PI Controllers, Zeros, Extra Poles, and Stability

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1 PI Controller Analysis

The transfer function of the response of a PI controller can be simplified to the form:

$$H_z(s) = \frac{\omega^2\left(\frac{s}{\alpha\xi\omega} + 1\right)}{s^2 + 2\xi\omega s + \omega^2}$$

By analyzing the response of a PI controller to the step function, this can be decomposed into a standard 2nd order impulse response and a standard 2nd order step response.

Specifically, if $\alpha \to \infty$, then it goes to the step response, and if $\alpha \to 0$, then it goes to $\infty\cdot$ the impulse response.

Combining these effects can lead to using k_i to adjust the speed of the response and using k_p to control the dampening of the oscillations.

2 Adding a Pole

By adding a pole at $s = -\alpha\xi\omega$ to the transfer function of a system, it can be decomposed into the linear combination of 1st and 2nd order systems.

For this, if $\alpha \to 0$, then the impulse response looks like a standard 1st order system. If $\alpha \to \infty$, then the impulse response looks like a standard 2nd order system.

When a real pole or a complex-conjugate pair of poles are an order of magnitude closer to the imaginary axis than all other poles, then they are dominant.

3 Stability

The impulse response of **all** linear time invariant systems is a linear combination of the types of functions analyzed. Further, since $\{\delta(t-\tau) | \tau \in \mathbb{R}\}$ is a basis for the set of all functions, the response of any LTI to any function $f(t)$ can be written as a convolution of $f(t)$ with a linear combination of the types of functions analyzed.

An LTI S is stable if $S(\delta(t))$ decays to 0.

An LTI S is unstable if $S(\delta(t))$ is unbounded.

An LTI S is marginally stable if $S(\delta(t))$ is bounded but does not decay to 0.

A transfer function is stable if the system it is a transfer function for is stable.

A transfer function is unstable if the system it is a transfer function for is unstable.

A transfer function is marginally stable if the system it is a transfer function for is marginally stable.

A transfer function is stable if all poles have a negative real part.

A transfer function is unstable if there is a pole with a positive real part or there is a 2nd order pole that has real part 0.

A transfer function is marginally stable if there are no poles with postive real parts or 2nd order poles with real part 0 and in addition at least order 1 pole with real part 0.

An LTI S is bounded-input, bounded-output (BIBO) stable if $S(f)$ is bounded for all bounded functions f.

An LTI system with a rational transfer function is BIBO stable if and only if its transfer function is both stable and proper.