Basic Control Theory

Arnav Gupta

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1 Basics of Closed Control Loops

Input can be split up into terms that can be controlled and terms that can't be controlled.

$$f(t) = u(t) + d(t)$$

where $u(t)$ is the control input and $d(t)$ is the disturbance input.

The goal here is find $u(t)$ such that the output follows some given reference, and this can be done with a controller.

1.1 Open Loop Controller

The cruise control velocity system can be controlled with an open loop controller, where:

- $U(s)$ is the Laplace transform of the control input

- $D(s)$ is the Laplace transform of the disturbance input
- $F(s)$ is the Laplace transform of the total forcing term
- $V(s)$ is the Laplace transform of the resulting velocity

and $F(s)$ and $V(s)$ are related by the transfer function.

These controllers only work well if we know the disturbance the system undergoes, if the system always needs a constant known output, or if the user can make adjustments as needed.

1.2 Closed Loop Controller

Output is fed back into controller and error is calculated from this based on difference from the reference. A controller is also added to dynamically adjust $U(s)$ based on feedback.

The new goal is to find a transfer function $C(s)$ such that the error goes to 0.

2 Proportional and Integral Control Systems

2.1 Proportional Control

2.1.1 Idea

Let $u(t) = k_p e(t)$ for $k_p \in \mathbb{R}_{>0}$, then

- if $e(t) > 0$, increase $u(t)$
- if $e(t) < 0$, decrease $u(t)$,
- if $e(t) = 0$ do nothing

For this controller, k_p is the transfer function $C(s)$.

For the cruise control velocity controller (with no disturbance), the transfer function for the controlled system becomes

$$H_{RV}(s) = \frac{k_p/(b + k_p)}{[m/(b + k_p)]s + 1}$$

which is the transfer function for a first order system with a DC gain of

$$\kappa = \frac{k_p}{b + k_p}$$

and a time constant of

$$\tau = \frac{m}{b + k_p}$$

This is the same as the uncontrolled transfer function, but the DC gain and time constant can be adjusted through k_p.

For a system with disturbance, transfer function is $H_{DV}(s)$ with the same system. This gives the net response to be

$$V(s) = H_{RV}(s)R(s) + H_{DV}(s)D(s)$$

For larger values of k_p, τ becomes smaller which leads to faster decay to the final value of ths system.

By changing k_p, can make κ obtain any value between 0 and 1.

2.1.2 Limitations

If $e(t)$ is ever 0, then $u(t) = k_p r(t) = 0$. Thus, in the absence of any disturbance to the system, the input into the system $f(t)$ also vanishes, and so no system input means $v(t)$ will drop since $b > 0$.

This means p controllers can never achieve perfect asymptotic velocity.

2.2 Integral Controllers

Let the error term be proportional to the integral of the error

$$c(t) = k_i \int_0^t e(\tau) d\tau$$

which goes to 0 exactly when average error is 0, and so it can bring the asymptotic error to exactly 0.

The transfer function for the integral term is $C(s) = \frac{k_i}{s}$ and so

$$V(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} R(s)$$

and so the new system has a transfer function of

$$H(s) = \frac{k_i/m}{s^2 + \frac{b}{m}s + \frac{k_i}{m}}$$

which is the transfer function for a second order system.

The DC gain is $H(0) = 1$.

2.2.1 Limitations

Since we adjust based on the integral in the error, integral controllers do strongly quickly adjust to changes.

Leads to potential overshoot of the goal, which is in part because solutions generally come in conjugate pairs and so the solution will have sinusoidal terms that will decay to 0.