

# ECE110: Electrical Fundamentals (Lecture Notes)

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# 1 Electricity

## 1.1 Lecture 01

What is this course about?

- Fundamental physics of electrical engineering
- Basic electronic circuits
- Electronics are everywhere
- Textbook is a custom textbook on Wiley, made for this course

### Electric Charges

- Come in positive and negative – opposites attract
- Static charges – stationary electric charges
- Crushing wintergreen lifesavers
- Charges move through friction – excess charges

## 1.2 Lecture 02

### Electric charges recap

- Negative charges transferred from rod to silk
- Negative charges transferred from fur to rod
- **We only deal with ideal conductors and insulators in this course**

### Conductors & Insulators

- Both conductors and insulators are able to carry excess charge

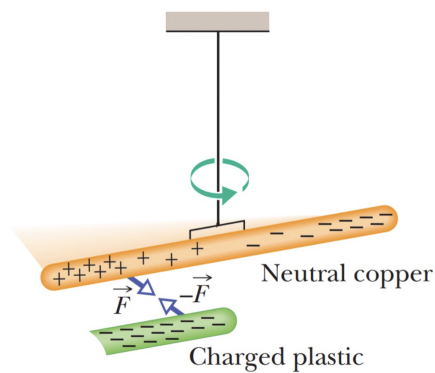


Figure 1: Induced Charge

## Coulomb's Law

- Force between two electrically charged particles is called the electrostatic force
  - Quantified by Charles-Augustine de Coulomb in 1785
- SI unit of charge is the coulomb (C)
- We do not consider shape when discussing particles

$$\vec{F} = \frac{kq_1q_2}{r^2}\hat{r} \quad (1)$$

## Electrostatic Constant

- $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 Nm^2/C^2$
- $\epsilon_0$  is the permittivity constant of free space:  $8.85 \times 10^{-12} C^2/Nm^2$
- $F_{AB}$  means force exerted on A by B
- Force will be radially towards or exactly away from other particle

## Multiple Forces

- If multiple forces are exerted on one particle,  $F_{NET}$  is the vector sum of all forces on the said particle
- If  $\sum F = 0$ , then we have equilibrium

## Shell Theories – proved later by Gauss' Law

- Theory 1: charged particle outside a shell with charge uniformly distributed on its surface is attracted/repelled as if shell's charge is concentrated at the centre of the shell
  - Assumption: presence of particle has negligible effect on distribution of charge of shell
- Theory 2: charged particle inside a shell with a charge distributed evenly on its surface has no net force due to the shell
  - Assumption: presence of particle has negligible effect on distribution of charge of shell

## Charge is Quantized

- Electric charges come in set quantities – proved by the Millikan Oil Drop Experiment
- Any  $q = ne$ , where  $e = 1.602 \times 10^{-19}$
- Net electric charge of any isolated system is always conserved

## 1.3 Lecture 03

### Electric Fields

- Region in which an electric charge experiences an electric force
  - Coined by Micheal Faraday to describe a force at a distance (without physical contact)
- Electric fields always compared to a **small, positive test charge**.

$$\vec{E} = \frac{\vec{F}}{q_0} \quad (2)$$

## Electric Field Lines

- ALWAYS drawn positive to negative
- When there's two particles, the field line drawn at any point is the tangent of the net field at that point

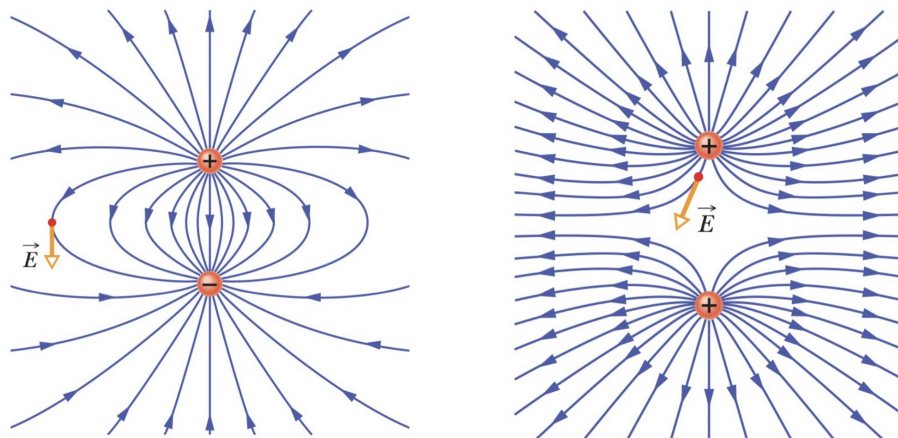


Figure 2: Electric Field Lines

## Electric Field Due to a Charged Particle

- Also called a point charge

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (3)$$

- We can take the vector sum of a group of fields to find a net electric field
- If particle with charge  $q$  placed in a field  $\vec{E}$ , then force experienced is  $q\vec{E}$

## Millikan's Oil Drop Experiment

- Allowed Millikan to find charge to mass ratio
- Charged oil droplets were placed into a charged field and the voltage was varied until the droplets were floating w/ no net acceleration
- Data analysis showed that charge is quantized
  - Jumped by  $1.902 \times 10^{-19}$  gave rise to the elementary charge

## 1.4 Lecture 04

- Gauss' Law – surface integral of electric field, AKA electric flux of  $\vec{E}$  over a closed surface  $s$ 
  - Equals total charge (w/ sign) enclosed in  $s$ , divided by  $\epsilon_0$
  - Assumes that charges can reside in a vacuum.

## Flux of Electric Field

- Orientation of surface to field matters
- $\phi = \int E \cdot dA$
- One way to account for surface orientation is to define area of a vector
  - Area vector is vector drawn orthogonal to the surface
- How to evaluate a double integral

$$\int_{y=c}^{y=d} \int_{x=a}^{x=b} f(x)g(y) \cdot dx \cdot dy = \int_{y=c}^{y=d} g(y) \cdot dy \cdot \int_{x=a}^{x=b} f(x) \cdot dx = G(y) \Big|_c^d \cdot F(x) \Big|_a^b \quad (4)$$

## Charge Densities

- In many cases the number of charges is so large, we should consider it as a continuous, not discrete distribution of charge
  - Line charge density  $\lambda \rightarrow C/m$
  - Surface charge density  $\sigma \rightarrow C/m^2$
  - Volume charge density  $\delta \rightarrow C/m^3$

## 1.5 Lecture 05

### Electric Flux

- Amount of  $\vec{E}$  piercing a small square patch w/ area  $\Delta A$  defined to be  $\Delta\phi$

$$\phi = \sum E \cdot \Delta A \quad (5)$$

- Special case of Gauss' Law: consider a particle w/ charge  $+q$  is surrounded by an imaginary concentric sphere

$$\phi = \oint E \cdot dA = E \oint dA = \frac{q}{\epsilon_0} \quad (6)$$

- Gauss' Law relates the net flux of an electric field through a closed surface and net charge enclosed by the surface:

$$\epsilon_0 \phi = q_{enc} \quad (7)$$

- Gaussian surface must be a closed surface
- A charge outside of a Gaussian surface contributed zero net flux through the system
- Thus, Gauss' Law can also be written in terms of the electric field piercing the enclosing Gaussian surface

$$\epsilon_0 \oint E \cdot dA = q_{enc} \quad (8)$$

## 1.6 Lecture 06

### Electric Potential Energy

- Defined as  $U = -W_\infty$ , where  $W_\infty$  is the work that would be done by the electric force on bringing the object from an infinite distance
- Electric potential defined as:

$$V = \frac{W_\infty}{q_0} = \frac{U}{q_0} \quad (9)$$

- If a particle with charge  $q$  is placed at a point where the electric potential of a charged object is  $V$ , the electric potential energy  $U$  is:  $U = qV$
- Particle moving from  $i$  to  $f$ , the electric potential change is  $\Delta V = V_f - V_i$
- Conservation of energy applies to any such closed system

### Equipotential Surfaces

- Adjacent points that have the same electric potential form an equipotential surface, which can be either an imaginary surface or a real physical force
  - Like a contour surface map
- A family of EP surfaces associated w/ the electric field due to some distribution of charges
  - There cannot be a component of  $\vec{E}$  that is along the equipotential surface
- In a uniform field:

$$\Delta V = -E\Delta x \quad (10)$$

## 1.7 Lecture 07

- Positive  $q$  – positive PD and vice versa

### PD Due to a Group of Particles

•

$$V = \sum_{i=1}^n V_i = k \sum_{i=1}^n \frac{q_i}{r_i} \quad (11)$$

- For 2 particles:  $U = k \frac{q_1 q_2}{r}$
- Total  $E_P$  for a system of particles is the sum of the potential energies for every pair of particles in the system

### Electric Potential Energy of a Two Particle System

- EPE system is equal to work required to assemble system w/ particle initially at rest and infinitely distant from each other

$$U = qV \quad (12)$$

- Total PE of system is sum of potential energies for EVERY PAIR of particles in the system

## 1.8 Lecture 08

### Capacitors

- Two conductors, electrically isolated from each other, and from surroundings, forms a capacitor
  - Surface of conductor is an equipotential system in static conditions
  - The electric field  $\vec{E}$  just outside surface of a conductor is perpendicular to surface

### Capacitance

- When a capacitor is charged, charges of conductors have same magnitude but opposite charge
- If PD between two plates is  $V$ , charge  $q \propto V$  – this proportionality is called capacitance

$$q = CV \quad (13)$$

- SI unit of capacitance is called farad (F)

## Parallel Plate Capacitor

- PPC consists of two parallel conducting plates w/ area  $A$  separated by distance  $d$
- $\vec{E}$  is uniform in central region b/w the plates, but near the edges we observe “fringing.”

## Charging a Capacitor

- Device that maintains a certain PD between terminals
- We assume capacitors can retain a charge indefinitely
- Until given opportunity to discharge

## Calculating Capacitance

1. Assume a  $q$  on plates
2. Calculate  $\vec{E}$  between plates:

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q \quad (14)$$

3. Calculate PD:

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s} \quad (15)$$

4. Calculate  $C$  using  $q = CV$

## 1.9 Lecture 09

### Capacitance of a Parallel Plate Capacitor

- Area  $A$  and spacing of  $d$  –  $C = \frac{\epsilon_0 A}{d}$
- Capacitance of cylindrical capacitor –  $C = \frac{2\pi\epsilon_0 L}{\ln b/a}$
- Capacitance of a spherical capacitor –  $C = 4\pi\epsilon_0 \frac{ab}{b-a}$
- Capacitance of an isolated sphere –  $C = 4\pi\epsilon_0 R$

### Capacitors in Parallel

$q_1 = C_1 V$ ,  $q_2 = C_2 V$ ,  $q_3 = C_3 V$  – Total charge  $\rightarrow q_{tot} = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$

$$C_{eq} = \frac{q_{tot}}{V} = \frac{(C_1 + C_2 + C_3)V}{V} = C_1 + C_2 + C_3 \quad (16)$$

$$C_{eq} = \sum_{j=1}^n C_j \quad (17)$$

### Capacitors in Series

$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3} \quad (18)$$

$$V = V_1 + V_2 + V_3 = q \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \quad (19)$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^n \frac{1}{C_j} \quad (20)$$



### Potential Energy of a Charged Capacitor

$$U = \frac{1}{2}CV^2 \quad (21)$$

### Energy Density

- Energy density of a capacitor is the potential energy stored per unit volume

$$u = \frac{1}{2}\epsilon_0 E^2 \quad (22)$$

## 1.10 Lecture 10

### Capacitor with a Dielectric

- Must be insulating material with a constant  $\kappa$ 
  - $\epsilon = \kappa\epsilon_0$
  - $\kappa > 1$  for any dielectric material

### Electric Current

- Defined as  $\frac{dq}{dt}$
- $i = \int dq = \int_0^t dq$
- Conventional current is a positive charge movement
- $i_0 = i_1 + i_2$

### Current Density

- Magnitude of  $J$  is equal to current per unit of the cross section of a conductor
- Direction of  $J$  is same as velocity of the moving charge if they are positive
- If current is uniform across surface and parallel to  $dA$  then

$$i = \int D \cdot dA = J \int dA = JA \quad (23)$$

### Drift Spread of Charge Carriers

- When a conductor has a current through it, the conduction  $e^-$  move with a drift speed  $v_d$

$$\vec{J} = (ne)\vec{v}_d \quad (24)$$

## 1.11 Lecture 11

### Resistance

- Resistance  $R$  is given by  $R = \frac{V}{i}$ , unit is ohm  $\Omega$

### Resistivity

- Resistivity  $\rho$  of a material we define as  $\rho = \frac{E}{J}$ 
  - Unit is ohm meter  $\Omega \cdot m$
  - Vector equation is  $\vec{E} = \rho\vec{J}$

### Resistance of a Conducting Wire

- Resistance  $R$  of conducting wire of length  $L$  and uniform cross section  $A$  is:

$$R = \rho \frac{L}{A} \quad (25)$$

### Conductivity

- Conductivity  $\sigma$  of a material is defined as  $\sigma = \frac{1}{\rho}$
- $\vec{J} = \sigma \vec{E}$

### Change of $\rho$ with Temperature

- Relation between  $\rho$  and temp  $T$  is appointed by:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0) \quad (26)$$

### Ohm's Law

- Assertion that current through a device is always directly proportional to PD applied to the device

$$V = iR \quad (27)$$

### Power in Electric Current

- Power  $P$  is rate of energy transfer
- $P = iV$
- For a resistor w/ resistance  $R$  the electrical energy dissipation due to a resistance is:

$$P = i^2 R = \frac{V^2}{R} \quad (28)$$

## 2 Magnetism

### 2.1 Lecture 12

#### Magnetic Field $\vec{B}$

- Magnetic fields can produce magnetic force
- Who produces a magnetic field?
  - Magnetic charges – monopoles theorized to exist but not proven
  - Permanent magnet – intrinsic magnetic field around an object
  - Moving charged particles
- SI unit for  $\vec{B}$  is tesla (T) – Gauss (G)
- Array of DOTS represents a magnetic field coming OUT of plane
- Array of ARROWS represents a magnetic field going INTO the plane

#### Magnetic Force

- When charged particles move in a field, a magnetic field is given by

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad (29)$$

## Magnetic Field Lines

- Always go from North to South
- Direction of tangent line gives directions of field vector at that point
- Spacing of lines represents the magnitude of  $\vec{B}$

## Magnetic Dipole

- Field lines emerge from North pole and enter into South pole
- Magnets always come in dipoles – never just North or South

## 2.2 Lecture 13

### Magnetic Force on a Current Carrying Wire

- Wire exposed to a magnetic field will experience a force

$$\vec{F} = i\vec{L} \times \vec{B} \quad (30)$$

- If the wire is not straight:

$$d\vec{F}_b = i\vec{L} \times \vec{B} \quad (31)$$

### Magnetic Field Due to a Current

- Moving charged particles produce a magnetic field

$$dB = \frac{\mu_0}{4\pi} \frac{i \cdot ds \sin \theta}{r^2} \quad (32)$$

- Biot-Savart Law

### Magnetic Field due to a Long Straight Wire

- DIRECTION – use right hand rule for current in a wire
- Same points can be applied to analyze a wire of any shape and length

### Field Due to Current in a Circular Arc

$$B = \frac{\mu_0 i \phi}{4\pi R} \quad (33)$$

## 2.3 Lecture 14

### Force Between Two Parallel Currents

- Parallel currents attract and opposite currents repel

$$F = \frac{\mu_0 L i_a i_b}{2\pi d} \quad (34)$$

- Rail Guns!!!
- Ampere's Law of field around a wire

$$B = \frac{\mu_0 i}{2\pi R} \quad (35)$$

## Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \quad (36)$$

- This line integral represents the total amount current enclosed within an Amperian Loop

## Ampere's Law Inside of Long Straight Wire

$$B = \frac{\mu_0 i r}{2\pi R^2} \quad (37)$$

## Solenoids and Currents

$$B = \mu_0 i n \quad (38)$$

## 2.4 Lecture 15

### Ampere's Law

$$\oint \vec{B} d\vec{s} = \mu_0 i_{enc} \quad (39)$$

- Solenoid – magnetic version of capacitor –  $B = \mu_0 i n$

### Faraday's Law

#### First Experiment

- Conducting loop connected to ammeter – should read zero
- If we move a bar magnet toward to loop, it will read some current
  - If stopped will read zero again
  - Moved backwards, and will read negative current
- EMF – electromotive force, **not a real force**

$$\varepsilon = \frac{dW}{dq} \quad (40)$$

- Difference between emf and electric potential – emf is PD not produced by an electric charge
- Discoveries:
  - Current only when relative motion
  - Faster motion – more current

#### Second Experiment

- Second loop added attached to circuit
- When switch closed, a brief current will be produced in a second loop
  - If kept closed, no current is registered
  - If opened again, then brief current in opposite direction

### Key Takeaway

- An emf and a current can be induced in a loop by changing amount of magnetic field passing through the loop

– MAGNETIC FLUX!!!

- Magnetic Flux –  $\phi_B = \int \vec{B} \cdot d\vec{A}$ , unit is weber (Wb)
- Edge case – loop lies in plane and  $\vec{B}$  is perpendicular to plane

$$\phi_B = \vec{B} \cdot \vec{A} \quad (41)$$

### Faraday's Law

- Faraday's Law of Induction:

$$\varepsilon = -\frac{d\phi_B}{dt} \quad (42)$$

- If we change the loop to a coil of  $N$  turns:

$$\varepsilon = -N \frac{d\phi_B}{dt} \quad (43)$$

### Lenz's Law

- An induced current will have a direction such that  $\vec{B}$  produced by current opposed change in  $\phi_B$

## 2.5 Lecture 16

### Induction

- Rectangular loop of wire  $L$  has one end in a uniform external magnetic field directed perpendicularly

$$\vec{F}_B = i\vec{L} \times \vec{B} \quad (44)$$

- To find current, we apply Faraday's Law

$$\phi_B = BA \quad (45)$$

$$F = \frac{B^2 L^2 v}{R^2} \quad (46)$$

### Work by Induction

- $W = Fd$
- Power:

$$P = \frac{B^2 L^2 v^2}{R} \quad (47)$$

### Eddy Current Loop

- Same phenomenon occurs when we have a solid conducting plane

## 2.6 Lecture 17

### Inductors

- A device that is used to produce a known magnetic field in a specified region
- We typically use a solenoid

## Inductance

- $L = \frac{N\phi_B}{i}$  – unit is henry (H)
  - $N\phi_B$  – magnetic field linkage
  - Inductance is a measure of the flux linkage produced by inductor per unit current

## Inductance of a Solenoid

- Consider a solenoid of length  $l$  and area  $A$ 
  - $B = \mu_0 i n$
  - $L = \mu_0 n^2 l A$
  - $L \propto l, L \propto A, L \propto n^2$

## Self-Inductance

- An induced  $\varepsilon_L$  appears in any coil in which the current is changing

$$\varepsilon_L = -\frac{d(N\phi_B)}{dt} \quad (48)$$

- Self induced  $\varepsilon$  has an orientation such that it opposes the change in current  $i$ .

$$V_L = L \frac{di}{dt} \quad (49)$$

## Energy Stored in Magnetic Field

- If an inductor  $L$  carries current  $i$ , magnetic field is given by:

$$U_B = \frac{1}{2} L i^2 \quad (50)$$

## Inductors in Series

$$L_{eq} = \sum_{j=1}^n L_j \quad (51)$$

## Inductors in Parallel

$$\frac{1}{L_{eq}} = \sum_{j=1}^n \frac{1}{L_j} \quad (52)$$

## 2.7 Lecture 18

Magnetism review for midterm! I'll skip it as it's all already covered.

## 3 Basics of Circuit Analysis

### 3.1 Lecture 19

$$i(t) = \frac{dq(t)}{dt} \rightarrow q(t) = \int_{-\infty}^t i(x) \cdot dx \quad (53)$$

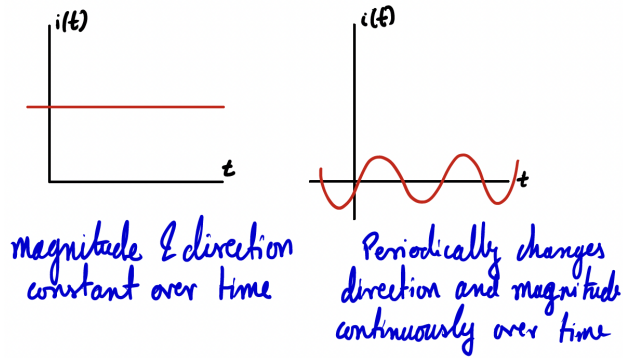


Figure 3: DC and AC Current

### Passive Sign Convention

- If product of current and voltage represents magnitude and sign of power dissipated
  - Positive is power absorbed, and
  - Negative is power supplied

### Independent Voltage Source

- An independent voltage source is a 2-terminal element that maintains a specified voltage regardless of the current going through it

### Independent Current Source

- Sources of current that maintains same current regardless of voltage applied

### Dependent Sources

- V/C determined by another point in the circuit
- Four types:
  - voltage controlled voltage source:  $\mu$ ,
  - current controlled voltage source:  $r$ ,
  - voltage controlled current source:  $g$ , and
  - current controlled current source:  $\beta$ .

### Tellegen's Theorem

- By the conservation of energy, the power supplied by an energy network equals the power absorbed by the network.

$$P_1 + P_2 + P_3 + \dots + P_n = 0 \text{ OR } \sum P_i = 0 \quad (54)$$

### Ohm's Law and Resistance

- Independent sources do NOT obey Ohm's Law
- For any Ohmic resistor:

$$V = iR \quad (55)$$

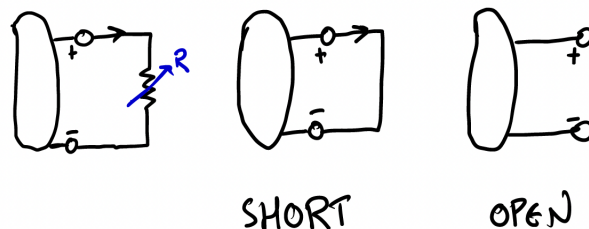


Figure 4: Open and Short Circuits

### Conductance

$$G = \frac{1}{R} \quad (56)$$

$$i = GV \quad (57)$$

### Power Using Ohm's Law

- General:

$$P = iV \quad (58)$$

- For Ohmic resistors:

$$P = Ri^2 = \frac{V^2}{R} \quad (59)$$

$$P = \frac{i^2}{G} = GV^2 \quad (60)$$

### Node, Loop, and Branches

- A node is simply a point in connection of 2/+ circuit elements
- A loop is any closed path through the circuit in which no node is encountered more than once
- A branch is a portion of a circuit containing only a simple element and nodes at each end of the element

## 3.2 Lecture 20

### Kirchoff's Current Law (KCL)

- Algebraic sum of all current entering a node is zero

$$\sum_{j=1}^N i_j(t) = 0 \quad (61)$$

### Kirchoff's Voltage Law (KVL)

- The algebraic sum of all the voltages around any loop is zero

$$\sum_{j=1}^N V_j(t) = 0 \quad (62)$$

### Voltage Division Rule

$$V_{P_1} = \frac{R_1}{R_1 + R_2} V(t) \text{ and } V_{P_2} = \frac{R_2}{R_1 + R_2} V(t) \quad (63)$$



## Current Division Rule

$$i_{P_1} = \frac{R_2}{R_1 + R_2} i(t) \text{ and } i_{P_2} = \frac{R_1}{R_1 + R_2} i(t) \quad (64)$$

## 3.3 Lecture 21

### Reference Node or Ground

- Node voltages are often defined with respect to a common point in the circuit
  - This is commonly called the ground
- Usually the node with the largest number of branches connected to it

$$i = \frac{V_m - V_N}{R} \quad (65)$$

### Nodal Analysis 1/4 – Circuits Containing Only Independent Current Sources

- If we have  $N$  nodes, we need to write  $N - 1$  KCL equations
- Write a KCL equation for every non-reference node
- Apply Ohm's Law

$$\begin{bmatrix} 1 & 1-2 & 1-3 \\ 1-2 & 2 & 2-3 \\ 1-3 & 2-3 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \text{Current terms (depend on equations)}$$

where the diagonal is the sum of the conductances connected to each node, and non-diagonal values are negatives of conductance between nodes.

### Nodal Analysis 2/4 – Circuits Containing Dependent Current Sources

- Same as above, except now we have an extra constraint equation that describes one current in terms of another.

### Nodal Analysis 3/4 – Circuits Containing Independent Voltage Sources

- If source is connected to reference node, then node voltage is already given
- If source is connected to two non-reference nodes, then need to use a supernode
  - Write a KCL equation for the full supernode, and
  - Write a constraint equation for the supernode itself

## 3.4 Lecture 22

### Nodal Analysis 4/4 – Circuits With Dependent Voltage Sources

- Same as the above, except with additional constraint equations for the voltage source

## 3.5 Lecture 23

### Loop Analysis

- Just as nodal analysis utilizes KCL equations and node voltages, loop analysis utilizes KVL equations and loop currents
- A mesh is a current loop that doesn't contain another loop within it
- There are  $B - N + 1$  linearly independent equations for any network

### Loop Analysis 1/3 – Circuits With Only Independent Voltage Sources

- Apply KVL to each mesh
- Apply Ohm's Law
- To determine the sign of the voltage source:
  - Assume positive and if it aids the assumed direction of the current's flow.

### Loop Analysis 2/3 – Circuits With Independent Current Sources

- Same thing as above except now we are given the loop current from the beginning

## 3.6 Lecture 24

### Loop Analysis 3/3 – Circuits Containing Dependent Sources

- Same thing as above, except may need supermesh equations, and will have additional constraint equations

### How to Know Which Type of Analysis to Use?

- Look at question and givens carefully, may also depend on what kind of value is ultimately needed as the final answer
- Count needed equations:
  - Nodal –  $N - 1$
  - Loop –  $B - N + 1$

### Equivalence

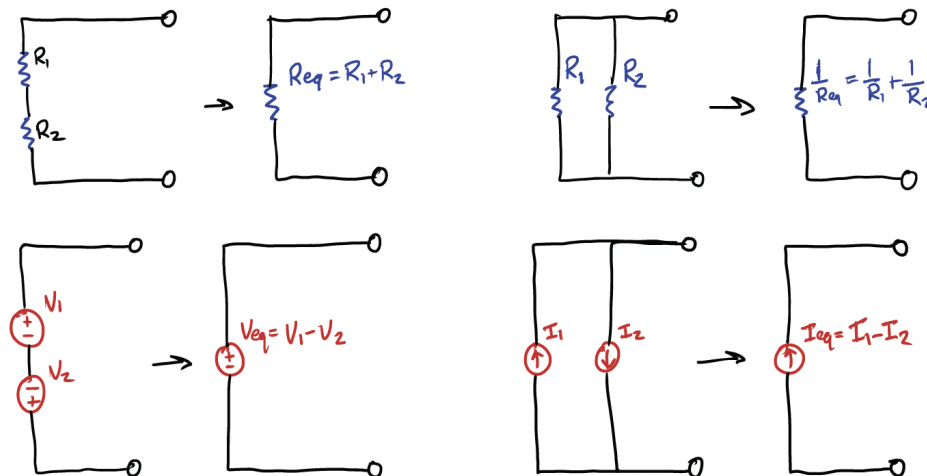


Figure 5: Equivalent Circuits

### Linearity

- A linear system satisfies additivity and homogeneity

## Superposition

- Current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone

## 3.7 Lecture 25

Lecture 25 was just examples covering previously discussed topics.

## 3.8 Lecture 26

### Thevenin's and Norton's Theorem

- Thevenin's -  $V_0 = V_{OC} - R_{Th}i$
- Norton's -  $i = -\frac{V_0}{R_{Th}} + i_{SC}$

### Circuits Containing Only Independent Sources

- Find the open circuit voltage  $V_{OC}$
- Find the Thevenin resistance  $R_{Th}$

## 3.9 Lecture 27

### Circuits Containing Only Dependent Sources

- An independent voltage or current source is applied at the open terminals and the corresponding current/voltage is measured.
- Voltage/current ratio at the terminal is Thevenin equivalent resistance
- Remember  $V_{OC} = 0$  as there is no energy source

## 3.10 Lecture 28

### Circuits Containing Both Independent and Dependent Sources

- Find  $V_{OC}$
- Find  $i_{SC}$
- Calculate  $R_{Th}$

### Maximum Power Transfer

Occurs when  $R_L = R$ :

$$P_L = \frac{V^2}{4R_L} \quad (66)$$

## 4 Transient Response of Circuits

### 4.1 Lecture 29

Also has review of Basics of Circuit Analysis.

- In DC analysis, a capacitor creates an open circuit so we don't consider it
- Inductors are short circuits in DC analysis

## 4.2 Lecture 30

### Capacitor/Inductor Review

- Capacitor – open circuit

$$V = V_{t_0} + \frac{1}{C} \int_{t_0}^t i(t) \cdot dt \quad (67)$$

- Inductor – short circuit

$$i = i_{t_0} + \frac{1}{L} \int_{t_0}^t V(t) \cdot dt \quad (68)$$

- Voltage across capacitor cannot change instantaneously
- Current in inductor cannot change instantaneously

### First Order Transient Circuit

- First order – single energy storage element – either capacitor or inductor

### First-Order Differential Equation

$$\frac{dx(t)}{dt} + ax(t) = A \quad (69)$$

has solution

$$x(t) = x_c(t) + x_p(t) \quad (70)$$

where  $x_p(t)$  is the forced response and  $x_c(t)$  is the natural response.

- We assume  $x_p(t) = K_1$
- $x_c(t) = K_2 e^{-at} - \tau = \frac{1}{a}$  called the time constant
- Therefore we can generalize solution as:

$$x(t) = K_1 + K_2 e^{-t/\tau} \quad (71)$$

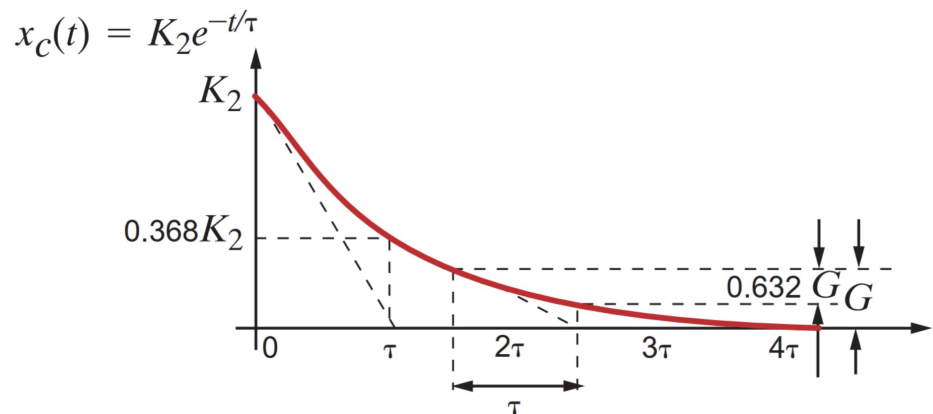


Figure 6: Transient Response

- Larger  $\tau$  means it takes longer for the circuit to settle

### 4.3 Lecture 31

#### 5 Steps to solve a circuit equation

1. Solution is always  $x(t) = K_1 + K_2 e^{-t/\tau}$
2. Solve for voltage across capacitor,  $V_C(0^-)$  or current through inductor  $i_L(0^-)$  before the switch is thrown
3. Replace capacitor w/ voltage source  $x(0^-) = V_C(0^+)$
4. Solve for steady state 2

$$x(t) \Big|_{t > 5\tau} \doteq x(\infty) \quad (72)$$

5. The time constant for:

- Capacitor:

$$\tau = R_{Th} \cdot C \quad (73)$$

- Inductor:

$$\tau = \frac{L}{R_{Th}} \quad (74)$$

6. Combine and evaluate constants:

$$x(0^+) = K_1 + K_2 \quad (75)$$

$$x(\infty) = K_1 \quad (76)$$

### 4.4 Lecture 32

Lecture 32 was just examples covering previously discussed topics.

## 5 AC Circuit Analysis

### 5.1 Lecture 33

#### Sinusoids

- $x(t) = X_M \sin(\omega t)$
- $\omega = \frac{2\pi}{T} = 2\pi f$
- There is also the phase angle –  $x(t) = X_M \sin(\omega t + \theta)$
- Positive phase angle moves signal earlier
- For  $x_1(t) = X_M \sin(\omega t + \theta)$  and  $x_2(t) = X_M \sin(\omega t + \phi)$ 
  - $x_1(t)$  leads by  $(\theta - \phi)$  rads
  - $x_2(t)$  lags by  $(\phi - \theta)$  rads
- If  $x(t) = X_M \sin(\omega t + \frac{\pi}{2}) = \cos(\omega t)$

#### Complex Numbers

- Polar –  $A = z \angle \theta$
- Rectangular –  $A = x + jy$
- Exponential –  $A = z e^{j\theta}$
- Euler's Identity –  $e^{j\theta} = \cos \theta + j \sin \theta$

## 5.2 Lecture 34

- If we apply a sinusoidal forcing function to a linear network
  - State voltages and currents in the network will also be sinusoidal

$$V(t) = A \sin(\omega t + \theta) \longrightarrow i(t) = B \sin(\omega t + \phi) \quad (77)$$

- Every steady state voltage/current in a linear network will have same form and same frequency.

### Phasors

- Phase angles are based on cosine functions

$$\mathbb{V} = V_M \angle \theta \quad (78)$$

- This complex representation is called a phasor

### Phasor Relationship for RLC Elements

- For resistors,  $V_M$  and  $I_M$  are always in phase
- For inductors, voltage always leads current by 90
- For capacitors, current leads voltage by 90

## 5.3 Lecture 35

### Impedance

- Essentially a complex version of resistance – with same unit Ohm
  - Now we are dealing with phasor voltage and current
  - Impedance is not a phasor

- $Z = \frac{\mathbb{V}}{\mathbb{I}}$

$$Z = \frac{V_M \angle \theta_V}{I_M \angle \theta_i} \quad (79)$$

- Expressed in rectangular form as  $\mathbb{Z}(\omega) = R(\omega) + jX(\omega)$
- Impedance of:

$$R \rightarrow \mathbb{Z} = R \quad (80)$$

$$C \rightarrow \mathbb{Z} = \frac{1}{j\omega C} \quad (81)$$

$$L \rightarrow \mathbb{Z} = j\omega L \quad (82)$$

## 5.4 Lecture 36

Lecture 36 was just examples covering previously discussed topics.

## 5.5 Lecture 37

Lecture 37 was just examples covering previously discussed topics.

## 5.6 Lecture 38

Lecture 38 was a final exam review lecture, no new content was discussed.