Arnav Dani - Period 3 - Project 1

Defining Functions

```
r[t_{-}] = \{t, t^{2}, t^{3}\};
(*speed, velocity and tangent*)
v[t_] = r'[t];
speed[t_] = \sqrt{(v[t].v[t])};
uTan[t] = v[t] / speed[t];
absDuTan[t_] = \sqrt{(uTan'[t].uTan'[t])};
(*normal and binormal*)
uNorm[t_] = uTan'[t] / absDuTan[t];
biNorm[t_] = Cross[uTan[t], uNorm[t]];
(*curvature*)
kurv[t_] = absDuTan[t] / speed[t];
evolute[t_] = r[t] + uNorm[t] * 1 / kurv[t]
(*defining planes using polygon*)
(*osculating plane is defined by osc vector -
 means that tangent and normal vector are on that plane*)
oscpl[t_] = Polygon[{(r[t] + 15 * uNorm[t]),}
    (r[t] + 15 * uTan[t]), (r[t] - 15 * uNorm[t]), (r[t] - 15 * uTan[t]) \}];
(*rectifying plane is defined by normal vector -
 means that tangent and binormal vector are on that plane*)
rectpl[t_] = Polygon[{ (r[t] + 10 * biNorm[t]),
     (r[t] + 10 * uTan[t]), (r[t] - 10 * biNorm[t]), (r[t] - 10 * uTan[t]) \}];
(*normal plane is defined by the tangent vector -
 means that noral and binormal vector are on that plane*)
norpl[t_] = Polygon[{(r[t] + 5 * biNorm[t]),}
     (r[t] + 5 * uNorm[t]), (r[t] - 5 * biNorm[t]), (r[t] - 5 * uNorm[t]) \}];
```

Graphing

```
In[*]:= Manipulate[
      (*use graphicsgrid to plot 2 graphs
       and simlutaneously control them with 1 manipulate*)
      GraphicsGrid[
       {
        {
         Show [
          ParametricPlot3D[r[t], \{t, -3, 3\}, PlotStyle \rightarrow \{Red, Thick\}, Axes \rightarrow True,
            AxesOrigin \rightarrow {0, 0, 0}, PlotRange \rightarrow 15], (*plots the function*)
           (*plotting the vectors and point on the function*)
           (*using q to go with the manipulate
            and get a single vector instead of a vector function*)
          Graphics3D[\{Black, Arrowheads[0.03], Arrow[\{r[q], (r[q] + 5 * uTan[q])\}]\}],
           Graphics3D[{Purple, Arrowheads[0.03], Arrow[{r[q], (r[q] + 5 * uNorm[q])}]}],
           Graphics3D[{Blue, Arrowheads[0.03], Arrow[\{r[q], (r[q] + 5 * biNorm[q])\}]}],
          Graphics3D[{PointSize → 0.025, Blue, Point[r[q]]}],
           (*plotting the relevant planes*)
           Graphics3D[{Red, Opacity[0.17], norpl[q]}],
          Graphics3D[{Purple, Opacity[0.17], rectpl[q]}],
          Graphics3D[{Blue, Opacity[0.17], oscpl[q]}]
         ]
        },
        {
         Show [
           (*2nd graph with curvature -
           this graph is in 2d since curvature at any t value is a scalar value*)
           Plot[kurv[t], \{t, -3, 3\}, PlotStyle \rightarrow \{Red, Thick\},
            Axes \rightarrow True, AxesOrigin \rightarrow {0, 0}, PlotRange \rightarrow 4],
           (*attaching moving point*)
          Graphics[{PointSize → 0.025, Blue, Point[{q, kurv[q]}]}]
         1
        }
       }
      ],
      \{q, -3, 3\}
```

