Assignment 1

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Question 1.

Consider Fig.1., which describes a 2-dof robot arm. The Euler-Lagrange dynamics will be of the following form (as

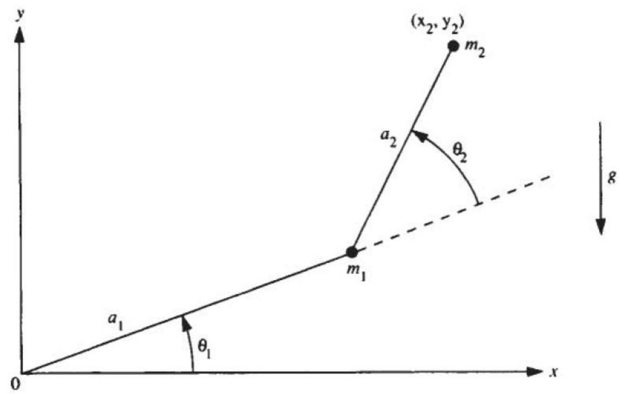

Figure 1: Two-Link Planar Elbow Arm

discussed in the class).

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau \tag{1}$$

where $q(t) = [\theta_1(t), \theta_2(t)]^T$ is the position vector (joint angles) and $\tau(t) \in \mathbb{R}^2$ is the external torque input (control input). The expression of different components in the model is given below.

$$M(q) = \begin{bmatrix} (m_1+m_2)a_1^2 + m_2a_2^2 + 2m_2a_1a_2cos\theta_2 & m_2a_2^2 + m_2a_1a_2cos\theta_2 \\ m_2a_2^2 + m_2a_1a_2cos\theta_2 & m_2a_2^2 \end{bmatrix}$$

$$V(q,\dot{q}) = \begin{bmatrix} -m_2a_1a_2(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)sin\theta_2 \\ m_2a_1a_2\theta_1^2 sin\theta_2 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} (m_1+m_2)ga_1cos\theta_1 + m_2ga_2cos(\theta_1+\theta_2) \\ m_2ga_2cos(\theta_1+\theta_2) \end{bmatrix}$$

Choose all the paprameters such as mass, link-length as unity. Choose gravity $g = 10$ unit. (Your task is to code these exact expressions of the different components $M(q)$, $V(q,\dot{q})$ and $G(q)$ for the given manipulator.)

case a) Desired trajectory : $q_d = [\frac{\pi}{3}, \frac{\pi}{4}]^T$, (set-point regulation)

case b) Desired trajectory : $q_d = [\frac{\pi}{4}sin(t), \frac{\pi}{5}cos(t)]^T$ (sinusoidal tracking)[1]

Consider the following proportional derivative (PD) control input[2]

$$\tau(t) = -k_1e(t) - k_2\dot{e}(t) \tag{2}$$

[1]Note that desired trajectory is an outcome of a trajectory planning algorithm depending on the application at hand, however, here some test trajectories are chosen for the assignment.

[2]The rational behind the designed controller will be discusses later in the class.

where $e(t) \triangleq q(t) - q_d(t)$ is tracking error and $k_1 > 0$ and $k_2 > 0$ are controller gains. Consider the initial conditions as $q(0) = [0,0]^T$ and $\dot{q}(0) = [0,0]^T$, assuming $t = 0$ as the initial time and choose the controller gains as $k1 = k2 = 1$ (Note that if the tracking performance is not satisfactory, you may have to tune the gains appropriately!).

Simulate the dynamics and the controller for both of the above mentioned cases of desired trajectories. (You can use Simulink or Matlab/Python/C++ script with appropriate ODE-solver depending on your preference.) Also visualize the motion of the manipulator using Peter Corke's Toolbox or any other visualization tool corresponding to both the cases.

What you have to submit :

1> Simulink file or matlab/python/C++ code (including the visualization part; you may consider a separate code for visualization if required!)

2> Plot of time evolution of the following variables for both the cases during the time-span $t \in [0, 10]$.

a> e-vs-t,

b> $\dot{e}$-vs-t

c> τ-vs-t.

with a brief description in a pdf file.

3> A video file comprising the animation of both the cases.

Put all of these in a zip file and upload it in the google classroom.