Lecture 25B – Modelling Using Dirac-Distribution Functions

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Objectives of this lecture are to:

1. Understand the definition of a Dirac-distribution function and how it is represented in 1d, 2d, and 3d and specific coordinate systems, and
2. Compute double integrals (flux calculations) and triple integrals (compute total charge enclosed) whose integrands contain Dirac-distribution functions.

1 Properties of Scalar Densities and Flux Densities

- ρ is a scalar density ont he right-hand side of the Divergence Theorem

$$\oiint_S \vec{F} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{F} dV = \iiint_V \rho dV$$

- $\vec{J}$ is a flux density on the right-hand side of Stokes' Theorem

$$\oint_C \vec{F} \cdot d\vec{S} = \iiint_S \nabla \times \vec{F} \cdot d\vec{S} = \iint_S \vec{J} \cdot d\vec{S}$$

2 Distribution Functions

Consider cases in which a scalar function or vector valued function is infinite or undefined at specific points and is zero elsewhere. We need to invoke distribution functions, otherwise known as Dirac-Delta functions. We use the symbol $\sigma(x)$ to represent a 1d Dirac-Delta function in a Cartesian coordinate system.

3 Gaussian Surfaces

The closed surface S associated with the left-hand side of the Divergence Theorem

$$\oiint_S \vec{F} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{F} dV = \iiint_V \rho dV \tag{1}$$

is commonly referred to as the Gaussian surface.