Systems Described by Linear Constant-Coefficient Difference Equations

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The Difference Equation

This laboratory report constantly refers to the difference equation in (1)

$$y(n) = y(n-1) - 0.8y(n-2) + x(n) \tag{1}$$

We can represent (1) as a z-plane transfer function through the following steps:

$$\begin{aligned} Z\{y(n)\} &= Z\{y(n-1) - 0.8y(n-2) + x(n)\} \\ Y &= z^{-1}Y - 0.8z^{-2}Y + X \\ Y - z^{-1}Y + 0.8z^{-2}Y &= X \\ Y\left(1 - z^{-1} + 0.8z^{-2}\right) &= X \\ \frac{Y}{X} &= \frac{1}{1 - z^{-1} + 0.8z^{-2}} \end{aligned}$$

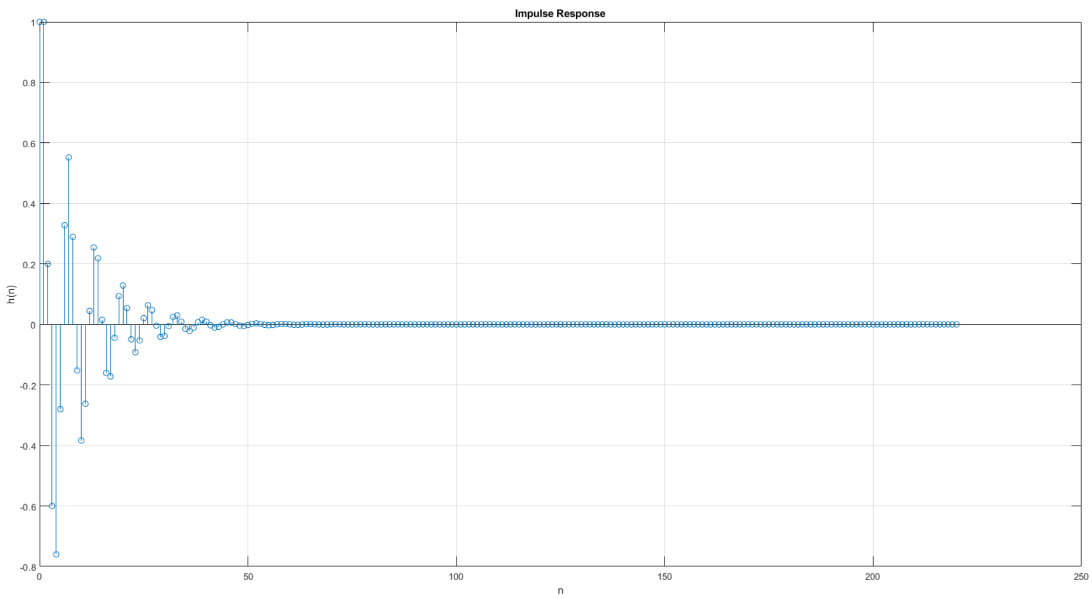
Essentially we can represent this difference equation as a transfer function with numerator coefficients $b = [1]$, and denominator coefficients $a = [1, -1, 0.8]$. This is what is implemented at the top of every `MATLAB` script.

Impulse Response

In order to calculate and plot the Impulse Response, the following `MATLAB` script was created.

```
%% QUESTION 1
clear;

% define the difference equation
b = 1; % numerator (output) x coeffs
a = [1, -1, 0.8]; % denominator (input) y coeffs

% finding impulse resp
t = -20:200;
[h, n] = impz(b, a, length(t));

% plotting impulse resp
stem(n, h);
grid on;
title('Impulse Response');
xlabel('n');
ylabel('h(n)');
```

This script produced the following output plot for the impulse response



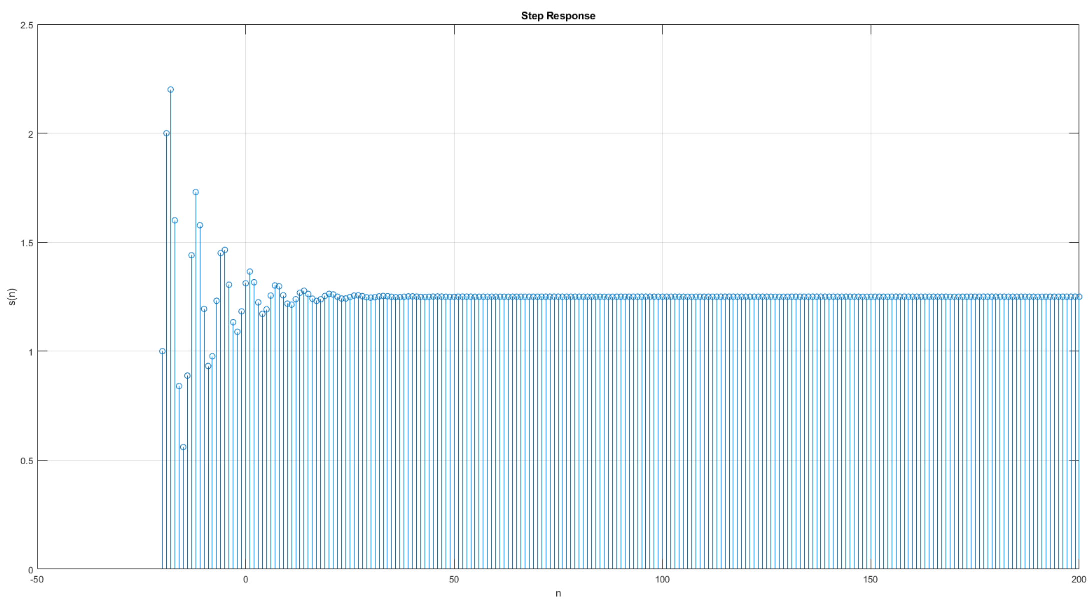
Step Response

In order to calculate and plot the Step Response the following `MATLAB` script was created.

```
%% QUESTION 2
clear;

% define the difference equation
b = 1; % numerator (output) x coeffs
a = [1, -1, 0.8]; % denominator (input) y coeffs

% find step resp
t = -20:200; % time scale (x-axis)
u = ones(size(t));
[s, final] = filter(b, a, u);
% filters data in 't' according to num 'b' and den 'a', outputs this in 'y'

% plot step resp
stem(t,s);
grid on;
title('Step Response');
```

```
ylabel('s(n)');
xlabel('n');
```

This script produced the following output plot for the step response



System Stability

In order to test system stability, we must check if sum of the absolute values of the impulse response converges. In other words we must establish that (2) holds true:

$$\sum_{n=0}^{\infty} |h(n)| < \infty \tag{2}$$

To test that, the following `MATLAB` script was created.

```
%% QUESTION 3

% define the difference equation
b = 1; % numerator (output) x coeffs
a = [1, -1, 0.8]; % denominator (input) y coeffs
```

```
% finding impulse resp
t = -20:200;
[h, n] = impz(b, a, length(t));

% check stability criterion , does the summation converge?
val = sum(abs(h));
stable = val < inf; % this is a boolean 1=True, 0=False
disp(['System Stable? : ' num2str(stable)]);
disp(['Converges to : ' num2str(val) ' on -20:200']);
```

This script produced the following output for the system stability.

Note: Within this script a '1' means Stable, and a '0' means unstable.

```
>> lab4_3351_3
System Stable? : 1
Converges to : 7.6193 on -20:200
>>
```