

Question 1:

$A = \{ 'a', 'd', 'c', 'b' \}$ $B = \{ 'v', 'z', 'w', 'x', 'y' \}$ $f = \{ ('a', 'z'), ('b', 'y'), ('d', 'w'), ('c', 'x') \}$

Is a function

Is injective

Not surjective

Not bijective therefore no inverse

 $A = \{ 'a', 'd', 'c', 'b' \}$ $B = \{ 'z', 'x', 'y' \}$ $f = \{ ('a', 'z'), ('b', 'y'), ('c', 'x'), ('d', 'z') \}$

Is a function

Not injective

Is surjective

Not bijective therefore no inverse

 $A = \{ 'a', 'd', 'c', 'b' \}$ $B = \{ 'w', 'z', 'x', 'y' \}$ $f = \{ ('a', 'z'), ('b', 'y'), ('d', 'w'), ('c', 'x') \}$

Is a function

Is injective

Is surjective

Is bijective Inverse of f is $\{ ('x', 'c'), ('w', 'd'), ('z', 'a'), ('y', 'b') \}$

 $A = \{ 'a', 'd', 'c', 'b' \}$ $B = \{ 1, 2, 3, 4, 5 \}$ $f = \{ ('c', 1), ('a', 4), ('b', 5), ('d', 3) \}$

Is a function

Is injective

Not surjective

Not bijective therefore no inverse

 $A = \{ 'a', 'c', 'b' \}$ $B = \{ 1, 2, 3, 4 \}$ $f = \{ ('a', 3), ('b', 4), ('c', 1) \}$

Is a function

Is injective

Not surjective

Not bijective therefore no inverse

 $A = \{ 'a', 'd', 'c', 'b' \}$ $B = \{ 1, 2, 3 \}$ $f = \{ ('c', 3), ('b', 1), ('d', 2), ('a', 2) \}$

Is a function

Not injective

Is surjective

Not bijective therefore no inverse

 $A = \{ 'a', 'd', 'c', 'b' \}$ $B = \{ 1, 2, 3, 4 \}$ $f = \{ ('a', 4), ('c', 3), ('b', 1), ('d', 2) \}$

Is a function

Is injective

Is surjective

Is bijective Inverse of f is $\{ (1, 'b'), (2, 'd'), (3, 'c'), (4, 'a') \}$

 $A = \{ 'a', 'd', 'c', 'b' \}$ $B = \{ 1, 2, 3, 4 \}$ $f = \{ ('a', 2), ('c', 2), ('b', 1), ('d', 3) \}$

Is a function

Not injective

Not surjective

Not bijective therefore no inverse

 $A = \{ 'a', 'c', 'b' \}$ $B = \{ 1, 2, 3, 4 \}$ $f = \{ ('a', 4), ('c', 3), ('b', 1), ('a', 2) \}$

Not a function therefore not injective, surjective or bijective

Question 2:

Steps for gcd(414,662)

$$662/414 = 1 \text{ R } 248$$

$$414/248 = 1 \text{ R } 166$$

$$248/166 = 1 \text{ R } 82$$

$$166/82 = 2 \text{ R } 2$$

$$82/2 = 41 \text{ R } 0$$

$$\text{GCD}(414, 662) = 2$$

Steps for gcd(6,14)

$$14/6 = 2 \text{ R } 2$$

$$6/2 = 3 \text{ R } 0$$

$$\text{GCD}(6, 14) = 2$$

Steps for gcd(24,36)

$$36/24 = 1 \text{ R } 12$$

$$24/12 = 2 \text{ R } 0$$

$$\text{GCD}(24, 36) = 12$$

Steps for gcd(12,42)

$$42/12 = 3 \text{ R } 6$$

$$12/6 = 2 \text{ R } 0$$

$$\text{GCD}(12, 42) = 6$$

Steps for gcd(252,198)

$$252/198 = 1 \text{ R } 54$$

$$198/54 = 3 \text{ R } 36$$

$$54/36 = 1 \text{ R } 18$$

$$36/18 = 2 \text{ R } 0$$

$$\text{GCD}(252, 198) = 18$$

Question 3:

3a:

Product Sum format:

$$662 = 414*1 + 248$$

$$414 = 248*1 + 166$$

$$248 = 166*1 + 82$$

$$166 = 82*2 + 2$$

$$82 = 2*41 + 0$$

Euclid backwards format:

$$2 = 1 * 166 - 2 * 82$$

$$2 = 1 * 166 - 2 * (248 - 1*166)$$

$$2 = 3 * 166 - 2 * 248$$

$$2 = 3 * (414 - 1*248) - 2 * 248$$

$$2 = 3 * 414 - 5 * 248$$

$$2 = 3 * 414 - 5 * (662 - 1*414)$$

$$2 = 8 * 414 - 5 * 662$$

$$\text{GCD}(662, 414) = 8 * 414 - 5 * 662$$

3b:

Product Sum format:

$$14 = 6*2 + 2$$

$$6 = 2*3 + 0$$

Euclid backwards format:

$$2 = 1 * 14 - 2 * 6$$

$$\text{GCD}(14, 6) = 1 * 14 - 2 * 6$$

3c:

Product Sum format:

$$36 = 24*1 + 12$$

$$24 = 12*2 + 0$$

Euclid backwards format:

$$12 = 1 * 36 - 1 * 24$$

$$\text{GCD}(36, 24) = 1 * 36 - 1 * 24$$

3d:

Product Sum format:

$$42 = 12*3 + 6$$

$$12 = 6*2 + 0$$

Euclid backwards format:

$$6 = 1 * 42 - 3 * 12$$

$$\text{GCD}(42, 12) = 1 * 42 - 3 * 12$$

3e:

Product Sum format:

$$252 = 198*1 + 54$$

$$198 = 54*3 + 36$$

$$54 = 36*1 + 18$$

$$36 = 18*2 + 0$$

Euclid backwards format:

$$18 = 1 * 54 - 1 * 36$$

$$18 = 1 * 54 - 1 * (198-3*54)$$

$$18 = 4 * 54 - 1 * 198$$

$$18 = 4 * (252-1*198) - 1 * 198$$

$$18 = 4 * 252 - 5 * 198$$

$$\text{GCD}(252, 198) = 4 * 252 - 5 * 198$$

Question 4:

4a:

$q1 = 1$	$q2 = 1$	$q3 = 1$	$q4 = 2$	$q5 = 41$
$s0 = 1$	$s1 = 0$	$s2 = s0 - s1*q1 = 1 - 0*1 = 1$		$s3 = s1 -$
$s2*q2 = 0 - 1*1 = -1$		$s4 = s2 - s3*q3 = 1 - -1*1 = 2$		$s5 = s3 -$
$s4*q4 = -1 - 2*2 = -5$				
$t0 = 0$	$t1 = 1$	$t2 = t0 - t1*q1 = 0 - 1*1 = -1$		$t3 =$
$t1 - t2*q2 = 1 - -1*1 = 2$		$t4 = t2 - t3*q3 = -1 - 2*1 = -3$		$t5 =$
$t3 - t4*q4 = 2 - -3*2 = 8$				
$2 = -5*662 + 8*414$				
$\text{GCD}(662, 414) = -5*662 + 8*414$				

4b:

$$\begin{array}{ll}
q_1 = 2 & q_2 = 3 \\
s_0 = 1 & s_1 = 0 \\
t_0 = 0 & t_1 = 1 \\
2 = 1 \cdot 14 + -2 \cdot 6 \\
\text{GCD}(14, 6) = 1 \cdot 14 + -2 \cdot 6
\end{array}
\qquad
\begin{array}{l}
s_2 = s_0 - s_1 \cdot q_1 = 1 - 0 \cdot 2 = 1 \\
t_2 = t_0 - t_1 \cdot q_1 = 0 - 1 \cdot 2 = -2
\end{array}$$

$$\begin{array}{ll}
q_1 = 1 & q_2 = 2 \\
s_0 = 1 & s_1 = 0 \\
t_0 = 0 & t_1 = 1 \\
12 = 1 \cdot 36 + -1 \cdot 24 \\
\text{GCD}(36, 24) = 1 \cdot 36 + -1 \cdot 24
\end{array}
\qquad
\begin{array}{l}
s_2 = s_0 - s_1 \cdot q_1 = 1 - 0 \cdot 1 = 1 \\
t_2 = t_0 - t_1 \cdot q_1 = 0 - 1 \cdot 1 = -1
\end{array}$$

$$\begin{array}{ll}
q_1 = 3 & q_2 = 2 \\
s_0 = 1 & s_1 = 0 \\
t_0 = 0 & t_1 = 1 \\
6 = 1 \cdot 42 + -3 \cdot 12 \\
\text{GCD}(42, 12) = 1 \cdot 42 + -3 \cdot 12
\end{array}
\qquad
\begin{array}{l}
s_2 = s_0 - s_1 \cdot q_1 = 1 - 0 \cdot 3 = 1 \\
t_2 = t_0 - t_1 \cdot q_1 = 0 - 1 \cdot 3 = -3
\end{array}$$

$$\begin{array}{llll}
q_1 = 1 & q_2 = 3 & q_3 = 1 & q_4 = 2 \\
s_0 = 1 & s_1 = 0 & s_2 = s_0 - s_1 \cdot q_1 = 1 - 0 \cdot 1 = 1 & s_3 = s_1 - \\
s_2 \cdot q_2 = 0 - 1 \cdot 3 = -3 & & s_4 = s_2 - s_3 \cdot q_3 = 1 - -3 \cdot 1 = 4 & \\
t_0 = 0 & t_1 = 1 & t_2 = t_0 - t_1 \cdot q_1 = 0 - 1 \cdot 1 = -1 & t_3 = \\
t_1 - t_2 \cdot q_2 = 1 - -1 \cdot 3 = 4 & & t_4 = t_2 - t_3 \cdot q_3 = -1 - 4 \cdot 1 = -5 & \\
18 = 4 \cdot 252 + -5 \cdot 198 & & & \\
\text{GCD}(252, 198) = 4 \cdot 252 + -5 \cdot 198 & & &
\end{array}$$