19.2 Taylor Series

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0.0.1 Theorem 19.2.1 – Continuity

A power series $\sum_{k=0}^{\infty} a_k(z-z_0)^k$ represents a continuous function f within it's circle of convergence $|z-z_0| = R$.

0.0.2 Theorem 19.2.2 – Term-by-Term Integration

A power series $\sum_{k=0}^{\infty} a_k(z-z_0)^k$ can be integrated term by term within its circle of convergence $|z-z_0| = R$ for every contour C lying entirely within the circle of convergence.

0.0.3 Theorem 19.2.3 – Term-by-Term Differentiation

A power series $\sum_{k=0}^{\infty} a_k(z-z_0)^k$ can be differentiated term by term within it's circle of convergence $|z-z_0| = R$.

1 Taylor Series

A power series can represent an analytic function within it's circle of convergence. The series:

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z-z_0) + \frac{f''(z_0)}{2!}(z-z_0)^2 + ... = \sum_{k=0}^{\infty} \frac{f^{(k)}(z_0)}{k!}(z-z_0)^k \tag{1}$$

is called the Taylor series for f centered at z_0. When the Taylor series is taken at $z_0 = 0$, then we may call it a Mclaurin series (though we don't really do that).

1.0.1 Theorem 19.2.4 – Taylor's Theorem

Let f be analytic within a domain D and let z_0 be a point D. Then f has the series representation

$$f(z) = \sum_{k=0}^{\infty} \frac{f^{(k)}(z_0)}{k!}(z-z_0)^k \tag{2}$$

valid for the largest circle C with centre at z_0 and radius that lies entirely within D.