Signals Intro

Arnav Gupta

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1 Bode Plots

Bode plots show how a system responds to input signals of the form $\sin(\omega t)$. Can be used to quickly tell what type of filtering/amplifying the LTI does to the sine wave.

A low pass filter is an LTI that removes the high frequency waves, by reducing the amplitude of all waves with a frequency larger than some cutoff ω. All stable systems with a single pole and no zeroes are low pass filters.

For a high pass filter, the amplitude curve must go to 0dB at some cutoff point. The standard high pass filter transfer function is

$$T(s) = \frac{as}{s + \omega_0}$$

for $a, \omega_0 > 0$.

For a medium band filter, add another pole later

$$T(s) = \frac{as}{(s + \omega_0)(s + \omega_1)}$$

for $a, \omega_0, \omega_1 > 0$.

2 Fourier Series

Fourier series are useful for working with systems and signals.

Taylor's Theorem: let $k \geq 1$ be an integer and let f be a real valued function that is differentiable at least k times at some point $a \in \mathbb{R}$, there then exists a real valued function h_k such that

$$f(x) = \left(\sum_{i=0}^{k} \frac{f^{(i)}(a)}{i!}(x-a)^i \right) + h_k(x)(x-a)^{k+1}$$

and $\lim_{x \to a} h_k(x) = 0$.

For infinitely differential functions f, this becomes

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

Similar to how the polynomials are used here, sinusoidals can be used as a basis instead.