19.4 Zeros and Poles

Arnav Patil

University of Toronto

1 Classification of Isolated Singular Points

An isolated singular point $z = z_0$ of a complex function f is given a classification depending on whether the principal part of it's Laurent expansion contains zero, a finite number, or an infinite number of terms.

1. If the principal part is zero, or, all coefficients a_k are zero, then z_0 is called a **removable singularity**.
2. If the principal part contains a finite number of nonzero terms, then z_0 is called a **pole**. If the last nonzero coefficient is a_n, $n \geq 1$, then we say that z_0 is a pole of order n. If z_0 is a pole of order 1, then the pol is called a **simple pole**.
3. If the principal part contains infinitely many nonzero terms, then z_0 is called an **essential singularity**.

2 Zeros

2.0.1 Theorem 19.4.1 – Pole of Order n

If the functions f and g are analytic at z_0 and f has a zero of order n at z_0 and $g(z_0) \neq 0$ then the function $F(z) = g(z)/f(z)$ has a pole of order n at z_0.