4.4 Additional Operational Properties

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1 Derivatives of Transforms

1.1 Multiplying a Function by t^n

1.1.1 Theorem 4.4.1 – Derivatives of Transforms

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

2 Transforms of Integrals

2.1 Convolution

$$f * g = \int_0^t f(\tau) g(t-\tau) d\tau$$

2.1.1 Theorem 4.4.2 – Convolution Theorem

$$\mathcal{L}\{f * g\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\} = F(s)G(s)$$

2.1.2 Transform of an Integral

$$\mathcal{L}\{\int_0^t f(\tau) d\tau\} = \frac{F(s)}{s} \tag{1}$$

The convolution theorem is useful in solving other types of equations where an unknown function appears under an integral sign. For example we have the Volterra integral equation

$$f(t) = g(t) + \int_0^t f(\tau) h(t-\tau) d\tau$$

3 Transform of a Periodic Function

3.0.1 Theorem 4.4.3 – Transform of a Periodic Function

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T f(t) dt$$