16.2 Double Integrals over General Regions

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1 General Regions of Integration

Review of 16.1.

2 Iterated Integrals

Double integrals over non-rectangular regions are also evaluated using iterated integrals, but in this case, the order of integration is critical.

Step-by-step procedure for iterated integrals:

1. Convert to an iterated integral,
2. Evaluate inner integral with respect to y,
3. Simplify,
4. Evaluate outer integral with respect to x.

$$\begin{aligned}\int\int 2x^2y\ dA &= \int_{-2}^{2}\int_{3x^2}^{16-x^2} 2x^2y\ dy\ dx \\ &= \int_{-2}^{2} x^2y^2\Big|_{3x^2}^{16-x^2} dx \\ &= \int_{-2}^{2} x^2((16-x^2)^2-(3x^2)^2)dx \\ &= \int_{-2}^{2}(-8x^6-32x^4+256x^2)dx \\ &\approx 663.2\end{aligned}$$

2.0.1 Theorem 16.2 – Double Integrals over Non-Rectangular Regions

Let R be a region bounded below and above by the graphs of the continuous functions $y = g(x)$ and $y = h(x)$, respectively, and by the lines $x = a$ and $x = b$. If f is continuous on R, then:

$$\int\int f(x,y)dA = \int_a^b\int_{g(x)}^{h(x)} f(x,y)\ dy\ dx$$

If R is instead a region bounded by the continuous functions $x = g(y)$ and $x = h(y)$, respectively, and the lines $y = c$ and $y = d$, then:

$$\int\int f(x,y)dA = \int_c^d\int_{g(y)}^{h(y)} f(x,y)\ dx\ dy$$

3 Choosing and Changing the Order of Integration

Just examples.

4 Regions Between Two Surfaces

Volume of the solid between the surfaces is:

$$V = \int\int (f(x,y) - g(x,y))dA$$

5 Decomposition of Regions

By partitioning regions and using Riemann sums, it can be shown that:

$$\int\int_R f(x,y)dA = \int\int_{R_1} f(x,y)dA + \int\int_{R_2} f(x,y)dA$$

6 Finding Area by Double Integrals

Just examples.