In[1]:=
```mathematica
f[x_, y_] = 9 - x^2 - y^2;
fx[x_, y_] = D[f[x, y], x];
fy[x_, y_] = D[f[x, y], y];
(*the point*)
fp[x_, y_] = {x, y, f[x, y]};
(*directional*)
vx[x_, y_] = {1, 0, fx[x, y]};
vy[x_, y_] = {0, 1, fy[x, y]};

tanplane[x_, y_] = {2 * vx[x, y] + fp[x, y],
    2 * vy[x, y] + fp[x, y], -2 * vx[x, y] + fp[x, y], -2 * vy[x, y] + fp[x, y]};
dz[theta_, x_, y_] = fx[x, y] * Cos[theta] + fy[x, y] * Sin[theta];
gradient[{x_, y_}] = {fx[x, y], fy[x, y]};


cp = ContourPlot3D[z == f[x, y], {x, -5, 5}, {y, -5, 5}, {z, -10, 10}];
plot = ContourPlot[f[x, y], {x, -5, 5}, {y, -5, 5}];

gr[{x_, y_}] = {fx[x, y] / Norm[gradient[{x, y}]],
    fy[x, y] / Norm[gradient[{x, y}]], Norm[gradient[{x, y}]]};
```

```
In[13]:= Manipulate[
          GraphicsGrid[
           {
            {Show[
              cp,
              (*x tangent*)
              ParametricPlot3D[vx[a, b] * t + {a, b, f[a, b]}, {t, -2, 2}, PlotStyle → {Pink}],
              (*y tangent*)
              ParametricPlot3D[vy[a, b] * t + {a, b, f[a, b]}, {t, -2, 2}, PlotStyle → {Cyan}],
              (*tangent plane*)
              Graphics3D[{LightGreen, Polygon[tanplane[a, b]]}],

              (*point*)
              Graphics3D[{Black, PointSize[0.03], Point[fp[a, b]]}],
              (*red arrow is dz in alpha direction*)
              Graphics3D[{Red, Thickness[.01],
                Arrow[{{fp[a, b], fp[a, b] + {Cos[alpha], Sin[alpha], dz[alpha, a, b]}}}]}],

              (*purple arrow is gradient*)
              Graphics3D[{Purple, Thickness[.01], Arrow[{fp[a, b], fp[a, b] + gr[{a, b}]}]}]
             ]},
            {Show[
              (*plots the same vectors on a level curve*)
              plot,
              Graphics[{Black, PointSize[0.03], Point[{a, b}]}],
              Graphics[
               {Red, Thickness[.01], Arrow[{{a, b}, {a, b} + {Cos[alpha], Sin[alpha]}}]}],
              Graphics[{Purple, Thickness[.01], Arrow[
                  {{a, b}, {a, b} + gradient[{a, b}] / Norm[gradient[{a, b}]]}]}]
             ]}
           }
          ]
          , {a, -3, 3}
          , {b, -3, 3}
          , {alpha, 0, 2 π}
         ]
```

a

b

alpha

Out[13]=