Additional Information on a Case Study

A. Preliminaries on TPF and MCDA

Three-Phase Power Flow (TPF) is a tool to evaluate operating states of three-phase distribution networks [1] by modeling the steady-state behavior of grid equipment. The output states from TPF include currents and power across cables and transformers, and voltages at various three-phase nodes in the network [2]. TPF can be formulated either using power mismatch equations [3] or current mismatch equations [4]. Current-mismatch formulation is found to be more robust than power mismatch formulation for analysis of three-phase unbalanced networks [5]. Therefore, we will build on and use the current-mismatch formulation described in [1].

Multi-Criteria Decision Analysis (MCDA) is used to evaluate tradeoffs between investment decisions [6]. By allowing different criteria to remain in their original units, MCDA allows practitioners to separate the process of evaluating options by each criteria, from the process of determining how much importance any particular criteria is given when coming to a decision [7]. We acknowledge that there are multiple methods for MCDA [8]–[10]. The method employed in this research is the weighted sum model [11], which uses pre-determined weights of how important particular criteria are to a practitioner, to determine the single optimal option for them.

B. Technology Generation Timeseries Creation

For one SOFC unit, the timeseries $\mathcal{T}_{SOFC}$ of power output of the SOFC unit is defined for the outage duration d as per (1), where r_{SOFC} is the rated output of the unit.

$$\mathcal{T}_{SOFC} = r_{SOFC} : h \in [1, d] \tag{1}$$

We assume that fuel is readily available, such as from a natural gas pipeline, for the duration of the outage. If the gas pipelines were unavailable, we would assume that there is additional infrastructure damage in the region, and that residents would have evacuated. For SOFC units, the variable cost is calculated using data gathered from various sources. With an expected energy intensity of 3.02 MJ consumed / MJ delivered [12], a conversion rate of 3.412142 MMBTU to 1000 kWh, and a natural gas price at the October 2022 Henry Hub price of \$5.66 per MMBTU [13], we calculate that the cost per kWh by (2).

$$\begin{aligned} 5.66\frac{\$}{MMBTU} &\times \frac{3.412142 MMBTU}{1000 kWh} \\ &\times 3.02\frac{MJconsumed}{MJdelivered} \\ &\approx 0.0583\frac{\$}{kWh} \end{aligned} \tag{2}$$

For one diesel generator, the timeseries $\mathcal{T}_{diesel}$ of power output of the diesel unit is defined by (3). The diesel unit is modeled as running at its full rated output r_{diesel} until either the end of the outage duration at hour d, or until it runs out of fuel. We assume that fuel is stored onsite in a unit that begins by storing its maximum amount of fuel $S^{t=1}_{diesel} = S^{max}_{diesel}$, and that each of the n_{diesel} units has an hourly fuel consumption of c_{diesel}. We assume that in the final hour of fuel availability, the diesel unit will run on whatever fuel still remains in storage after the previous hours of consumption, and generates power proportional to what it would have generated at its typical hourly fuel consumption.

$$\mathcal{T}^t_{diesel} = \begin{cases} r_{diesel} \\ \qquad \text{if } t \in [1, min(d, ceil(\dfrac{S^{max}_{diesel}}{c_{diesel} \times n_{diesel}}))] \\ r_{diesel} \times \dfrac{S^{max}_{diesel} - (t \times c_{diesel} \times n_{diesel})}{c_{diesel}} \\ \qquad \text{if } t = min(d, ceil(\dfrac{S^{max}_{diesel}}{c_{diesel} \times n_{diesel}}) \end{cases} \tag{3}$$

Next we determine the variable cost of diesel. The cost of diesel in Richmond, CA is assumed to be \$6.006 per gallon as per EIA prices for 11/28/2022 in California [14]. With a diesel generator expected to consume 0.2461 liters per kWh generated [15], this results in a price per kWh defined by (4).

$$\begin{aligned} 6.006\frac{\$}{gallon} &\div 3.78541\frac{liter}{gallon} \\ &\times 0.2461\frac{liter}{kWh} \\ &= 0.3905\frac{\$}{kWh} \end{aligned} \tag{4}$$

For a solar-plus-battery-storage unit, the solar power generation is defined by $\mathcal{T}_{solar}$, a pre-computed timeseries of power output for a single unit of solar at the Resilience Hub location during the outage using the NREL PVWatts tool [16]. The usage of the total battery storage is modeled dynamically, to attempt to balance out supply and demand at every timestamp it is able to do so. Battery storage is modeled as if all units are operating as one large unit. $S_{battery}$ is the maximum storage possible for a battery unit, which each unit is assumed to begin with. The backup power system's total possible battery storage $S^{max}_{batteries}$, as well as the amount of stored energy the system is defined as starting with $S^0_{batteries}$, is described by (5).

$$S^{max}_{batteries} = S_{battery} \times n_{solar} = S^0_{batteries} \tag{5}$$

The flow of power either in or out of the batteries at any given hour t is $\mathcal{T}^t_{batteries}$. The direction of flow depends on whether the required power output in that hour $\mathcal{T}^t_{\mathcal{P}}$ is greater or less than the power output of the generators in the system in that hour $\mathcal{T}^t_{gen}$, which is defined by (6).

$$\begin{aligned} \mathcal{T}_{gen} =& (\mathcal{T}_{SOFC} \times n_{SOFC}) \\ &+ (\mathcal{T}_{diesel} \times n_{diesel}) \\ &+ (\mathcal{T}_{solar} \times n_{solar}) \end{aligned} \tag{6}$$

The magnitude of the flow is the difference between these two values, limited by the batteries' collective maximum rate of charging or discharging $r_{batteries}$ as per (7).

$$r_{batteries} = r_{battery} \times n_{solar} \tag{7}$$

The timeseries of battery energy outflow is defined in (8).

$$\mathcal{T}^t_{batteries} = \begin{cases} min \quad (\mathcal{T}^t_{\mathcal{P}} - \mathcal{T}^t_{gen}, r_{batteries}, S^{t-1}_{batteries}) \\ \qquad \text{if } \mathcal{T}^t_{\mathcal{P}} > \mathcal{T}^t_{gen} \\ \qquad S^{t-1}_{batteries} > 0 \\ -min(\mathcal{T}^t_{gen} - \mathcal{T}^t_{\mathcal{P}}, r_{batteries}) \\ \qquad \text{if } \mathcal{T}^t_{gen} > \mathcal{T}^t_{\mathcal{P}} \\ \qquad S^0_{batteries} < S^{max}_{batteries} \\ 0 \qquad \text{if } \mathcal{T}^t_{\mathcal{P}} = \mathcal{T}^t_{gen} \end{cases} \tag{8}$$

The amount of energy stored in the battery after this charge or discharge occurs is defined by (9). The batteries are not allowed to discharge beyond their previously held capacity $S^{t-1}_{batteries}$, and are not allowed to charge beyond their maximum possible capacity $S^{max}_{batteries}$.

$$S^t_{batteries} = min(S^{t-1}_{batteries} - \mathcal{T}^{t-1}_{batteries}, S^{max}_{batteries}) \tag{9}$$

Using this, we get the total power output of a backup power system alternative by (10).

$$\mathcal{T}_s = \mathcal{T}_{gen} + \mathcal{T}_{batteries} \tag{10}$$

C. Generated Backup Power System Alternatives

The process of generating and filtering down the backup power system alternatives can be described by (11) through (14).

$$\mathcal{S}_{all} = \{s = (n_{\text{solar}}, n_{\text{SOFC}}, n_{\text{diesel}})\} \tag{11}$$
$$n_o = \text{rand}(0, n_{\max})$$

$$\mathcal{S}_{aff} = \{s = (n_{solar}, n_{SOFC}, n_{diesel}) \in \mathcal{S}_{all}\} \tag{12}$$
$$\sum_{o \in O} (n_o \times f_o) < b$$

$$\mathcal{T}_s = \sum_{o \in O} (n_o \times \mathcal{T}_o) : n_o \in s \tag{13}$$

$$\mathcal{S}_f = \{s \in \mathcal{S}_{aff}\} \tag{14}$$
$$CNSP_s < CNSP_{acc}$$

where the fixed cost of a technology o is f_o

D. Case Study Outage Scenarios

The dates for the four outage scenarios in the case study were selected by identifying the peak loads in each of four seasons of the year of load data provided for the Taxonomy Feeder in [17]. See Table I for details.

TABLE I
CASE STUDY SEASONS AND OUTAGE DATES AT SEASONAL PEAK LOADS

Season	Winter	Spring	Summer	Fall
Dates	Dec 1-Mar 1	Mar 1-Jun 1	Jun 1-Sep 1	Sep 1-Dec 1
Peak Load	Dec 8	May 17	Jul 19	Sep 1

Table II contains the weights for the 5 criteria under consideration.

TABLE II
EXAMPLE CRITERIA WEIGHTS UNDER DIFFERENT WEIGHTING SCHEMES

Weighting	Economic Cost	CO2	PM	CNSP	EWOMP
w_{eq}	0.2	0.2	0.2	0.2	0.2
w_{om}	0.2	0.1	0.1	0.1	0.5

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