Compare the CDF of $\text{Exp}(1/2)$ with the CDF of the sum of two independent $\text{Exp}(1)$ RV(s).

Note that the expected value of $\text{Exp}(1/2)$ is 2. The expected value of the sum is also 2.

Assuming the RVs model power, we have an average power of 2 Watts in both cases.

However, what is the probability that the power is > the expected value for when we have power distributed as $\text{Exp}(1/2)$?

What is this prob for the sum of two independent $\text{Exp}(1)$ RV(s)?

Now consider the sum of $n > 2$ $Exp(n/2)$ independent RV(s). What is the expected value of the sum?

As $n \to \infty$, what is the prob that sum takes a value > than the expected value?

The example illustrates why sums of RVs may be useful in practice.

The sum in our example can be thought of as the sum of powers received by multiple (n) antennas.

As you can see, we can keep the average received power the same while increasing the prob of obtaining power gt than average by using more antennas.

Probability and Random Processes

Sanjit Kaul

INDRAPRASTHA INSTITUTE *of*
INFORMATION TECHNOLOGY
DELHI

Statistics

- We won't assume knowledge of the probabilistic *model*
- We will soon use *data* to calculate *statistics*
  - *Data comes from* _______________
- *Any statistic is function of data*
- A statistic could, for example, be an estimate of the parameter of a PDF

Parameter Estimation

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Sampling From a Finite Population

- Creating the data
- Total population m
- Sample size n
- Sample size is often much smaller than the total population
  - Often infeasible to measure all in total population

Sampling From a Finite Population

- Suppose we want to calculate the fraction of people in Delhi that are taller than 5 feet 6 inches.

- To calculate the **exact** fraction (probability *p*) we must measure the heights of all m people in Delhi
  - Total number of people taller than 5 feet 6 inches is ______________

- Note that *p* is unknown

Sampling From a Finite Population

- Suppose we want to calculate the fraction of people in Delhi that are taller than 5 feet 6 inches.
- To calculate the exact fraction (probability p) we must measure the heights of all m people in Delhi
- Note that p is unknown
- We want to estimate p using the sample of size n

Sampling From a Finite Population

- The *n* samples are chosen randomly
- Are they independent samples?

Sampling From a Finite Population

- The *n* samples are chosen randomly
- Are they independent samples?
- The height of a sample is a random variable of the _____________ family.

Sampling From a Finite Population

- The n samples are chosen randomly
- Are they independent samples?
- Think RVs that model the heights of the first two samples....
  - Are they independent RVs?

Sampling From a Finite Population

- We will assume samples are iid

Definition 7.1 Sample Mean

For iid random variables $X_1, \ldots, X_n$ *with PDF* $f_X(x)$, *the* sample mean *of* X *is the random variable*

$$M_n(X) = \frac{X_1 + \cdots + X_n}{n}.$$

$E[M_n(X)]$

$\mathrm{Var}[M_n(X)]$

Theorem 7.1

The sample mean $M_n(X)$ has expected value and variance

$$E[M_n(X)] = E[X], \qquad \mathrm{Var}[M_n(X)] = \frac{\mathrm{Var}[X]}{n}.$$

Quiz 7.1

Let X be an exponential random variable with expected value 1. Let $M_n(X)$ denote the sample mean of n independent samples of X. How many samples n are needed to guarantee that the variance of the sample mean $M_n(X)$ is no more than 0.01?

$$\mathrm{Var}[M_n(X)] = \frac{\mathrm{Var}[X]}{n} \leq 0.01$$

$$n \geq \frac{\mathrm{Var}[X]}{0.01}$$

Theorem 7.2 Markov Inequality

For a random variable X such that $P[X < 0] = 0$ and a constant c,

$$P\left[X \geq c^2\right] \leq \frac{E[X]}{c^2}.$$

$$P[X \geq c^2] = \int_{c^2}^{\infty} f_X(x)\,dx = \frac{1}{c^2}\int_{c^2}^{\infty} c^2 f_X(x)\,dx$$

$$\leq \frac{1}{c^2}\int_{c^2}^{\infty} x f_X(x)\,dx \leq \frac{E[X]}{c^2}$$