Code obfuscation through Mixed Boolean-Arithmetic expressions

Arnau Gàmez i Montolio

FURA LABS

About

Hacker, Reverse Engineer & Mathematician.

About

Hacker, Reverse Engineer & Mathematician.

Senior Malware Reverse Engineer @ Panda (WatchGuard)

About

Hacker, Reverse Engineer & Mathematician.

Senior Malware Reverse Engineer @ Panda (WatchGuard)

Founder, Security Researcher & Trainer @ Fura Labs

About

Hacker, Reverse Engineer & Mathematician.

Senior Malware Reverse Engineer @ Panda (WatchGuard)

Founder, Security Researcher & Trainer @ Fura Labs

@arnaugamez

@FuraLabs

Agenda

1. Code obfuscation
2. Preliminary MBA concepts
   - Introduction and Motivation
   - Obfuscation vs Cryptography
   - Definitions
     - Polynomial MBA expressions
     - Linear MBA expressions
3. Obfuscation with MBA expressions
   - MBA rewriting
   - Insertion of identities
   - Opaque constants

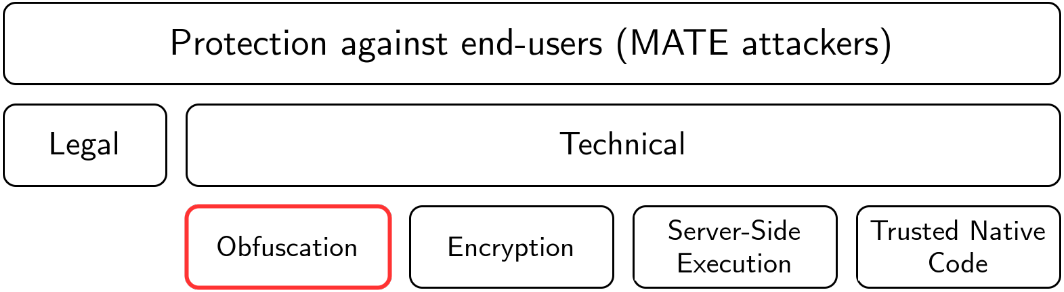
Code obfuscation

Context

Technical protection against Man-At-The-End (MATE) attacks, where the attacker/analyst has an instance of the program and completely controls the environment where it is executed.

Context

Technical protection against Man-At-The-End (MATE) attacks, where the attacker/analyst has an instance of the program and completely controls the environment where it is executed.



Idea

Code obfuscation is the process of transforming an input program P into a functionally equivalent program P' which is harder to analyze and to extract information that from P.

$$P \longrightarrow \boxed{\text{Obfuscation}} \longrightarrow P'$$

Idea

Code obfuscation is the process of transforming an input program P into a functionally equivalent program P' which is harder to analyze and to extract information that from P.

$$P \longrightarrow \boxed{\text{Obfuscation}} \longrightarrow P'$$

<u>Motivation:</u> *~~prevent~~ complicate reverse engineering.*

Presence

Software protection:

Presence

Software protection:

Intellectual property

Presence

Software protection:

- Intellectual property
- Digital Rights Management (DRM)

Presence

Software protection:

- Intellectual property
- Digital Rights Management (DRM)
- Anti-cheating

Presence

Software protection:

- Intellectual property
- Digital Rights Management (DRM)
- Anti-cheating

Malware threats:

Presence

Software protection:

- Intellectual property
- Digital Rights Management (DRM)
- Anti-cheating

Malware threats:

- Avoid automatic signature detection

Presence

Software protection:

- Intellectual property
- Digital Rights Management (DRM)
- Anti-cheating

Malware threats:

- Avoid automatic signature detection
- Slow down analysis $\rightarrow$ time $\rightarrow$ money

Methodology

Apply a **transformation** to mess (complicate) the program's control-flow and/or data-flow at **different abstraction levels** (source code, compiled binary or an intermediate representation) and affecting **different target units** (whole program, function, basic block or instruction).

Methodology

Apply a **transformation** to mess (complicate) the program's control-flow and/or data-flow at **different abstraction levels** (source code, compiled binary or an intermediate representation) and affecting **different target units** (whole program, function, basic block or instruction).

<u>Remark:</u> *many "weak" techniques can be combined to create a "hard" obfuscation transformation.*

Preliminary MBA concepts

Introduction

In a nutshell, a Mixed Boolean-Arithmetic (MBA) expression is composed of integer arithmetic operators, e.g. $(+, -, \times)$ and bitwise operators, e.g. $(\wedge, \vee, \oplus, \neg)$.

Introduction

In a nutshell, a Mixed Boolean-Arithmetic (MBA) expression is composed of integer arithmetic operators, e.g. $(+, -, \times)$ and bitwise operators, e.g. $(\wedge, \vee, \oplus, \neg)$.

$$E = (x \oplus y) + 2(x \wedge y)$$

Introduction

MBA expressions can be leveraged to **obfuscate the data-flow** of code by iteratively applying rewriting rules and function identities that complicate (obfuscate) the initial expression while **preserving its semantic behavior**.

Introduction

MBA expressions can be leveraged to **obfuscate the data-flow** of code by iteratively applying rewriting rules and function identities that complicate (obfuscate) the initial expression while **preserving its semantic behavior**.

Combination of operators from these different fields **do not interact well together**: we have no rules (distributivity, factorization...) or general theory to deal with this mixing of operators.

Obfuscation vs Cryptography

In **cryptography**, the MBA expression is the direct result of the algorithm description. The resulting cryptosystem has to verify a set of properties (e.g. non-linearity, high algebraic degree) from a *black-box* point of view.

Obfuscation vs Cryptography

In **cryptography**, the MBA expression is the direct result of the algorithm description. The resulting cryptosystem has to verify a set of properties (e.g. non-linearity, high algebraic degree) from a *black-box* point of view.

The complex form of writing is directly related to some kind of intrinsic computational (semantic) complexity for the resulting function: one wants the inverse computation to be difficult to deduce (without knowing the key).

Obfuscation vs Cryptography

In **obfuscation**, the MBA expression is the result of rewriting iterations from a simpler expression which can have very simple *black-box* characteristics.

Obfuscation vs Cryptography

In **obfuscation**, the MBA expression is the result of rewriting iterations from a simpler expression which can have very simple *black-box* characteristics.

There is no direct relation between the complex form of writing and any intrinsic computational (semantic) complexity of the resulting expression.

Obfuscation vs Cryptography

In **obfuscation**, the MBA expression is the result of rewriting iterations from a simpler expression which can have very simple *black-box* characteristics.

There is no direct relation between the complex form of writing and any intrinsic computational (semantic) complexity of the resulting expression.

On the contrary, when obfuscating simple expressions, one knows that the complex form of writing is related to a semantically simpler expression.

Obfuscation vs Cryptography

We will be focusing on MBA expressions in the context of code (de)obfuscation.

Definitions

We choose to define MBA expressions by **explicitly describing the different building blocks** (operators) that compose them and how they are bundled together.

Definitions

We choose to define MBA expressions by **explicitly describing the different building blocks** (operators) that compose them and how they are bundled together.

Two main categorizations are considered and studied in literature:

Definitions

We choose to define MBA expressions by **explicitly describing the different building blocks** (operators) that compose them and how they are bundled together.

Two main categorizations are considered and studied in literature:

Linear MBA expressions

Definitions

We choose to define MBA expressions by **explicitly describing the different building blocks** (operators) that compose them and how they are bundled together.

Two main categorizations are considered and studied in literature:

Linear MBA expressions $\subset$

Definitions

We choose to define MBA expressions by **explicitly describing the different building blocks** (operators) that compose them and how they are bundled together.

Two main categorizations are considered and studied in literature:

$$\text{Linear MBA expressions} \subset \text{Polynomial MBA expressions}$$

Polynomial MBA expressions

A **polynomial MBA expression** consists of a **sum of terms**, each one composed by an **n-bit constant a_i** times the **product** of several **bitwise expressions** on **a number t of n-bit variables**.

Polynomial MBA expressions

A **polynomial MBA expression** consists of a **sum of terms**, each one composed by an n**-bit constant** a_i times the **product** of several **bitwise expressions** on **a number t of n-bit variables**.

$$E = \sum_{i \in I} a_i \cdot \left(\prod_{j \in J_i} e_{i,j}(x_1, \ldots, x_t) \right)$$

Polynomial MBA expressions

$$E = \sum_{i \in I} a_i \cdot \left(\prod_{j \in J_i} e_{i,j}(x_1, \ldots, x_t) \right)$$

Example

$$E = 43 (x \wedge y \vee z)^2 ((x \oplus y) \wedge z \vee t) + 2x + 123 (x \vee y) z t^2$$

Linear MBA expressions

A polynomial MBA expression of the form

$$E = \sum_{i \in I} a_i \cdot e_i(x_1, \ldots, x_t)$$

is called a linear MBA expression.

Linear MBA expressions

A polynomial MBA expression of the form

$$E = \sum_{i \in I} a_i \cdot e_i(x_1, \ldots, x_t)$$

is called a linear MBA expression.

They are defined by imposing **just one bitwise expression for each term** instead of a product of an arbitrary number of them.

Linear MBA expressions

A polynomial MBA expression of the form

$$E = \sum_{i \in I} a_i \cdot e_i(x_1, \ldots, x_t)$$

is called a linear MBA expression.

They are defined by imposing **just one bitwise expression for each term** instead of a product of an arbitrary number of them.

Note: *In practice, you can vaguely think of linearity as a restriction not allowing variables to end up being multiplied together.*

Linear MBA expressions

$$E = \sum_{i \in I} a_i \cdot e_i(x_1, \ldots, x_t)$$

Example

$$E = (x \oplus y) + 2(x \wedge y)$$

Linear MBA expressions

Notice that, assuming variables of the same *bit size*, the previous MBA expression example $E = (x \oplus y) + 2(x \wedge y)$ simplifies to $E_{simp} = x + y$.

Linear MBA expressions

Notice that, assuming variables of the same *bit size*, the previous MBA expression example $E = (x \oplus y) + 2(x \wedge y)$ simplifies to $E_{simp} = x + y$.

Namely, E is a more complex expression than E_{simp} syntactically speaking, but they are semantically equivalent.

Linear MBA expressions

We can easily verify this equivalence with an SMT solver like Z3.

Linear MBA expressions

We can easily verify this equivalence with an SMT solver like Z3.

```
from z3 import *
x = BitVec('x', 8)
y = BitVec('y', 8)
E = (x ^ y) + 2 * (x & y)    # E = (x ⊕ y) + 2(x ∧ y)
E_simp = x + y               # E_simp = x + y
prove (E == E_simp)          # E ≟ E_simp
```

Linear MBA expressions

We can easily verify this equivalence with an SMT solver like Z3.

```
from z3 import *
x = BitVec('x', 8)
y = BitVec('y', 8)
E = (x ^ y) + 2 * (x & y)     # E = (x ⊕ y) + 2(x ∧ y)
E_simp = x + y                # E_simp = x + y
prove (E == E_simp)           # E ≟ E_simp
```

```
$ python eq.py
proved
```

Obfuscation with MBA expressions

Obfuscation idea

The previous example already suggests a basic obfuscation idea: we could replace the arithmetic sum $+$ of two variables in our code by the more complex expression involving $\oplus$ and $\wedge$ boolean operators, while preserving the code semantics.

Obfuscation idea

Given an MBA expression E_1, we are interested in generating a **semantically equivalent** expression E_2 which is **syntactically more complex** than the initial expression E_1.

Obfuscation idea

Given an MBA expression E_1, we are interested in generating a **semantically equivalent** expression E_2 which is **syntactically more complex** than the initial expression E_1.

The process relies on two differentiated components, which can be used either alone or combined:

Obfuscation idea

Given an MBA expression E_1, we are interested in generating a **semantically equivalent** expression E_2 which is **syntactically more complex** than the initial expression E_1.

The process relies on two differentiated components, which can be used either alone or combined:

MBA rewriting & Insertion of identities

MBA rewriting

A chosen operator is rewritten with an equivalent MBA expression.

MBA rewriting

A chosen operator is rewritten with an equivalent MBA expression.

Example

$$x + y \rightarrow (x \oplus y) + 2 \times (x \wedge y)$$

Insertion of identities

Let e be any subexpression of the target expression being obfuscated. Then, we can write e as $f^{-1}(f(e))$ with f being any invertible function on n-bits.

Insertion of identities

Let e be any subexpression of the target expression being obfuscated. Then, we can write e as $f^{-1}(f(e))$ with f being any invertible function on n-bits.

The function f is often an affine function.

Insertion of identities

Let e be any subexpression of the target expression being obfuscated. Then, we can write e as $f^{-1}(f(e))$ with f being any invertible function on n-bits.

The function f is often an affine function.

<u>Note:</u> *For our usage, you can vaguely think of affine functions as those with the form $f(e) = a \cdot e + b$, where a, b are n-bit constants and e is our MBA subexpression.*

Insertion of identities

Example

Let $E_1 = x + y$, and the following functions f and f^{-1} on 8 bits:

$$f : x \mapsto 39x + 23 \qquad f^{-1} : x \mapsto 151x + 111$$

Insertion of identities

Example

Let $E_1 = x + y$, and the following functions f and f^{-1} on 8 bits:

$$f : x \mapsto 39x + 23 \qquad f^{-1} : x \mapsto 151x + 111$$

Consider now E_2 obtained by applying the previous rewriting rule to E_1:

$$E_2 = (x \oplus y) + 2 \times (x \wedge y)$$

Insertion of identities

Example

Let $E_1 = x + y$, and the following functions f and f^{-1} on 8 bits:

$$f : x \mapsto 39x + 23 \qquad f^{-1} : x \mapsto 151x + 111$$

Consider now E_2 obtained by applying the previous rewriting rule to E_1:

$$E_2 = (x \oplus y) + 2 \times (x \wedge y)$$

Then apply the insertion of identities produced by f and f^{-1}:

$$\begin{aligned} E_{tmp} &= f(E_2) = 39 \times E_2 + 23 \\ E_3 &= f^{-1}(E_{tmp}) = 151 \times E_{tmp} + 111 \end{aligned}$$

Insertion of identities

Example

Let $E_1 = x + y$, and the following functions f and f^{-1} on 8 bits:

$$f : x \mapsto 39x + 23 \qquad f^{-1} : x \mapsto 151x + 111$$

Consider now E_2 obtained by applying the previous rewriting rule to E_1:

$$E_2 = (x \oplus y) + 2 \times (x \wedge y)$$

Then apply the insertion of identities produced by f and f^{-1}:

$$E_{tmp} = f(E_2) = 39 \times E_2 + 23$$
$$E_3 = f^{-1}(E_{tmp}) = 151 \times E_{tmp} + 111$$

Finally, expand E_3 to observe the final obfuscated expression:

$$E_3 = 151 \times (39 \times ((x \oplus y) + 2 \times (x \wedge y)) + 23) + 111$$

Practical demo

Let:

$$
\begin{aligned}
E_1 &= x + y \\
E_2 &= (x \oplus y) + 2 \times (x \wedge y) \\
E_3 &= 151 \times (39 \times ((x \oplus y) + 2 \times (x \wedge y)) + 23) + 111
\end{aligned}
$$

Practical demo

Let:

$$
\begin{aligned}
E_1 &= x + y \\
E_2 &= (x \oplus y) + 2 \times (x \wedge y) \\
E_3 &= 151 \times (39 \times ((x \oplus y) + 2 \times (x \wedge y)) + 23) + 111
\end{aligned}
$$

Task:

1 Compare syntactic complexity of E_1, E_2 and E_3.

Practical demo

Let:

$$E_1 = x + y$$
$$E_2 = (x \oplus y) + 2 \times (x \wedge y)$$
$$E_3 = 151 \times (39 \times ((x \oplus y) + 2 \times (x \wedge y)) + 23) + 111$$

Task:

1 Compare syntactic complexity of E_1, E_2 and E_3.
2 Observe semantic equivalence of E_1, E_2 and E_3.

Practical demo

Let:

$$
\begin{aligned}
E_1 &= x + y \\
E_2 &= (x \oplus y) + 2 \times (x \wedge y) \\
E_3 &= 151 \times (39 \times ((x \oplus y) + 2 \times (x \wedge y)) + 23) + 111
\end{aligned}
$$

Task:

1 Compare syntactic complexity of E_1, E_2 and E_3.
2 Observe semantic equivalence of E_1, E_2 and E_3.
3 Prove semantic equivalence of E_1, E_2 and E_3.

Practical demo

Task 1: Compare syntactic complexity of E_1, E_2 and E_3.

Practical demo

Task 1: Compare syntactic complexity of E_1, E_2 and E_3.

```c
uint8_t E1(uint8_t x, uint8_t y)
{
  return x+y;
}
```

```asm
movzx edx, byte [var_4h]
movzx eax, byte [var_8h]
add   eax, edx
```

Practical demo

Task 1: Compare syntactic complexity of E_1, E_2 and E_3.

```c
uint8_t E1(uint8_t x, uint8_t y)
{
  return x+y;
}
```

```
movzx edx, byte [var_4h]
movzx eax, byte [var_8h]
add   eax, edx
```

```c
uint8_t E2(uint8_t x, uint8_t y)
{
  return (x^y)+2*(x&y);
}
```

```
movzx eax, byte [var_4h]
xor   al, byte [var_8h]
mov   edx, eax
movzx eax, byte [var_4h]
and   al, byte [var_8h]
add   eax, eax
add   eax, edx
```

Practical demo

Task 1: Compare syntactic complexity of E_1, E_2 and E_3.

```c
uint8_t E1(uint8_t x, uint8_t y)
{
  return x+y;
}
```

```asm
movzx edx, byte [var_4h]
movzx eax, byte [var_8h]
add   eax, edx
```

```c
uint8_t E2(uint8_t x, uint8_t y)
{
  return (x^y)+2*(x&y);
}
```

```asm
movzx eax, byte [var_4h]
xor   al, byte [var_8h]
mov   edx, eax
movzx eax, byte [var_4h]
and   al, byte [var_8h]
add   eax, eax
add   eax, edx
```

```c
uint8_t E3(uint8_t x, uint8_t y)
{
  return 151*(39*((x^y)+2*(x&y))+23)+111;
}
```

```asm
movzx eax, byte [var_4h]
xor   al, byte [var_8h]
movzx edx, al
movzx eax, byte [var_4h]
and   al, byte [var_8h]
movzx eax, al
add   eax, eax
add   eax, edx
imul  eax, eax, 0x27
add   eax, 0x17
mov   edx, 0xffffff97
imul  eax, edx
add   eax, 0x6f
```

Practical demo

Task 2: Observe semantic equivalence of E_1, E_2 and E_3.

Practical demo

Task 2: Observe semantic equivalence of E_1, E_2 and E_3.

```c
#include <stdio.h>
#include <stdint.h>
#include <stdlib.h>

uint8_t E1(uint8_t x, uint8_t y)
{ return x + y; }
uint8_t E2(uint8_t x, uint8_t y)
{ return (x ^ y) + 2 * (x & y); }
uint8_t E3(uint8_t x, uint8_t y)
{ return 151 * (39 * ((x ^ y) + 2 * (x & y)) + 23) + 111; }

int main(int argc, char* argv[])
{
  uint8_t x = (uint8_t) atoi (argv[1]);
  uint8_t y = (uint8_t) atoi (argv[2]);
  printf ("%s(%d, %d) = %d\n", "E1", x, y, E1(x, y));
  printf ("%s(%d, %d) = %d\n", "E2", x, y, E2(x, y));
  printf ("%s(%d, %d) = %d\n", "E3", x, y, E3(x, y));
  return 0;
}
```

Practical demo

Task 2: Observe semantic equivalence of E_1, E_2 and E_3.

```c
#include <stdio.h>
#include <stdint.h>
#include <stdlib.h>

uint8_t E1(uint8_t x, uint8_t y)
{ return x + y; }
uint8_t E2(uint8_t x, uint8_t y)
{ return (x ^ y) + 2 * (x & y); }
uint8_t E3(uint8_t x, uint8_t y)
{ return 151 * (39 * ((x ^ y) + 2 * (x & y)) + 23) + 111; }

int main(int argc, char* argv[])
{
  uint8_t x = (uint8_t) atoi (argv[1]);
  uint8_t y = (uint8_t) atoi (argv[2]);
  printf ("%s(%d, %d) = %d\n", "E1", x, y, E1(x, y));
  printf ("%s(%d, %d) = %d\n", "E2", x, y, E2(x, y));
  printf ("%s(%d, %d) = %d\n", "E3", x, y, E3(x, y));
  return 0;
}
```

```
$ gcc linear_mba.c -o linear_mba
$ ./linear_mba 1 2
E1(1, 2) = 3
E2(1, 2) = 3
E3(1, 2) = 3
$ ./linear_mba 23 89
E1(23, 89) = 112
E2(23, 89) = 112
E3(23, 89) = 112
```

Practical demo

Task 3: Prove semantic equivalence of E_1, E_2 and E_3.

Practical demo

Task 3: Prove semantic equivalence of E_1, E_2 and E_3.

$$
\begin{tikzcd}
E_1 \arrow[dr, "\equiv"] & \\
 & E_2 \arrow[dl, "\equiv"] \\
E_3 \arrow[uu, "\equiv"] &
\end{tikzcd}
$$

Practical demo

Task 3: Prove semantic equivalence of E_1, E_2 and E_3.

$$
\begin{tikzcd}
E_1 \arrow[dr, "\equiv"] & \\
& E_2 \arrow[dl, "\equiv"] \\
E_3 \arrow[uu, "\equiv"] &
\end{tikzcd}
$$

```python
from z3 import *
x = BitVec('x', 8)
y = BitVec('y', 8)

E1 = x + y
E2 = (x ^ y) + 2 * (x & y)
E3 = 151 * (39 * ((x ^ y) + 2 * (x & y)) + 23) + 111

prove (E1 == E2)
prove (E2 == E3)
prove (E3 == E1)
```

Practical demo

Task 3: Prove semantic equivalence of E_1, E_2 and E_3.

$$
\begin{tikzcd}
E_1 \arrow[dr, "\equiv"] & \\
 & E_2 \arrow[dl, "\equiv"] \\
E_3 \arrow[uu, "\equiv"] &
\end{tikzcd}
$$

```python
from z3 import *
x = BitVec('x', 8)
y = BitVec('y', 8)

E1 = x + y
E2 = (x ^ y) + 2 * (x & y)
E3 = 151 * (39 * ((x ^ y) + 2 * (x & y)) + 23) + 111

prove (E1 == E2)
prove (E2 == E3)
prove (E3 == E1)
```

```
$ python prove.py
proved
proved
proved
```

Opaque constants

This technique allows to hide a target constant K.

Opaque constants

This technique allows to hide a target constant K.

It combines the power of MBA expressions with *permutation polynomials*.

Opaque constants

This technique allows to hide a target constant K.

It combines the power of MBA expressions with *permutation polynomials*.

A permutation polynomial is a polynomial that acts as a permutation of the elements of the set they apply to (in our case, n-bit values), i.e. they define a *1-to-1* map (bijection).

Opaque constants

This technique allows to hide a target constant K.

It combines the power of MBA expressions with *permutation polynomials*.

A permutation polynomial is a polynomial that acts as a permutation of the elements of the set they apply to (in our case, n-bit values), i.e. they define a *1-to-1* map (bijection).

Thus, for any permutation polynomial P, there exists another one Q that defines the inverse map, i.e., for all n-bit X values we have that:

$$P(Q(X)) = X$$

Opaque constants

Let:

K be an n-bit target constant to hide,

Opaque constants

Let:

K be an n-bit target constant to hide,

P and Q polynomials with n-bit coefficients and acting as inverse *1-to-1* maps, i.e. $P(Q(X)) = X$ for all X,

Opaque constants

Let:

K be an n-bit target constant to hide,

P and Q polynomials with n-bit coefficients and acting as inverse *1-to-1* maps, i.e. $P(Q(X)) = X$ for all X,

E be an MBA expression of n-bit variables non-trivially equal to zero, i.e. $E(x_1, \ldots, x_t) = 0$ for any input variables $x_1, \ldots, x_t$.

Opaque constants

Let:

K be an n-bit target constant to hide,

P and Q polynomials with n-bit coefficients and acting as inverse *1-to-1* maps, i.e. $P(Q(X)) = X$ for all X,

E be an MBA expression of n-bit variables non-trivially equal to zero, i.e. $E(x_1, \ldots, x_t) = 0$ for any input variables $x_1, \ldots, x_t$.

Then, the constant K can be replaced by $P(E + Q(K))$ for any values taken by $(x_1, \ldots, x_t)$.

Practical demo

Working on 8-bit values, let:

$$P(X) = 97X + 248X^2$$
$$Q(X) = 161X + 136X^2$$
$$E(x, y) = x - y + 2(\neg x \wedge y) - (x \oplus y)$$

Practical demo

Working on 8-bit values, let:

$$P(X) = 97X + 248X^2$$
$$Q(X) = 161X + 136X^2$$
$$E(x,y) = x - y + 2(\neg x \wedge y) - (x \oplus y)$$

Task:

1 Check that P and Q define inverse maps, i.e. $P(Q(X)) = X$ for all X.

Practical demo

Working on 8-bit values, let:

$$P(X) = 97X + 248X^2$$
$$Q(X) = 161X + 136X^2$$
$$E(x, y) = x - y + 2(\neg x \wedge y) - (x \oplus y)$$

Task:

1 Check that P and Q define inverse maps, i.e. $P(Q(X)) = X$ for all X.
2 Check that E defines a non-trivially equal to zero MBA expression, i.e. $E(x, y) = 0$ for all x, y.

Practical demo

Working on 8-bit values, let:

$$P(X) = 97X + 248X^2$$
$$Q(X) = 161X + 136X^2$$
$$E(x, y) = x - y + 2(\neg x \wedge y) - (x \oplus y)$$

Task:

1 Check that P and Q define inverse maps, i.e. $P(Q(X)) = X$ for all X.
2 Check that E defines a non-trivially equal to zero MBA expression, i.e. $E(x, y) = 0$ for all x, y.
3 Create an opaque constant function using P, Q and E to hide the constant $K = 123$.

Practical demo

Working on 8-bit values, let:

$$P(X) = 97X + 248X^2$$
$$Q(X) = 161X + 136X^2$$
$$E(x, y) = x - y + 2(\neg x \wedge y) - (x \oplus y)$$

Task:

1 Check that P and Q define inverse maps, i.e. $P(Q(X)) = X$ for all X.
2 Check that E defines a non-trivially equal to zero MBA expression, i.e. $E(x, y) = 0$ for all x, y.
3 Create an opaque constant function using P, Q and E to hide the constant $K = 123$.
4 Check the previous opaque constant.

Practical demo

Task 1: Check that P and Q define inverse maps (bruteforce).

Practical demo

Task 1: Check that P and Q define inverse maps (bruteforce).

```c
#include <stdio.h>
#include <stdint.h>
#include <stdlib.h>

uint8_t P(uint8_t x) { return 97*x + 248*x*x; }
uint8_t Q(uint8_t x) { return 161*x + 136*x*x; }

int main(int argc, char* argv[])
{
  uint8_t i = 0;
  do
  {
      if (P(Q(i)) != i) { printf("P(Q(X)) != X)\n"); return -1; }
      i++;
  } while (i != 0);
  printf("P(Q(X)) = X\n");
  return 0;
}
```

Practical demo

Task 1: Check that P and Q define inverse maps (bruteforce).

```c
#include <stdio.h>
#include <stdint.h>
#include <stdlib.h>

uint8_t P(uint8_t x) { return 97*x + 248*x*x; }
uint8_t Q(uint8_t x) { return 161*x + 136*x*x; }

int main(int argc, char* argv[])
{
  uint8_t i = 0;
  do
  {
      if (P(Q(i)) != i) { printf("P(Q(X)) != X)\n"); return -1; }
      i++;
  } while (i != 0);
  printf("P(Q(X)) = X\n");
  return 0;
}
```

```
$ gcc check_poly.c -o check_poly
$ ./check_poly
P(Q(X)) = X
```

Practical demo

Task 1: Check that P and Q define inverse maps (SMT).

Practical demo

Task 1: Check that P and Q define inverse maps (SMT).

```
from z3 import *
X = BitVec('X', 8)

def P(X): return 97*X + 248*X*X
def Q(X): return 161*X + 136*X*X

prove(P(Q(X)) == X)
```

Practical demo

Task 1: Check that P and Q define inverse maps (SMT).

```
from z3 import *
X = BitVec('X', 8)

def P(X): return 97*X + 248*X*X
def Q(X): return 161*X + 136*X*X

prove(P(Q(X)) == X)
```

```
$ python check_poly.py
proved
```

Practical demo

Task 2: Check that E is non-trivially equal to zero (bruteforce).

Practical demo

Task 2: Check that E is non-trivially equal to zero (bruteforce).

```c
#include <stdio.h>
#include <stdint.h>
#include <stdlib.h>

uint8_t E(uint8_t x, uint8_t y) { return x-y + 2*(~x&y) - (x^y); }

int main(int argc, char* argv[])
{
  uint8_t i = 0; uint8_t j = 0;
  do
  {
      do
      {
          if (E(i, j) != 0) { printf("E(x, y) != 0)\n"); return -1; }
          j++;
      } while (j != 0);
      i++;
  } while (i != 0);
  printf("E(x, y) = 0\n"); return 0;
}
```

Practical demo

Task 2: Check that E is non-trivially equal to zero (bruteforce).

```c
#include <stdio.h>
#include <stdint.h>
#include <stdlib.h>

uint8_t E(uint8_t x, uint8_t y) { return x-y + 2*(~x&y) - (x^y); }

int main(int argc, char* argv[])
{
  uint8_t i = 0; uint8_t j = 0;
  do
  {
      do
      {
          if (E(i, j) != 0) { printf("E(x, y) != 0)\n"); return -1; }
          j++;
      } while (j != 0);
      i++;
  } while (i != 0);
  printf("E(x, y) = 0\n"); return 0;
}
```

```
$ gcc check_mba.c -o check_mba
$ ./check_mba
E(x, y) = 0
```

Practical demo

Task 2: Check that E is non-trivially equal to zero (SMT).

Practical demo

Task 2: Check that E is non-trivially equal to zero (SMT).

```
from z3 import *
x = BitVec('x', 8)
y = BitVec('y', 8)

def E(x, y): return x-y + 2*(~x&y) - (x^y)

prove(E(x, y) == 0)
```

Practical demo

Task 2: Check that E is non-trivially equal to zero (SMT).

```
from z3 import *
x = BitVec('x', 8)
y = BitVec('y', 8)

def E(x, y): return x-y + 2*(~x&y) - (x^y)

prove(E(x, y) == 0)
```

```
$ python check_mba.py
proved
```

Practical demo

Task 3: Create an opaque constant function.

Practical demo

Task 3: Create an opaque constant function.

```
from z3 import *

X = BitVec('X', 8)
def P(X): return 97*X + 248*X*X
def Q(X): return 161*X + 136*X*X

x = BitVec('x', 8)
y = BitVec('y', 8)
def E(x, y): return x-y + 2*(~x&y) - (x^y)

K = BitVecVal(123, 8)

# Opaque Constant
OC = P(E(x,y) + Q(K))

# Apply basic simplification rules
print (simplify(OC))
```

Practical demo

Task 3: Create an opaque constant function.

```python
from z3 import *

X = BitVec('X', 8)
def P(X): return 97*X + 248*X*X
def Q(X): return 161*X + 136*X*X

x = BitVec('x', 8)
y = BitVec('y', 8)
def E(x, y): return x-y + 2*(~x&y) - (x^y)

K = BitVecVal(123, 8)

# Opaque Constant
OC = P(E(x,y) + Q(K))

# Apply basic simplification rules
print (simplify(OC))
```

```
$ python create_oc.py
195 +
97*x +
159*y +
194*~(x | ~y) +
159*(x ^ y) +
(163 + x + 255*y + 2*~(x | ~y) + 255*(x ^ y))*
(232 + 248*x + 8*y + 240*~(x | ~y) + 8*(x ^ y))
```

Practical demo

Task 4: Check an opaque constant function (bruteforce).

Practical demo

Task 4: Check an opaque constant function (bruteforce).

```
#include <stdio.h>
#include <stdint.h>
#include <stdlib.h>

uint8_t OC(uint8_t x, uint8_t y)
{
    return 195 + 97*x + 159*y +
    194*~(x | ~y) + 159*(x ^ y) +
    (163 + x + 255*y + 2*~(x | ~y) + 255*(x ^ y))*
    (232 + 248*x + 8*y + 240*~(x | ~y) + 8*(x ^ y));
}

int main(int argc, char* argv[])
{
    uint8_t i = 0; uint8_t j = 0;
    do
    {
/* ... */
```

```
        do
        {
            if (OC(i, j) != 123)
            {
                printf("OC(x, y) != 123)\n");
                return -1;
            }
            j++;
        } while (j != 0);
        i++;
    } while (i != 0);
    printf("OC(x, y) = 123\n"); return 0;
}
```

Practical demo

Task 4: Check an opaque constant function (bruteforce).

```c
#include <stdio.h>
#include <stdint.h>
#include <stdlib.h>

uint8_t OC(uint8_t x, uint8_t y)
{
    return 195 + 97*x + 159*y +
    194*~(x | ~y) + 159*(x ^ y) +
    (163 + x + 255*y + 2*~(x | ~y) + 255*(x ^ y))*
    (232 + 248*x + 8*y + 240*~(x | ~y) + 8*(x ^ y));
}

int main(int argc, char* argv[])
{
    uint8_t i = 0; uint8_t j = 0;
    do
    {
/* ... */
```

```c
        do
        {
            if (OC(i, j) != 123)
            {
                printf("OC(x, y) != 123)\n");
                return -1;
            }
            j++;
        } while (j != 0);
        i++;
    } while (i != 0);
    printf("OC(x, y) = 123\n"); return 0;
}
```

```
$ gcc check_oc.c -o check_oc
$ ./check_oc
OC(x, y) = 123
```

Practical demo

Task 4: Check an opaque constant function (SMT).

Practical demo

Task 4: Check an opaque constant function (SMT).

```
from z3 import *

x = BitVec('x', 8)
y = BitVec('y', 8)

def OC(x, y):
    return 195 + 97*x + 159*y +\
    194*~(x | ~y) + 159*(x ^ y) +\
    (163 + x + 255*y + 2*~(x | ~y) + 255*(x ^ y))*\
    (232 + 248*x + 8*y + 240*~(x | ~y) + 8*(x ^ y))

prove(OC(x, y) == 123)
```

Practical demo

Task 4: Check an opaque constant function (SMT).

```
from z3 import *

x = BitVec('x', 8)
y = BitVec('y', 8)

def OC(x, y):
    return 195 + 97*x + 159*y +\
    194*~(x | ~y) + 159*(x ^ y) +\
    (163 + x + 255*y + 2*~(x | ~y) + 255*(x ^ y))*\
    (232 + 248*x + 8*y + 240*~(x | ~y) + 8*(x ^ y))

prove(OC(x, y) == 123)
```

```
$ python check_oc.py
proved
```

Summary

We have seen how to apply several MBA obfuscation techniques: MBA rewriting, insertion of identities and opaque constants.

Summary

We have seen how to apply several MBA obfuscation techniques: MBA rewriting, insertion of identities and opaque constants.

For this purpose, we have used: rewrite rules, affine functions, non-trivially equal to zero MBA expressions and permutation polynomials.

What would come next?

Learn methods to generate:

What would come next?

Learn methods to generate:

- Non-trivially equal to zero MBA expressions

What would come next?

Learn methods to generate:

- Non-trivially equal to zero MBA expressions
- Linear MBA rewrite rules

What would come next?

Learn methods to generate:

- Non-trivially equal to zero MBA expressions
- Linear MBA rewrite rules
- Pairs of inverse affine functions

What would come next?

Learn methods to generate:

- Non-trivially equal to zero MBA expressions
- Linear MBA rewrite rules
- Pairs of inverse affine functions
- Pairs of inverse permutation polynomials

References

Code deobfuscation by program synthesis-aided simplification of Mixed Boolean-Arithmetic expressions[1] – Arnau Gàmez i Montolio

Obfuscation with Mixed Boolean Arithmetic Expressions[2] – Ninon Eyrolles

Information Hiding in Software with Mixed Boolean-Arithmetic Transforms[3] – Y. Zhou et al.

[1] `https://github.com/arnaugamez/tfg`
[2] `https://tel.archives-ouvertes.fr/tel-01623849/document`
[3] `https://link.springer.com/chapter/10.1007/978-3-540-77535-5_5`

Materials

https://github.com/arnaugamez/talks/tree/master/2021/00_intent

To know more

Blog

Trainings

Get in touch

@arnaugamez

@FuraLabs