# Operators and Measurement

Arnav Gupta

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## Contents



## 1 Operators, Eigenvalues, and Eigenvectors

**Operator**: mathematical object that acts on a ket and transforms it into a new ket

**Postulate 2**: a physical observable is represented mathematically by an operator $A$ that acts on a ket

**Postulate 3**: the only possible result of a measurement of an observable is one of the eigenvalues $a_n$ of the corresponding operator $A$

An operator is always diagonal in its own basis. Eigenvectors are unit vectors in their own basis.

### 1.1 Matrix Representation of Operators

Each matrix element can be found as a bra multiplied by an operator multiplied by a ket.

## 2 New Operators

### 2.1 Hermetian Operators

An operator $A$ is Hermetian if it is equal to its Hermetian adjoint.

### 2.2 Projection Operators

**Closure/Completeness**: the sum of the projector operators for all eigenstates is the identity operator.

When a projector operator for an eigenstate acts on a state, it produces a new ket that is aligned along the eigenstate and has a magnitude equal to the amplitude (including the phase) for the state to be in that eigenstate.

**Postulate 5**: after a measurement of $A$ that yields the result $a_n$, the quantum system is in a new state that is the normalized projection of the original system ket onto the ket or kets corresponding to the result of the measurement, giving

$$\psi' = \frac{P_n \psi}{\sqrt{\psi P_n \psi}}$$

Quantum measurements cannot be made without disturbing the system (except where input and output are the same), this leads to collapse.

## 3 Measurement

The mean (expected value) measurement for an operator $A$ on a state $\psi$ is

$$\langle A \rangle = \psi A \psi = \sum_n a_n P_{a_n}$$

The standard deviation is defined as:

$$\Delta A = \sqrt{\langle (A - \langle A \rangle)^2 \rangle} = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$$

which is derived from root-mean-square.

## 4 Commuting Observables

The commutator of two operators is $[A, B] = AB - BA$.

If the commutator is 0, the operators commute.

Commuting operators share common eigenstates.

The commutation relations for the spin component operators are:

$$[S_x, S_y] = i\hbar S_z, [S_y, S_z] = i\hbar S_x, [S_z, S_x] = i\hbar S_y$$

## 5 Uncertainty Principle

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|$$

One spin component can be known absolutely but not 2 or simultaneously.