SOFC Model

Grymes

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0.1 Mathematical Formulation

Sets		
$\mathcal{K}$	Technology cost segments	
$\mathcal{J}$	Power producing technologies	
$\mathcal{M}$	Months of year	
$\mathcal{T}$	Time steps	
$\mathcal{L}$	Locations to construct microgrid	
Subsets and indexed Sets		
$\mathcal{J}^{\mathrm{S}} \subseteq \mathcal{J}$	Solid oxide fuel cell technologies	
$\mathcal{J}^{\mathrm{CHP}} \subseteq \mathcal{J}$	Combined heat and power technologies	
$\mathcal{J}^{\mathrm{R}} \subseteq \mathcal{J}$	Renewable technologies	
$\mathcal{J}^{\mathrm{B}} \subseteq \mathcal{J}$	Heat-only producing technologies	
$\mathcal{J}^{\mathrm{E}} \subseteq \mathcal{J}$	Electrical producing technologies	
$\mathcal{T}^{\mathrm{fa}}$	Time steps needed for feasibility analysis	
$\mathcal{T}_m \subseteq \mathcal{T}$	Time steps in month m	
$\mathcal{T}^{\mathrm{g}} \subseteq \mathcal{T}$	Time steps when the utility is available	
Time and demand parameters		
Δ	Demand time steps	[hours]
d^{h}_{lt}	Heating load in time step t for the campus/building at location l	[kW]
d^{p}_{lt}	Electric load in time step t	[kW]
Cost and emission parameters		
κ_j	Annualized variable capital cost of technology j	[$/unit]
κ^{a}_{jk}	Annualized fixed installation cost of technology j in size segment k	[$]
κ^{b}	Annualized variable capital cost of electric battery	[$/kWh]

κ^{w}	Annualized variable capital cost of water storage	[$/gal]
c_j^{om}	Operation and maintenance cost of technology j	[$/kWh]
c_t^{p}	Utility energy cost (including emissions penalty) in time step t	[$/kWh]
c_t^{s}	Utility energy purchase price in time step t	[$/kWh]
c_t^{g}	Utility gas cost (including emissions penalty) in time step t	[$/kWh]

Power generation and storage parameters

$\bar{b}_{jk}$	Maximum power rating of technology j in cost segment k	[kW]
$\overline{\eta}_j^{\mathrm{e}}$	Maximum electricity efficiency for technology j	[fraction]
$\underline{\eta}_j^{\mathrm{e}}$	Minimum electricity efficiency for technology j	[fraction]
$\overline{\eta}^{\mathrm{b}}$	Maximum electricity efficiency for technology electrical storage	[fraction]
$\hat{k}_j$	Power rating of technology j	[kW/unit]
f_j^{b}	y-intercept for fuel of technology j	[unitless]
f_j^{m}	Fuel burn slope of technology j	[unitless]
f_{jt}^{p}	Production factor of technology j in time step t	[fraction]
μ_j	Maximum turn-down of technology j	[fraction]
ψ_j	Amount of fuel needed to start up technology j	[kWh/unit]
$\underline{s}$	Minimum capacity of electrical storage system	[fraction]
$\overline{s}$	Maximum capacity of electrical storage system	[fraction]
σ_j	Start-up time for each technology j to reach maximum turn-down (μ_j)	[hours]

Heat generation and storage parameters

α	Ambient heat loss for water	[fraction]
ϵ	Arbitrary temperature for which there is no thermal loss	[°C]
η_j^{h}	Thermal efficiency for technology j	[fraction]
γ_j	Exhaust gas output for technology j	[kg/kWh]
h^{e}	Specific heat of exhaust	[kWh/(kg °C)]
h^{w}	Specific heat of water	[kWh/(gal °C)]
$\overline{\nu}$	Maximum water storage capacity	[gal]
$\underline{\nu}$	Minimum water storage capacity	[gal]

$\hat{\tau}_j$	Average exhaust temp from hot-thermal-producing technology j	[°C]
$\check{\tau}$	Average return water temperature to water storage tank	[°C]
$\overline{\tau}$	Maximum allowed temperature of water in the system	[°C]
$\underline{\tau}$	Minimum allowed temperature of water in the system	[°C]

Continuous variables

X_l^{w}	Volume of water storage tank at location l	[gal]
X_l^{ba}	Amount of electrical storage procured at location l	[kWh]
$\hat{X}_t^{\mathrm{u}}$	Power purchased from the utility in time step t	[kW]
$\check{X}_t^{\mathrm{u}}$	Power sold to the utility in time step t	[kW]
$\bar{X}_m^{\mathrm{u}}$	Peak power purchased from the utility in month m	[kW]
X_{jlt}^{p}	Power produced by each technology j at location l in time step t	[kW]
$\check{X}_{lt}^{\mathrm{b}}$	Power into electrical storage system at location l in time step t	[kW]
$\hat{X}_{lt}^{\mathrm{b}}$	Power out of electrical storage system at location l in time step t	[kW]
X_{lt}^{bsc}	State of charge of electrical storage system at location l in time step t	[kWh]
X_{jlt}^{ef}	Electric efficiency of each technology j at location l in time step t	[fraction]
X_{jlt}^{f}	Fuel consumed by technology j at location l in time step t	[kW]
$\check{X}_{jlt}^{\mathrm{fl}}$	Flow rate of fluid into thermal storage from technology j at location l in time step t	[kg/hour]
$\hat{X}_{lt}^{\mathrm{fl}}$	Flow rate of water out of thermal storage at location l in time step t	[gal/hour]
X_{lt}^{t}	Temperature of water in storage at location l in time step t	[°C]

Integer variables

Y_{jl}^{a}	Number of each technology j procured and emplaced at location l	[units]
Y_{jlt}^{op}	Number of each technology j at location l operating in time step t	[units]
Y_{jlt}^{to}	Increased number of each technology j operating at location l from $t-1$ to t	[units]

Binary variables

Z^{ak}_{jk}	1 if generating technology j in segment k is procured, 0 otherwise	[binary]
Z^{w}_{l}	1 if additional water storage capacity is procured at location l, 0 otherwise	[binary]
$\check{Z}^{\mathrm{t}}_{lt}$	1 if water storage tank is above $(\check{\tau}+\epsilon)$ at location l in time step t, 0 otherwise	[binary]
$\hat{Z}^{\mathrm{t}}_{lt}$	1 if water storage tank is above $(\hat{\tau}_{\mathrm{boiler}})$ at location l in time step t, 0 otherwise	[binary]

Objective function

$$
\begin{aligned}
(\mathcal{P}') \quad \text{minimize} \quad & \underbrace{\sum_{j\in\mathcal{J},k\in\mathcal{K}} \kappa^{\mathrm{a}}_{jk} Z^{\mathrm{ak}}_{jk} + \sum_{l\in\mathcal{L}} \Bigg\{ \kappa^{b} X^{\mathrm{ba}}_{l} + \sum_{j\in\mathcal{J}} \kappa_j Y^{\mathrm{a}}_{jl} + \kappa^{\mathrm{w}}(X^{\mathrm{w}}_{l} - \underline{\nu})}_{\text{Capital Costs}} \\
& + \underbrace{\Delta \sum_{j\in\mathcal{J}^{\mathrm{E}},t\in\mathcal{T}} c^{\mathrm{om}}_{j} X^{\mathrm{p}}_{jlt}}_{\text{O\&M Costs}} + \underbrace{\sum_{j\in\mathcal{J}^{\mathrm{S}},t\in\mathcal{T}} c^{\mathrm{g}}_{t}(\psi_j Y^{\mathrm{to}}_{jlt} + \Delta X^{\mathrm{f}}_{jlt})}_{\text{Fuel Costs}} \\
& + \underbrace{\Delta \sum_{j\in\mathcal{J}^{\mathrm{B}},t\in\mathcal{T}} (\eta^{\mathrm{h}}_{j} c^{\mathrm{om}}_{j} + c^{\mathrm{g}}_{t}) X^{\mathrm{f}}_{jlt} \Bigg\}}_{\text{Existing Boiler Cost}} \\
& + \underbrace{\Delta \sum_{t\in\mathcal{T}} c^{\mathrm{p}}_{t} \hat{X}^{\mathrm{u}}_{t} + \sum_{m\in\mathcal{M}} c^{\mathrm{d}}_{m} \bar{X}^{\mathrm{u}}_{m}}_{\text{Grid Purchase}} - \underbrace{\Delta \sum_{t\in\mathcal{T}} c^{\mathrm{s}}_{t} \check{X}^{\mathrm{u}}_{t}}_{\text{Grid Sales}} \qquad (1)
\end{aligned}
$$

Load balancing

$$\sum_{l\in\mathcal{L}} \left((\bar{\eta}^{\mathrm{b}} \hat{X}^{\mathrm{b}}_{lt} - \check{X}^{\mathrm{b}}_{lt}) + \sum_{j\in\mathcal{J}^{\mathrm{E}}} X^{\mathrm{p}}_{jlt} \right) + (\hat{X}^{\mathrm{u}}_{t} - \check{X}^{\mathrm{u}}_{t}) = \sum_{l\in\mathcal{L}} d^{\mathrm{p}}_{lt} \quad \forall t \in \mathcal{T}^{\mathrm{g}} \qquad (2a)$$

$$(\bar{\eta}^{\mathrm{b}} \hat{X}^{\mathrm{b}}_{lt} - \check{X}^{\mathrm{b}}_{lt}) + \sum_{j\in\mathcal{J}^{\mathrm{E}}} X^{\mathrm{p}}_{jlt} = d^{\mathrm{p}}_{lt} \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \setminus \mathcal{T}^{\mathrm{g}} \qquad (2b)$$

$$h^{\mathrm{w}}(\hat{\tau}_j - \check{\tau}) \hat{X}^{\mathrm{fl}}_{lt} \left[\left(1 - \left[1 - \frac{\hat{\tau}_j - \underline{\tau}}{X^{\mathrm{t}}_{lt} - \underline{\tau}} \right] \hat{Z}^{\mathrm{t}}_{lt} \right)^{-1} \right] = d^{\mathrm{h}}_{lt} \quad \forall j \in \mathcal{J}^{\mathrm{B}}, l \in \mathcal{L}, t \in \mathcal{T} \qquad (2c)$$

Utility operations

$$\bar{X}^{\mathrm{u}}_{m} \geq \hat{X}^{\mathrm{u}}_{t} \quad \forall m \in \mathcal{M}, t \in \mathcal{T}_m \qquad (3a)$$

$$\sum_{t\in\mathcal{T}_m} \check{X}^{\mathrm{u}}_{t} \leq \sum_{t\in\mathcal{T}_m} \hat{X}^{\mathrm{u}}_{t} \quad \forall m \in \mathcal{M} \qquad (3b)$$

Power capacity

$$\sum_{l\in\mathcal{L}} X^{\mathrm{p}}_{jlt} \leq f^{\mathrm{p}}_{jt} \hat{k}_j \sum_{l\in\mathcal{L}} Y^{\mathrm{a}}_{jl} \quad \forall j \in \mathcal{J}^{\mathrm{R}}, t \in \mathcal{T} \qquad (4a)$$

$$\mu_j \hat{k}_j Y^{\mathrm{op}}_{jlt} \le X^{\mathrm{p}}_{jlt} \le \hat{k}_j Y^{\mathrm{op}}_{jl} \quad \forall j \in \mathcal{J}^{\mathrm{E}} \setminus \mathcal{J}^{\mathrm{R}}, l \in \mathcal{L}, t \in \mathcal{T} \tag{4b}$$

$$Y^{\mathrm{op}}_{jlt} \le Y^{\mathrm{a}}_{jl} \quad \forall j \in \mathcal{J}^{\mathrm{S}}, l \in \mathcal{L}, t \in \mathcal{T} \tag{4c}$$

$$\hat{k}_j \sum_{l \in \mathcal{L}} Y^{\mathrm{a}}_{jl} \le \bar{b}_{jk} Z^{\mathrm{ak}}_{jk} \quad \forall j \in \mathcal{J}, k \in \mathcal{K} \tag{4d}$$

$$\sum_{k \in \mathcal{K}} Z^{\mathrm{ak}}_{jk} \le 1 \quad \forall j \in \mathcal{J} \tag{4e}$$

Electricity efficiency

$$X^{\mathrm{ef}}_{jlt} = \left(\frac{\bar{\eta}^{\mathrm{e}}_j - \mu_j \underline{\eta}^{\mathrm{e}}_j}{1 - \mu_j} \right) - \left(\frac{\bar{\eta}^{\mathrm{e}}_j - \underline{\eta}^{\mathrm{e}}_j}{\hat{k}_j (1 - \mu_j)} \right) \left(\frac{X^{\mathrm{p}}_{jlt}}{Y^{\mathrm{op}}_{jlt}} \right) \quad \forall j \in \mathcal{J}^{\mathrm{S}}, l \in \mathcal{L}, t \in \mathcal{T} \tag{5}$$

Fuel consumption

$$X^{\mathrm{f}}_{jlt} = \frac{X^{\mathrm{p}}_{jlt}}{X^{\mathrm{ef}}_{jlt}} \quad \forall j \in \mathcal{J}^{\mathrm{S}}, l \in \mathcal{L}, t \in \mathcal{T} \tag{6a}$$

$$\sum_{l \in \mathcal{L}} X^{\mathrm{f}}_{jlt} = \sum_{l \in \mathcal{L}} \left(f^{\mathrm{b}}_j \hat{k}_j Y^{\mathrm{op}}_{jl} + f^{\mathrm{m}}_j X^{\mathrm{p}}_{jlt} \right) \quad \forall j = \mathcal{J}^{\mathrm{E}} \setminus (\mathcal{J}^{\mathrm{S}} \cup \mathcal{J}^{\mathrm{R}}), t \in \mathcal{T} \tag{6b}$$

$$X^{\mathrm{f}}_{jlt} = \frac{h^{\mathrm{w}} \hat{X}^{\mathrm{fl}}_{lt} (\hat{\tau}_j - X^{\mathrm{t}}_{lt})(1 - \hat{Z}^{\mathrm{t}}_{lt})}{\eta^{\mathrm{h}}_j} \quad \forall j \in \mathcal{J}^{\mathrm{B}}, l \in \mathcal{L}, t \in \mathcal{T} \tag{6c}$$

Start-up

$$Y^{\mathrm{op}}_{j,l,t+\sigma_j} - Y^{\mathrm{op}}_{jlt} \le Y^{\mathrm{to}}_{j,l,t+\sigma_j} \quad \forall j \in \mathcal{J}^{\mathrm{S}}, l \in \mathcal{L}, t \in \mathcal{T} : t < |\mathcal{T}| - \sigma_j \tag{7}$$

Power storage

$$X^{\mathrm{bsc}}_{l,t+1} - X^{\mathrm{bsc}}_{lt} = \Delta (\bar{\eta}^{\mathrm{b}} \check{X}^{\mathrm{b}}_{lt} - \hat{X}^{\mathrm{b}}_{lt}) \quad \forall l \in \mathcal{L}, t \in \mathcal{T} : t < |\mathcal{T}| \tag{8a}$$

$$\underline{s} X^{\mathrm{ba}}_l \le X^{\mathrm{bsc}}_{lt} \le \bar{s} X^{\mathrm{ba}}_l \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{8b}$$

$$X^{\mathrm{bsc}}_{l,1} = X^{\mathrm{bsc}}_{l,|\mathcal{T}|} \forall l \in \mathcal{L} \tag{8c}$$

Heat capacity

$$\check{X}^{\mathrm{fl}}_{jlt} \le \gamma_j X^{\mathrm{f}}_{jlt} \quad j \in \mathcal{J}^{\mathrm{CHP}}, l \in \mathcal{L}, t \in \mathcal{T} \tag{9}$$

Heat storage

$$X^{\mathrm{t}}_{l,t+1} - (1 - \alpha \check{Z}^{\mathrm{t}}_{lt}) X^{\mathrm{t}}_{lt} = \frac{\sum_{j \in \mathcal{J}^{\mathrm{CHP}}} (\Delta \eta^{\mathrm{h}}_j h^{\mathrm{e}} \check{X}^{\mathrm{fl}}_{jlt} (\hat{\tau}_j - X^{\mathrm{t}}_{lt})) - \Delta h^{\mathrm{w}} \hat{X}^{\mathrm{fl}}_{lt} (X^{\mathrm{t}}_{lt} - \check{\tau})}{h^{\mathrm{w}} X^{\mathrm{w}}_l} \quad \forall l \in \mathcal{L}, t \in \mathcal{T} : t < |\mathcal{T}| \tag{10a}$$

$$X^{\mathrm{t}}_{lt} - \check{\tau} \le (\bar{\tau} - \check{\tau}) Z^{\mathrm{w}}_l \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{10b}$$

$$\epsilon \check{Z}^{\mathrm{t}}_{lt} \le X^{\mathrm{t}}_{lt} - \check{\tau} \le \epsilon + (\bar{\tau} - \check{\tau} - \epsilon) \check{Z}^{\mathrm{t}}_{lt} \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{10c}$$

$$(\check{\tau} - \hat{\tau}_j)(1 - \hat{Z}^{\mathrm{t}}_{lt}) \leq X^{\mathrm{t}}_{lt} - \hat{\tau}_j \leq (\bar{\tau} - \hat{\tau}_j)\hat{Z}^{\mathrm{t}}_{lt} \quad \forall j \in \mathcal{J}^{\mathrm{B}}, l \in \mathcal{L}, t \in \mathcal{T} \tag{10d}$$

$$\underline{\nu} \leq X^{\mathrm{w}}_l \leq \bar{\nu} \quad l \in \mathcal{L} \tag{10e}$$

$$Z^{\mathrm{w}}_l \leq \sum_{j \in \mathcal{J}^{\mathrm{CHP}}} Y^{\mathrm{a}}_{jl} \leq \left\lceil \frac{\max_{t \in \mathcal{T}} \{d^{\mathrm{P}}_{lt}\}}{\min_{j \in \mathcal{J}^{\mathrm{CHP}}} \{\hat{k}_j\}} \right\rceil Z^{\mathrm{w}}_l \quad \forall l \in \mathcal{L} \tag{10f}$$

Non-negativity and integrality

$$X^{\mathrm{w}}_l, X^{\mathrm{ba}}_l \geq 0 \quad \forall l \in \mathcal{L} \tag{11a}$$

$$X^{\mathrm{f}}_{jlt}, X^{\mathrm{p}}_{jlt}, X^{\mathrm{ef}}_{jt}, \check{X}^{\mathrm{fl}}_{jt} \geq 0 \quad \forall j \in \mathcal{J}, l \in \mathcal{L}, t \in \mathcal{T} \tag{11b}$$

$$\bar{X}^{\mathrm{u}}_m \geq 0 \quad \forall m \in \mathcal{M} \tag{11c}$$

$$\hat{X}^{\mathrm{u}}_t, \check{X}^{\mathrm{u}}_t, \check{X}^{\mathrm{b}}_t, \hat{X}^{\mathrm{b}}_t, X^{\mathrm{bsc}}_t, \hat{X}^{\mathrm{fl}}_t, X^{\mathrm{t}}_t \geq 0 \quad \forall t \in \mathcal{T} \tag{11d}$$

$$Y^{\mathrm{a}}_{lj} \geq 0, \text{integer} \quad \forall l \in \mathcal{L}, j \in \mathcal{J} \tag{11e}$$

$$Y^{\mathrm{op}}_{jlt}, Y^{\mathrm{to}}_{jlt} \geq 0, \text{integer} \quad \forall j \in \mathcal{J}, l \in \mathcal{L}, t \in \mathcal{T} \tag{11f}$$

$$Z^{\mathrm{w}}_l \text{ binary} \quad \forall l \in \mathcal{L} \tag{11g}$$

$$\check{Z}^{\mathrm{t}}_{lt}, \hat{Z}^{\mathrm{t}}_{lt} \text{ binary} \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \tag{11h}$$

$$Z^{\mathrm{ak}}_{jk} \text{ binary} \quad j \in \mathcal{J}, k \in \mathcal{K} \tag{11i}$$