Homework 6

Thursday, October 16, 2025 7:27 PM

1. bare spherical fuel 1 in diameter

pressure 2500 psia $\quad 1\cdot 10^7 \frac{Btu}{hr\cdot ft^3}$ heat generation rate. q'''

fuel thermal conductivity k_f is $10 \frac{Btu}{hr\cdot ft\cdot °F}$ maximum possible temp in fuel w/ no boiling

uniform properties

$\nabla k \nabla T + q''' = \rho c_p \cancel{\frac{dT}{dt}} = 0$ b/c SS.

$k\nabla^2 T = -q'''$

$\nabla^2 T = \frac{-q'''}{k} = \frac{10^7}{10} \frac{\frac{Btu}{hr\cdot ft^3}}{\frac{Btu}{hr\cdot ft\cdot °F}}$

∇^2 for spherical, assuming symmetry

$\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr}$ → only radial changes.

$\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr} T(r) = -10^6 \frac{°F}{ft^2}$

$r^2 \frac{d}{dr} T(r) = -10^6 \frac{r^3}{3} + C_1$

$\frac{d}{dr} T(r) = -10^6 \frac{r}{3} + \frac{C_1}{r^2}$

$T(r) = \frac{-10^6}{6} r^2 - \frac{C_1}{r} + C_2 \quad °F$

at $r=0$ $\frac{dT(r)}{dr} = 0$ bc of symmetry

so $\frac{C_1}{0^2} \to 0$ which means $C_1 = 0$

at fuel surface, $T(2500\,psia) = 668.31°F$

$R = 0.5$ in or 0.0417 ft

$T(0.0417ft) = \frac{-10^6 \frac{°F}{ft^2}}{6} \cdot (0.0417ft)^2 + C_2 = 668.31°F$

$C_2 = 668.31 + 289.35 = \boxed{957.66°F}$

max T at $r=0$

so $\boxed{T(0) = 957.66°F}$ → maximum possible temperature in the fuel element.

2. 6 in

Gamma radiation of $10^{14} \frac{photon}{sec\cdot cm^2}$ & $5 \frac{MeV}{photon}$ → $5\cdot 10^{14} \frac{MeV}{sec\cdot cm^2} \cdot \frac{1.519\cdot 10^{-16} Btu}{1 MeV} \cdot \frac{1 cm^2}{0.155 in^2} \cdot \frac{3600 s}{1 hr} \cdot \frac{144 in^2}{ft^2} = 254016 \frac{Btu}{hr\, ft^2}$

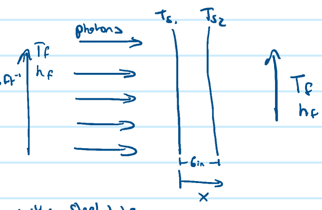
forced convection

both sides $1000 \frac{Btu}{hr\cdot ft^2 \cdot °F} = h$ at 300°F

2 surface temps

max temp within plate

$k = 28 \frac{Btu}{hr\cdot ft\cdot °F}$, $\mu = \mu_m = 0.245 cm^{-1} = 7.4676 ft^{-1}$



$q''' = \cancel{B}\cdot \mu \cdot I_0 \cdot e^{-\mu x}$ (1)

$q''' = 1.8969\cdot 10^6 \cdot e^{-7.4676\cdot x} \frac{Btu}{hr\cdot ft^3}$

within steel plate

uniform

$\nabla k \nabla T + q''' = \rho c_p \cancel{\frac{dT}{dt}} = 0$

$\nabla^2 T = \frac{-q'''}{k}$

∇^2 → only in x direction → $\frac{d^2}{dx^2}$

$h(T(0) - 300°F) = k \frac{dT}{dx}\Big|_{x=0}$

$-k\frac{dT}{dx}\Big|_{x=0.5ft} = h(T(0.5) - 300°F)$

$\frac{d^2T}{dx^2} = \frac{-1.8969\cdot 10^6}{28} \cdot e^{-7.4676\cdot x} \frac{°F}{ft^2}$

$\frac{d^2T}{dx^2} = -67746.1 \cdot e^{-7.4676x} \frac{°F}{ft^2}$

$\frac{dT}{dx} = -\frac{67746.1}{-7.4676} e^{-7.4676x} + C_1$ → $\frac{dT}{dx} = 9072 e^{-7.4676x} + C_1$

$T(x) = \frac{-67,746.1}{(7.4676)^2} e^{-7.4676x} + C_1 x + C_2$

$T(x) = -1214.85 e^{-7.4676x} + C_1 x + C_2 \quad °F$

$1000 \frac{Btu}{hr\cdot ft^2 °F}\left(-1214.85 + C_2 - 300\right) = 28 \frac{Btu}{hr\cdot ft \cdot °F}\left(9072 + C_1\right)$

$\underbrace{-28 \frac{Btu}{hr\cdot ft \cdot °F}\left(216.84 + C_1\right) = 1000 \frac{Btu}{hr\cdot ft^2 °F}\left(-29.0871 + 0.5C_1 + C_2 - 300\right)}$

$C_1 = -2600.54 \frac{°F}{ft}$

$C_2 = 1696.05 °F$

a) $T(0) = -1214.85 + 1696.05 °F = \boxed{481.2 °F}$

$T(0.5) = -29.087 + 0.5\cdot(-2600.54) + 1696.05 °F = \boxed{366.74 °F}$

b) max temp at $\frac{dT}{dx} = 0$ within plane

$9072\, e^{-7.4676x} = 2600.54$

$x = 0.16732$ ft

less than 0.5ft ✓

$T(0.16732\,ft) = -124.856\, e^{-7.4676 \cdot 0.16732} + -2600.54 \cdot 0.16732 + 1696.05\,°F$

$\boxed{T(0.16732 ft) = 912.686°F}$ ← max temperature

3

Clad OD = 12.52 mm

Clad thickness = 0.86 m

gap = 0.230 mm

Solid pellet w/ density = 88%

$k_c = 17 \frac{W}{mK}$

$k_g = 4300 \frac{W}{m^2 K}$

$k_f = 2.7 \frac{W}{m \cdot K}$ at 95%

clad outside = 295°C

$q' = 44 \frac{kW}{m} \rightarrow 44 \frac{kW}{m} \cdot \frac{1}{\pi (5.17 \cdot 10^{-3})^2} = 523.988 \frac{MW}{m^3}$

uniform

$\nabla k \nabla T + q''' = \rho c_p \cancel{\frac{dT}{dt}}$ 0

I. Fuel

$\frac{1}{r}\frac{d}{dr} r \frac{dT_I}{dr} = -\frac{q'''}{k_f}$

$r \frac{dT_I}{dr} = \frac{-q'''}{k_f}\frac{r^2}{2} + C_1$

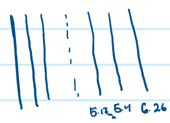
$\frac{dT_I}{dr} = \frac{-q'''}{k_f}\frac{r}{2} + \frac{C_1}{r} \rightarrow \frac{-q''' r}{k_f 2} = \frac{dT_I}{dr}$

$T_I(r) = \frac{-q'''}{k_f}\frac{r^2}{4} + C_1 \ln(r) + C_2 \rightarrow T_I(r) = \frac{-q'''}{k_f}\frac{r^2}{4} + C_2$

q'''

$\frac{dT_I}{dr}\Big|_{r=0} = \frac{C_1}{0} = 0$

So $C_1 = 0$



II Clad

uniform

$\nabla k \nabla T + \cancel{q'''} = \rho c_p \cancel{\frac{dT}{dt}}$ 0 0

$\frac{1}{r}\frac{d}{dr} r \frac{dT_{II}}{dr} = 0$

$\frac{dT_{II}}{dr} = \frac{C_3}{r}$

$T_{II}(r) = C_3 \ln(r) + C_4$

$T_{II}(6.26mm) = 295°C = C_3 \cdot \ln(6.26mm) + C_4$

$-k_f \frac{dT_I}{dr}\Big|_{r=5.17mm} = -k_c \frac{dT_{II}}{dr}\Big|_{r=5.4mm} = h_g \left(T_I(r=5.17mm) - T_{II}(r=5.40mm)\right)$

$\frac{523.988 \frac{MW}{m^3} \cdot 5.17mm}{2} = \frac{-17W}{mK} \cdot \frac{C_3}{5.4mm}$ $\cancel{-k_f}$ / $\cancel{-k_f}$

$\frac{523.988 \frac{MW}{m^3} \cdot 5.17mm}{2} = 4300 \frac{W}{m^2 K}\left(\frac{-523.988 \frac{MW}{m^3}}{2.485 \frac{W}{mK}} \frac{(5.17 \cdot 10^{-3} m)^2}{4} + C_2 - \underbrace{\left(-430.256 \cdot \ln(5.4 \cdot 10^{-3}) - 1614.79K\right)}\right)$

$\boxed{C_3 = -430.256K}$

$\boxed{T_{clad\,inner} \rightarrow 631.73K \rightarrow 358.6°C}$

$315.002\, K = -1447.76 + C_2 - 631.73K$

$\boxed{C_2 = 2394.49\,K}$

$T_{II} = C_3 \ln(r) + C_4$

$T_{II}\Big|_{r=6.26mm} = -430 \cdot \ln(6.26 \cdot 10^{-3}) + C_4 = 568.15K$

$\boxed{C_4 = -1614.79K}$

$\boxed{T_{max\, I}\Big|_{r=0} = C_2 = 2394.49K = 2121.34°C}$

$T_{fuel\,outer} = 315.00 + 631.73\,K$

$\boxed{T_{fo} = 673.582°C}$ (946.73K)

$T_{avg} = \frac{\int_0^z \int_0^{2\pi} \int_0^r T(r)\, r\, dr\, d\theta\, dz}{\pi r^2 \cdot z}$

$T_{avg} = \frac{2}{R^2} \int_0^R -4.8517 \cdot 10^7 r^3 + 2394.49 r \, dr$

$T_{avg} = 1746.08K$ or $\boxed{1472.93°C}$

4. 2in ball at 850°F (T_i) put in 200°F environment (T_∞)

$h = 2 \frac{Btu}{hr \cdot ft^2 \cdot °F}$ time needed to cool to 300°F

$c_p = 0.122 \frac{Btu}{lbm\, °F}$ $\rho = 490 \frac{lbm}{ft^3}$

$r = 1in = 0.0833$ ft

$A_c = 0.087266\, ft^2$

$V_c = 0.002424\, ft^3$

$\cancel{\dot{E}_{in}} - \dot{E}_{out} + \cancel{\dot{E}_{gen}} = \dot{E}_{st}$ 0 0

$-h \cdot A_s (T_s - T_\infty) = \rho \cdot c_p \cdot V_c \frac{dT}{dt}$

$\theta = T - T_\infty \quad \frac{d\theta}{dt} = \frac{dT}{dt}$

$-h \cdot A_s \cdot \theta = \rho \cdot c_p \cdot V_c \frac{d\theta}{dt}$

$\frac{-h \cdot A_s}{\rho c_p V_c} dt = \theta^{-1} d\theta$

$\int -1.20442\, dt = \int \theta^{-1} d\theta$

$-1.2042\, hr \cdot t = \ln(\theta) - \ln(\theta_i)$

$\theta_i = T_i - T_\infty = 850°F - 200°F$

$e^{-1.2042t} = \frac{\theta}{650°F}$

at $T = 300°F$

$\theta = 100°F$

$e^{-1.2042t} \cdot 650°F = 100°F$

$\boxed{t = 1.554 hr}$

Sphere.

5. 1 inch diameter $\rightarrow$ 0.041667 ft, r $\rightarrow$ $A_s = 0.021817\ ft^2$, $V_c = 3.0301 \cdot 10^{-4}\ ft^3$

$$\frac{0.04\ Btu}{lbm \cdot {}^\circ F} = c_p, \quad \rho = 700\ \frac{lbm}{ft^3}, \quad h = 200\ \frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}$$

$$-h A_s (T - T_\infty) = \rho c_p V_c \frac{dT}{dt}$$

$$\Theta = T - T_\infty, \quad \frac{d\Theta}{dt} = \frac{dT}{dt}$$

$$\int_0^t \frac{-h A_s}{\rho \cdot c_p \cdot V_c}\, dt = \int_0^t \frac{1}{\Theta}\, d\Theta$$

$$-514.286\, t = \ln\left(\frac{\Theta(t)}{\Theta_i}\right)$$

$\leftarrow T_i - T_\infty$

100 - 500 = -400

$$\frac{1}{514.286} = \boxed{\tau = 0.001944\ hr}$$ a)

$T(t_a) = 499^\circ F \rightarrow \Theta = -1^\circ F$

b)

$$t = \frac{\ln\left(\frac{-1}{-400}\right)}{-514.286} = \boxed{0.01165\ hr}$$