HW1

Friday, August 29, 2025 10:06 AM

1. In a thermal reactor the energy is released in the form of fission fragments, fast neutrons and prompt gammas. The fission fragments carry 80.5% of the energy and deposit their energy in the fuel. They are also usually radioactive and decay, releasing delayed neutrons, β particles, γs, and neutrinos. These neutrinos account for 5% of the total power, and their energy is unrecoverable. Neutrons, both prompt and delayed, deposit their energy in the moderator through scattering collisions. These collisions also release gammas. Once scattered the slow neutrons can be absorbed and in the case of capture, as opposed to fission, gammas and βs are also released. The gammas and βs from all of the different avenues deposit their energy in the fuel and the surrounding structural materials.

2. T-K 3-1

a) rate of 3083 MW w/ 193 assemblies w/ 517.4 kg UO_2 each

3.25% U^{235} avg neutron flux?

$M_{UO_2} = M_U + 2M_O$

$M_U = aM_{25} + (1-a)M_{28}$ where a is the atomic fraction

enrichment (r):

$$r = \frac{aM_{25}}{M_U} = \frac{aM_{25}}{aM_{25} + (1-a)M_{28}}$$

$M_{25} = 235.044$ g/mol

$M_{28} = 238.051$ g/mol

$\rightarrow a = 0.032902$

$M_U = 237.952$ g/mol

$\boxed{M_{UO_2} = 269.952 \text{ g/mol}}$

$\sigma_f^{25} = 350\text{ b}$

$\chi_f = 140 \frac{\text{MeV}}{\text{fission}}$

$3.04 \cdot 10^{-11} \frac{\text{J}}{\text{fission}}$

$$\phi_1(\vec{r}) = \frac{q'''(\vec{r})}{\chi_f \cdot \Sigma_f}$$

$$\phi_1(\vec{r}) = \frac{q'''(\vec{r})}{\chi_f \cdot \sigma_f^{25} \cdot N^{25}} \cdot \frac{V_{UO_2}}{V_{UO_2}} = \frac{\dot{Q}}{\chi_f \cdot \sigma_f^{25} \cdot N^{25} V_{UO_2}}$$

$$N^{25} = a \cdot N^0 = a \cdot \frac{Av \cdot \rho_{UO_2}}{M_{UO_2}}$$

$$N^{25} \cdot V_{UO_2} = a \cdot \frac{Av \cdot m_{UO_2}}{M_{UO_2}} = 0.032902 \cdot \frac{6.022 \cdot 10^{23} \cdot 9.9858 \cdot 10^{7}\text{ g}}{269.952 \text{ g/mol}} = \boxed{7.330 \cdot 10^{27} \text{ atoms } U^{25}}$$

$$m_{UO_2} = 193 \cdot 517.4\text{ kg} \cdot \frac{1000\text{ g}}{\text{kg}} = 9.9858 \cdot 10^{7}\text{ g}$$

$$\langle \phi_1(\vec{r}) \rangle = \frac{3083\text{ MW} \cdot 0.95 \cdot 10^6 \frac{\text{W}}{\text{MW}}}{3.04 \cdot 10^{-11} \frac{\text{J}}{\text{fission}} \cdot 350 \cdot 10^{-24}\text{ cm}^2 \cdot 7.330 \cdot 10^{27}\text{ atoms}}$$

$$\boxed{\langle \phi_1(\vec{r}) \rangle = 3.7554 \cdot 10^{13} \frac{\text{neutrons}}{\text{cm}^2 \cdot \text{s}}}$$

b) Power density in MW/m³

fuel is 90% of theoretical density

Theoretical density → 10.96 g/cm³

9.864 g/cm³ → fuel density → ρ_{UO_2}

$$\langle \phi_1 \rangle = \frac{\langle q''' \rangle}{\chi_f \sigma_f^{25} \cdot N^{25}}$$

$$N^{25} = a \cdot \frac{Av \cdot \rho_{UO_2}}{M_{UO_2}}$$

$$N^{25} = 0.032902 \cdot \frac{6.022 \cdot 10^{23} \cdot 9.869 \text{ g/cm}^3}{269.952 \text{ g/mol}} = 7.24924 \cdot 10^{20} \frac{\text{atoms}}{\text{cm}^3}$$

$$\langle q''' \rangle = 3.7554 \cdot 10^{13} \frac{\text{n}}{\text{cm}^2 \cdot \text{s}} \cdot 3.04 \cdot 10^{-11} \frac{\text{J}}{\text{fission}} \cdot 350 \cdot 10^{-24} \frac{\text{cm}^2}{\text{atom} \cdot \text{n}} \cdot 7.244 \cdot 10^{20} \frac{\text{atoms}}{\text{cm}^3}$$

$$\langle q''' \rangle = 288.66 \frac{\text{W}}{\text{cm}^3} \text{ or } \boxed{288.66 \frac{\text{MW}}{\text{m}^3}}$$

c) Average linear power

264 per assembly, rod length is 3810 mm

$$\langle q' \rangle = \frac{\gamma \dot{Q}}{N \cdot L} = \frac{0.95\,(3083\text{ MW}) \cdot \frac{1000\text{ kW}}{1\text{ MW}}}{264 \frac{\text{rods}}{\text{assembly}} \cdot 3.810 \frac{\text{m}}{\text{rod}} \cdot 193 \text{ assemblies}}$$

$$\boxed{\langle q' \rangle = 19.2417\text{ kW/m}}$$

d)

$$\langle q'' \rangle = \frac{\gamma \dot{Q}}{NL \pi \cdot 2 \cdot R} = \frac{\langle q' \rangle}{\pi D}$$

$$\frac{19.2417 \frac{\text{kW}}{\text{m}}}{\pi \cdot 0.01\text{ m}} = \boxed{612.484 \frac{\text{kW}}{\text{m}^2}}$$

3. T-K 3-3

a) Total power generation if its 4m high

Example 3-1 → heat generation rate

$\underbrace{1.333_m - 1.206_m}_{12.7cm}$

$S = 10^{14} \frac{\gamma}{cm^2 \cdot s}$, $B_{Fe} = 4.212$, $E_0 = 2MeV$, $\phi_{fast} = 10^{14} \frac{n}{cm^2 \cdot s}$

neutron energy $= 0.6MeV$, $\sigma_T = 3b$, $f(2MeV) = 0.1$, $\mu_a = 0.182$ for iron at 2MeV, $\mu = 0.333$ for iron

$q_\gamma''' = S \cdot B \cdot \mu_a \cdot E_0 \cdot e^{\mu x}$

$= 10^{14} \cdot 4.212 \cdot 0.182 \cdot 2 \cdot e^{-0.333 \cdot 12.7} = 2.23 \cdot 10^{12}$ MeV/cm³·s → $\dot{Q}_\gamma = \int q'''(r)\, dV$

$= 2\pi h \cdot S \cdot B \cdot \mu_a \cdot E_0 \int_{120.6cm}^{133.3cm} r \cdot e^{-\mu(r)} dr$

$q_{el}''' = \xi_1 E \cdot \Sigma_{s,el} \cdot \phi$ & $q_{i}''' = f(E) \vec{E} \Sigma_{s,i} \cdot \phi$

$r_0 = 120.6cm$

$\xi_1 = \frac{1}{2}\left(1 - \left(\frac{A-1}{A+1}\right)^2\right) = \boxed{0.034591}$ $\Sigma_T = \frac{S \cdot A_r}{M_{Fe}} \sigma_T = 0.254 cm^{-1}$ $\dot{Q}_\gamma = \underbrace{2\pi \cdot 400cm \cdot 10^{14} \frac{\gamma}{cm^2 s} \cdot 4.212 \cdot 0.182 cm^{-1} \cdot 2MeV}$ $\cdot \int_{120.6cm}^{133.3cm} r \cdot e^{-\mu(r-r_0)} dr$

$A = 55.80$

$\Sigma_{i} = \Sigma_{el} = 0.127 cm^{-1}$

$\dot{Q}_\gamma = 3.8533 \cdot 10^{17} \cdot 356.218 \frac{MeV}{s}$ $1.6 \cdot 10^{-13} \frac{J}{MeV} \cdot \frac{1MW}{10^6 W}$

$\boxed{\dot{Q}_\gamma = 22.5166\ MW}$

$q_{el}''' = 0.0346 \cdot 0.6MeV \cdot 0.127 \cdot 10^{14}$ $q_{i}''' = 0.1 \cdot 0.6MeV \cdot 0.127 \cdot 10^{14}$

$= 0.26 \cdot 10^{12}$ MeV/cm³·s $= 0.76 \cdot 10^{12}$ MeV/cm³·s

$q_n''' = 1.02 \cdot 10^{12}$ MeV/cm³·s →

$V = 400cm \cdot \left(\pi(133.3^2 - 120.6^2)\right) = 4.05206 \cdot 10^6 cm^3$

$\boxed{\dot{Q}_n = q_n''' \cdot V = 0.662\ MW}$ → $\boxed{\dot{Q} = 23.1786\ MW}$

b) if total thickness was 15cm

based on the last problem

$V = 400cm \cdot \pi\left((133.3+2.3)^2 - 120.6^2\right) = 4.829 \cdot 10^6$

$\dot{Q}_\gamma = \underbrace{2\pi h \cdot S \cdot B \cdot \mu_a \cdot E_0} \cdot \int_{120.6cm}^{135.6cm} r \cdot e^{-\mu(r-r_0)} dr$

new $\dot{Q}_n = q_n \cdot V = \boxed{0.778 MW}$

$\dot{Q}_\gamma = 3.8533 \cdot 10^{17} \cdot 368.362 \frac{MeV}{s} = 1.419 \cdot 10^{20} \frac{MeV}{s}$

$\sim 1.02 \cdot 10^{12} \frac{MeV}{cm^3 \cdot s}$

$\boxed{\dot{Q}_\gamma = 22.71 MW}$

$\boxed{\dot{Q} = \dot{Q}_n + \dot{Q}_\gamma = 23.4985 MW}$

4. 3-4

$\underbrace{3000\ MWth\ LWR \quad 1\ yr\ at\ 75\%\ total\ power}_{2250}$

b) 1 dy = 86400 sec $\int_0^{86400} 148.5\left(t^{-0.2} - (t + (8760 \cdot 3600))^{-0.2}\right) dt$

$\tau_s = 1_{yr} = 8760$ hrs → 31,536,000 s

$P = 1.2455 \cdot 10^6$ MJ so $\boxed{1.246 TJ}$

$\frac{P}{P_0} = 0.066\left(t_s^{-0.2} - (t_s + \tau_s)^{-0.2}\right)$

c) 1 Month = $2.592 \cdot 10^6$ $\int_0^{2.592 \cdot 10^6} 148.5\left(t^{-0.2} - (t + (8760 \cdot 3600))^{-0.2}\right) dt$

$P = \int 2250\ MWth \cdot 0.066\left(t_s^{-0.2} - (t_s + \tau_s)^{-0.2}\right) dt_s$

$P = 1.30108 \cdot 10^7$ MJ so $\boxed{13.011 TJ}$

a) 1 hr = 3600 sec → $\int_0^{3600} 148.5\left(t^{-0.2} - (t + (8760 \cdot 3600))^{-0.2}\right) dt$

$P = 113006$ MJ so $\boxed{0.11301 TJ}$

d) They would be higher as Eq 3-71, it includes the decay heat of actinides as well