4.1 Definition of the Laplace Transform

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1 Integral Transform

$$\int_0^\infty K(s,t)f(t)dt = \lim_{b\to\infty}\int_0^b K(s,t)f(t)dt \tag{1}$$

If the limit in (1) exists, we say the integral exists, or is convergent; else, it does not exist and is divergent.

2 A Definition

The function $K(s,t)$ is said to be the kernel of the transform. The choice of $K(s,t) = e^{-st}$ as the kernel gives an important integral transform.

2.0.1 Laplace Transform

$$F(s) = \int_0^\infty e^{-st}f(t)dt \tag{2}$$

is said to be the Laplace transform of f. The domain of $F(s)$ is the set of values for s for which the improper integral (2) converges.

We may denote the Laplace transform in many ways:

$$\mathcal{L}\{f(t)\} = F(s), \mathcal{L}\{g(t)\} = G(s), \mathcal{L}\{i(t)\} = I(s)$$

3 Linearity of the Laplace Transform

$$\mathcal{L}\{\alpha f(t) + \beta g(t)\} = \alpha\mathcal{L}\{f(t)\} + \beta\mathcal{L}\{g(t)\} \tag{3}$$

4 Sufficient Conditions for Existence of $\mathcal{L}\{f(t)\}$

4.0.1 Definition – Exponential Order

A function f is said to be of exponential order is there are constants $c, M > 0$ and $T > 0$ such that $|f(t)| \leq Me^{ct}$ for all $t > T$.

4.0.2 Theorem 4.1.2 – Sufficient Conditions for Existence

If $f(t)$ is piecewise continuous on the interval $[0,\infty)$ and of exponential order, then $\mathcal{L}\{f(t)\}$ exists for $s > c$.