MGMTMSA 408 – Operations Analytics – Spring 2024

Final Exam – Answer Sheet - Q2

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Please follow all instructions on the Final Exam question sheet.

Q2 - Bike-share network design

Part 1: Minimizing average distance

a)

Part 1a

$$
\begin{aligned}
\text{minimize} \quad & \frac{1}{m}\sum_{j=1}^{m}\sum_{i=1}^{n} t_{i,j} y_{i,j} \quad && \text{[Objective Function]} \\
\text{subject to} \quad & \sum_{i=1}^{n} y_{i,j} = 1, \quad \forall j \in \{1,\ldots,m\}, && \text{[Serve each Customer Constraint]} \\
& y_{i,j} \le x_i, \quad \forall i \in \{1,\ldots,n\},\ j \in \{1,\ldots,m\}, && \text{[Assignment Constraint]} \\
& \sum_{i=1}^{n} x_i \le 10, && \text{[10 Stations Constraint]} \\
& x_i \in \{0,1\}, \quad \forall i \in \{1,\ldots,n\}, && \text{[Binary Constraint]} \\
& y_{i,j} \in \{0,1\}, \quad \forall i \in \{1,\ldots,n\},\ j \in \{1,\ldots,m\}. && \text{[Binary Constraint]}
\end{aligned}
$$

where decision variables are given as follows:

$t_{i,j}$ is the walktime from customer j to docking station i in seconds

$y_{i,j}$ is a binary variable to decide if customer j is assigned to docking station i or not

x_i is a binary variable to decide if docking station i is open or not

to minimize average walktime for all customers.

b) The optimal objective value is 487.81 seconds.

c) The stations that are opened are candidates 4, 8, 9, 11, 19, 24, 25, 27, 28, 38.

Part 2: Minimizing maximum distance

a)

b)

The optimal objective value is 1268.0 seconds.

c)

The stations that are opened are candidates 5, 13, 15, 20, 22, 25, 26, 28, 30, 42.

Part 2a

$$
\begin{aligned}
\text{minimize} \quad & r \quad \text{[Objective Function]} \\
\text{subject to} \quad & r \geq \sum_{i=1}^{n} t_{i,j} y_{i,j}, \quad \forall j \in \{1, \ldots, m\}, \quad \text{[Maximum Walktime Constraint]} \\
& y_{i,j} \leq x_i, \quad \forall i \in \{1, \ldots, n\},\ j \in \{1, \ldots, m\}, \quad \text{[Assignment Constraint]} \\
& \sum_{i=1}^{n} x_i = 10, \quad \text{[10 Stations Constraint]} \\
& x_i \in \{0, 1\}, \quad \forall i \in \{1, \ldots, n\}, \quad \text{[Binary Constraint]} \\
& y_{i,j} \in \{0, 1\}, \quad \forall i \in \{1, \ldots, n\},\ j \in \{1, \ldots, m\}. \quad \text{[Binary Constraint]}
\end{aligned}
$$

where decision variables are given as follows:

$t_{i,j}$ is the walktime from customer j to docking station i in seconds

$y_{i,j}$ is a binary variable to decide if customer j is assigned to docking station i or not

x_i is a binary variable to decide if docking station i is open or not

r is the maximum walktime

to minimize maximum walktime for all customers.

Part 3: Maximizing coverage

a)

b) The optimal objective value is 496.0 seconds.

c) The stations that are opened are candidates 8, 9, 12, 15, 19, 24, 25, 28, 34, 37.

Part 3a

$$
\begin{aligned}
\text{maximize} \quad & \sum_{j=1}^{m} z_j \quad \text{[Objective Function]} \\
\text{subject to} \quad & z_j \leq \sum_{i=1}^{n} a_{i,j} x_i, \quad \forall j \in \{1, \ldots, m\}, \quad \text{[Coverage Constraint]} \\
& \sum_{i=1}^{n} x_i = 10, \quad \text{[10 Stations Constraint]} \\
& x_i \in \{0, 1\}, \quad \forall i \in \{1, \ldots, n\}, \quad \text{[Binary Constraint]} \\
& z_j \in \{0, 1\}, \quad \forall\, j \in \{1, \ldots, m\}. \quad \text{[Binary Constraint]}
\end{aligned}
$$

where decision variables are given as follows:

$a_{i,j}$ is the coverage parameter obtained from problem data whether docking station i covers customer j or not

z_j is a binary variable to decide if we serve customer j or not

x_i is a binary variable to decide if docking station i is open or not

to maximize coverage for all customers.

Part 4: Walking time preferences and capacity

a)

b)

c)

d)