MGMTMSA 408 – Operations Analytics – Spring 2024

Final Exam – Answer Sheet - Q4

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Please follow all instructions on the Final Exam question sheet.

Q4 - Hotel revenue management

Part 1: Understanding the data

a) The probability that the hotel receives a reservation for a Queen room (with any check-in / check-out dates) in a given (4 hour) period is 0.32.

b) The probability that the hotel receives a reservation that checks in on Thursday 9/14/2023 and checks out on Sunday 9/17/2023 (with any kind of room) in a given (4 hour) period is 0.01.

c) The probability that there is no request in a given (4 hour) period is 0.26.

d) The expected value of the number of requests for a California King room, with check-in on Friday 9/15/2023 (d = 5) and check-out on Saturday 9/16/2023 (d = 6) is 6.00.

Part 2: Determining an optimal static allocation

a)

Part 2a

$$\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{N} r_i x_i \quad \text{[Objective Function]} \\
\text{subject to} \quad & \sum_{i=1}^{N} x_i \cdot 1_{(\ell \in \text{itineraries}_i) \wedge (r \geq \text{din}_i) \wedge (r < \text{dout}_i)} \leq B_\ell \quad \forall d \in \{1, \ldots, 7\} \quad \text{[Everyday Occupancy Constraint]} \\
& x_i \geq 0, \quad \forall i \in \{1, \ldots, N\}. \quad \text{[Non-Negativity Constraint]}
\end{aligned}$$

where decision variables are given as follows:

r_i is the revenue from accepting a request of type i

x_i is the number of requests of type i that are accepted

B_r is the total capacity of room type r

to maximize the total revenue.

b)

The optimal revenue is $172140.0017788.

c)

The top five request types in terms of the number of accepted requests in the optimal solution are: (K, 2, 3), (K, 3, 4), (K, 4, 5), (Q, 1, 2), (K, 2, 4)

d)

The optimal values of the dual variables for the resource constraints are:
RoomType: Q, Day: 1, DualVariable: 0.0
RoomType: Q, Day: 2, DualVariable: 200.0
RoomType: Q, Day: 3, DualVariable: 200.0
RoomType: Q, Day: 4, DualVariable: 230.0
RoomType: Q, Day: 5, DualVariable: 229.99999999999997
RoomType: Q, Day: 6, DualVariable: 0.0
RoomType: Q, Day: 7, DualVariable: 0.0
RoomType: K, Day: 1, DualVariable: 0.0
RoomType: K, Day: 2, DualVariable: 250.0
RoomType: K, Day: 3, DualVariable: 250.0
RoomType: K, Day: 4, DualVariable: 287.5
RoomType: K, Day: 5, DualVariable: 0.0
RoomType: K, Day: 6, DualVariable: 0.0
RoomType: K, Day: 7, DualVariable: 0.0
RoomType: C, Day: 1, DualVariable: 0.0
RoomType: C, Day: 2, DualVariable: 300.0
RoomType: C, Day: 3, DualVariable: 300.0
RoomType: C, Day: 4, DualVariable: 345.0
RoomType: C, Day: 5, DualVariable: 345.0
RoomType: C, Day: 6, DualVariable: 345.0
RoomType: C, Day: 7, DualVariable: 0.0

e)

The predicted change in revenue that would result from this conversion is $725.00.

Predicted Revenue Change = New Profit - Old Profit
Predicted Revenue Change = 10 * [Sum(Queen Dual Variables for all days) - Sum(King Dual Variables for all days)
Predicted Revenue Change = 10 * (860 - 787.5)
Predicted Revenue Change = 725

Part 3: Determining an optimal dynamic allocation policy

a)

The average revenue of this policy over the 100 random requests sequences is $147472.025

b)

For a request of type (K, 2, 5), the opportunity cost is calculated as the difference in revenue if we accept the request versus if we don't accept the request. It is calculated as follows:
Opportunity Cost = Dual(K, 2) + Dual(K, 3) + Dual(K, 4) = 787.5

In general, for a request of type (r, din, dout), we can calculate the opportunity cost as the sum of dual variables for a particular room type through all nights requested.
Opportunity Cost(r, din, dout) = Sum(Dual(r, d)) from d = din to d = dout-1.

c)

The average revenue of this policy over the 100 random request sequences is $163673.975.

d)

The dynamic allocation policy in (c) performs better than the hotel's current policy in (a) due to its strategic handling of resource allocation based on arrival probabilities and expected arrivals. High-probability request types, which are common, can quickly deplete resources under policy (a) as it likely accepts requests without considering future demand. This leads to resource exhaustion and missed opportunities for high-value bookings later.

In contrast, policy (c) dynamically allocates resources, anticipating future high-probability and high-value requests. It may reject or defer less valuable requests early to preserve resources for more profitable ones. This strategic approach prevents premature resource depletion, ensuring availability for high-value bookings, and optimizes overall resource utilization and revenue.

In summary, policy (c) aligns resource allocation with demand patterns, maximizing occupancy and profitability, whereas policy (a) risks inefficient resource use and missed revenue opportunities by not accounting for future high-demand requests.