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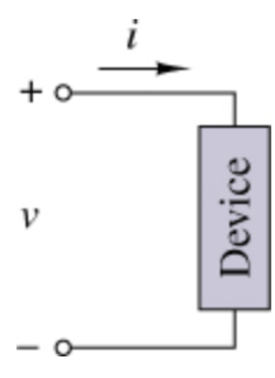
Linear Resistive Circuits

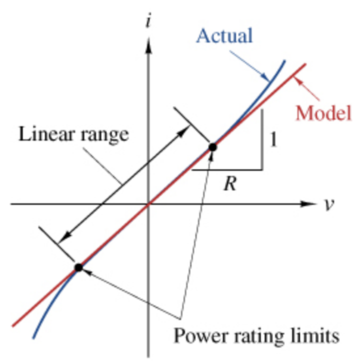
2 Basic Circuit Analysis

2.1 Element Constraints

- A circuit is a collection of interconnected electrical devices. An electrical device is a component treated as a separate entity.
- Two terminal device is described by it's $i - v$ characteristic.
- TO distinguish between an actual device and it's model, we model the circuit **element**.



Linear Resistor



- Equation describing linear resistor is Ohm's Law – $v = iR$
- **Conductance** G is a parameter with unit **siemens** S

$$G = \frac{1}{R}$$

- Linear means the characteristic is a straight line through the origin
- Power associated with the resistor is

$$p = i^2 R = \frac{v^2}{R}$$

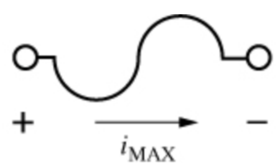
- By the **passive sign convention**, the linear resistor always absorbs or consumes power.

Open and Short Circuits

- Open circuit – $i = 0$ and $R = \infty$
- Short circuit – $v = 0$ and $R = 0$

Fuses

- A fuse is a safety device designed to protect from high current
- Consists of a metal strip mounted between a pair of electrical terminals
    - Housed in noncombustible housing like glass
- If current goes too high, temperature increase will melt the conducting material thus opening the circuit



The Ideal Switch

- Off – $i = 0$, v can take any value
- On – $v = 0$, i can take any value

Ideal Sources

- Two main elements, **voltage sources** and **current sources**
- Ideal voltage source – $v = v_S$ and i can take any value
- Ideal current source – $i - i_S$ and v can take any value
- Voltage or current produced by an ideal source is called a **forcing or driving function** because it represents an input that causes a circuit response

2.2 Connection Constraints

- Laws governing circuit behaviour (**Kirchoff's Laws**) are based on work of German scientist Gustav Kirchoff
    - These are called connection constraints because they are based only on circuit connections, not devices in the circuit
- Circuit — interconnection of electrical devices
- Node — electrical juncture of two or more devices
- Loop — Closed path formed by tracing through an ordered sequence of nodes without passing through a node more than once.

Kirchoff's Current Law

> 💡 The algebraic sum of the currents entering a node is 0 at every instant.

- We can alternatively express this as "the sum of the currents entering a node equals the sum of the currents leaving the node."
- In a circuit containing N nodes, there are only $N-1$ independent KCL connection equations.

Kirchoff's Voltage Law

> 💡 The algebraic sum of all the voltages around a loop is 0 at every instant.

- In a circuit containing E two-terminal elements and N nodes, there are only $E-N+1$ independent KVL connection equations.

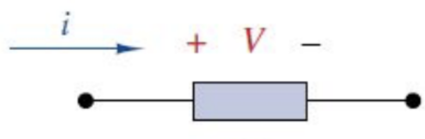
Parallel and Series Connections

- Two elements are in parallel if one can form a loop containing no other elements.
    - Alternatively, they share the same two nodes at both terminals.
- Two elements are in series when they have one common node.

2.3 Combined Constraints

Assigning Reference Marks

- If you pick the plus (+), then the current arrow (—>) must point to the plus.
- If you pick the current arrow (—>), then it must point to the plus (+).



2.4 Equivalent Circuits

- Two circuits are called **equivalent** if they have identical $i - v$ characteristics at a specified pair of terminals.

Current Division

$$i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

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3 Circuit Analysis Techniques

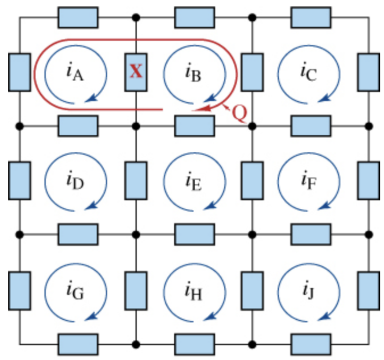
3.1 Node-Voltage Analysis

1. Select a reference node (ground). Identify a node voltage at each of the remaining $N - 1$ nodes and a current with every element in the circuit.
2. Write KCL connection constraints in terms of element currents at $N - 1$ non-reference nodes.
3. Use the $i - v$ relationships of the elements and the fundamental property of node analysis to express the element currents in terms of the node voltages.
4. Substitute the element constraints from step 3 into the KCL connection constraints from step 2 and arrange the resulting $N - 1$ equations in a standard form.

- When voltage sources are involved, we can use supernodes to simplify analysis.

3.2 Mesh-Current Analysis

- A planar circuit can be drawn on a flat surface without crossovers in the window pane.
- To define a set of variables, we associate a mesh current with each of the window panes and assign a reference direction.



1. Identify a mesh current with every mesh and a voltage across every circuit element.
2. Write KVL connection constraints in terms of the element voltages around every mesh.
3. Use KCL and $i - v$ relationships of the elements to express the element voltages in terms of the mesh currents.
4. Substitute the element constraints from step 3 into the connection constraints from step 2 and arrange the resulting equations in standard form.

- If current sources are involved, we can use super-meshes to simplify our analysis.

3.3 Linearity Properties

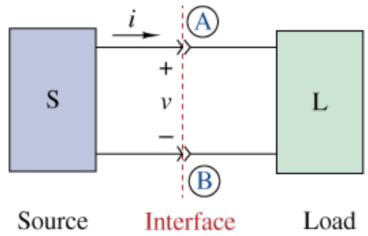
$$f(Kx) = Kf(x) \quad \text{(homogeneity)}$$
$$f(x_1 + x_2) = f(x_1) + f(x_2) \quad \text{(additivity)}$$

Superposition Principle

1. Set all independent sources except one to zero and find the output of the circuit due to that source alone.
2. Repeat step 1 for all the independent sources.
3. The total output is the sum of the contribution of each source acting independently.

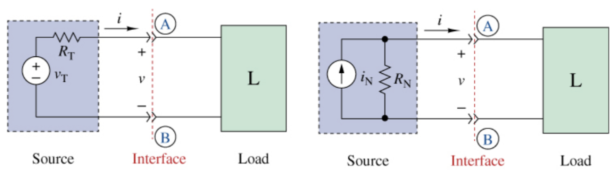
3.4 Thévenin and Norton Equivalent Circuits

- An interface is the connection between two circuits
  - We think of one as the source (S) and the other as the load (L)



- Conditions under which Thévenin and Norton equivalent circuits exist can be stated as a theorem

> 💡 If the source circuit in a two-terminal interface is linear, then the interface signals v and i do not change when the source circuit is replaced by it's Thévenin or Norton equivalent circuit.

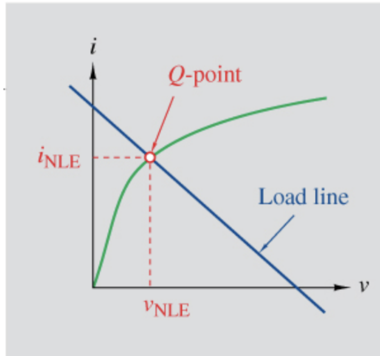


Application to Nonlinear Loads

- The $i - v$ relationship of the Thevenin equivalent can be written with interface current as the dependent variable

$$i = \left(-\frac{1}{R_T}\right)v + \left(\frac{v_T}{R_T}\right)$$

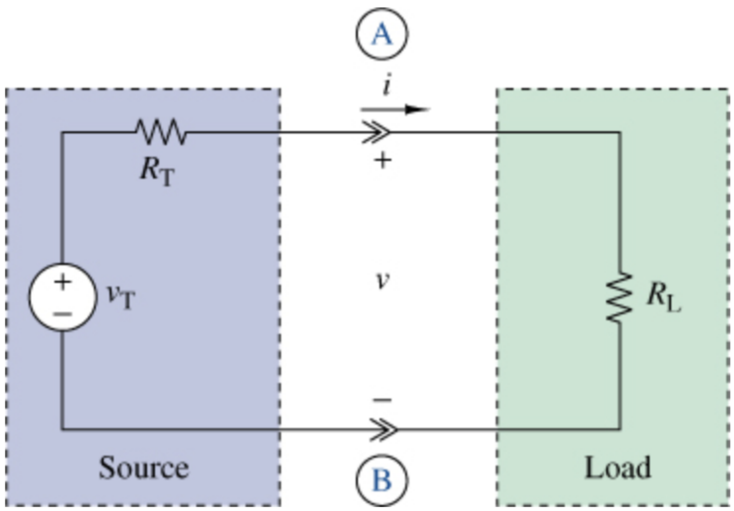
- This is the equation of a straight line which intersects the y-axis at i_{SC} and the x-axis at v_{OC}
    - While this should be called the source line because it is determined by the Thevenin equivalent of the source circuit, electrical engineers call it the **load line**.
- If we graph this load line with the $i - v$ characteristic of the nonlinear device, we can solve for the intersection point called the **operating point, quiescent point, or Q-point**.
- We usually can't solve these Q-points algebraically, instead we use approximation or nowadays, computational tools.



3.5 Maximum Signal Transfer

- We define the maximum voltage, current, and power available at an interface between a fixed source and an adjustable load.

For the circuit below:



the interface voltage is

$$v = \frac{R_L}{R_L + R_T} v_T$$

The voltage is a max if R_L is made very large, ideally infinite. In that case $v_{MAX} = v_T = V_{OC}$.

The current delivered at the interface is:

$$i = \frac{v_T}{R_L + R_T}$$

It then follows that the maximum possible current is then the short-circuit current i_{SC}.

Then,

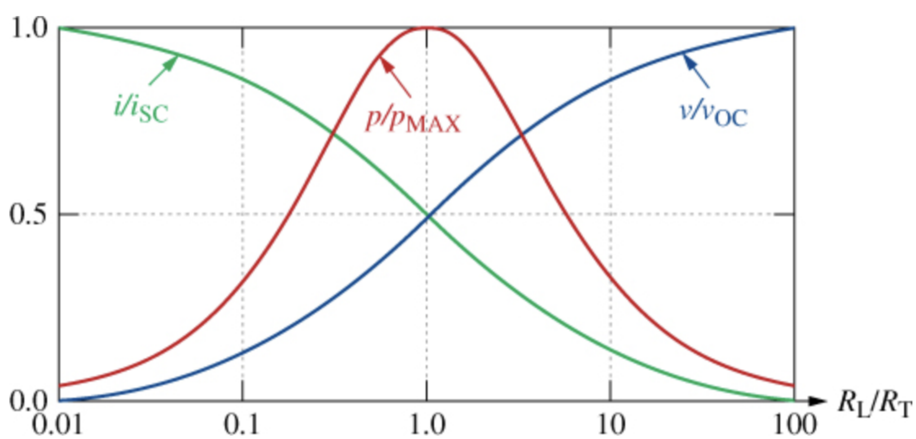
$$p = v \times i = \frac{R_L v_T^2}{(R_L + R_T)^2}$$

For a given source, the conditions for max voltage and current both give zero power (as either voltage or current go to zero). The value of R_L that maximizes the power can be found by differentiating the above equation and solve for R_L where $dp/dR_L = 0$.

$$\begin{aligned} \frac{dp}{dR_L} &= \frac{[(R_L + R_T)^2 - 2R_L(R_L + R_T)]v_T^2}{(R_L + R_T)^4} \\ &= \frac{R_T - R_L}{(R_L + R_T)^3} v_T^2 \\ &= 0 \end{aligned}$$

By solving further, we find that the condition for maximum power transfer is when the load resistance equals the Thevenin resistance, or $R_T = R_L$ (this called **matching**). Then,

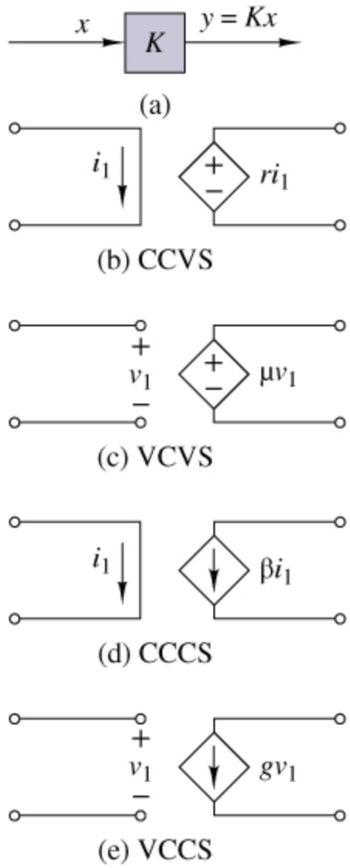
$$p_{MAX} = \frac{v_T^2}{4R_T} = \frac{i_N^2 R_T}{4} = \left[\frac{v_{OC}}{2}\right]\left[\frac{i_{SC}}{2}\right]$$



4 Active Circuits

4.1 Linear Dependent Sources

- An **active device** requires an external power supply to operate, and an **active circuit** is one that contains one or more active devices.
- Four possible types of dependent sources:



4.2 Analysis of Circuits with Dependent Sources

- No matter what analysis we do, we must not lose track of the signal(s) that drives the dependent source(s).
    - Methods like node and mesh analysis can be adapted to include dependent sources as well.
- Most techniques/content covered in examples so I won’t be including here.