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Digital Systems & Signal Processing

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Preface

The purpose of this document is to act as a comprehensive note for my understanding on the subject matter. I may also use references aside from the lecture material to further develop my understanding, and these references will be listed here.

This document should eventually serve as a standalone reference for learning or review of the subject matter. There is also a lot of organization within these documents, please refer to the table of contents within your PDF viewer for ease of navigation.

References

- Course Material & Provided Lecture Notes
- Applied Digital Signal Processing - G. Manoalkis, K. Ingle

Introduction to Signal Processing

Signals & Systems

The study of signal processing involves **signals** [1] and **systems** [2].

- Real-life signals contain **information** , and **noise** [3]
- Information can be encoded within a signal through modulating its amplitude, frequency, phase or spectral content.

Signal processing is basically the study of extracting the encoded information within an **analog** or **digital** signal.

- Analog signals vary continuously with respect to their independent variables
- Digital signals are discrete-time and discrete-valued signals

Digital Signals have some amazing benefits[4], and drawbacks[5] too. Overall the benefits outweight the drawbacks.

[1] A signal is a measurable or detectable physical quantity (air-pressure, voltage, current, etc.) that carries some information

[2] A system is an object that performs an operation on a signal

[3] noise is just the part of the signal we are not interested in

[4] Noise robustness, software implementation, ease of storage and transmitting, and independent of external factors like temperature

[5] Increased system complexity due to A/D and D/A conversion, Limited Frequency Range, Increased power consumption - requires active devices

Signal Classification

Signals possess several characteristics that allow us to classify them further, below is a table of the characteristics and a short description of each.

Signal Category	Description
Discrete Time	Defined on a set of numbers - a sequence
Continuous Time	Defined on all real numbers - a function
Discrete Valued	Take on values from a finite set
Continuous Valued	Takes on a continuous range of values
Single Channel	From One Source, Represented by a scalar function
Multi-Channel	From Many Sources, Represented by a vector function
Single Dimension	Functions of one independent variable (usually time)
Multi-Dimension	Functions of more than one independent variable
Deterministic	Signal is exactly predictable with past values only
Random	Signal is NOT exactly predictable with past values only

Consider the picture of a color TV. The signal would be a three channel, three dimensional signal. Which we could represent mathematically as below.

$$\mathbb{I}\left(x,y,t\right) = \begin{bmatrix} I_r\left(x,y,t\right) \\ I_g\left(x,y,t\right) \\ I_b\left(x,y,t\right) \end{bmatrix}$$

Sampling

The Notion of Frequency

Both types of time-based signals[6]. Have the properties of **frequency** and **period**.

Through the basic definition of a sinusoidal-CT-signal[7] one can obtain both definitions of frequency

- ω [rad/sec] defines radial frequency
- $\omega = 2\pi f$, here f [cycles/sec] defines temporal frequency

Similarly through the basic definition of a sinusoidal-DT-signal[8] we can get more definitions that deal with how often we take samples.

- ω_s is the radial sampling frequency [samples/rad]
- $\omega_s = 2\pi f_s$, here f_s is the temporal sampling frequency [samples/sec]

If we take the inverse of frequency, we obtain the period $T = 1/f$. The period deals with how long it takes per cycle, T [sec/cycle] or how long we have in-between samples (sampling period) T_s [sec/sample].

To make notation clear, every time frequency is represented with a base symbol of

[6] Continuous-Time (CT) and discrete-time (DT)

[7] Sinusoidal CT Signals can be defined as $x(t) = A\cos(\omega t + \phi)$

[8] Sinusoidal DT Signals can be defined as $x(n) = A\cos(\omega_s n + \phi)$

ω it represents radial frequencies, frequency with a base unit of f will represent temporal frequencies.

Properties of Discrete-Time Signals

Analog-to-Digital Conversion

Conversion of a signal from analog into digital form occurs in three steps[9]. After this process our signal can be represented as a sequence of binary values.

We can perform **uniform sampling** (periodic sampling) with **sampling period** T_s[10] on a CT signal $x_{ct}(t)$ by constructing a sequence out of it:

$$x(n) =: x_{ct}(nT_s),\ n \in \mathbb{Z}$$

[9] Sampling, Quantization, and Encoding. Sampling produces a DT signal, and Quantization produces a Discrete-Valued (DV) signal.

[10] Recall: this also means that we have a sampling frequency of $f_s = 1/T_s$