Name: ______________________________

READ THE FOLLOWING INSTRUCTIONS.

- You may use one sheet a handwritten notes.
- **Show all your work unless otherwise indicated.** Write your answers clearly! Include enough steps for the grader to be able to follow your work.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- You will be given exactly 50 minutes for this exam.

Fill in the Blanks.

1. (5 points) For any integer $n \geq 2$ the number of permutations of $1, 2, \ldots, n$ that have 1 in the first position and 2 in the last position (e.g. $1, 3, 5, 4, 2$ for $n = 5$) is $(n-2)!$.

2. (5 points) For any integer $n > 0$ the value of the sum $\sum_{k=0}^{n} 2^k \binom{n}{k}$ is 3^n.

3. (5 points) The number of solutions to $x + y + z + w = 11$ where each variable is a nonnegative integer is $\left(\binom{4}{11}\right) = \binom{14}{11}$.

Extra Work Space.

4. (5 points) The negation of $\forall x, \exists n, n > x$ is $\exists x, \forall n, n \leq x$ (your answer should be simplified so to not include the negation symbol $\neg$).

5. (5 points) In terms of logical operators $\wedge, \vee, \neg$ an expression logically equivalent to $\neg(p \to q)$ is $p \wedge \neg q$.

6. (5 points) A domain (subset of real numbers) so that the statement $\forall x, x^2 \leq x$ is true is $(0, 1)$.

Extra Work Space.

Standard Response Questions. Partial credit available. Must show all steps.

7. Answer the following about unlabeled balls and unlabeled bins. (In general this is a difficult counting problems, but in this question we will only use 2 unlabeled bins.)

   (a) (8 points) You have 10 unlabeled balls and 2 unlabeled bins. How many ways are there to distributed the 10 balls into the bins?

   (b) (8 points) You have N unlabeled balls and 2 unlabeled bins. How many ways are there to distributed the N balls into the bins?

Solution: There are 6 ways:

$$10, 9+1, 8+2, 7+3, 6+4, 5+5$$

There are $1 + \lfloor \frac{N}{2} \rfloor$ ways:

$$N, (N-1)+1, (N-2)+2, (N-3)+3, \ldots, \left\lceil \frac{N}{2} \right\rceil + \left\lfloor \frac{N}{2} \right\rfloor$$

8. (16 points) Recall a *bit flip* changes a single position of a binary string from a 0 to a 1 or vice versa (e.g. 001100 to 001000). How many bit strings can be obtained from 001100 by performing one or two bit flips? List all such bit strings.

Solution: We can choose either one or two of the 6 positions to flips gives $\binom{6}{1} + \binom{6}{2} = 6 + 15 = 21$ such strings.

101100	111100	010100	000000	001010
011100	100100	011000	000110	001001
000100	101000	011110	000101	001111
001000	101110	011101		
001110	101101			
001101				

9. (16 points) Construct a truth table for $p \to (q \wedge r)$.

Solution:

p	q	r	$q \wedge r$	$p \to (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

10. (18 points) Determine if the following statements are true or false

$$
\begin{aligned}
(i) &\quad \forall a, \forall b, \forall, c, \exists x, ax^2 + bx + c = 0 \\
(ii) &\quad \exists a, \forall b, \forall, c, \exists x, ax^2 + bx + c = 0 \\
(iii) &\quad \forall a, \exists b, \exists, c, \exists x, ax^2 + bx + c = 0
\end{aligned}
$$

where a, b, c, x are taken from the domain of real numbers. Justify your answer.

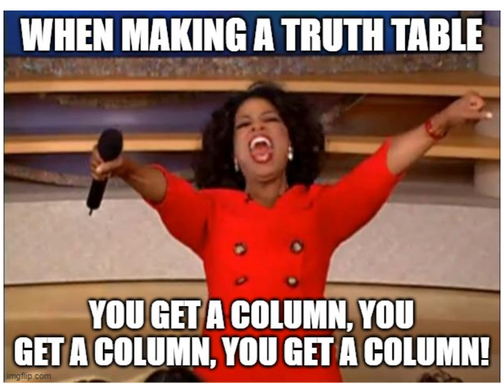
Solution: The first is false. Take for example $a = 1, b = 0, c = 1$ then $x^2 + 1$ has no real number solutions.

The second is false. For $a = 0$ take $b = 0$, and $c = 1$ then there is no solution. For any $a \neq 0$ there is no solution when $b^2 - 4ac < 0$ so take $b = 0$ and c so that $-4ac < 0$ (a and c same sign).

The third is true. For any a take $b = 0$ and $c = -a$ then $ax^2 - a = 0$ has a solution $x = 1$.

Making points add to 100.

11. (4 points) Circle your favorite meme.



Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity.

When you are completely happy with your work please turn in your exam on Canvas.

Page	Points	Score
2	15	
3	15	
4	16	
5	16	
6	16	
7	18	
8	4	
Total:	100	