Final project Quantitative Macroeconomics: SOME INTERESTING ANALYSES ABOUT THE EFFECTS OF COVID-19 PANDEMIC ON THE NEOCLASSICAL MODEL OF GROWTH

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INTRODUCTION.

Between last quarter 2019 and first quarter 2020, the appearance of the new virus SARS-CoV-2 (cause of Covid-19 disease) and its global expansion as a pandemic, generated an unprecedented unexpected shock on global economy with lockdowns, drop of global trade, etc, followed by a posterior period of instability with restrictions to stop the new outbreaks In addition, as an alternative to this new situation, the labor from home (telework) increased in many sectors of the economy. The relevance of this Covid-19 topic in current days is what motivated me to do this final project.

At the beginning my idea was to analyze a multi-period extension of the Covid-19 model that we saw in class the first week, but when trying to model it, I realized that it was not fully convenient for what I wanted to do. The two main reasons for this are:

- Under certain utility assumptions (like log utility which is the one I wanted to use) it was not trivial to computationally aggregate the individual utilities into a single social welfare function.
- I wanted to set up the Ramsey optimal taxation problem, and, under the multi-period extension of the model seen in class that I had in my mind, this implied to introduce survival risk dynamics at the individual level, which was overcomplicating the model.

For this reason, I decided to work with a more tractable environment, which is the representative agent, neoclassical model of growth with some variations that I will explain in each section. The project is divided in 4 sections. The sections obviously share a common topic, but are not necessarily related between them. Rather than trying to link them, my work consisted more in applying the knowledge that we acquired in class and in the TA sessions to analyze some facts related with the Covid-19 topic that I find relevant from the point of view of economic analysis. An exception to this are Sections 3 and 4, in which there exists connection between them.

Sections are organized as follows. In Section 1, as a preface to start thinking about the dynamics of a pandemic, I introduce a simple extension with deaths and social distancing to the usual SIR (Susceptible, Infected and Removed) epidemiological model, and try to assign parameter values to it in a manner such that simulates the Covid-19 first outbreak evolution. In Section 2 I introduce a labor productivity shock which follows a five state Markov chain that simulates the dynamics of the pandemic in the simple neoclassical model of growth with (still) inelastic labor supply and I simulate that economy under that shock structure. My goal here is to check how does economical instability derived from the pandemic affect the main variables of the model. In Section 3, I extend the simple neoclassical growth model by introducing two types of labor (at workplace and from home) which can be choosed elastically, and I simulate the transition of the economy from the steady state before the pandemic to the steady state during the pandemic. To do so, some assumptions are needed, like that the pandemic is long enough to allow the economy to reach a "Covid steady state". Finally, in Section 4, I take the same environment of Section 3 (that's why they are related) but I introduce finite horizon (only 6 periods), and I solve the Ramsey optimal taxation problem with commitment assumption under a pandemic. To simulate the pandemic, what I do is to generate an exogenous sequence of public expenditure that increases when infection rate increases (because of public health financing, transfers to the households and firms affected economically by the restrictions, etc), and then see what are the optimal taxes on capital, labor at workplace, and labor at home to finance this expenditure. I'm not sure if I fully succeeded in this one, because I think that there must be some little typo in the algebra or in the code but it still produces interesting results.

You can find the code associated to each section in the Python file with the name of that section.

SECTION 1: Extension of SIR model with social distancing policies and deaths.

The SIR (Susceptible Infected and Removed) model is an epidemiological model used to simulate the spread of infectious diseases. It was developed by Ronald Ross, Hilda Hudson, William Hamer, A.G. McKendrick and W.O.Kermack, among others, in the beginning of twentieth century. The aim of adding this model at the beginning of this project is to introduce the topic, and see what are the dynamics of an epidemic/pandemic, and see how the lockdown policies are effective to flat the infections curves. The original SIR model (expressed in fractions of the total population N) is composed by the following system of three differential equations (which are extracted from Kermack and McKendrick (1927). The idea of this equations is to express the law of motion between groups (or compartments) along time when an initial (usually small) proportion of the population is originally infected. The equations are:

$$\frac{dS}{dt} = -c\frac{SI}{N} \tag{1}$$

$$\frac{dI}{dt} = c\frac{SI}{N} - \gamma I \tag{2}$$

$$\frac{dR}{dt} = \gamma I \tag{3}$$

Where (1) is the susceptibles equation (those who can can be infected), (2) the infected equation, and (3) the removed equation. Removed are those that either have recovered and are immune, have died, were originally immune, etc. The original model does not distinguish between this different possibilities. Parameter c represents a rate of contact between infected and susceptible (the higher is c, the faster is the spread of the epidemic).t is a measure of time, usually in days. Parameter γ is the removal rate. In a simple environment with a non-mortal disease, we could think about it the recovery rate (what fraction of the currently infected population recovers per day). Note that we are assuming that it is not possible to get reinfected. Notice also that the model assumes that the studied period is short enough so that to assume that population N is constant, i.e., nobody is added to susceptible group (no births, no migration). Notice, however, that this assumption will not prevent me to introduce deaths from the disease (which is what I'm going to do next) because what I will be doing will be to split the Removed group in 2: Recovered and Dead, and not to assume that N reduces with deaths. In other words, mathematically, what I will be assuming is that dead, are another group of the population that cannot infect.

Let me, hence, present my extension of the original model presented above, introducing deaths, and social distancing measures to control the spread of the epidemic. The model consists now in the following system of 4 differential equations:

$$\frac{dS}{dt} = -(1-\alpha)c\frac{SI}{N} \tag{4}$$

$$\frac{dI}{dt} = (1-\alpha)c\frac{SI}{N} - \delta I - \epsilon D \tag{5}$$

$$\frac{dRec}{dt} = \delta I \tag{6}$$

$$\frac{dD}{dt} = \epsilon D \tag{7}$$

where things don't change a lot from the original model. I just split the group of Removed into the groups of Recovered (Rec) and dead (D), which implies also to distinguish a recovery rate (δ) and a rate of mortality (ϵ). On the other hand, I also allow government social distancing policy $\alpha \in [0, 1]$. This could also be done by simply reducing c, but I preferred to consider the contact c as something given (the government cannot directly reduce c) and introduce this α parameter as a more concrete measure of government intervention.

In the following lines I present my results. You can find the associated code in the Python file: $Section1\_Final\_project.py$. Also there you can find the parameter values that I choosed. I tired to assign parameter values approximately similar to the ones that we can observe for the Covid-19 (for example, I assume an average recovery time of 14 days, which implies that $\delta = \frac{1}{14}$, or I assume a mortality rate ϵ of 0.02 (2%). What I do is first simulate the model without social distancing policies ($\alpha = 0$), and then introduce social distancing policies ($\alpha = 0.3$). The effects of this social distancing policies can be seen in the graphs below:

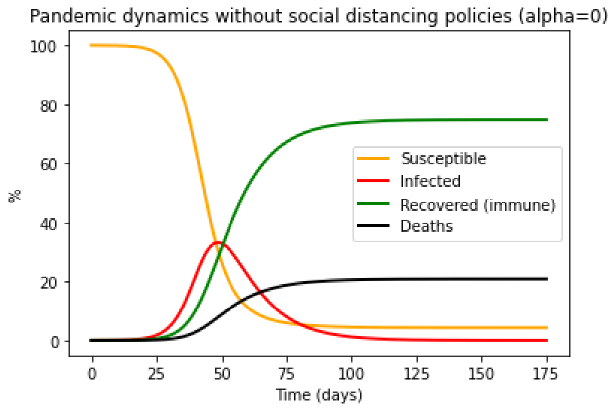

Figure 1:

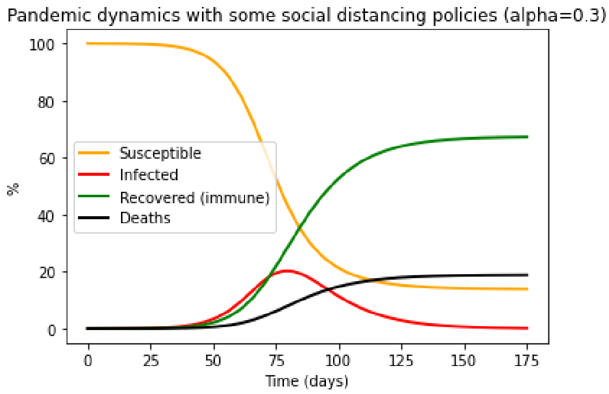

Figure 2:

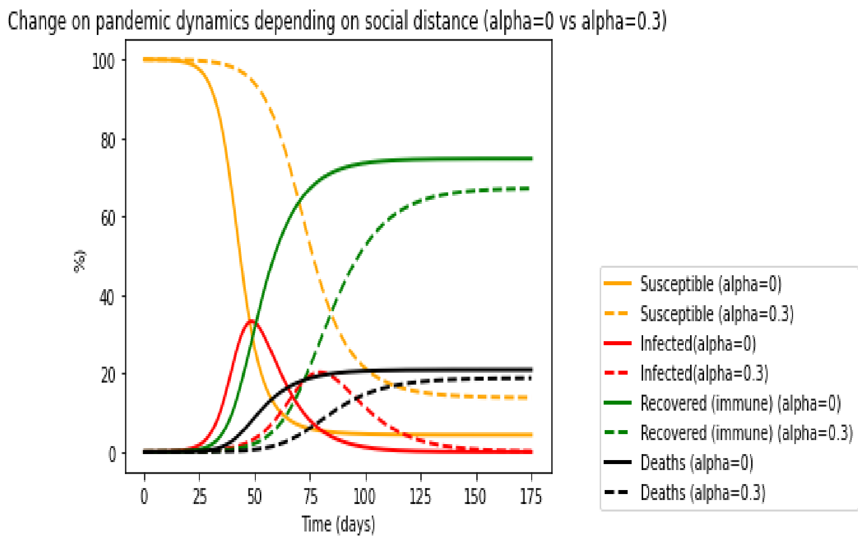

Figure 3:

The usual dynamics of an epidemic can be observed in previous graphs. At the beginning there are few infected population and many susceptible. As the time runs and more people gets the infection and recovers and gets immune (or dies), the susceptible group starts to rapidly decrease, and when the susceptible group is small enough, the infections curve reaches its peak and starts to exponentially decrease until the outbreak gets controlled.

More interestingly, we can see in Figure 3 how social distancing policies clearly flat the infections and deaths curves, which is good to avoid that the hospitals become overwhelmed. However we also know that this policies have huge economical and social costs. An interesting extension for future work could be to use this epidemiological model combined with some macroeconomic model in order to (under certain assumptions) investigate what is the optimal level of social distancing.

SECTION 2: Neoclassical growth model with inelastic labor supply and stochastic labor productivity shock.

Consider the usual neoclassical growth model with a closed economy with a representative agent and a representative firm (the firm is for now irrelevant because we are going to restrict our attention to Social Planner problem). Time is discrete and lasts forever t=0,1,2...∞. At each period agent owns one unit of time which is inelastically used in the following manner $h = 0.5$ for all t and $l = 0.5$ for all t where h and l are labor and leisure respectively. Production is done with labor and capital with the usual Cobb-Douglas production technology. Agent lifetime utility is expressed by:

$$U = \sum_{t=0}^{\infty} \beta^t \left[ln(c_t) - \frac{\omega h^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right] \tag{8}$$

Under this usual set up we know that the social planner problem is:

$$\max_{[c_t, k_{t+1}]_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[ln(c_t) - \frac{\omega h^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right] \tag{9}$$

subject to:

$$c_t + k_{t+1} - (1-\delta)k_t = k_t^{1-\alpha}(\gamma_t h)^{\alpha} \tag{10}$$

$c_t \geq 0$

where γ_t is the labor productivity parameter. Notice that I index it by time because it is the parameter that will make randomly fluctuate to simulate the instability generated by the Covid 19 pandemic and the associated lockdown and restriction policies. I assign this unstable parameter to the labor productivity to capture phenomenon like the fact that having some skilled workers in quarantine may reduce labor productivity, mobility restrictions may block human capital flows, etc. In the following lines I will explain more details about how I make this parameter fluctuate.

Taking and combining the FOC's of previous problem we obtain the following Euler equation:

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}}[(1-\delta) + (1-\alpha)k_{t+1}^{-\alpha}(\gamma_{t+1}h)^{\alpha}] \tag{11}$$

Now, what I want to do is to use this Euler equation to simulate this economy for T=150 periods, in which there are 20 periods of stability at the beginning (γ_t fixed to 3), 80 periods of instability in the middle, with (γ_t) following a five state Markov chain, and 50 periods of stability again at the end of the simulation with gamma returning to its original value of 3. For this 80 periods of instability, I try to build a five state Markov chain that simulates the ups and downs produced by the pandemic. That is I need a Markov chain with an associated transition matrix such that if I simulate this Markov chain for 80 periods I obtain realizations of γ that follow approximately the inverse path of the infections curve seen in

Section 1 (when infections increase γ falls, and the opposite when infections decrease). The states are
γ_t=[3, 2.5, 2, 1.5, 1] and the transition matrix is:

$$\Pi_\gamma = \begin{bmatrix} 0.2 & 0.8 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.95 & 0.05 \end{bmatrix}$$

where I try to follow the dynamics of a pandemic in the sense that when you are in an state is probable that tomorrow you will be worst, except when, in the case of infections, you reach the peak (in the case of our productivity parameter γ this means to hit the rock bottom, i.e, $\gamma = 1$). Then is very likely to improve.

In my Python code, which you can find in *Section2_Final_project.py*, what I do to simulate this economy in the manner described above is to follow this simple 3 steps:

- **STEP 1:** Create the transition matrix and simulate the Markov chain for γ_t for 80 periods. Add to the resulting array of realizations, the 20 periods of $\gamma_t = 3$ at the beginning and the 50 periods of $\gamma_t = 3$ at the end.
- **STEP 2:** Compute the capital stock at the steady state of the economy for $\gamma = 3$. From (11) we know that the k^* is given by:

$$k^* = \left(\frac{(1-\alpha)(\gamma h)^\alpha}{\frac{1}{\beta} - (1-\delta)} \right) \tag{12}$$

- **STEP 3:** Use k^* as initial an final value for the simulation. So that the simulation consists in that the economy is at the steady state corresponding to $\gamma = 3$, then there is a batch periods of volatility of γ in which the economy can not reach any steady state because γ is always changing, and, after that, γ gets stabilized again to 3. To run the simulation, I use the Euler equation in (11) written only in terms of capital stock. To obtain it, one must use the feasibility constraint in (10) to substitute for consumption in (11) obtaining:

$$\frac{1}{k_t^{1-\alpha}(\gamma_t h)^\alpha + (1-\delta)k_t - k_{t+1}} = \frac{\beta}{k_{t+1}^{1-\alpha}(\gamma_{t+1} h)^\alpha + (1-\delta)k_{t+1} - k_{t+2}} [(1-\delta)+(1-\alpha)k_{t+1}^{-\alpha}(\gamma_{t+1}h)^\alpha] \tag{13}$$

Solving this equation for all the periods in the simulation (150) I get the path for capital stock, which determines the path of the other relevant variables of the model.

In the graphs below I present the simulated paths for all the relevant variables of the model.In Figure 8 the welfare measure that I use is instantaneous welfare, i.e. welfare of each period t, which is obtained from:

$$u_t = ln(c_t) - \frac{\omega h^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \tag{14}$$

See the graphs below. You can find the parameter values that I used in the Python file associated to this section: *Section2_Final_project.py* .

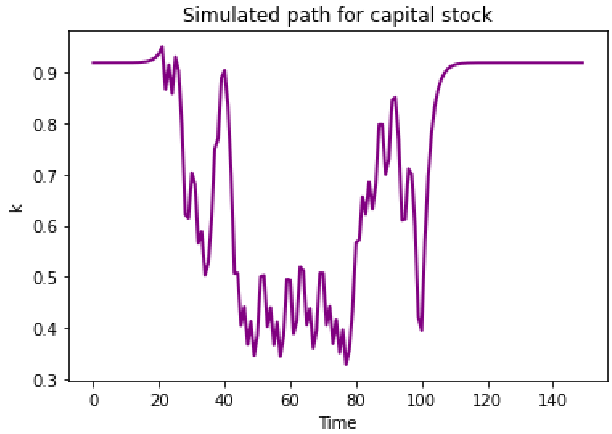

Figure 4:

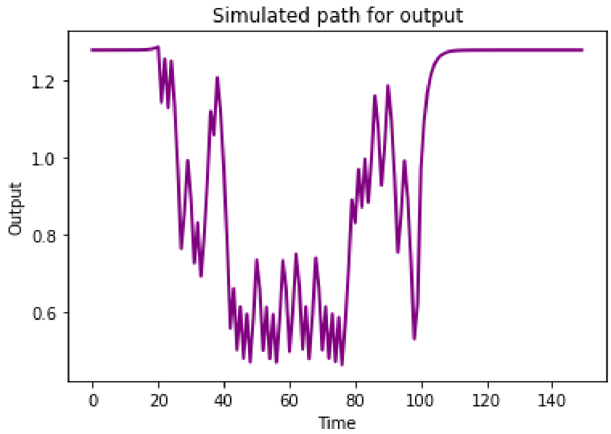

Figure 5:

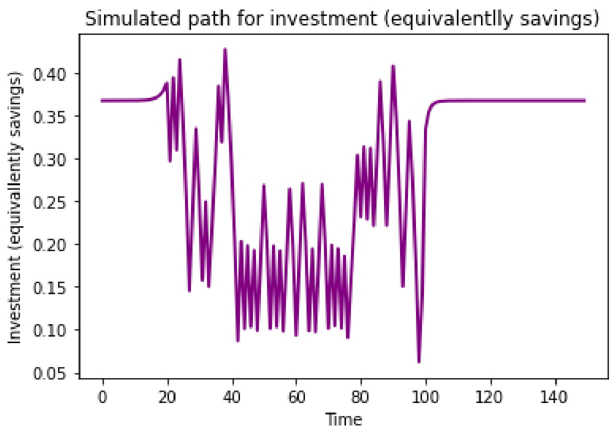

Figure 6:

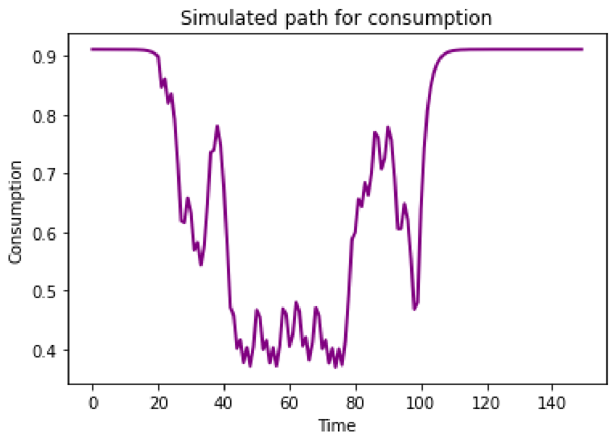

Figure 7:

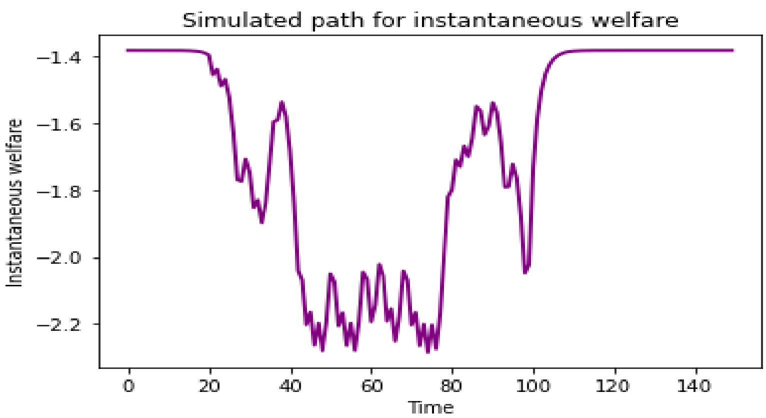

Figure 8:

As expected, all the variables experiment a long cycle of volatility associated to the variability of γ. The most interesting fact is how near period 20, between the end of the stability period and the beginning of the volatility period consumption starts to decrease, and investment (equivalently savings) and capital stock to increase. This could be due to the fact that social planner anticipates that more unstable times are coming, and, since under usual assumptions is optimal for agents to smooth consumption along the lifetime, the planner tries to save (accumulate capital) in order to obtain the as smoothed consumption profile as possible in the following periods of volatility. From Figure 7 we can conclude that the levels of volatility are so big that the planner does not fully succeed in his aim, but, still, the consumption path is more smooth than the paths for the other variables.

SECTION 3: Computing transitions in neoclassical growth model with two types of labor (at workplace and from home)

In this section I extend the usual representative agent neoclassical growth model by adding a third production factor which is labor from home, and a productivity and welfare shock i which is the infection rate. What I do in this section is to try to simulate what happened in March/April (depending on the country) when ,suddenly, the Covid-19 outbreak appeared generating an unexpected shock that made the economy to fall towards a new "Covid steady state". My aim here is to study the transition from the pre-covid economy to the covid economy. Specifically, the element in which I'm mostly interested is in how is the transition path for labor at workplace and labor at home for the model that I present below. Is it a fast transition, or is it more slow? What we observed in March/April in the actual world, was a fast transition, but we will see at the end of this section if the model is able to produce the same results or not.

Let me now present in detail the theoretical model that I use in this Section. Again it is a representative agent closed economy in which time is discrete and lasts forever $t = 0, 1, 2, \infty$. There is also a representative firm , which for now is irrelevant because we are going to restrict our assumption to Social Planner problem, but that in Section 4 will become important to set up the Ramsey government problem. At each period t, the agent owns one unit of time $h_t = 1$ which can elastically decide to split it in labor at workplace h_t^w or from home h_t^h, but, importantly, there is no leisure, so that $h_t^w + h_t^h = 1$ must hold for all t. In this economy production is done with the following merge of Cobb-Douglas and CES production functions which is used in Ottaviano and Peri (2007) in a paper about the effects of migration on wages. Although the topic that they analyze there is very different to mine, is straightforward to adapt that production function to this situation. The production function is, hence:

$$F(k_t, h_t^w, h_t^h) = k_t^{1-\alpha}\left[\left((A_w - i)(h_t^w)^z + \phi A_h (h_t^h)^z\right)^{\frac{1}{z}}\right]^{\alpha} \tag{15}$$

where A_w and A_h are productivity parameters related, respectively, with labor at workplace and from home. In the code I use $A_w < A_h$ which seems reasonable because, for example human contact spillovers are missing when working from home. This assumption also allows me to make more evident the trade of between production and health under the pandemic. We also have ϕ, which is an additional parameter measuring productivity looses of working from home. In this case, this parameter is more related to the possibility that, at home, workers can cheat and do other activities at working time, etc. The parameter i is the most important parameter here, and it is the infection rate. We will be considering two cases, one with $i = 0$, i.e. the pre-Covid situation, and one with $i > 0$ (I choose i=0.2 in the code) the Covid situation, and computing the transition between the steady state induced by each setting. Notice that as can be seen in (15), an $i > 0$ is reducing the productivity of labor at workplace. Here what I wanted to capture is the fact that applying all the safety and prevention measures, combined with the fact that some workers my get infected and quarantined, may reduce the productivity of labor at workplace, whereas it does not affect labor from home, because I assume that this does not happen at home. Another important parameter in (15) is z, which is the substitution parameter and is defined as $z = \frac{\rho-1}{\rho}$ where ρ is the elasticity of substitution between labor at workplace and from home. Finally the parameter $\alpha \in [0,1]$ is the usual labor share in the production function.

Regarding the representative agent preferences, the lifetime utility is given by:

$$U = \sum_{t=0}^{\infty} \beta^t \left[ln(c_t) - \frac{(\omega + qi)(h_t^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - \frac{\omega(h_t^h)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right] \tag{16}$$

where notice that an $i > 0$ generates additional disutility of working at workplace. q is just an augmenting factor to increase the effect of this disutility, because as $i \in [0,1]$, the effects would be small otherwise. **Also, don't get confused with the notation of ω. ω (omega) is the usual labor disutility, and is different form the index w (double v) that I use to refer to labor at workplace.** Sorry if this is a bit messy, but I realized at the end I couldn't change it.

With all this information we can set up the Social Planner (SP) problem. The fact that we are restricting our attention to Social Planner problem all the time, is that by the First Welfare theorem it is equivalent to analyze the Competitive Equilibrium (CE), and for the type of analysis that I'm doing, the CE would not be contributing with extra information. This will change in next section, in which we will also be analyzing firms side. Therefore, after this parenthesis, let's focus now on the SP problem in the current environment. Taking into account the constraint $h_t^w + h_t^h = 1$, the planner problem can be written as:

$$\max_{[c_t, k_{t+1}, h_t^w]_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big[ln(c_t) - \frac{(\omega + qi)(h_t^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - \frac{\omega(1-h_t^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \Big] \tag{17}$$

subject to:

$$c_t + k_{t+1} - (1-\delta)k_t = k_t^{1-\alpha} \Bigg[\Big((A_w - i)(h_t^w)^z + \phi A_h (1-h_t^w)^z \Big)^{\frac{1}{z}} \Bigg]^{\alpha} \tag{18}$$

$c_t \geq 0$

The lagrangian is:

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \Big[ln(c_t) - \frac{(\omega + qi)(h_t^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - \frac{\omega(1-h_t^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \Big] \\ & - \sum_{t=0}^{\infty} \lambda_t \Bigg[c_t + k_{t+1} - (1-\delta)k_t - k_t^{1-\alpha} \Bigg[\Big((A_w - i)(h_t^w)^z + \phi A_h (1-h_t^w)^z \Big)^{\frac{1}{z}} \Bigg]^{\alpha} \Bigg] \end{aligned} \tag{19}$$

Taking the FOC:

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \frac{\beta^t}{c_t} = \lambda_t \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial c_{t+1}} = 0 \Rightarrow \frac{\beta^{t+1}}{c_{t+1}} = \lambda_{t+1} \tag{21}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial h_t^w} = 0 \Rightarrow & \beta^t \big[-(\omega + qi)(h_t^w)^{\frac{1}{\nu}} + \omega(1-h_t^w)^{\frac{1}{\nu}} \big] \\ & + \lambda_t \Bigg[\frac{\alpha k_t^{1-\alpha} \Big((A_w - i)z(h_t^w)^{z-1} - A_h \phi z (1-h_t^w)^{z-1} \Big) \Bigg(\Big((A_w - i)(h_t^w)^z + A_h \phi (1-h_t^w)^z \Big)^{\frac{1}{z}} \Bigg)^{\alpha}}{z \Big((A_w - i)(h_t^w)^z - A_h \phi (1-h_t^w)^z \Big)} \Bigg] = 0 \end{aligned} \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \Rightarrow \lambda_{t+1} \Bigg[(1-\delta) + (1-\alpha) k_{t+1}^{-\alpha} \Bigg(\Big((A_w - i)(h_{t+1}^w)^z + A_h \phi (1-h_{t+1}^w)^z \Big)^{\frac{1}{z}} \Bigg)^{\alpha} \Bigg] = \lambda_t \tag{23}$$

And now we can combine this FOC's. Combining (20), (21) and (23) gives the Euler equation:

$$\frac{\beta}{c_{t+1}} \Bigg[(1-\delta) + (1-\alpha) k_{t+1}^{-\alpha} \Bigg(\Big((A_w - i)(h_{t+1}^w)^z + A_h \phi (1-h_{t+1}^w)^z \Big)^{\frac{1}{z}} \Bigg)^{\alpha} \Bigg] = \frac{1}{c_t} \tag{24}$$

whereas combining (20) and (22) we get the labor condition:

$$\frac{1}{c_t}\left[\frac{\alpha k_t^{1-\alpha}\Big((A_w-i)z(h_t^w)^{z-1}-A_h\phi z(1-h_t^w)^{z-1}\Big)\Big(\Big((A_w-i)(h_t^w)^z+A_h\phi(1-h_t^w)^z\Big)^{\frac{1}{z}}\Big)^{\alpha}}{z\Big((A_w-i)(h_t^w)^z-A_h\phi(1-h_t^w)^z\Big)}\right]$$
$$=(\omega+qi)(h_t^w)^{\frac{1}{\nu}}-\omega(1-h_t^w)^{\frac{1}{\nu}} \tag{25}$$

Once the theoretical model is presented and the solution characterized by the equations (24) and (25), now I'm going to explain in detail what I did in my Python code *Section3_Final_project.py* to compute the transition paths of the main variables of the model from the pre-covid situation to the Covid situation. As explained before the parameter that suddenly changes with the appearance of the pandemic is the infection rate i. Specifically, in my code I use $i = i_1 = 0$ in the pre-Covid situation and $i = i_2 = 0.2$ in the Covid situation, I compute the steady state in both settings, and, after that, I compute the transition from the first steady state to the second one. To do so I follow this steps:

- **STEP 1:** Solve the non-linear system composed by (24) and (25) evaluated at the steady state under $i = i_1 = 0$. This gives me the capital stock and the labor at workplace at the steady state in the pre-Covid situation. With this capital stock and labor at workplace at pre-covid steady state I can compute all the rest of relevant variables of the model at the pre-Covid steady state (labor from home, investment (equivalently savings), output, consumption...).

- **STEP 2:** Repeat Step 1 under $i = i_2 = 0.2$. This gives me the "pandemic steady state".

- **STEP 3:** Compute the transition. This was the most challenging part, and deserves some explanation. I'm going to split it in sub-steps.

  - Step 3.1: Solve the recursive formulation of the problem **under the Covid setting ($i = i_2 = 0.2$)**. Substituting for consumption in the utility function (using the feasibility constraint in (18)), and, again, taking advantage of the fact that $h_t^w + h_t^h = 1$ for all t, the recursive formulation can be written as:

$$V(K)=\max_{k'\in\left[0,F[k,h^w,1-h_w]+(1-\delta)k\right),h^w\in[0,1]}\left[ln\Big(F[k,h^w,1-h_w]+(1-\delta)-k'\Big)-\frac{(\omega+qi_2)(h^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}-\frac{\omega(1-h^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}+\beta V(k')\right] \tag{26}$$

    where F is the production function in (15).

WARNING: I solve this recursive problem by brute force value function iteration, that's why the code takes near 5 minutes to run. Sorry for the inconvenience.

Once this recursive problem is solved, it gives me (apart of the value function and the consumption policy function), three relevant objects: the policy functions for k, and for h^w (and implicitly h^h) under Covid ($i = i_2 = 0.2$). The policy function for capital, $g_{k'}(k)$ gives me, under the Covid setting ($i = i_2 = 0.2$), the optimal choice of capital stock in the following period (k'), given its value in current period (k). And the policy functions for labor at workplace and from home $g_{hw}(k)$, $g_{hh}(k)$ give me (again under the Covid setting) the optimal choices of this two variables in current period, given capital stock value in current period k. You can see the value function and the policy functions that I obtain below:

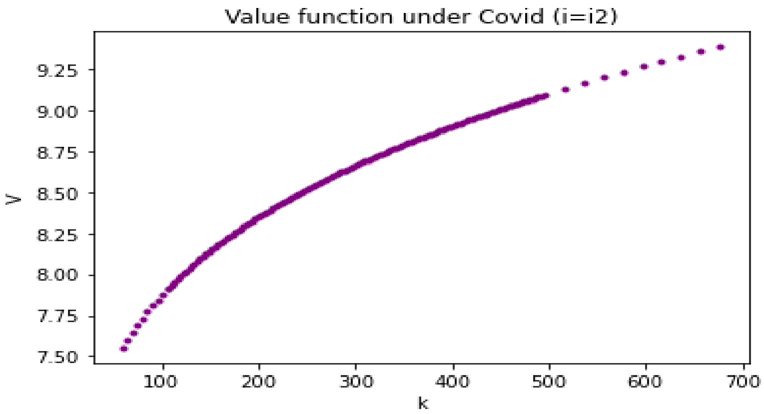

Figure 9:

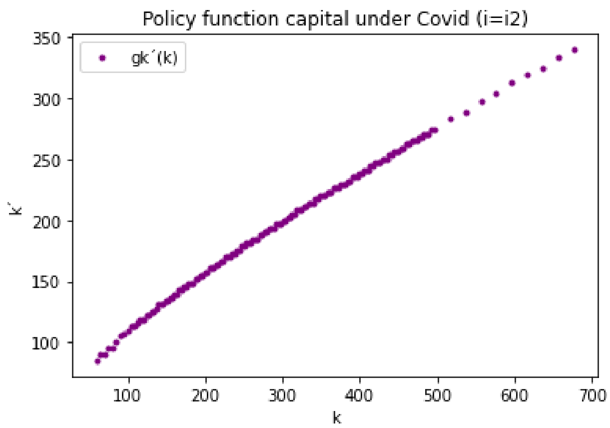

Figure 10:

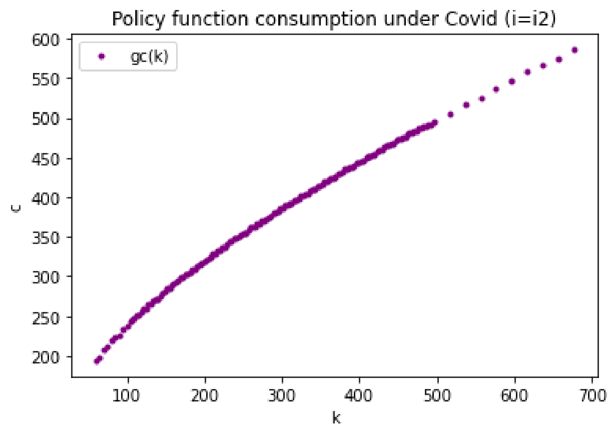

Figure 11:

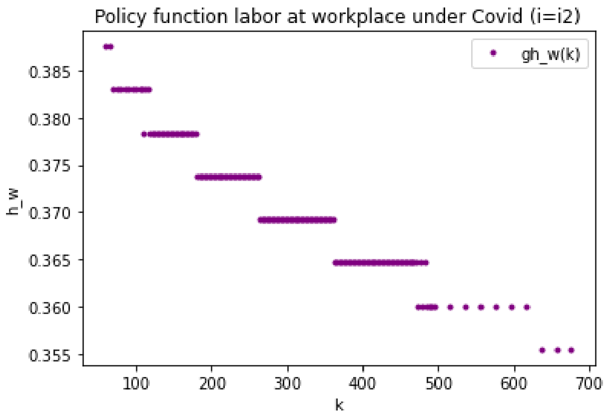

Figure 12:

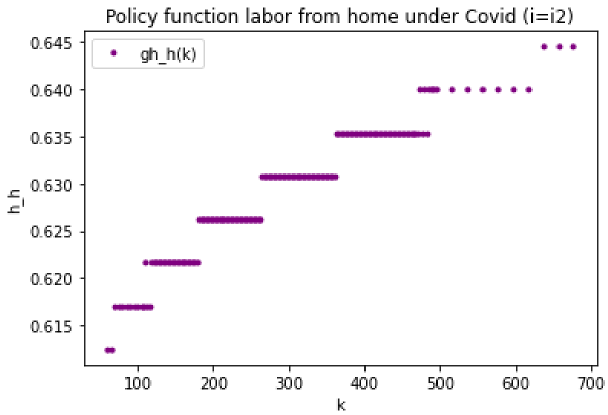

Figure 13:

Therefore, my strategy to compute the transition is to use this policy functions (specifically, the policy functions for k, h^w and h^h), introducing in them the values of variables at the Pre-Covid steady state, and iterating on them (in a simple manner that I will explain in the next sub step) until reaching the Covid steady state. Let's see it in more detail below.

– Step 3.2: Notice that the policy functions for Covid situation that I obtain from previous step for capital and labor at workplace (and implicitly for labor from home), which are the relevant ones in the sense that the rest of the variables are determined by them, are nothing more than scatter plots with each point associated to one point in the grid that I created for capital. When I refer to "iterate" in this environment, I mean to introduce a value in the policy function, and get the value for the following period, and so on until reaching the new steady state. Hence, in order to be able to introduce here the values of the pre-covid steady state for k, h^w and h^h and get the whole transition by iterating on this functions I need something more than isolated points, I need continuity because many points that take place in the transition will probably not be in the grid of capital.

The way that I used to obtain this "continuity" is to run three OLS regressions (for capital, for labor at workplace and for labor from home) of the values of the respective policy functions ($g_{k'}(k)$,$g_{hw}(k)$, $g_{hh}(k)$) on the grid of capital. To illustrate better what I do see the graphs below:

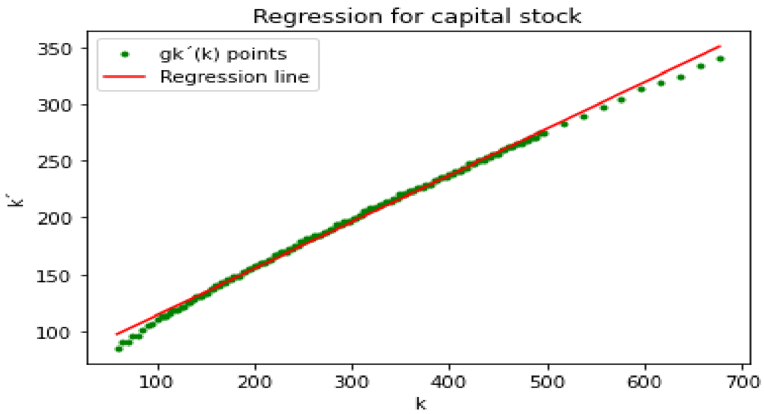

Figure 14:

Figure 15:

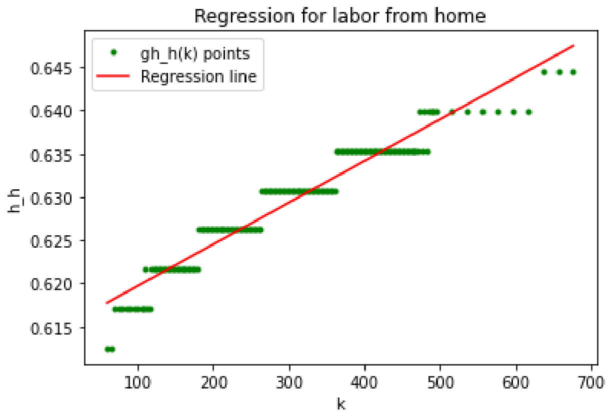

Figure 16:

– <u>Step 3.3</u>: The coefficients of the OLS regressions above allow me to, given any k, obtain, not an exact, but a very good approximation for what is the optimal choice of k' for the following period, and h^w and h^h for current period under the Covid setting $i = i_2 = 0.2$. Hence, if I introduce to the regression equations the variables of k, h^w and h^h at the pre-Covid steady state, and keep iterating and storing the values until the values converge to a value near (not exact because of regression residuals) the Covid steady state computed in <u>Step 2</u>, I will have computed the whole transition. Once the transition paths for k, h^w and h^h are computed, is straightforward to obtain the transition for output, investment (equivalently savings), consumption, etc. This concludes the process.

Let me now do three comments before showing and explaining the results. The first one is that, in order to simulate what happened in actual world, one could give to each period in the model a factual month interpretation, so that if we interpret period 0 as January 2020, and period 2 as March 2020, one can get a good flavour of what happened from March onwards, using this simple model. The second comment is that in Figure 23, I again use instantaneous welfare (i.e, welfare at each period t) as welfare measure. The last comment is that you can check all the parameter values that I used in the Python file. Below I present my results for the transitions:

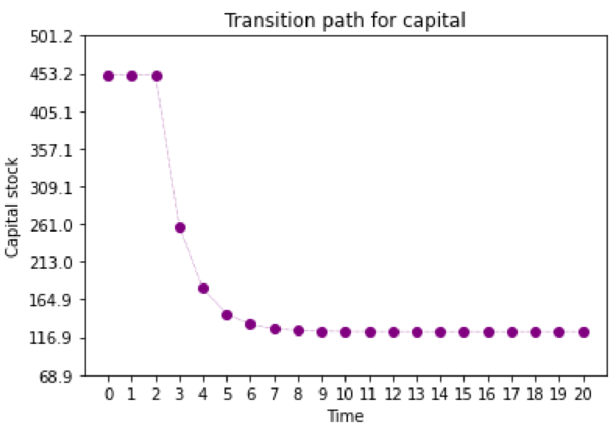

Figure 17:

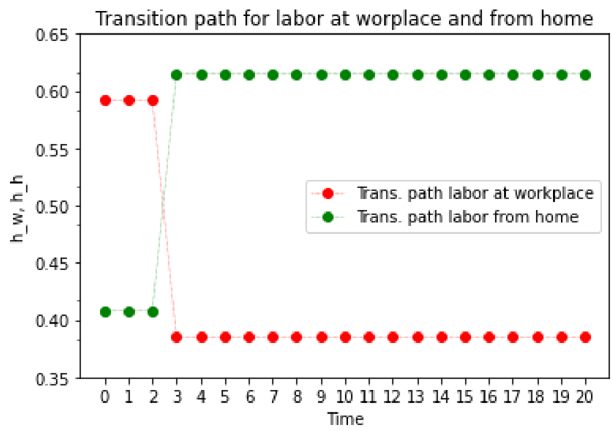

Figure 18:

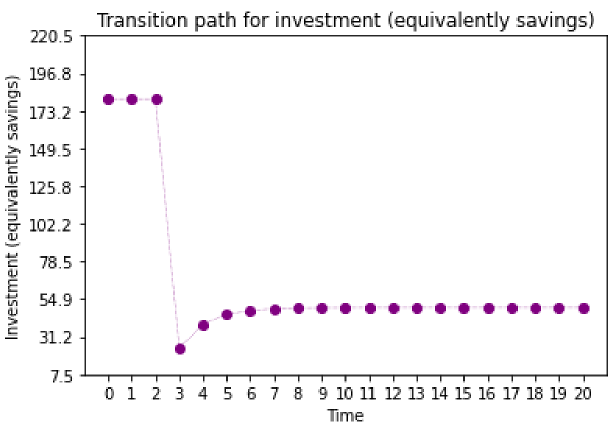

Figure 19:

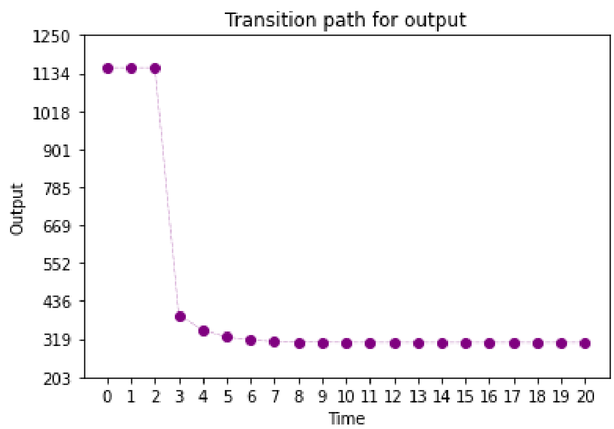

Figure 20:

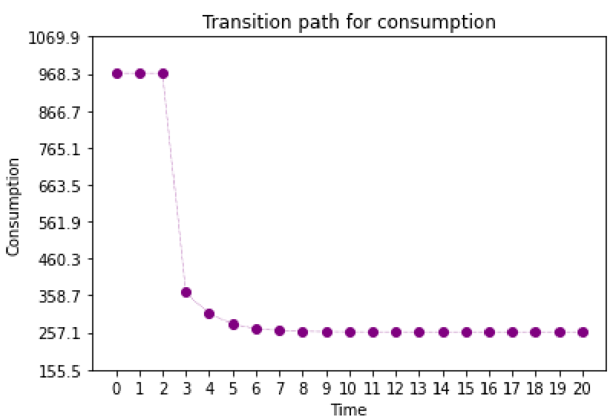

Figure 21:

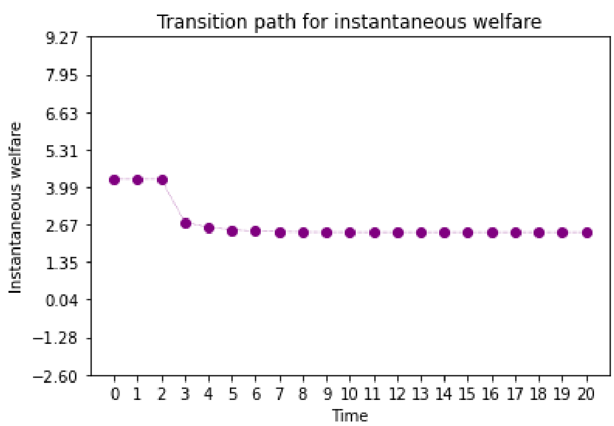

Figure 22:

Looking at the transition graphs above induced by the increase of i, the first conclusion is that the model predicts a fast transition, with abrupt falls of capital stock, output, consumption, investment and labor at workplace. Notice that near 5 periods after the Covid shock, all the variables are very close, to the Covid steady state. Focusing in labor market, which is probably the most interesting aspect to analyze, notice that the change is impressive. In just one period the transition is completed. Labor at workplace decreases in more that 20 percentual points, and labor from home increases in the same quantity (obviously given the $h_t w + h_t^h = 1$ assumption) in just one period. In the case of welfare, notice that the decrease does not seem so abrupt. A possible explanation for this is that the fast adjustment of working at workplace to working at home, could, partly, have prevented higher welfare loses with the increase of i.

This fast convergences to the new steady state that we observe in the model, specially for labor, actually fit what we could see in last spring, when, at least in Spain, from Friday 13 March to Monday 16 March, many people became changed to "telework". Similar arguments hold for output, investment, etc, it was an unexpected shock, generating huge and fast changes in all the macroeconomic variables.

SECTION 4: Ramsey government optimal taxation problem under the assumption of commitment in a 6 period finite-horizon economy with a pandemic.

It seems clear that during a pandemic as the Covid-19, government expenditure increases in order to pay all the public healthcare system services, all the transfers to the sectors and families more affected by the economic crisis derived from the health crisis, etc. That's why I thought that an interesting question was to know how is the optimal way of funding this increase in public expenditure in the economy described in Section 3. Should the increase in the expenditure be funded with an increase of taxes on capital? on labor at workplace?, on labor from home?.

Optimal taxation problems are, often, no trivial, so that I tried to introduce it in a simple way. That's why I decided to do it for only 6 periods (finite horizon), i.e t=0 ,1, 2, 3, 4, 5, under the assumption of government commitment and without uncertainty. In addition, I use an approach similar to the one in Chamley (1986), which makes the algebra simpler than using the usual Primal approach.

As said above, I use the same model as in Section 3, but now introducing a government that finances an exogenous stream of government expenditure $\{g_t\}_{t=0}^{t=T=5}$ with distortionary taxes on capital (τ_t^k),on labor at workplace (τ_t^w),and on labor from home (τ_t^h) and issuing debt (B_{t+1}). Also the firm problem associated to the environment in Section 3 will become important in this section to determine prices of factors, which affect government revenue. In order to simulate the pandemic, I construct an stream of government expenditure g_t such that keeps constant in periods 0 and 1 (without pandemic), suddenly increases in periods 2 and 3 (pandemic) and then returns to the original values in periods 5 and 6 (pandemic is over). At the same time, I link this public expenditures with the infection rate, i.e., I set i to be $i = i_1 = 0$ for periods 0 and 1, then be $i = i_2 = 0.2$ for periods 2 and 3, and, after that, return to $i = i_1 = 0$ in periods 4 and 5.

I'm going to proceed as follows: First I present representative household and firm problem and derive the FOC, then present government budget constraint, and finally set up and explain the Ramsey optimal taxation problem, which is what I solve in Python.

Notice that now we are leaving the Social Planner problem and solving for competitive equilibrium. Taking the same economy as in Section 3, but now from the competitive equilibrium perspective, with a government, and reducing time to 6 periods, the representative household problem is:

$$\max_{[c_t,k_{t+1},h_t^w,ht^h,b_{t+1}]_{t=0}^{t=T=5}} \sum_{t=0}^{t=T=5} \beta^t \left[ln(c_t) - \frac{(\omega+qi)(h_t^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - \frac{\omega(h_t^h)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + g_t \right] \tag{27}$$

subject to:

$$c_t + k_{t+1} - (1-\delta)k_t + \Upsilon_t b_{t+1} = (1-\tau_t^k)r_t k_t + (1-\tau_t^w)w_t h_t^w + (1-\tau_t^h)p_t h_t^h + b_t \tag{28}$$

$c_t \geq 0$
where b are government bounds that households purchases at price Υ, whereas τ_t^k, τ_t^w and τ_t^h are the taxes on the factors (capital, labor at workplace and labor from home respectively), and r_t, w_t and p_t the prices of those factors, also respectively. Notice hence that we are assuming that labor at workplace and labor from home do not have necessarily the same price, and that also can be taxed differently by the government. Also note in the lifetime utility function that household derives positive utility from the government expenditure. However she perceives it as something exogenous, that does not depend on her individual decisions. Finally, let me recall that condition $h_t^w + h_t^h = 1$ still holds, but that I will use it later, because in this case, to derive the FOC is more convenient to work with all the variables fully defined. Also let me remind again that ω expresses disutility of labor, and w are wages, don't get confused by the similarity of the letters.

The lagrangian corresponding to the problem stated above is:

$$\mathcal{L} = \sum_{t=0}^{t=T=5} \beta^t \left[ln(c_t) - \frac{(\omega + qi)(h_t^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - \frac{\omega(h_t^h)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + g_t \right]$$
$$- \sum_{t=0}^{t=T=5} \lambda_t \left[c_t + k_{t+1} - (1-\delta)k_t + \Upsilon_t b_{t+1} - (1-\tau_t^k)r_t k_t - (1-\tau_t^w)w_t h_t^w - (1-\tau_t^h)p_t h_t^h - b_t \right] \tag{29}$$

Take the FOC:

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \frac{\beta^t}{c_t} = \lambda_t \tag{30}$$

$$\frac{\partial \mathcal{L}}{\partial c_{t+1}} = 0 \Rightarrow \frac{\beta^{t+1}}{c_{t+1}} = \lambda_{t+1} \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial h_t^w} = 0 \Rightarrow \beta^t(\omega + qi)(h_t^w)^{\frac{1}{\nu}} = \lambda_t\big[(1-\tau_t^w)w_t\big] \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial h_t^h} = 0 \Rightarrow \beta^t\omega(h_t^h)^{\frac{1}{\nu}} = \lambda_t\big[(1-\tau_t^h)p_t\big] \tag{33}$$

$$\frac{\partial \mathcal{L}}{\partial b_{t+1}} = 0 \Rightarrow \lambda_t \Upsilon_t = \lambda_{t+1} \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \Rightarrow \lambda_{t+1}\big[(1-\tau_{t+1}^k)r_{t+1} + (1-\delta)\big] = \lambda_t \tag{35}$$

And combine them. (30), (31) and (35) give the Euler equation:

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}}\big[(1-\tau_{t+1}^k)r_{t+1} + (1-\delta)\big] \tag{36}$$

(34) and (35) give the no-arbitrage condition

$$\Upsilon_t = \frac{1}{\big[(1-\tau_{t+1}^k)r_{t+1} + (1-\delta)\big]} \tag{37}$$

(30) and (32) give labor at workplace condition:

$$(\omega + qi)(h_t^w)^{\frac{1}{\nu}} = \frac{1}{c_t}\big[(1-\tau_t^w)w_t\big] \tag{38}$$

and (30) and (33) give labor from home condition:

$$\omega(h_t^h)^{\frac{1}{\nu}} = \frac{1}{c_t}\big[(1-\tau_t^h)p_t\big] \tag{39}$$

Once household problem is characterized, let's now focus on firms. The representative firm problem is:

$$\max_{K_t,H_t^w,H_t^h} K_t^{1-\alpha}\left[\left((A_w-i)(H_t^w)^z+\phi A_h(H_t^h)^z\right)^{\frac{1}{z}}\right]^{\alpha}-r_tK_t-w_tH_t^w-p_tH_t^h \tag{40}$$

The FOC of the firm are:

$$K_t:\quad (1-\alpha)K_t^{-\alpha}\left(\left((A_w-i)(H_t^w)^z+A_h\phi(H_t^h)^z\right)^{\frac{1}{z}}\right)^{\alpha}=r_t \tag{41}$$

$$H_t^w:\quad \frac{\alpha(A_w-i)K_t^{1-\alpha}(H_t^w)^{z-1}\left(\left((A_w-i)(H_t^w)^z+A_h\phi(H_t^h)^z\right)^{\frac{1}{z}}\right)^{\alpha}}{(A_w-i)(H_t^w)^z+A_h\phi(H_t^h)^z}=w_t \tag{42}$$

$$H_t^h:\quad \frac{\alpha A_hK_t^{1-\alpha}\phi(H_t^h)^{z-1}\left(\left((A_w-i)(H_t^w)^z+A_h\phi(H_t^h)^z\right)^{\frac{1}{z}}\right)^{\alpha}}{(A_w-i)(H_t^w)^z+A_h\phi(H_t^h)^z}=p_t \tag{43}$$

Regarding the government budget constraint, is given by:

$$g_t+\Upsilon_tB_{t+1}=\tau_t^kr_tk_t+\tau_t^ww_th_t^w+\tau_t^hp_th_t^h+B_t \tag{44}$$

As a final necessary step before defining the Ramsey optimal taxation problem, let me briefly expose what is a competitive equilibrium in this economy:

A competitive equilibrium is a sequence of allocations for household $x^h=\{c_t,k_{t+1},h_t^w,h_t^h,b_{t+1}\}_{t=0}^{t=T=5}$ for firm $x^f=\{K_t,H_t^w,H_t^h\}_{t=0}^{t=T=5}$ and government policies $x^g=\{\tau_t^k,\tau_t^w,\tau_t^h,B_{t+1}\}_{t=0}^{t=T=5}$ and prices $x^p=\{r_t,w_t,p_t,\Upsilon_t\}_{t=0}^{t=T=5}$ such that:

- Given prices x^p , x^h solves HH problem.
- Given prices x^p , x^f solves firm problem for all t=0,1,2,3,4,5.
- Given prices x^p, x^g makes the government budget constraint to be balanced for all t=0,1,2,3,4,5.
- All markets clear for all t=0,1,2,3,4,5:
  1. Capital market: $k_t=K_t$
  2. Labor at workplace market: $h_t^w=H_t^w$
  3. Labor from home market: $h_t^h=H_t^h$
  4. Bonds market: $b_t^h=-B_t$
  5. Goods market (feasibility constraint):
     $c_t+g_t+k_{t+1}-(1-\delta)k_t=k_t^{1-\alpha}\left[\left((A_w-i)(h_t^w)^z+\phi A_h(h_t^h)^z\right)^{\frac{1}{z}}\right]^{\alpha}$

Now, the Ramsey optimal taxation problem, consists in, given k_0, b_0, and the exogenous government expenditure stream $\{g_t\}_{t=0}^{t=T=5}$ choose a competitive equilibrium (as the described above) that maximizes agent utility in (27). In order to set up the lagrangian corresponding to this optimal taxation problem, some algebra manipulations with the conditions obtained above are needed. The objective of this manipulations is to get rid of prices. For all this manipulations, since we are doing nothing more than characterizing a competitive equilibrium, I will already be assuming that market clearing conditions described above hold. See the manipulations below:

First, using the no-arbitrage condition in (37) and the FOC's of firm in (40), (41) and (42), substitute for prices in government budget constraint in (44), obtaining:

$$\begin{aligned}
& g_t + \frac{1}{\left[(1-\tau_{t+1}^k)(1-\alpha)k_{t+1}^{-\alpha}\left(\left((A_w-i)(h_{t+1}^w)^z + A_h\phi(h_{t+1}^h)^z\right)^{\frac{1}{z}}\right)^{\alpha} + (1-\delta)\right]} b_{t+1} \\
& = \tau_t^k(1-\alpha)k_t^{-\alpha}\left(\left((A_w-i)(h_t^w)^z + A_h\phi(h_t^h)^z\right)^{\frac{1}{z}}\right)^{\alpha} k_t \\
& + \tau_t^w \frac{\alpha(A_w-i)k_t^{1-\alpha}(h_t^w)^{z-1}\left(\left((A_w-i)(h_t^w)^z + A_h\phi(h_t^h)^z\right)^{\frac{1}{z}}\right)^{\alpha}}{(A_w-i)(h_t^w)^z + A_h\phi(h_t^h)^z} h_t^w \\
& + \tau_t^h \frac{\alpha A_h K_t^{1-\alpha}\phi(h_t^h)^{z-1}\left(\left((A_w-i)(h_t^w)^z + A_h\phi(h_t^h)^z\right)^{\frac{1}{z}}\right)^{\alpha}}{(A_w-i)(h_t^w)^z + A_h\phi(h_t^h)^z} h_t^h + b_t
\end{aligned} \tag{45}$$

Now, similarly, also use the FOC's of firms to get rid of prices in household's conditions (36) (38) and (39), obtaining:

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}}\left[(1-\tau_{t+1}^k)(1-\alpha)k_{t+1}^{-\alpha}\left(\left((A_w-i)(h_{t+1}^w)^z + A_h\phi(h_{t+1}^h)^z\right)^{\frac{1}{z}}\right)^{\alpha} + (1-\delta)\right] \tag{46}$$

$$(\omega + qi)(h_t^w)^{\frac{1}{\nu}} = \frac{1}{c_t}\left[(1-\tau_t^w)\frac{\alpha(A_w-i)k_t^{1-\alpha}(h_t^w)^{z-1}\left(\left((A_w-i)(h_t^w)^z + A_h\phi(h_t^h)^z\right)^{\frac{1}{z}}\right)^{\alpha}}{(A_w-i)(h_t^w)^z + A_h\phi(h_t^h)^z}\right] \tag{47}$$

$$\omega(h_t^h)^{\frac{1}{\nu}} = \frac{1}{c_t}\left[(1-\tau_t^h)\frac{\alpha A_h k_t^{1-\alpha}\phi(h_t^h)^{z-1}\left(\left((A_w-i)(h_t^w)^z + A_h\phi(h_t^h)^z\right)^{\frac{1}{z}}\right)^{\alpha}}{(A_w-i)(h_t^w)^z + A_h\phi(h_t^h)^z}\right] \tag{48}$$

Now we have all the information needed to set up the Ramsey problem. The problem consists in maximizing agent lifetime utility in (27), subject to the rewritten version of government budget constraint in (45), the rewritten HH conditions in (46), (47) and (48), and subject to the feasibility constraint described above when defining the competitive equilibrium. Imposing now the condition $h_t^w + h_t^h = 1$, which implies that $h_t^h = 1 - h_t^w$, and since we got rid of prices, the variables of choice of the Ramsey government in this environment are: $\{c_t, k_{t+1}, h_t^w, b_{t+1}, \tau_t^k, \tau_t^w, \tau_t^h\}_{t=0}^{t=T=5}$. In the following lines I directly pose the lagrangian corresponding to this problem, which is what I solve in my Python code. The lagrangian is formulated in an slightly different way as usual, because I also multiply the discount factor β by the lagrange multipliers. This way is equivalent to the usual one, but is more convenient for this type of problems. Therefore the lagrangian is:

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{t=T=5} \beta^t \Bigg\{ & \left[ln(c_t) - \frac{(\omega + qi)(h_t^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - \frac{\omega(1-h_t^w)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + g_t \right] \\
& + \lambda_t \Bigg\langle \tau_t^k (1-\alpha) k_t^{-\alpha} \left(\left((A_w - i)(h_t^w)^z + A_h \phi (h_t^h)^z \right)^{\frac{1}{z}} \right)^{\alpha} k_t \\
& + \tau_t^w \frac{\alpha (A_w - i) k_t^{1-\alpha} (h_t^w)^{z-1} \left(\left((A_w - i)(h_t^w)^z + A_h \phi (h_t^h)^z \right)^{\frac{1}{z}} \right)^{\alpha}}{(A_w - i)(h_t^w)^z + A_h \phi (h_t^h)^z} h_t^w \\
& + \tau_t^h \frac{\alpha A_h K_t^{1-\alpha} \phi (h_t^h)^{z-1} \left(\left((A_w - i)(h_t^w)^z + A_h \phi (h_t^h)^z \right)^{\frac{1}{z}} \right)^{\alpha}}{(A_w - i)(h_t^w)^z + A_h \phi (h_t^h)^z} h_t^h + b_t \\
& - g_t - \frac{1}{\left[(1-\tau_{t+1}^k)(1-\alpha) k_{t+1}^{-\alpha} \left(\left((A_w - i)(h_{t+1}^w)^z + A_h \phi (h_{t+1}^h)^z \right)^{\frac{1}{z}} \right)^{\alpha} + (1-\delta) \right]} b_{t+1} \Bigg\rangle \\
& + \mu_t \Bigg\langle \frac{1}{c_t} - \frac{\beta}{c_{t+1}} \left[(1-\tau_{t+1}^k)(1-\alpha) k_{t+1}^{-\alpha} \left(\left((A_w - i)(h_{t+1}^w)^z + A_h \phi (h_{t+1}^h)^z \right)^{\frac{1}{z}} \right)^{\alpha} + (1-\delta) \right] \Bigg\rangle \\
& + \sigma_t \Bigg\langle (\omega + qi)(h_t^w)^{\frac{1}{\nu}} - \frac{1}{c_t} \left[(1-\tau_t^w) \frac{\alpha (A_w - i) k_t^{1-\alpha} (h_t^w)^{z-1} \left(\left((A_w - i)(h_t^w)^z + A_h \phi (h_t^h)^z \right)^{\frac{1}{z}} \right)^{\alpha}}{(A_w - i)(h_t^w)^z + A_h \phi (h_t^h)^z} \right] \Bigg\rangle \\
& + \iota_t \Bigg\langle \omega (h_t^h)^{\frac{1}{\nu}} - \frac{1}{c_t} \left[(1-\tau_t^h) \frac{\alpha A_h k_t^{1-\alpha} \phi (h_t^h)^{z-1} \left(\left((A_w - i)(h_t^w)^z + A_h \phi (h_t^h)^z \right)^{\frac{1}{z}} \right)^{\alpha}}{(A_w - i)(h_t^w)^z + A_h \phi (h_t^h)^z} \right] \Bigg\rangle \\
& + \chi_t \Bigg\langle k_t^{1-\alpha} \left[\left((A_w - i)(h_t^w)^z + \phi A_h (h_t^h)^z \right)^{\frac{1}{z}} \right]^{\alpha} - c_t - g_t - k_{t+1} + (1-\delta) k_t \Bigg\rangle \Bigg\}
\end{aligned} \tag{49}
$$

where $\lambda, \mu, \sigma, \iota, \xi$ are the Lagrange multipliers. Notice that by writing the lagrangian, what we do is nothing more than transforming a constrained maximization problem to an unconstrained one, in which we add the lagrange multipliers to the choice variables, so that now the choice variables are: $\{c_t, k_{t+1}, h_t^w, b_{t+1}, \tau_t^k, \tau_t^w, \tau_t^h, \lambda_t, \mu_t, \sigma_t, \iota_t, \chi_t\}_{t=0}^{t=T=5}$. Also notice that when solving a finite horizon problem, the initial and final conditions acquire importance. Specifically I will be assuming:

- <u>As initial conditions:</u> $k_0 = k^{*pre}$ which is the capital at the steady state in the pre-covid situation from Section 3 (to do so I obviously kept exactly the same parameter values as in Section 3). I also set $b_0 = 30$ and $\tau_0^k = 0$ to rule out lump sum taxation

- <u>As final conditions:</u> Basically the usual final coditions in finite horizon, i.e: $k_{T+1} = k_6 = 0$, $c_{T+1} = c_6 = 0$, $h^w_{T+1} = h^w_6 = 0$, $h^h_{T+1} = h^h_6 = 0$, $b_{T+1} = b_6 = 0$, $h^w_{T+1} = h^w_6 = 0$

In the Python file *Section4_Final_project.py* you can find my solution to this lagrangian. As you will see the way that I solve it is not very automatic, but I was running out of time and I was not able to program it in a more efficient way. You will also see that in the code I provide bounds to all the variables when maximizing. This is because the optimization routine that I used requires it. As you can see in the code I set upper bounds large enough to make sure that the variables are not restricted by those bounds. Regarding to the lower bounds, in most of the variables (e.g. consumption, capital stock, Lagrange multipliers, etc.) I use the 0 lower bound as a non-negativity constraint.

Regarding to the results, the most interesting thing is to present the evolution of the different optimal taxes (on capital, on labor at workplace, and on labor from home) that I obtain solving the problem, and see how do they evolve when at period 2 the pandemic appears. Hence, I present two figures below. In Figure 23 I show the exogenous sequence of public expenditure $\{g_t\}_{t=0}^{t=T=5}$ that I generated to simulate the pandemic in periods 2 and 3. In Figure 24 I show what is the optimal taxation plan of the government obtained from solving the lagrangian above. I think that there must be some small typo in the algebra or when writing the code, because a $\tau^k = 0.94$ in period 2 seems too high, but anyway, disregarding the concrete quantities, the shapes obtained in my results make sense, that's why I found interesting to show them:
Note: Recall that in order to make the simulation of the pandemic more realistic, in the periods of the increase of the government expenditure (periods 2 and 3), I make also increase the infection rate i in those two periods by setting it to the level in the Covid situation of Section 3, i.e. $i = i_2 = 0.2$. After that, in periods 4 and 5 I return it to 0.

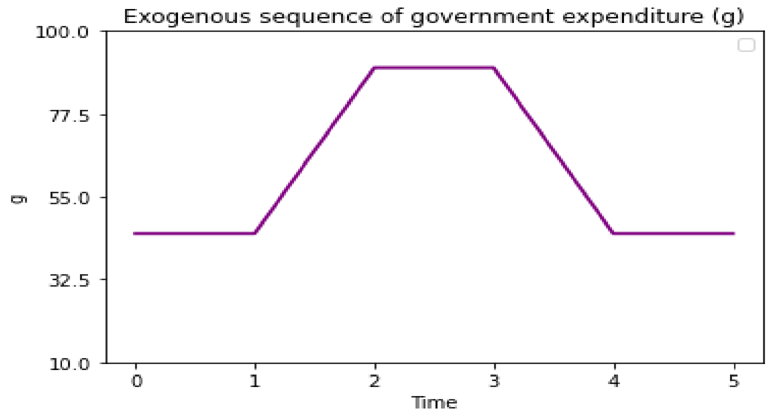

Figure 23:

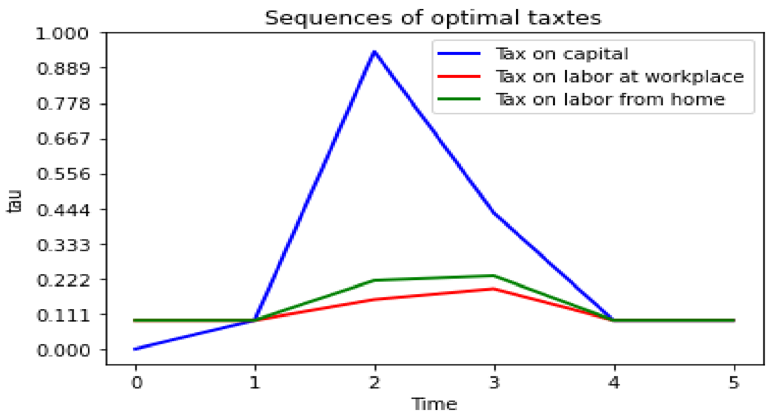

Figure 24:

Ignoring the quantities, which as said before, probably due to some small typo in the algebra, seem excessive, specially in the taxation of capital, we can take some interesting conclusions form the shape of Figure 24:

- The first conclusion, is that, obviously, during the pandemic, the optimal taxes on the three inputs increase to fund a higher government expenditure.

- The second conclusion is that the optimal taxation is higher on capital than on the two types of labor, **but only during the periods in which the pandemic takes place**, whereas in the periods in which there is no pandemic, it is equal, or, even smaller. In future work it could be interesting to see if this also happens when there is only one type of labor.

- The third interesting conclusion is that the Ramsey government decides to set higher optimal taxes on labor from home than on labor at workplace during the periods of pandemic, whereas without pandemic they are equal. This can be due to the fact that, as the Ramsey government is benevolent by definition, and, during the pandemic, household derives more disutility from labor at workplace, Ramsey government decides to balance disutility sources by setting a lower optimal tax on those type of labor that generates more disutility by itself during the pandemic (labor at workplace), and a higher optimal tax on that one that generates less disutilility by itself during the pandemic (labor from home).

About possible extensions and future work:

The first thing that I would like to be able to do in future work is to be able to solve the Ramsey government problem in a more automatic way and for a larger range of time (say 500 periods for example).

Apart from that, an interesting way of extending this project that come up to my mind could be the following one:

In the lifetime utility of the agent in (27), I assume that she derives positive utility from the public expenditure g_t, but, as she perceives it as something exogenous, it is not included in the FOC. It could be interesting to repeat the exercise but endogenizing the public expenditure in the agent utility function. When I say endogenizing, I basically mean substituting for g_t in (27) using the government budget constraint in (44), and see how do the results change. Would then she be willing to pay higher taxes? If yes, always?, only during the pandemic?
Also in this more complicated environment it could be also interesting to split the government expenditure in two: expenditures in public health and the rest of the expenditures, and endogenize (also) the infection rate making it depend on government expenditures in health. Finally, also the possibility of no commitment of the government about his announced taxation plan, and sources of aggregate and idiosyncratic uncertainty could be added to complicate the model even more.

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Finally, another important guide, specially in the Ramsey problem part, where my class notes of Macro 3 from last year.