Arnau Mas

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- Extract many radiomic features from scans
- Look for correlations between radiomic features and treatment response

# What is radiomics?

## Benefits

- Non-invasive
- More systematic
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- Non-invasive
- More systematic
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### Drawbacks

- May be hard to reproduce
- High dimensional feature spaces
- Limited number of cases

# TOPiomics

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- Main aim: early detection of patients with strange radiomic signatures, *outliers*, for whom standard treatments could fail.
- Use *topological data analysis*, hence TOPiomics (topological radiomics).

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- Idea: look at *every* parameter value.

This is persistence

# The elevator pitch

Algebraic topology studies topological spaces by computing algebraic invariants, namely with functors

$$\mathsf{Top} \to \mathsf{Grp}$$
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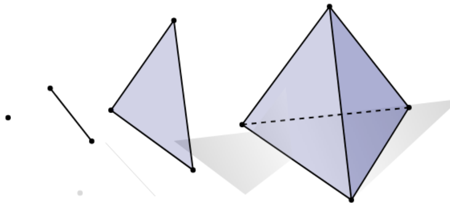
They come in many flavours, the simplest one is *simplicial homology*, which works for *simplicial complexes*.

# The ingredients of homology

- *Simplex*. The convex hull of a set of points.



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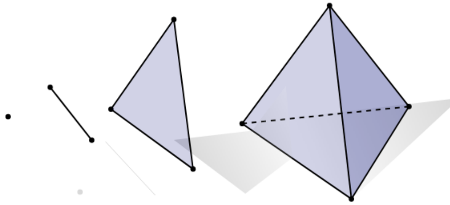

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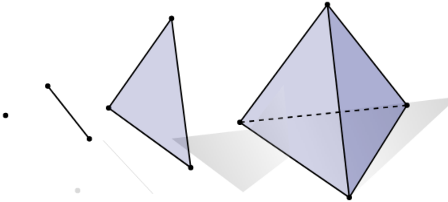

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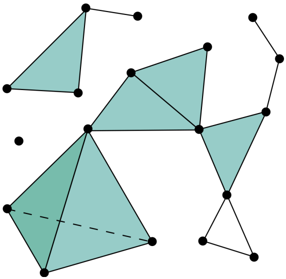
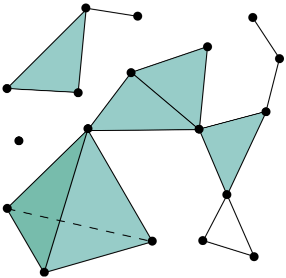
- *Simplicial complex*, $K$. Topological space assembled out of simplices glued along their faces.



# The ingredients of homology

- *Simplex*. The convex hull of an (ordered) set of (geometrically independent) points.
- *Simplicial complex*, $K$. Topological space assembled out of simplices glued along their faces.
- *Chain groups*, $C_n(K)$. Free abelian groups generated by the $n$-simplices of a complex.



## Remark

In the combinatorial picture, the geometrical requirements are dropped and one speaks of *abstract* simplices, *abstract* simplicial complexes, etc.

# The boundary morphism

### Orientation

The simplex generated by $p_0, \ldots, p_n$ is written $[p_0, \ldots, p_n]$. The ordering determines an orientation, and we require that flipping orientation implies a change of sign in $C_n(K)$, e.g.

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The chain groups are not really free!

# The boundary morphisms

Define the boundary morphism $\partial_n \colon C_n(K) \to C_{n-1}(K)$ by

$$\partial_n[p_0, \ldots, p_n] := \sum_{k=0}^{n} (-1)^k [p_0, \ldots, \hat{p}_k, \ldots, p_n]$$

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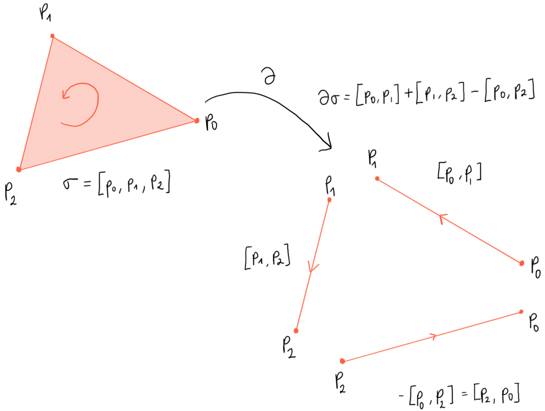

# Boundaries and cycles

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## Fact

$$\partial_n \circ \partial_{n+1} = 0$$

so boundaries are always cycles.

# Homology groups

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The key insight: Homology groups measure cycles which are not boundaries. They tell us about the number of $n$-dimensional holes in our space.

# What is persistence?

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The steps of the filtration can be thought of as time

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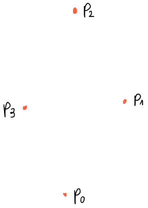
$$f_i^j : H_d^i(K) \to H_d^j(K).$$

At each step, classes can be born (not in the image of $f_i^{i+1}$), die (in the kernel of $f_i^{i+1}$) or merge (have the same image through $f_i^{i+1}$).

# An example




$[p_0], [p_1], [p_2], [p_3]$ are born.

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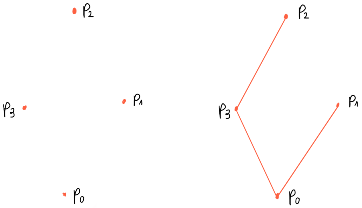


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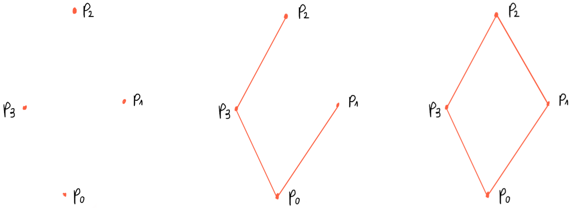


The cycle $[p_0, p_1] + [p_1, p_2] + [p_2, p_3] + [p_3, p_4]$ is born.

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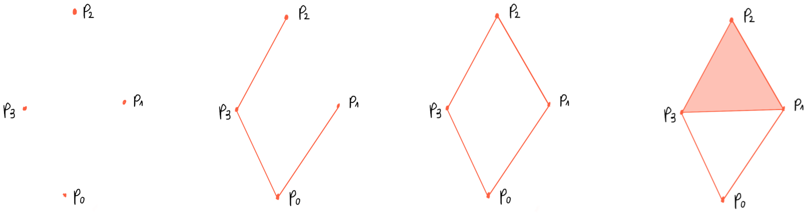
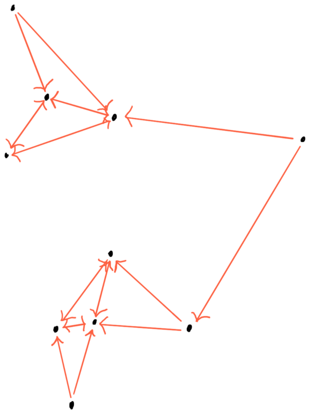
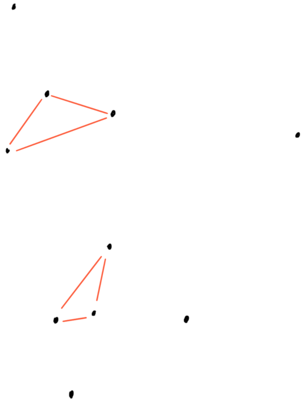



The cycle does not die.

# Why the M$k$NN graph?

It is good for detecting clusters in a data set.





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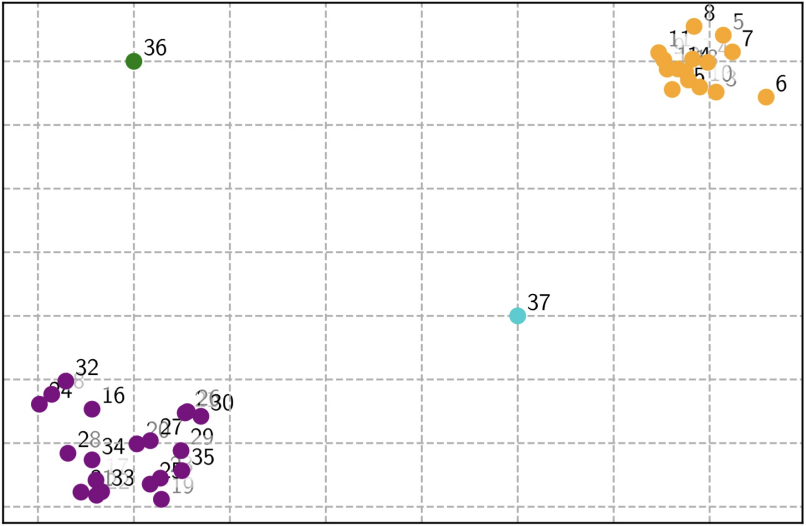
- The class `Cloud` stores a point cloud and computes its M$k$NN graph.
- The class `Filtration` builds up the filtration.
- The class `Homology` computes the homology.
- The persistence data of each class is gathered and plotted to detect the outliers.

# The class `Cloud`

# A run with toy data



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