Transfer Functions, Responses, and the Standard First-Order System

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1 Transfer Functions

If a system is able to be modelled by a linear DE with constant coefficients, then the transfer function can be obtained from the differential equations.

In general, $Y(s) = H(s)F_{ap}(s)$ so $y(t) = \mathscr{L}^{-1}\{H(s)F_{ap}(s)\}$.

In general, $F_{ap}(s)$ can have poles of its own and thus the system response will reflect the poles of both the transfer function $H(s)$ and the Laplace Transform of the forcing term $F_{ap}(s)$.

The effects of the transfer function are present for any input so it is particularly important to understand the effects of the poles (and zeros) of $H(s)$.

2 Responses

The response to the unit impulse is $Y(s) = H(s)$.

The response to the unit step impulse is $Y(s) = \frac{1}{s}H(s)$.

The step response is the integral of the impulse response.

In the real world, often easier to physically generate a unit step function than a unity impulse.

All polynomials with real valued coefficients can be factored into a product of linear and quadratic terms, so to understand the system response of any system, it is sufficient to understand how 1st and 2nd order linear systems respond.

3 Standard 1st Order System

The standard 1st order system has transfer function

$$H(s) = \frac{\kappa}{s\tau + 1}$$

where $\kappa, \tau > 0$, κ is called the **DC gain** and τ is called the **time constant**.

Since $H(s)$ has a pole at $s = -\frac{1}{\tau}$, the impulse response of the standard 1st order system is:

$$h(t) = \frac{\kappa}{\tau} e^{-t/\tau} u(t)$$

The standard 1st order system is causal.

Changing τ changes the value of $h(0)$ inversely and changes the growth rate.

The step response of the standard 1st order system is

$$Y(s) = H(s)\frac{1}{s} = \kappa/(s(s\tau + 1)), y(t) = \kappa[1 - e^{-t/\tau}]u(t)$$

Changing κ increases the steady state value of $y(t)$ and $H(0) = \kappa$.