hw3

1. $$\lambda f.\,(\lambda x.\,f\ (x\ x))(\lambda x.\,f\ (x\ x))\ (\lambda f.\,\lambda x.\,\text{if (isnil x) then 0 else (head x) + (f tail x))}$$

2. Prove |FV(t)| $\leq$ size(t)
   Note: all of the proofs will assume $\forall t,\ size(t) \geq 1$ which is obvious
   Case 1: t = value ie $t \in \{true, false, 0\}$
   $|FV(t)| = 1$ and $size(t) = 1$ therefore holds
   Case 2: isZero(t), succ(t), pred(t)
   $\frac{t \in Nat}{\text{isZero : Nat ; succ(t) : Nat ; pred(t) : Nat}}$ so then $|FV(isZero(t))|\ =\ |FV(Nat)| = 1$, same proof applies to succ and pred so we statement holds
   Case 3: if $t_1\ t_2\ t_3$
   Let $t_1 \to^* t_1'$ and $t_2 \to^* t_2'$, by definition of if, the if statement must evaluate to one of those 2. By the induction hypothesis $|FV(t_1')| = 1$ and $|FV(t_2')| = 1$ so
   $|FV(\text{if } t_1\ t_2\ t_3)| = 1 \leq size(\text{if } t_1\ t_2\ t_3)$
   Thus the statement holds
   Another proof would be all terms evaluate to 1 value therefore $\forall t, |FV(t)| = 1$ so the statement must be true (proof by logic)

I just found out that this should be for lambda calc so finish this later
Prove |FV(t)| $\leq$ size(t)
Case 1: t = x. This case is obvious, the |FV(t)| = 1 = size(t)
Case 2: $t = \lambda x.\,t_1$. |FV(t)| = 1 = size(t) since functions are values
Case 3: $t = (t_1\ t_2)$ $|FV(t)| = |FV(t_1) \cup FV(t_2)| \leq |FV(t_1)| + |FV(t_2)|$ which by induction hypothesis $\leq size(t_1) + size(t_2) = size(t)$
So for all cases, the statement holds
3.
Lazy eval: Delete the first/second and replace the 3rd with the following: $(\lambda x.\,t_1)t_2 \to [x \to t_2]t_1$
Beta reduction: replace the first two terms with: $\frac{t_1 \to v_1 \quad t_2 \to v_2}{\lambda t_1 t_2 \to v_1 v_2}$
4.

$$\frac{t_1 \to wrong}{t_1\ t_2 \to wrong} \quad \frac{t_2 \to wrong}{t_1\ t_2 \to wrong} \quad \frac{v_1 \to wrong}{(\lambda.\,t_{12})v_1 \to wrong}$$