# ID2S Password-Authenticated Key Exchange Protocols

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Abstract—In a two-server password-authenticated key exchange (PAKE) protocol, a client splits its password and stores two shares of its password in the two servers, respectively, and the two servers then cooperate to authenticate the client without knowing the password of the client. In case one server is compromised by an adversary, the password of the client is required to remain secure. In this paper, we present two compilers that transform any two-party PAKE protocol to a two-server PAKE protocol on the basis of the identity-based cryptography, called ID2S PAKE protocol. By the compilers, we can construct ID2S PAKE protocols which achieve implicit authentication. As long as the underlying two-party PAKE protocol and identity-based encryption or signature scheme have provable security without random oracles, the ID2S PAKE protocols constructed by the compilers can be proven to be secure without random oracles. Compared with the Katz et al.'s two-server PAKE protocol with provable security without random oracles, our ID2S PAKE protocol can save from 22 to 66 percent of computation in each server.

Index Terms—Password-authenticated key exchange, identity-based encryption and signature, Diffie-Hellman key exchange, decisional Diffie-Hellman problem

## 1 INTRODUCTION

To secure communications between two parties, an authenticated encryption key is required to agree on in advance. So far, two models have existed for authenticated key exchange. One model assumes that two parties already share some cryptographically-strong information: either a secret key which can be used for encryption/authentication of messages, or a public key which can be used for encryption/signing of messages. These keys are random and hard to remember. In practice, a user often keeps his keys in a personal device protected by a password/PIN. Another model assumes that users, without help of personal devices, are only capable of storing "human-memorable" passwords.

Bellovin and Merritt [4] were the first to introduce password-based authenticated key exchange (PAKE), where two parties, based only on their knowledge of a password, establish a cryptographic key by exchange of messages. A PAKE protocol has to be immune to on-line and off-line dictionary attacks. In an off-line dictionary attack, an adversary exhaustively tries all possible passwords in a dictionary in order to determine the password of the client on the basis of the exchanged messages. In an on-line dictionary attack, an

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adversary simply attempts to login repeatedly, trying each possible password. By cryptographic means only, none of PAKE protocols can prevent on-line dictionary attacks. But on-line attacks can be stopped simply by setting a threshold to the number of login failures.

Since Bellovin and Merritt [4] introduced the idea of PAKE, numerous PAKE protocols have been proposed. In general, there exist two kinds of PAKE settings, one assumes that the password of the client is stored in a single server and another assumes that the password of the client is distributed in multiple servers.

PAKE protocols in the single-server setting can be classified into three categories as follows.

Password-only PAKE: Typical examples are the "encrypted key exchange" (EKE) protocols given by Bellovin and Merritt [4], where two parties, who share a password, exchange messages encrypted by the password, and establish a common secret key. The formal model of security for PAKE was firstly given in [3], [8]. Based on the security model, PAKE protocols [1], [2], [5], [10], [11], [16], [20], [22] have been proposed and proved to be secure.

*PKI-based PAKE*: PKI-based PAKE protocol was first given by Gong et al. [17], where the client stores the server's public key in addition to share a password with the server. Halevi and Krawczyk [18] were the first to provide formal definitions and rigorous proofs of security for PKI-based PAKE.

*ID-based PAKE*: ID-based PAKE protocols were proposed by Yi et al. [32], [33], where the client needs to remember a password in addition to the identity of the server, whereas the server keeps the password in addition to a private key related to its identity. ID-based PAKE can be thought as a trade-off between password-only and PKI-based PAKE.

In the single-server setting, all the passwords necessary to authenticate clients are stored in a single server. If the server is compromised, due to, for example, hacking or even insider attacks, passwords stored in the server are all disclosed. This is also true to Kerberos [12], where a user authenticates against the authentication server with his username and password and obtains a token to authenticate against the service server.

To address this problem, the multi-server setting for PAKE was first suggested in [15], [19], where the password of the client is distributed in n servers. PAKE protocols in the multi-server setting can be classified into two categories as follows.

Threshold PAKE: The first PKI-based threshold PAKE protocol was given by Ford and Kaliski [15], where n servers, sharing the password of the client, cooperate to authenticate the client and establish independent session keys with the client. As long as n-1 or fewer servers are compromised, their protocol remains secure. Jablon [19] gave a protocol with similar functionality in the password-only setting. MacKenzie et al. proposed a PKI-based threshold PAKE protocol which requires only t out of t servers to cooperate in order to authenticate the client. Their protocol remains secure as long as t-1 or fewer servers are compromised. Di Raimondo and Gennaro [26] suggested a password-only threshold PAKE protocol which requires fewer than t0 of the servers to be compromised.

Two-server PAKE: Two-server PKI-based PAKE was first given by Brainard [9], where two servers cooperate to authenticate the client and the password remains secure if one server is compromised. A variant of the protocol was later proved to be secure in [27]. A two-server passwordonly PAKE protocol was given by Katz et al. [23], in which two servers symmetrically contribute to the authentication of the client. The protocol in the server side can run in parallel. Efficient protocols [21], [29], [30], [31] were later proposed, where the front-end server authenticates the client with the help of the back-end server and only the front-end server establishes a session key with the client. These protocols are asymmetric in the server side and have to run in sequence. Yi et al. gave a symmetric solution [34] which is even more efficient than asymmetric protocols [21], [29], [30], [31]. Recently, Yi et al. constructed an ID2S PAKE protocol with the identity-based encryption scheme (IBE) [35].

In this paper, we will consider the two-server setting for PAKE only. In two-server PAKE, a client splits its password and stores two shares of its password in the two servers, respectively, and the two servers then cooperate to authenticate the client without knowing the password of the client. Even if one server is compromised, the attacker is still unable to pretend any client to authenticate against another server.

A typical example is the two-server PAKE protocol given by Katz et al. [23], which is built upon the two-party PAKE protocol (i.e., the KOY protocol [22]), where two parties, who share a password, exchange messages to establish a common secret key. Their basic two-server protocol is secure against a passive (i.e., "honest-but-curious") adversary who has access to one of the servers throughout the protocol execution, but cannot cause this server to deviate from its prescribed behavior. In [23], Katz et al. also showed how to modify their basic protocol so as to achieve security against an active adversary who may cause a corrupted server to deviate arbitrarily from the protocol. The core of their protocol is the KOY protocol. The client looks like running two KOY protocols with two servers in parallel. However, each

server must perform a total of roughly 80 exponentiations (i.e., each server's work is increased by a factor of roughly six as compared to the basic protocol [23]).

In [35], a security model for ID2S PAKE protocol was given and a compiler that transforms any two-party PAKE protocol to an ID2S PAKE protocol was proposed on the basis of the Cramer-Shoup public key encryption scheme [13] and any identity-based encryption scheme, such as the Waters' scheme [28].

Our contribution. In this paper, we propose a new compiler for ID2S PAKE protocol based on any identity-based signature scheme (IBS), such as the Paterson et al.'s scheme [25]. The basic idea is: The client splits its password into two shares and each server keeps one share of the password in addition to a private key related to its identity for signing. In key exchange, each server sends the client its public key for encryption with its identity-based signature on it. The signature can be verified by the client on the basis of the identity of the server. If the signature is genuine, the client submits to the server one share of the password encrypted with the public key of the server. With the decryption keys, both servers can derive the same one-time password, by which the two servers can run a two-party PAKE protocol to authenticate the client.

In addition, we generalize the compiler based on IBE in [35] by replacing the Cramer-Shoup public key encryption scheme with any public key encryption scheme. Unlike the compiler based on IBS, the compiler based on IBE assumes that each server has a private key related to its identity for decryption. In key exchange, the client sends to each server one share of the password encrypted according to the identity of the server. In addition, a one-time public key encryption scheme is used to protect the messages (containing the password information) from the servers to the client. The one-time public key is generated by the client and sent to the servers along with the password information in the first phase.

In the identity-based cryptography, the decryption key or the signing key of a server is usually generated by a Private Key Generator (PKG). Therefore the PKG can decrypt any messages encrypted with the identity of the server or sign any document on behalf of the server. As mentioned in [7], using standard techniques from threshold cryptography, the PKG can be distributed so that the master-key is never available in a single location. Like [35], our strategy is to employ multiple PKGs which cooperate to generate the decryption key or the signing key for the server. As long as one of the PKGs is honest to follow the protocol, the decryption key or the signing key for the server is known only to the server. Since we can assume that the two servers in two-server PAKE never collude, we can also assume that at least one of the PKGs do not collude with other PKGs.

Based on this assumption, we provide a rigorous proof of security for our compilers. The two compilers do not rely on the random oracle model as long as the underlying primitives themselves do not rely on it. For example, by using the KOY protocol [22] and the Paterson et al.'s IBS scheme [25] and the Cramer-Shoup public key encryption scheme [13], the compiler based on IBS can construct an ID2S PAKE protocol with provable security in the standard model. By using the KOY protocol [22] and the Waters IBE scheme [28] and the Cramer-Shoup public key encryption scheme [13],

the compiler based on IBE can construct an ID2S PAKE protocol with provable security in the standard model.

We also compare our ID2S PAKE protocols with the Katz et al.'s two-server PAKE protocol [23] with provable security in the standard model. The Katz et al.'s protocol is password-only, where the client needs to remember the password only and refer to common public parameters, and each server, having a public and private key pair, and keeps a share of the password. Our protocols are identity-based, where the client needs to remember the password in addition to the meaningful identities of the two servers, and refer to common public parameters, including the master public key, and each server, having a private key related to his identity, keeps a share of the password.

In terms of the setting and the client performance, the Katz et al.'s protocol is superior to our protocols. However, in the Katz et al.'s protocol, each server performs approximately six times the amount of the work as the KOY protocol, whereas in our protocols, each server performs the same amount of work as the KOY protocol in addition to one identity-based decryption (or signature) and one public key encryption (or decryption).

We have implemented our ID2S PAKE protocols. Our experiments show that our protocols save from 22 to 66 percent of computation in each server, compared with the Katz et al.'s protocol. The server performance is critical to the performance of the whole protocol when the servers provide services to a great number of clients concurrently. In addition, our experiments show that less than one second is needed for the client to execute our protocols.

*Organization*. In Section 2, we describe the security model for ID2S PAKE protocol given in [35]. In Section 3, we present our new ID2S PAKE compilers. After that, in Section 4, a rigorous proof of security for our protocols is provided. In Section 5, we analyze the performance of our protocols and compare them with the Katz's protocol by experiments. We conclude this paper in Section 6.

#### 2 DEFINITIONS

A formal model of security for two-server PAKE was given by Katz et al. [23] (based on the MacKenzie et al.'s model for PKI-based PAKE [24]). Boneh and Franklin [7] defined chosen ciphertext security for IBE under chosen identity attack. Combining the two models, a model for ID2S PAKE protocol was given in [35] and can be described as follows.

Participants, initialization and passwords. An ID2S PAKE protocol involves three kinds of protocol participants: (1) A set of clients (denoted as Client), each of which requests services from servers on the network; (2) A set of servers (denoted as Server), each of which provides services to clients on the network; (3) A group of Private Key Generators, which generate public parameters and corresponding private keys for servers.

We assume that ClientServerTriple is the set of triples of the client and two servers, where the client is authorized to use services provided by the two servers, Client  $\bigcap$  Server =  $\emptyset$ , User = Client  $\bigcup$  Server, any PKG  $\notin$  User, and ClientServerTriple  $\subseteq$  Client  $\times$  Server  $\times$  Server.

Prior to any execution of the protocol, we assume that an initialization phase occurs. During initialization, the PKGs cooperate to generate public parameters for the protocol,

which are available to all participants, and private keys for servers, which are given to the appropriate servers. The user may keep the public parameter in a personal device, such as a smart card or a USB flash drive. When the PKGs generate the private key for a server, each PKG generates and sends a private key component to the server via a secure channel. The server then derives its private key by combining all private key components from all PKGs. We assume that at least one of PKGs is honest to follow the protocol. Therefore, the private key of the server is known to the server only.

For any triple  $(C,A,B) \in \mathsf{ClientServerTriple}$ , we assume that the client C chooses its password  $\mathsf{pw}_C$  independently and uniformly at random from a "dictionary"  $\mathcal{D} = \{\mathsf{pw}_1, \mathsf{pw}_2, \cdots, \mathsf{pw}_N\}$  of size N, where  $\mathcal{D} \subset Z_q$ , N is a fixed constant which is independent of any security parameter, and q is a large prime. The password is then split into two shares  $\mathsf{pw}_{C,A}$  and  $\mathsf{pw}_{C,B}$  and stored at the two servers A and B, respectively, for authentication. We assume that the two servers never collude to determine the password of the client. The client C needs to remember  $\mathsf{pw}_C$  to log into the servers A and B.

For simplicity, we assume that each client C shares its password  $pw_C$  with exactly two servers A and B. In this case, we say that servers A and B are associated with C. A server may be associated with multiple clients.

Execution of the protocol. In the real world, a protocol determines how users behave in response to input from their environments. In the formal model, these inputs are provided by the adversary. Each user is assumed to be able to execute the protocol multiple times (possibly concurrently) with different partners. This is modeled by allowing each user to have unlimited number of instances (please refer to [3]) with which to execute the protocol. We denote instance i of user U as  $U^i$ . A given instance may be used only once. The adversary is given oracle access to these different instances. Furthermore, each instance maintains (local) state which is updated during the course of the experiment. In particular, each instance  $U^i$  is associated with the following variables, initialized as NULL or FALSE (as appropriate) during the initialization phase.

- $\operatorname{sid}_U^i$ ,  $\operatorname{pid}_U^i$  and  $\operatorname{sk}_U^i$  are variables containing the session identity, partner identity, and session key for an instance  $U^i$ , respectively. Computation of the session key is, of course, the ultimate goal of the protocol. The session identity is simply a way to keep track of the different executions of a particular user U. Without loss of generality, we simply let this be the (ordered) concatenation of all messages sent and received by instance  $U^i$ . The partner identity denotes the identity of the user with whom  $U^i$  believes it is interacting. For a client C,  $\operatorname{sk}_C^i$  consists of a pair  $(\operatorname{sk}_{C,A}^i, \operatorname{sk}_{C,B}^i)$ , which are the two keys shared with servers A and B, respectively.
- $\operatorname{acc}_U^i$  and  $\operatorname{term}_U^i$  are boolean variables denoting whether a given instance  $U^i$  has been accepted or terminated, respectively. Termination means that the given instance has done receiving and sending messages, acceptance indicates successful termination. In our case, acceptance means that the instance is sure that it has established a session key with its intended partner; thus, when an instance  $U^i$  has been accepted,  $\operatorname{sid}_{U^i}^i\operatorname{pid}_U^i$  and  $\operatorname{sk}_U^i$  are no longer NULL.

- state  $_{U}^{i}$  records any state necessary for execution of the protocol by  $U^{i}$ .
- $\mathbf{used}_U^i$  is a boolean variable denoting whether an instance  $U^i$  has begun executing the protocol. This is a formalism which will ensure each instance is used only once.

The adversary  $\mathcal{A}$  is assumed to have complete control over all communications in the network (between the clients and servers, and between servers and servers) and the adversary's interaction with the users (more specifically, with various instances) is modelled via access to oracles. The state of an instance may be updated during an oracle call, and the oracle's output may depend upon the relevant instance. The oracle types include:

- Send(C, i, A, B, M) This sends message M to a client instance  $C^i$ , supposedly from two servers A and B. Assuming  $\mathsf{term}_C^i = \mathsf{FALSE}$ , this instance runs according to the protocol specification, updating state as appropriate. The output of  $C^i$  (i.e., the message sent by the instance) is given to the adversary, who receives the updated values of  $\mathsf{sid}_C^i$ ,  $\mathsf{pid}_C^i$ ,  $\mathsf{acc}_C^i$ , and  $\mathsf{term}_C^i$ . This oracle call models the active attack to a protocol. If M is empty, this query represents a prompt for C to initiate the protocol.
- Send(S, i, U, M) This sends message M to a server instance  $S^i$ , supposedly from a user U (either a client or a server). Assuming  $\mathsf{term}_S^i = \mathsf{FALSE}$ , this instance runs according to the protocol specification, updating state as appropriate. The output of  $S^i$  (i.e., the message sent by the instance) is given to the adversary, who receives the updated values of  $\mathsf{sid}_S^i$ ,  $\mathsf{pid}_S^i$ ,  $\mathsf{acc}_S^i$ , and  $\mathsf{term}_S^i$ . If S is corrupted, the adversary also receives the entire internal state of S. This oracle call also models the active attack to a protocol.
- Execute(C, i, A, j, B, k) If the client instance  $C^i$  and the server instances  $A^j$  and  $B^k$  have not yet been used (where  $(C, A, B) \in \text{ClientServerTriple}$ ), this oracle executes the protocol between these instances and outputs the transcript of this execution. This oracle call represents passive eavesdropping of a protocol execution. In addition to the transcript, the adversary receives the values of sid, pid, acc, and term for client and server instances, at each step of protocol execution. In addition, if  $S \in \{A, B\}$  is corrupted, the adversary is given the entire internal state of S.
- Corrupt(S) This sends the private key of the server S in addition to all password information stored in the server S to the adversary. This oracle models possible compromising of a server due to, for example, hacking into the server.
- Corrupt(C) This query allows the adversary to learn the password of the client C, which models the possibility of subverting a client by, for example, witnessing a user typing in his password, or installing a "Trojan horse" on his machine.
- Reveal(U, U', i) This outputs the current value of session key  $\mathsf{sk}^i_{U,U'}$  held by instance  $U^i$  if  $\mathsf{acc}^i_U = \mathsf{TRUE}$ , where  $U' \in \mathsf{pid}^i_U$ . This oracle call models possible

- leakages of session keys due to, for example, improper erasure of session keys after use, compromise of a host computer, or cryptanalysis.
- Test(U,U',i) This oracle does not model any real-world capability of the adversary, but is instead used to define security. Assume  $U' \in \operatorname{pid}_U^i$ , if  $\operatorname{acc}_U^i = \operatorname{TRUE}$ , a random bit b is generated. If b = 0, the adversary is given  $\operatorname{sk}_{U,U'}^i$ , and if b = 1 the adversary is given a random session key. The adversary is allowed only a single Test query, at any time during its execution.

Partnering. Let  $(C,A,B) \in \mathsf{ClientServerTriple}$ . For the client instance  $C^i$ , let  $\mathsf{sid}_C^i = (\mathsf{sid}_{C,A}^i, \mathsf{sid}_{C,B}^i)$ , where  $\mathsf{sid}_{C,A}^i$  (resp.,  $\mathsf{sid}_{C,B}^i)$ ) denotes the ordered sequence of messages sent to / from the client C and the server A (resp., server B). For the server instance  $A^j$ , let  $\mathsf{sid}_A^j = (\mathsf{sid}_{A,C}^j, \mathsf{sid}_{A,B}^j)$ , where  $\mathsf{sid}_{A,C}^j$  denotes the ordered sequence of messages sent to / from the server A and the client C, and  $\mathsf{sid}_{A,B}^j$  denotes the ordered sequence of message sent to / from the server A and  $A^j$  are partnered if (1)  $\mathsf{sid}_{C,A}^i = \mathsf{sid}_{A,C}^j \neq \mathsf{NULL}$  and (2)  $A \in \mathsf{pid}_C^i$  and  $C \in \mathsf{pid}_A^j$ . We say that instances  $A^j$  and  $A^j$  are partnered if (1)  $\mathsf{sid}_{A,B}^j = \mathsf{sid}_{B,A}^j \neq \mathsf{NULL}$  and (2)  $A \in \mathsf{pid}_B^k$  are partnered if (1)

*Correctness.* To be viable, a key exchange protocol must satisfy the following notion of correctness: If a client instance  $C^i$  and server instances  $A^j$  and  $B^k$  run an honest execution of the protocol with no interference from the adversary, then  $\mathrm{acc}_C^i = \mathrm{acc}_A^j = \mathrm{acc}_B^k = \mathsf{TRUE}$ , and  $\mathrm{sk}_{C,A}^i = \mathrm{sk}_{A,C}^j$ ,  $\mathrm{sk}_{C,B}^i = \mathrm{sk}_{B,C}^k$  and  $\mathrm{sk}_{C,A}^i \neq \mathrm{sk}_{C,B}^i$ . Note that a correct protocol may not be secure. The security of a protocol is defined as follows.

Freshness. To formally define the adversary's success we need to define a notion of freshness for a session key, where freshness of a key is meant to indicate that the adversary does not trivially know the value of the session key. We say a session key  $\mathrm{Sk}^i_{U,U'}$  is fresh if (1) both U and U' are not corrupted; (2) the adversary never queried  $\mathrm{Reveal}(U,U',i)$ ; (3) the adversary never queried  $\mathrm{Reveal}(U',U,j)$  where  $U^i$  and  $U'^j$  are partnered.

Advantage of the adversary. Informally, the adversary succeeds if it can guess the bit b used by the Test oracle. We say an adversary  $\mathcal A$  succeeds if it makes a single query  $\mathsf{Test}(U,U',i)$  to a fresh instance  $U^i$ , with  $\mathsf{acc}_U^i = \mathsf{TRUE}$  at the time of this query, and outputs a single bit b' with b' = b (recall that b is the bit chosen by the Test oracle). We denote this event by  $\mathsf{Succ}$ . The advantage of adversary  $\mathcal A$  in attacking protocol P is then given by  $\mathsf{Adv}_{\mathcal A}^P(k) = 2 \cdot \mathsf{Pr}[\mathsf{Succ}] - 1$ , where the probability is taken over the random coins used by the adversary and the random coins used during the course of the experiment (including the initialization phase).

An adversary can always succeed by trying all passwords one-by-one in an on-line impersonation attack. A protocol is secure if this is the best an adversary can do. The on-line attacks correspond to Send queries. Formally, each instance for which the adversary has made a Send query counts as one on-line attack. Instances with which the adversary interacts via Execute are not counted as on-line attacks. The number of on-line attacks represents a bound on the number of passwords the adversary could have tested in an on-line fashion.

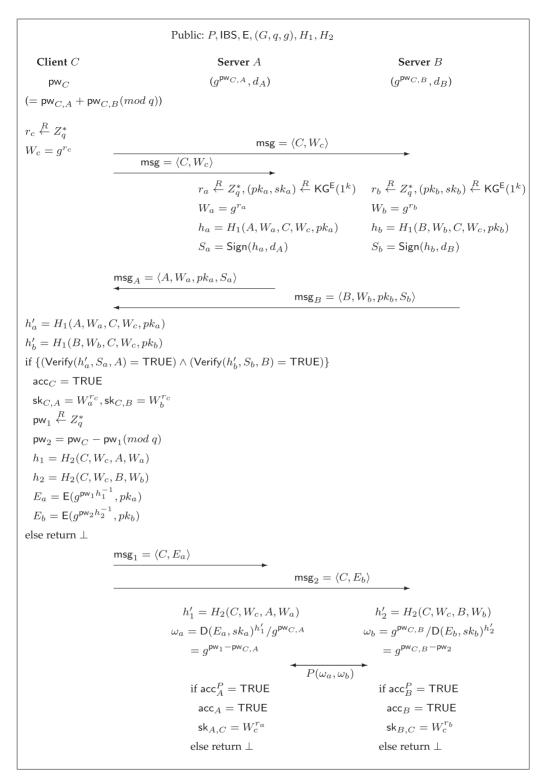


Fig. 1. ID2S PAKE protocol P' based on IBS.

**Definition 1.** Protocol P is a secure ID2S PAKE protocol if, for all dictionary size N and for all PPT adversaries A making at most Q(k) on-line attacks, there exists a negligible function  $\varepsilon(\cdot)$  such that  $\mathsf{Adv}^P_A(k) \leq Q(k)/N + \varepsilon(k)$ .

#### 3 ID2S PAKE PROTOCOLS

In this section, we present two compilers transforming any two-party PAKE protocol P to an ID2S PAKE protocol P' with identity-based cryptography. The first compiler is built

on identity-based signature and the second compiler is based on identity-based encryption.

#### 3.1 ID2S PAKE Based on IBS

### 3.1.1 Protocol Description

We need an identity-based signature scheme as our cryptographic building block. A high-level description of our compiler is given in Fig. 1, in which the client C and two servers A and B establish two authenticated keys, respectively. If

we remove authentication elements from our compiler, our key exchange protocol is essentially the Diffie-Hellman key exchange protocol [14]. We present the protocol by describing initialization and execution.

**Initialization.** Given a security parameter  $k \in \mathbb{N}$  (the set of all natural number), the initialization includes:

Parameter Generation: On input k, (1) m PKGs cooperate to run  $Setup^P$  of the two-party PAKE protocol P to generate system parameters, denoted as params<sup>P</sup>. (2) m PKGs cooperate to run Setup<sup>IBS</sup> of the IBS scheme to generate public system parameters for the IBS scheme, denoted as params<sup>IBS</sup> (including a subgroup G of the additive group of points of an elliptic curve), and the secret master-key<sup>IBS</sup>. (3) m PKGs choose a public key encryption scheme E, e.g., [13], whose plaintext group is a large cyclic group G with a prime order gand a generator g and select two hash functions,  $H_1: \{0,1\}^* \to Z_n^*$ , where n is the order of  $\mathbb{G}$ , and  $H_2: \{0,1\}^* \to Z_q^*$ , from a collision-resistant hash family. The public system parameters for the protocol P' is params = params $^{P,lBS,E}$   $\bigcup$   $\{(G,q,g),(H_1,H_2)\}$  and the secret master-key<sup>IBS</sup> is secretly shared by the PKGs in a manner that any coalition of PKGs cannot determine master-key<sup>IBS</sup> as long as one of the PKGs is honest to follow the protocol.

**Remark 1.** Taking the Paterson-Schuldt IBS scheme [25] for example, m PKGs agree on randomly chosen  $\mathcal{G}, \mathcal{G}_2 \in \mathbb{G}$  and each PKG randomly chooses  $\alpha_i \in Z_p$  and broadcast  $\mathcal{G}^{\alpha_i}$  with a zero-knowledge proof of knowing  $\alpha_i$  and a signature. Then we can set  $\mathcal{G}_1 = \mathcal{G}^{\sum_i \alpha_i}$  as the public master key and the secret master-key<sup>IBS</sup>  $= \mathcal{G}_2^{\sum_i \alpha_i}$ . The secret master key is privately shared among m PKGs and unknown to anyone even if m-1 PKGs maliciously collude.

Key Generation: On input the identity S of a server  $S \in Server$ , params<sup>IBS</sup>, and the secretly sharing master-key<sup>IBS</sup>, PKGs cooperate to run Extract<sup>IBS</sup> of the IBS scheme and generate a private (signing) key for S, denoted as  $d_S$ , in a manner that any coalition of PKGs cannot determine  $d_S$  as long as one of the PKGs is honest to follow the protocol.

**Remark 2.** In the Paterson-Schuldt IBS scheme with m PKGs , each PKG computes one component of the private key for a server S, i.e.,  $(\mathcal{G}_2^{\alpha_i}H(S)^{r_i},\mathcal{G}^{r_i})$ , where H is the Waters' hash function, and sends it to the server via a secure channel. Combining all components, the server can construct its private key  $d_S = (\mathcal{G}_2^{\sum_i \alpha_i} H(S)^{\sum_i r_i}, \mathcal{G}_i^{\sum_i r_i})$ , which is known to the server only even if m-1 PKGs maliciously collude. In addition, the identity of a server is public, meaningful, like an e-mail address, and easy to remember or keep. Anyone can write down the identity of a server on a note.

Password Generation: On input a triple  $(C,A,B) \in ClientServerTriple$ , a string  $pw_C$ , the password, is uniformly drawn from the dictionary  $\mathcal{D} = \{pw_1, pw_2, \ldots, pw_N\}$  by the client C, and randomly split into  $pw_{C,A}$  and  $pw_{C,B}$  such that  $pw_{C,A} + pw_{C,B} = pw_C \pmod{q}$ .  $g^{pw_{C,A}}$  and  $g^{pw_{C,B}}$  are stored in the servers A and B, respectively. We implicitly assume that N < min(n,q), which will certainly be true in practice.

*Protocol Execution.* Given a triple (C,A,B) ∈ ClientServerTriple, the client C (knowing its password  $pw_C$ ) runs the protocol P' with the two servers A (knowing  $g^{pw_{C,A}}$  and its private key  $d_A$ ) and B (knowing  $g^{pw_{C,B}}$  and its private key  $d_B$ ) to establish two session keys, respectively, as shown in Fig. 1.

At first, the client C randomly chooses an integer  $r_c$  from  $Z_q^*$  and computes  $W_c = g^{r_c}$  and broadcasts  $\mathsf{msg} = \langle C, W_c \rangle$  to the servers A and B.

**Remark 3.** To facilitate the communications between the client and two servers, a gateway may be used to forward messages between the client and the two servers. In this case, the client needs to communicate with the gateway only.

After receiving msg from the client, the server X (either A or B) randomly generates a public and private key pair  $(pk_x, sk_x)$  (where x is either a or b) for the public key encryption scheme E, and randomly chooses an integer  $r_x$  from  $Z_q^*$  and computes

$$W_x = g^{r_x}, h_x = H_1(X, W_x, C, W_x, pk_x), S_x = \text{Sign}(h_x, d_X),$$

where  $S_x$  denotes an identity-based signature of X on  $h_x$ . Then the server X replies to the client C with  $\mathsf{msg}_X = \langle X, W_x, pk_x, S_x \rangle$ .

**Remark 4.** The public and private key pair for the public key encryption scheme E can be generated once only and used repeatedly.

When getting the responses  $\mathsf{msg}_A$ ,  $\mathsf{msg}_B$  from the servers A and B, the client C computes

$$h'_a = H_1(A, W_a, C, W_c, pk_a), h'_b = H_1(B, W_b, C, W_c, pk_b),$$

and verifies the two signatures  $S_a, S_b$  on the basis of the identities of the servers A and B. If

$$(\mathsf{Verify}(h'_a, S_a, A) = \mathsf{TRUE}) \land (\mathsf{Verify}(h'_b, S_b, B) = \mathsf{TRUE}),$$

the client C sets  $acc_C = TRUE$  and computes two session keys  $sk_{C,A} = W_a^{r_c}$ ,  $sk_{C,B} = W_b^{r_c}$ .

Furthermore, the client C randomly chooses  $\mathsf{pw}_1$  from  $Z_q^*$  and computes

$$\mathsf{pw}_2 = \mathsf{pw}_C - \mathsf{pw}_1(mod\ q)$$

$$h_1 = H_2(C, W_c, A, W_a), h_2 = H_2(C, W_c, B, W_b).$$

Next, according to the public keys  $pk_a$ ,  $pk_b$  from the servers A and B, the client C performs the public key encryptions

$$E_a = \mathsf{E}(g^{\mathsf{pw}_1 h_1^{-1}}, pk_a), E_b = \mathsf{E}(g^{\mathsf{pw}_2 h_2^{-1}}, pk_b),$$

where E denotes the encryption algorithm. Then, the client sends  $\mathsf{msg}_1 = \langle C, E_a \rangle$  and  $\mathsf{msg}_2 = \langle C, E_b \rangle$  to the two servers A and B, respectively.

After receiving  $msg_1$  from C, the server A computes

$$h'_1 = H_2(C, W_c, A, W_a), \omega_a = \mathsf{D}(E_a, sk_a)^{h'_1}/g^{\mathsf{pw}_{C,A}},$$

where D denotes the decryption algorithm.

After receiving  $msg_2$  from C, the server B computes

$$h'_2 = H_2(C, W_c, B, W_b), \omega_b = g^{\mathsf{pw}_{C,B}}/\mathsf{D}(E_b, sk_b)^{h'_2}.$$

Because  $\mathsf{pw}_C = \mathsf{pw}_{C,A} + \mathsf{pw}_{C,B} (mod \ q)$  and  $\mathsf{pw}_C = \mathsf{pw}_1 + \mathsf{pw}_2 (mod \ q)$ , we have  $\omega_a = g^{\mathsf{pw}_1 - \mathsf{pw}_{C,A}} = g^{\mathsf{pw}_{C,B} - \mathsf{pw}_2} = \omega_b$ .

Using  $\omega_a$  and  $\omega_b$  as one-time password, the servers A and B run a two-party PAKE protocol P to establish a session key. If the server A accepts the session key as an authenticated key according to P (i.e.,  $\mathrm{acc}_A^P = \mathrm{TRUE}$ ), the server A sets  $\mathrm{acc}_A = \mathrm{TRUE}$  and computes the session key  $\mathrm{sk}_{A,C} = W_c^{r_a}$ . If the server B accepts the session key as an authenticated key according to P (i.e.,  $\mathrm{acc}_B^P = \mathrm{TRUE}$ ), the server B sets  $\mathrm{acc}_B = \mathrm{TRUE}$  and computes the session key  $\mathrm{sk}_{B,C} = W_c^{r_b}$ .

#### 3.1.2 Correctness

Assume that a client instance  $C^i$  and server instances  $A^j$  and  $B^k$  run an honest execution of the protocol P' with no interference from the adversary and the two-party PAKE P has the correctness property.

With reference to Fig. 1, we have  $h_a'=h_a, h_b'=h_b$  and the signatures are genuine. Therefore, the client C computes two session keys, i.e.,  $\mathbf{sk}_{C,A}=W_a^{r_c}, \mathbf{sk}_{C,B}=W_b^{r_c}$ , and lets  $\mathbf{acc}_C^i=\mathsf{TRUE}$ .

With reference to Fig. 1, the server instances  $A^j$  and  $B^k$  are able to derive the same one-time password  $\omega_a$  (=  $\omega_b$ ). Because P has the correctness property, after running P based on  $\omega_a$  and  $\omega_b$ , the server instances  $A^j$  and  $B^k$  accept the established session key as an authenticated key. This indicates that the client C has provided a correct password  $\mathrm{pw}_C$ . Next, the server instances  $A^j$  and  $B^k$  compute the session keys with the client C, i.e.,  $\mathrm{sk}_{A,C} = W_c^{r_a}$  and  $\mathrm{sk}_{B,C} = W_c^{r_b}$ , and let  $\mathrm{acc}_A^j = \mathrm{TRUE}$  and  $\mathrm{acc}_B^k = \mathrm{TRUE}$ .

Since  $W_c = g^{rc}$ ,  $W_a = g^{ra}$ ,  $W_b = g^{rb}$ , we have  $\mathsf{sk}_{C,A} = W_a^{rc} = g^{rarc} = W_c^{ra} = \mathsf{sk}_{A,C}$  and  $\mathsf{sk}_{C,B} = W_b^{rc} = g^{rbrc} = W_c^{rb} = \mathsf{sk}_{B,C}$ . In addition, because  $r_a, r_b$  are chosen randomly, the probability of  $\mathsf{sk}_{C,A} = \mathsf{sk}_{C,B}$  is negligible. Therefore, our protocol has the correctness property.

### 3.1.3 Explicit Authentication

By verifying the signatures  $S_a, S_b$  with the identities of the servers A and B, the client C can make sure that its intended servers A and B are authentic and the computed session keys  $\mathbf{S}\mathbf{k}_{C,A} = W_a^{r_c}, \mathbf{S}\mathbf{k}_{C,B} = W_b^{r_c}$  are authentic.

By running the two-party PAKE protocol P based on  $w_a$  (derived by  $\mathsf{pw}_{C,A}$ ) and  $w_b$  (derived by  $\mathsf{pw}_{C,B}$ ), the two servers A and B can verify if the client C provides a password  $\mathsf{pw}_C$  such that  $\mathsf{pw}_C = \mathsf{pw}_{C,A} + \mathsf{pw}_{C,B}(mod\ q)$ . This shows that when  $\mathsf{acc}_A^j = \mathsf{TRUE}$ , the server A knows that its intended client C and server B are authentic. Our protocol achieves the implicit authentication. Using the hash function like [6], [35], however, it is easy to add explicit authentication to any protocol achieving implicit authentication.

**Remark 5.** After the execution of our protocol, the password in the client cache must be deleted like SSL-based Internet banking.

#### 3.2 ID2S PAKE Based on IBE

## 3.2.1 Protocol Description

A high-level description of our compiler based on identity-based encryption is given in Fig. 2. We present the protocol by describing initialization and execution.

*Initialization*. Given a security parameter  $k \in \mathbb{N}$ , the initialization includes:

Parameter Generation: On input k, (1) m PKGs cooperate to run Setup<sup>P</sup> of the two-party PAKE protocol P to generate system parameters, denoted as params<sup>P</sup>. (2) m PKGs cooperate to run Setup<sup>IBE</sup> of the IBE scheme to generate public system parameters for the IBE scheme, denoted as params<sup>IBE</sup>, and the secret master-key<sup>IBE</sup>. Assume that  $\mathcal{G}$  is a generator of IBE plaintext group  $\mathbb{G}$  with an order n. (3) mPKGs choose a public key encryption scheme E, e.g., [13], whose plaintext group is a large cyclic group G with a prime order q and a generator g and select two hash functions,  $H_1: \{0,1\}^* \to Z_n^*$  and  $H_2: \{0,1\}^* \to Z_q^*$ , from a collisionresistant hash family. The public system parameters for the  $protocol \ P' \ is \ params = params^{P, \text{IBE}, \text{E}} \ \bigcup \ \{(\mathbb{G}, \mathcal{G}, n), (G, q, g), \}$  $(H_1, H_2)$  and the secret master-key<sup>IBE</sup> is secretly shared by the PKGs in a manner that any coalition of PKGs cannot determine master-keyIBE as long as one of the PKGs is honest to follow the protocol.

Key Generation: On input the identity S of a server  $S \in Server$ , params<sup>IBE</sup>, and the secretly sharing master-key<sup>IBE</sup>, PKGs cooperate to run Extract<sup>IBE</sup> of the IBE scheme and generate a private (decryption) key for S, denoted as  $d_S$ , in a manner that any coalition of PKGs cannot determine  $d_S$  as long as one of the PKGs is honest to follow the protocol.

Password Generation: On input a triple (C,A,B) ∈ ClientServerTriple, a string  $pw_C$ , the password, is uniformly drawn from the dictionary  $\mathcal{D} = \{pw_1,pw_2,\ldots,pw_N\}$  by the client C, and randomly split into  $pw_{C,A}$  and  $pw_{C,B}$  such that  $pw_{C,A} + pw_{C,B} = pw_C (mod\ n)$ , and  $pw_{C,A}^*$  and  $pw_{C,B}^*$  such that  $pw_{C,A}^* + pw_{C,B}^* = pw_C (mod\ q)$ . ( $\mathcal{G}^{pw_{C,A}}$ ,  $g^{pw_{C,A}^*}$ ) and ( $\mathcal{G}^{pw_{C,B}}$ ,  $g^{pw_{C,B}^*}$ ) are stored in servers A and B, respectively. We implicitly assume that N < min(n,q), which will certainly be true in practice.

*Protocol Execution.* Given a triple  $(C, A, B) \in$  ClientServerTriple, the client C (knowing its password  $pw_C$ ) runs the protocol P' with the two servers A (knowing  $\mathcal{G}^{pw_{C,A}}$ ,  $g^{pw_{C,A}^*}$ ,  $g^{pw_{C,A}^*}$  and its private key  $d_A$ ) and B (knowing  $\mathcal{G}^{pw_{C,B}}$ ,  $g^{pw_{C,B}^*}$  and its private key  $d_B$ ) to establish two session keys, respectively, as shown in Fig. 2.

At first, the client randomly chooses  $pw_1$  from  $Z_n^*$  and computes  $pw_2 = pw_C - pw_1 \pmod{n}$ . Next the client C randomly generates a one-time public and private key pair (pk, sk) for the public key encryption scheme E, and randomly chooses an integer  $r_c$  from  $Z_q^*$  and computes

$$W_c = q^{r_c}, h = H_1(C, W_c, pk).$$

Next, according to the identities of the two servers A and B, the client C performs the identity-based encryptions

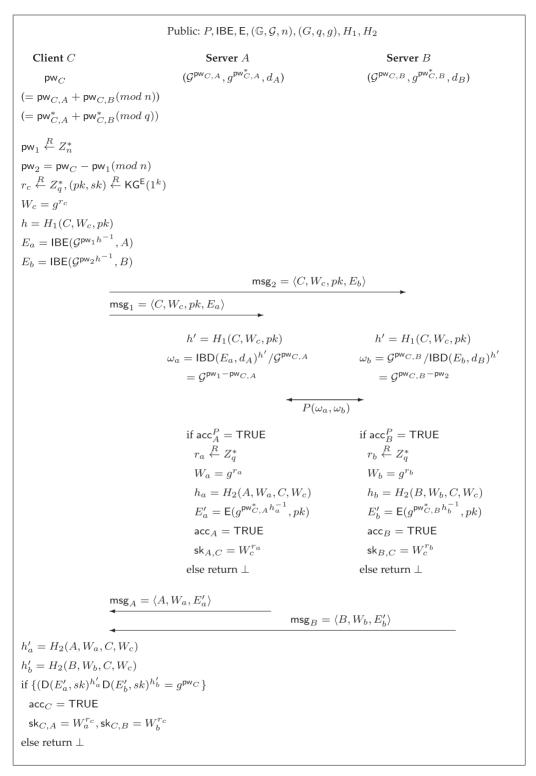


Fig. 2. ID2S PAKE protocol P' based on IBE.

$$E_a = \mathsf{IBE}(\mathcal{G}^{\mathsf{pw}_1 h^{-1}}, A), E_b = \mathsf{IBE}(\mathcal{G}^{\mathsf{pw}_2 h^{-1}}, B).$$

Then, the client sends  ${\sf msg}_1 = \langle C, W_c, pk, E_a \rangle$  and  ${\sf msg}_2 = \langle C, W_c, pk, E_b \rangle$  to the two servers A and B, respectively.

After receiving  $msg_1$  from C, the server A computes

$$h' = H_1(C, W_c, pk), \omega_a = \mathsf{IBD}(E_a, d_A)^{h'}/\mathcal{G}^{\mathsf{pw}_{C,A}},$$

where IBD denotes identity-based decryption.

After receiving  $msg_9$  from C, the server B computes

$$h' = H_1(C, W_c, pk), \omega_b = \mathcal{G}^{\mathsf{pw}_{C,B}}/\mathsf{IBD}(E_a, d_B)^{h'},$$

Because  $\mathsf{pw}_C = \mathsf{pw}_{C,A} + \mathsf{pw}_{C,B} (mod\ n)$  and  $\mathsf{pw}_C = \mathsf{pw}_1 + \mathsf{pw}_2 (mod\ n)$ , we have  $\omega_a = \mathcal{G}^{\mathsf{pw}_1 - \mathsf{pw}_{C,A}} = \mathcal{G}^{\mathsf{pw}_{C,B} - \mathsf{pw}_2} = \omega_h$ .

Using  $\omega_a$  and  $\omega_b$  as one-time password, the servers A and B run a two-party PAKE protocol P to establish a session key. If the server X (either A or B) accepts the

session key as an authenticated key according to P (i.e.,  $\mathtt{acc}_X^P = \mathsf{TRUE}$ ), it randomly chooses an integer  $r_x$  (where x is either a or b) from  $Z_q^*$  and computes

$$W_x = g^{r_x}, h_x = H_2(X, W_x, C, W_c),$$

$$E'_{x} = \mathsf{E}(g^{\mathsf{pw}_{C,X}^*h_{x}^{-1}}, pk), \mathsf{sk}_{X,C} = W_{c}^{r_{x}},$$

where  $\operatorname{sk}_{X,C}$  is the session key between X and C and  $E_x'$  is the encryption of  $g^{\operatorname{pw}_{C,X}^*h_x^{-1}}$ . Then the server X sets  $\operatorname{acc}_X = \operatorname{TRUE}$  and replies to the client C with  $\operatorname{msg}_X = \langle X, W_x, E_x' \rangle$ . Finally, after receiving  $\operatorname{msg}_A$  and  $\operatorname{msg}_{B'}$ , C computes

$$h'_a = H_2(A, W_a, C, W_c), h'_b = H_2(B, W_b, C, W_c),$$

and checks if  $\mathsf{D}(E_1,sk)^{h'_a}\mathsf{D}(E_2,sk)^{h'_b}=g^{\mathsf{DW}_C}.$  If so, the client C sets  $\mathsf{acc}_C=\mathsf{TRUE}$  and computes two session keys  $\mathsf{sk}_{C,A}=W_a^{r_c},\mathsf{sk}_{C,B}=W_b^{r_c}.$ 

**Remark 6.** Our IBE-based protocol needs less rounds of communication than our IBS-based protocol. This can be seen by comparing Figs. 1 and 2.

#### 3.2.2 Correctness

Assume that a client instance  $C^i$  and server instances  $A^j$  and  $B^k$  run an honest execution of the protocol P' with no interference from the adversary and the two-party PAKE P has the correctness property.

With reference to Fig. 2, the server instances  $A^j$  and  $B^k$  are able to derive the same one-time password  $\omega_a$  (=  $\omega_b$ ). Because P has the correctness property, after running P based on  $\omega_a$  and  $\omega_b$ , the server instances  $A^j$  and  $B^k$  accept the established session key as an authenticated key. This indicates that the client C has provided a correct password  $\mathrm{pw}_C$ . Next, the server instances  $A^j$  and  $B^k$  compute the session keys with the client C, i.e.,  $\mathrm{sk}_{A,C} = W_c^{r_a}$  and  $\mathrm{sk}_{B,C} = W_c^{r_b}$ , and let  $\mathrm{acc}_A^j = \mathrm{TRUE}$  and  $\mathrm{acc}_B^k = \mathrm{TRUE}$ .

With reference to Fig. 2, we have  $h'_a = h_a$ ,  $h'_b = h_b$ , and

$$\begin{split} \mathsf{D}(E_1, sk)^{h'_1} \mathsf{D}(E_2, sk)^{h'_b} &= (g^{\mathsf{pw}_{C,A}^* h_a^{-1}})^{h'_a} (g^{\mathsf{pw}_{C,B}^* h_b^{-1}})^{h'_b} \\ &= g^{\mathsf{pw}_{C,A}^* + \mathsf{pw}_{C,B}^*} = g^{\mathsf{pw}_C}. \end{split}$$

Therefore, the client instance  $C^i$  computes two session keys, i.e.,  $\mathsf{sk}_{C,A} = W_a^{r_c}, \mathsf{sk}_{C,B} = W_b^{r_c}$ , and lets  $\mathsf{acc}_c^i = \mathsf{TRUE}$ . Since  $W_c = g^{r_c}, W_a = g^{r_a}, W_b = g^{r_b}$ , we have  $\mathsf{sk}_{C,A} = W_a^{r_c} = g^{r_a r_c} = W_c^{r_a} = \mathsf{sk}_{A,C}$  and  $\mathsf{sk}_{C,B} = W_b^{r_c} = g^{r_b r_c} = W_c^{r_b} = \mathsf{sk}_{B,C}$ . In addition, because  $r_a, r_b$  are chosen randomly, the probability of  $\mathsf{sk}_{C,A} = \mathsf{sk}_{C,B}$  is negligible. Therefore, our protocol has the correctness property.

## 3.2.3 Explicit Authentication

By running the two-party PAKE protocol P based on  $w_a$  (derived by  $\mathsf{pw}_{C,A}$ ) and  $w_b$  (derived by  $\mathsf{pw}_{C,B}$ ), the two servers A and B can verify if the client C provides a password  $\mathsf{pw}_C$  such that  $\mathsf{pw}_C = \mathsf{pw}_{C,A} + \mathsf{pw}_{C,B} (mod\ n)$ . In addition, by checking that  $\mathsf{D}(E_1,sk)^{h'_a}\mathsf{D}(E_2,sk)^{h'_b} = g^{\mathsf{pw}_C}$  (involving  $\mathsf{pw}_C$ ), the client C can verify if the two servers provide two shares of the password,  $\mathsf{pw}_{C,A}^*$  and  $\mathsf{pw}_{C,B}^*$ , such that  $\mathsf{pw}_C = \mathsf{pw}_{C,A}^* + \mathsf{pw}_{C,B}^* (mod\ q)$ . This shows that

when  $\operatorname{acc}_A^j = \mathsf{TRUE}$ , the server A knows that its intended client C and server B are authentic, and when  $\operatorname{acc}_C^i = \mathsf{TRUE}$ , the client C knows that its intended servers A and B are authentic. Our protocol achieves the implicit authentication. Using the hash function like [6], [35], however, it is easy to add explicit authentication to any protocol achieving implicit authentication.

## 4 PROOF OF SECURITY

Based on the security model defined in Section 2, we provide a rigorous proof of security for our compilers in this section.

## 4.1 Security of ID2S PAKE Protocol Based on IBS

**Theorem 1.** Assuming that (1) the identity-based signature scheme is existentially unforgeable under an adaptive chosenmessage attack; (2) the public key encryption scheme E is secure against the chosen-ciphertext attack; (3) the decisional Diffie-Hellman problem is hard over (G,g,q); (4) the protocol P is a secure two-party PAKE protocol with explicit authentication; (5) P1, P2 are collision-resistant hash functions, then the protocol P1 illustrated in Fig. 1 is a secure ID2S PAKE protocol according to Definition 1.

**Proof.** Given an adversary A attacking the protocol, we imagine a simulator S that runs the protocol for A.

First of all, the simulator S initializes the system by generating params= params<sup>P,IBS,E</sup>  $\bigcup \{(G,q,g),(H_1,H_2)\}$  and the secret master-key<sup>IBS</sup>. Next, Client, Server, and ClientServerTriple sets are determined. Passwords for clients are chosen at random and split, and then stored at corresponding servers. Private keys for servers are computed using master-key<sup>IBS</sup>.

The public information is provided to the adversary. Considering  $(C,A,B) \in \mathsf{ClinetServerTriple}$ , we assume that the adversary  $\mathcal A$  chooses the server B to corrupt and the simulator  $\mathcal S$  gives the adversary  $\mathcal A$  the information held by the corrupted server B, including the private key of the server B, i.e.,  $d_B$ , and one share of the password of the client C,  $g^{\mathsf{pw}_{C,B}}$ . After computing the appropriate answer to any oracle query, the simulator  $\mathcal S$  provides the adversary  $\mathcal A$  with the internal state of the corrupted server B involved in the query.

We view the adversary's queries to its **Send** oracles as queries to five different oracles as follows:

- Send(C, i, A, B) represents a request for instance  $C^i$  of client C to initiate the protocol. The output of this query is  $msg = \langle C, W_c \rangle$ .
- Send $(A, j, C, \mathsf{msg})$  represents sending message msg to instance  $A^j$  of the server A from C. The output of this query is  $\mathsf{msg}_A = \langle A, W_a, pk_a, S_a \rangle$ .
- Send $(C, i, A, B, \mathsf{msg}_A|\mathsf{msg}_B)$  represents sending the message  $\mathsf{msg}_A|\mathsf{msg}_B$  to instance  $C^i$  of the client C. The output of this query is either  $\mathsf{msg}_1 = \langle C, E_a \rangle|\mathsf{msg}_2 = \langle C, E_b \rangle$  or  $\bot$ .
- Send $(A, j, C, \mathsf{msg}_1)$  represents sending message  $\mathsf{msg}_1$  to instance  $A^j$  of the server A from C. The output of this query is either  $\mathsf{acc}_A = \mathsf{TRUE}$  or  $\bot$ .
- Send<sup>P</sup>(A, j, B, M) represents sending message M to instance  $A^j$  of the server A, supposedly by the

server *B*, in the two-party PAKE protocol *P*. The input and output of this query depends on the protocol *P*.

When  $\mathcal{A}$  queries the Test oracle, the simulator  $\mathcal{S}$  chooses a random bit b. When the adversary completes its execution and outputs a bit b', the simulator can tell whether the adversary succeeds by checking if (1) a single Test query was made regarding some fresh session key  $\mathsf{Sk}^i_{U,U'}$ , and (2) b' = b. Success of the adversary is denoted by event Succ. For any experiment P', we denote  $\mathsf{Adv}^{P'}_{\mathcal{A}} = 2 \cdot \mathsf{Pr}[\mathsf{Succ}] - 1$ , where  $\mathsf{Pr}[\cdot]$  denotes the probability of an event when the simulator interacts with the adversary in accordance with experiment P'.

We will use some terminology throughout the proof. A given message is called oracle-generated if it was output by the simulator in response to some oracle query. The message is said to be adversarially-generated otherwise. An adversarially-generated message must not be the same as any oracle-generated message.

We refer to the real execution of the experiment, as described above, as  $P_0$ . We introduce a sequence of transformations to the experiment  $P_0$  and bound the effect of each transformation on the adversary's advantage. We then bound the adversary's advantage in the final experiment. This immediately yields a bound on the adversary's advantage in the original experiment.

Experiment  $P_1$ : In this experiment, the simulator interacts with the adversary as  $P_0$  except that the adversary does not succeed, and the experiment is aborted, if any of the following occurs: 1) At any point during the experiment, an oracle-generated message (e.g., msg, msg<sub>1</sub>, msg<sub>2</sub>, msg<sub>A</sub>, or msg<sub>B</sub>) is repeated; 2) At any point during the experiment, a collision occurs in the hash function  $H_1$  or  $H_2$  (regardless of whether this is due to a direct action of the adversary, or whether this occurs during the course of the simulator's response to an oracle query).

It is immediate that events 1 occurs with only negligible probability, event 2 occurs with negligible probability assuming  $H_1, H_2$  as collision-resistant hash functions. Put everything together, we are able to see that

Claim 1. If  $H_1$  and  $H_2$  are collision-resistant hash functions,  $|\mathsf{Adv}_A^{P_0}(k) - \mathsf{Adv}_A^{P_1}(k)|$  is negligible.

Experiment  $P_2$ : In this experiment, the simulator interacts with the adversary  $\mathcal{A}$  as in experiment  $P_1$  except that the adversary's queries to Execute oracles are handled differently: in any Execute(C,i,A,j,B,k), where the adversary  $\mathcal{A}$  has not queried  $\operatorname{corrupt}(A)$ , but may have queried  $\operatorname{corrupt}(B)$ , the plaintext  $g^{\operatorname{pw}_1h_1^{-1}}$  in the public key encryption  $E_a$  is replaced with a random element in G.

The difference between the current experiment and the previous one is bounded by the probability that an adversary breaks the semantic security of the public key encryption E. More precisely, we have

Claim 2. If the public key encryption scheme E is semantically secure,  $|\mathsf{Adv}_{\mathcal{A}}^{P_1}(k) - \mathsf{Adv}_{\mathcal{A}}^{P_2}(k)|$  is negligible.

**Remark 7.** If a public key encryption scheme is secure against the chosen-ciphertext attack (CCA), it is secure against the chosen-plaintext attack (CPA) (i.e., it is semantically secure).

Experiment  $P_3$ : In this experiment, the simulator interacts with the adversary  $\mathcal{A}$  as in experiment  $P_2$  except that: for any  $\mathsf{Execute}(C,i,A,j,B,k)$  oracle, where the adversary  $\mathcal{A}$  has not queried  $\mathsf{corrupt}(A)$ , but may have queried  $\mathsf{corrupt}(B)$ , the session keys  $\mathsf{sk}_{C,A}$  and  $\mathsf{sk}_{A,C}$  are replaced with a same random element in the group G.

The difference between the current experiment and the previous one is bounded by the probability to solve the decisional Diffie-Hellman (DDH) problem over (G, g, q). More precisely, we have

Claim 3. If the decisional Diffie-Hellman (DDH) problem is hard over (G,q,g),  $|\mathsf{Adv}_{\mathcal{A}}^{P_2}(k) - \mathsf{Adv}_{\mathcal{A}}^{P_3}(k)|$  is negligible.

If  $|\mathsf{Adv}_{\mathcal{A}}^{P_2}(k) - \mathsf{Adv}_{\mathcal{A}}^{P_3}(k)|$  is non-negligible, we show that the simulator can use  $\mathcal{A}$  as a subroutine to solve the DDH problem with non-negligible probability as follows.

Given a DDH problem  $(g^{\alpha},g^{\beta},Z)$ , where  $\alpha,\beta$  are randomly chosen from  $Z_q^*$  and Z is either  $g^{\alpha\beta}$  or a random element z from G, the simulator replaces  $W_c$  with  $g^{\alpha}$ , and  $W_a$  with  $g^{\beta}$ , and the session keys  $\mathrm{Sk}_{C,A}$ ,  $\mathrm{Sk}_{A,C}$  with Z. When  $Z=g^{\alpha\beta}$ , the experiment is the same as the experiment  $P_2$ . When Z is a random element z in  $\mathbb{G}$ , the experiment is the same as the experiment  $P_3$ . If the adversary can distinguish the experiments  $P_2$  and  $P_3$  with non-negligible probability, the simulator can solve the DDH problem with non-negligible probability. Assuming that the DDH problem is hard, Claim 3 is true.

In experiment  $P_3$ , the adversary's probability of correctly guessing the bit b used by the Test oracle is exactly 1/2 when the Test query is made to a fresh client instance  $C^i$  or a fresh server instance  $A^j$  invoked by an Execute (C,i,A,j,B,k) oracle, even if the adversary queried corrupt(B) (i.e., the adversary corrupted the server B). This is so because the session keys  $\mathrm{Sk}_{C,A}$  and  $\mathrm{Sk}_{A,C}$  for such instances in  $P_3$  are chosen at random from G, and hence there is no way to distinguish whether the Test oracle outputs a random session key or the "actual" session key (which is just a random element, anyway). Therefore, all passive adversaries cannot win the game, even if they can query Corrupt(B) oracles.

The rest of the proof concentrates on the instances invoked by Send oracles.

Experiment  $P_4$ : In this experiment, we modify the simulator's responses to  $Send(C, i, A, B, msg_A|msg_B)$  and  $Send(A, j, C, msg_1)$  queries.

Before describing this change we introduce some terminology. For a query  $\operatorname{Send}(C,i,A,B,\operatorname{msg}_A|\operatorname{msg}_B)$ , where  $\operatorname{msg}_A|\operatorname{msg}_B$  is adversarially-generated, if  $\operatorname{acc}_C^i = \operatorname{TRUE}$ , then  $\operatorname{msg}_A|\operatorname{msg}_B$  is said to be valid. Otherwise, it is said to be invalid. Similarly, for a query  $\operatorname{Send}(A,j,C,\operatorname{msg}_1)$ , where  $\operatorname{msg}_1$  is adversarially-generated, if  $\operatorname{acc}_A^j = \operatorname{TRUE}$ , then  $\operatorname{msg}_1$  is said to be valid. Otherwise,  $\operatorname{msg}_1$  is said to be invalid. Informally, valid messages use correct passwords while invalid messages do not. Given this terminology, we continue with our description of experiment  $P_4$ . When the adversary makes oracle query  $\operatorname{Send}(C,i,A,B,\operatorname{msg}_A|\operatorname{msg}_B)$ , the simulator examines  $\operatorname{msg}_A|\operatorname{msg}_B$ . If the message is adversarially-generated and valid, the simulator halts and  $\operatorname{acc}_C^i$  is assigned the special value  $\nabla$ . In any other case, (i.e.,

 $\mathsf{msg}_A|\mathsf{msg}_B$  is oracle-generated, or adversarially-generated but invalid), the query is answered exactly as in experi-ment  $P_3$ . When the adversary makes oracle query  $\mathsf{Send}(A,j,C,\mathsf{msg}_1)$ , the simulator examines  $\mathsf{msg}_1$ . If it is adversarially-generated and valid, the simulator halts and  $\mathsf{acc}_A^j$  is assigned the special value  $\nabla$ . In any other case, (i.e.,  $\mathsf{msg}_1$  is oracle-generated, or adversarially-generated but invalid), the query is answered exactly as in  $P_3$ .

Now, we change the definition of the adversary's success in  $P_4$ . At first, we define that a client instance  $C^i$  is fresh if the adversary has not queried  $\operatorname{Corrupt}(C)$  and a server instance  $A^j$  is fresh if the adversary has not queried  $\operatorname{Corrupt}(A)$ . If the adversary ever queries  $\operatorname{Send}(C,i,A,B,\ \operatorname{msg}_A|\operatorname{msg}_B)$  oracle to a fresh client instance  $C^i$  with  $\operatorname{acc}_C^i = \nabla$  or  $\operatorname{Send}(A,j,C,\operatorname{msg}_1)$  oracle to a fresh server instance  $A^j$  with  $\operatorname{acc}_A^j = \nabla$ , the simulator halts and the adversary succeeds. Otherwise the adversary's success is determined as in experiment  $P_3$ .

The distribution on the adversary's view in experiments  $P_3$  and  $P_4$  are identical up to the point when the adversary queries  $\mathsf{Send}(C,i,A,B,\mathsf{msg}_A|\mathsf{msg}_B)$  oracle to a fresh client instance with  $\mathsf{acc}_C^i = \nabla$  or  $\mathsf{Send}(A,j,C,\mathsf{msg}_1)$  oracle to a fresh server instance with  $\mathsf{acc}_A^j = \nabla$ . If such a query is never made, the distributions on the view are identical. Therefore, we have

Claim 4.  $\operatorname{Adv}_{A}^{P_3}(k) \leq \operatorname{Adv}_{A}^{P_4}(k)$ .

Experiment  $P_5$ : In this experiment, the simulator interacts with the adversary  $\mathcal A$  as in experiment  $P_4$  except that the adversary's queries to  $\mathsf{Send}(A,j,C,\mathsf{msg}_1)$  oracles are handled differently: in any  $\mathsf{Send}(A,j,C,\mathsf{msg}_1)$ , where the adversary  $\mathcal A$  has not queried  $\mathsf{corrupt}(A)$ , but may have queried  $\mathsf{corrupt}(B)$ , the plaintext  $g^{\mathsf{pw}_1h_1^{-1}}$  in  $E_a$  is replaced with a random element in the group G.

As we prove Claims 2, we can prove

Claim 5. If the public key encryption scheme E is semantically secure,  $|\mathsf{Adv}_{\mathcal{A}}^{P_4}(k) - \mathsf{Adv}_{\mathcal{A}}^{P_5}(k)|$  is negligible.

In experiment  $P_5$ ,  $\mathsf{msg}_1$  from Execute and Send oracles become independent of the password  $\mathsf{pw}_C$  used by the client C in the view of the adversary  $\mathcal{A}$ , even if  $\mathcal{A}$  may require  $\mathsf{Corrupt}(B)$ . In addition, although the adversary who has corrupted the server B is able to obtain  $g^{\mathsf{pw}_2}$ ,  $g^{\mathsf{pw}_{C,B}}$ , they are independent of the password  $\mathsf{pw}_C$  in the view of the adversary because the reference  $\mathsf{msg}_1$  is independent of the password in the view of the adversary. In view of this, any off-line dictionary attack cannot succeed.

The adversary  $\mathcal{A}$  succeeds only if one of the following occurs: (1) the adversary queries  $\operatorname{Send}(C,i,A,B,\operatorname{msg}_A|\operatorname{msg}_B)$  oracle to a fresh client instance  $C^i$  for adversarially-generated and valid  $\operatorname{msg}_A|\operatorname{msg}_B$ , that is,  $\operatorname{acc}_C^i = \nabla$  (let  $\operatorname{Succ}_1$  denote this event); (2) the adversary queries  $\operatorname{Send}(A,j,C,\operatorname{msg}_1)$  oracle to a fresh server instance  $A^j$  for adversarially-generated and valid  $\operatorname{msg}_1$ , that is,  $\operatorname{acc}_A^j = \nabla$  (let  $\operatorname{Succ}_2$  denote this event); (3) neither  $\operatorname{Succ}_1$  nor  $\operatorname{Succ}_2$  happens, the adversary wins the game by a Test query to a fresh instance  $C^i$  or a server instance  $A^j$ .

Claim 6. If the identity-based signature scheme is existentially unforgeable under an adaptive chosen-message attack and the hash function  $H_1$  is collision-resistant,  $\mathsf{Pr}[\mathsf{Succ}_1]$  is negligible.

 $\operatorname{\mathsf{msg}}_A$  contains a signature  $S_a$  of the server A on  $H_1(A,W_a,C,W_c,pk_a)$ . If the adversary  $\mathcal A$  is able to find  $W_a'$   $(\neq W_a)$  or  $pk_a'$   $(\neq pk_a)$  such that  $H_1(A,W_a',C,W_c,pk_a') = H_1(A,W_a,C,W_c,pk)$  for given  $W_c$ , he may make  $\operatorname{\mathsf{acc}}_C^i = \nabla$  with the same  $S_a$ . However, this probability is negligible because we assume that the hash function  $H_1$  is collision-resistant. If the adversary can forge a new signature  $S_a'$  of the server A on  $H_1(A,W_a',C,W_c,pk_a')$  for chosen  $W_a'$   $(\neq W_a)$  or  $pk_a'$   $(\neq pk_a)$ , he may also make  $\operatorname{\mathsf{acc}}_C^i = \nabla$ . However, this probability is also negligible because we assume that the identity-based signature scheme is existentially unforgeable under an adaptive chosen-message attack. Therefore, Claim 6 is true.

Claim 7. If (1) P is a secure two-party PAKE protocol with explicit authentication; (2) the public key encryption E is secure against the chosen-ciphertext attack; (3)  $H_2$  is collision-resistant hash functions, then  $\Pr[\operatorname{Succ}_2] \leq Q(k)/N + \varepsilon(k)$ , where Q(k) denotes the number of on-line attacks and  $\varepsilon(k)$  is a negligible function.

To evaluate  $Pr[Succ_2]$ , we assume that the adversary A has corrupted the server B and consider three cases as follows.

Case 1. If the adversary  $\mathcal{A}$  is able to find  $W_c'$  ( $\neq W_c$ ) such that  $H_2(C, W_c', A, W_a) = H_2(C, W_c, A, W_a)$  for given  $W_a$ , he may make  $\mathtt{acc}_A^j = \nabla$  with the same  $E_a$  in  $\mathsf{msg}_1$ . However, this probability is negligible because we assume that the hash function  $H_2$  is collision-resistant.

Case 2. If the adversary  $\mathcal{A}$  is able to replace the plaintext  $g^{\mathsf{pw}_1h_1^{-1}}$  in  $E_a$  with  $g^{\mathsf{pw}_1h_1^{*-1}}$  (like homomorphic encryption), where  $h_1^* \neq h_1$ , he may make  $\mathsf{acc}_A^j = \nabla$  with  $E_a'$  transformed from  $E_a$ . However, this probability is negligible because we assume that the public key encryption  $\mathsf{E}$  is secure against the chosen-ciphertext attack.

Case 3. The adversary  $\mathcal{A}$  forges  $\mathsf{msg'} = \langle C, W'_c \rangle$  and  $\mathsf{msg'}_1 = \langle C, E'_a \rangle$  where  $E'_a = \mathsf{E}(g^{\mathsf{pw}_1^*h_1^{*-1}}, pk_a)$  and  $h_1^* = H_2$   $(C, W'_c, A, W_a)$  and  $\mathsf{pw}_1^*$  is chosen by the adversary. After receiving  $\mathsf{msg'}_1$ , the server A derives  $\omega_a = g^{\mathsf{pw}_1^* - \mathsf{pw}_{C,A}} = g^{\mathsf{pw}_1^* + \mathsf{pw}_{C,B} - \mathsf{pw}_C} = g^{\mathsf{pw}_1^* + \mathsf{pw}_{C,B}}/g^{\mathsf{pw}_C}$  and then runs two-party PAKE protocol P with the server B (controlled by the adversary) on the basis of  $\omega_a$ . In the view of the adversary,  $\omega_a$  can be anyone of  $\{g^{\mathsf{pw}_1^* + \mathsf{pw}_{C,B}}/g^{\mathsf{pw}}|\mathsf{pw} \in \mathcal{D}\}$  even if he knows  $\mathsf{pw}_1^* + \mathsf{pw}_{C,B}$ . Without knowing  $\omega_a$ , this probability of  $\mathsf{acc}_A^P = \mathsf{TRUE}$  is less than  $Q^P(k)/N + \varepsilon(k)$ , where  $Q^P(k)$  denotes the number of on-line attacks in the protocol P, because we assume that P is a secure two-party PAKE protocol with explicit authentication.

In experiment  $P_5$ , the adversary's probability of success when neither  $Succ_1$  nor  $Succ_2$  occurs is 1/2. The preceding discussion implies that

 $\mathsf{Pr}_{\mathcal{A}}^{P_6}[\mathsf{Succ}] \leq Q(k)/N + \varepsilon(k) + 1/2 \cdot (1 - Q(k)/N - \varepsilon(k)),$  and thus the adversary's advantage in experiment  $P_5$ 

$$\begin{split} \mathsf{Adv}^{P_5}_{\mathcal{A}}(k) &= 2\mathsf{Pr}^{P_5}_{\mathcal{A}}[\mathsf{Succ}] - 1 \\ &\leq 2Q(k)/N + 2\varepsilon(k) + 1 - Q(k)/N - \varepsilon(k) - 1 \\ &= Q(k)/N + \varepsilon(k), \end{split}$$

for some negligible function  $\varepsilon(\cdot).$  The sequence of claims proved above show that

$$\mathrm{Adv}_{\mathcal{A}}^{P_0}(k) \leq \mathrm{Adv}_{\mathcal{A}}^{P_5}(k) + \varepsilon(k) \leq Q(k)/N + \varepsilon(k),$$

for some negligible function  $\varepsilon(\cdot)$ . This completes the proof of the theorem.  $\qed$ 

## 4.2 Security of ID2S PAKE Protocol Based on IBE

**Theorem 2.** Assuming that (1) the identity-based encryption scheme is secure against the chosen-ciphertext attack; (2) the public key encryption scheme E is secure against the chosen-ciphertext attack; (3) the decisional Diffie-Hellman problem is hard over (G,g,q); (4) the protocol P is a secure two-party PAKE protocol with explicit authentication; (5)  $H_1, H_2$  are collision-resistant hash functions, then the protocol P' illustrated in  $Fig.\ 2$  is a secure  $ID2S\ PAKE$  protocol according to Definition 1.

**Proof.** Given an adversary  $\mathcal{A}$  attacking the protocol, a simulator  $\mathcal{S}$  runs the protocol for  $\mathcal{A}$ . First of all, the simulator  $\mathcal{S}$  initializes the system by generating params = params  $^{P,IBE,E} \bigcup \{(\mathbb{G},\mathcal{G},n),(G,q,g),(H_1,H_2)\}$  and the secret master-key  $^{IBE}$ . Next, Client, Server, and ClientServerTriple sets are determined. Passwords for clients are chosen at random and split, and then stored at corresponding servers. Private keys for servers are computed using master-key  $^{IBE}$ .

The public information is provided to the adversary. Considering  $(C, A, B) \in \mathsf{ClinetServerTriple}$ , we assume that the adversary  $\mathcal{A}$  chooses the server B to corrupt and the simulator  $\mathcal{S}$  gives the adversary  $\mathcal{A}$  the information held by the corrupted server B, including the private key of the server B, i.e.,  $d_B$ , and one share of the password of the client C,  $\mathcal{G}^{\mathsf{pw}_{C,B}}$  and  $g^{\mathsf{pw}_{C,B}^*}$ . After computing the appropriate answer to any oracle query, the simulator  $\mathcal{S}$  provides the adversary  $\mathcal{A}$  with the internal state of the corrupted server B involved in the query.

We view the adversary's queries to its **Send** oracles as queries to four different oracles as follows:

- Send(C, i, A, B) represents a request for instance  $C^i$  of client C to initiate the protocol. The output of this query is  $msg_1 = \langle C, W_c, pk, E_a \rangle$  and  $msg_2 = \langle C, W_c, pk, E_b \rangle$ .
- $\operatorname{Send}(A,j,C,\operatorname{msg}_1)$  represents sending message  $\operatorname{msg}_1$  to instance  $A^j$  of the server A. The output of this query is either  $\operatorname{msg}_A = \langle A, W_a, E_1 \rangle$  or  $\bot$ .
- $\operatorname{Send}(C, i, A, B, \operatorname{msg}_A|\operatorname{msg}_B)$  represents sending the message  $\operatorname{msg}_A|\operatorname{msg}_B$  to instance  $C^i$  of the client C. The output is either  $\operatorname{acc}_C^i = \operatorname{TRUE}$  or  $\bot$ .
- Send<sup>P</sup>(A, j, B, M) represents sending message M to instance A<sup>j</sup> of the server A, supposedly by the server B, in the two-party PAKE protocol P. The input and output of this query depends on the protocol P.

We refer to the real execution of the experiment, as described above, as  $P_0$ .

Experiment  $P_1$ : In this experiment, the simulator interacts with the adversary as  $P_0$  except that the adversary does not succeed, and the experiment is aborted, if any of the following occurs:

- 1) At any point during the experiment, an oracle-generated message is repeated.
- 2) At any point during the experiment, a collision occurs in the hash function  $H_1$  or  $H_2$

Claim 1. If  $H_1$  and  $H_2$  are collision-resistant hash functions,  $|\mathsf{Adv}_{\mathcal{A}}^{P_0}(k) - \mathsf{Adv}_{\mathcal{A}}^{P_1}(k)|$  is negligible.

Experiment  $P_2$ : In this experiment, the simulator interacts with the adversary  $\mathcal{A}$  as in  $P_1$  except that the adversary's queries to Execute oracles are handled differently: in any Execute(C,i,A,j,B,k), where the adversary  $\mathcal{A}$  has not queried corrupt(A), but may have queried corrupt(B), (1) the plaintext  $\mathcal{G}^{\mathsf{pw}_1h^{-1}}$  in  $E_a$  is replaced with a random element in  $\mathbb{G}$ ; (2) the plaintext  $g^{\mathsf{pw}_{C,A}^*h_a^{-1}}$  in  $E_a'$  is replaced by a random element in G; (3) the session keys  $\mathsf{sk}_{C,A}$  and  $\mathsf{sk}_{A,C}$  are replaced with a same random element in G.

Claim 2. If (1) the identity-based encryption scheme is secure against the chosen-ciphertext attack; (2) the public key encryption scheme E is secure against the chosen-ciphertext attack; (3) the decisional Diffie-Hellman problem is hard over (G,g,q),  $|\mathsf{Adv}_{\mathcal{A}}^{P_1}(k)|$  Adv $_{\mathcal{A}}^{P_2}(k)$  is negligible.

Experiment  $P_3$ : In this experiment, we modify the simulator's responses to  $\operatorname{Send}(A,j,C,\operatorname{msg}_1)$  and  $\operatorname{Send}(C,i,A,B,\operatorname{msg}_A|\operatorname{msg}_B)$  queries. If the adversary ever queries  $\operatorname{Send}(A,j,C,\operatorname{msg}_1)$  oracle to a fresh server instance  $A^j$  with an adversarially-generated and valid  $\operatorname{msg}_1$  (denoted as  $\operatorname{acc}_A^j = \nabla$ ) or  $\operatorname{Send}(C,i,A,B,\operatorname{msg}_A|\operatorname{msg}_B)$  oracle to a fresh client instance  $C^i$  with an adversarially-generated and valid  $\operatorname{msg}_A|\operatorname{msg}_B$  (denoted as  $\operatorname{acc}_C^i = \nabla$ ), the simulator halts and the adversary succeeds. Otherwise the adversary's success is determined as in experiment  $P_2$ .

Claim 3.Adv<sub>A</sub><sup>P<sub>2</sub></sup> $(k) \leq Adv_A^{P_3}(k)$ .

Experiment  $P_4$ : In this experiment, the simulator interacts with the adversary  $\mathcal A$  as in experiment  $P_3$  except that the adversary's queries to  $\mathsf{Send}(A,j,C,\mathsf{msg}_1)$  and  $\mathsf{Send}(C,i,A,B,\mathsf{msg}_A|\mathsf{msg}_B)$  oracles are handled differently: in any  $\mathsf{Send}(A,j,C,\mathsf{msg}_1)$  or  $\mathsf{Send}(C,i,A,B,\mathsf{msg}_A|\mathsf{msg}_B)$  oracles where the adversary  $\mathcal A$  has not queried  $\mathsf{corrupt}(A)$ , but may have queried  $\mathsf{corrupt}(B)$ , the plaintext  $\mathcal G^{\mathsf{pw}_1h^{-1}}$  in  $E_a$  is replaced with a random element in  $\mathbb G$  and the plaintext  $g^{\mathsf{pw}_{C,A}^*h_a^{-1}}$  in  $E_a'$  (if any) is replaced with a random element in G.

Claim 4. If both the IBE scheme and the public key encryption scheme E are semantically secure,  $|\mathsf{Adv}_{\mathcal{A}}^{P_3}(k) - \mathsf{Adv}_{\mathcal{A}}^{P_4}(k)|$  is negligible.

The adversary  $\mathcal{A}$  succeeds only if one of the following occurs: (1) the adversary queries  $\mathsf{Send}(A,j,C,\mathsf{msg}_1)$  oracle to a fresh server instance  $A^j$  with  $\mathsf{acc}_A^j = \nabla$  (let  $\mathsf{Succ}_1$  denote this event); (2) the adversary queries  $\mathsf{Send}(C,i,A,B,\;\mathsf{msg}_A|\mathsf{msg}_B)$  oracle to a fresh client instance  $C^i$  with  $\mathsf{acc}_C^i = \nabla$  (let  $\mathsf{Succ}_2$  denote this event); (3) neither  $\mathsf{Succ}_1$  nor  $\mathsf{Succ}_2$  happens, the adversary wins the game by a Test query to a fresh instance  $C^i$  or a server instance  $A^j$ .

Katz et al. Protocol [23] Our IBS-based Protocol Our IBE-based Protocol Public Keys Client: None Client: None Client: None Sever A: Public Key  $pk_A$ Server A: AServer A: AServer B: B Server B: BSever B: Public Key  $pk_B$ Client:  $pw_C$ Client:  $pw_C$ Private Keys Client: pw<sub>C</sub> Server  $A: \mathcal{G}^{\mathsf{pw}_{C,A}}, \overset{\cdot}{g^{\mathsf{pw}_{C,A}^*}}, d_A$ Server  $A: g^{\mathsf{pw}_{C,A}}, d_A$ Sever A:  $pw_{C,A}$ , Private Key  $sk_A$ Server  $B: \mathcal{G}^{\mathsf{pw}_{C,B}}, g^{\mathsf{pw}^*_{C,B}}, d_B$ Server  $B: g^{\mathsf{pw}_{C,B}}, d_B$ Sever  $B: q^{\mathsf{pw}_{C,B}}$ , Private Key  $sk_B$ Client: 21(exp.)+1(Sign) Client: 23(Exp.)+6(Pair) Client: 23(Exp.) Computation Complexity Server: about 6(KOY) Server: about 1(KOY)+9(Exp)Server: about 1(KOY)+2(Pair)+9(Exp.)Communication Client/Server: 27(exp.)+1(Sign) Client/Server: 22(Exp.) Client/Server: 24(Exp.) Server/Server: about 1(KOY) Complexity Server/Server: about 2(KOY) Server/Server: about 1(KOY)

TABLE 1
Performance Comparison of Katz et al. Protocol and Our Protocols

To evaluate  $Pr[Succ_1]$  and  $Pr[Succ_2]$ , we assume that the adversary A has corrupted the server B and consider two as follows.

Case 1. The adversary  $\mathcal{A}$  forges  $\operatorname{msg}_1' = \langle C, W_c', pk', E_a' \rangle$  where  $E_a' = \operatorname{IBE}(\mathcal{G}^{\operatorname{pw}_1^*h^{*-1}}, A), h^* = H_1(C, W_c', pk'), W_c', pk',$   $\operatorname{pw}_1'$  are chosen by the adversary, and sends  $\operatorname{msg}_1'$  to the server A. After receiving  $\operatorname{msg}_1'$ , the server A derives  $\omega_a = \mathcal{G}^{\operatorname{pw}_1^*-\operatorname{pw}_{C,A}} = \mathcal{G}^{\operatorname{pw}_1^*+\operatorname{pw}_{C,B}-\operatorname{pw}_C} = \mathcal{G}^{\operatorname{pw}_1^*+\operatorname{pw}_{C,B}}/\mathcal{G}^{\operatorname{pw}_C}$  and then runs two-party PAKE protocol P with the server B (controlled by the adversary) on the basis of  $\omega_a$ . In the view of the adversary,  $\omega_a$  can be anyone of  $\{\mathcal{G}^{\operatorname{pw}_1^*+\operatorname{pw}_{C,B}}/\mathcal{G}^{\operatorname{pw}}|\operatorname{pw}\in\mathcal{D}\}$ . If P is a secure two-party PAKE protocol with explicit authentication,  $\operatorname{Pr}[\operatorname{Succ}_1] \leq Q^P(k)/N + \varepsilon(k)$ .

Case 2. Given  $C, W_c$ , for X = A or B, x = a or b, the adversary  $\mathcal{A}$  forges  $\mathsf{msg}'_X = \langle X, W'_x, E'_x \rangle$ , where  $E'_x = \mathsf{E}(g^{\mathsf{pw}^*_x h^{*-1}_x}, pk)$ ,  $h^*_x = H_2(X, W'_x, C, W_c)$ , and  $W'_x, \mathsf{pw}^*_x$  are chosen by the adversary. Then  $\mathcal{A}$  sends  $\mathsf{msg}'_A | \mathsf{msg}'_B$  to the client. In this case, the event  $\mathsf{Succ}_2$  occurs if and only if  $\mathsf{pw}^*_a + \mathsf{pw}^*_b = \mathsf{pw}_C$ . Therefore,  $\mathsf{Pr}[\mathsf{Succ}_2] \leq Q^C(k)/N$ , where  $Q^C(k)$  denotes the number of on-line attacks to the client instance  $C^i$ .

The above discussion shows that

Claim 5. If (1) P is a secure two-party PAKE protocol with explicit authentication; (2) the IBE scheme and the public key encryption scheme are secure against the chosen-ciphertext attack; then  $\Pr[\operatorname{Succ}_1 \vee \operatorname{Succ}_2] \leq Q(k)/N + \varepsilon(k)$ , where Q(k) denotes the number of on-line attacks.

In  $P_4$ , the adversary's probability of success when neither  $Succ_1$  nor  $Succ_2$  occurs is 1/2. This implies that

$$\Pr_A^{P_4}[\operatorname{Succ}] \leq Q(k)/N + \varepsilon(k) + 1/2 \cdot (1 - Q(k)/N - \varepsilon(k)),$$

and thus the adversary's advantage in experiment  $P_0$ 

$$\operatorname{Adv}_{\mathcal{A}}^{P_0}(k) \leq \operatorname{Adv}_{\mathcal{A}}^{P_4}(k) + \varepsilon(k) \leq Q(k)/N + \varepsilon(k),$$

TABLE 2
Execution Time of Protocols (in Milliseconds)

	Katz et al. Protocol [23]	Our IBS-based Protocol	Our IBE-based Protocol
Client	1.26	5.26	7.08
Server A	5.31	4.14	2.08
Server B	5.31	3.82	1.76

for some negligible function  $\varepsilon(\cdot)$ . This completes the proof.

## 5 Performance Analysis

The efficiency of the compiled protocols using our compilers depends on performance of the underlying protocols.

In our IBS-based protocol, if we use the KOY two-party PAKE protocol [22], the Paterson et al.'s IBS scheme [25] and the Cramer-Shoup public key encryption scheme [13] as cryptographic building blocks, the performance of our IBS-based protocol can be shown in Table 1. In our IBEbased protocol, if we use the KOY two-party PAKE protocol [22], the Waters IBE scheme [28] and the Cramer-Shoup public key encryption scheme [13] as cryptographic building blocks, the performance of our IBE-based protocol can also be shown in Table 1. In addition, we compare our protocols with the Katz et al. two-server PAKE protocol [23] (secure against active adversary) in Table 1. In Table 1, exp., Sign. and Pair for computation represent the computation complexities of a modular exponentiation over an elliptic curve, a modular exponentiation over  $Z_p$ , a signature generation and a pairing, respectively, and Exp., exp. and Sign. in communication denote the size of the modulus and the size of the signature, and KOY stands for the computation or communication complexity of the KOY protocol.

In Table 1, different operations are computed in different protocols. For example, some modular exponentiations in our protocols are over an elliptic curve group, while the modular exponentiations in the Katz et al.'s protocol are over  $\mathbb{Z}_p$  only. Our protocols need to compute pairings while the Katz et al.'s protocol does not. In order to further compare their performance, we implement our two protocols.

To realize the modular exponentiation  $\mathcal{G}^x$  over an elliptic curve group  $\mathbb{G}$  and the pairing map  $e:\mathbb{G}\times\mathbb{G}\to\mathbb{G}_T$  in our protocols, we build our implementation on top of the PBC pairing-based cryptography library, whereas the multiplicative group over the prime integer p is based on the GNU MP library. Moreover, the elliptic curve we use is the A512 ECC in which the first two groups are the same, i.e., a symmetric pairing. Another library mbed TLS³ is adopted due to the invocations of AES and SHA-512 for the one-time signature in KOY. All the experiments were conducted in

<sup>1.</sup> https://crypto.stanford.edu/pbc/download.html

<sup>2.</sup> https://gmplib.org/

<sup>3.</sup> https://tls.mbed.org/

Ubuntu 14.04 running on a computer equipped with an Intel i7-4770HQ CPU and 16 GBytes of memory. When implementing our protocols, we also performed optimization when applicable. For example, we compute the Waters' hash function by parallel computation.

The execution time of our two protocols compared with the Katz et al.'s protocol can be shown in Table 2. From Table 2, we can see that the client performance in Katz et al.' s protocol is better than our protocols, but the execution times for client in the three protocols are all less than 10 ms. The server performance in our protocols is better than the Katz et al.'s protocol, saving from 22 to 66 percent of computation. When the servers provide services to a great number of clients concurrently, the server performance is critical to the performance of the whole protocol. For example, assume that Servers A and B provide services to 100 clients concurrently and there is no communication delay, the longest waiting time with respect to a client for our IBE-based protocol is around 7.08+208+176=391.08 ms while the Katz et al.'s protocol takes about 1.26+531+531=1,063.26 ms. The difference is 672.18 ms.

In terms of communication complexity, the size of a group element over elliptic curve (denoted as Exp.) in our protocols can be 512 bits, while the size of a group element over  $\mathbb{Z}_p$  in the Katz et al.'s protocol [23] has to be 1,024 bits. From Table 1, we can see that the communication complexity of our protocols is about a half of the Katz et al.'s protocol [23]. For a more comprehensive comparison between ours and Katz et al.'s protocol, please refer to our supplementary material for further details.

#### 6 Conclusion

In this paper, we present two efficient compilers to transform any two-party PAKE protocol to an ID2S PAKE protocol with identity-based cryptography. In addition, we have provided a rigorous proof of security for our compilers without random oracle. Our compilers are in particular suitable for the applications of password-based authentication where an identity-based system has already been established. Our future work is to construct an identity-based multiple-server PAKE protocol with any two-party PAKE protocol.

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