

Lecture 25D – Superposition for Vector Fields

Arnav Patil

University of Toronto

1 Superposition Principle

Consider the total field $\vec{F}_1 + \vec{F}_2$ and the linear operators $\nabla \cdot$ (divergence) and $\nabla \times$ (curl). Applying them to the total field gives us two identities.

$$\nabla \cdot [\vec{F}_1 + \vec{F}_2] = \nabla \cdot \vec{F}_1 + \nabla \cdot \vec{F}_2 = \rho_1 + \rho_2 \quad (1)$$

$$\nabla \times [\vec{F}_1 + \vec{F}_2] = \nabla \times \vec{F}_1 + \nabla \times \vec{F}_2 = \vec{J}_1 + \vec{J}_2 \quad (2)$$

2 Applying the Principles of Superposition

A volume charge distribution has spherical symmetry if the charge depends only on the distance from a point in space and not direction.

A volume charge distribution has cylindrical symmetry if the charge density depends only on the distance measured in a direction orthogonal to the cylinder's axis and not the position along the axis at which the measurement was made.

A volume charge distribution has planar symmetry if the charge density depends only on the distance measured in a direction orthogonal to the plane and not the position on the plane at which the measurement was made.

A surface current flux density distribution has cylindrical symmetry if the surface current flux density depends only on the distance measured in a direction orthogonal to the cylinder's axis. The surface current flux density can be distributed in two different ways: in a circumferential direction with respect to the cylinder's axis, or parallel to the cylinder's axis.

A surface current flux density distribution has toroidal symmetry if the surface current flux density forms a closed loop parallel to the exterior portion of a donut-shaped object, and if the direction of the density is parallel to the plane defined by the toroid's axis and the direction radial to toroid's axis.

A volumetric current flux density has planar symmetry if the current flux density depends only on the distance measured in a direction orthogonal to the plane and is oriented in a direction parallel to the plane.