

hw4

Original language:

$$\begin{aligned} t &::= \lambda x : T. t \mid (t \ t) \mid x \\ v &::= \lambda x : T. t \\ T &::= T \rightarrow T \end{aligned}$$

1. New language:

$$\begin{aligned} t &::= \lambda x : T. t \mid (t \ t) \mid x \mid \text{True} \mid \text{False} \mid \text{if } t_1 \ t_2 \ t_3 \mid \text{AND } t_1 \ t_2 \\ v &::= \lambda x : T. t \mid \text{True} \mid \text{False} \\ T &::= T \rightarrow T \mid \text{Bool} \end{aligned}$$

New rules:

This one just says True and False are of type Booleans

$$\frac{}{\Gamma \vdash \text{True} : \text{Bool}} \quad \frac{}{\Gamma \vdash \text{False} : \text{Bool}}$$

This is basic typing

$$\frac{(x \mapsto \text{Bool}) \in \Gamma}{\Gamma \vdash x : T} T_{\text{Bool}}$$

These are for evaluating (IFApp 1, 2, 3)

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \ t_2 \ t_3} \quad \frac{t_2 \rightarrow t'_2}{\text{if } v_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } v_1 \ t'_2 \ t_3}$$

$$\frac{}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } v_1 \ t_2 \ t_3} \quad \frac{}{\text{if } v_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } v_1 \ t'_2 \ t_3}$$

The IF true and IF false are trivial, not going to list them. This one is for actually type checking

$$\frac{}{\Gamma \vdash \text{if } v_1 \text{ then } t_2 \text{ else } t_3 : T} \quad \frac{}{\Gamma \vdash \text{if } v_1 \text{ then } t_2 \text{ else } t_3 : T} \text{ TIf}$$

Not going to write down the eval rules for AND, pretend they exist (very similar) with the name ANDApp1, 2

$$\Gamma \vdash \text{AND } t_1 \ t_2 : \text{Bool}$$

Assume progress for \$t_1\$ and \$t_2\$. If \$t_1\$ or \$t_2\$ are not values then ANDApp1 and 2 will apply (respectively).

$$\frac{}{\text{Let } x = v_1 \text{ in } t_2 \rightarrow [x \mapsto v_1] t_2} \text{ ELetV}$$

$$\frac{}{\text{Let } x = v_1 \text{ in } t_2 \rightarrow [x \mapsto v_1] t_2}$$

$$\frac{t_1 \rightarrow t_1'}{\text{Let } x = t_1 \text{ in } t_2 \rightarrow \text{Let } x = t_1' \text{ in } t_2} ; \text{ELet}$$

Typerule

$$\frac{\Gamma \vdash t_1 : T_1 \quad (x \mapsto T_1), \Gamma \vdash t_2 : T_2}{\Gamma \vdash \text{Let } x = t_1 \text{ in } t_2 : T_2} ; \text{TLet}$$

4. *Proof Progress* : $\$ \Gamma \vdash \text{Let } x = t_1 \text{ in } t_2 : T_2 \$ Assume progress for t_1 and t_2 t_1 not a value then $ELet$ applies to $t_1$$