# Lecture 25C – Vectors and Coordinate Transformations

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## 1 3D Coordinate Systems

### 1.1 Cartesian

$$\begin{aligned}\hat{x} \times \hat{y} &= \hat{z},\\ \hat{y} \times \hat{z} &= \hat{x},\\ \hat{z} \times \hat{x} &= \hat{y}\end{aligned}$$

Any vector can be represented in Cartesian coordinates by:

$$\vec{r} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z}$$

The differential elements in the 3d Cartesian case are:

$$\begin{aligned}dl &= dx\hat{x} + dy\hat{y} + dz\hat{z}\\ d\vec{S}_1 &= dydz\hat{x}\\ d\vec{S}_2 &= dxdz\hat{y}\\ d\vec{S}_3 &= dxdy\hat{z}\\ dV &= dxdydz\end{aligned}$$

### 1.2 Cylindrical

Any vector can be expressed in cylindrical coordinates as:

$$\vec{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_z\hat{z}$$

The differential elements in a cylindrical coordinate system are:

$$\begin{aligned}\vec{dl} &= dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z}\\ d\vec{S}_r &= rd\theta dz\hat{r}\\ d\vec{S}_\theta &= drdz\hat{\theta}\\ d\vec{S}_z &= rdrd\theta\hat{z}\\ dV &= rdrd\theta dz\end{aligned}$$

### 1.3 Spherical

Any vector may be represented in spherical coordinates as

$$v = v_\rho\hat{\rho} + v_\phi\hat{\phi} + v_\theta\hat{\theta}$$