Homework 7

Thursday, October 23, 2025 7:34 PM

1.

I atm pressure, water, 6cm diameter tube at $\dot{m}$ 0.01 $\frac{kg}{s}$ & 20°C

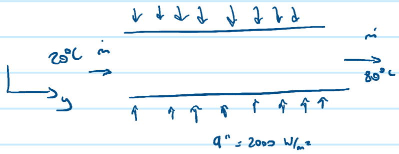
2000 W/m² = q'' exit temperature of 80°C

$\dot{m} = \rho \cdot v \cdot A$ $\quad v = \frac{\dot{m}}{\rho \cdot A} = \frac{0.01\,\frac{kg}{s}}{998.2\,\frac{kg}{m^3} \cdot (0.03^2 \pi)\,m^2} = 3.537 \cdot 10^{-3}\,\frac{m}{s}$

$Re = \frac{1000\,\frac{kg}{m^3} \cdot 3.53 \cdot 10^{-3}\,\frac{m}{s} \cdot 0.06\,m}{1.002 \cdot 10^{-3}\,\frac{kg}{m\,s}} = 211.783$ ← Laminar

$Re = \frac{\rho v D}{\mu}$ $\quad \mu$ for water at 20°C $= 1.002 \cdot 10^{-3}\,\frac{N\,s}{m^2}$

ν at 20°C $= 1.004 \cdot 10^{-6}\,\frac{m^2}{s}$



Energy conservation

$\cancel{\frac{d}{dt}\rho h} + \frac{d}{dy}\rho v h = \frac{q'' \wp_w}{A} + \cancel{\frac{dP}{dt}} + \cancel{q'''}$ $\quad$ at $y = 0$

$\rho v \frac{dh}{dy} = \frac{q'' \wp_w}{A}$ $\quad T(0) = 20°C = 293.15K = C$

$\frac{dh}{dy} = \frac{dT c_p}{dy}$ $\quad T(y) = \frac{q'' \wp_w}{A \cdot \rho v \cdot c_p} + 293.15$

$T(y) = \frac{q'' \wp_w}{A \cdot \rho v \cdot c_p} y + C$ $\quad T(L) = 353.15K = 293.15K + \frac{2000\,\frac{W}{m^2} \cdot \cancel{\pi} \cdot 0.06\,m \cdot L\,m}{\cancel{\pi}(0.03)^2\,m^2 \cdot 1000\,\frac{kg}{m^3} \cdot 3.53 \cdot 10^{-3}\,\frac{m}{s} \cdot 4200\,\frac{J}{kg\,K}}$

a) $\boxed{L = 6.672\,m}$

q'' constant so $Nu = 4.36$

k_f is $669.5 \cdot 10^{-3}\,\frac{W}{m \cdot K}$ for 353.15 K

$L_c = D = 0.06\,m$

$h = \frac{Nu \cdot k}{L_c} = 48.65\,\frac{W}{m^2 K}$ at 80°C (353.15K)

$h(T_s - 353.15) = 2000\,\frac{W}{m^2}$

$T_s = 394.26\,K = \boxed{121.11\,°C}$ b)

c) $\frac{x}{D} \approx 0.05\,Re_D$

where x is the length to become FD

$x \approx 0.05 \cdot Re_D \cdot D$

$x = 0.6353\,m << L$ so a fully developed assumption $\boxed{\text{is valid}}$

2. $10^6\,\frac{W}{m^3} \cdot \frac{\cancel{\pi} \cdot \left((0.02)^2 - (0.01)^2\right)}{2 \cdot \cancel{\pi} \cdot 0.01} = 15{,}000\,\frac{W}{m^2}$

$\rho v \frac{dh}{dy} = \frac{q'' \wp_w}{A}$ $\quad T(0) = C_1 = 293.15$

$\frac{dT}{dy} = \frac{q'' \wp_w}{A \cdot \rho v \cdot c_p}$ $\quad T(L) = 333.15K = \frac{15{,}000 \cdot 2\pi \cdot 0.01}{0.1\,kg/s \cdot 4.2\,\frac{kJ}{kg \cdot K}} \cdot L + 293.15K$

a) $\boxed{L = 17.825\,m}$

$T(y) = \frac{q'' \wp_w}{\dot{m} \cdot c_p} y + C_1$

b)

$q'' = h(T_s - T_0)$ at outlet $\rightarrow \frac{q''}{(70 - 60)} \rightarrow \boxed{h = 1{,}500\,\frac{W}{m^2 K}}$

3. $0.15m = D$ hot air $0.05 kg/s$, 1 atm

$103°C \rightarrow 77°C$ after $L = 5m$

376 K 350 K

$0°C$ air with $h_2 = 6 \frac{W}{m^2 \cdot K}$

C_p at 360 K is $1012 \frac{J}{kgK}$

μ at 360K is $213 \cdot 10^{-7} \frac{kg}{ms}$

k at 360K is $30.7 \cdot 10^{-3} \frac{W}{mK}$

q'' is a function of y

$$q'' = h(T_{(y)} - T_f) = \frac{6W}{m^2K}(\overset{T_\infty}{T_f} - 273.15)$$

$$\dot{m} = \rho v \cdot \frac{\pi D^2}{4}$$

$$\frac{4\dot{m}}{\pi D} = \rho v D$$

$$Nu = 0.023 \cdot Re^{0.8} Pr^{0.3} = 53.0233 \rightarrow Nu = \frac{h \cdot D}{k}$$

$$h_1 = 10.8521$$

$$Pr = \frac{\mu \cdot c_p}{k} = 0.70214$$

$$Re = \frac{\rho v D}{\mu} = \frac{4\dot{m}}{\pi D \cdot \mu} = 19925.5$$

$$6(T_f - 273.15) = 10.8521(350K - T_f)$$

$$16.8521 \cdot T_f = 5437.14$$

$$T_f(5m) = 322.638\,K \text{ or } \boxed{49.488°C} \text{ b)}$$

q (total heat lost) $\rightarrow \dot{m} \cdot c_p \cdot \Delta T$ a)

$$q = 0.05 \frac{kg}{s} \cdot 1012 \frac{J}{kgK} \cdot 26K = \boxed{1315.6W}$$

$$q'' = 6(322.6384 - 273.15) = \boxed{296.931 \frac{W}{m^2}} \text{ c)}$$

4. Oil enters a tube of $D = 0.05m$, $L = 25m$, $T_{sp} = 150°C$ or $423.15K$

$\dot{m} = 0.5 kg/s$ & $T_{(0)} = 20°C = 293K \rightarrow \rho = 852 \frac{kg}{m^3}$, $k_f = 0.138 \frac{W}{mK}$, $\mu = 0.032 \frac{kg}{ms}$

$c_p = 2131 \frac{J}{kgK}$

N_u for pipe with constant surface T $\rightarrow 3.66$

a) fluid outlet temperature

$$\frac{3.66 \cdot k_f}{D} = h \Rightarrow 10.102 \frac{W}{m^2K}$$

$$\rho v \frac{dh}{dy} = \frac{q'' \cdot \rho_w}{A}$$

$$q'' = h(T_s - T_{(y)})$$

$$\frac{dh}{dy} = \frac{dT c_p}{dy} \quad \frac{dT}{dy} = \frac{q'' \cdot \rho_w}{\dot{m} c_p}$$

$$\frac{dT}{dy} = \frac{h \cdot \rho_w}{\dot{m} \cdot c_p}(T_s - T_{(y)})$$

$$\frac{dT}{T_s - T_{(y)}} = \frac{h \cdot \rho_w}{\dot{m} \cdot c_p}$$

$$-\ln(T_s - T_{(y)}) = \frac{h \cdot \rho_w}{\dot{m} \cdot c_p} y + C_0$$

$$\hookrightarrow \frac{10.102 \cdot \pi \cdot 0.05}{0.5 kg \cdot 2131 \frac{J}{kgK}}$$

$$423.15K - T_{(y)} = C_1 \cdot e^{-1.489 \cdot 10^{-3} \cdot y}$$

at $y = 0$, $T_{(0)} = 293.15K$

$C_1 = 130$

$$423.15K - T_{(25)} = 130 \cdot e^{-1.489 \cdot 10^{-3} \cdot 25m}$$

$$T_{(25m)} = 423.15K - 125.25K = \boxed{297.901K}$$

b)

$$T_{(y)} = 423.15 - 130e^{-1.489 \cdot 10^{-3} y}$$

$$q'' = h(T_s - T_{(y)})$$

$$q'' = 10.102 \frac{W}{m^2 \cdot K} \cdot 130\, e^{-1.484 \cdot 10^{-3} y}$$

$$1313.26 \int_0^{25m} e^{-1.489 \cdot 10^{-3} \cdot y}\, dy \cdot \pi \cdot 0.05m = \boxed{5062.34W}$$

5. 2kg/s through 0.04m tube

25°C to 75°C surface temp at 100°C

$$\cancel{\frac{d\rho v}{dt}} + \frac{d}{dz}\rho v^2 = -\frac{dP}{dz} - \tau_f \frac{\wp_w}{A} - \rho g \sin\theta$$

$\theta = 0$

$$\rho v \frac{dh}{dy} = \frac{q'' \wp_w}{A}$$

$q'' = h(T_s - T_{(y)})$ $c_p = 4181\ J/kgK$

μ at 323.15K $= 552 \cdot 10^{-6} \frac{kg}{ms}$

$$\frac{dh}{dy} = \frac{dTc_p}{dy}$$

ρ at 323.15 $= \frac{1}{1.012 \cdot 10^{-3}} = 988.142 \frac{kg}{m^3}$

$$\frac{dT}{dy} = \frac{h(T_s - T_{(y)})\wp_w}{\dot{m} \cdot c_p}$$

$Re = \frac{\dot{m} \cdot 4}{\mu \cdot \pi D} = 115,330 \rightarrow$ turbulent

k_f at 323.15K $= 642 \cdot 10^{-3} \frac{W}{mK}$

$$-\frac{dP}{dy} = G^2 \overbrace{\frac{d}{dy}\frac{1}{\rho}}^{\text{constant}} + \frac{1}{2} \cdot f \cdot \frac{G^2}{\rho} \frac{\wp_w}{A}$$

$\frac{\dot{m}^2}{A^2}$ $A = \frac{\pi D^2}{4}$ πD

$$-\ln(T_s - T_{(y)}) = \frac{h \cdot \wp_w}{\dot{m} \cdot c_p} y + C_0$$

$Pr = \frac{\mu \cdot c_p}{k} = 3.595$

$Nu = 0.023 \cdot (115,330)^{0.8} \cdot (3.595)^{0.4}$

$Nu = 430.09$

$$\Delta P = L \cdot \frac{1}{2} f \cdot \frac{\dot{m}^2}{\rho} \cdot \frac{\pi D}{\frac{\pi^3 D^6}{64}} \Rightarrow \frac{L}{2} \cdot f \cdot \frac{\dot{m}^2}{D^5} \cdot \frac{64}{\pi^2 \rho}$$

turbulent $f = \frac{0.079}{Re^{0.25}}$

$$T_s - T_{(y)} = C_1 e^{-\frac{h \cdot \wp_w}{\dot{m} \cdot c_p} y}$$

at $y = 0$ $75 = C_1$

$h = \frac{378.43 \cdot 642 \cdot 10^{-3} \frac{W}{mK}}{0.04m} = 6902.87 \frac{W}{m^2K}$

$$\frac{32L}{\pi^2 \cdot \rho} \cdot \frac{\dot{m}^2}{D^5} \cdot \frac{0.079}{\left(\frac{\dot{m} \cdot 4}{\mu \pi D}\right)^{0.25}}$$

at $\dot{m} = 2kg/s$ $D = 0.04m$

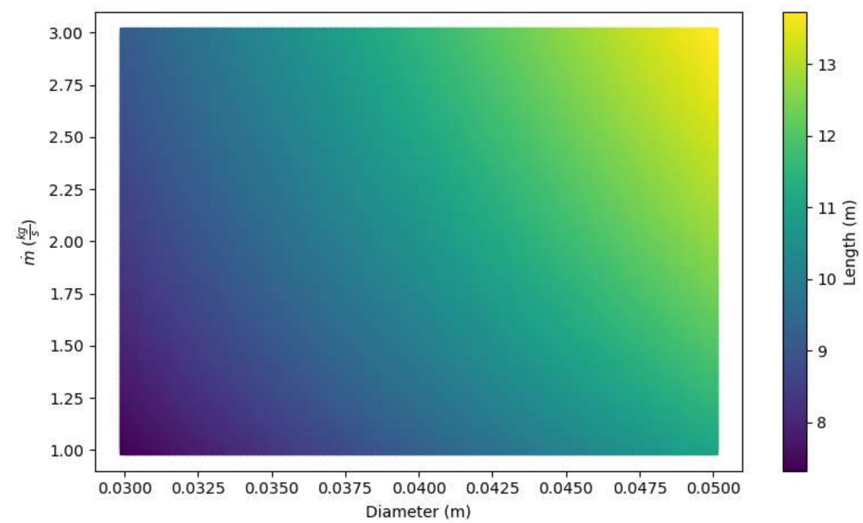
$\boxed{\Delta P = 5818.99 Pa}$

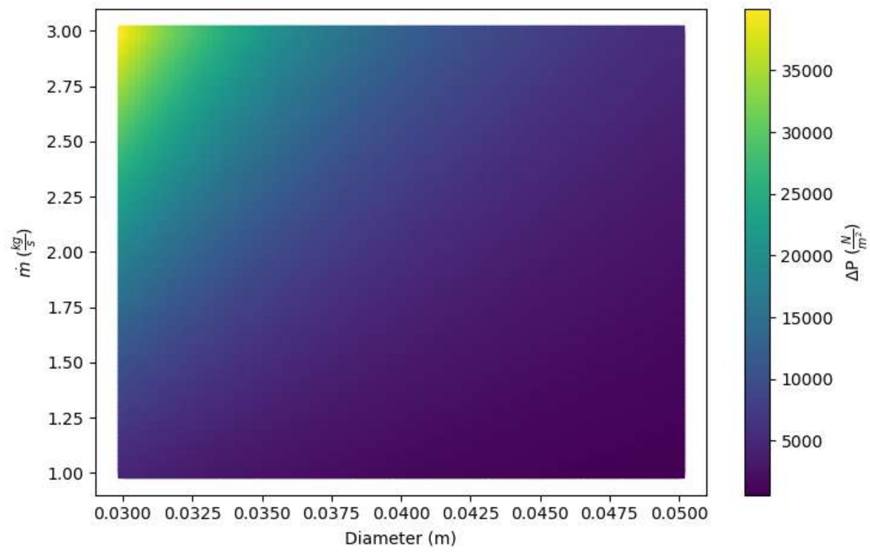
at $y = L$

$T_s - T(L)$

$$25K = 75 \cdot e^{-((6902.87 \cdot \pi \cdot 0.04m)/(2kg/s \cdot 4181\ J/kgK)) \cdot L}$$

$\boxed{L = 10.590\ m}$





6. Length L, diameter D bare fuel rod in a concentric insulated tube

water in between $\dot{m}$

$$q'''(z) = q_0''' \cdot \sin\left(\frac{\pi z}{L}\right)$$

h is inuniform

a) heat flux out of fuel $q''(z)$

$$q'''(z) = q_0''' \sin\left(\frac{\pi z}{L}\right)$$

$$q_0''' \cdot \sin\left(\frac{\pi z}{L}\right) \cdot \pi r^2 \cdot dz = q'' \cdot 2\pi r \, dz$$

$$\boxed{\frac{q_0''' \cdot \sin\left(\frac{\pi z}{L}\right) \cdot r}{2} = q''(z)}$$

c) $$\frac{dT}{dz} = \frac{q_0''' \cdot \sin\left(\frac{\pi z}{L}\right) \cdot \pi r^2}{\dot{m} \cdot c_p}$$

$\frac{dT}{dz} = 0$ at maximum so

$$0 = \frac{q_0''' \cdot \pi r^2}{\dot{m} \cdot c_p} \cdot \sin\left(\frac{\pi z}{L}\right)$$

$\sin\left(\frac{\pi z}{L}\right) = 0$ so $\frac{\pi z}{L} = 0$ or π

$\therefore z = 0$ or $L \rightarrow$ max surface temperature of rod.

b) $$\cancel{\frac{d}{dt}\rho h} + \frac{d}{dz}\rho v h = \frac{q'' \xi_w}{A} + \cancel{\frac{dP}{dt}} + \cancel{q'''}$$

$$\frac{dh}{dz} = \frac{q'' \xi_w}{\dot{m}} \qquad h = c_p \cdot T$$

$$\frac{dT}{dz} = \frac{q'' \cdot \xi_w}{\dot{m} \cdot c_p} = \frac{q_0''' \sin\left(\frac{\pi z}{L}\right) \pi r^2}{\dot{m} c_p}$$

$$\int_0^L \frac{dT}{dz} = \frac{1}{L} \cdot \frac{q_0''' \cdot \pi r^2}{\dot{m} \cdot c_p} \int_0^L \sin\left(\frac{\pi z}{L}\right)$$

$$T_{(L)} = T_{(0)} + \frac{q_0''' \cdot \pi r^2}{\dot{m} \cdot L \cdot c_p}\left(-\cos\left(\frac{\pi z}{L}\right) \cdot \frac{L}{\pi} \Big|_0^L\right)$$

$$T_{(L)} = T_0 + \frac{q_0''' \cdot \pi r^2}{\dot{m} \, L \cdot c_p}\left(1 \cdot \frac{L}{\pi} - -\frac{L}{\pi}\right)$$

$$T_{(L)} = T_0 + \frac{q_0''' \cdot 2 r^2}{\dot{m} \cdot c_p}$$

$$\frac{T_{(L)} + T_{(0)}}{2} = T_{avg} \text{ so } \boxed{T_{avg} = T_0 + \frac{q_0''' \cdot r^2}{\dot{m} \cdot c_p}}$$