1.2 Initial Value Problems

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1 Initial-Value Problems

Given some interval I containing the domain value x_0, the problem:

$$\text{Solve: } \frac{d^n y}{dx^n} = f(x, y, y', ..., y^{(n-1)})$$
$$\text{Subject to: } y(x_0) = y_0, y'(x_0) = y_1, ..., y^{(n-1)}(x_0) = y_{n-1}$$

where all $y_0, ..., y_{n-1}$ values are specified constants, is called an initial-value problem. Each point $y(x_0) = y_0$ is called an initial condition.

2 First- and Second-Order IVPs

The problem described above is known as a n**th order initial-value problem**. First- and second-order IVPs are easy to interpret in geometric terms.

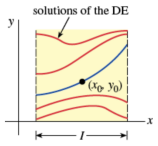

FIGURE 1.2.1 First-order IVP

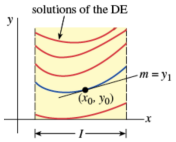

FIGURE 1.2.2 Second-order IVP

Figure 1: First- and Second-Order IVPs Visualized

3 Existence and Uniqueness

In every initial-value problem we have two fundamental questions: Does a solution exist? If so, is it unique?

We have to be careful in using the words “a solution” versus “the solution“ because there may be multiple solutions, a single solution, or none at all.