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Digital Systems & Signal Processing

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Preface

The purpose of this document is to act as a comprehensive note for my understanding on the subject matter. I may also use references aside from the lecture material to further develop my understanding, and these references will be listed here.

This document should eventually serve as a standalone reference for learning or review of the subject matter. There is also a lot of organization within these documents, please refer to the table of contents within your PDF viewer for ease of navigation.

References

- Course Material & Provided Lecture Notes
- Applied Digital Signal Processing - G. Manoalkis, K. Ingle

Introduction

Signals & Systems

As we begin the study of **signal processing** we should start by defining what a **signal** actually is.

DEFINITION A **signal** is a measurable or detectable physical quantity (air-pressure, voltage, current, etc.) that carries some information.

The study of signal processing involves passing a signal through a **system**. With the goal of extracting something (usually the information) from the signal

DEFINITION A **system** is a physical device that performs an operation on a signal.

Information & Noise

Signals will always contain two things. **Information** and **Noise**. The information is the part we are interested in, and the Noise is considered everything else. For example when dealing with speech recognition, background music would be considered noise.

Information can be encoded within a signal through the some of the following methods:

- Signal Amplitude
- Signal frequency or Spectral Content
- Signal Phase

Now we can see that signal processing deals with extracting information by essentially 'interpreting' its used encoding method.

Analog & Digital Signals

Signals can be found as **analog signals** or as **digital signals** .

DEFINITION An **analog signal** is a signal that varies continuously with respect to its independent variables.

Most real life signals are analog signals, and analog signals can either be processed directly or converted into digital signals for processing before being reconverted into analog signals.

DEFINITION A **digital signal** is a signal that represents data as a sequence of discrete and quantized values. In other words, it is a discrete-valued AND discrete-time signal.

Digital signals have some pretty big benefits:

- Noise robustness - either a 0 or 1
- Software Implementation - can be processed through code instead of circuits
- Easy to store and transmit
- Fairly independent of external parameters like temperature

They have some drawbacks too:

- Increased system complexity - Need A/D and D/A conversion and complex digital circuitry
- Limited Range of Frequencies - Sampling rate must be twice the value of the highest frequency present within the signal
- Power Consumption - requires active devices, rather than analog signal processors which can be passive.

Overall, the advantages matter more than the drawbacks which is why digital signals are so commonplace.

Classifying Signals

Signals can be further classified based on the information they provide. Namely to classify a signal we choose an option among the 5 characteristics below.

- Continuous-Time OR Discrete-Time

- Continuous-Valued OR Discrete-Valued
- One-Channel OR Multi-Channel
- One-Dimensional OR Multi-Dimensional
- Deterministic OR Random

Continuous & Discrete-Time Signals

Continuous-time signals are defined on some interval (a, b), thus it can be represented as a function of a continuous variable $f(t), t \in [a, b]$

Discrete-time signals are defined on a set of integer numbers (indices) $\{0, 1, 2, \ldots\}$ and each index has a corresponding value $\{f_0, f_1, f_2, \ldots\}$. This means that we can represent them as sequences: $f[n] = \{f_0, f_1, f_2, \ldots\}$

Continuous & Discrete-Valued Signals

Continuous-valued signals can take on all-possible values from its continuous range, while **discrete-valued signals** take on values from a finite set. The process of converting a continuous-valued signal into a discrete-valued one is called **quantization**

Signal Channels

single channel signals are usually generated by one source and can be represented by *scalar* functions, **multi-channel signals** are usually generated by multiple sources/sensors and are represented by *vector functions*

Signal Dimensions

One-dimensional signals are a function of one independent variable (usually time). while **N-dimensional signals** are a function of N independent variables (time, x-pos, y-pos, etc.)

Deterministic & Random Signals

If we can exactly predict the future values of a signal by using its past values, we say that the signal is a **deterministic signal**. On the other hand, if we cannot predict it exactly we say that the signal is a **random signal**. There is no sharp distinction between the two due to the presence of noise, but these serve the purpose of broad categories.

Consider the picture of a color TV. The signal would be a three channel, three dimensional signal. Which we could represent mathematically as below.

$$\mathbb{I}\left(x,y,t\right) = \begin{bmatrix} I_r\left(x,y,t\right) \\ I_g\left(x,y,t\right) \\ I_b\left(x,y,t\right) \end{bmatrix}$$

Sampling

The Notion of Frequency

Both types of time-based signals[1]. Have the properties of **frequency** and **period**.

[1] Continuous-Time (CT) and discrete-time (DT)

Through the basic definition of a sinusoidal-CT-signal[2] one can obtain both definitions of frequency

[2] Sinusoidal CT Signals can be defined as $x(t) = A\cos(\omega t + \phi)$

- ω [rad/sec] defines radial frequency
- $\omega = 2\pi f$, here f [cycles/sec] defines temporal frequency

Similarly through the basic definition of a sinusoidal-DT-signal[3] we can get more definitions that deal with how often we take samples.

[3] Sinusoidal DT Signals can be defined as $x(n) = A\cos(\omega_s n + \phi)$

- ω_s is the radial sampling frequency [samples/rad]
- $\omega_s = 2\pi f_s$, here f_s is the temporal sampling frequency [samples/sec]

If we take the inverse of frequency, we obtain the period $T = 1/f$. The period deals with how long it takes per cycle, T [sec/cycle] or how long we have in-between samples (sampling period) T_s [sec/sample].

To make notation clear, every time frequency is represented with a base symbol of

ω it represents radial frequencies, frequency with a base unit of f will represent temporal frequencies.

Properties of Discrete-Time Signals

Analog-to-Digital Conversion

Conversion of a signal from analog into digital form occurs in three steps[4]. After this process our signal can be represented as a sequence of binary values.

We can perform **uniform sampling** (periodic sampling) with **sampling period** T_s[5] on a CT signal $x_{ct}(t)$ by constructing a sequence out of it:

$$x(n) =: x_{ct}(nT_s),\ n \in \mathbb{Z}$$

[4] Sampling, Quantization, and Encoding. Sampling produces a DT signal, and Quantization produces a Discrete-Valued (DV) signal.

[5] Recall: this also means that we have a sampling frequency of $f_s = 1/T_s$