BH++: A Black Hole Geodesics Integrator in C++

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***Abstract*—The short abstract (50-80 words) is intended to give the reader an overview of the work.**

I. Introduction

GENERAL

II. The integrator algorithm

III. Testing the algorithm

In this section, we initialize our solver with initial conditions for known sceneraios: the existence of a circular orbit for photons at $R = \frac{3}{2}R_s$, and the planetary system formed by the Sun, the Earth and Jupiter (ignoring the effects of the planets to the metric).

A. Photon geodesics

The effective potential for massless particles is

$$V(R) = \frac{L^2}{2R^2} - \frac{GML^2}{c^2R^3}, \tag{1}$$

which has an extremum at

$$\frac{dV}{dR} = 0\ ;\ R = \frac{3GM}{c^2} = \frac{3}{2}R_s, \tag{2}$$

independently of its angular momentum. This corresponds to the circular photon orbit, which is unstable as we can clearly see from the shape of the effective potential in figure 1.

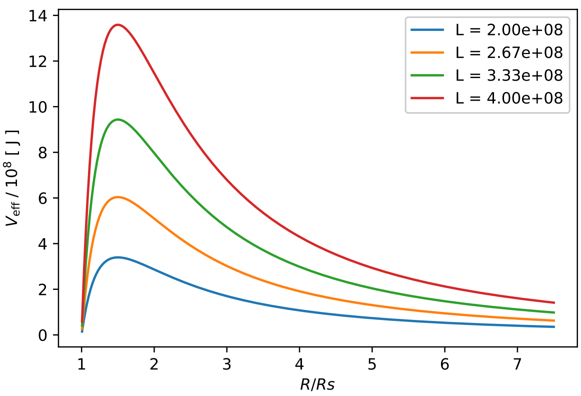

Fig. 1. Effective potential for a photon close to the black hole.

We initialize a set of 15 photons, one of them at the expected circular orbit, with an arbitrarily chosen specific angular momentum $L = 10^8$ m^2/s. The results are shown in figure 2. As we can see, the photon with $R0 = 1.5R_s$ traces a circular orbit as expected, while the outer photons escape from the black hole and the inner ones get trapped. This is consistent with the extremum being a maximum, hence the circular orbit is unstable. Our integrator gives the same result even after thousands of revolutions, confirming its numerical stability.

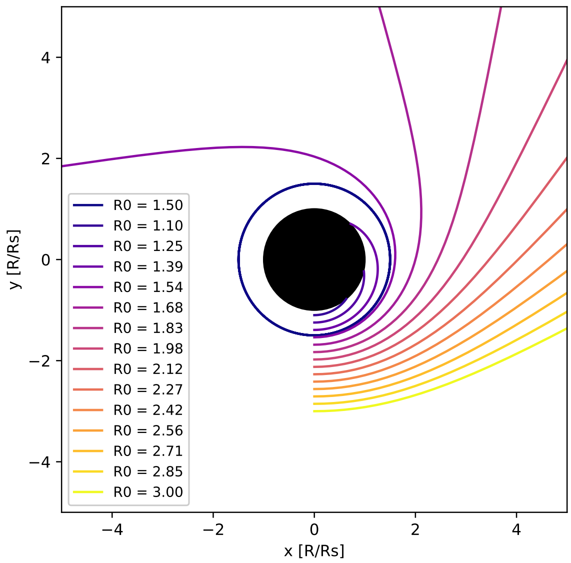

Fig. 2. Set of 15 photon orbits initialized at different radii. $R0 = 1.5$ confirms the photon circular orbit.

B. Massive particle geodesics

From [REF] the effective potential for massive particles is

$$V(R) = \frac{c^2}{2} - \frac{GM}{R} + \frac{L^2}{2R^2} - \frac{GML^2}{c^2R^3}, \tag{3}$$

which differs from the Newtonian one by the last factor $\sim r^{-3}$ and the rest energy of the particle. We find the circular orbits at the extrema of the effective potential,

$$\frac{dV}{dr} = 0, \tag{4}$$

$$R_\pm = \frac{L^2 \pm \sqrt{L^4 - \frac{12G^2M^2L^2}{c^2}}}{2GM}, \tag{5}$$

that now depend on angular momenta as opposed to the massless case. The shape of the effective potential for different L is plotted in figure 3 We initialize 6 massive particles with different angular momentum at the extrema position. We see that they indeed form circular orbits in figure 4.

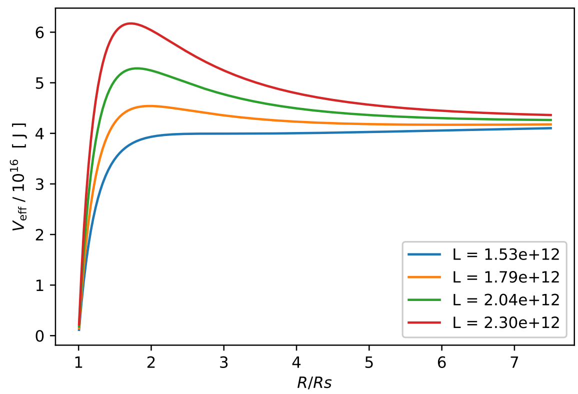

Fig. 3. Effective potential of massive particles for different values of the angular momentum.

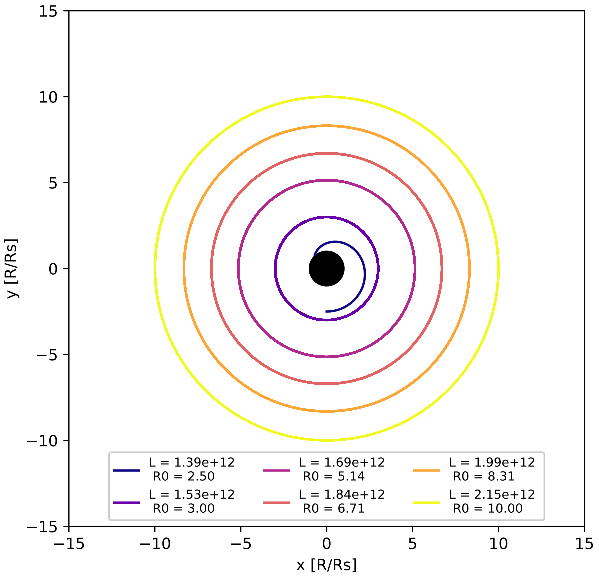

Fig. 4. Set of 6 massive particle orbits initialized at different radii and angular momenta. $R_0 = 3$ confirms the innermost circular orbit.

There is a characteristic radius below which no circular orbits exist,

$$L^4 < \frac{12G^2M^2L^2}{c^2} \to L < \sqrt{12}GM, \quad (6)$$

which corresponds to $r < 3R_s$. No particle can orbit in a circular fashion inside this radius, in contrast to the Newtonian case.

C. Newtonian limit: The Solar System

The last check we perform is initializing the solver with a planteray system consisting of the Earth, Jupiter and the Sun as seen in figure 5. The eccentricity of the orbits is $\varepsilon = 0.0034$ for the Earth and $\varepsilon = 0.123$ for Jupiter. The disagreement on Jupiter's eccentricity (compared to the observed one) is due to ignoring the gravitational fields from other planets.

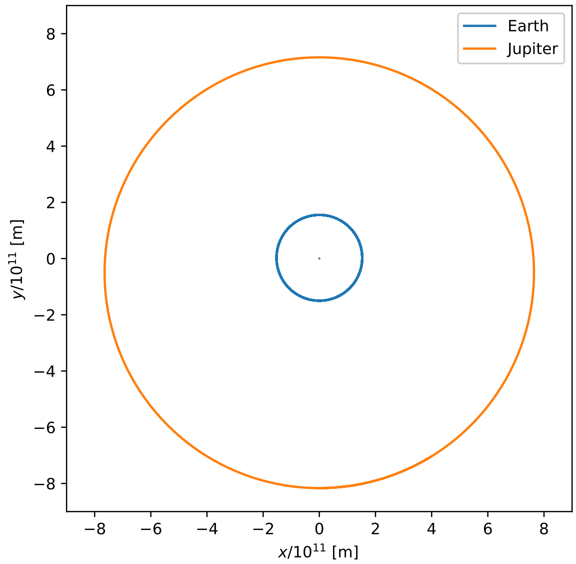

Fig. 5. Orbits of the Earth and Jupiter around the sun.

IV. Conclusion

This section summarizes the paper.

References

[1] J. Hagenauer, E. Offer, and L. Papke. Iterative decoding of binary block and convolutional codes. *IEEE Trans. Inform. Theory*, vol. 42, no. 2, pp. 429-445, Mar. 1996.
[2] T. Mayer, H. Jenkac, and J. Hagenauer. Turbo base-station cooperation for intercell interference cancellation. *IEEE Int. Conf. Commun. (ICC)*, Istanbul, Turkey, pp. 356–361, June 2006.
[3] J. G. Proakis. *Digital Communications*. McGraw-Hill Book Co., New York, USA, 3rd edition, 1995.
[4] F. R. Kschischang. Giving a talk: Guidelines for the Preparation and Presentation of Technical Seminars. http://www.comm.toronto.edu/frank/guide/guide.pdf.
[5] IEEE Transactions LaTeXand Microsoft Word Style Files. http://www.ieee.org/web/publications/authors/transjnl/index.html