# Lecture 22 – Overview of Hemholtz Decomposition Theorem, Divergence Theorem, and Stokes' Theorem

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## 1 Hemholtz' Decomposition Theorem

$$\mathbf{F} = \mathbf{F}_{flux} + \mathbf{F}_{circ} = -\nabla\phi + \nabla \times \mathbf{A} \tag{1}$$

$\mathbf{A}$ is defined as the vector potential and $\phi$ as the scalar potential.

## 2 Divergence and Stokes' Theorem

THe Divergence Theorem relates a volume integral to a closed surface integral whereas Stokes' Theorem relates an open surface integral to a closed contour integral.

#### 2.0.1 Divergence Theorem in 3d

$$\oiint_S \mathbf{F} \cdot \mathbf{n}_S dS = \iiint_V \nabla \cdot \mathbf{F} dV \tag{2}$$

$\mathbf{n}_s$ is the unit vector normal to a differential surface area element on the closed surface $S$. It points outwards from the surface $S$ that encloses the volume $V$.

#### 2.0.2 Stokes' Theorem in 3d

$$\oint_C \mathbf{F} \cdot \mathbf{T} ds = \iint_{S_1} \nabla \times \mathbf{F} \cdot \mathbf{n}_{S_1} dS_1 = \iint_{S_2} \nabla \times \mathbf{F} \cdot \mathbf{n}_{S_2} dS_2 \tag{3}$$