Bode Plots

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Contents

1 Frequency Response

If $S : f \to y$ is an LTI with transfer function $H(s)$, then for any $s \in \mathbb{C}$,

$$e^{st} \xrightarrow{S} H(s)e^{st}$$

If $s = j\omega$, then $H(j\omega)$ can be written in polar form. $H(j\omega)$ is then the frequency response.

If S is an LT with transfer function $H(s)$, then

$$\sin(\omega t) \xrightarrow{S} |H(j\omega)| \sin(\omega t + \angle_{H(j\omega)})$$

This means that the system reponse of an LTI to a sin wave of frequency ω:

- has an amplitude scaled by $H(j\omega)$
- has the same frequency
- has a phase shifted by $\angle_{H(j\omega)}$

If S is an LT with transfer function $H(s)$, then as $t \to \infty$

$$\sin(\omega t)u(t) \xrightarrow{S} |H(j\omega)| \sin(\omega t + \angle_{H(j\omega)})$$

Fourier series/transforms allow decomposing functions as a sum/integral of sin and cos waves.

2 Bode Plots

Bode plots: a graphical representation of the frequency response

One plot for magnitude and one for phase.

2.1 Decibels

$|H(j\omega)|$ in decibels is $20 \log_{10}(|H(j\omega)|)$

This means magnitude curves for multiplied frequency responses can be found by adding magnitude curves for each factor.

2.2 Finding

If given a transfer function $H(s) = H_1(s) \cdot H_2(s) \cdots H_k(s)$, then the Bode plot for $H(j\omega)$ is found by

- finding the magnitude and phase curves for each $H_i(j\omega)$
- adding the magnitude and phase curves