System Representation & Mason's Rules

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Control Systems - Note 3

Contents

Chapter 1

System Representation

Previously we studied transfer functions of control systems and how we relate the input and the ouput. We can now represent this graphically through the use of block diagrams.

$$C(s) = G(s) \cdot R(s)$$

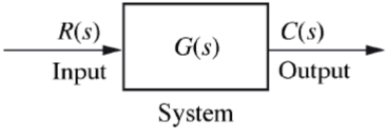

Figure 1.1: A Simple Block Diagram

1.1 *Cascade Form*

When multiple blocks are **connected in cascade form**, it is quite obvious to see that their total transfer function is a product of each transfer function.

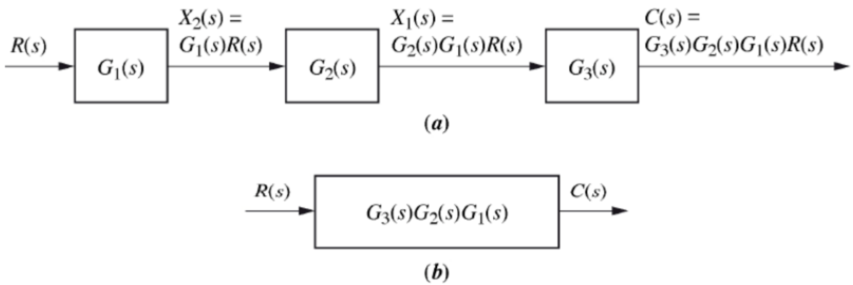

Figure 1.2: Cascade Form (a), being converted into an equivalent block (b)

$$G_{\text{eq}}(s) = \prod_i G_i(s) \tag{1.1}$$

1.2 *Parallel Form*

When multiple block's outputs are **connected in parallel** and joined through a summing junction, The equivalent transfer function is the sum of the transfer functions connected in parallel.

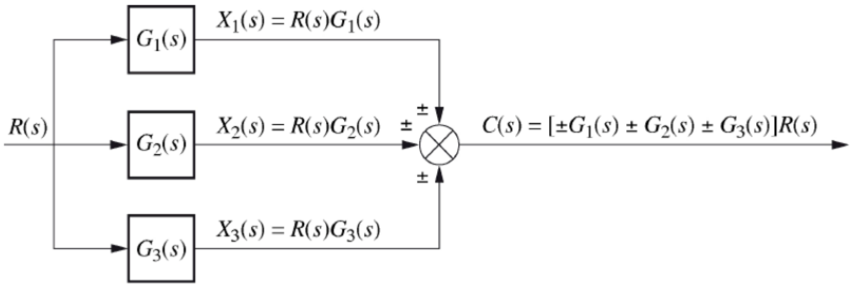

Figure 1.3: Parallel connection and its equivalent transfer function

$$G_{\text{eq}}(s) = \sum_i \pm G_i(s) \tag{1.2}$$

1.3 *Feedback Form*

Here is the basic layout for something in **feedback form**. It has an equivalent transfer equal to the one showed on the diagram.

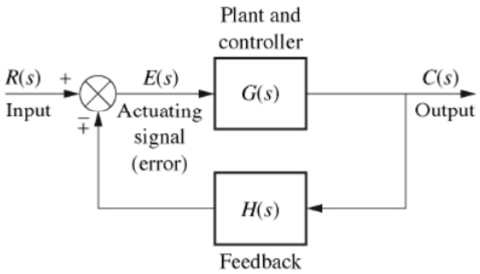

Figure 1.4: Feedback Form

$$G_{\text{eq}}(s) = \frac{G(s)}{1 \pm G(s) \cdot H(s)} \tag{1.3}$$

1.4 Signal Flow Graphs

We can write block diagrams in a different way through **signal flow graphs** which are nodes connected through gains.

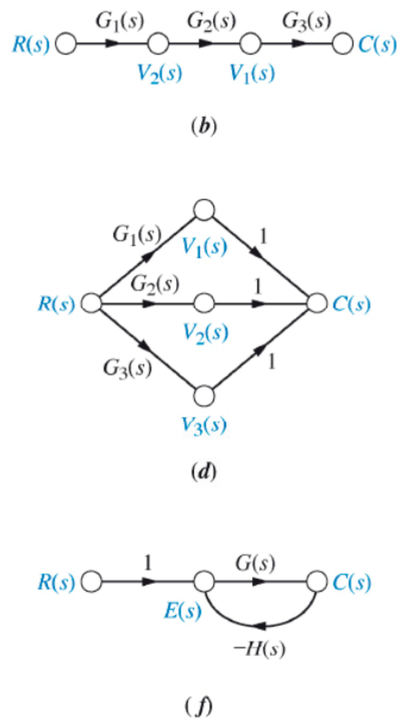

Figure 1.5: Signal Flow Graphs for: Cascade (b), Parallel (d), and Feedback (f) Forms.

Chapter 2

Mason's Rules

Often we can derive the transfer functions for simple systems algebraically, or through equations 1.1, 1.2, and 1.3 derived in Chapter 1. However, this might not always be feasible. For extremely complicated systems we follow **Mason's Rules**, which help us get right to the transfer function for the input of a system and any output we desire. Consider the diagram below. Throughout this chapter we will be using mason's rules to derive the transfer function for the signal flow diagram in Figure 2.1

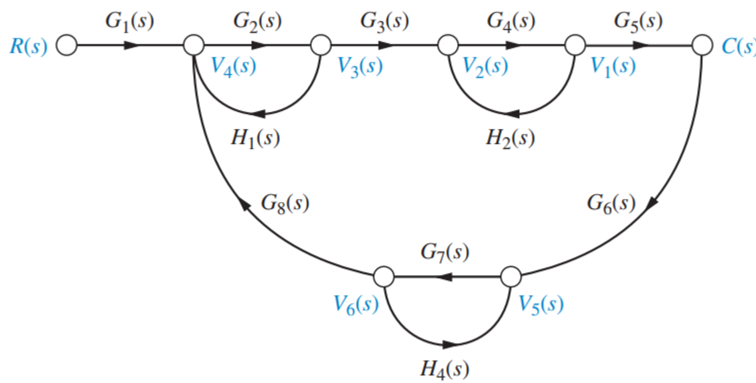

Figure 2.1: Find the transfer function $C(s)/R(s)$ for the following diagram

2.1 *Mason's Rule*

Mason's rule states that the transfer function $G(s)$ between the specified input on the diagram $R(s)$, and some output point we choose $C(s)$. Can be found through the following equation.

$$G(s) = \frac{\sum_k T_k \Delta_k}{\Delta}$$

where:

- k= number of forward paths
- T_k = the k'th forward path gain

- Δ = 1 - \sum loop gains + \sum nontouching loop gains taken two at a time - \sum nontouching loop gains taken 3 at a time ...
- Δ_k = loop gains after removing the k'th forward path

This makes no sense, so lets define these terms, and then derive the transfer function in Figure 2.1

2.2 Definitions

Loop gain - The product of branch gains found by traversing a path that starts at a node and ends at the same node, without passing through any other node more than once, all while following the direction of signal flow. We can think of this as *mesh* gains.

Forward path gain - The product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

Nontouching loops - Loops that do nore have any nodes in common.

2.3 Example

Lets start by determining the number of **forward paths**, in this case there is only one thus $k = 1$. It's corresponding **forward path gain** is

$$G_1G_2G_3G_4G_5$$

Next we should find the **loop gains**.

1. G_2H_1
2. G_4H_2
3. G_7H_4

4. $G_2G_3G_4G_5G_6G_7G_8$

$$\text{LG} = (1) + (2) + (3) + (4)$$

Next lets take the nontouching loops two at a time. We can take loops 1 and 2, 1 and 3, and then 2 and 3.

- $(G_2H_1)(G_4H_2)$
- $(G_2H_1)(G_7H_4)$
- $(G_4H_2)(G_7H_4)$

$$\text{LG-2} = (1)(2) + (1)(3) + (2)(3)$$

Next we take nontouching loops three at a time. We can take loops 1 2 and 3 here.

- $(G_2H_1)(G_4H_2)(G_7H_4)$

$$\text{LG-3} = (1)(2)(3)$$

There are only three nontouching loops, thus every other term in Δ will be zero.

$$\Delta = 1 - (\text{LG}) + (\text{LG-2}) - (\text{LG-3})$$

Now we can find Δ_k, in this case $\Delta_1$by removing forward path 1, and then taking the delta of the remaining graph.

$$\Delta_1 = 1 - G_7H_4$$

Thus we can write the whole transfer function

$$G(s) = \frac{T_1\Delta_1}{\Delta} = \frac{(G_1G_2G_3G_4G_5)(1 - G_7H_4)}{1 - (\text{LG}) + (\text{LG-2}) - (\text{LG-3})}$$