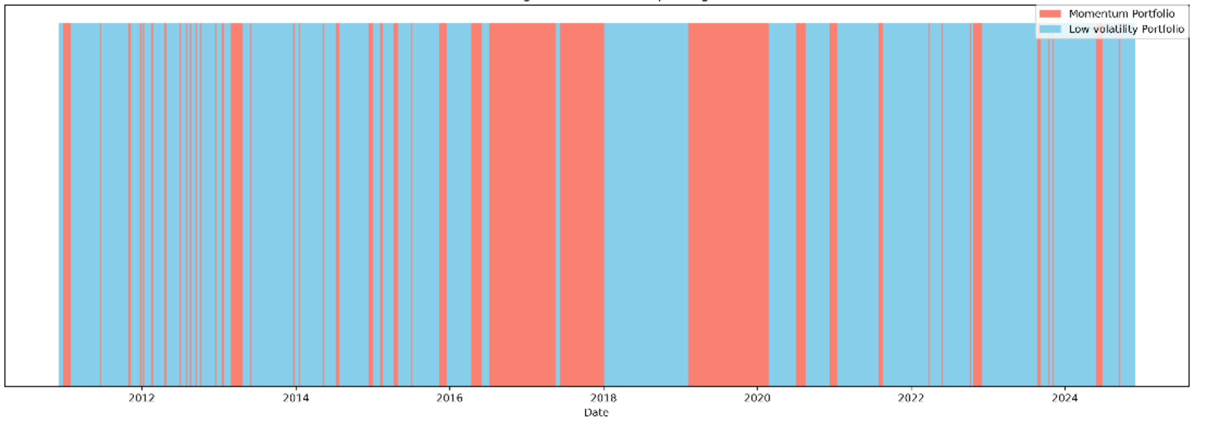

Figure 6: Activation of Regime Over Time - Expanding Windows

The Expanding Window method achieves 80 switches, a strong Sharpe ratio of 1.06, and a good regime precision. Figure 6 shows longer trends than the Rolling Window; a comparison with Figure 4 highlights similar regime allocations. Regimes align 580 out of 720 times, giving an 80% precision. Training on sufficient data at each step avoids forward bias and delivers strong performance, making it the preferred choice for the final model.

Low-Signal Regime Transitions

In regime-switching models, it is essential to evaluate the strength of regime selection to determine if the allocation is a firm conviction or an undefined boundary. This chapter examines the possibility of low-signal regime switches and evaluates their impact on the model's effectiveness. The model is changed to have stricter regime classifications to address the potential issue of low-signal switches. As the transition matrix can attest, the regime-switching model already has clear and distinct boundaries. Therefore, a high probability threshold of 0.9 has been set, meaning that a regime change is recognized only when the probability of switching exceeds this value. After implementing this threshold, it is observed that only a small number of transitions are excluded. Figure 7 illustrates the regime classifications and the excluded neutral zone. The minimal impact of the high threshold adjustment reflects the firm conviction of the regime selection. No further constraints are then needed.

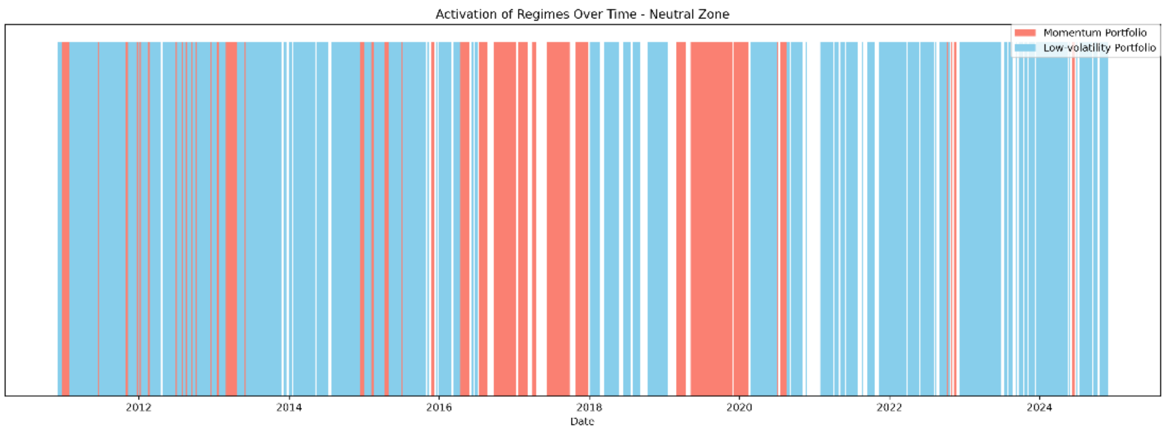

Figure 7: Activation of Regime Over Time - Neutral Zone Addition

To evaluate the impact of the Neutral Switch Zone, the excluded transitions are forward-filled with the last allocated regime (Figure 8). The adjusted model demonstrates a drop in the Sharpe ratio to 0.93, highlighting that the disregarded switches significantly contribute to performance.

This suggests that the excluded switches correspond to meaningful regime changes, and their removal diminishes the effectiveness of the allocation strategy. Given these findings, the Neutral Switch Zone is not incorporated into the final model.

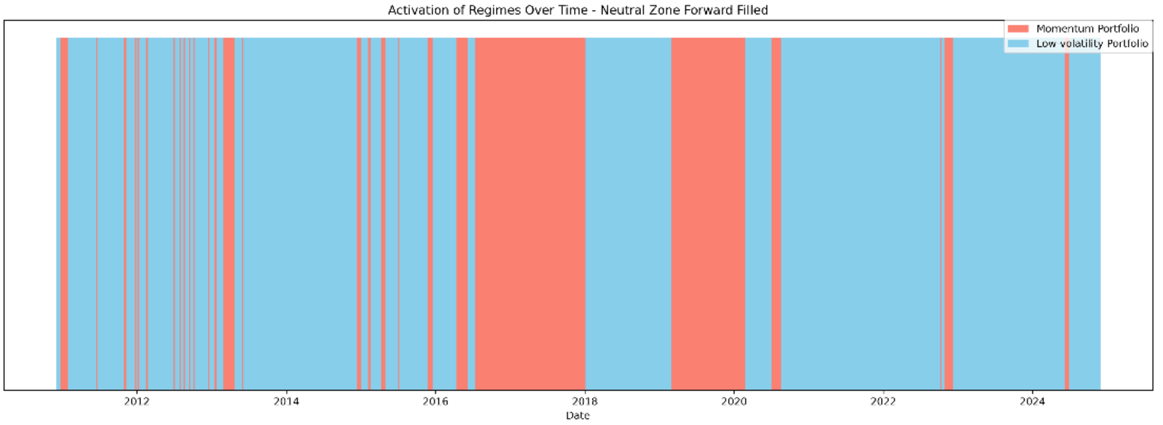

Figure 8: Activation of Regime Over Time - Neutral Zone Forward Filled

Results & Discussion

Before presenting the results, the final model employed is briefly defined. The Expanding Window Model was selected for its ability to eliminate forward bias and mitigate small data bias through an increasing dataset. Fitted on the 11 selected variables (Table 8), the model identified 82 regime switches over nearly 14 years, achieving an annualized mean return of 8.7%, a standard deviation of 8.2%, and a Sharpe Ratio of 1.06. These results validate the model's ability to optimize risk-adjusted returns effectively. The following sections examine the model performance and regime dynamics by analysing the model evaluation metrics, the regime allocation tendencies, and the switching efficiency. The analysis then transitions to assessing the strategy's overall performance and resilience and finally discusses its practical applications in the financial industry.

Model Performance & Regime Dynamics

Model Evaluation

The first part of this section is dedicated to analysing the model's overall quality. Table 12 gives the key statistics of the fitted model, while Table 13 is the conversion table, linking the model's output to the variable's associate name.

The first set of key statistics in Table 12 are the AIC of 1907.2 and the BIC of 2035.8, two evaluation variables already discussed in the chapter Feature Selection & Optimization. These metrics were optimized during the variable selection for a good model fit. The next part of the table provides the coefficient statistics for Regime 0 (Momentum) and Regime 1 (Minimum Volatility). These statistics include the coefficient values, standard deviations, z-statistics, p-values, and 95% confidence intervals. To start, the p-values indicate the statistical significance of each coefficient. All variables in the model are statistically significant for at least one regime, as their p-values are below the 0.05 threshold. However, some variables, such as Credit Spread Log Return (x_6) and Liquidity (x_{10}), show significance in only one regime, suggesting a regime-specific effect.

```
                        Markov Switching Model Results
==============================================================================
Dep. Variable:          MOM_log_return   No. Observations:                 1037
Model:                MarkovRegression   Log Likelihood                -927.607
Date:                 Fri, 27 Dec 2024   AIC                           1907.215
Time:                         16:35:50   BIC                           2035.761
Sample:                     01-16-2005   HQIC                          1955.985
                          - 11-24-2024
Covariance Type:                approx
                             Regime 0 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.5968      0.046     12.884      0.000       0.506       0.688
x1            -0.3794      0.027    -13.978      0.000      -0.433      -0.326
x2             0.0526      0.027      1.947      0.052      -0.000       0.106
x3            -0.1277      0.038     -3.343      0.001      -0.203      -0.053
x4            -0.0863      0.025     -3.428      0.001      -0.136      -0.037
x5             0.3445      0.042      8.142      0.000       0.262       0.427
x6             0.0128      0.030      0.421      0.674      -0.047       0.072
x7             0.0903      0.043      2.082      0.037       0.005       0.175
x8            -0.5583      0.108     -5.160      0.000      -0.770      -0.346
x9             0.1990      0.036      5.597      0.000       0.129       0.269
x10           -0.0002      0.032     -0.007      0.994      -0.064       0.063
sigma2         0.1261      0.013      9.733      0.000       0.101       0.151
                             Regime 1 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0287      0.029     -0.977      0.329      -0.086       0.029
x1            -0.6397      0.029    -21.785      0.000      -0.697      -0.582
x2             0.2243      0.028      8.032      0.000       0.170       0.279
x3            -0.2277      0.033     -6.919      0.000      -0.292      -0.163
x4            -0.0785      0.028     -2.772      0.006      -0.134      -0.023
x5             0.0314      0.032      0.970      0.332      -0.032       0.095
x6            -0.0594      0.027     -2.181      0.029      -0.113      -0.006
x7            -0.0647      0.026     -2.527      0.011      -0.115      -0.015
x8             0.0314      0.025      1.232      0.218      -0.019       0.081
x9            -0.1718      0.044     -3.907      0.000      -0.258      -0.086
x10            0.1397      0.040      3.451      0.001       0.060       0.219
sigma2         0.4299      0.024     18.231      0.000       0.384       0.476
                         Regime transition parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
p[0->0]        0.9402      0.018     52.992      0.000       0.905       0.975
p[1->0]        0.0232      0.008      3.040      0.002       0.008       0.038
==============================================================================
```

Table 12: Markov Regime Switching Model Evaluation

VARIABLES CONVERSION

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
VARIABLES	VIX Log Return	Crude Oil Log Return	PUT/ CALL	IR Return	IR	CS Log Return	CPI	GDP	VIX	Liquidity

Table 13: Variable Conversion

Then, coefficients provide additional insights into the direction and magnitude of the relationships between the variables and the regimes. Not all variables will be discussed; the analysis is focused only on the largest magnitudes. Initially, it is important to consider that the coefficient direction or magnitude could be unexpected, and this is for two main reasons. First, the simultaneous inclusion of 10 variables in the model could induce interactions between variables and residual multicollinearity. The interpretation of individual coefficients could then be modified. While the Elastic Net and best AIC methods help reduce multicollinearity, they cannot suppress interactions between variables in the dataset, as is possible in Figure 2. Second, the coefficient values might reflect specific economic or market conditions, such as post-COVID recovery or periods of abnormal volatility period. These two points highlight the importance of cautiousness with coefficient analysis in a multiple-variable model. It is then important to consider both the economic context and the potential interdependence of variables.

Starting with the most impactful coefficients, VIX Log Return (x_1) shows negative coefficients for both momentum (-0.37) and minimum volatility (-0.63). This result aligns with the understanding that increasing market volatility generally hurts the market, affecting both strategies. However, the weaker impact on the momentum strategy is surprising, as momentum strategies are typically more sensitive to volatility. GDP (x_8) has a negative coefficient (-0.56) for momentum, which contradicts the expectation that economic growth is favourable for upward market trends. This result could reflect specific periods, such as the market correction in 2021 associated with the high GDP growth or the significant market rebound in 2020 despite the low GDP levels. Conversely, GDP has a negligible effect (0.03) on the minimum volatility strategy, which is consistent with the stability of the strategy. Interest Rate (x_5) shows a positive coefficient (+0.34) for momentum, which diverges from the traditional expectation that rising interest rates slow down growth and market optimism. This anomaly could be due to specific periods in the dataset, such as the post-COVID recovery. The rising interest rates of this period coincided with positive market sentiment and strong momentum. Similarly, VIX (x_9) exhibits a positive coefficient (+0.2) for momentum, which is counterintuitive initially. However, this result could indicate that momentum strategies perform well during recovery periods following volatility spikes. The negative impact of VIX on minimum volatility further supports this interpretation.

The regime transition parameters at the end of Table 12 further validate the model's robustness. The model has a high probability of remaining in the same regime, $P[0 \rightarrow 0] = 0.94$ and $P[1 \rightarrow 1] = 0.98$, the regime can catch long term tendencies. Conversely, lower probabilities of switching between regimes, $P[0 \rightarrow 1] = 0.06$ and $P[1 \rightarrow 0] = 0.02$, suggest precise selection of regime switch. These parameters are advantageous for portfolio management, because they minimize transaction costs and captures long-term trends rather than short-term fluctuations.

In conclusion, the MRSM shows a strong fit due to its statistically significant variables, well-defined transition probabilities, and meaningful coefficients. However, some coefficients show unexpected relationships; these can be attributed to interactions between variables and specific dataset factors. The previous arguments validate the Elastic Net AIC selection and underlying assumptions, confirming the model's ability to provide strong results for implementing a dynamic portfolio allocation. These variables' significance and associated transition probabilities underscore the model's ability to adapt to market conditions. These features directly inform the regime allocations, which will be explored in the next section.

Regime Allocation

The allocation results provided by the Markov Regime Switching Model are represented in Figure 9. It shows the activation of regimes over time, with Minimum Volatility and Momentum represented in blue and red, respectively.

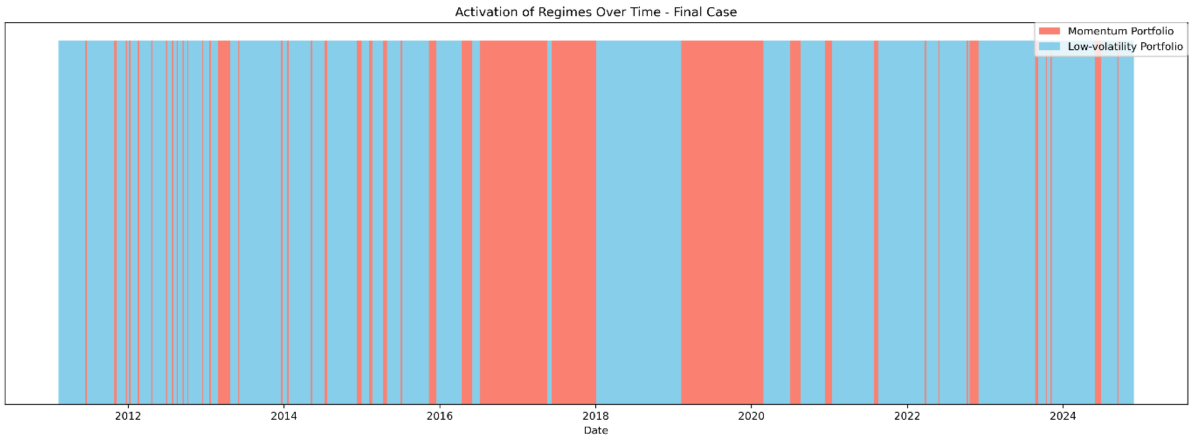

Figure 9: Activation of Regime Over Time - Final Case

The model identified 82 switches over the 14-year horizon, with 513 weeks in the Minimum Volatility portfolio and 217 weeks in the Momentum portfolio. This means that the defensive characteristics of the low-volatility strategy are preferred to the risk exposition of the momentum strategy. The two following figures will be used for the regime allocation analysis. Figure 10 highlights relative performance on a 10-week average, allowing for trend analysis instead of week-by-week fluctuations. Figure 11 gives the cumulative performance, revealing the strategy's reactions to market movements.

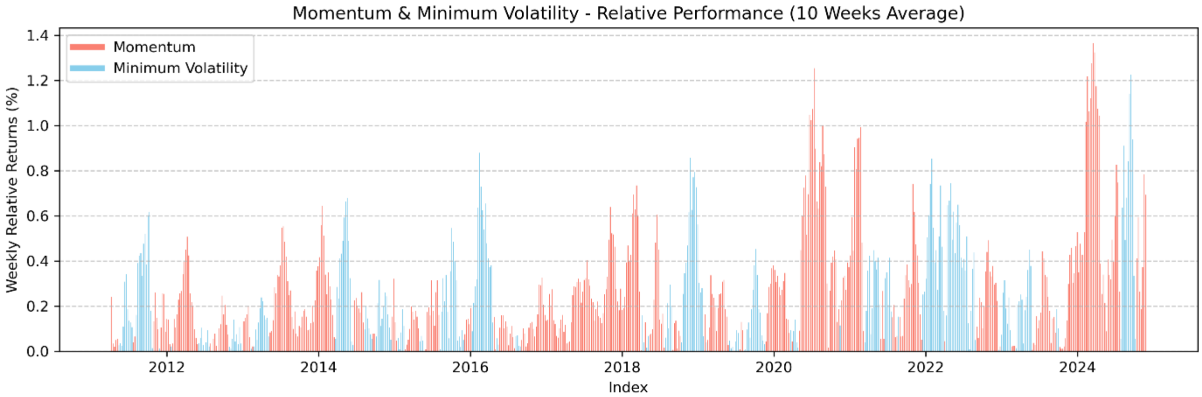

Figure 10: Momentum & Minimum Volatility – Relative Performance (10 Weeks Average)

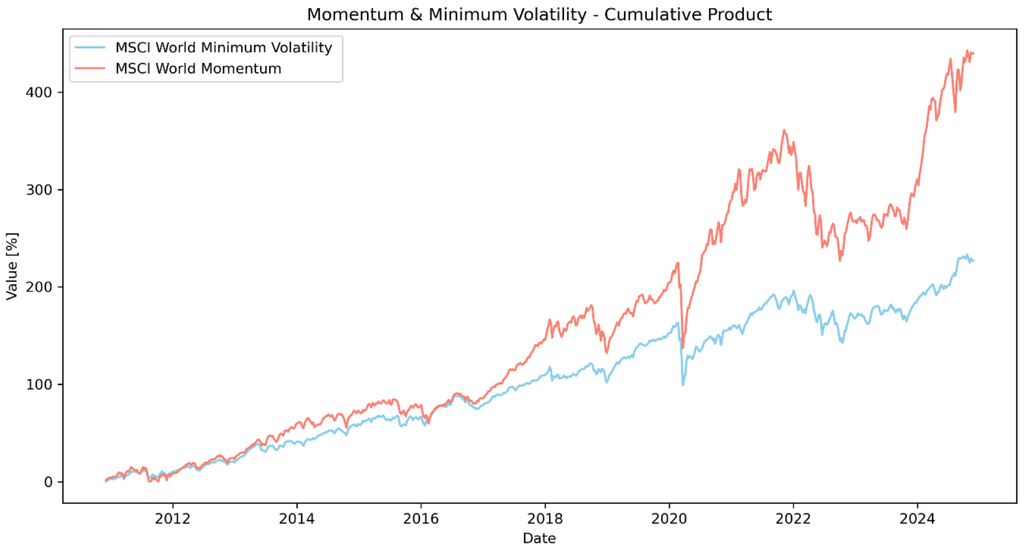

Figure 11: Momentum & Minimum Volatility - Cumulative Performances

From 2011 to 2016, the model was mainly allocated to Minimum Volatility, while Momentum was occasionally activated. Figure 11 underscores the absence of clear trends by highlighting the comparable performance of the two strategies during this period. Allocating to the correct strategy in the period following no trend is complicated; this is why the model probably adopts the Minimum Volatility strategy, which is more defensive. This cautious approach helped minimize drawdowns during turbulent times while promoting stable growth, demonstrating the model's risk-averse tendencies. Furthermore, Figure 11 highlights that allocating between Momentum or Minimum Volatility gives the same cumulative product during this period.

Between 2016 and 2018, the model shifted toward Momentum, capitalizing on the global economic upswing, pro-growth policies in the United States, and rising equity markets. During this period, Momentum benefited from stable upward trends. Figure 10 reflects the relative outperformance of the Momentum strategy during the entire period, while Figure 11 shows Momentum diverging from the Minimum Volatility strategy starting in 2016. By late 2018, the model returned to Minimum Volatility in response to escalating global uncertainties, and this adjustment helped mitigate the market declines seen toward the end of 2018 (World Economic & Financial Survey, 2018).

The 2019-2020 period is characterized by greater exposure to Momentum. However, this period lacked identifiable trends, as shown in Figure 10. On March 1, 2020, the model effectively captured the significant market drawdown caused by the COVID-19 pandemic. The model switches to Minimum Volatility before the performance release. Following the pandemic recession, the model maintained its defensive position in the low volatility. However, this induced missed opportunities, such as the large performance that began in the second half of 2023. These missed opportunities can be attributed to elevated volatility during this period, which favoured Minimum Volatility over Momentum. This conservative approach supports strong risk-adjusted performance; however, sometimes, the model misses substantial gains, as in this instance.

An examination of Figure 9 from a broader perspective reveals a notable decline in the frequency of switches after 2016. This decline is likely due to the expanding window approach, which increased the dataset size and enabled the model to better identify long-term trends. This reduction aligns with the model's objective of minimizing transaction costs and focusing on lasting trends rather than reacting to short-term market fluctuations. This underscores the importance of a large amount of data to obtain a strong identification of regimes by the model.

Lastly, Figure 12, which gives the "perfect switching" scenario, offers valuable insights. This idealized case assumes that the investor knows future market performance, making it unattainable. Nevertheless, it underscores the importance of capturing longer-term trends, as the high frequency of switches of such a model is difficult to replicate without overfitting to random market fluctuations. The goal is to identify periods of sustained outperformance, which may be influenced by distinct market conditions and not market noise.

Figure 12: Activation of Regimes Over Time - Perfect Case

Switch Efficiency

Building on the regime allocation analysis, this section evaluates how efficiently the model selects the best-performing strategy during switches. Switch efficiency directly impacts overall performance, as correct allocation decisions maximize returns while incorrect switches result in opportunity costs. This section evaluates the model's ability to allocate effectively during each switch by measuring the relative performance of the chosen strategies. The cumulative impact of correct and incorrect switches is then assessed to understand the model's value-add over time.

The regime-switching model allocates to the better-performing strategy in 53 out of 83 switches. Figure 13 illustrates the annualized relative returns of each switch, with positive bars indicating successful allocations and negative bars representing errors. Correct switches often result in higher returns, with two outliers achieving over 100% annualized relative returns. However, the switch annualized return does not represent the real outperformance added to the strategy because it does not consider the switch allocation time. Overall, the model shows significant improvement over time; in the last 35 switches, 82% of the switches were correct. The model improved decision-making accuracy as the dataset expanded.

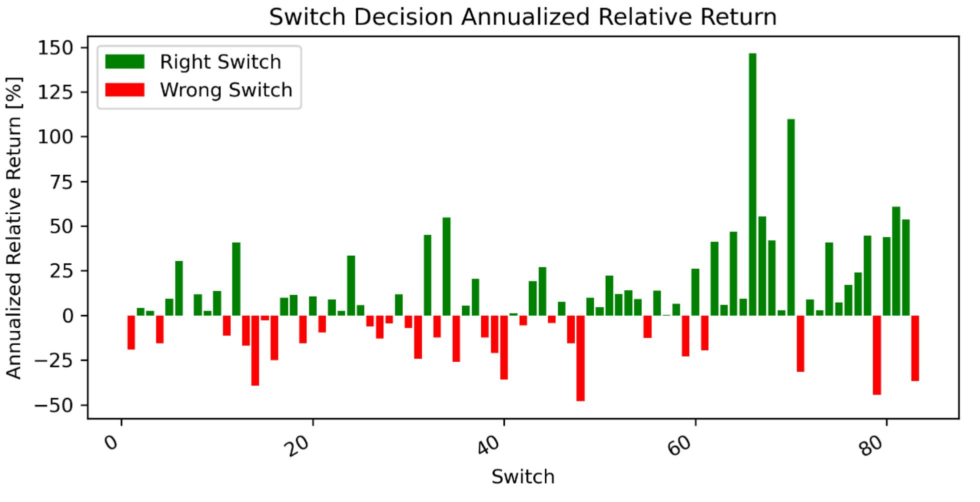

Figure 13: Switch Decision Annualized Relative Return

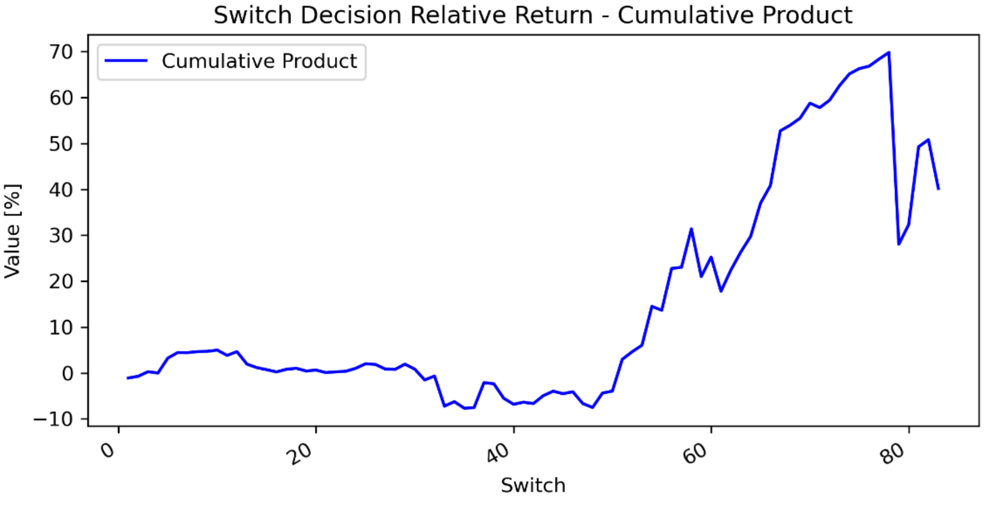

Figure 14: Switch Decision Relative Return - Cumulative Product

The cumulative product plotted in Figure 14 shows the impact of these switches on the overall performance. Firstly, it is possible to see the lack of relative performance during the first 50 switches until November 2015. This is probably due to the difficulties of trend identification, explained by the limited data available in the early phase of the expanding window and the absence of clear market tendencies during the period. The second argument is highlighted in Figure 10 of the Regime Allocation chapter. Despite the initial lag, the cumulative relative return rose to a peak of 70%, indicating that later switches significantly boosted performance. This illustrates the model's increasing ability to improve regime selection as more data becomes available. In addition, the duration of the regime allocation increases, reflecting stronger conviction in regime allocation. Despite the improvement over time, penalties from incorrect switches remain a limitation, especially in the early period with limited data and unclear trends. These missed opportunities suggest space for refinement. However, the model's consistent alignment with the superior strategy and growing precision over time underscore its strength, contributing to 40% relative overperformance.

The model performance and regime dynamics analysis confirm the model's adaptability to evolving economic conditions, with correct regime identification over time. The model's frequent allocation to Minimum Volatility underscores its preference for stability. While Momentum activations are sporadic, they often benefit from targeted growth and market recovery phases. It demonstrates the model's capacity to catch signals from changing market dynamics. Finally, the model's adaptability delivers relative performance through well-timed regime switches. However, the model occasionally misses opportunities during Momentum phases, such as those observed in 2023-2024. The next step is the dynamic portfolio allocation strategy performances, which would provide a deeper understanding of strategy performances and resilience in market conditions.

Dynamic Portfolio Allocation Strategy Performances

The dynamic portfolio allocation strategy performance section is divided into three main topics. It starts by comparing the main metrics, then moves on to evaluate the short-term investment horizon with a 3-year rolling window assessment, and finally, it evaluates the strategy's resilience by identifying the most significant drawdowns during the analysed period.

	MSCI WORLD MINIMUM VOLATILITY	MSCI WORLD MOMENTUM	MSCI WORLD	REGIME SWITCHING STRATEGY
ANNUALIZED MEAN RETURN	7.6%	11.2%	8.9%	8.7%
ANNUALIZED SD	10.4%	14.4%	14.1%	8.2%
SHARPE RATIO	0.73	0.78	0.63	1.06

Table 14: Strategy Performance Comparison

Performances Comparison

Table 14 summarizes the performance metrics of the Markov Regime Switching Strategy and the static allocation strategies. Figure 15 shows a bar plot of the annualized mean return and annualized standard deviation, and Figure 16 shows a bar plot representing each strategy's Sharpe ratio.

Figure 15: Strategies Comparison - Return & Volatility

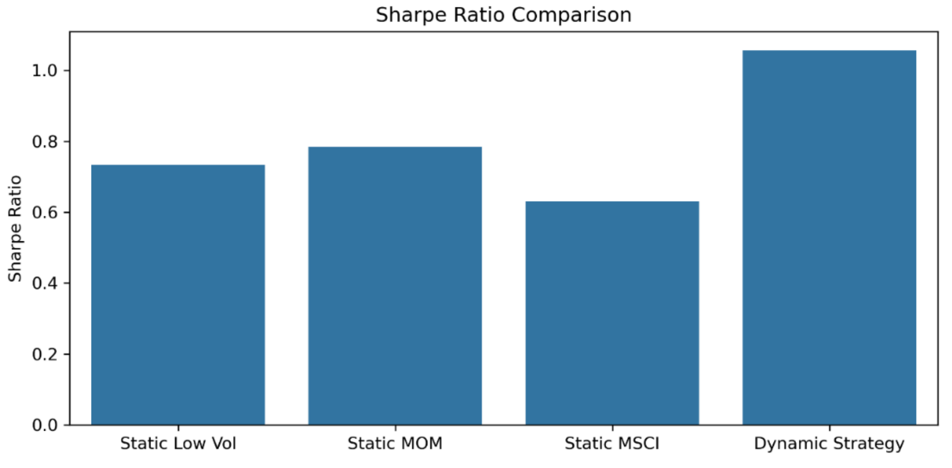

Figure 16: Sharpe Ratio Comparison

First, the Annualized Mean Return of the Regime Switching Strategy of 8.7% is comparable to the MSCI benchmark and lower than the Momentum strategy. However, it surpasses the Low Volatility strategy, demonstrating a balance between the risky profile of Momentum and the low-risk profile of Minimum Volatility. Second, the Regime Switching Strategy's standard deviation of 8.2% is significantly lower than static strategies. For instance, its standard deviation is approximately 6% lower than MSCI World and MSCI World Momentum. The volatility is even lower than the minimum-volatility portfolio one, renowned in the financial industry as a reference for low-risk equity investing. The two previous elements are represented in Figure 15. Finally, the Regime Switching Strategy's Sharpe ratio is the highest among all strategies, with a value of 1.06. Figure 16 highlights that its risk-adjusted return outperforms Momentum, Low Volatility, and MSCI World. The regime switching strategy effectively leverages the regime allocation to enhance performance while managing risk.

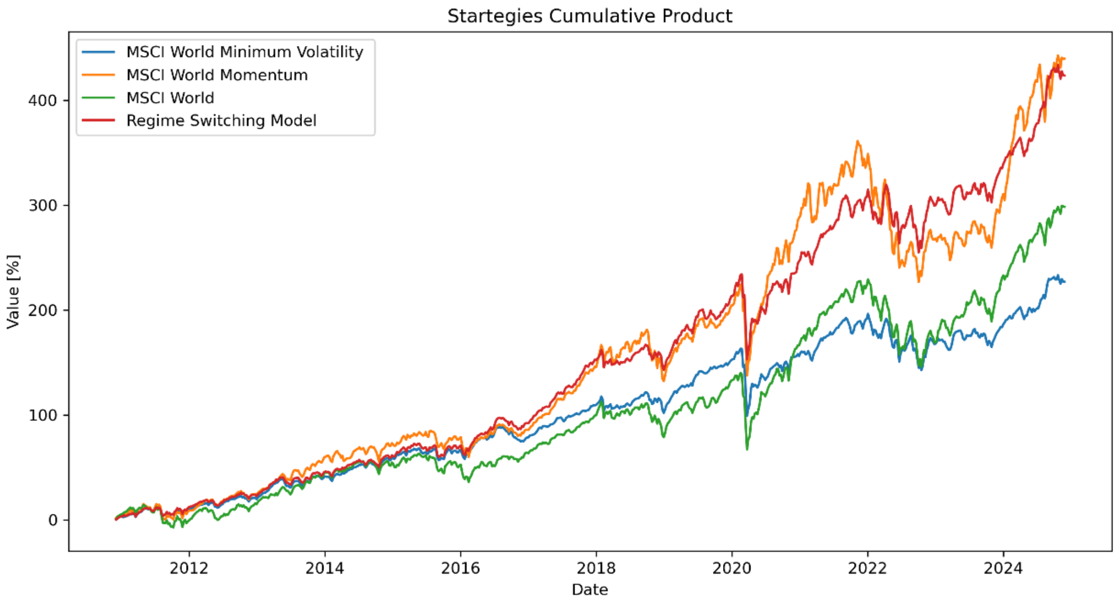

Figure 17: Strategies Comparison - Cumulative Product

Overall, the results demonstrate that the Regime Switching Strategy provides a well-balanced approach, outperforming the static strategies in terms of risk-adjusted returns and volatility control, making it a robust alternative for dynamic portfolio management. The graph in Figure 17 represents the cumulative product return of all the strategies over the 14 years. The first element noticeable is that the MSCI World Momentum and the Regime Switching Strategies outperform other strategies. Both have similar cumulative performances; however, the Regime Switching Model exhibits relatively stable returns compared to the more turbulent performance of the Momentum strategy. It is well represented during the troubled period of 2022-2024, where momentum has a more considerable decrease followed by a more vigorous recovery phase too.

Regime Switching strategy, in contrast, is less affected by the market volatility of the period. Regarding the benchmark, the cumulated performance is lower than the two best performers in terms of return and seems to have an in-between volatility. The last performing strategy is, as expected, the Minimum Volatility one. However, the strategy shows incredible consistency in the period compared to other strategies.

Rolling Window Analysis

The rolling window analysis evaluates the cumulative returns of the four investment strategies across overlapping time frames to assess their consistency in performance for a shorter investment horizon. It will assess if the strategy benefits from "lucky" timing or if the strategy performs disregarding the investment starting date. The 3-year rolling window analysis gives the cumulative returns for 3 years of each strategy for different starting years. As illustrated in Figure 18, the MSCI World Momentum emerges as the best performer, outperforming its peers in 7 out of the last 13 starting periods. The Regime Switching Strategy closely follows, ranking as the second-best performer with the highest returns in 6 of the same periods. In contrast, the MSCI World and Minimum Volatility strategies consistently underperform. However, Minimum Volatility demonstrates greater consistency than the MSCI World, with cumulative returns ranging between 15% and 40% over three years. Momentum exhibits the highest variability in returns, with returns ranging from below 10% to as high as 90% during specific windows. The Regime Switching Strategy balances performance and risk effectively, delivering returns between 25% and 70%. It exhibits then less variability in return than the MSCI World and MSCI World Momentum strategies.

In conclusion, the rolling window analysis highlights the strong performance of the Regime Switching Strategy, even over shorter three-year horizons. Its adaptability to market conditions enables it to deliver consistent competitive returns, outperforming the MSCI World and Minimum Volatility strategies on average. While the MSCI World Momentum strategy achieves higher returns during periods with favourable market conditions, its reliability diminishes in adverse environments. Conversely, the Minimum Volatility strategy demonstrates stability across all periods but sacrifices return, leading to underperformance. As a result, the Regime Switching is a resilient and balanced option for investors with a shorter-term focus. In addition to short-term performance, it is significant to understand how the strategy navigates periods of considerable market stress. The following section examines the worst drawdown scenarios, providing insights into the strategy's resilience and recovery capabilities.

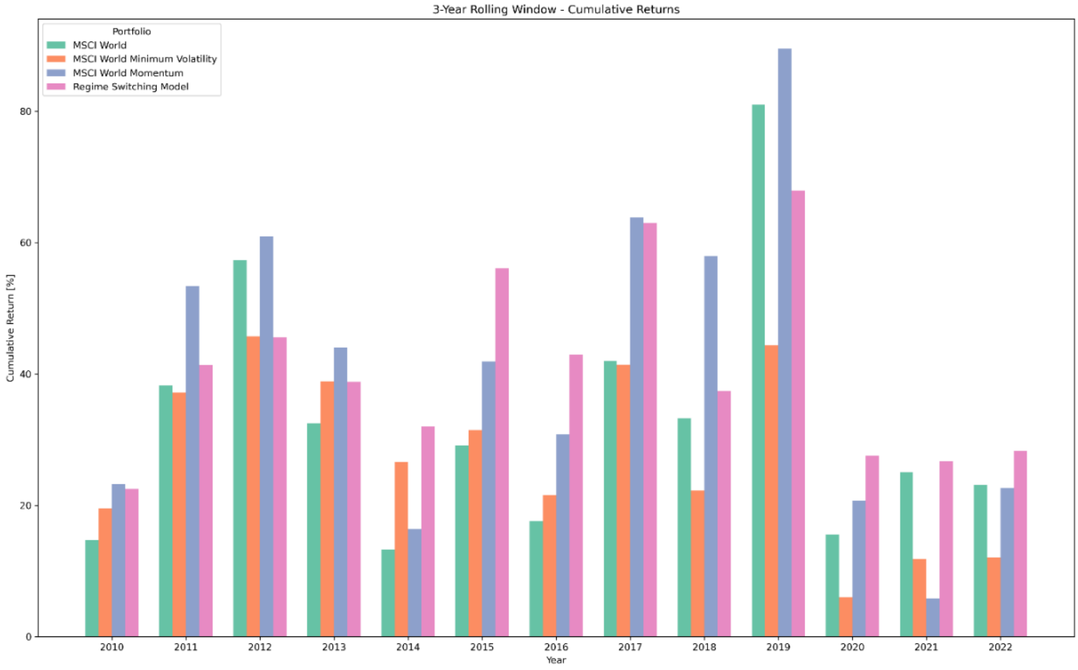

Figure 18: 3-Year Rolling Window - Cumulative Returns

Worst Drawdown & Risk Assessment

Drawdowns are the decline from a portfolio’s peak to its trough. They provide valuable insights into strategy resilience under stressful market conditions and the ability to profit from recovery periods. This section focuses on three significant drawdown periods: the 2011 U.S. Debt Ceiling crisis (Figure 19), the 2020 COVID-19 pandemic (Figure 20), and the post-pandemic drawdown (Figure 21). To facilitate the understanding of the model’s strategy selection, periods of regime allocation are summarized in Table 16 of the Appendix A: TablesAppendi section.

During the **2011 U.S. debt ceiling crisis** (Figure 19), markets faced increasing instability, and the benchmark experienced a maximum drawdown of -17% over 14 months. In contrast, the MRSM recovered within 6 months, with a maximum drawdown of only -6%. The strategy's resilience stemmed from its ability to switch to Minimum Volatility during the first four months of the drawdown. It permits the strategy to minimize the performance release, allowing a shorter recovery time. After the drawdown, the strategy is only partially exposed to the momentum strategy, losing a small quantity of extra performance. Overall, the behaviour of the strategy during this drawdown is greater than other strategies, attesting for a good risk management.

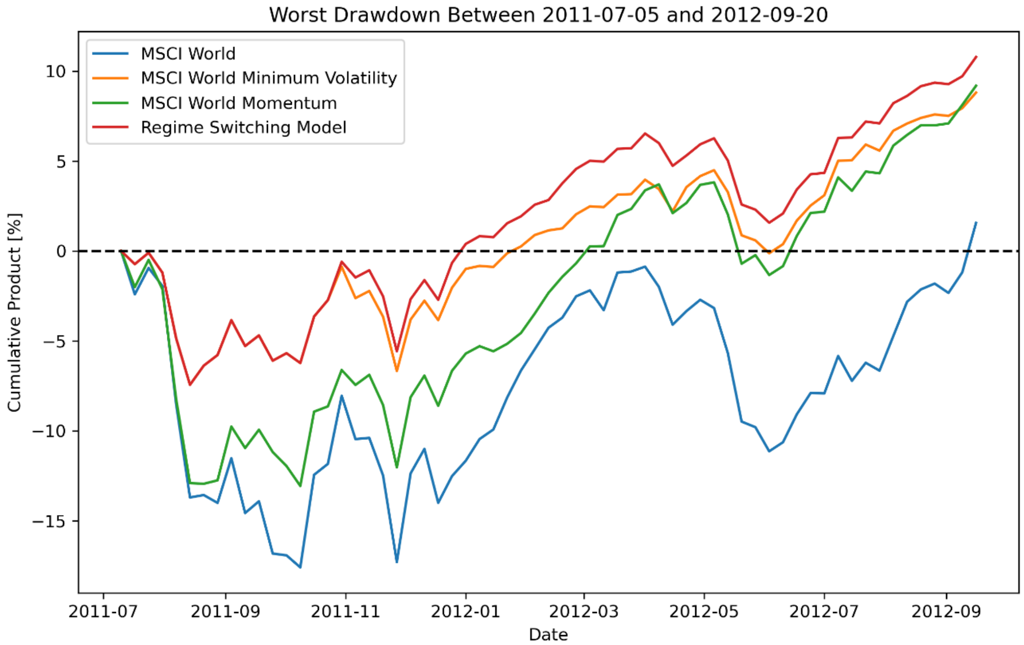

Figure 19: 2011 U.S. Debt Ceiling Crisis

The **COVID-19 pandemic** (Figure 20) is a sharp economic shock., inducing a benchmark performance loss of -30% over 7 months. During this period, the MRSM limited its drawdown to approximately -25%. However, the recovery time was longer than the benchmark, taking 9 months to recover fully. While the strategy initially benefitted from being in Minimum Volatility, it switched to Momentum too late to benefit from the steep recovery. The delay is probably due to the atypically rapid rebound following the crisis, which diverged from historical recovery patterns (Brooks, 2024). It could also be due to the atypical macroeconomic condition of the period, troubling the model to catch suitable regime identification.

The **post-pandemic drawdown** (Figure 21) presents unique challenges, including rising inflation, interest rates, and geopolitical tensions, such as the Russian invasion of Ukraine (ETH Zürich: KOF Swiss Economic Institute, 2021). The benchmark has experienced a prolonged drawdown of -28% over two years, whereas the MRSM mitigated losses with a maximum drawdown of -15% and recovered within 16 months. The strategy primarily allocated to minimum volatility during this period, which is a strategy that performs best during this period. The stability of the chosen strategy allows the model to keep a maximum of the previous performance by avoiding unnecessary volatility exposition during this period of uncertainty. The momentum underperforms in this drawdown period.

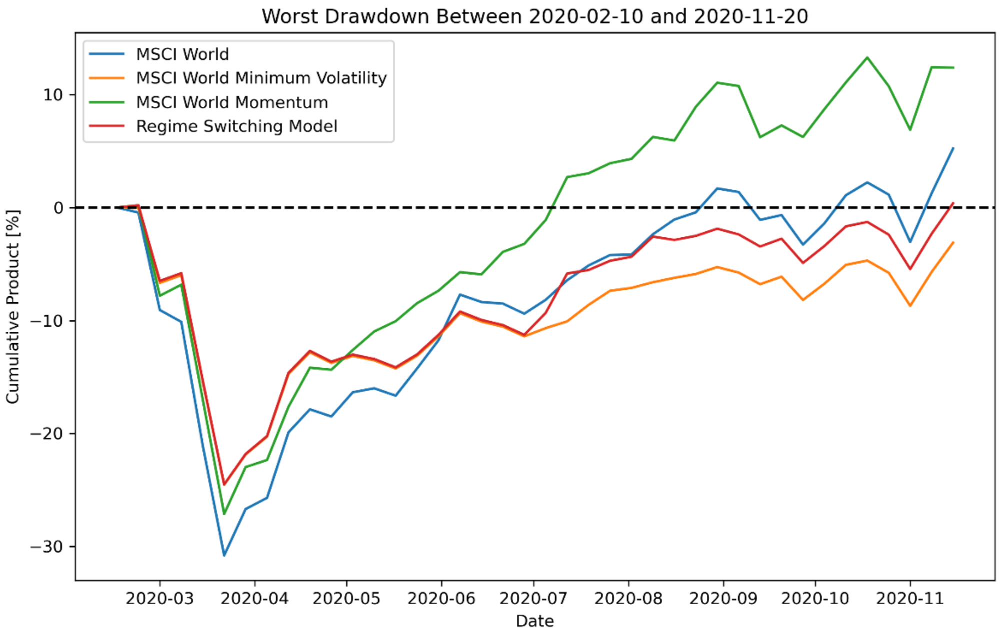

Figure 20: COVID-19 Pandemic

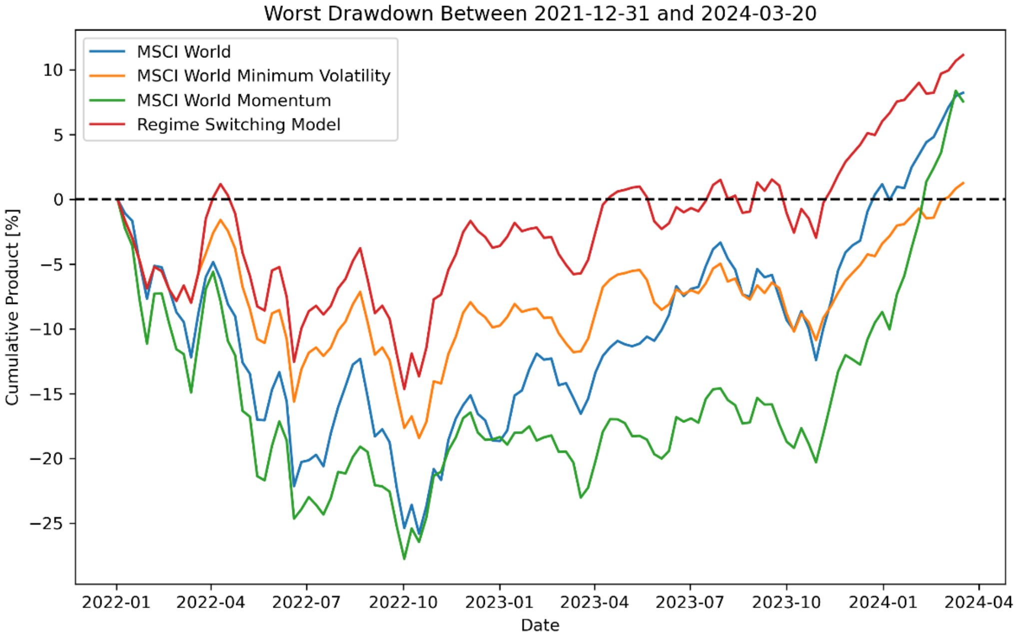

Figure 21: Post-pandemic Drawdown (2021–2024)

Across all three scenarios, the MRSM demonstrated good risk management by reducing the drawdown amplitude and minimizing the recovery time. The strategy outperformed static benchmarks by leveraging Minimum Volatility during downturns, underscoring its resilience in stressful market environments. However, the model occasionally lagged during rapid transitions to a momentum-exposed recovery. The COVID-19 recovery is an excellent example of a miss, highlighting opportunities for refinement. These findings align with the strategy's low-risk exposure, underscoring the dynamic strategy resilience and versatility across diverse market conditions.

Practical Applications

The regime-switching model demonstrates strong potential for dynamic portfolio allocation in finance. This is why, in this section, the model's practical applications will be discussed to identify the possibility of implementing the strategy in a financial institution.

Firstly, the strategy's design is practical for implementation. It only needs reallocation between two ETFs or internally managed strategies, by selling one part to buy the other. In addition, its low switch frequency minimizes transaction costs and portfolio turnover, supporting strategy returns and low operational costs. However, as highlighted in the “Regime Allocation” chapter, quantitative signals alone may lead to suboptimal outcomes during atypical economic or geopolitical stress periods. For example, the post-COVID recovery revealed the model’s limitations in responding to abnormal market dynamics. Integrating qualitative market experience alongside quantitative signals can enhance decision-making and improve resilience to unexpected events. Furthermore, the strategy could be incorporated into a broader investment framework to enhance diversification and risk management. Allocating partial exposure to this approach within a diversified portfolio, including bonds, real estate, or commodities, would mitigate risks associated with equity-only strategies. In addition, the exposure to the superior Sharpe ratios offered by Momentum and Minimum Volatility will be maintained. This integration would allow investors to leverage the dynamic allocation strategy as a high-performing core while benefiting from the stability and diversification of other asset classes.

In summary, the regime-switching model offers a practical, cost-efficient, and adaptable solution for dynamic portfolio allocation. While its low switch frequency and simplicity make it investor-friendly, the model needs qualitative integration to address atypical market conditions. In addition, the strategy could be refined by embedding this strategy within a diversified investment framework. Those arguments and precautions confirm the validity of the investment solution in an institutional framework.

Conclusion

This research explored the application of a dynamic portfolio allocation between Minimum Volatility and Momentum strategies using the Markov Regime Switching Model. The MRSM results highlight statistically significant coefficients and strong fitting metrics such as AIC and BIC, representing its good calibration. Regarding regime allocation, the model can identify the correct regime and capture long-term tendencies. The accuracy improves over time due to the expanding dataset. Indeed, the data available increases with time, avoiding the forward bias and reducing small-data bias with time. Notably, one of the key successes of the MRSM is the relative outperformance achieved with accurate regime selection. It enables the dynamic strategy to generate additional returns by tailoring investment decisions to varying market conditions.

The dynamic portfolio allocation strategy achieves impressive performance metrics. It has an annualized return of 8.7%, comparable to the MSCI World Index. In addition, the strategy maintains a low volatility level of 8.2%. Leveraging these attributes, the strategy reaches the highest Sharpe ratio among all strategies analysed. Its value of 1.06 highlights its impressive risk-adjusted performance. The strategy demonstrates consistent long-term returns, matching the MSCI World Momentum's performance over the period. Furthermore, the dynamic portfolio is pertinent for shorter-term horizons, as evidenced by 3-year rolling window analyses. The strategy exhibits significant resilience during worst drawdown scenarios, effectively mitigating losses and recovering faster than static benchmarks. Lastly, its practical design requires simple reallocations between two ETFs or internally managed strategies, minimizing transaction costs and portfolio turnover. This makes it easily implementable for investors seeking an efficient dynamic portfolio management solution.

Despite its promising results, the MRSM has some limitations. One of the most significant challenges is its difficulty in adapting to unconventional market environments. For instance, the post-COVID-19 crisis macroeconomic conditions deviate significantly from historical patterns, troubling regime allocation of the model. Additionally, the model optimizes the risk-adjusted returns, which emphasize missed opportunities due to low-risk exposure. The avoided significant return contribution of 2023-204 highlights the previous argument. Another limitation is the dataset itself. The small-data bias at the beginning of the expanding window remains a problem.

The model troubles to identify substantial regime allocation with the few observations available in the first iterations. Another point is the interpretability of the model. Despite the variable's statistical significance, the direction and magnitude of their coefficients are not always economically intuitive. It probably comes from the interaction of multiple variables; however, further improvement could be made in this field.

Future research could improve the model and potentially address these limitations. The first exploration could be the inclusion of a third regime by incorporating an in-between risk profile strategy, for example. The third portfolio strategy will improve the model's adaptability and could result in a better fit for varying market conditions. The model could, for example, allocate the third strategy when regime conviction is weak. In this way, the model could diminish missed opportunities influence on the total performance. The refinement of the binary regime switching model could also be explored by incorporating additional explanatory variables such as macroeconomic or monetary policy indicators. Additional variables could further enrich decision-making and improve accuracy by allocating new dependencies in the model. Another approach would be the expansion of the dataset, either by extending the timeframe or using daily data. This could enhance the model's accuracy and avoid small-data bias at the beginning of the expanding window. However, if the data is transformed into a daily format, care must be taken to avoid amplifying short-term market fluctuations that may reduce long-term regime tendencies. Lastly, alternative regime-switching models, such as hidden Markov models, neural networks, or Bayesian approaches, could be investigated to further improve allocation accuracy and adaptability to evolving market conditions.

Finally, this study expands the application of Markov Regime Switching Models in dynamic portfolio allocation by effectively combining the stability of Minimum Volatility with the growth potential of Momentum strategies. It demonstrates a practical and efficient approach to dynamic portfolio management, delivering superior risk-adjusted returns. Overall, this research introduces a novel MRSM application, providing a foundation for future studies in adaptive investment strategies and valuable insights for both academic research and professional portfolio management.

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Appendix A: Tables

NAME	TICKER
MSCI World Index	M1WO Index
MSCI World Minimum Volatility Index	M1WOMVOL Index
MSCI World Momentum Index	M1WOMOM Index
CBOE Volatility Index (VIX)	VIX Index
Credit spread 10y	BICLB10Y Index
Global GDP	GDPGAWLD Index
US Generic Govt 10y	USGG10YR Index
Consumer Price Index	CPI YOY Index
Real Liquidity	GFSIRLIQ Index
PUT/CALL Ratio	PCUSEQTR Index
Crude Oil	CL1 Comdty

Table 15: Variables & Associated Tickers

SWITCH	START DATE	END DATE	REGIME	SWITCH	START DATE	END DATE	REGIME
1	05.12.2010	19.12.2010	LOW	**43**	04.01.2015	01.02.2015	LOW
2	26.12.2010	23.01.2011	MOM	**44**	08.02.2015	15.02.2015	MOM
3	30.01.2011	12.06.2011	LOW	**45**	22.02.2015	05.04.2015	LOW
4	19.06.2011	19.06.2011	MOM	**46**	12.04.2015	26.04.2015	MOM
5	26.06.2011	23.10.2011	LOW	**47**	03.05.2015	28.06.2015	LOW
6	30.10.2011	06.11.2011	MOM	**48**	05.07.2015	05.07.2015	MOM
7	13.11.2011	18.12.2011	LOW	**49**	12.07.2015	08.11.2015	LOW
8	25.12.2011	25.12.2011	MOM	**50**	15.11.2015	13.12.2015	MOM
9	01.01.2012	01.01.2012	LOW	**51**	20.12.2015	10.04.2016	LOW
10	08.01.2012	08.01.2012	MOM	**52**	17.04.2016	29.05.2016	MOM
11	15.01.2012	12.02.2012	LOW	**53**	05.06.2016	03.07.2016	LOW
12	19.02.2012	19.02.2012	MOM	**54**	10.07.2016	14.05.2017	MOM
13	26.02.2012	15.04.2012	LOW	**55**	21.05.2017	04.06.2017	LOW
14	22.04.2012	22.04.2012	MOM	**56**	11.06.2017	31.12.2017	MOM
15	29.04.2012	24.06.2012	LOW	**57**	07.01.2018	03.02.2019	LOW
16	01.07.2012	01.07.2012	MOM	**58**	10.02.2019	23.02.2020	MOM
17	08.07.2012	22.07.2012	LOW	**59**	01.03.2020	28.06.2020	LOW
18	29.07.2012	29.07.2012	MOM	**60**	05.07.2020	16.08.2020	MOM
19	05.08.2012	12.08.2012	LOW	**61**	23.08.2020	06.12.2020	LOW
20	19.08.2012	19.08.2012	MOM	**62**	13.12.2020	10.01.2021	MOM
21	26.08.2012	09.09.2012	LOW	**63**	17.01.2021	25.07.2021	LOW
22	16.09.2012	16.09.2012	MOM	**64**	01.08.2021	15.08.2021	MOM
23	23.09.2012	30.09.2012	LOW	**65**	22.08.2021	20.03.2022	LOW
24	07.10.2012	07.10.2012	MOM	**66**	27.03.2022	27.03.2022	MOM
25	14.10.2012	09.12.2012	LOW	**67**	03.04.2022	22.05.2022	LOW
26	16.12.2012	16.12.2012	MOM	**68**	29.05.2022	29.05.2022	MOM
27	23.12.2012	13.01.2013	LOW	**69**	05.06.2022	02.10.2022	LOW
28	20.01.2013	20.01.2013	MOM	**70**	09.10.2022	09.10.2022	MOM

29	27.01.2013	24.02.2013	LOW	**71**	16.10.2022	16.10.2022	LOW
30	03.03.2013	21.04.2013	MOM	**72**	23.10.2022	27.11.2022	MOM
31	28.04.2013	26.05.2013	LOW	**73**	04.12.2022	20.08.2023	LOW
32	02.06.2013	02.06.2013	MOM	**74**	27.08.2023	03.09.2023	MOM
33	09.06.2013	15.12.2013	LOW	**75**	10.09.2023	08.10.2023	LOW
34	22.12.2013	22.12.2013	MOM	**76**	15.10.2023	15.10.2023	MOM
35	29.12.2013	12.01.2014	LOW	**77**	22.10.2023	29.10.2023	LOW
36	19.01.2014	19.01.2014	MOM	**78**	05.11.2023	05.11.2023	MOM
37	26.01.2014	04.05.2014	LOW	**79**	12.11.2023	26.05.2024	LOW
38	11.05.2014	11.05.2014	MOM	**80**	02.06.2024	23.06.2024	MOM
39	18.05.2014	06.07.2014	LOW	**81**	30.06.2024	08.09.2024	LOW
40	13.07.2014	20.07.2014	MOM	**82**	15.09.2024	15.09.2024	MOM
41	27.07.2014	07.12.2014	LOW	**83**	22.09.2024	24.11.2024	LOW
42	14.12.2014	28.12.2014	MOM				

Table 16: Dates & Regime Allocation Over Time

Appendix B: Code Accessibility

To ensure transparency and facilitate reproducibility, the full implementation of the Markov Regime Switching Model, including data preparation, model implementation, and performance evaluation, is available in an open-access GitHub repository.

GitHub Repository: https://github.com/arnaudfelber/MRSM_Thesis

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