4.5 Dirac Delta Function

Arnav Patil

University of Toronto

1 Unit Impulse

If we graph the piecewise function

$$\delta_a(t-t_0) = \begin{cases} 0, 0, \leq t < t_0 - a \\ \frac{1}{2a}, t_0 - 1 \leq t < t_0 + a \\ t \geq t_0 + a \end{cases} \tag{1}$$

we get the following: The function is called a unit impulse since we have defined it such that:

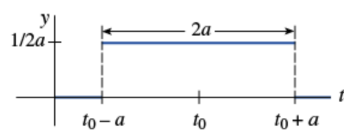

Figure 1: Unit Impulse

$$\int_0^\infty (t-t_0)dt = 1$$

2 The Dirac Delta Function

2.0.1 Theorem 4.5.1 – Transform of the Dirac Delta Function

$$\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0}$$

We can conclude from the above theorem that:

$$\mathcal{L}\{\delta(t)\} = 1$$

2.1 Alternative Definitions

If f is a continuous function, then

$$\int_0^\infty f(t)\delta(t-t_0)dt = f(t_0)$$

This result of $\delta(t-t_0)$ is known as the sifting property.