Definition

A B-tree of order m, if non empty, is an m-way search tree in which:

(i) the root has at least two child nodes and at most m child nodes

(ii) the internal nodes except the root have at least $\left\lceil \frac{m}{2} \right\rceil$ child nodes and at most m child nodes

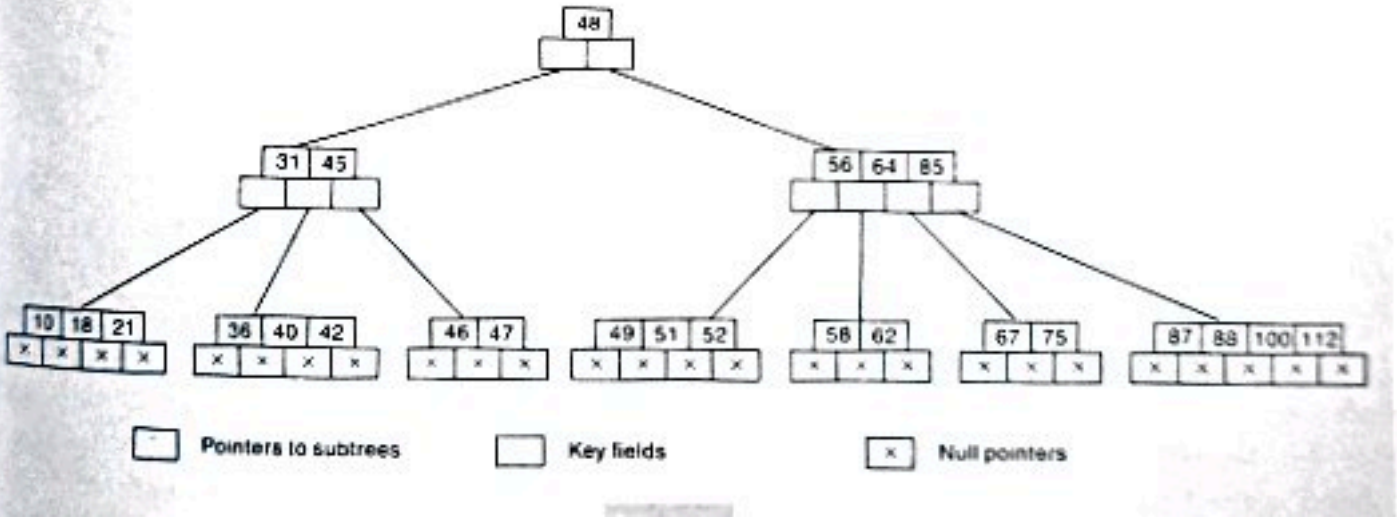
(iii) the number of keys in each internal node is one less than the number of child nodes and these keys partition the keys in the subtrees of the node in a manner similar to that of m-way search trees.

(iv) all leaf nodes are on the same level

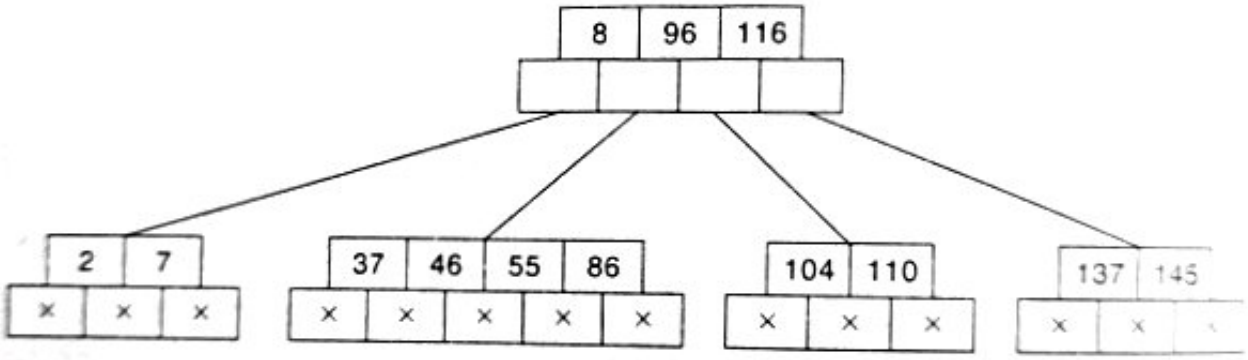
A B-tree of order 3 is referred to as 2–3 tree since the internal nodes are of degree 2 or 3 only.

Example 7.30

The tree shown in Fig. 7.57 is a B tree of order 5. All the properties of the B tree can be verified on the tree.



Consider the B-tree of order 5 shown in Fig. 7.58. Insert the elements 4, 5, 58, 6 in the order given.



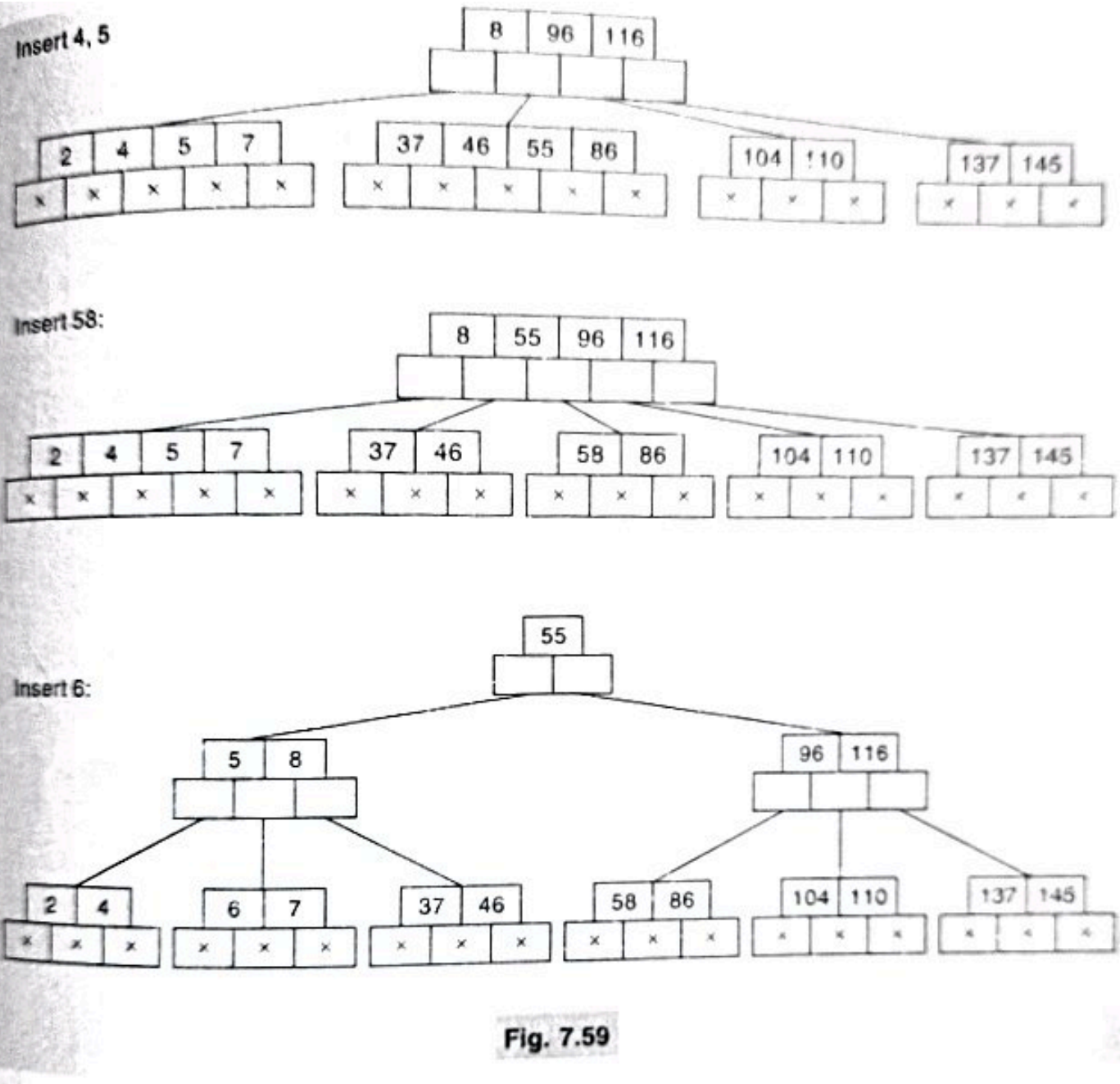
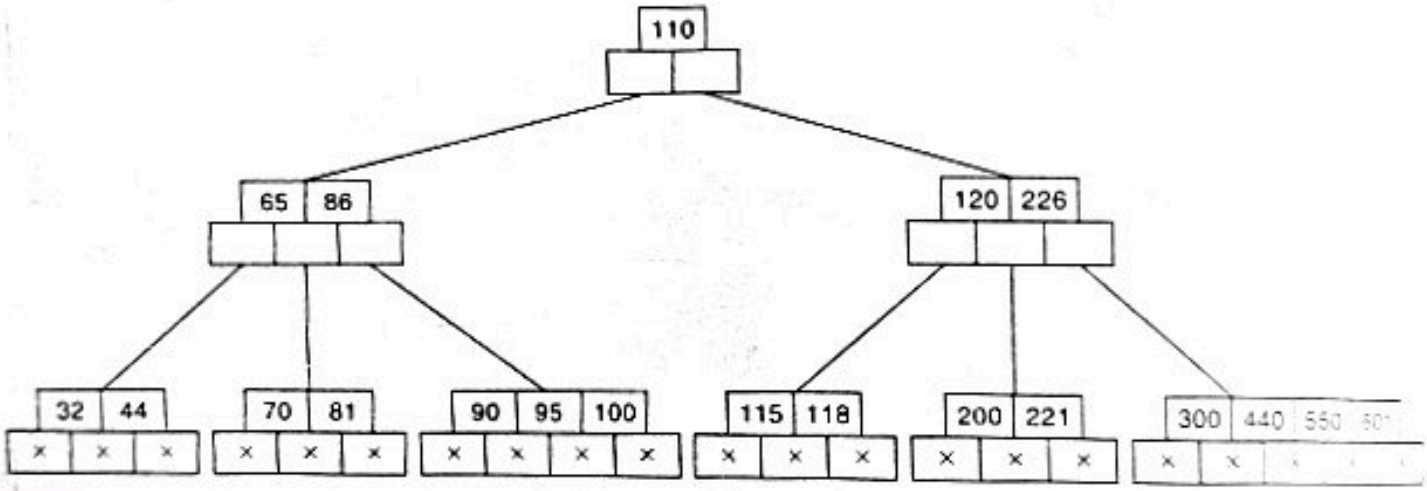

Insert 4, 5
8
96
116
2
4
5
7
37
46
55
86
104
110
137
145
Insert 58:
8
55
96
116
2
4
5
7
37
46
58
86
104
110
137
145
Insert 6:
55
5
8
96
116
2
4
6
7
37
46
58
86
104
110
137
145

Fig. 7.59

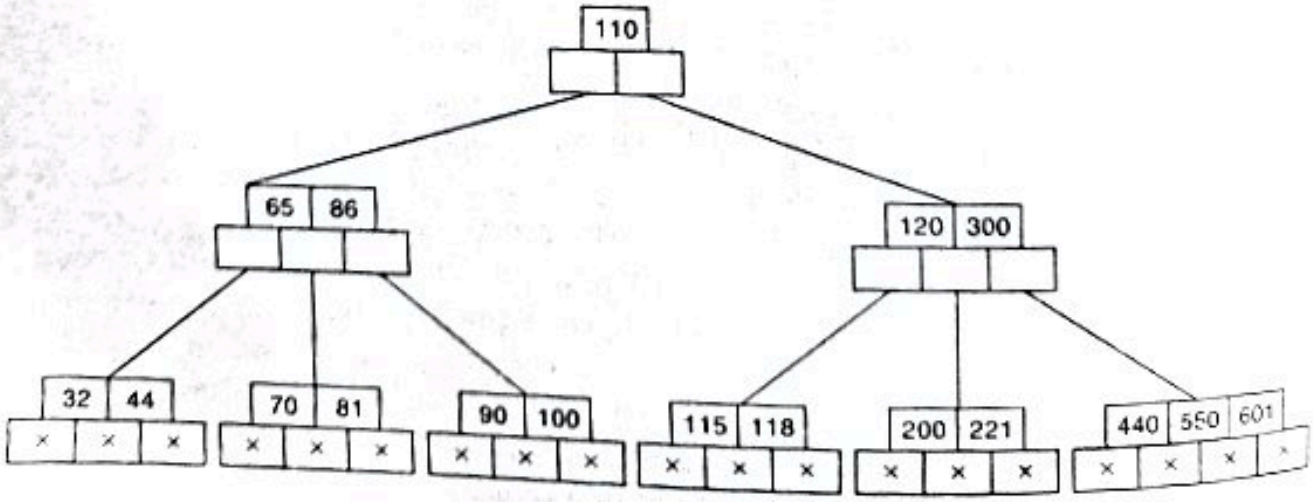
Example 7.32

Deletion of keys 95, 226, 221 and 70 on a given B-tree of order 5 is shown in Fig. 7.60. The deletion of key 95 is simple and straight since the leaf node has more than the minimum number of elements. To delete 226, the internal node has only the minimum number of elements and hence borrows the immediate successor viz., 300 from the leaf node which has more than the minimum number of elements. Deletion of 221 calls for the hauling of key 440 to the parent node and pulling down of 300 to take the place of the deleted entry in the leaf. Lastly the deletion of 70 is a little more involved in process. Since none of the adjacent leaf nodes can afford lending a key, two of the leaf nodes are combined with the intervening element from the parent to form a new leaf node, viz., [32, 44, 65, 81] leaving 86 alone in the parent node. This is not possible since the parent node is now running low on its minimum number of elements. Hence we once again proceed to combine the adjacent sibling nodes of the specified parent node with a median element of the parent which is the root. This yields the node [86, 110, 120, 440] which is the new root. Observe the reduction in height of the B-tree.

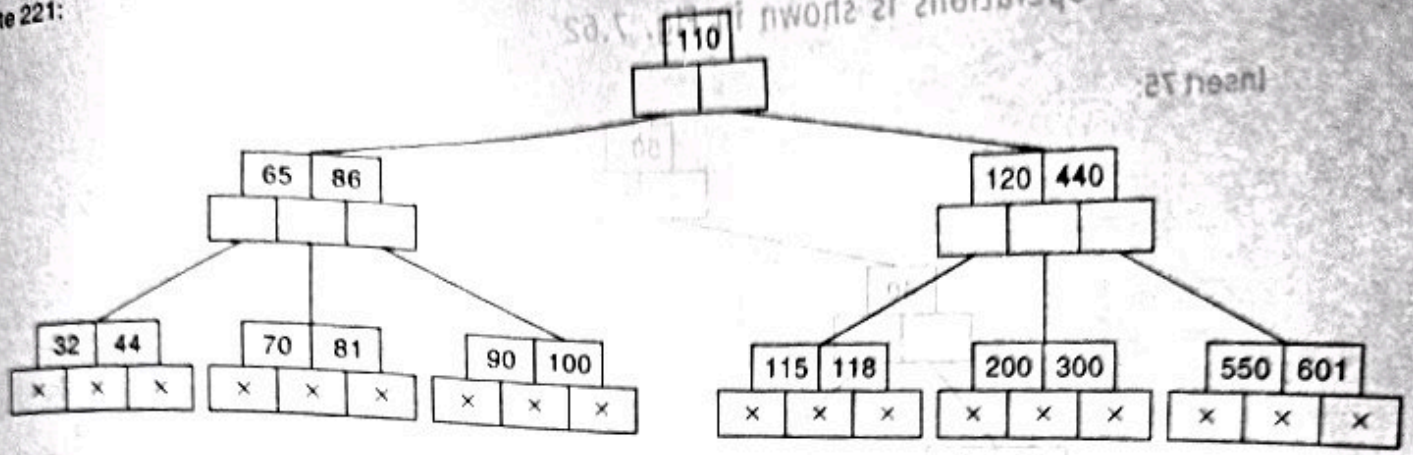
B-tree of order 5:



Delete 95, 226:



Delete 221:



Delete 70:

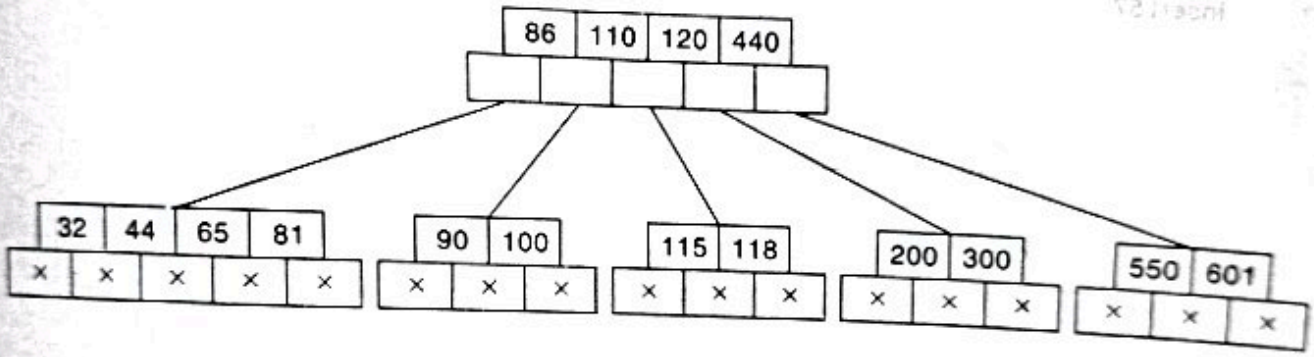


Fig. 7.60

Example 7.33

On the B-tree of order 3 shown in Fig. 7.61, perform the following operations in the order of their appearance:

Insert 75, 57 Delete 35, 65

B-tree of order 3:

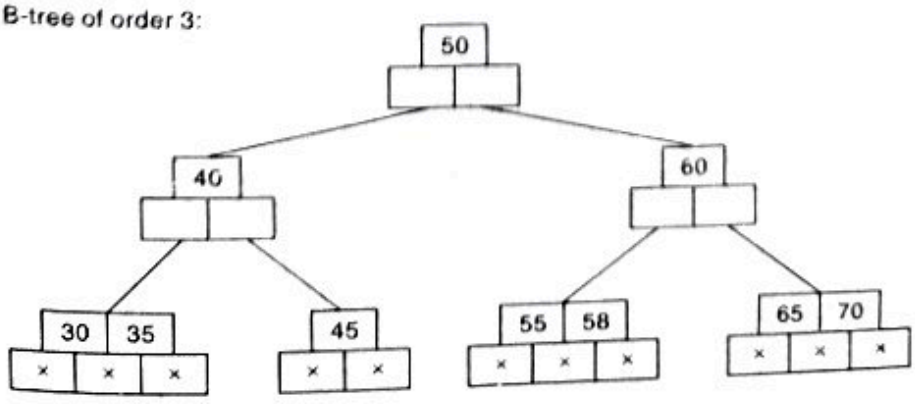
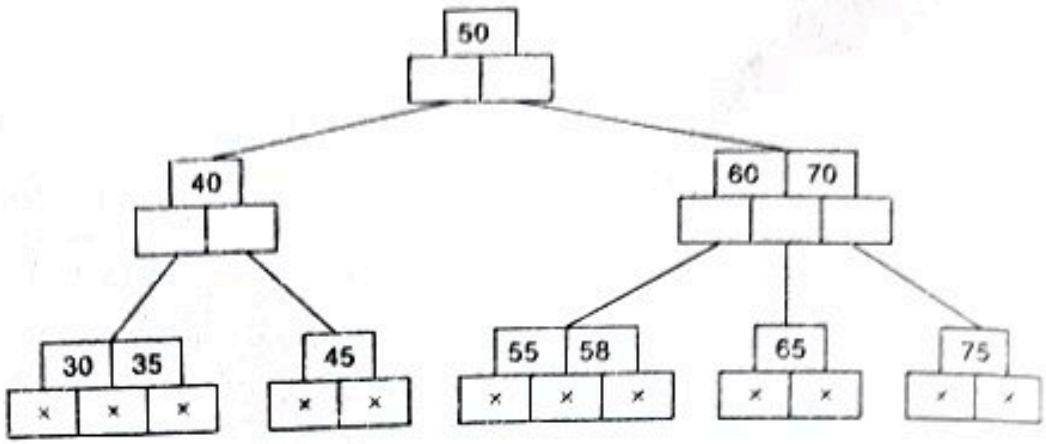


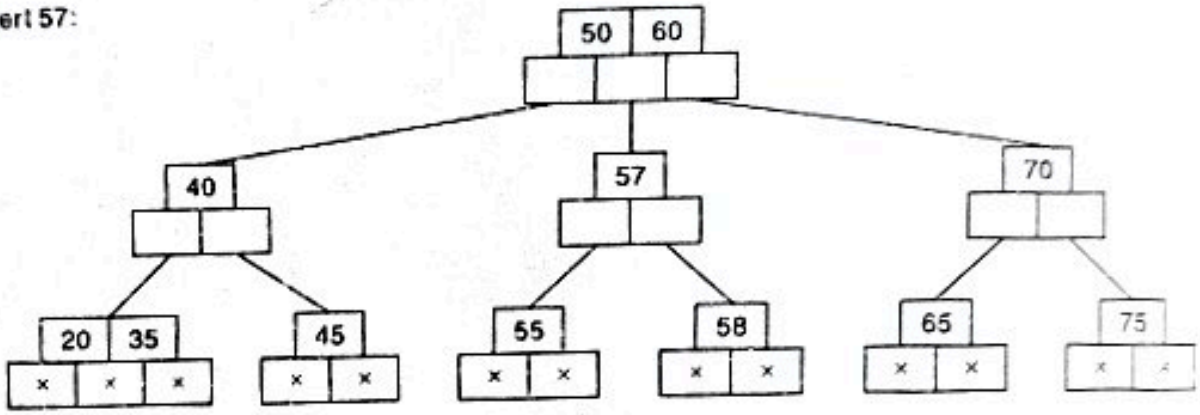
Fig. 7.61

The B-tree after the operations is shown in Fig. 7.62

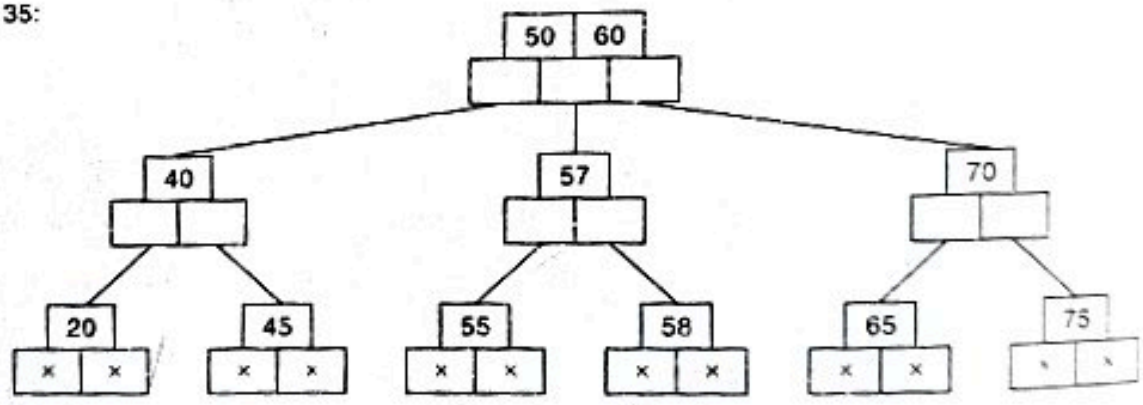
Insert 75:



Insert 57:



Delete 35:



Delete 65:

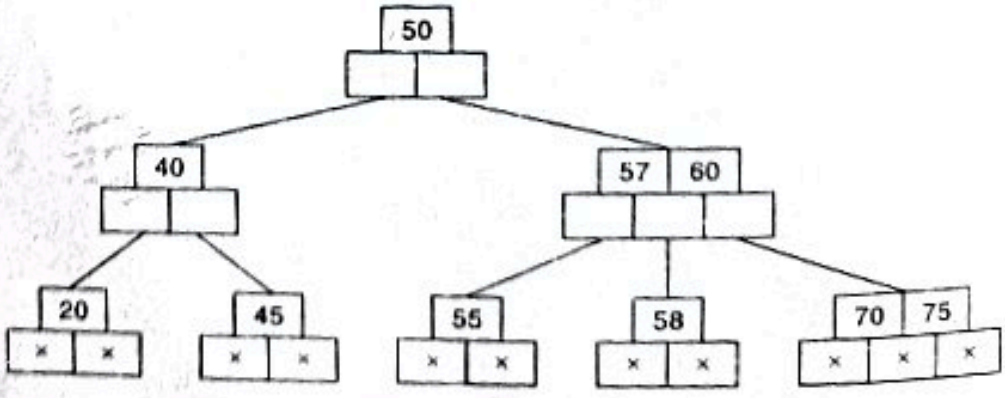


Fig. 7.62