HW 6

Q1) (a) $\{x/A, y/A, z/B\}$

(b) One does not exist. This is because y cannot be $G(A,B)$ and $G(x,x)$ at the same time.

(c) $\{x/B, y/A\}$

(d) $\{x/John, y/John\}$

(e) One does not exist. This is because x cannot be both y and $Father(y)$ at the same time.

Q2a Converting sentences to FOL:

- $\forall_X\ Food(X) \Rightarrow Likes(John, x)$
- $Food(Apples)$
- $Food(Chicken)$
- $\forall_X \forall_Y\ Eat(x,y) \wedge \neg Killed(y,x) \Rightarrow Food(y)$
- $\forall_X \forall_Y\ Killed(y,x) \Rightarrow \neg Alive(x)$
- $Eats(Bill, Peanuts) \wedge Alive(Bill)$
- $\forall_X\ Eats(Bill, x) \Rightarrow Eats(Sue, x)$

Q2b Convert formulas to CNF

1. $\forall_X\ Food(X) \Rightarrow Likes(John, x)$	1. $(\neg Food(x) \vee Likes(John, x))$
2. $Food(Apples)$	2. $Food(Apples)$
3. $Food(Chicken)$	3. $Food(Chicken)$
4. $\forall_X \forall_Y\ Eat(x,y) \wedge \neg Killed(y,x) \Rightarrow Food(y)$	4. $\neg Eats(x, F(x)) \vee Killed(F(x), x) \vee Food(x)$
5. $\forall_X \forall_Y\ Killed(y,x) \Rightarrow \neg Alive(x)$	5. $\neg Killed(F(x), x) \vee \neg Alive(x)$
6. $Eats(Bill, Peanuts) \wedge Alive(Bill)$	6.1 $Eats(Bill, Peanuts)$ 6.2 $Alive(Bill)$
7. $\forall_X\ Eats(Bill, x) \Rightarrow Eats(Sue, x)$	7. $\neg Eats(Bill, x) \vee Eats(Sue, x)$

Q2c Prove using Resolution : Likes (John, Peanuts) $\rightarrow \alpha$

$\therefore$ We need to prove that $\Delta \wedge \neg\alpha$ is unsatisfiable

$\sim\alpha = \sim$Likes (John, Peanuts)

8. Killed (Peanuts, Bill) v Food (Peanuts) after resolving 4, 6.1 and unifying x with Bill and $F(x)$ with Peanuts

9. ¬ Killed (G(Bill), Bill) after resolving 5, 6.2 and unifying Bill with x

10. Food (Peanuts) after resolving 8, 9 and unifying Peanuts with G(Bill)

11. Likes (John, Peanuts) after resolving 1, 10 and unifying x with Peanuts

$\therefore$ John likes Peanuts

Q2d 12. Eats (Sue, Peanuts) after resolving 6.1, 7 and unifying x with Peanuts

$\therefore$ Sue eats Peanuts

This is the only resolution that can be performed since Sue is only in one of the CNF clauses.

Q2e If we swap sentence 6 with these new sentences, we essentially lose out only the only piece of knowledge that could help determine what Sue eats. This is especially because we lack the knowledge to make any inference since there are no longer any sentences/axioms that tell us what Bill eats, and hence, what Sue eats.

Q3.1 With 3 colors, graph 1 is unsatisfiable

Q3.2 With 4 colors, graph 1 is satisfiable

Q3.3 The answers indicate that if the instance is satisfiable, then the graph coloring problem is satisfiable and vice-versa.

RSAT generates:

v -1 -2 -3 4 -5 -6 7 -8 -9 10 -11 -12 -13 -14 15 -16 17 -18 -19 -20 -21 22 -23 -24 25 -26 -27 -28 0

From this, we can see that the nodes can be colored in the following way:

Node 1 : Color 4
Node 2 : Color 3
Node 3 : color 2
Node 4 : color 3
Node 5 : color 1
Node 6 : color 2
Node 7 : color 1

Q3.4 The minimum number of colors required for Graph 2 is 8.