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ECS 20 Midterm 1

This test has 6 problems.
You will use **your y** variable which is calculated by taking just the last digit of your student **id** ~~[illegible]~~ 6 then **adding 4** to it.
Everything in the test is self-explanatory. If you are confused, state your assumption and answer the question. We will grade with your assumption in mind. **We cannot answer any questions related to your interpretation.**

What is your y ? 10

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

Question 1: (20pts) Use your y from previous page.

20×100
$= 2000$

What is it your y 10

Expand the following notation

Assuming $i = 100$ $[(2\times10)(100) - 1]$

a) $\sum_{i=1}^{i=3}((2y)(i) - 1) =$ $[2\times10\times(3)-1]$, $[2\times10\times(2)-1]$, $[2\times10\times(1)-1]$ $= 59, 39, 19 \ldots\ldots 1999$

b) $\frac{(n+y)!}{(n+y-2)!} = \frac{(n+10)!}{(n+10-2)!} = \frac{(n+10)!}{(n+8)!} = \frac{(n+10)\times(n+9)\times\ldots\ldots(n+1)}{(n+8)(n+7)\ldots\ldots(n+1)} =$

$(n+8)$ to $(n+1)$ cancels $\therefore$ $= (n+10)(n+9)$

Question 2: (30pts)

Determine if the following statements are propositions if yes, determine the truth value, if no explain why

a) $(2 + 7 = 9) \to (1 + 1 = 3)$ Proposition, False

b) There exists a number y that is less than x, where $(x, y) \in \mathbb{R}$. Proposition, True

c) X+3=1 is a proposition Not a proposition, a proposition can be true or false but not both. when -2 is substituted its true, otherwise its false $\therefore$ it's not a proposition.

d) $\sqrt{2}$ is a rational number proposition, True

e) X+3=2 is solvable if x is in the domain of real numbers Proposition, true

f) 2+3=2 if and only if 1+1=3 Proposition, false

Question 3 (15 pts)

Let a, b, and c be three propositions. Show that this is a tautology, using a truth table. Decide if it is satisfiable- explain your answer.

The last column is all True. So, its a tautology. It is satisfiable.

$$(a \oplus b) \wedge (b \vee c) \to (\neg a \vee c)$$

$\neg a$	a	b	c	$a \oplus b$	$(b \vee c)$	$(a\oplus b)\wedge(b\vee c)$	$(\neg a \vee c)$	$(a\oplus b)\wedge(b\vee c) \to (\neg a \vee c)$
F	T	T	T	F	T	F	T	T
F	T	T	F	F	T	F	F	T
F	T	F	T	T	T	T	T	T
F	T	F	F	T	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	T	F	T	F	T	T
T	F	F	F	F	F	F	T	T

Question 4: (15pts)

Restate a propositions in English:
p= "I love cats and dogs"
q= "I love all animals."

a.) $(\neg p) \rightarrow (\neg q)$: If I don't love cats and dogs, then I don't love all animals.

b.) $(\neg p) \leftrightarrow q$: If and only if I don't love cats and dogs then I love all animals.

c.) $p \wedge (\neg q)$: I love cats and dogs and I don't love all animals.

Question 5. Puzzle (10pts)

This exercise relate to the inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described. If you cannot determine what these two people are, can you draw any conclusions?

A says "We are both the same"

B says "A is a knight."

∴ A is a knight and B is a knight

A	B	Truth value	Satisfiable?
Knight	Knight	A(✓) B(✓)	✓
Knight	Knave	A(X) B(X)	X
Knave	Knight	A(X) B(X)	X
Knave	Knave	A(X) B(X)	X

∴ ~~A is a B and B~~

Question 6: Proofs (40pts)

a) Prove $\exists n \in \mathbb{N}\ \ 4 + n = 13$

b) Prove $\neg(\forall n \in \mathbb{N}\ \ 4 + n = 8)$

c) Prove that if n is a positive integer, then n is even if and only if 5n + 8 is even

(a)

Statement	Reason
$n \in N$	given
$4 \in N$	
$4 + 6 = 10$	by def of addition
$10 \neq 13$	by def of order

Hypothesis: $\neg(\exists n \in N\ \ 4 + n = 13)$

∴ we have prooved that our hypothesis is correct and the original statement is wrong.

Hypothesis $(\forall n \in N\ \ 4 + n = 8)$

Statement	Reason
$n = 8$	by def of substitution
$4 + n = 8$	given
$4 + 8 = 12$	addition
$12 \neq 8$	by def of order

We have shown our hypothesis is wrong.

∴ The original statement is true.

If n is odd, 5n+8 is odd

$n = 2k+1$

$5n + 8 = 5(2k+1) + 8$ substitution

$10k + 5 + 8$ expansion of brackets

$10k + 12 + 1$

$2(5k+6) + 1$ factoring

Let $5k+6$ be j

$2j + 1$ is in the form $2k+1$, so it's odd and ∴ $5n+8$ is odd.

Proof by contrapositive used.

d.) Let n be a natural number and let $a_1, a_2, \ldots, a_n$ be a set of n real numbers. Prove that at least one of these numbers is more than, or equal to the average of these numbers.

Let $a_1, a_2 \cdots a_n$ be a set of n real numbers and let $\bar{a}$ be their average.

$\bar{a} = \dfrac{a_1 + a_2 + \ldots + a_n}{n}$ by definition of average

$a_1 + a_2 + \cdots + a_n = n \times \bar{a}$ by multiplication

$a_1 + a_2 + \cdots + a_n > a_1 + a_2 + \cdots a_n$ by substitution

A number cannot be strictly greater than itself, so we have contradicted the original statement.

Pt 2

If $5n+8$ is odd, then n is odd

Let $5n + 8 = 2b + 1$ where b is a positive integer.

$5n = 2b - 7$; subtracting an integer (7) from even number makes the whole number odd ∴ $2b - 7$ is odd.

If $2b - 7$ is odd, $5n$ is odd.

Since n is odd & odd × odd = odd, 5n is odd.

e.)

Let n be an integer. Show that if $n^2 + 1$ is even, then n is odd, using: a proof by contradiction

If n^2+1 is even, then n is even: Statement

Let $n = 2k$ by def of even

$(2k)^2 + 1 = n^2 + 1$ by substitution

$= 4k^2 + 1$

$= 4k^2 + 1$ is in the form $2k+1$ and $2k+1$ is an odd integer.

Thus, we have contracted our original statement

q.e.d.