ECE110: Electrical Fundamentals (Textbook Notes)

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Preface and Acknowledgements

ECE110: Electrical Fundamentals is a course taken by some first-year undergraduate students at the University of Toronto's Faculty of Applied Science and Engineering. This course is taken by students in ECE, Mech, Indy, MSE, and TrackOne. This course provides a review of topics in electricity and magnetism that students may have learned in high school, and introduces topics such as: basic circuit analysis, RC and RL circuits, transient response, and AC steady-state analysis.

This document, which are my textbook notes, are one out of a two-part notes package for this course. The required texts for this course are *Fundamentals of Physics (12th edition)* by Halliday & Resnick, and *Basic Engineering Circuit Analysis (12th edition)* by Irwin & Nelms. The other document is my lecture notes taken in class.

I took this course in Winter 2024, under Professor Xilin Liu, and earned an A+. This was by far my favourite and the most engaging I course I took in first-year. While the content was extensive and challenging, it is easy to do well in this course if you're willing to put your nose to the grindstone. ECE110 also has a significant lab component, which introduces students to equipment they'll use in upper years.

A Word of Caution!

This notes package is in no way a complete representation of the course. That being said, I wrote these notes (originally with pen & paper) as part of my final exam review, to ensure I had covered everything.

This study guide was written in LaTeX, using Overleaf; I configured the `tcolorbox` library and some custom commands to create the blue boxes. These boxes are to highlight important concepts and ideas. The source project can be found here [click me!]. If you find any errors (or just want to provide feedback) feel free to reach out to arnav.patil@mail.utoronto.ca!

1 Coulomb's Law

Summary

- The strength of a particle's electrical interactions depend on its electric charge.
- Law of Electrostatic Forces $\rightarrow$ like charges repel, opposites attract.
- Electric current is defined as the rate at which charge passes through a point.

Coulomb's Law

Describes the electrostatic force between two charged particles. See equation 2.

- Electrostatic force vector on one particle always acts towards or away from other articles.
- Electric charge is quantized to particular values, which are multiples of the elementary charge. This was proven by the Millikan Oil Drop Experiment.

Shell Theorems

1. A charged particle outside a shell with a uniformly distributed charge is attracted or repelled as if the charge is concentrated at the centre of the shell.
2. A charged particle inside a shell with a uniformly distributed charge experiences no net force acting on it.

Conservation of Charge

The net charge in any closed system is always conserved.

Relevant Formulae

1. Electric current:
$$i = \frac{dq}{dt} \tag{1}$$
2. Coulomb's Law:
$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \tag{2}$$
where $\varepsilon_0 = 8.85 \times 10^{-12} C/N \cdot m^2$ is the permittivity of free space and $\frac{1}{4\pi\varepsilon_0}$ is replaced with $k = 8.99 \times 10^9 N \cdot m^2/C^2$, or Coulomb's constant.
3. The elementary charge $e = 1.602 \times 10^{-19} C$.

2 Electric Fields

Summary

- We assume that each charge sets up an electric field around itself to analyze the forces acting on electric charges.

Electric Field Lines

Provide a means of visualizing the direction and magnitude of an electric field. The electric field vector at any point is tangent to a field line through the point. The density of field lines in any region is proportional to the magnitude of the electric field present in the region. Field lines originate on positive charges and terminate on negative charges.

- Test charges are always positive and carry a small charge such that we can measure the force exerted on the test charge but the test charge creates no electric field of its own.
- The electric field due to a continuous charge distribution is found by treating charge elements as point charges and then summing the electric field vectors produced by all of the charges to find a net electric field vector.

Relevant Formulae

1. Electric field:
$$E = \frac{F}{q_0} \tag{3}$$
where electric field has the unit N/C or V/m.
2. Electric field due to a point charge:
$$E = k\frac{|q|}{r^2} \tag{4}$$

3 Gauss' Law

Summary

Gauss' Law

Gauss' Law and Coulomb's Law are different ways of describing the relationship between charge and electric fields in static situations. Coulomb's Law can be derived from Gauss' Law.

Relevant Formulae

1. Gauss' Law:
$$\varepsilon_0 \Phi = q_{enc} \tag{5}$$
where q_{enc} is the net charge enclosed in an imaginary closed surface (a Gaussian surface) and Φ is the net flux of the electric field through the surface. The units of electric flux is $V \cdot m$.
2. External electric field near the surface of a charged conductor perpendicular to the surface:
$$E = \frac{\sigma}{\varepsilon_0} \tag{6}$$
3. Electric field at any point due to a line of charge:
$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \tag{7}$$
4. Electric field at any point due to an infinite non-conducting sheet:
$$E = \frac{\sigma}{2\varepsilon_0} \tag{8}$$

5. Electric field outside a spherical shell of charge:

$$E = k\frac{q}{r^2} \tag{9}$$

6. Electric field inside a uniform sphere of charge is directed radially and has magnitude:

$$E = \frac{q}{4\pi\varepsilon_0 R^3}r \tag{10}$$

4 Electric Potential

Summary

- Electric potential V at a point P in the electric field of a charged object is V where W_∞ is the work that would be done by the electric force on a small positive test charge were it brought from an infinite distance to P. U is the potential energy that would be stored in the test charge-object system.
- All points on an equipotential surface have the same electric potential. The work done on a test charge in moving it along an equipotential line is zero.
- The electric field E is always directed perpendicularly to corresponding equipotential surfaces.
- An excess charge placed on a conductor will be located entirely on the outer surface of the conductor in the equilibrium state. The charge will distribute itself so that the following occurs:
  - The entire conductor, including all interior points, is at a uniform potential.
  - At every internal point, the electric field due to the charge cancels the external electric field that would have been there.
  - The net electric field at every point on the surface is perpendicular to the surface.

Electric Potential Energy

To find the electric potential energy of a system of charged particles, find the potential energy of each pair of particles and add them together.

Relevant Formulae

1. Electric potential V at a point P in the electric field of a charged object is:

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0} \tag{11}$$

2. Electric potential V at a point P in the electric field of a charged object is:

$$U = q \cdot V \tag{12}$$

$$\Delta U = q\Delta V = q(V_f - V_i) \tag{13}$$

3. If a particle moves through a change ΔV in electric potential without an applied force acting on it, applying the conservation of mechanical energy gives the change in kinetic energy:

$$\Delta K = -q\Delta V \tag{14}$$

4. If, instead, an applied force acts on the particle, doing work W_{app}, the change in kinetic energy is:

$$\Delta K = -q\Delta V + W_{app} \tag{15}$$

5. Electric potential difference between two points i and f is:

$$V_f - V_i = -\int_i^f E \cdot ds \tag{16}$$

6. In the special case of a uniform field of magnitude E, the potential chage between two adjacent equipotential lines separated by distance Δx is:

$$\Delta V = -E\Delta x \tag{17}$$

7. Potential due to a charged particle:

$$V = k\frac{q}{r} \tag{18}$$

where the potential due to a collection of charged particles is:

$$V = k\sum_{i=1}^{n}\frac{q}{r} \tag{19}$$

8. Potential due to a continuous charge distribution:

$$V = k\int\frac{dq}{r} \tag{20}$$

5 Capacitance

Summary

- We determine the capacitance of a particular capacitor configuration by:
  - Assuming a charge q to have been placed on the plates,
  - Finding the electric field E due to this charge,
  - Evaluating the potential difference V, and
  - Calculating C from $q = CV$.
- The electric potential energy U of a charged capacitor is equal to the work required to charge the capacitor. This energy can be associated with the capacitor's electric field E.

Dielectrics

If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ.

Relevant Formulae

1. Capacitance C of a capacitor given by:

$$q = CV \tag{21}$$

2. A parallel-plate capacitor has capacitance:

$$C = \frac{\varepsilon_0 A}{d} \tag{22}$$

3. A cylindrical capacitor has capacitance:

$$C = 2\pi\varepsilon_0\frac{L}{\ln(b/a)} \tag{23}$$

4. A spherical capacitor has capacitance:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \tag{24}$$

5. An isolated sphere has capacitance:

$$C = 4\pi\varepsilon_0 R \tag{25}$$

6. Capacitors in parallel:

$$C_{eq} = \sum_{j=1}^{n} C_j \tag{26}$$

7. Capacitors in series:

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \tag{27}$$

8. Potential energy of a charged capacitor:

$$U = \frac{q^2}{2C} = \frac{CV^2}{2} \tag{28}$$

9. Energy density within an electric field E:

$$u = \frac{1}{2}\varepsilon_0 E^2 \tag{29}$$

10. When a dielectric is present, Gauss' Law may be generalized to:

$$\varepsilon_0 \oint \kappa E \cdot dA = q \tag{30}$$

6 Current and Resistance

Summary

Ohm's Law

A given device obeys Ohm's Law if its resistance R is independent of the applied potential difference V. A given material obeys Ohm's Law if its resistivity is independent of the magnitude and direction of the applied electric field E.

- Semiconductors are materials that have few conduction electrons but can become conductors when they are doped with other atoms that contribute charge carriers.
- Superconductors are materials that lose all electrical resistance at low temperatures. Some materials are superconducting at surprisingly high temperatures.

Relevant Formulae

1. Current is related to current density by:

$$i = \int J \cdot dA \tag{31}$$

2. Drift speed of the charge carriers:

$$J = (ne) \cdot v_d \tag{32}$$

3. Resistance R of a conductor is defined by:
$$R = \frac{V}{i} \tag{33}$$
4. We define resistivity ρ and conductivity σ as:
$$\rho = \frac{1}{\sigma} = \frac{E}{J} \tag{34}$$
5. Resistance R of a conducting wire is given by:
$$R = \rho \frac{L}{A} \tag{35}$$
6. Change of resistivity with temperature is given by:
$$\rho - \rho_0 = \rho_0 \alpha (T - T_0) \tag{36}$$
7. Resistivity of a metal is given by:
$$\rho = \frac{m}{e^2 n \tau} \tag{37}$$
8. Power, or rate of energy transfer, is given by:
$$P = iV \tag{38}$$
9. Resistive dissipation is given by:
$$P = i^2 R = \frac{V^2}{R} \tag{39}$$

7 Magnetic Fields

Summary

The Hall Effect

The Hall Effect is when a conducting strip carrying a current i is placed in a uniform field B, some charge carriers build up on one side of the conductor, creating a potential difference V across the strip.

Relevant Formulae

1. A magnetic field B is defined in terms of the force F_B acting on a test particle with charge q moving through it with a velocity v:
$$F_B = q(v \times B) \tag{40}$$
where the unit for B is the tesla $T = 1N/(A \cdot m)$.
2. A charged particle circulating in a magnetic field is governed by:
$$|q|vB = \frac{mv^2}{r} \tag{41}$$
from which we can find the radius:
$$r = \frac{mv}{|q|B} \tag{42}$$
and gather that the frequency of the revolution of the particle is:
$$f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{|q|}{B2\pi m} \tag{43}$$
3. A straight wire carrying a current in a uniform magnetic field experiences a sideways force:
$$F_B = iL \times B \tag{44}$$

8 Magnetic Fields Due to Currents

Summary

Ampere's Law

Ampere's Law states that the the magnetic field integrated along a closed line known as an Amperian loop is equal to the permeability of free space times the the current enclosed within the loop.

Relevant Formulae

1. The Biot-Savart Law asserts that the contribution dB to the field produced by a current-length element ids at a point P is:
$$dB = \frac{\mu_0}{4\pi} = \frac{ids \times \hat{r}}{r^2} \tag{45}$$
2. Magnetic field of a long straight wire:
$$B = \frac{\mu_0 i}{2\pi R} \tag{46}$$
3. Magnetic field of a circular arc:
$$B = \frac{\mu_0 i \varphi}{4\pi R} \tag{47}$$
4. Force between parallel currents is given by:
$$F_{ba} = i_b L B_a \sin 90 = \frac{\mu_0 L i_a i_b}{2\pi d} \tag{48}$$
5. Ampere's Law:
$$\oint B \cdot ds = \mu_0 i_{enc} \tag{49}$$
6. Magnetic field inside of a solenoid and a toroid:
$$B = \mu_0 i n \textbf{ (solenoid)} \tag{50}$$
$$B = \frac{\mu_0 i N}{2\pi r} \textbf{ (toroid)} \tag{51}$$

9 Induction and Inductance

Summary

Faraday's Law of Induction

Faraday's Law of Induction states that if the magnetic flux through an area bounded by a closed conducting loop changes over time, a current and emf are produced. This process is called induction.

Lenz' Law

Lenz's Law states that an induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that causes the current. This induced emf has the same direction as the induced current.

- An inductor is a device that can be used to produce a known magnetic field in a specified region. If a current i is established through each of the N windings of an inductor, a magnetic flux Φ_B links those windings.

- If a current i in a coil changes with time, an emf is induced in the coil.
- This self-induced emf acts to oppose the change that produces it.

Energy Stored in an Inductor

If an inductor L carries a current i, the inductor's magnetic field stores an energy given by:

$$U_B = \frac{1}{2}Li^2 \tag{52}$$

- Mutual induction is when a changing current in one coil can induce an emf in the other.

Relevant Formulae

1. Magnetic flux Φ_B is defined as:
$$\Phi_B = \int B \cdot dA \tag{53}$$
2. Faraday's Law:
$$emf = -\frac{d\Phi_B}{dt} = -N\frac{f\Phi_B}{dt} \tag{54}$$
3. Emf and the induced electric field:
$$emf = \oint E \cdot ds \tag{55}$$
$$\oint E \cdot ds = -\frac{d\Phi_B}{dt} \tag{56}$$
4. The inductance L of the inductor is:
$$L = \frac{N\Phi_B}{i} \tag{57}$$
$$\frac{L}{l} = \mu_0 i n^2 A \ \textbf{(solenoid)} \tag{58}$$
5. Self-induced emf is given by:
$$emf_L = -L\frac{di}{dt} \tag{59}$$
6. Series RL circuits:
$$i = \frac{emf}{R}(1 - e^{-t/\tau_L}) \ \textbf{(rise of current)} \tag{60}$$
$$i = i_0 e^{-t/\tau_L} \ \textbf{(decay of current)} \tag{61}$$
7. Density of stored magnetic energy is given by:
$$u_B = \frac{B^2}{2\mu_0} \tag{62}$$

10 Basic Concepts

Summary

- The passive sign convention states that the product of voltage and current associated with an element determines the magnitude or sign of the power.
  - If the product is positive, power is absorbed by the element.
  - If the product is negative, power is being supplied by the element.
- An ideal independent voltage source is a two-terminal element that maintains a specified voltage between its terminals, regardless of the current across through the element.
- Dependent or controlled sources generate a voltage or current that is determined by the voltage or current at a specified location in the circuit.

Conservation of Energy

The electric circuits under investigation satisfy the conservation of energy.

Tellegen's Theorem

The sum of the powers absorbed by all the elements in an electrical network is zero.

Relevant Formulae

1. Relationship between current and charge:

$$i(t) = \frac{d1(t)}{dt} \textbf{ or } q(t) = \int_{-\infty}^{i} i(x)\,dx \tag{63}$$

2. Relationships among power, energy, current, and voltage:

$$p = \frac{dw}{dt} = vi \tag{64}$$

$$\Delta w = \int_{t^1}^{t^2} p \cdot dt = \int_{t^1}^{t^2} vi \cdot dt \tag{65}$$

11 Resistive Circuits

Summary

- Ohm's Law: $V = iR$
- Passive sign convention with Ohm's Law states that current enters the resistor terminal with positive voltage reference.

Kirchoff's Current Law

Algebraic sum of currents leaving a node is zero.

Use with Ohm's Law to solve a single-node-pair circuit.

Kirchoff's Voltage Law

Algebraic sum of voltage around a closed path is zero.

Use with Ohm's Law to solve a single-loop circuit.

- The voltage-division rule → The voltage is divided between two series resistors in direct proportion to their resistance.
- The current-division rule → The current is divided between two parallel resistors in reverse proportion to their resistance.
- Short circuit → Zero resistance and voltage, current is determined by the remainder of the circuit.
- Open circuit → Zero conductance, zero current, the voltage across the open terminals is determined by the rest of the circuit.

12 Nodal and Loop Analysis Techniques

Summary

Nodal Analysis for an N-node Circuit

1. Determine the number of nodes in the circuit. Select one node as the reference node.
   (a) Assign a node voltage between each reference and non-reference node. All node voltages are assumed positive with respect to the reference node.
   (b) For an N-node circuit, there are $N-1$ node voltages and $N-1$ linearly independent node equations must be written.
2. Write a constraint equation for each voltage source in terms of the assigned node voltages using KVL.
   (a) Each constraint equation represents one of the necessary linearly independent equations and N_v voltage sources yield N_v equations.
   (b) For each dependent voltage source, express the controlling variable for that source in terms of the node voltages.
3. A voltage source may be connected between a non-reference node and reference node, or between two non-reference nodes. A supernode is formed by a voltage source and its two connecting non-reference nodes.
4. Use KCL to formulate the remaining $N-1-N_v$ equations. First, apply KCL at each reference node not connected to a voltage source.
   (a) Second, apply KCL at each supernode.
   (b) Treat dependent current sources like independent current sources when formulating the KCL equations.
   (c) For each dependent current source, express the controlling variable in terms of the node voltages.

Loop Analysis for an N-loop Circuit

1. Determine the number of independent loops in the circuit. Assign a loop current to each independent loop. For an N-loop circuit, there are N-loop currents.
   (a) As a result, there are N linearly independent equations that must be solved.
2. If current sources are present in the circuit, either of the two techniques can be employed.

(a) In the first case, one loop current is selected to pass through one of the current sources. The remaining loop currents are determined by open-circuiting the current sources in the circuit and using this modified circuit to select them.

(b) In the second case, a current is assigned to each mesh in the circuit.

3. Write a constraint equation for each current source—independent or dependent—in the circuit in terms of the assigned loop currents using KCL. Each constraint equation represents one of the necessary linearly independent equations, and N1 current sources yield N1 linearly independent equations.

   (a) For each dependent current source, express the controlling variable for that source in terms of the loop currents.

4. Use KVL to formulate the remaining N - N1 linearly independent equations. Treat dependent voltage sources like independent voltage sources when formulating the KVL equations.

   (a) For each dependent voltage source, express the controlling variable in terms of the loop currents.

13 Additional Analysis Techniques

Summary

- **Linearity**: This property requires both **additivity and homogeneity**.
  - We can determine the voltage/current somewhere in a network by assuming a specific value for the variable, then determining what source value is required to produce it.
  - The ratio $\frac{\textbf{specified source value}}{\textbf{assumed value}}$ can be used to obtain a solution.
- In a linear network containing multiple independent sources, the **principle of superposition** allows us to compute any current or voltage in the network as the algebraic sum of the individual contributions of each source acting alone.
- Superposition does not apply to power dissipation.
- Using **Thevenin's theorem**, we can replace some portion of a network at a pair of terminals with:
  - A voltage source V_{OC}, which is the **open-circuit voltage** at the terminals, and
  - The **Thevenin equivalent** resistance R_{Th} obtained by setting all independent sources to 0.
- Using **Norton's theorem**, we can replace some portion of a network at a pair of terminals with a current source I_{SC} in parallel with a resistor.
  - I_{SC} is the short-circuit current at the terminals.
- Maximum power transfer can be achieved by selecting the load R_L to be equal to R_{Th} found by looking into the network from the load terminals.

14 First-Order Transient Circuits

Summary

- An RC or RL is said to be first-order only if it contains a single capacitor or a single inductor. The voltage of current anywhere in the network can be obtained by solving a first-order differential equation.
- The function $e^{-t\tau}$ decays to a value that is less than 1% of its initial value after a period 5τ. Therefore, the time constant determines the time required to reach steady state.

Relevant Formulae

1. The form of a first-order differential equation with a constant forcing function is:

$$\frac{dx(t)}{dt} + \frac{x(t)}{\tau} = A \tag{66}$$

   and the solution to this ODE is:

$$x(t) = K_1 + K_2 e^{-t/\tau} \tag{67}$$

2. The time constant is calculated by:

$$\tau = R \cdot C \textbf{ for capacitors} \tag{68}$$

$$\tau = \frac{L}{R} \textbf{ for inductors} \tag{69}$$

15 AC Steady-State Analysis

Summary

- **The sinusoidal function definition**: The sinusoidal function $x(t) = X_M \sin(\omega t + \theta)$ has an amplitude of X_M, a radian frequency of ω, a period of $2\pi/\omega$, and a phase angle of θ.
- **The phase lead and lag definition**: If $x_1(t) = X_{M_1} \sin(\omega t + \theta)$ and $x_2(t) = X_{M_2} \sin(\omega t + \Phi)$, then $x_1(t)$ leads $x_2(t)$ by $\theta - \Phi$ radians, and $x_2(t)$ lags $x_1(t)$.
- **The phasor definition**: The sinusoidal voltage $v(t) = V_M \cos(\omega t + \theta)$ can be written in exponential form as $v(t) = Re[V_M e^{j(\omega t + \theta)}]$ and in phasor form as $V = V_M \theta$.
- The phasor relationship in θ_v and θ_i for elements R, L, and C:
  - θ_i and θ_v if the element is a resistor.
  - θ_i lags θ_v by 90 if the element is an inductor.
  - θ_i leads θ_v by 90 if the element is a capacitor.
- **The impedances of R, L, and C**: Impedance, Z is defined as the ratio of the phasor voltage, V, to the phasor current, I, where $Z = R$ for a resistor, $Z = j\omega l$ for an inductor, and $Z = 1/j\omega C$ for a capacitor.
- **The phasor diagrams**: Phasor diagrams can be used to display the magnitude and phase relationships of various voltages and currents in a network.
- Frequency domain analysis:
  - Represent all voltages and currents as phasors and represent all passive elements by their impedance or admittance.
  - Solve for the unknown phasors in the frequency ω domain.
  - Transform the now-known phasors back to the time domain.

Relevant Formulae

1. Sinusoidal function:

$$x(t) = X_M \sin(\omega t + \theta) \tag{70}$$

2. Impedance:

$$\begin{cases} Z = R & \textbf{resistors} \\ Z = j\omega l & \textbf{inductors} \\ Z = 1/j\omega l & \textbf{capacitors} \end{cases} \tag{71}$$