19.5 Residue Theorem

Arnav Patil

University of Toronto

1 Residue

The coefficient a_{-1} of $1/(z - z_0)$ in the Laurent series given above is called the **residue** of the function f at the isolated singularity z_0. We shall use the notation

$$a_{-1} = \text{Res}(f(z), z_0) \tag{1}$$

1.0.1 Theorem 19.5.1 – Residue at a Simple Pole

If f has a simple pole at z_0, then

$$\text{Res}(f(z), z_0) = \lim_{z \to z_0} (z - z_0) f(z) \tag{2}$$

1.0.2 Theorem 19.5.2 – Residue at a Pole of Order n

If f has a pole of order n at z_0, then

$$\text{Res}(f(z), z_0) = \frac{1}{(n-1)!} \lim_{z \to z_0} \frac{d^{n-1}}{dz^{n-1}} (z - z_0)^n f(z) \tag{3}$$

Here is an alternative method for computing a residue at a *simple pole*.

$$\text{Res}(f(z), z_0) = \frac{g(z_0)}{h'(z_0)} \tag{4}$$

2 Residue Theorem

2.0.1 Theorem 19.5.3 – Cauchy's Residue Theorem

Let D be a simply connected domain and C a simple closed contour lying entirely within D. If a function f is analytic on and within C, except at a finite number of singular points z_1, z_2, ..., z_n within C, then

$$\oint_C f(z) dz = 2\pi i \sum_{k=1}^{n} \text{Res}(f(z), z_k) \tag{5}$$