# 17.5 Cauchy-Riemann Equations

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#### 0.0.1 Theorem 17.5.1 – Cauchy-Riemann Equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \tag{1}$$

#### 0.0.2 Theorem 17.5.2 – Criterion for Analyticity

If $u(x, y)$ and $v(x, y)$ satisfy the Cauchy-Riemann equations at all points of $D$, then the complex function $f(z) = u(x, y) + iv(x, y)$ is analytic in $D$. $u$ and $v$ are also then harmonic functions.

### 0.1 Harmonic Conjugate Functions

If $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain $D$, then $u$ and $v$ are harmonic in $D$. Sometimes, it is possible to find another function $v(x, y)$ that is harmonic in $D$ so that $u(x, y) + iv(x, y)$ is harmonic in $D$. Function $v$ is then called a harmonic conjugate function of $u$.