Schrödinger Time Evolution

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April 17, 2024

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1 Schrödinger Equation

Postulate 6: the time evolution of a quantum system is determined by the Hamiltonian or total energy operator $H(t)$ through the Schrödinger equation

$$i\hbar\frac{d}{dt}\psi(t) = H(t)\psi(t)$$

The Hamiltonian is an observable so it is a Hermetian operator. The energy eigenvalue equation is

$$HE_n = E_nE_n$$

The basis of eigenvectors of the Hamiltonian is the **energy basis**.

If the initial state of the system at time 0 is

$$\psi(0) = \sum_n c_nE_n$$

then the time evolution of this state under the action of the time-independent Hamiltonian H is

$$\psi(t) = \sum_n c_ne^{-iE_nt/\hbar}E_n$$

For an observable A whose eigenstates consist of superpositions of energy eigenstates, where $a_1 = \alpha_1 E_1 + \alpha_2 E_2$. Then for $psi(0) = c_1 E_1 + c_2 E_2$, \$\$ $\mathrm{P}_{\mathrm{a}_1} = |\alpha_1|^2\ |\mathrm{c}_1|^2 + |\alpha_2|^2\ |\mathrm{c}_2|^2$

- $2\mathrm{Re}(\alpha_1\ \mathrm{c}_1{}^{\backslash *}\ \alpha_2{}^{\backslash *}\ \mathrm{c}_2\ \mathrm{e}^{\text{-i(E}_2\text{ - E}_1\text{)t}/\hbar}$

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$\omega_{21} = \{\overline{E_2 - E_1\}\{\hbar\}\$\$isthe\textbf{Bohr frequency}.}$

1.1 Recipe for Solving Time-Dependent Quantum Mechanics

Given a Hamiltonian H and an initial state $\psi(0)$, find the probability that the eigenvalue a_j of the observable A is measured at time t.

1. Diagonalize H to find the eigenvalues and eigenvectors for the energies.
2. Write $\psi(0)$ in terms of the energy eigenstates E_n.
3. Multiply each eigenstate coefficient by $e^{-iE_n t/\hbar}$ to get $\psi(t)$.
4. Calculate the probability $P_{a_j} = |a_j \mid \psi(t)|^2$.

2 Spin Precession

For the magnetic dipole

$$\mu = g\frac{q}{2m_e}S$$

the Hamiltonian is

$$H = -g\frac{q}{2m_e}S \cdot B$$

2.1 Magnetic Field in the z-Direction

If $B = B_0\hat{z}$, then $H = \omega_0 S_z$ for $\omega_0 = -g\frac{q}{2m_e}B_0$.

Spin precession: where probabilities change for certain spins since the Hamiltonian is defined for other spins

Larmor precession: the precession of the spin vector

Larmor frequency: the frequency of precession

Ehrenfest's Theorem: quantum mechanical expectation values obey classical laws

2.2 Magnetic Field in a General Direction

Spin flip: where a state flips to the orthogonal state

The probability of a spin flip is

$$P_{+\to-} = \frac{\omega_1^2}{\omega_0^2 + \omega_1^2} \sin^2 \left(\frac{\sqrt{\omega_0^2 + \omega_1^2}}{2} t \right)$$

which is Rabi's formula.