19.3 Laurent Series

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If a complex function f is not analytic at a point $z = z_0$, then we call that point a **singularity** or **singular point** of the function.

1 Isolated Singularities

Suppose $z = z_0$ is a singularity of a complex function f. The point $z = z_0$ is said to be an **isolated singularity** of the function f if there exists some deleted neighbourhood or punctured disk of z_0 throughout which f is analytic.

2 A New Kind of Series

Let us start with

$$f(z) = \sum_{k=1}^{\infty} a_{-k}(z - z_0)^{-k} + \sum_{k=0}^{\infty} a_k(z - z_0)^k \tag{1}$$

where the left-hand sum is called the **principal part** of the series and the right-hand sum is called the **analytic part** of the series.
The sum of the two parts converges when z is in an annular domain defined by $r < |z - z_0| < R$.

2.0.1 Theorem 19.3.1 – Laurent's Theorem

Let f be analytic within the annular domain D defined by $r < |z - z_0| < R$. Then, f has the series representation

$$f(z) = \sum_{k=-\infty}^{\infty} a_k(z - z_0)^k \tag{2}$$

valid for $r < |z - z_0| < R$. The coefficients a_k are given by

$$a_k = \frac{1}{2\pi i} \oint_C \frac{f(s)}{(s - z_0)^{k+1}} ds \tag{3}$$