Ans-1 To Show

Find a recursive formula for the number of triangulations C_{n+1} of a convex polygon with $n+3$ sides.

Given

① Let C_n be the number of triangulations of a polygon in the plane with $n+2$ sides.

② C_1 are triangulations of the triangle and there is only one, so $C_1 = 1$.

③ $C_2 = 2$

④ $C_3 = 5$ [$(n+2)$ gons which represent 5 sides]

⑤ $C_4 = 14$.

Proof

Now, say we are triangulating a polygon with $n+3$ sides. Choose one vertex of the polygon as a reference point. Now we can connect this to (n) other vertices (excluding itself) ~~and its immediate neighbours because loop~~

Now, suppose the chosen vertex for connection splits the polygon into smaller polygon one with $(k+2)$ vertices and the other with $(n-k)+2$ vertices, where k is the number of vertices btw the reference point and the other vertex on the side.

So, the number of ways to triangulate will be C_k and C_{n-k}.

So, we can say that closed formula for it can be :-

$$C_{n+1} = \sum_{k=0}^{n} C_k \cdot C_{n-k}$$ as it is the sum of ~~multi~~ multiplication of number of triangulations of the two polygons over all valid k.

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To verify this formula let's take C_5.

$$C_5 = C_0 C_4 + C_1 C_3 + C_2 C_2 + C_3 C_1 + C_4 C_0$$

Let's substitute the values, $C_3 = 5$, $C_4 = 14$, $C_1 = 1$, $C_2 = 2$, $C_0 = 1$

$$C_5 = (1)(14) + (1)(5) + (2)(2) + (5)(1) + (14)(1) = \boxed{42}$$

So, this closed formula verifies this.

So, <u>Hence proved</u> $\square$

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H-W-5

Ans=2 (a) Prove

(1) Given sets A, B and C. Show that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

To-prove

We need to show that each set is a subset of each other

$A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$.

Proof

Let (x, y) be an arbitrary element of $A \times (B \cup C)$. This means that $x \in A$ and $y \in B$ or $y \in C$. Now, there are 2 cases :-

(1) If $y \in B$, then $(x, y) \in A \times B$, so $(x, y) \in (A \times B) \cup (A \times C)$.

(2) If $y \in C$, then $(x, y) \in A \times C$, so, $(x, y) \in (A \times B) \cup (A \times C)$.

Since $(x, y) \in (A \times B) \cup (A \times C)$ in both cases, we conclude that $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$.

So, Hence proved

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(ii). Prove

Let $B - C = \{x \mid x \in B, x \notin C\}$. For sets A, B, C show that $A \times (B - C) = (A \times B) - (A \times C)$.

Proof

Let (x, y) be arbitrary element of $(A \times B) - (A \times C)$. This means that $(x, y) \in A \times B$ and $(x, y) \notin A \times C$. There will be 2 cases.

1. If $x \in A$ and $y \in B$ by $y \notin C$, then $(x, y) \in A \times (B - C)$.
2. If $(x, y) \in A \times B$ but $(x, y) \notin A \times C$, then $(x, y) \in A \times (B - C)$.

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So, since (x, y) belongs to $\boxed{A \times (B - C)}$ in both cases, we conclude that $(A \times B) - (A \times C) \subseteq A \times (B - C)$

So, Hence proved

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ANS-3 To-prove

$(A \times C) \cup (B \times D) \subseteq (A \cup B) \times (C \cup D)$

① Proof

Let (x, y) be an arbitrary element of $(A \times C) \cup (B \times D)$. This means that (x, y) belongs to either $(A \times C)$ or $(B \times D)$.

- If $(x, y) \in A \times C$, it means $x \in A$ and $y \in C$, Since $x \in A$, so, ~~$(A \cup B) \in$~~ $x \in (A \cup B)$ and since $y \in C$, so $y \in (C \cup D)$. Therefore $(x, y) \in (A \cup B) \times (C \cup D)$.

- If $(x, y) \in (B \times D)$, it means $x \in B$ and $y \in D$, Since $x \in B$, so, $x \in (A \cup B)$ and since $y \in D$, so $y \in (C \cup D)$, so, $(x, y) \in (A \cup B) \times (C \cup D)$.

Since $(x, y) \in (A \cup B) \times (C \cup D)$ in both cases, we conclude that $(A \times C) \cup (B \times D) \subseteq (A \cup B) \times (C \cup D)$.

So, Hence proved

② Disprove

$(A \times C) \cup (B \times D) \supseteq (A \cup B) \times (C \cup D)$

Let's take $A = \{1\}$, $B = \{2\}$, $C = \{3\}$, $D = \{4\}$.

So, Taking LHS

$(A \times C) = (1, 3)$ $(B \times D) = (2, 4)$.

$(A \times C) \cup (B \times D) = \{(1, 3), (2, 4)\}$ — ①

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Solving RHS

~~$(A \cup B) = \{(1,3),(1,4),(2,3),(2,4)\}$~~

$(C \cup D) = \{3,4\}$.

$(A \cup B) = \{1,2\}$

$$\boxed{(A \cup B) \times (C \cup D) = \{(1,3),(1,4),(2,3),(2,4)\}} \quad (2)$$

Comparing (1) and (2)

We can prove that $\boxed{(A \times C) \cup (B \times D) \neq (A \cup B) \times (C \cup D)}$

Hence proved $\square$.

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Ans 4 To-Show

$\{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid x \equiv y \pmod{10}\} \subseteq \{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid x \equiv y \pmod{5}\}$

Proof

Taking $\{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid x \equiv y \pmod{10}\}$.

So we can say $x - y = 10k$, this means $(x-y)$ is divisible by $\boxed{10}$.

Now, Taking $\{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid x \equiv y \pmod{5}\}$.

So we can say $x - y = 5k$, this means $(x-y)$ is divisible by 5.

So since from the first expression we can say $x-y$ is divisible by $10k$, which can be written as $x - y \mid 2 \cdot 5k$, and this show $(2 \cdot 5k)$ shows that first set belongs to second set.

So, Hence proved $\blacksquare$

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$\boxed{\text{Ans} = 5}$

Given

$R = \{(x,y) \in R \times R \mid x - y \in Z\}$ on R.

To-prove

Reflexive, Symetric, transitive.

$\boxed{\text{Proof}}$

Case 1 : $\boxed{\text{Reflexive}}$

For every $x \in R$, $x - x = 0 \in Z$, therefore $(x,x) \in R$ $\forall x \in R$ making R reflexive.

Case 2 : $\boxed{\text{Symetric}}$

If $(x,y) \in Z$, then $x - y \in Z$ and $y - x \in Z$. ~~Thus (y,x)~~ Thus $(y,x) \in R$, and R is symmetric

Case 3 : $\boxed{\text{Transitive}}$

If $(x,y) \in R$ and $(y,z) \in R$, So $x - y \in Z$, and $y - z \in$ and then $x - z \in Z$, so this shows $(x,z) \in R$ and R is transitive.

$\boxed{\text{Hence proved}}$

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HW5 - Q(a) To - prove

5^{100} is not divisible by 3.

Proof

Let's say $5^{100} = 3$ so, we can do $5 \bmod (3) = 2$

So, now, it will be 2^{100} and now, we can say =

$= 3\ 2^1 = 2$

$= 3\ 2^2 = 1$

$= 3\ 2^3 = 2$

So, from here we can see that it can come to 2 or 1.

So, now If I write $((2)^2)^{50}$ and now If I do 4 mod/3

So, it will come to 1 and now $(1)^{50} = 1$ which is not divisible by 3, so, that's how we can show that it is not divisible by 3.

~~So, HP~~ So, Hence-proved □.

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(iii) $7 \mid 8^{m+2} + 7m^2 + 3 \cdot 8^m - 4$

Now, $8^{m+2} + 7m^2 + 3 \cdot 8^m - 4$ divided by 7

$\Rightarrow 8^m \cdot 8^2 + 7m^2 + 3 \cdot 8^m - 4$

$64 \bmod (7) = \boxed{1}$

$7 \bmod 7 = \boxed{0}$

$4 \bmod (7) = 4$

$3 \bmod (7) = \boxed{3}$.

$\Rightarrow 8^m + 0 \cdot m^2 + 3 \cdot 8^m - 4$

$\Rightarrow 8^m + 3 \cdot 8^m - 4$

$\Rightarrow \boxed{4(8^m - 1)}$

Now, from here we can prove using induction :-

Base-Case.

Let $\boxed{m=1}$

$4(8^1 - 1) = 4(7) = \boxed{32}$ this is divisible by 7

Let $\boxed{m=2}$

$4(8^2 - 1) = 4(64 - 1) = \boxed{4(63)}$ this is divisible by 7

Induction Hypothesis

Let $\boxed{m=k}$

Let, $4(8^k - 1)$ this is divisible by 7

Induction Step

Let $\boxed{m=k+1}$

Let $4(8^{k+1} - 1)$

$4(8^k \cdot 8 - 1)$.

$4[8^k(7+1) - 1]$.

$4[8^k \cdot 7 + 8^k - 1]$.

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$4 \cdot 7 \cdot 8^k + 4 \cdot 8^k - 4$

$\boxed{\text{Apply mod}(7)}$

$4 \cdot 7 \cdot 8^k + 4 \cdot 8^k - 4$

$\rightarrow (7) \bmod (7)$. $\rightarrow 4 \bmod (7)$ $\rightarrow 4 \bmod (7)$

$0 \cdot 8^k + 4 \cdot 8^k - 4$

$\boxed{4 \cdot 8^k - 4}$

So, from this $(4 \cdot 8^k - 4)$ we know from base-case that it is equal to 32 which is divisible by 7

$\boxed{\text{So, hence proved}}$ $\square$.

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