

17.3 Sets in the Complex Plane

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1 Terminology

Suppose $z_0 = x_0 + iy_0$. Since $|z - z_0| = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ is the distance between points $z = x + iy$ and $z_0 = x_0 + iy_0$ that satisfy $|z - z_0| = \rho$ for $\rho > 0$ lie on a disk of radius ρ centered at z_0 .

The points that satisfy $|z - z_0| < \rho$ are within the disk but not on it. This set is called the neighbourhood of z_0 or an open disk. An **interior point** of a set S is a point for which there exists some neighbourhood of z_0 that lies entirely within S . If every point in S is an interior point, then S is an **open set**.

If every neighbourhood of z_0 has at least one point of S and one point not in S , then it is a **boundary point**.

If any pair of points z_1 and z_2 in an open set S can be connected by a polygon line that lies entirely within the set, then the set is called **connected**. A **region** is a domain in the complex plane with all, some, or none of its boundary points. A region containing all of its boundary bounds is labelled **closed**.