17.4 Functions of a Complex Variable

Arnav Patil

University of Toronto

A **function** f from a set A to a set B is a 'rule of correspondence' that assigns to each element in A one and only one element in B. If an element in A called a is assigned to b in B then b is the **image** of a. The set of all images b is called the **range** of the function.

1 Functions of a Complex Variable

A function of a complex variable of simply complex function is defined as:

$$w = f(z) = u(x, y) + iv(x, y)$$

2 Limits and Continuity

2.0.1 Definition 17.4.1 – Limit of a Function

Suppose the function f is defined in some neighbourhood of z_0 except possibly at z_0 itself. Then, f is said to possess a limit, written as:

$$\lim_{z \to z_0} f(z) = L$$

2.0.2 Definition 17.4.2 – Continuity at a Point

A function f is continuous at a point z_0 if:

$$\lim_{z \to z_0} f(z) = f(z_0)$$

3 Derivatives

The derivative of a complex function is defined as:

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{f(z_0 + \Delta z) - f(z_0)}{\Delta z}$$

If the above limit exists, then the function is said to be differentiable at z_0. As in real variables, differentiability implies continuity: "If f is differentiable at z_0, then f is continuous at z_0."