## hw5

## Straight from class notes

$$egin{align} rac{}{\langle v_1,v_2
angle.1 \,
ightarrow\,v_1} & ext{(PairLeft)} \ \hline \ rac{\langle v_1,v_2
angle.2 \,
ightarrow\,v_2}{\langle v_1,v_2
angle\, 
ightarrow\, \langle t_1',t_2
angle} & ext{(EPairL)} \ \hline \ rac{t_2 
ightarrow t_2'}{\langle v_1,t_2
angle \,
ightarrow\, \langle v_1,t_2'
angle} & ext{(EPairR)} \ \hline \ rac{t 
ightarrow\, t'}{t.1 \,
ightarrow\, t'.1} & ext{(EProjL)} \ \hline \ rac{t 
ightarrow\, t'}{t.2 \,
ightarrow\, t'.2} & ext{(EProjR)} \ \hline \ \end{array}$$

Type rules

$$egin{aligned} rac{\Gamma dash t_1: T_1 \quad \Gamma dash t_2: T_2}{\Gamma dash \langle t_1, t_2 
angle: T_1 imes T_2} \quad ext{(TPair)} \ & rac{\Gamma dash t: T_1 imes T_2}{\Gamma dash t. 1: T_1} \quad ext{(TProj1)} \ & rac{\Gamma dash t: T_1 imes T_2}{\Gamma dash t: T_2 imes T_2} \quad ext{(TProj2)} \end{aligned}$$

1.

$$egin{aligned} \{t_1,t_2\}: T_1 imes T_2 \equiv \lambda c.\, c\,\, t_1\,\, t_2 \ \{t_1,t_2\}.1: T_1 \equiv \lambda c.\, c\,\, t_1\,\, t_2\,\, true \ \{t_1,t_2\}.2: T_1 \equiv \lambda c.\, c\,\, t_1\,\, t_2\,\, false \end{aligned}$$

## 2. Proof by induction:

Case 1: Pair

Also assume that l and r are values corresponding to t1 and t2, .1/.2 is instead True or False respectively in accordance with the inference rules. Then

PairLeft

$$\{v_1,v_2\}: T_1 imes T_2.1 \equiv \ \lambda c.\ c\ l\ r\ True \ \Rightarrow \ {
m True}\ l\ r \Rightarrow \lambda x.\ \lambda y.\ x\ l\ r \Rightarrow l$$
 Since  $l \equiv t_1$  (stated in assumptions),  $l \equiv t_1$  so PairLeft holds

**PairRight** 

 $\{v_1, v_2\} : T_1 \times T_2.2 \equiv$ 

 $\lambda c. c \ l \ r \ False \ \Rightarrow \ False \ l \ r \Rightarrow \lambda x. \lambda y. \ y \ l \ r \Rightarrow r$ 

Since  $r\equiv t_2$  (stated in assumptions),  $r\equiv t_2$  so PairRight holds

Case 2: EPair

Assumptions:  $t_1 o t_1'$  (given, same for t2) and  $l=t_1$  and  $r=t_2$ 

**EPairL** 

 $\langle t_1,t_2 
angle \equiv$ 

 $\lambda c.~c~l~r = \lambda c.~c~t_1~r \Rightarrow \lambda c.~c~t_1'~r$  (by inductive hypothesis since we assume the rest is correct)  $\equiv \langle t_1', t_2 \rangle$ 

**EPairR** 

For this one assume  $l=v_1$  so I can't be evald further

 $\langle v_1,t_2 
angle \equiv$ 

 $\lambda c.~c~l~r=\lambda c.~c~l~t_2\Rightarrow \lambda c.~c~l~t_2'$  (by inductive hypothesis since we assume the rest is correct)  $\equiv \langle v_1,t_2' \rangle$ 

This satisfies EPair

Case 3: EProj

Assumptions:  $t \rightarrow t'$  (given)

For neatness sake, I will remove typing. Let t:T

EProjL

 $t.1 \equiv$ 

 $\lambda t.\,t\,True = \lambda t.\,t'\,True$  (by inductive hypothesis since we assume the rest is correct)

 $\equiv t'.1$ 

**EProjR** 

 $t.2 \equiv$ 

 $\lambda t.\,t\,False = \lambda t.\,t'\,False$  (by inductive hypothesis since we assume the rest is correct)  $\equiv t'.2$ 

This satisfies EProj