

Problem 1

Suppose users share a 100 Mbps link. Also suppose each user requires 10 Mbps when transmitting, but each user transmits only 25% of the time.

- When circuit switching is used, how many users can be supported?
- For the remainder of the problem, suppose packet switching is used. Find the probability that a given user is transmitting.
- Suppose there are 100 users. Find the probability that at any given time, exactly n users are transmitting simultaneously. (Hint: Use the binomial distribution)
- Find the probability that there are 21 or more users transmitting simultaneously (Hint: only the formula is required).

(a) 100 Mbps link ; Each user uses 10 Mbps when transmitting .
 Maximum users that can be supported at the same time is $\frac{100 \text{ Mbps}}{10 \text{ Mbps/user}} = 10 \text{ users}$.

(b) Since a user only transmits 25% of the time, if we assume T to be the event representing a user transmitting data, then

$$P(T) = \frac{25\%}{100\%} = 0.25$$

(c) $N = \text{No. of users} = 100$
 $X = \text{No. of users transmitting simultaneously}$
 This represents a binomial distribution:

$$P(X=x) = \binom{N}{x} p^x (1-p)^{N-x}$$

$$\therefore P(X=n) = \binom{N}{n} p^n (1-p)^{N-n}$$

$$= \binom{100}{n} (0.25)^n (0.75)^{100-n}$$

(d) $P(X \geq 21) = 1 - P(X < 21)$

$$= 1 - P(X=0) - P(X=1) - \dots - P(X=20)$$

$$= 1 - [P(X=0) + P(X=1) + \dots + P(X=20)]$$

$$= 1 - \sum_{k=0}^{20} \binom{100}{k} (0.25)^k (0.75)^{100-k}$$

Problem 2

Queuing delay.

- (a) Suppose N packets arrive simultaneously to a link at which no packets are currently being transmitted or queued. Each packet is of length L and the link has transmission rate R . What is the average queuing delay for N packets?
- (b) Now suppose that a batch of packets arrives to the link every $\frac{LN}{R}$ seconds and each batch has N packets. What is the average queuing delay of a packet?

- (a) Packet 1 $d_{\text{queue}} = 0$ [No other packets being transmitted]
 The second packet now needs to wait in the queue while the first packet is being transmitted, which takes L/R .
 \therefore Packet 2 $d_{\text{queue}} = L/R$
 Packet 3 $d_{\text{queue}} = 2L/R$
 \vdots
 Packet N $d_{\text{queue}} = \frac{(N-1)L}{R}$
 \therefore Average queuing delay = $\frac{\text{sum of delays}}{N}$

$$= \frac{0 + \frac{L}{R} + \frac{2L}{R} + \dots + \frac{(N-1)L}{R}}{N} = \frac{\frac{L}{R}(0+1+2+\dots+(N-1))}{N} = \frac{\frac{L}{R} \left(\frac{(N-1)N}{2} \right)}{N} = \frac{(N-1)L}{2R}$$

$1+2+3+\dots+N = \frac{N(N+1)}{2}$
- (b) 1 batch $\rightarrow N$ packets
 Batches arrive every $\frac{LN}{R}$ seconds
 Transmission time for 1 batch with N packets = $N \times \frac{L}{R} = \frac{NL}{R}$ seconds
 \therefore Transmission time is exactly equal to the arrival time of the next batch, indicating that the queue will be empty when the next batch arrives. Therefore, this does not increase the queuing delay calculated in part (a) for a single packet.
 \therefore The average queuing delay of a packet = Avg. queuing delay of N packets

$$= \frac{(N-1)L}{2R}$$

Problem 3

Review the car-caravan analogy in lecture #1 slides (for Chapter 1). Assume a propagation speed of 100 km/h.

- (a) Suppose the caravan (5 cars) travels 100 km, beginning in front of one tollbooth, passing through a second tollbooth, and finishing just after a third tollbooth. The distance between two tollbooths is 50 km. Each car takes 12 sec to serve. The caravan can only dispatch a tollbooth after all cars in the caravan are served. What is the end-to-end delay (from when the caravan is lined up before 1st tollbooth till the caravan is served by the 3rd tollbooth)?
- (b) Repeat (a), now assuming that there are 8 cars in the caravan instead of 5.

(a) Time to serve = 12 sec |----- 50km -----|----- 50km -----|

Delay to serve all cars = $\frac{12 \text{ sec}}{\text{car}} \times 5 \text{ cars} = 60 \text{ s} = 1 \text{ min} / \text{toll booth}.$

Propagation Delay = $\frac{50 \text{ km}}{100 \text{ km/hr}} = \frac{1}{2} \text{ hour} = 30 \text{ min}$ from one toll booth to another

End-to-end delay (3 toll booths, 2 propagations)

$$= \left(3 \text{ booths} \times \frac{1 \text{ min}}{\text{booth}} \right) + \left(2 \text{ propagations} \times \frac{30 \text{ min}}{\text{propagation}} \right) = \boxed{63 \text{ min}}$$

(b) 8 cars will only affect delay to serve.

New delay = $\frac{12 \text{ sec}}{\text{car}} \times 8 \text{ cars} = 96 \text{ seconds} = 1.6 \text{ minutes} / \text{toll booth}$

\therefore New end-to-end delay = $(3 \times 1.6 \text{ minutes}) + (2 \times 30 \text{ min}) = \boxed{64.8 \text{ min}}$

Problem 4

In this problem, we consider sending real-time voice from Host A to Host B over a packet-switched network (VoIP). Host A converts analog voice to a digital 64 Kbps bit stream on the fly, which means it takes 1 second to create 64K bits from the analog signal. Host A then groups the bits into 56-byte packets. There is one link between Hosts A and B; its transmission rate is 2 Mbps and its propagation delay is 10 msec. As soon as Host A gathers a 56-byte packet, it sends it to Host B. As soon as Host B receives an entire packet, it converts the packet's bits to an analog signal. How much time elapses from the time the first bit of one packet is created (from the original analog signal at Host A) until the packet is received at Host B)?

Analog \rightarrow Digital 64 Kbps
 \hookrightarrow Takes 1 second to do this
 Groups of 56-byte packets
 Transmission rate = 2 Mbps
 $d_{\text{propagation}} = 10 \text{ msec}$

$$\text{Time for an entire packet to be created} = \frac{56 \text{ bytes} \times 8 \text{ bits/byte}}{64000 \text{ bits/s}} = \frac{7}{1000} \text{ sec} = 0.007 \text{ sec} = 7 \text{ msec}$$

$$\text{Time to transmit 1 packet} = \frac{56 \text{ bytes} \times 8 \text{ bits/byte}}{2 \times 10^6 \text{ bits/sec}} = \frac{224}{10^6} \text{ seconds} = 0.000224 \text{ sec} = 0.224 \text{ msec}$$

We know propagation delay is 10 msec.

Time from when 1st bit of one packet is created till the packet is received is

$$7 \text{ msec} + 10 \text{ msec} + 0.224 \text{ msec} = \boxed{17.224 \text{ msec or } 0.017224 \text{ seconds}}$$

Problem 5

Suppose you would like to urgently deliver 50 terabytes data from Boston to Los Angeles. You have available a 2 Gbps dedicated link for data transfer. Would you prefer to transmit the data via this link or to use FedEx overnight delivery instead? Explain your choice.

$$\begin{aligned}
 1 \text{ TB} &\rightarrow 1000 \text{ GB} \quad \therefore 50 \text{ TB} \rightarrow 50,000 \text{ GB} = 50000 \times \frac{8 \text{ bits}}{1 \text{ byte}} = 400,000 \text{ Gb} \\
 \text{Link speed: } &2 \text{ Gb/s (dedicated)} \\
 \therefore \text{Total duration of transfer over link:} \\
 &= \frac{400,000 \text{ Gb}}{2 \text{ Gb/s}} = 200,000 \text{ s} \approx 55.56 \text{ hours}
 \end{aligned}$$

Since 55.56 hours > 2 days, I would prefer to transmit the data via FedEx overnight delivery since it will reach earlier/faster.