hw5

0. Straight from class notes

$$\frac{}{\langle v_1, v_2\rangle.1 \;\rightarrow\; v_1} \quad \text{(PairLeft)}$$

$$\frac{}{\langle v_1, v_2\rangle.2 \;\rightarrow\; v_2} \quad \text{(PairRight)}$$

$$\frac{t_1 \rightarrow t_1'}{\langle t_1, t_2\rangle \;\rightarrow\; \langle t_1', t_2\rangle} \quad \text{(EPairL)}$$

$$\frac{t_2 \rightarrow t_2'}{\langle v_1, t_2\rangle \;\rightarrow\; \langle v_1, t_2'\rangle} \quad \text{(EPairR)}$$

$$\frac{t \;\rightarrow\; t'}{t.1 \;\rightarrow\; t'.1} \quad \text{(EProjL)}$$

$$\frac{t \;\rightarrow\; t'}{t.2 \;\rightarrow\; t'.2} \quad \text{(EProjR)}$$

Type rules

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash \langle t_1, t_2\rangle : T_1 \times T_2} \quad \text{(TPair)}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash t.1 : T_1} \quad \text{(TProj1)}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash t.2 : T_2} \quad \text{(TProj2)}$$

1.

$\{t_1, t_2\} : T_1 \times T_2 \equiv \lambda c.\, c\, t_1\, t_2$
$\{t_1, t_2\}.1 : T_1 \equiv \lambda c.\, c\, t_1\, t_2\; true$
$\{t_1, t_2\}.2 : T_1 \equiv \lambda c.\, c\, t_1\, t_2\; false$

2. Proof by induction:
   Case 1: Pair
   Also assume that l and r are values corresponding to t1 and t2, .1/.2 is instead True or False respectively in accordance with the inference rules. Then
   PairLeft
   $\{v_1, v_2\} : T_1 \times T_2.1 \equiv$
   $\lambda c.\, c\; l\; r\; True \;\Rightarrow\; \text{True}\; l\; r \Rightarrow \lambda x.\, \lambda y.\, x\; l\; r \Rightarrow l$
   Since $l \equiv t_1$ (stated in assumptions), $l \equiv t_1$ so PairLeft holds

PairRight

$\{v_1, v_2\} : T_1 \times T_2.2 \equiv$

$\lambda c.\, c\; l\; r\; False \;\Rightarrow\; \text{False}\; l\; r \Rightarrow \lambda x.\, \lambda y.\, y\; l\; r \Rightarrow r$

Since $r \equiv t_2$ (stated in assumptions), $r \equiv t_2$ so PairRight holds

Case 2: EPair

Assumptions: $t_1 \to t_1'$ (given, same for t2) and $l = t_1$ and $r = t_2$

EPairL

$\langle t_1, t_2 \rangle \equiv$

$\lambda c.\, c\; l\; r = \lambda c.\, c\; t_1\; r \Rightarrow \lambda c.\, c\; t_1'\; r$ (by inductive hypothesis since we assume the rest is correct)

$\equiv \langle t_1', t_2 \rangle$

EPairR

For this one assume $l = v_1$ so l can't be evald further

$\langle v_1, t_2 \rangle \equiv$

$\lambda c.\, c\; l\; r = \lambda c.\, c\; l\; t_2 \Rightarrow \lambda c.\, c\; l\; t_2'$ (by inductive hypothesis since we assume the rest is correct)

$\equiv \langle v_1, t_2' \rangle$

This satisfies EPair

Case 3: EProj

Assumptions: $t \to t'$ (given)

For neatness sake, I will remove typing. Let $t : T$

EProjL

$t.1 \equiv$

$\lambda t.\, t\; True = \lambda t.\, t'\; True$ (by inductive hypothesis since we assume the rest is correct)

$\equiv t'.1$

EProjR

$t.2 \equiv$

$\lambda t.\, t\; False = \lambda t.\, t'\; False$ (by inductive hypothesis since we assume the rest is correct)

$\equiv t'.2$

This satisfies EProj