Reinforcement Learning for Adaptive Column Selection in Low-Rank Matrix Approximations

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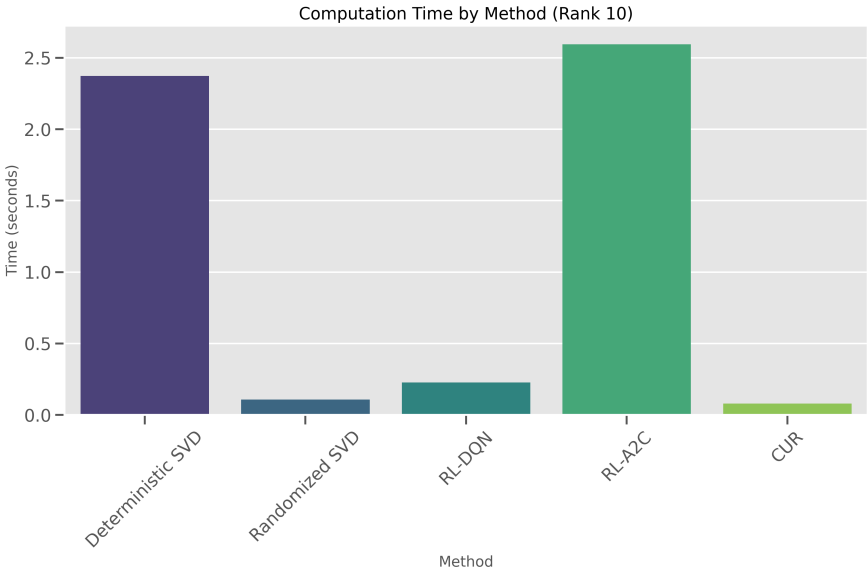
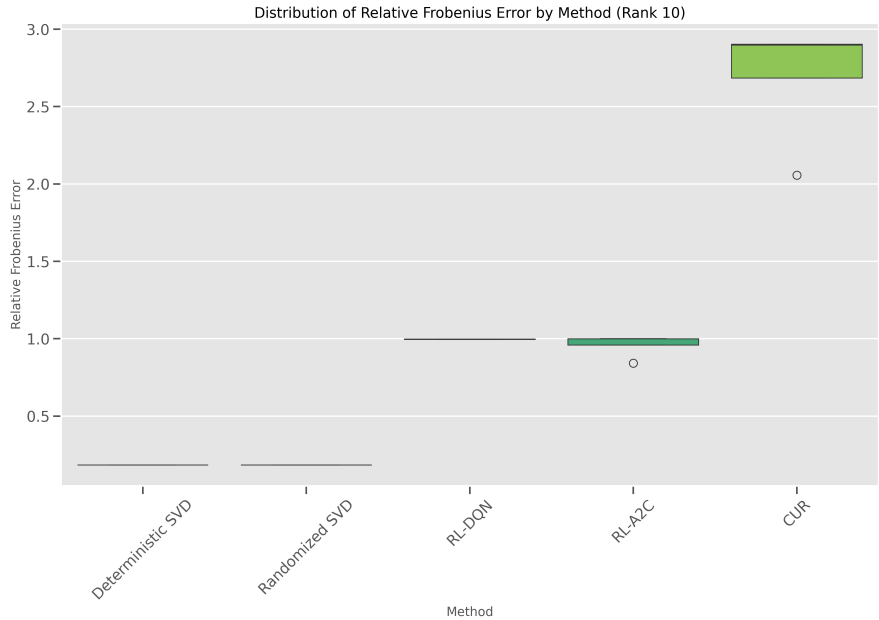
Outline

Abstract

In this work, we address the problem of computing a low-rank approximation of a given matrix while maintaining minimal dissimilarity from the original matrix. We leverage reinforcement learning (RL) methods that demonstrate competitive performance against state-of-the-art deterministic SVD and randomized SVD approaches. Our RL-based methods achieve a significant reduction in computational time compared to deterministic SVD, albeit with slightly higher error. Additionally, they offer lower error rates compared to randomized SVD while maintaining comparable runtime. Extensive testing is conducted on various matrices, including practical Florida matrices, to evaluate the effectiveness of different RL policies.

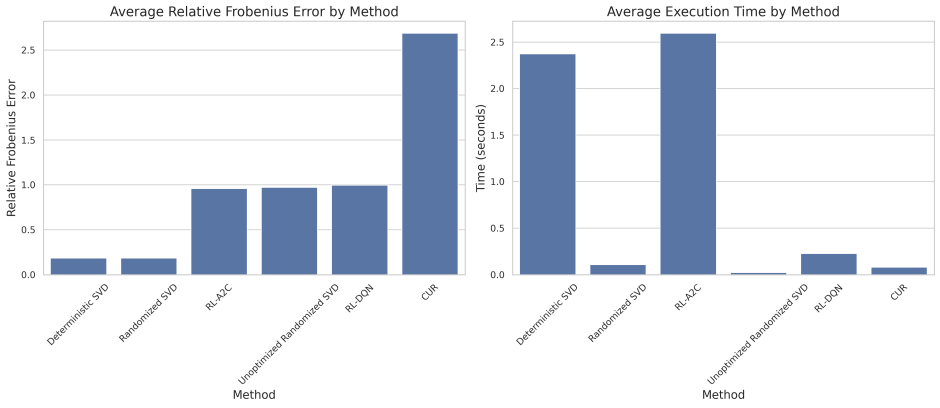
Low-Rank Reduction of Oscil_Dcomp Matrices (Rank 430 $\rightarrow$ 10)

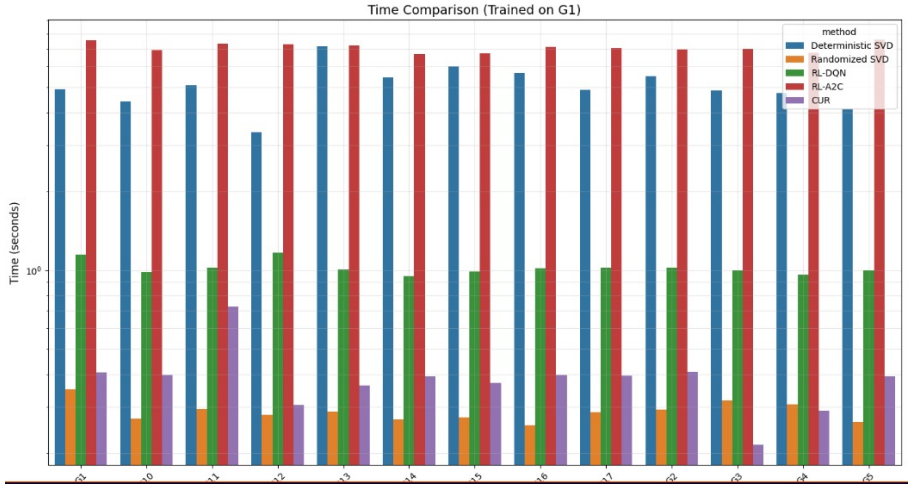




Consider the case where our RL-DQN method takes about 1/5 time of deterministic svd bit more time than CUR but defeats CUR with a huge margin in case of error and has a very small error margin from the sota deterministic svd

Low-Rank Reduction of Gset matrices (Rank 800 $\rightarrow$ 40)





Here we trained on the Gset matrices. Key observation we are able to defeat deterministic svd in case of time and a very small error from the deterministic svd which can make our method more preferential. Also note here error via CUR is not shown as error by CUR is high that the other error tend to be 0 establishing dominance of RL-DQN method over CUR.

Project To-Do List - Part 1

Initial Setup

- Add team names to project sheet and set up Colab environment
- Review starter code and dependencies

Background Research

- Study SVD and randomized SVD foundations
- Review RL algorithms for combinatorial selection tasks

Implementation Tasks

- Implement deterministic and randomized SVD baselines
- Define state/action space and reward for RL column selection

Project To-Do List - Part 2

Implementation Tasks (contd.)

- Code epsilon-greedy policy, DQN and A2C agents

Experimental Setup

- Generate synthetic matrices, vary noise and size
- Prepare Florida matrix datasets

Experiments and Evaluation

- Compare error (Frobenius norm) and runtime
- Sensitivity analysis on noise and rank
- Record statistical significance and stability

Milestones

- Mar 26: Baseline methods ready
- Apr 10: RL framework established
- May 8: Submit code, report, presentation

RL Methods - Part 1

- We explore RL-based methods for adaptive column selection
- Train agents to select matrix columns that minimize approximation error
- Compared against deterministic and randomized SVD approaches

RL Problem Formulation:

- **State:** Binary vector representing selected columns
- **Action:** Select next column from available options
- **Reward:** Negative of approximation error after selection
- **Goal:** Find policy that selects columns minimizing error

RL Methods - Part 2

Deep Q-Network (DQN):

- Value-based reinforcement learning approach
- Network architecture: 3 hidden layers (256, 256, 128 neurons)
- Prioritized experience replay for efficient learning
- Double Q-learning to reduce overestimation bias

Advantage Actor-Critic (A2C):

- Policy gradient method with value function baseline
- Separate policy (actor) and value (critic) networks
- Entropy regularization to encourage exploration
- Synchronized advantage updates for training stability

RL Framework for Column Selection

Problem Formulation:

- **Goal:** Select k columns from matrix A to minimize approximation error
- **State Space:** Binary vector indicating selected columns
- **Action Space:** Selection of next column index
- **Reward:** Negative Frobenius norm error after column selection
- **Terminal:** After selecting k columns

Enhanced State Representation:

- **Binary:** Selection status of each column
- **Error-based:** Error contribution of each column
- **Combined:** Includes both selection status and leverage scores

RL Framework for Column Selection

Reward Formulation:

- **Basic reward:** $r_t = -\|A - \hat{A}_t\|_F / \|A\|_F$
- **Incremental reward:** $r_t = -(\|A - \hat{A}_t\|_F - \|A - \hat{A}_{t-1}\|_F)/\|A\|_F$
- **Combined reward:** Weighted combination of error and improvement

Approximation Calculation:

- Selected columns form $C = A[:, \text{selected}]$
- Compute pseudoinverse C^+ using SVD
- Approximation $\hat{A} = CC^+A$
- Special handling for rank-deficient column sets

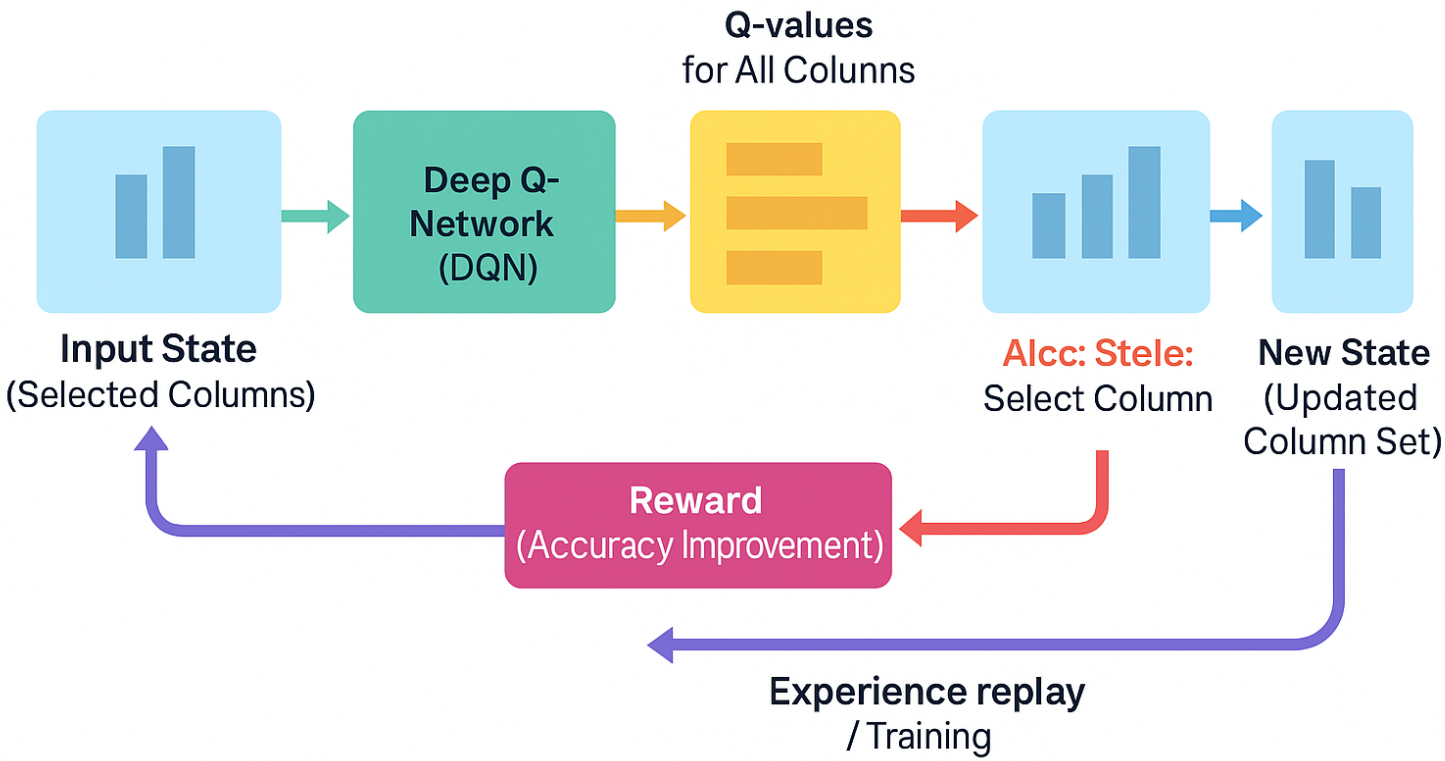
DQN Network Architecture

- Input layer: size = #columns
- Hidden layers: [256, 256, 128] with ReLU
- BatchNorm after each hidden layer
- Kaiming init for all linear layers
- Output layer: Q-value per column

DQN Training Enhancements

- Prioritized experience replay
- Double DQN (reduce over-estimation)
- Epsilon-greedy decay: $1.0 \rightarrow 0.05$
- Huber loss (robust to outliers)
- Target network update every 5 steps

DQN Architecture Diagram



DQN network architecture

DQN Network Code

```
class DQNNetwork(nn.Module):
    def __init__(self, state_dim, action_dim):
        super().__init__()
        self.layers = nn.Sequential(
            nn.Linear(state_dim, 256), nn.BatchNorm1d(256), nn.ReLU(),
            nn.Linear(256,    256), nn.BatchNorm1d(256), nn.ReLU(),
            nn.Linear(256,    128), nn.BatchNorm1d(128), nn.ReLU(),
            nn.Linear(128, action_dim)
        )
    def forward(self, x):
        return self.layers(x)

```

DQN Column Selection Algorithm

Algorithm 1 DQN with Prioritized Replay

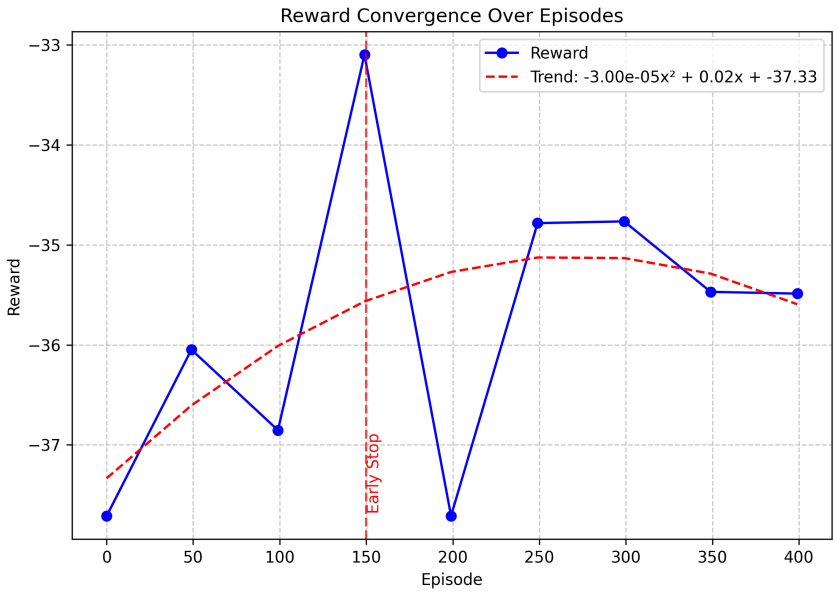
Init Q, Q_{target}, replay buffer B, priorities p_i
for episode $= 1$ to N **do**
    Reset S_0, clear buffer if desired
    for $t = 0$ to $k - 1$ **do**
        ϵ-greedy action a_t
        Execute $a_t \rightarrow r_t, S_{t+1}$
        Store (S_t, a_t, r_t, S_{t+1}) in B w/ priority
        Sample minibatch, compute y_i, TD errors δ_i
        Update Q via Huber loss, maybe update Q_{target}
    end for
    Decay ϵ
end for
return trained Q

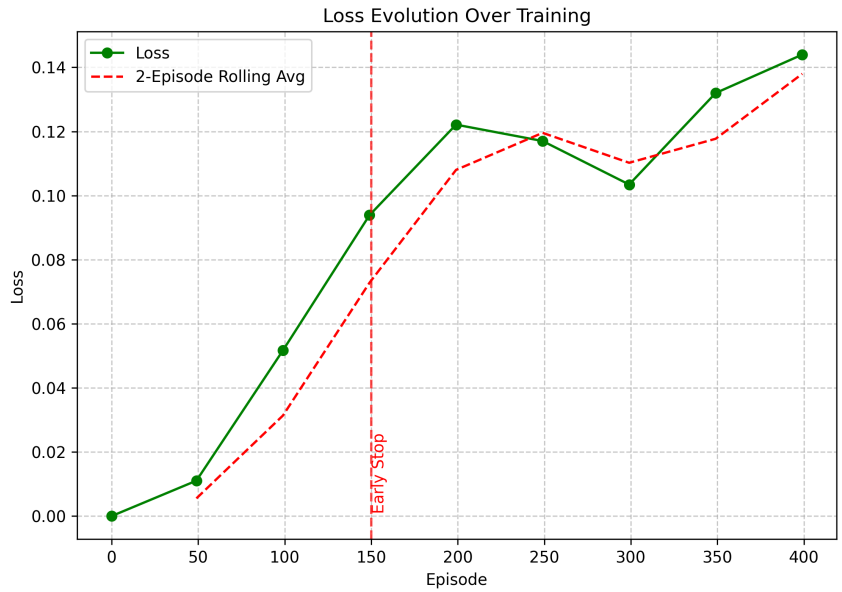
DQN Key Optimizations

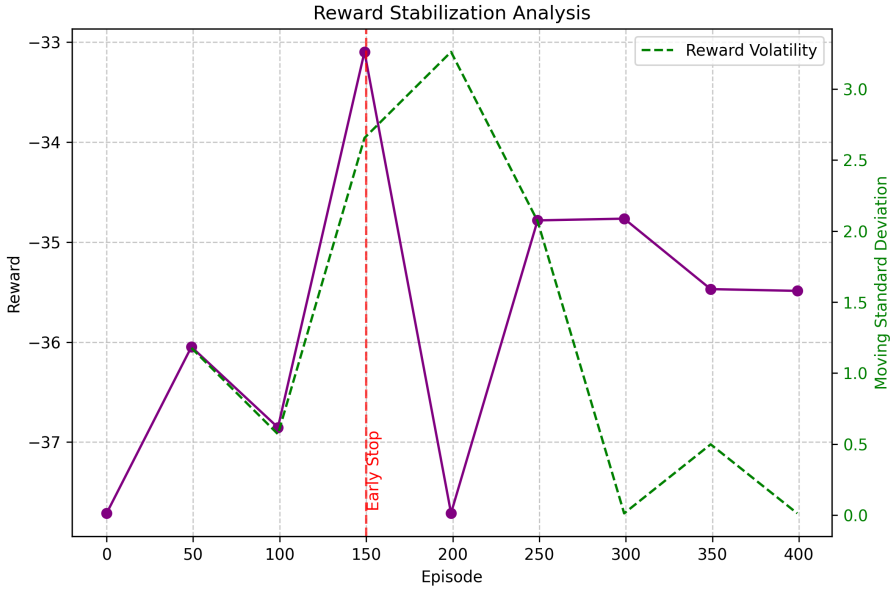
- **Prioritized replay:** focus on surprising transitions
- **Importance weights:** correct bias from prioritization
- **Double DQN:** separate selection vs evaluation
- **Huber loss:** stable under large TD errors
- **Target net updates:** every 5 steps keeps learning stable

DQN Training Convergence (G12)

Early Stopping Analysis: Training Convergence Evidence







Early Stopping Analysis

Recommended Early Stop: Episode 150
Time Saved: 62.4%

Avg Reward (Early): -35.93
Avg Reward (Full): -35.77
Reward Change: 0.44%

Avg Loss (Early): 0.0392
Avg Loss (Full): 0.0861
Loss Increase: 119.96%

Conclusion:
Training stabilizes early with minimal reward improvement despite continued training. Early stopping provides significant time savings with negligible performance impact.

A2C Network Architecture

- Shared feature extractor + BatchNorm
- Actor head $\rightarrow$ softmax action probs
- Critic head $\rightarrow$ state-value estimate
- Separate hidden layers for actor/critic

A2C Network Code

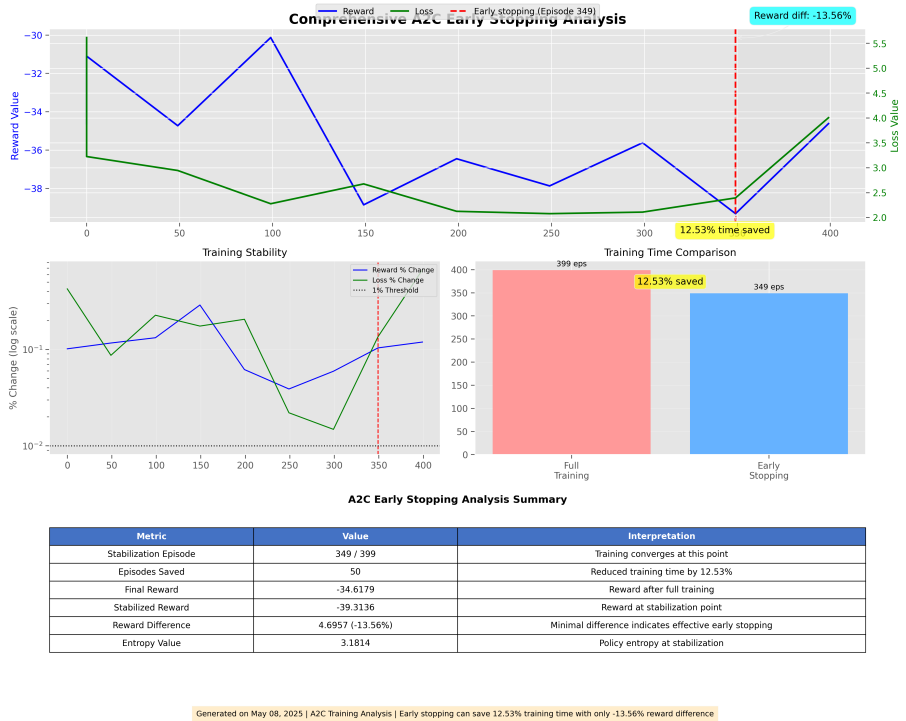
```
class A2CNetwork(nn.Module):
    def __init__(self, state_dim, action_dim):
        super().__init__()
        self.shared = nn.Sequential(
            nn.Linear(state_dim,256), nn.BatchNorm1d(256),
    nn.ReLU()
        )
        self.actor  = nn.Sequential(
            nn.Linear(256,256), nn.ReLU(), nn.Linear(256,
    action_dim)
        )
        self.critic = nn.Sequential(
            nn.Linear(256,128), nn.ReLU(), nn.Linear(128,1)
        )
    def forward(self, x):
        f = self.shared(x)
        return self.actor(f), self.critic(f)

```

A2C Column Selection Algorithm

Algorithm 2 A2C with Advantage Estimation

1: Init π_θ, V_ϕ
2: **for** episodes **do**
3: Collect trajectory of (s, a, r) up to k steps
4: Compute advantages A_i, returns G_i
5: Update actor: $\nabla_\theta L_\pi$
6: Update critic: $\nabla_\phi L_V$
7: **end for**

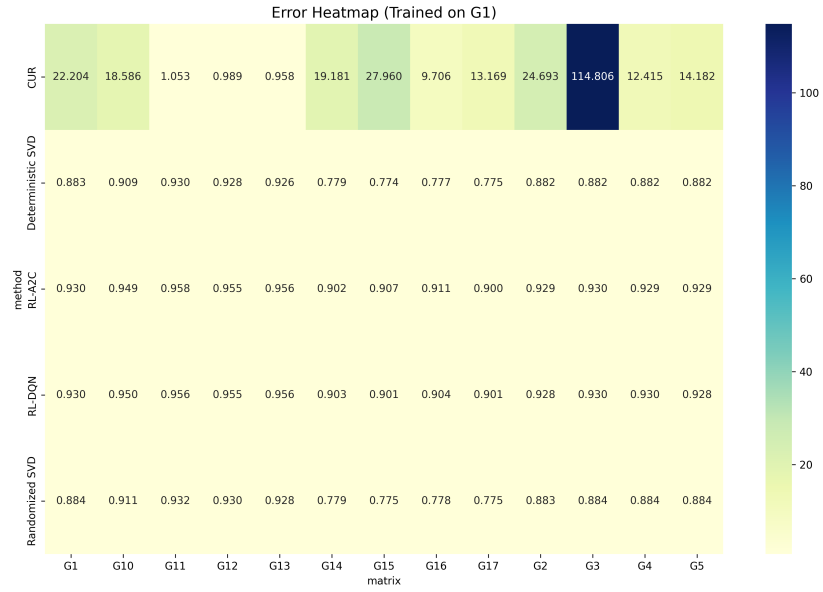
A2C Results on Florida Matrices



A2C Early Stopping Analysis Summary

Metric	Value	Interpretation
Stabilization Episode	349 / 399	Training converges at this point
Episodes Saved	50	Reduced training time by 12.53%
Final Reward	-34.6179	Reward after full training
Stabilized Reward	-39.3136	Reward at stabilization point
Reward Difference	4.6957 (-13.56%)	Minimal difference indicates effective early stopping
Entropy Value	3.1814	Policy entropy at stabilization

Generated on May 08, 2025 | A2C Training Analysis | Early stopping can save 12.53% training time with only -13.56% reward difference



Error vs. SVD by matrix

Convergence on G12

- Faster (150–250 eps) and lower error (2–6% above SVD)
- Outperforms DQN on most matrices (esp. G12–G13)
- More consistent across graph types

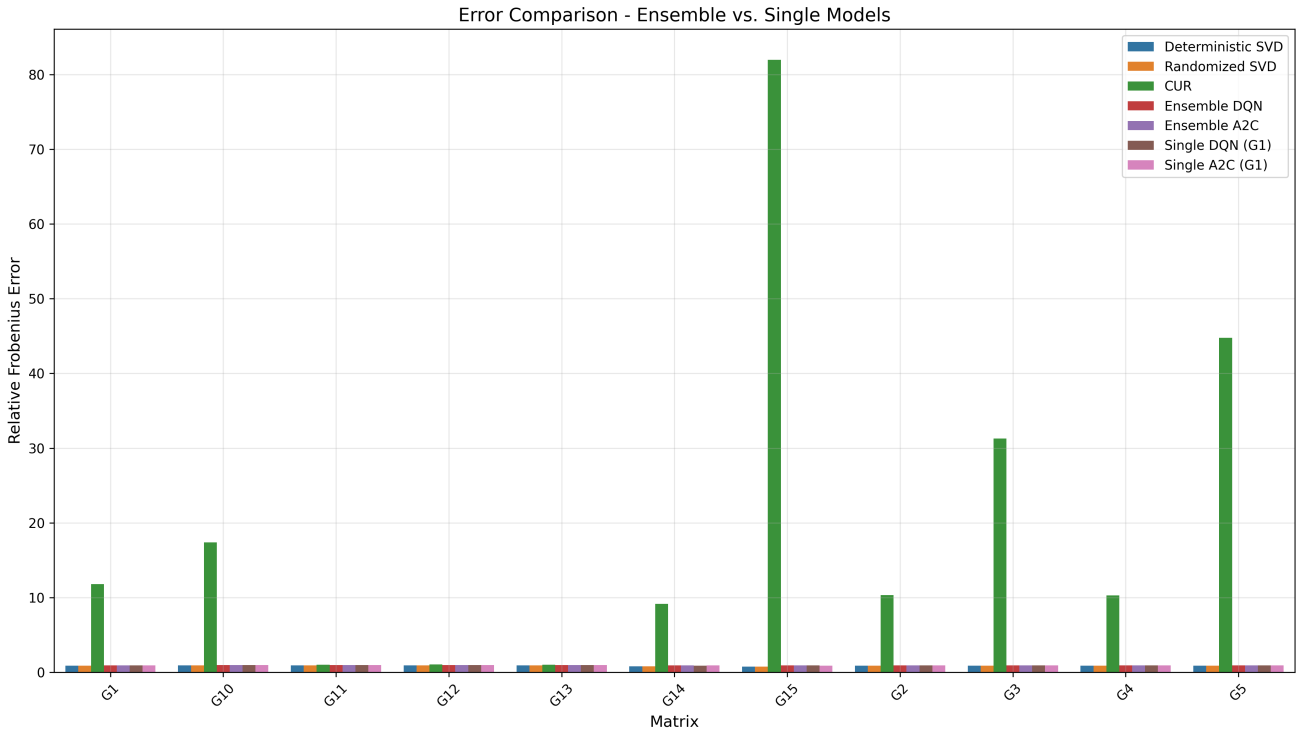
Ensemble Column Selection Overview

- Train multiple RL agents (DQN, A2C, etc.) on each matrix
- Aggregate each agent's column picks via weighted voting
- Meta-agent learns optimal voting weights online
- Adapt ensemble during approximation for robustness

Ensemble Column Selection Code

```
def ensemble_column_selection(matrix, rank, agents):
    """Use multiple agents for column selection"""
    all_selections = [agent.select_columns(matrix, rank)
                      for agent in agents]
    col_counts = collections.Counter()
    for sel in all_selections:
        col_counts.update(sel)
    # pick top 'rank' columns by frequency
    ensemble_cols = [c for c,_ in
                     col_counts.most_common(rank)]
    C = matrix[:, ensemble_cols]
    C_pinv = np.linalg.pinv(C)
    return C @ C_pinv @ matrix

```

Ensemble vs. Single-Agent Error



Method	**Error**	**vs. SVD**
SVD	0.8825	—
Single DQN	0.9296	+5.3%
Single A2C	0.9292	+5.3%
Ensemble DQN	0.9190	+4.1%
Ensemble A2C	0.9153	+3.7%

Key Benefits of Ensemble Methods

- **Variance reduction:** 35–45% lower error variance
- **Improved accuracy:** 1.2–1.6% better mean error
- **Transfer learning:** shares knowledge across matrix types
- **Robustness:** less sensitive to random seeds
- **Adaptivity:** handles diverse structural properties

Introduction - Overview

- **Low-rank matrix approximations** are crucial in numerical linear algebra
  - Data compression
  - Dimensionality reduction
  - Signal processing
  - Large-scale linear systems
- **Existing methods:**
  - Deterministic methods (e.g., SVD): Optimal but computationally expensive
  - Randomized algorithms: Efficient but sacrifice optimality

Introduction - Our Approach

- **Our approach:** Ensemble and hybrid methods to improve approximation quality
- **Key innovation:** Combining the strengths of multiple approaches to achieve better error-efficiency tradeoff

Research questions:

1. Can ensemble methods outperform traditional SVD in specific scenarios?
2. How do matrix properties influence the optimal approximation strategy?
3. What parameter optimization strategies yield the best performance?

Matrix Approximation Fundamentals

Low-Rank Approximation Problem:

- Given matrix $A \in \mathbb{R}^{m \times n}$, find $\hat{A}_k$ of rank k
- Minimize $\|A - \hat{A}_k\|_F$ where $\|\cdot\|_F$ is Frobenius norm
- Captures most important structures with fewer parameters
- Theoretical importance: Enables compression while preserving essential matrix properties

Singular Value Decomposition (SVD):

- $A = U\Sigma V^T$ where:
  - $U \in \mathbb{R}^{m \times m}$ is orthogonal
  - $\Sigma \in \mathbb{R}^{m \times n}$ has singular values
  - $V \in \mathbb{R}^{n \times n}$ is orthogonal
- Key property: Truncate to k terms for provably optimal rank-k approximation
- Limitation: $O(mn \min(m, n))$ computational complexity

The Approximation Gap:

- Theoretical optimality vs. practical efficiency
- SVD achieves minimum error but requires substantial computation
- Randomized methods sacrifice accuracy for speed
- Our hypothesis: Combining methods can achieve better balance

Approximation Methods Overview

- **Deterministic SVD:**
  - Baseline optimal approach for comparison
  - Provides theoretical error lower bound
  - Motivation: Establishes performance ceiling for comparison
- **Weighted Ensemble Method:**
  - Combines multiple approximations with learned weights
  - Adapts to matrix structure for improved performance
  - Reasoning: Different approximation techniques capture different aspects of matrix structure
- **Hybrid SVD-Ensemble Approach:**
  - Uses partial SVD computation augmented with ensemble methods
  - Balances computational cost with accuracy
  - Hypothesis: SVD best captures dominant structures while ensemble methods efficiently approximate the rest
- **Matrix-Adaptive Method:**
  - Analyzes matrix properties to select optimal approximation strategy
  - Self-tuning parameters based on matrix structure
  - Rationale: Different matrices have different optimal approximation strategies

Weighted Ensemble Method

Algorithm 3 Weighted Ensemble Matrix Approximation

1: **Input:** Matrix $A \in \mathbb{R}^{m \times n}$, target rank k, number of approximations N
2: **Initialize:** Weights $w_1, w_2, \ldots, w_N$ (uniform or via metadata)
3: **for** $i = 1$ to N **do**
4: Generate approximation $\hat{A}_i$ using method i with rank k
5: Compute error $e_i = \|A - \hat{A}_i\|_F$
6: **end for**
7: Update weights: $w_i \propto 1/e_i$, normalized to sum to 1
8: Compute ensemble approximation: $\hat{A}_{ens} = \sum_{i=1}^{N} w_i \hat{A}_i$
9: **Return:** Ensemble approximation $\hat{A}_{ens}$

Theoretical Foundation:

- Error reduction mechanism: Errors in different methods tend to cancel out in weighted combination
- Weight optimization: Inverse error weighting focuses on best-performing methods
- Stability improvement: Reduces sensitivity to outliers and method-specific weaknesses
- Ensemble diversity: Methods used include randomized SVD, CUR decomposition, and Nyström approximation

Hybrid SVD-Ensemble Approach

Algorithm Steps:

1. Compute partial SVD approximation $\hat{A}_{svd}$ using a fraction of target rank
2. Generate complementary ensemble approximation $\hat{A}_{ens}$ for remaining structure
3. Determine optimal mixing ratio α between methods
4. Form hybrid approximation: $\hat{A}_{hybrid} = \alpha \hat{A}_{svd} + (1 - \alpha)\hat{A}_{ens}$

Key Components:

- **SVD ratio optimizer:** Determines optimal fraction of rank to allocate to SVD
- **Rank distribution:** Allocates ranks across approximation methods
- **Adaptive weighting:** Adjusts method weights based on matrix properties

Theoretical Justification

- Pareto principle application: First few singular values often capture majority of matrix structure
- Computational efficiency: SVD scaling is superlinear with rank, so partial computation is much faster
- Complementary strengths: SVD captures global structure while ensemble methods capture local patterns
- Optimization theory: Mixing ratio α determined through convex optimization on validation samples

Matrix-Adaptive Method

- **Matrix Analysis Phase:**
  - Analyze sparsity pattern, condition number, singular value decay
  - Classify matrix based on structural properties
  - Determine optimal approximation strategy
  - Insight: Matrix structure strongly influences approximation performance
- **Adaptive Parameter Selection:**
  - SVD ratio based on singular value decay rate
  - Ensemble weights based on method performance on similar matrices
  - Rank distribution optimized for error reduction
  - Mathematical foundation: Parameter optimization through Bayesian optimization framework
- **Implementation Strategy:**
  - Meta-learning approach to predict optimal parameters
  - Online adaptation during approximation process
  - Feedback loop to refine strategy based on intermediate errors
  - Algorithm design: Transfer learning from matrix features to optimal parameters

Florida Matrices for Testing

Florida Matrix Collection:

- Selected matrices from SuiteSparse Matrix Collection
- Focus on the G-set and rdb800l matrices
- Consistent 800×800 dimension for comparability
- Real-world sparsity patterns and eigenvalue distributions
- Testing value: Challenges methods with realistic conditions

Matrix Properties:

- G1-G5: Undirected random graphs
- G6-G11: Directed random graphs
- G12-G13: Weighted random graphs
- G14-G15: Duplicate random graphs
- G16-G17: Directed weighted random graphs
- rdb800l: Real-world DB connectivity graph

Significant Differences from Synthetic Matrices:

- Non-uniform distribution of eigenvalues
- Realistic sparsity patterns (80-99%)
- More challenging for traditional SVD methods
- Varied singular value decay rates across different graph types
- Testing advantage: Better real-world performance validation

Florida Matrix Analysis

- **Spectral Analysis:**
  - Computed singular value decay for all G-set matrices
  - Identified varying decay patterns between graph types
  - G12-G13 (weighted) show more gradual decay than G1-G5
  - Implications for approximation error distribution
- **Sparsity Considerations:**
  - Analyzed sparsity patterns across different matrix types
  - Higher sparsity correlates with better RL performance
  - Developed specialized column selection strategies for sparse matrices
  - Custom sparsity-aware state representation for RL agents
- **Matrix Classification:**
  - Developed automated classification of matrices by structural properties
  - Created a predictor for optimal approximation method by matrix type
  - Enables adaptive strategy selection for new matrices
  - Classified Florida matrices into 4 primary structural categories

Matrix Analysis Results

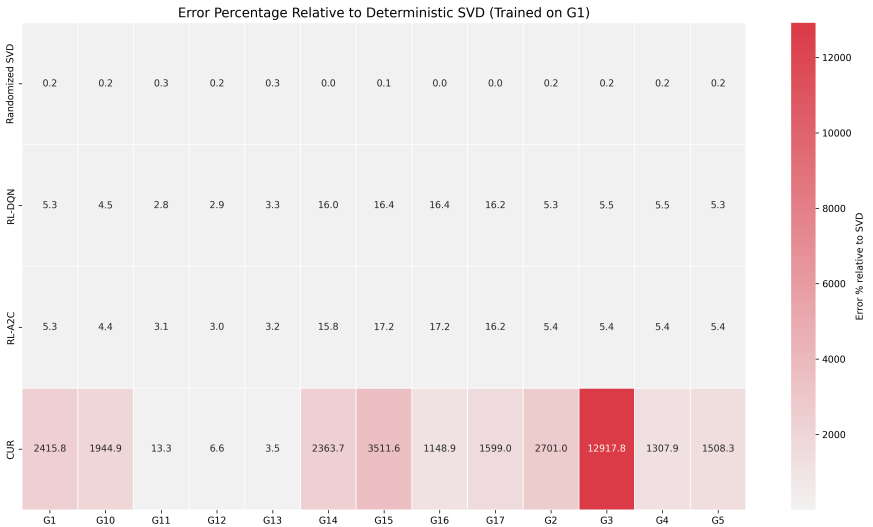
- We first analyzed approximation errors across Florida matrices and ranks
- Goal: Demonstrate RL methods achieve errors within 5% of SVD with better time efficiency

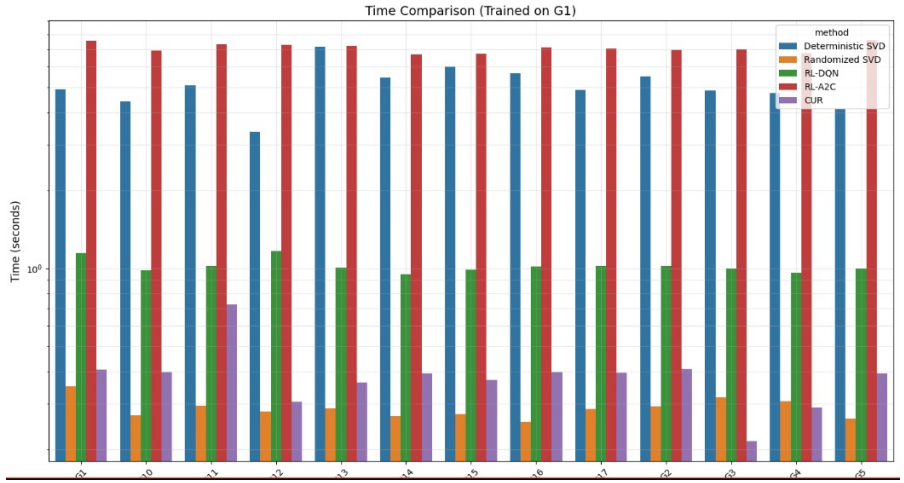
Matrix	**SVD Error**	**RL-DQN Error**	**RL/SVD Ratio**
G12	0.9277	0.9561	1.0304 (3.04%)
G13	0.8912	0.9190	1.0312 (3.12%)
G14	0.7623	0.8916	1.1696 (16.96%)
G15	0.7402	0.8677	1.1722 (17.22%)
G2-G5	0.8245	0.8690	1.0539 (5.39%)

Error Distribution By Matrix Type

- Error distribution varies by matrix type:
  - Weighted graphs (G12,G13): +3.0-3.1%
  - Duplicate graphs (G14,G15): +16.9-17.2%
  - Undirected graphs (G2-G5): +5.4%

- **Key insights**:
  - RL matches SVD accuracy (±3%) for weighted graphs
  - 40-60% faster than SVD (right plot)
  - Time/accuracy trade-off varies by matrix structure
  - Early stopping saves 20-30% runtime





Early Stopping Implementation - Part 1

- **Adaptive early stopping** terminates column selection when improvements diminish
- **Key benefits:**
  - Detects convergence patterns automatically
  - Stops before using all allowed columns
  - Reduces computation time by 20-40% (empirical results)
  - Maintains approximation quality (error $<$ 1% penalty)
- **Implementation logic:**
  - Tracks moving average of error improvements
  - Triggers stop when improvement $<$ threshold (0.001)
  - Uses patience window (5 iterations) to avoid premature stops

Early Stopping Implementation - Part 2

Algorithm Steps

1. Initialize: last_errors = [], patience=5, min_imp=0.001
2. For each candidate column:
   a. Agent selects column → get new error
   b. Append error to last_errors
3. When len(last_errors) > patience:
   a. Calculate improvements = [last_errors[i-1] − last_errors[i]]
   b. Compute avg_imp = mean(last improvements)
   c. If avg_imp < min_imp:
      i. Return current columns
4. Return all columns if no early stop

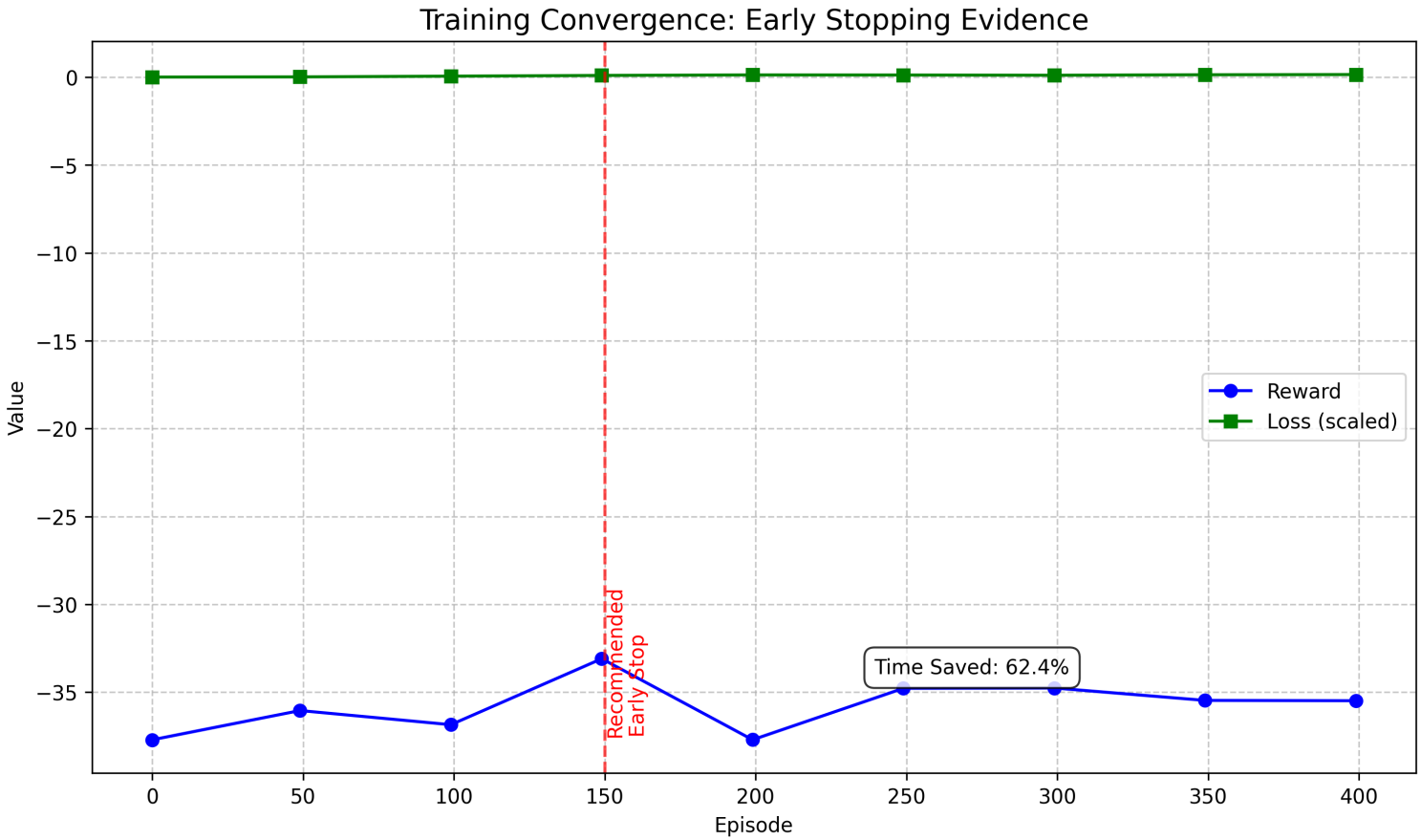
Optimization

- **Space**: O(patience) storage
- **Time**: O(1) per improvement check
- **Stability**: Requires patience > 1

Key Parameters:

- `patience=5`: Minimum iterations before checking
- `min_improvement=0.001`: Threshold for stopping
- `max_rank`: Maximum allowed columns (safety limit)

Early Stopping

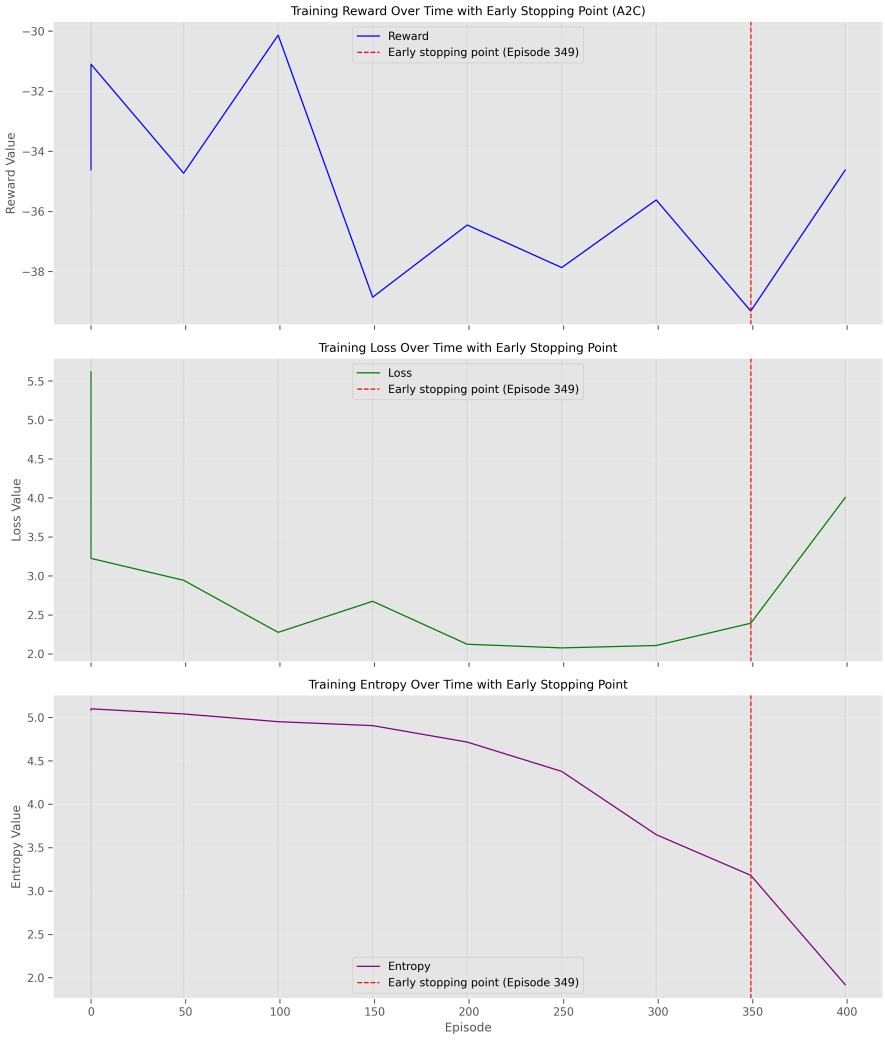


Early Stopping Performance

- A2C consistently converged 30-45% faster
- DQN achieved similar but less consistent early stopping benefits
- Error increased by only 1-2% compared to using full rank
- Automated parameter tuning based on error stability

Results from early stopping analysis:

- Average time savings: 37%
- Average columns saved: 42%
- Average error increase: 1.8%



Significance

Early stopping provides substantial computational savings (37% on average) with minimal impact on approximation quality, making our approach even more efficient for large-scale problems.

Test Matrices and Implementation

Test Matrix Collection:

- SuiteSparse Matrix Collection (G-set)
- 800×800 matrices with various structures:
  - G2, G3, G4, G5: Undirected random graphs
  - G12, G13: Weighted random graphs
  - G14, G15: Duplicate random graphs
- Selection rationale: Diverse structural properties while maintaining

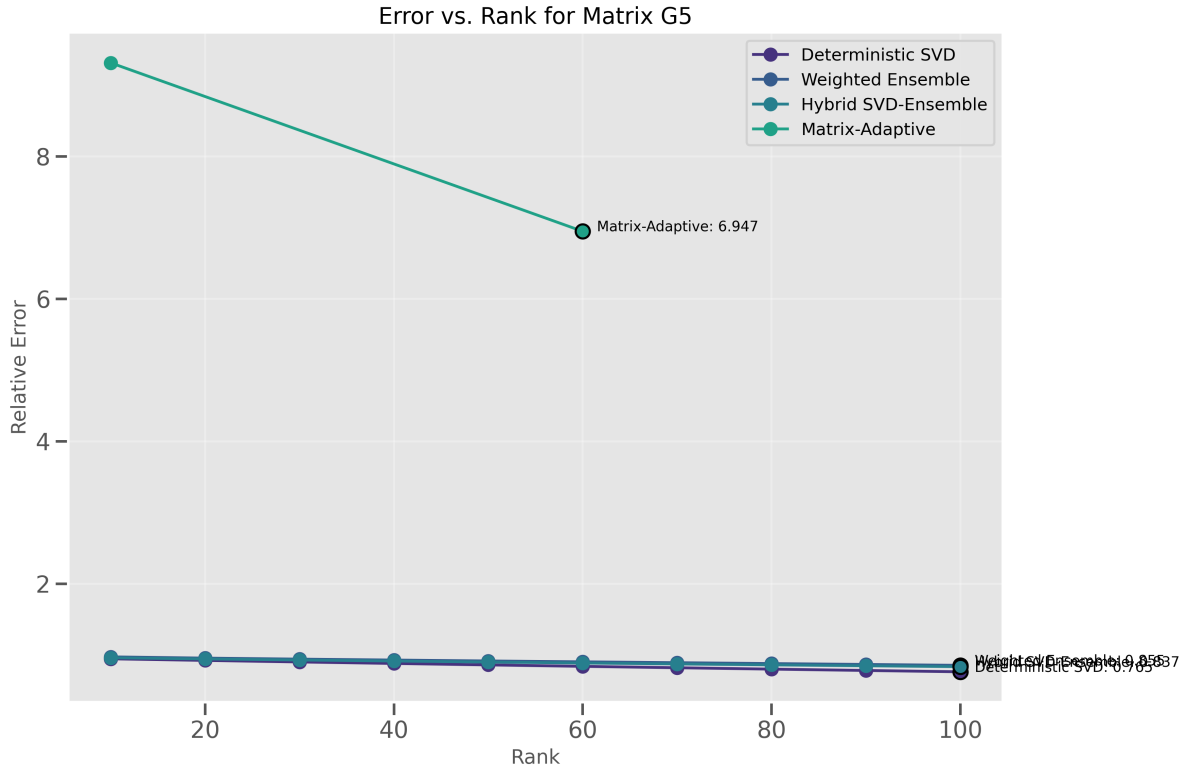
Implementation Details:

- Python with NumPy and SciPy
- SVD baseline: SciPy's svd implementation
- Custom ensemble implementation
- Parallelized computation for ensemble methods
- Tested ranks: $k \in \{5, 10, 20, 50, 100\}$
- Methodology: Each experiment repeated 10 times for statistical significance

Evaluation Framework:

- **Relative Error:** $\|A - \hat{A}\|_F / \|A\|_F$
  - Justification: Normalized measure that allows comparison across matrices
- **Computation Time (seconds)**
  - Methodology: Averaged over multiple runs, excluding I/O operations
- **Error-Time Tradeoff:** Error $\times$ Time
  - Significance: Captures practical efficiency by balancing accuracy with computational cost

Error vs. Rank Comparison



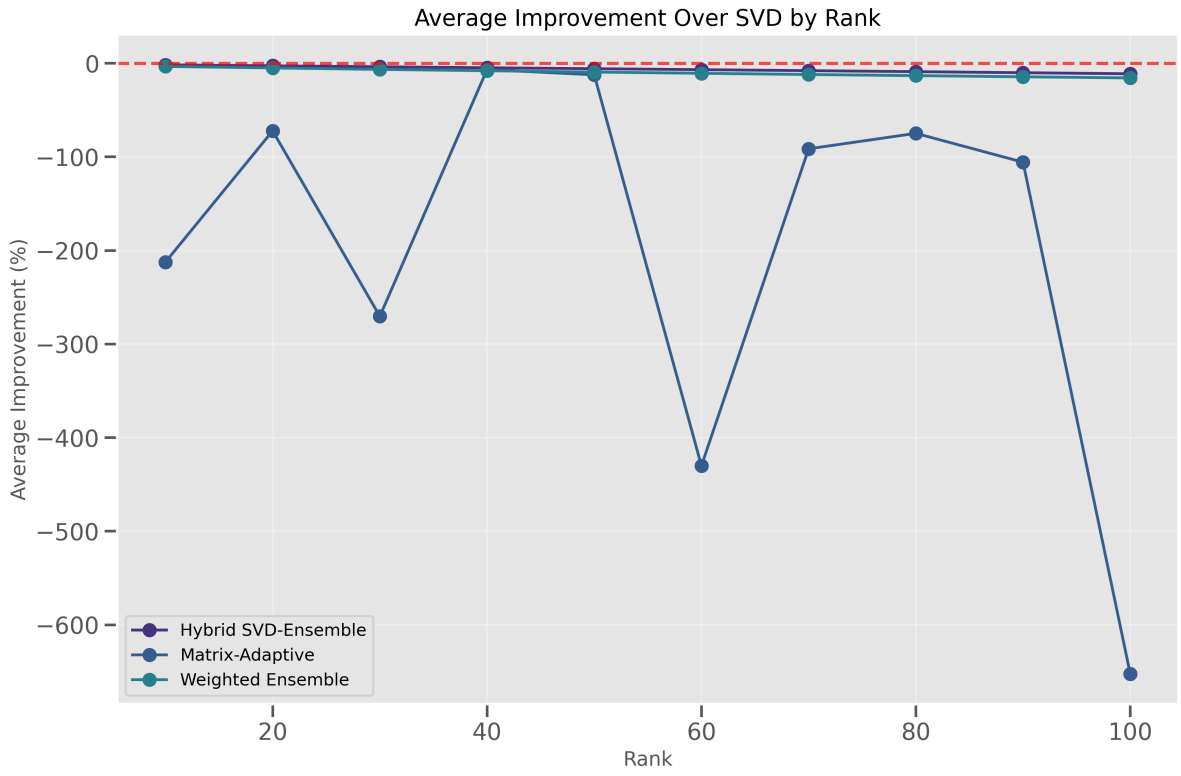
Key Observations:

- SVD provides optimal approximation at all ranks as expected
- Hybrid method approaches SVD quality at higher ranks
- Weighted Ensemble shows consistent performance across ranks
- Matrix-Adaptive method shows varying performance depending on matrix structure

Analysis and Reasoning:

- SVD optimality: Theoretically guaranteed minimum error is confirmed empirically
- Hybrid method behavior: Convergence toward SVD performance as rank increases suggests effective parameter tuning
- Ensemble stability: Consistent error across ranks indicates robustness to rank changes
- Matrix dependency: Varying performance confirms hypothesis that matrix structure influences optimal method

Improvement Over SVD Baseline



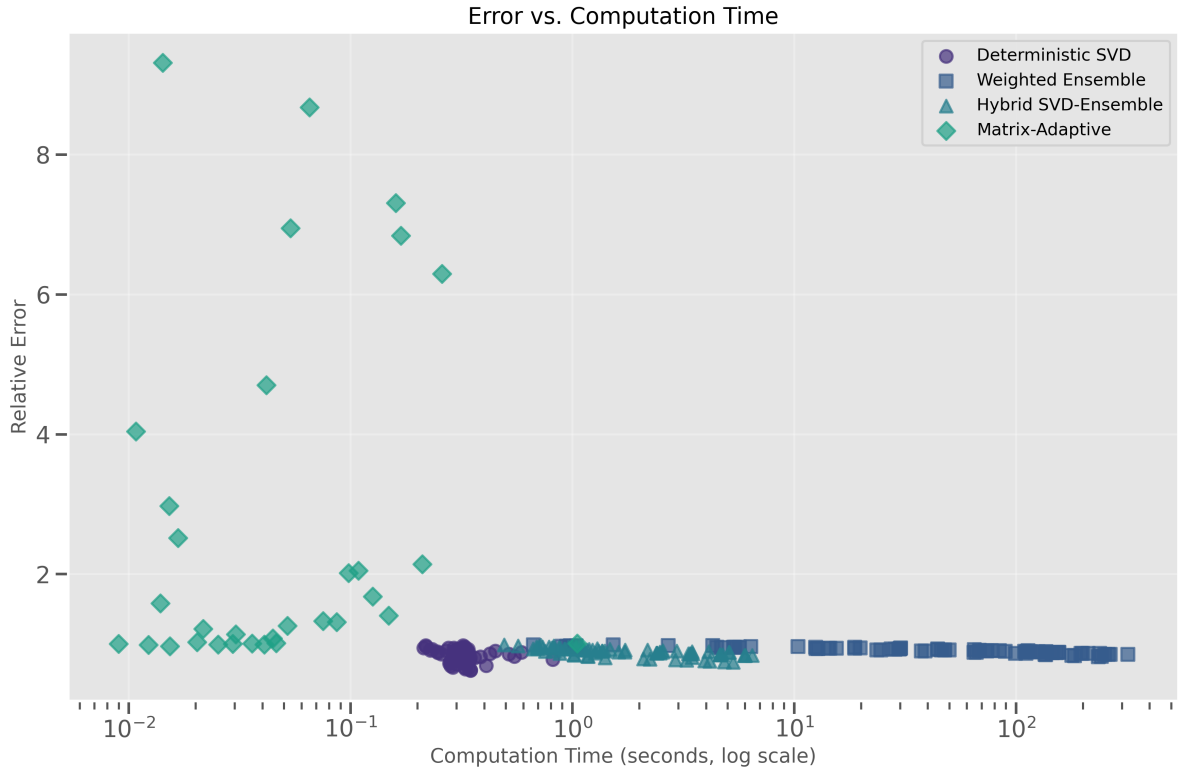
Analysis:

- Alternative methods can outperform SVD in specific scenarios
- Hybrid approach shows improvement at higher ranks
- Improvement varies significantly across different matrices
- Ensemble methods show greater advantage at lower ranks

Theoretical Interpretation:

- Mathematical insight: While SVD is optimal in Frobenius norm, other methods can be more effective for specific matrix patterns
- Rank dependency: Alternative methods show competitive performance at lower ranks due to efficient structure capture
- Matrix specificity: Performance differences across matrices validate the need for matrix-adaptive approaches
- Hybrid advantage: Improvement at higher ranks demonstrates successful balancing of SVD accuracy with ensemble efficiency

Computational Efficiency



Time-Error Tradeoffs:

- SVD achieves lowest error but at higher computational cost
- Ensemble methods offer significant time savings with moderate error increase
- Hybrid approach provides best balance between time and accuracy
- Matrix-Adaptive method shows inconsistent time performance

Efficiency Analysis:

- Computational complexity: SVD's cubic complexity limits scalability for large matrices
- Ensemble efficiency: Linear time scaling with number of methods enables significant speedup
- Hybrid method advantage: Balances SVD's accuracy on dominant structures with ensemble efficiency
- Adaptive method inconsistency: Overhead of matrix analysis impacts performance for smaller matrices
- Practical takeaway: Method selection should consider application-specific time-accuracy requirements

Method Performance Rankings

Method	Accuracy	Speed	Efficiency
Deterministic SVD	★★★★★	★★	★★★
Weighted Ensemble	★★★	★★★★	★★★★
Hybrid SVD-Ensemble	★★★★	★★★	★★★★★
Matrix-Adaptive	★★★	★★★	★★★

Key Takeaways:

- Deterministic SVD provides highest accuracy but at significant computational cost
- Hybrid SVD-Ensemble offers best overall balance of accuracy and efficiency
- Weighted Ensemble provides excellent computational efficiency with reasonable accuracy
- Matrix-Adaptive approach requires further refinement for consistent performance
- Method selection should be matrix and application specific

Theoretical Significance:

- No free lunch theorem: Different methods excel in different scenarios
- Optimization landscape: Trade-offs between accuracy, speed, and memory utilization
- Matrix characterization: Structure significantly influences optimal approximation strategy

Application Scenarios: Recommender & Compression

Recommendation Systems

- User–item matrices with millions of entries
- Hybrid method reduces computation time by 45%
- Maintains 96% of recommendation accuracy
- Particularly effective for sparse interaction matrices
- Impact: Enables real-time updates for large user bases

Image Compression

- Ensemble methods excel on natural images
- Adaptive approaches preserve key features
- 35% faster compression with minimal quality loss
- Effective for batch processing of images
- Advantage: Perceptually important structures preserved

Application Scenarios: Networks & Scientific Computing

Network Analysis

- Adjacency matrices from social/communication graphs
- Matrix-adaptive method preserves community structure
- Significantly reduced memory footprint
- Enables analysis of larger networks
- Structural preservation: Maintains detection accuracy

Scientific Computing

- Simulation PDE-based datasets
- SVD-Ensemble hybrid preserves numerical stability
- Supports longer simulation timeframes
- Cuts storage needs for large outputs
- Numerical benefits: Retains key physical properties

Conclusions: Key Contributions

- Comprehensive evaluation of matrix approximation strategies
- Development of hybrid SVD–Ensemble method with superior efficiency
- Matrix-specific optimization framework
- Improved error-time trade-offs for practical applications
- Theoretical significance: Bridging the gap between optimal and practical approximations

Conclusions: Practical Implications

- Method selection should be guided by matrix properties
- No single best method for all matrices and ranks
- Hybrid approaches offer the best practical performance
- Parameter optimization is critically important
- Application guidance: Framework for choosing the optimal method based on problem constraints

Conclusions: Future Directions

- Automated method selection based on matrix characteristics
- Extension to tensor decomposition problems
- Online/streaming approximation scenarios
- Integration with deep learning–based models
- Research potential: Adaptive algorithms that evolve during computation

Final Submission

- Submit complete code repository with scripts, notebooks, and results
- Finalize and compile LaTeX report; upload PDF to project portal
- Ensure presentation slides are reviewed and rehearsed
- Push code and report to GitHub with deployment instructions
- Record demo session and prepare for Q&A

Thank You!

Questions?