Homework 7

(1) $LHS = Pr(a_1, a_2, \ldots a_n / \beta)$

$RHS = Pr(a_1 / a_2, \ldots, a_n, \beta) * Pr(a_2 / a_3 \ldots a_n, \beta) * \ldots * Pr(a_n / \beta)$

$$= \frac{Pr(a_1, \ldots, a_n, \beta)}{Pr(\cancel{a_2, \ldots, a_n, \beta})} * \frac{Pr(\cancel{a_2, \ldots, a_n, \beta})}{Pr(\cancel{a_3, \ldots, a_n, \beta})} * \ldots * \frac{\cancel{Pr(a_n, \beta)}}{Pr(\beta)}$$

Almost all of the terms cancel out

$$= \frac{Pr(a_1, \ldots, a_n, \beta)}{Pr(\beta)}$$

$$= Pr(a_1, a_2, \ldots, a_n / \beta)$$

(2) $\mathbb{P}(\text{Oil}) = 0.5$

$\mathbb{P}(\text{Natural Gas}) = 0.2$

$\mathbb{P}(\sim\text{Oil} \,\&\, \sim\text{Natural Gas}) = 0.3$

$\mathbb{P}(\text{Positive}/\text{Oil}) = 0.9$

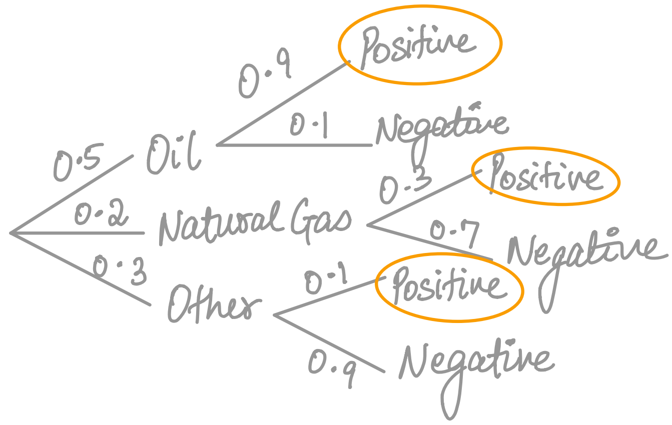
$\mathbb{P}(\sim\text{Positive}/\text{Oil}) = 0.1$

$\mathbb{P}(\text{Positive}/\text{Natural Gas}) = 0.3$

$\mathbb{P}(\sim\text{Positive}/\text{Natural Gas}) = 0.7$

$\mathbb{P}(\text{Positive}/\sim\text{Oil} \,\&\, \sim\text{Natural Gas}) = 0.1$

$\mathbb{P}(\sim\text{Positive}/\sim\text{Oil} \,\&\, \sim\text{Natural Gas}) = 0.9$



Need to find: $\mathbb{P}(\text{Oil}/\text{Positive})$

Bayes' Theorem: $$\mathbb{P}(A/B) = \frac{\mathbb{P}(B/A)\cdot\mathbb{P}(A)}{\mathbb{P}(B)}$$

$$\therefore \mathbb{P}(\text{Oil}/\text{Positive}) = \frac{\mathbb{P}(\text{Positive}/\text{Oil})\cdot\mathbb{P}(\text{Oil})}{\mathbb{P}(\text{Positive})}$$

$$\mathbb{P}(\text{Positive}) = \mathbb{P}(\text{Positive}/\text{Oil})\mathbb{P}(\text{Oil}) + \mathbb{P}(\text{Positive}/\text{Natural Gas})\mathbb{P}(\text{Natural Gas}) + \mathbb{P}(\text{Positive}/\sim\text{Oil}\,\&\,\sim\text{Natural Gas})\mathbb{P}(\sim\text{Oil}\,\&\,\sim\text{Natural Gas})$$

$$= (0.9\times0.5) + (0.3\times0.2) + (0.1\times0.3)$$

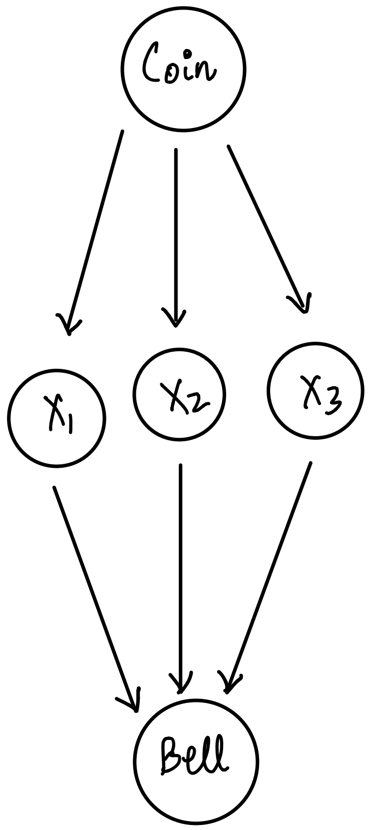
$$= 0.45 + 0.06 + 0.03 = 0.54$$

$$\therefore \mathbb{P}(\text{Oil}/\text{Positive}) = \frac{\cancel{0.9}\times0.5}{\cancel{0.54}\;0.6} = \frac{5}{6} \approx \boxed{0.8333}$$

3

1 coin picked

$Coin = \{a, b, c\}$
$X_1 = \{H, T\}$
$X_2 = \{H, T\}$
$X_3 = \{H, T\}$
$Bell = \{On, Off\}$



Flipped 3 times

Turns bell "on" if all outcomes are the same

Coins:

P(a)	P(b)	P(c)
1/3	1/3	1/3

X_1, X_2, X_3:

Coin	P(H)	P(T)
a	0.2	0.8
b	0.4	0.6
c	0.8	0.2

Bell:

X_1	X_2	X_3	On
H	H	H	1
H	H	T	0
H	T	H	0
H	T	T	0
T	H	H	0
T	H	T	0
T	T	H	0
T	T	T	1

4 a) Markov Assumption: A variable X is independent of every other variable (except X's effects) conditional on all of its direct causes.

Ind (A, {}, {B, E})
Ind (B, {}, {A, C})
Ind (C, {A}, {B, D})
Ind (D, {A, B}, {C, E})
Ind (E, {B}, {A, C, D, F, G})
Ind (F, {C, D}, {A, B, E})
Ind (G, {F}, {A, B, C, D, E, H})
Ind (H, {E, F}, {A, B, C, D, G})

(b) d_separated (A, F, E): False
- This is because if F is in not in Z, there are non-converging paths from A to E and E to A.

d_separated (G, B, E): True
- This is because if B is not in Z, there are no non-converging paths from G to E or E to G

d_separated (AB, CDE, GH): True
- This is because if CDE is not in Z, there are no non-converging paths from AB to GH or GH to AB

c) Chain Rule for a fully connected directed graph between $X_1, \ldots, X_n$

$$p(X) = \prod_{i=1}^{n} p(X_n / X_{n-1}, \ldots, X_1)$$

$$Pr(a,b,c,d,e,f,g,h) = Pr(a) * Pr(b) * Pr(c/a) * Pr(d/a,b) * Pr(e/b) * Pr(f/c,d) * Pr(g/f) * Pr(h/e,f)$$

d) $Pr(A=1, B=1) = Pr(A=1)\, Pr(B=1) = 0.2 \times 0.7 = \boxed{0.14}$

This is because they are d-separated

$Pr(E=0, A=0) = Pr(E=0)$ because they are d-separated

$$Pr(E=0) = P(E=0, B=0) * P(B=0) + P(E=0, B=1) * P(B=1)$$
$$= (0.1 * 0.3) + (0.9 * 0.7)$$
$$= 0.03 + 0.63 = \boxed{0.66}$$

5 $\alpha: A \Rightarrow B \equiv \neg A \vee B$

a)

w	A	B	$\alpha: A \Rightarrow B$
w_0	T	T	T
w_1	T	F	F
w_2	F	T	T
w_3	F	F	T

$\therefore$ The models of α are $\{w_0, w_2, w_3\}$

(b) Using the joint probability distributions

$$Pr(\alpha) = Pr(w_0) + Pr(w_2) + Pr(w_3)$$
$$= Pr(A=T, B=T) + P(A=F, B=T) + P(A=F, B=F)$$
$$= 0.3 + 0.1 + 0.4 = \boxed{0.8}$$

(c) $Pr(A, B / \alpha)$

A	B	$P(A,B/\alpha)$
T	T	$\frac{0.3}{0.8} = 0.375$
T	F	0
F	T	$\frac{0.1}{0.8} = 0.125$
F	F	$\frac{0.4}{0.8} = 0.5$

(d) $Pr(A \Rightarrow \neg B / \alpha) \equiv Pr(\neg A \vee \neg B / \alpha)$

Using the conditional probability distribution $Pr(A, B/\alpha)$

$$Pr(A \Rightarrow \neg B / \alpha) = Pr(A=T, B=F/\alpha) + Pr(A=F, B=T/\alpha) + Pr(A=F, B=F/\alpha)$$
$$= 0 + 0.125 + 0.5 = \boxed{0.625}$$