Lecture 24 – Stokes' Theorem in 3D

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In this set of notes we assume that:

- The vector tangent to the closed contour is continuous,
- The components of the vector valued function $\mathbf{F}$ at each point on the closed contour C are continuous,
- The closed contour encloses the entire open region S in 3d, and the open region is simply connected, and
- The partial derivatives of the vector valued function $\mathbf{F}$ at every point in the region is defined.

1 Stokes' Theorem

$$\oint_C \mathbf{F} \cdot d\mathbf{s} = \iint_{S_1} \nabla \times \mathbf{F} \cdot d\mathbf{S}_1 = \iint_{S_2} \nabla \times \mathbf{F} \cdot d\mathbf{S}_2 \tag{1}$$

where $d\mathbf{s} = \mathbf{T}\mathbf{ds}$, $\mathbf{T} = \frac{\dot{r}(t)}{|\dot{r}(t)|}$, $ds = |\dot{r}(t)|$ and thus $\mathbf{T}ds = \dot{r}(t)dt$.