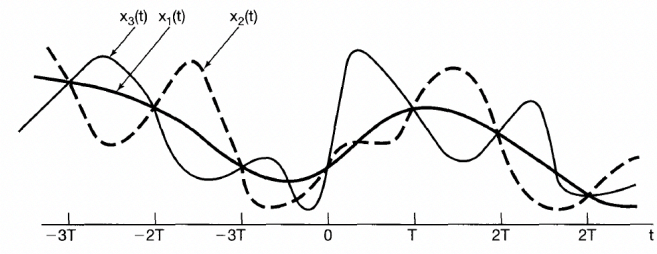
Sampling, Aliasing, and Interpolation

7 Sampling

7.1 Representation of a Continuous-Time Signals: The Sampling Theorem

Generally we would not expect that a signal could be uniquely specified by a sequence of equally spaced samples in the absence of additional conditions or information. However, if a signal is **band limited** (zero Fourier transform outside a finite band of frequencies) and if the samples are taken sufficiently close to the highest frequency, then the samples uniquely specify the signal and it **can be reconstructed perfectly**.

This result is the **sampling theorem**.



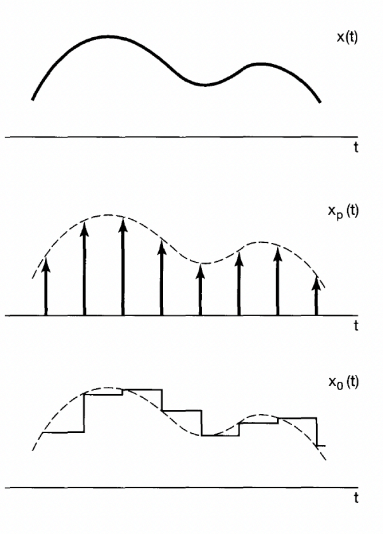
7.1.1 Impulse-Train Sampling

- We need a convenient way in which to represent the sampling of a CTS.
- We can do this by using a periodic impulse train multiplied by the CTS
  - This mechanism is called impulse-train sampling
  - Periodic impulse train $p(t)$ is called the sampling function
  - T is the sampling period
  - $\omega_s/2\pi/T$ is the sampling frequency

$$x_p(t) = x(t)p(t) \text{ where } p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

7.1.2 Sampling with a Zero-Order Hold

- In this system, a CTS $x(t)$ is sampled at a given instant, and its value is held until the next sample is taken.



7.2 Reconstruction of a Signal From its Samples Using Interpolation

- Fitting os a continuous signal to a set of example values is a commonly used procedure for reconstructing a function.
- One example is the zero-order hold discussed previously.
- Another example is to connect the sample points with a straight line.
- Interpolation using the impulse response of an ideal lowpass filter is also called band-limited interpolation, since it implements exact reconstruction of a CTS $x(t)$ if it’s band limited

7.3 The Effect of Undersampling: Aliasing

- When $\omega_s < 2\omega_M$, the spectrum of $x(t)$ is no longer replicated and is thus no longer recoverable by lowpass filtering.
- When aliasing occurs, the original frequency takes on the identity of a lower frequency $\omega_s - \omega_0$
- The sampling theorem explicitly requires that the sampling frequency be greater than twice the highest frequency in the signal.