3️⃣

Vector Analysis

3.2 Orthogonal Coordinate Systems

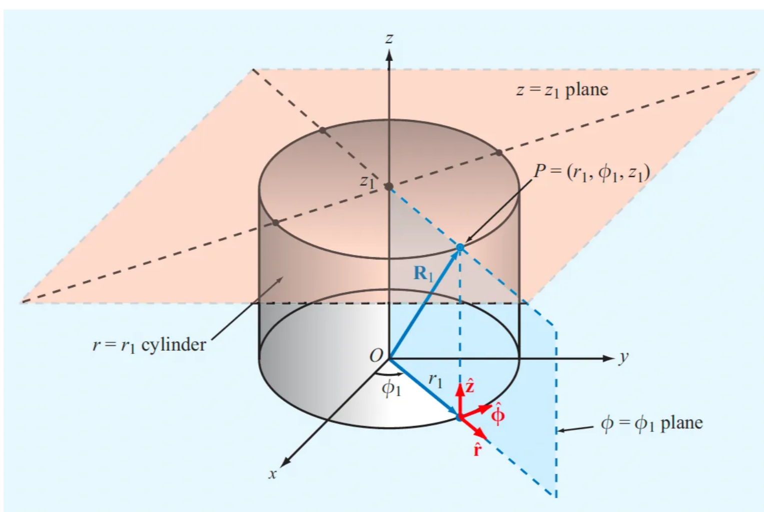
Cartesian Coordinates

$$d\mathbf{l} = \hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$$

Cylindrical Coordinates

- Measured in r, Φ, z.
  - Φ is the azimuth angle measured counterclockwise from the positive x axis in the x-y plane
- Differential volume element is given in:

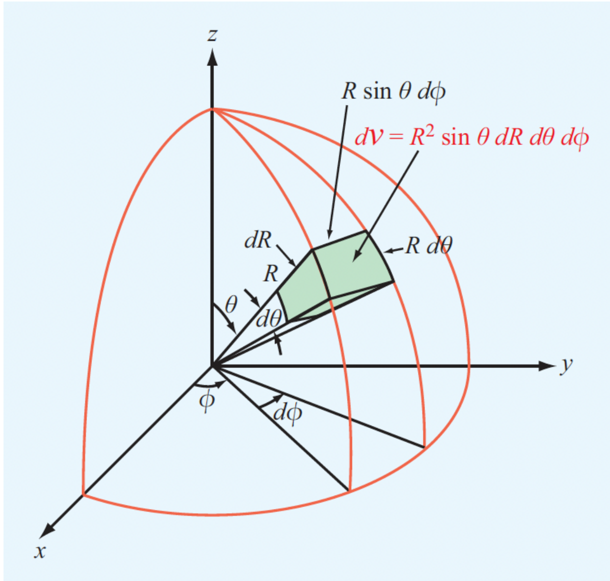
$$dV = r\ dr\ d\Phi\ dz$$



Cylindrical Coordinates

- Measured in R, θ, Φ
  - The zenith angle θ is measured from positive z-axis downwards
- Differential volume element is given by:

$$dV = R^2\ \sin\theta\ dR\,d\theta\,d\Phi$$



3.4 Gradient of a Scalar Field

Gradient

- In Cartesian coordinates

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

- In cylindrical coordinates

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \hat{\Phi}\frac{1}{r}\frac{\partial}{\partial \Phi} + \hat{z}\frac{\partial}{\partial z}$$

- In spherical coordinates

$$\nabla = \hat{R}\frac{\partial}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial}{\partial \theta} + \hat{\Phi}\frac{1}{R\sin\theta}\frac{\partial}{\partial \Phi}$$

Properties of the Gradient Operator

$$\nabla(U+V) = \nabla U + \nabla V$$

$$\nabla(UV) = U\nabla V + V\nabla U$$

$$\nabla V^n = nV^{n-1}\nabla V$$

3.7 The Laplacian Operator

- For a vector $\mathbf{E}$ specified in Cartesian coordinates, the Laplacian of $\mathbf{E}$ is given by:

$$\nabla^2\mathbf{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{E}$$

- Through direct substitution, it can also be shown that (relevance unknown)

$$\nabla^2\mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla \times (\nabla \times \mathbf{E})$$