ECE113- Basic Electronics

Lecture 8: Source equivalence, Thevenin and Norton’s Theorems

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Linearity and superposition

A function f is called linear if

- $f(x + y) = f(x) + f(y)$, additive
- $f(\alpha x) = \alpha f(x)$, homogenious

These two properties are called the superposition principle

The principle of superposition states that the response (a desired current or voltage) in a linear circuit having more than one independent source can be obtained by adding the responses caused by the separate independent sources acting alone. So that if input A produces response X and input B produces response Y then input (A + B) produces response (X + Y)

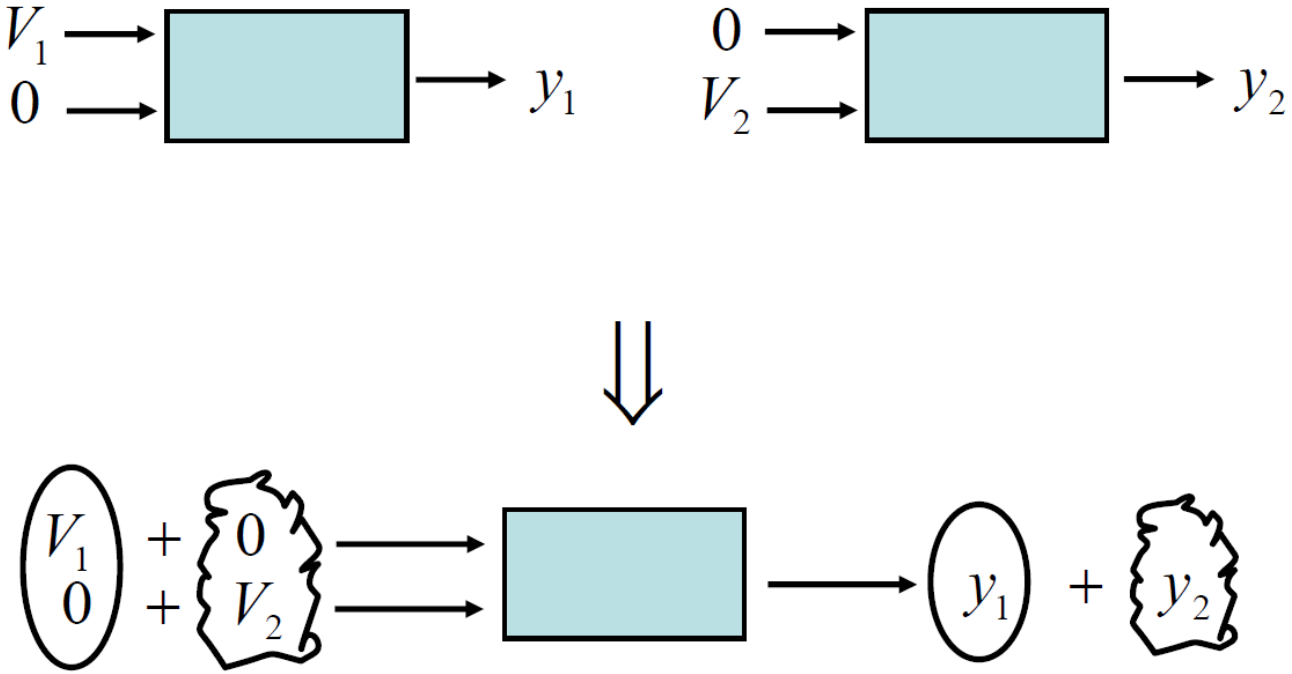
Superposition theorem for electrical circuits

- Linear elements are passive elements that have a linear voltage-current relationships

- We now define a Linear circuit as a circuit composed entirely of independent sources, linear dependent sources, and linear elements

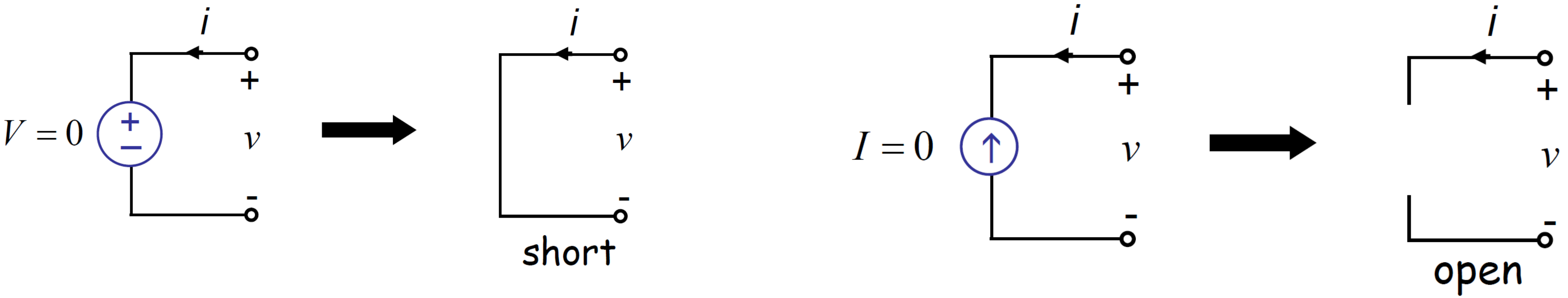
- In any linear resistive network, the voltage across or the current through any resistor or source may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits.

Superposition theorem for electrical circuits



Anant Agarwal and Jeffrey Lang, course materials for 6.002 Circuits and Electronics, Spring 2007. MIT OpenCourseWare(http://ocw.mit.edu/), Massachusetts Institute of Technology.

Superposition

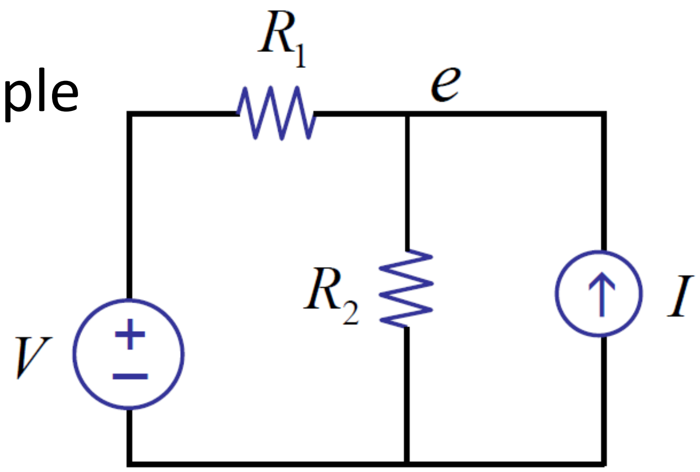


To ascertain the contribution of each individual source, all of the other sources first must be "turned off" (set to zero) by:

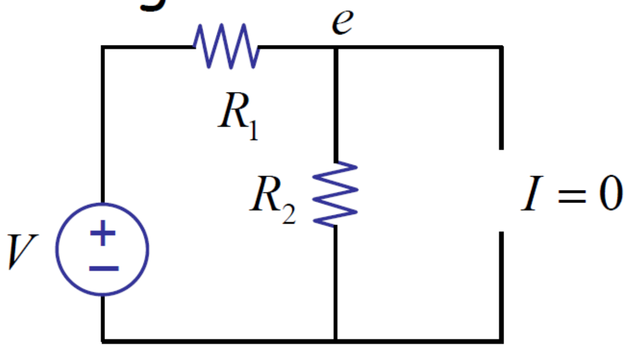
- Replacing all other **independent voltage sources** with a short circuit (thereby eliminating difference of potential i.e. V=0; internal impedance of ideal voltage source is zero (**short circuit**)).
- Replacing all other **independent current sources** with an open circuit (thereby eliminating current i.e. I=0; internal impedance of ideal current source is infinite (**open circuit**)).

An example

Find ***e*** using superposition principle

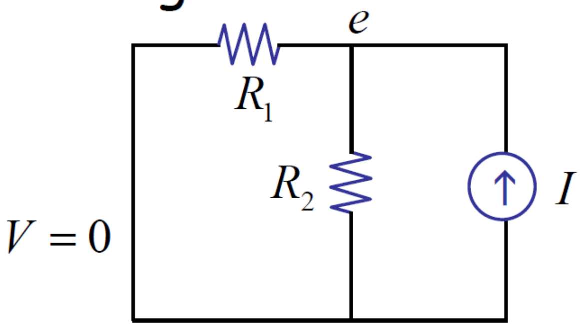


V acting alone



$$e_V = \frac{R_2}{R_1+R_2}V$$

I acting alone

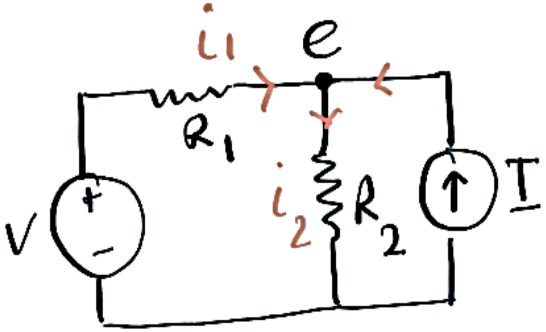


$$e_I = \frac{R_1R_2}{R_1+R_2}I$$

sum ⟶ superposition

$$e = e_V + e_I = \frac{R_2}{R_1+R_2}V + \frac{R_1R_2}{R_1+R_2}I$$

Proof



We have to find e, i.e., i_2R_2

we can write,

$$i_2 = i_1 + I \quad \text{— (1)}$$

and, $V = i_1R_1 + i_2R_2$

$$= i_1R_1 + i_1R_2 + IR_2$$

$$\Rightarrow i_1 = \frac{V - IR_2}{R_1 + R_2} \quad \text{— (2)}$$

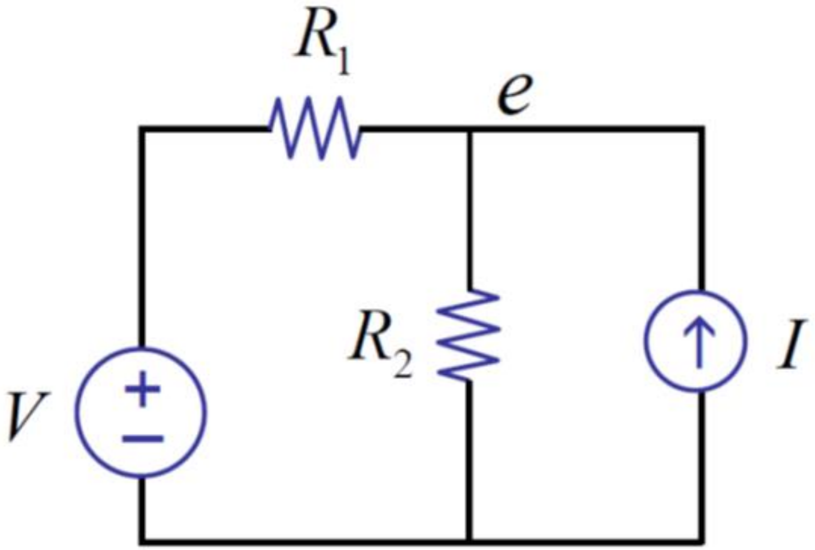
So, from (1),

$$i_2 = I + \frac{V - IR_2}{R_1 + R_2}$$

$$\therefore e = i_2R_2 = R_2\left(I + \frac{V - IR_2}{R_1 + R_2}\right)$$

$$\Rightarrow e = \frac{R_2}{R_1 + R_2}V + IR_2\left(1 - \frac{R_2}{R_1 + R_2}\right)$$

$$\Rightarrow \boxed{e = \frac{R_2}{R_1 + R_2}V + \frac{R_1R_2}{R_1 + R_2}}$$

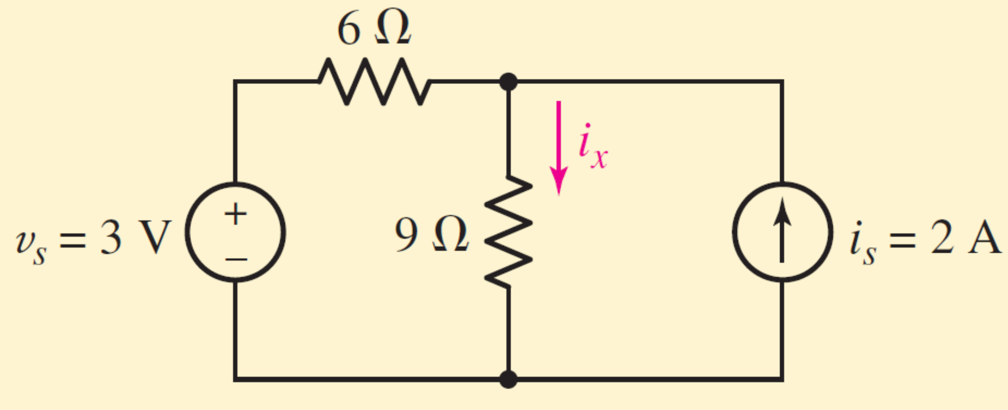


sum → superposition

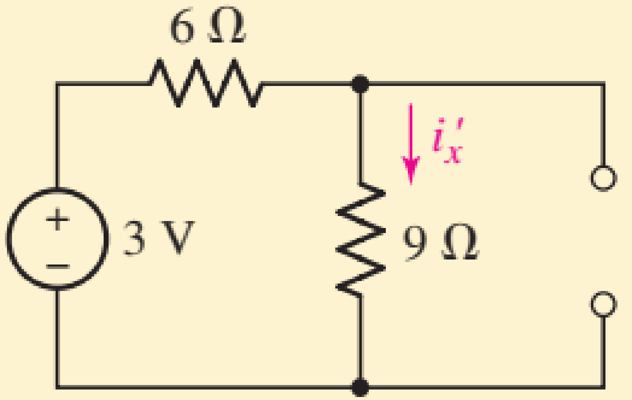
$$e = e_V + e_I = \frac{R_2}{R_1 + R_2}V + \frac{R_1R_2}{R_1 + R_2}I$$

Example 5.1

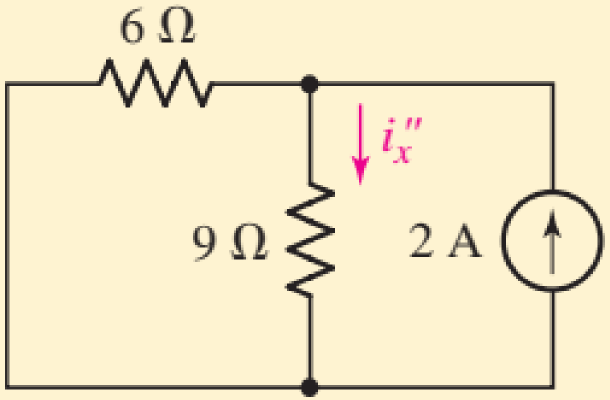
For the circuit of Fig., use superposition to determine the unknown branch current **ix**.



Ans: 1A



(*b*)



(*c*)

Now compute the total current i_x by adding the two individual components:

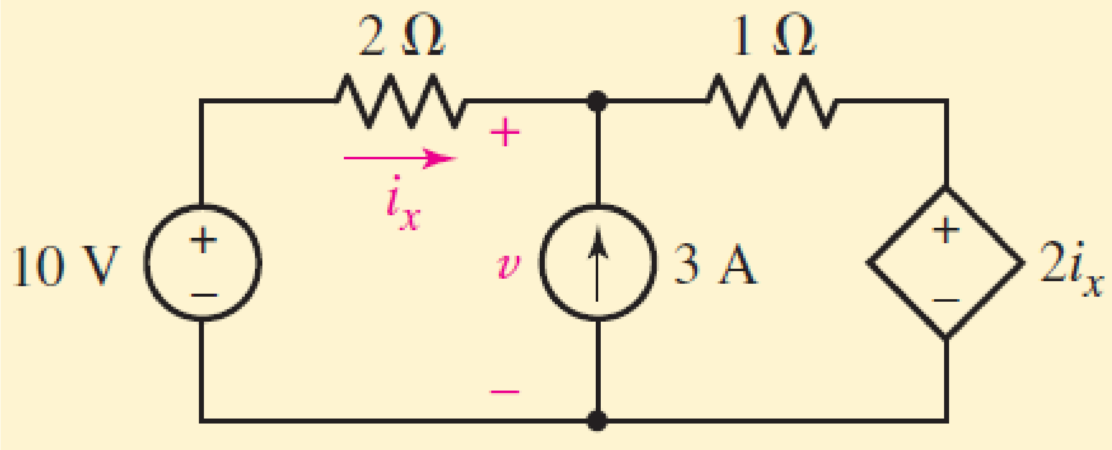
$$i_x = i_{x|_{3\text{ V}}} + i_{x|_{2\text{ A}}} = i_x' + i_x''$$

or

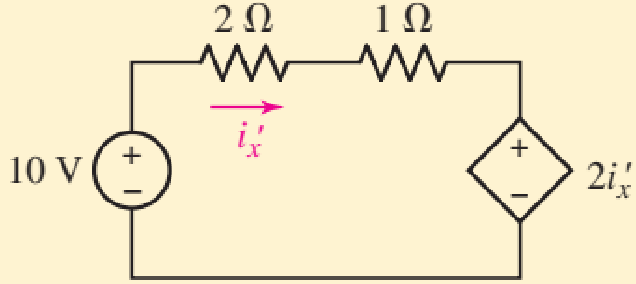
$$i_x = \frac{3}{6+9} + 2\left(\frac{6}{6+9}\right) = 0.2 + 0.8 = 1.0 \text{ A}$$

Example 5.3

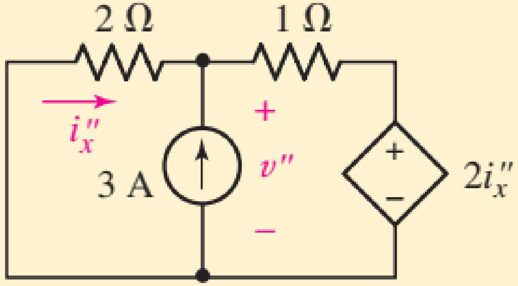
In the circuit of Fig., use the superposition principle to determine the value of ix



Ans: 1.4 A



(*b*)



(*c*)

First open-circuit the 3 A source (Fig. 5.6*b*). The single mesh equation is

$$-10 + 2i'_x + i'_x + 2i'_x = 0$$

so that

$$i'_x = 2 \text{ A}$$

Next, short-circuit the 10 V source (Fig. 5.6*c*) and write the single-node equation

$$\frac{v''}{2} + \frac{v'' - 2i''_x}{1} = 3$$

and relate the dependent-source-controlling quantity to v'':

$$v'' = 2(-i''_x)$$

Solving, we find

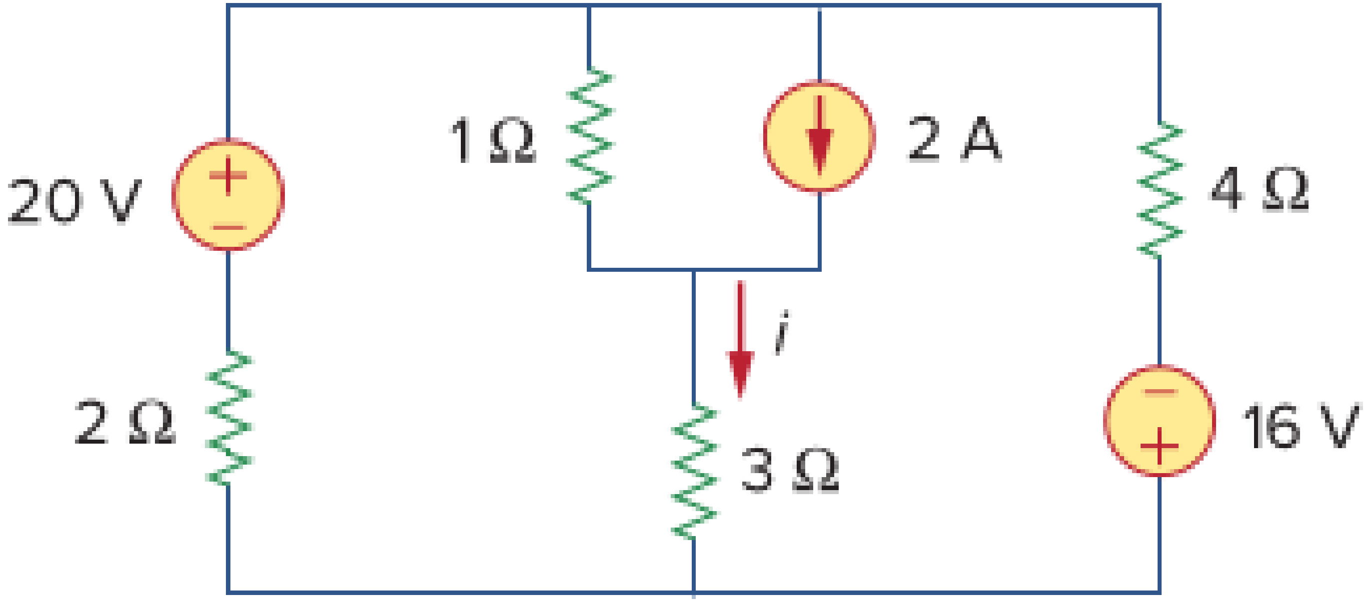
$$i''_x = -0.6 \text{ A}$$

and, thus,

$$i_x = i'_x + i''_x = 2 + (-0.6) = 1.4 \text{ A}$$

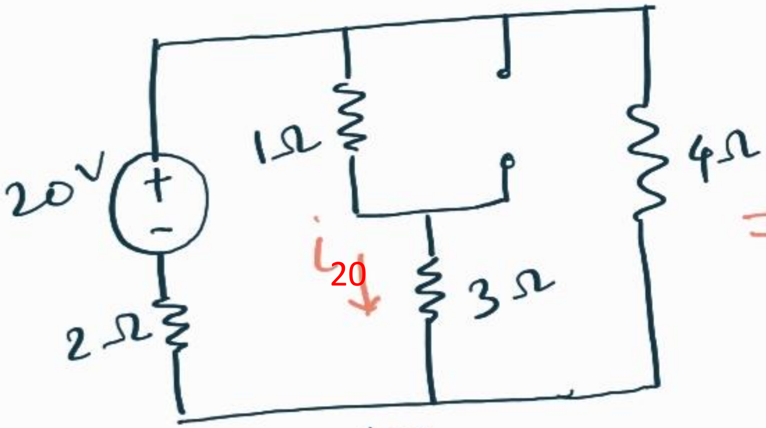
Example

In the circuit of Fig., use the superposition principle to determine the value of ***i***



Ans: i = 1.875A

Case I (i from 20 V)



(a)

(b)

Then $R_{tot} = [(1+3) \| 4] + 2$

$= 4\ \Omega$

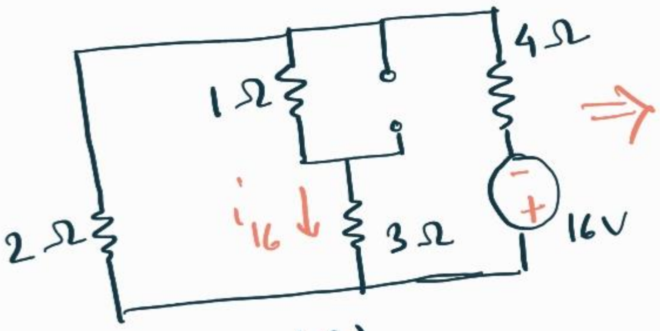
So, total current $I = \frac{20}{4} = 5$ A

From (b) we can get,

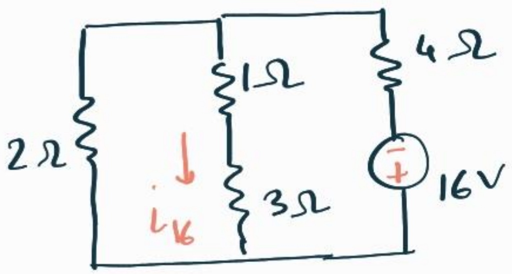
$$i_{20} = \frac{4}{4+4} I = \frac{1}{2} I$$

$$= 2.5 \text{ A}$$

Case II (current from 16 V)



(c)



(d)

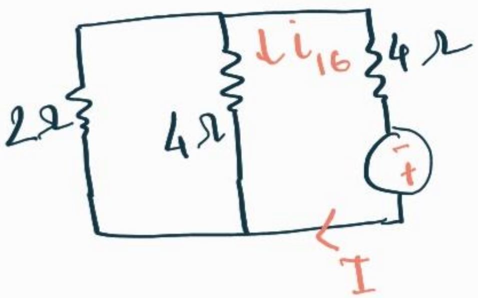
Then $R_{eq} = \left[(1+3) \parallel 2\right] + 4$

$$= \frac{8}{6} + 4$$

$$= \frac{32}{6} = \frac{16}{3}$$

Total current $I = \frac{16}{16/3} = 3A$

∴ from fig (d) →



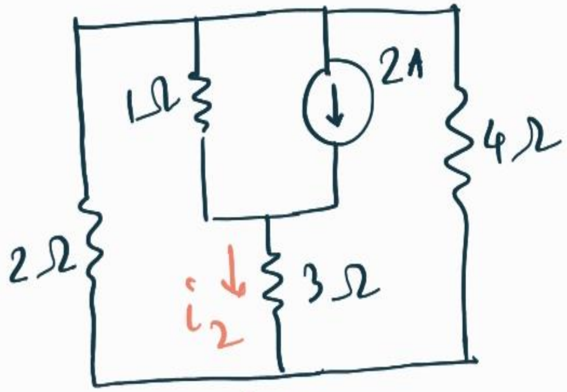
Then current i_{16}

$$i_{16} = - \frac{2}{2+4} I$$

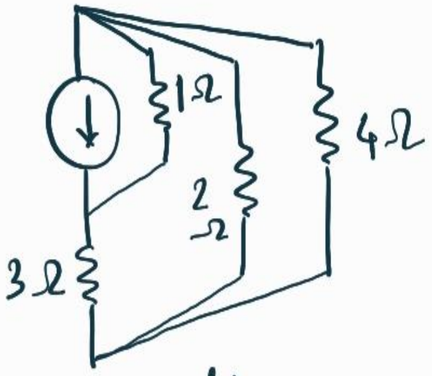
'–' for reverse polarity

$$i_{16} = -\frac{2}{6} \times 3 = -1\ A$$

Case $\underline{\text{III}}$ (current from 2A)



(e)



(f)

Here, $2 \parallel 4 = 8/6 = \frac{4}{3}\ \Omega$

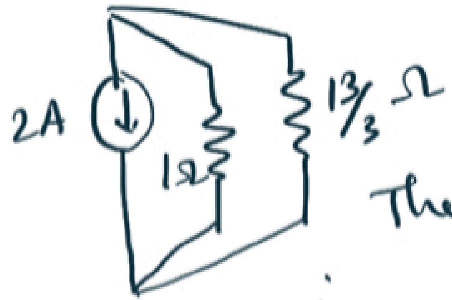
Then from (f) $\longrightarrow$

(g)

and here, 3 & $4/3$ in series,

so, $3 + 4/3 = 13/3\ \Omega$

$\therefore$ from (g) $\longrightarrow$



Therefore,

$$i_2 = 2 \cdot \frac{1}{1 + 13/3}$$

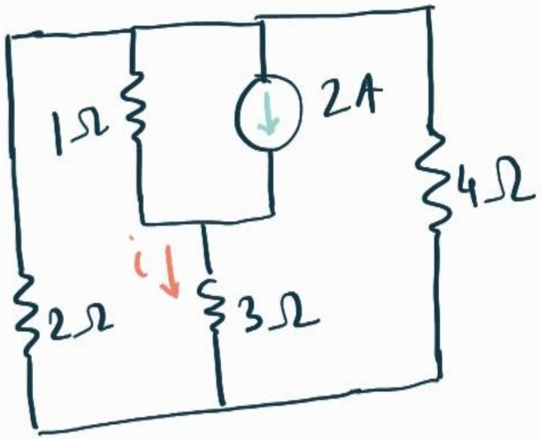
$$i_2 = \frac{2}{16/3} = \frac{6}{16} = 0.375\text{A}$$

Then the overall current through $3\ \Omega$,

$$i = i_{20} + i_{16} + i_2 = (2.5 - 1 + 0.375)\text{A}$$

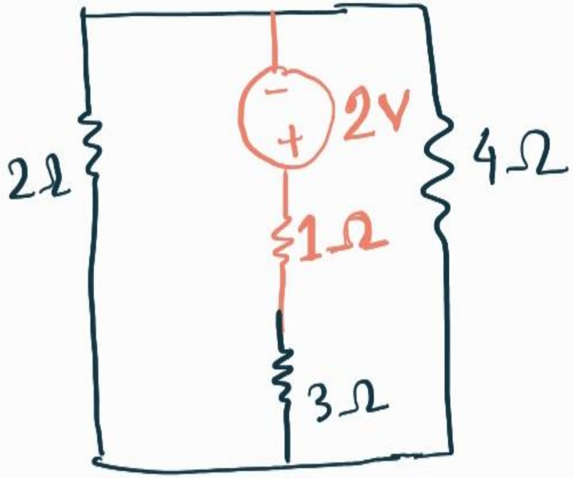
$$= 1.875\ \text{A}$$

Alternative approach for case III



(h)

⇒



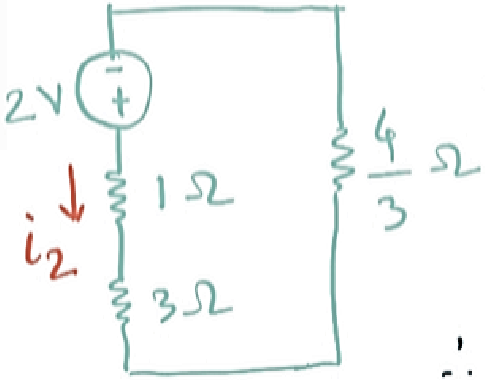
(i)

Here, $2 \| 4 \Rightarrow \frac{4}{3}\ \Omega$

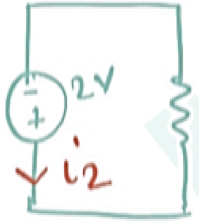
Using the concept of equivalent current and voltage source

convert the current source w/ parallel resistor into a voltage source with a series resistor

from fig (i) ⟶



⇒



$1+3+\frac{4}{3}$

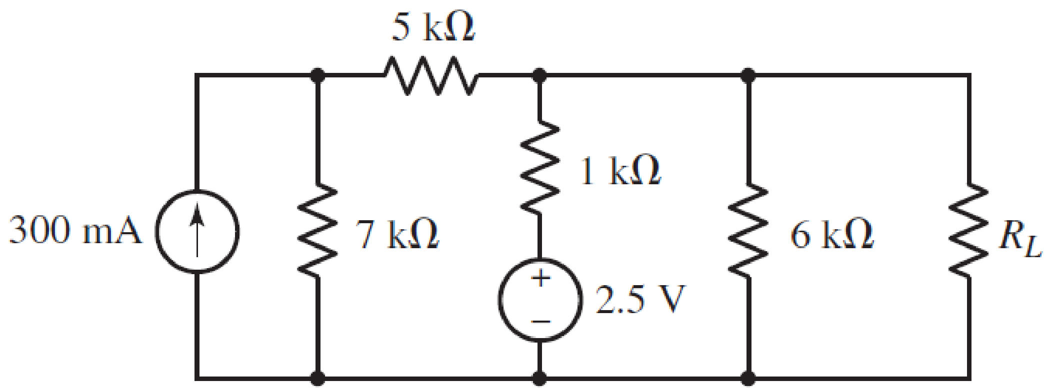
$= 16/3\ \Omega$

$\therefore\ i_2 = \frac{2}{16/3} = 3/8 = 0.375A$

We must constantly be aware of the limitations of superposition.

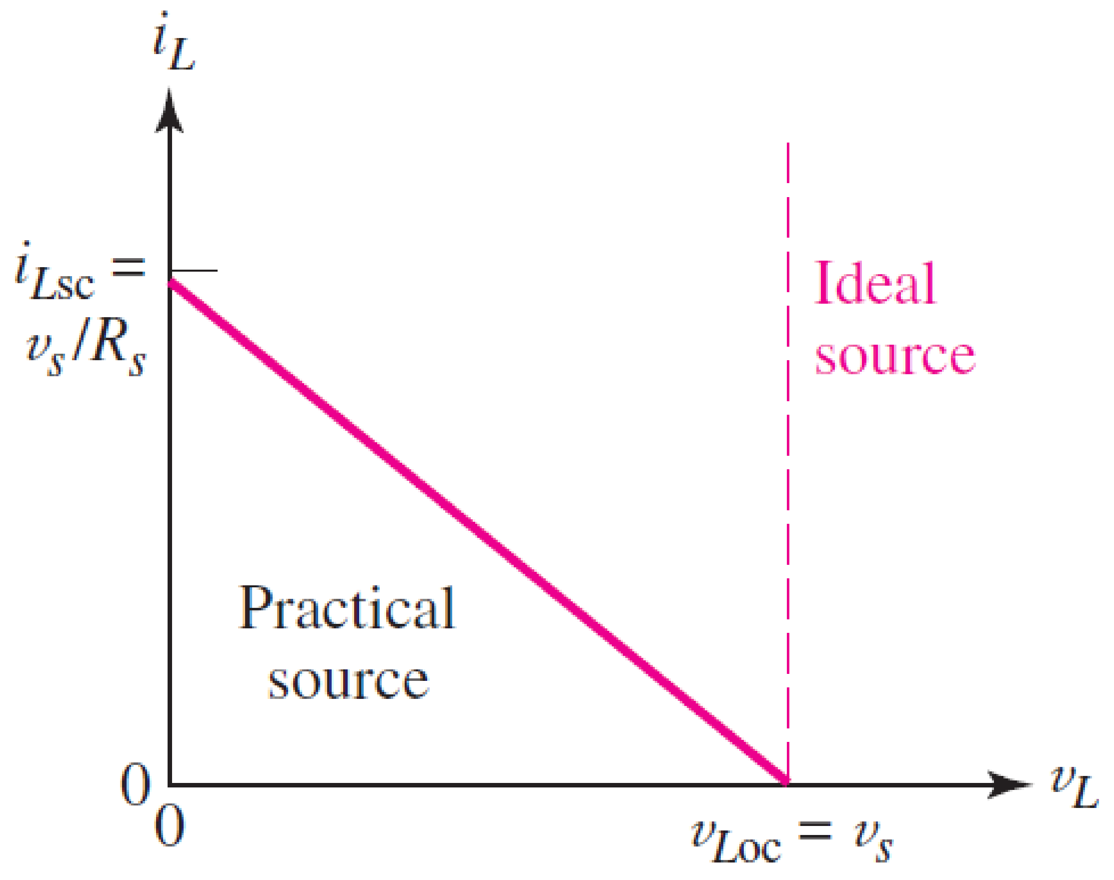
It is applicable only to linear responses, and thus the most common nonlinear response—power—is not subject to superposition. For example, consider two 1 V batteries in series with a 1 Ω resistor. The power delivered to the resistor is 4 W, but if we mistakenly try to apply superposition, we might say that each battery alone furnished 1 W and thus the calculated power is only 2 W. This is incorrect, but a surprisingly easy mistake to make.

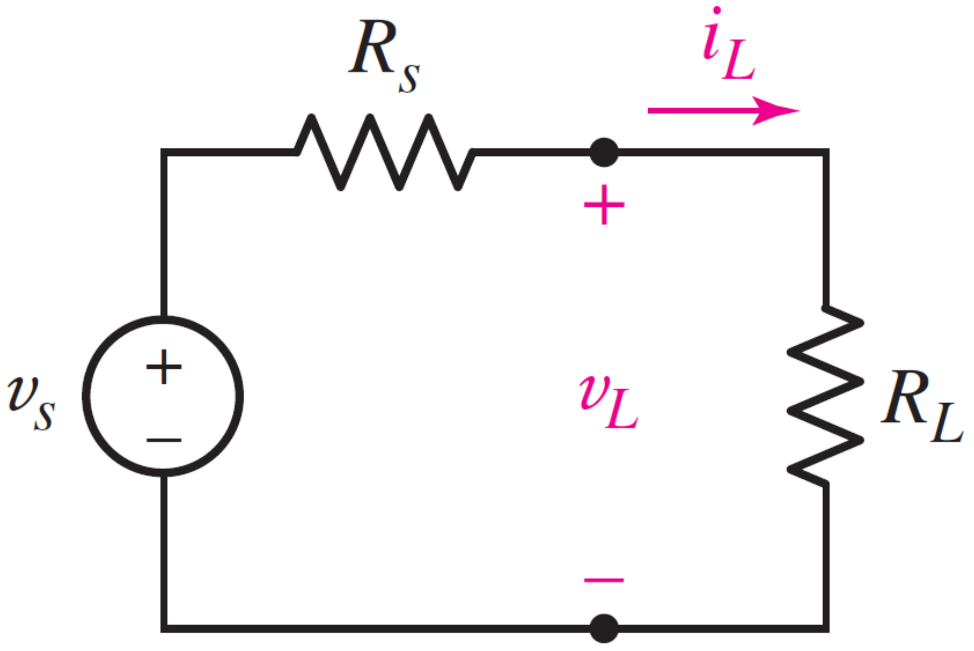
Motivation



- Some times the load resistance is subject to change
- Recalculation of circuit is necessary for each trial value of resistance

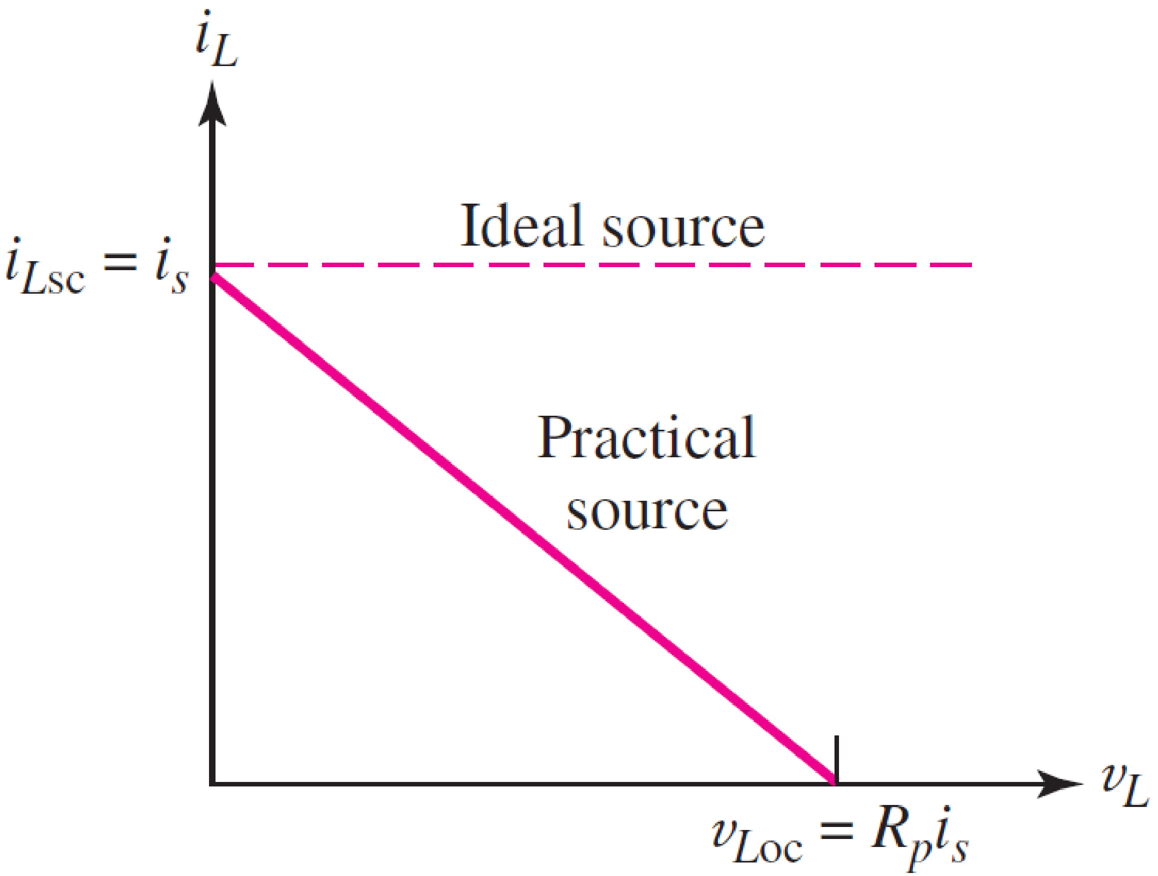
Practical voltage source

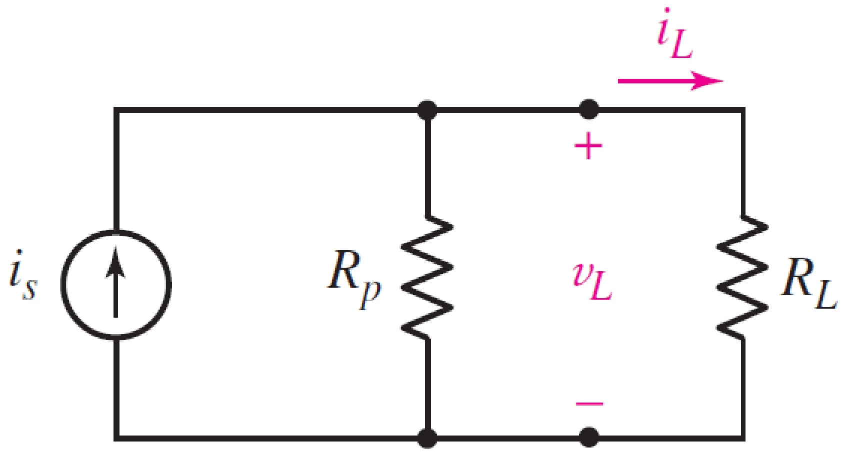




$$v_L = v_s - R_s i_L$$

Practical current source



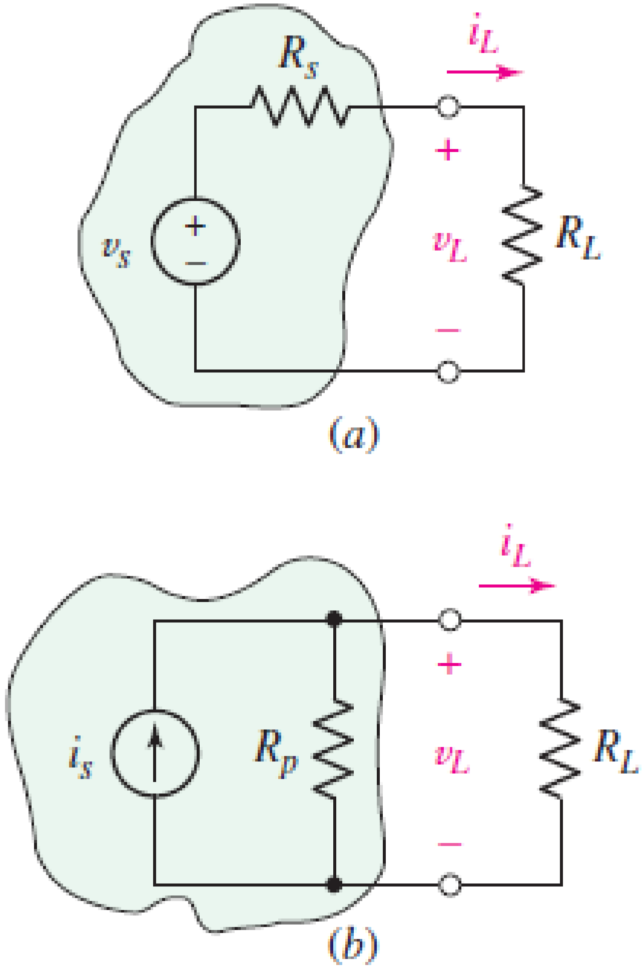


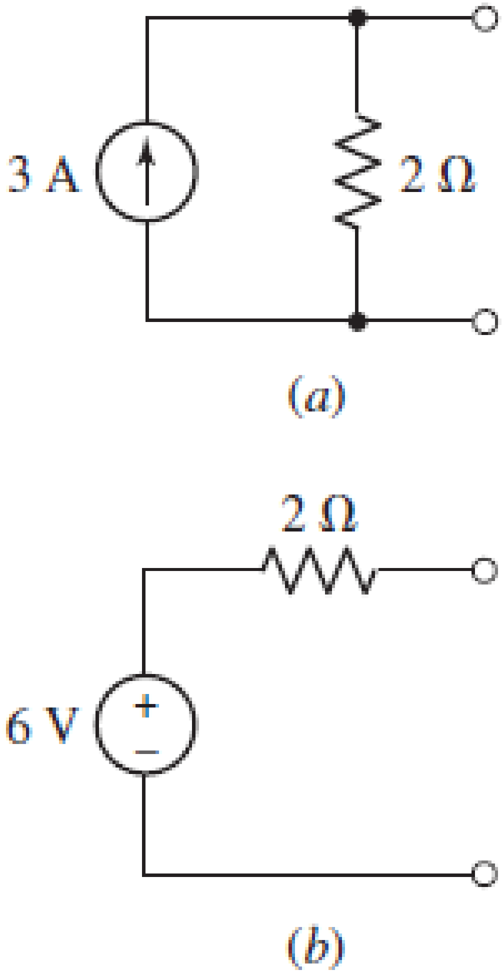
$$i_L = i_s - \frac{v_L}{R_p}$$

Source equivalence

$V_s = I_sR_s$ or I_sR_p

$R_s = R_p$

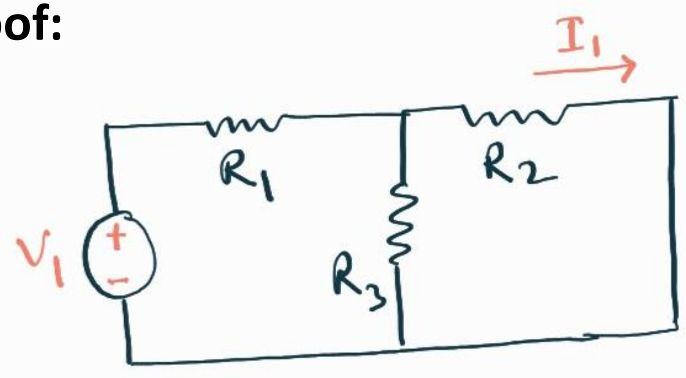




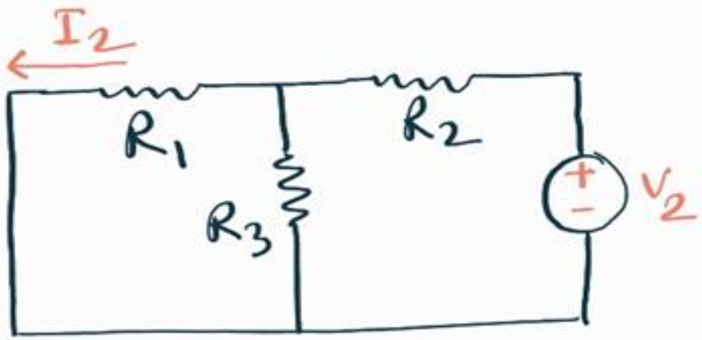
Reciprocity theorem

The current '*I*' in any branch of a network due to a single voltage source '*V*' anywhere else in the network will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current '*I*' was originally measured.

Proof:



(a)



(b)

For fig. (a),

$$I_1 = \frac{V_1}{R_1 + (R_2 \| R_3)} \cdot \frac{R_3}{R_2 + R_3} = \frac{V_1 R_3}{R_1R_2 + R_2R_3 + R_3R_1} \quad (1)$$

For Fig. (b),

$$I_2 = \frac{V_2}{R_2 + (R_1 \| R_3)} \cdot \frac{R_3}{R_1 + R_3} = \frac{V_2 R_3}{R_1R_2 + R_2R_3 + R_3R_1} \quad (2)$$

From eqn (1) & (2), we can see that

$$I_1 = I_2 \text{ if } V_1 = V_2$$

Conditions to be met for the application of reciprocity theorem

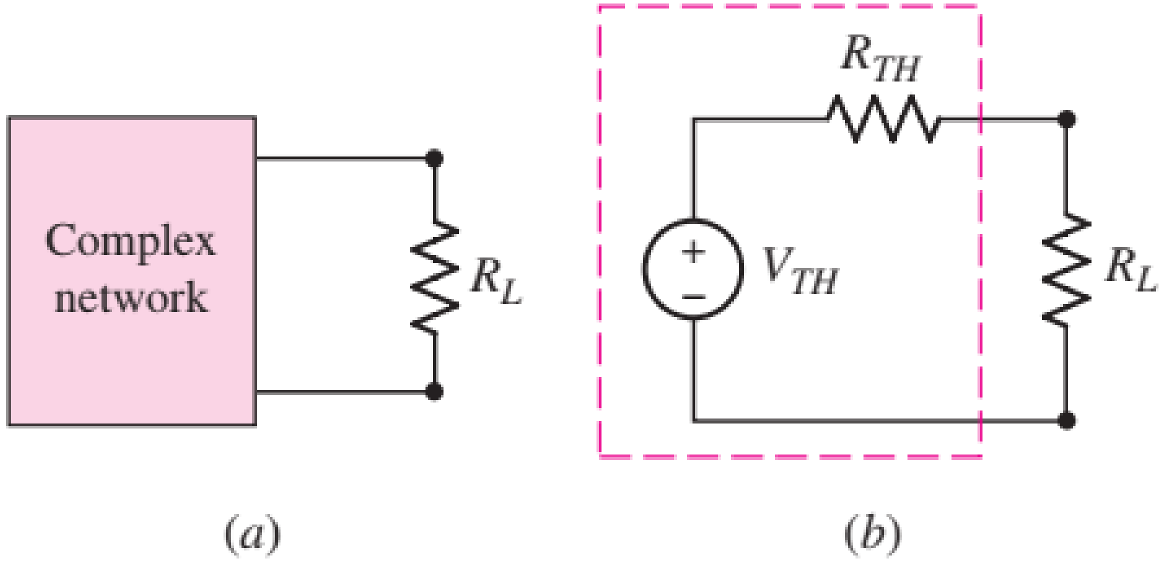
- The reciprocity theorem is applicable only to single-source networks. It is, therefore, not a theorem used in the analysis of multisource networks.

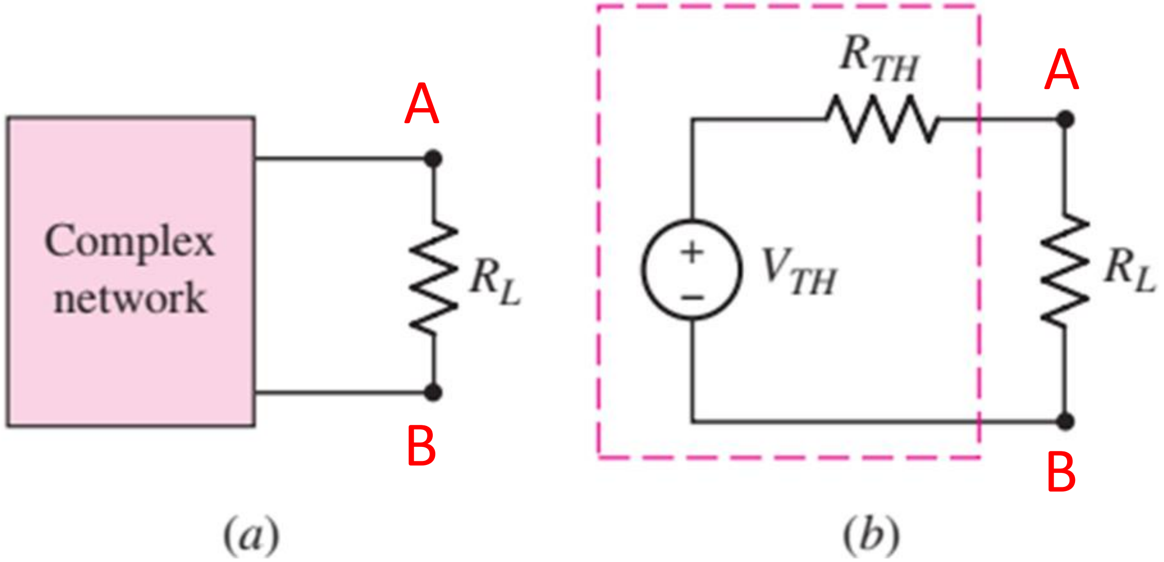
- The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.

- Dependent sources are excluded even if they are linear.

Thevenin's Theorem

Any two terminal linear network containing energy sources and resistances (or impedances) can be replaced by an equivalent circuit consisting of a voltage source V_{TH} in series with an resistance (or impedance) R_{TH}, where *V_{TH} is the open circuit voltage between the terminals* of the network and R_{TH} is the resistance (or impedance) measured between the terminals with all the energy sources replaced by their internal resistance (or impedance).





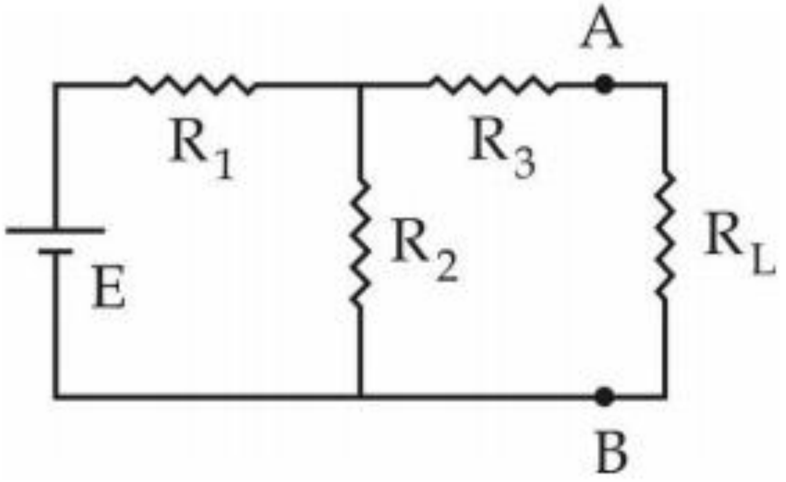
- The equivalent voltage V_{TH} is the voltage obtained at terminals A–B of the network with terminals A–B open circuited.
- The equivalent resistance R_{TH} is the resistance that the circuit between terminals A and B would have if all ideal voltage sources in the circuit were replaced by a short circuit and all ideal current sources were replaced by an open circuit. Non-ideal sources will be replaced by their internal resistance
- If terminals A and B are connected to one another, the current flowing from A to B will be V_{TH}/R_{TH}. This means that R_{TH} could alternatively be calculated as V_{th} divided by the short-circuit current between A and B when they are connected together.

Thevenin's Theorem

1. **Given any linear circuit, rearrange it in the form of two networks, *A* and *B*, connected by two wires.** Network *A* is the network to be simplified; *B* will be left untouched.
2. **Disconnect network *B*.** Define a voltage v_{oc} as the voltage now appearing across the terminals of network *A*.
3. **Turn off or "zero out" every independent source in network *A* to form an inactive network.** Leave dependent sources unchanged.
4. **Connect an independent voltage source with value v_{oc} in series with the inactive network.** Do not complete the circuit; leave the two terminals disconnected.
5. **Connect network *B* to the terminals of the new network *A*.** All currents and voltages in *B* will remain unchanged.

Thevenin's Theorem

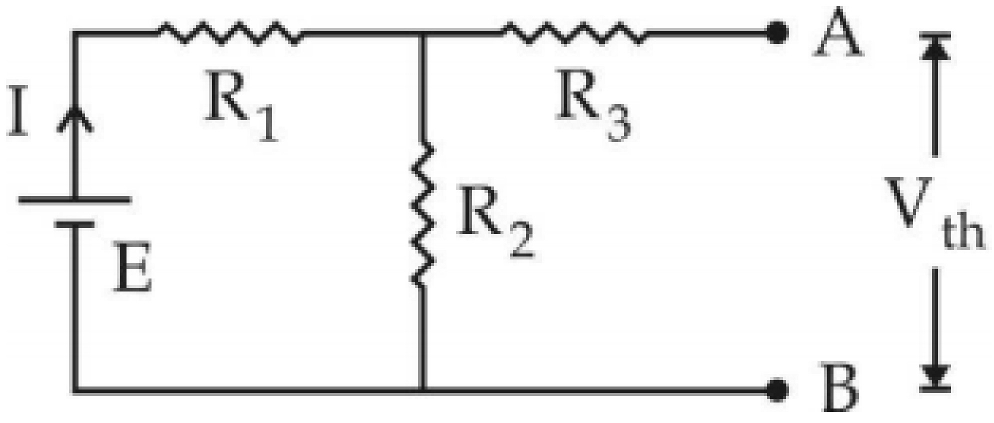
Explanation:



Consider a network or a circuit as shown. Let E be the emf of the cell having its internal resistance $r = 0$. $R_L \rightarrow$ load resistance across AB.

Thevenin's Theorem

To find V_{Th} :



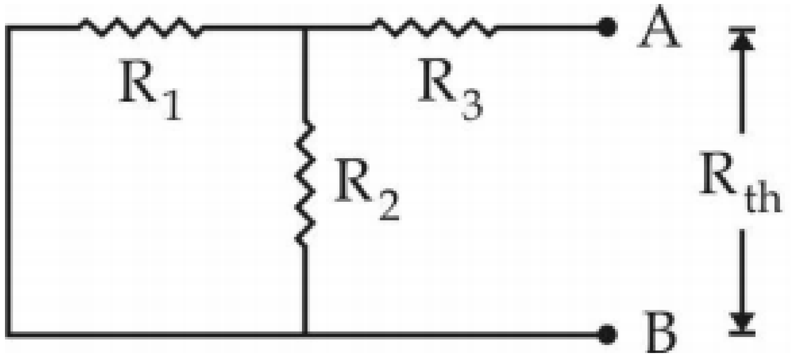
The load resistance R_L is removed. The current I in the circuit is $I = \dfrac{E}{R_1 + R_2}$.

The voltage across $AB =$ Thevenin's voltage V_{Th}.

$$V_{Th} = I R_2 \quad \Rightarrow \quad \boxed{V_{Th} = \frac{E R_2}{R_1 + R_2}}$$

Thevenin's Theorem

To find R_{Th} :



The load resistance R_L is removed. The cell is disconnected and the wires are short as shown.

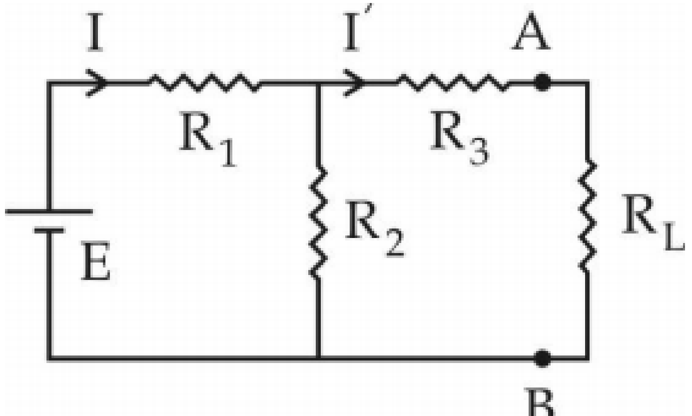
The effective resistance across $AB =$ Thevenin's resistance R_{Th}.

$\boxed{R_{Th} = R_3 + \dfrac{R_1 R_2}{R_1 + R_2}}$ [R_1 is parallel to R_2 and this combination in series with R_3]

If the cell has internal resistance r, then $V_{Th} = \dfrac{E R_2}{R_1 + R_2 + r}$ and $R_{Th} = R_3 + \dfrac{(R_1 + r) R_2}{R_1 + r + R_2}$.

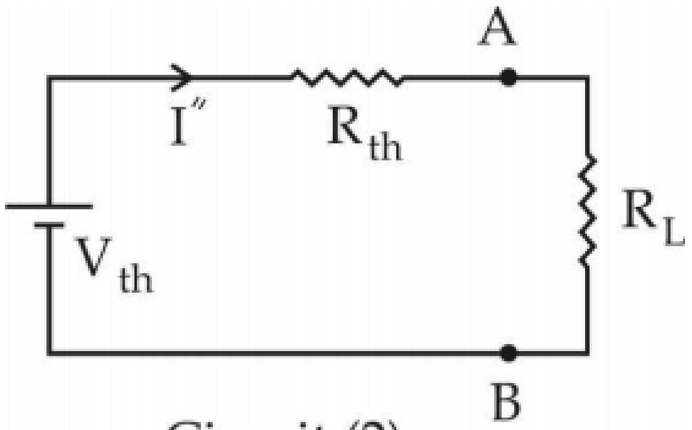
Proof of Thevenin's Theorem

Consider the network as shown below



Circuit (1)

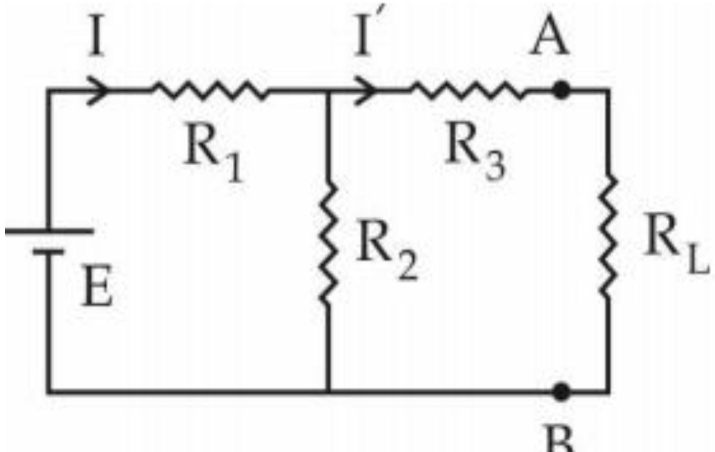
The equivalent circuit is given by



Circuit (2)

If we can show that I' and I" are same then, then we can say that these two circuits are equivalent and we can then verify Thevenin's theorem

Proof of Thevenin's Theorem



Circuit (1)

The effective resistance of the network in (1) is R_3 and R_L in series and this combination is parallel to R_2 which in turn is in series with R_1.

Thus, $R_{eff} = R_1 + \frac{R_2(R_3+R_L)}{R_2+R_3+R_L}$ -------------- (1)

The current I in the circuit is $I = \frac{E}{R_{eff}} = \frac{E}{R_1 + \frac{R_2(R_3+R_L)}{R_2+R_3+R_L}}$

or $I = \frac{E(R_2+R_3+R_L)}{R_1R_2+R_1R_3+R_1R_L+R_2R_3+R_2R_L}$ ----------- (2)

The current through the load resistance (I') is found using branch current method.

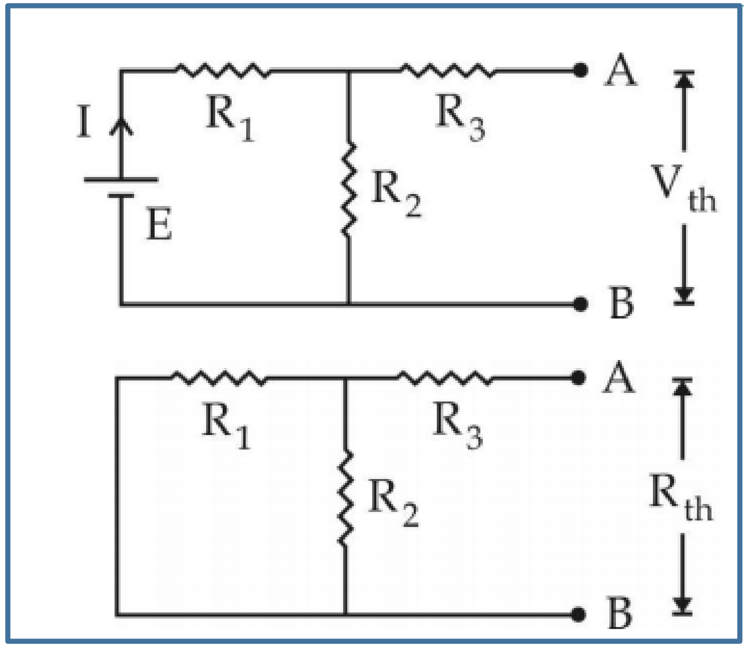
$I' = \frac{IR_2}{R_2+R_3+R_L}$ ----------- (3)

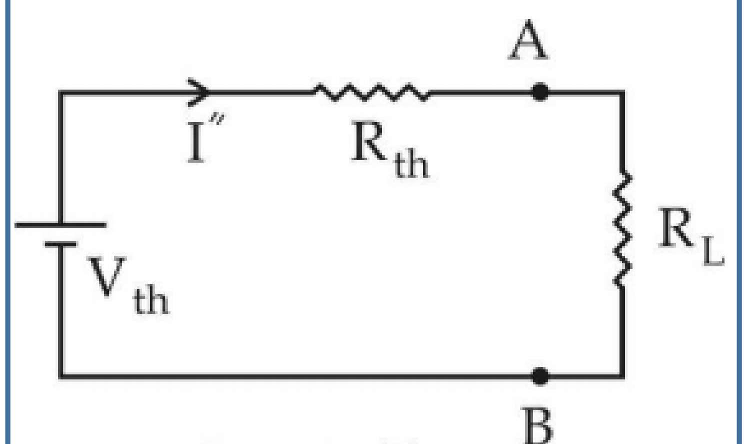
Substituting for I from (2) in (3)

$$I' = \frac{E(R_2+R_3+R_L)R_2}{(R_2+R_3+R_L)(R_1R_2+R_1R_3+R_1R_L+R_2R_3+R_2R_L)}$$

or $I' = \frac{ER_2}{R_1R_2+R_1R_3+R_1R_L+R_2R_3+R_2R_L}$ ------------ (4)

Proof of Thevenin's Theorem





Circuit (2)

Thevenin's voltage $V_{Th} = \dfrac{ER_2}{R_1 + R_2}$ ----------- (5)

Thevenin's resistance $R_{Th} = R_3 + \dfrac{R_1 R_2}{R_1 + R_2}$ ---------- (6)

Consider the equivalent circuit (circuit (2))

The current I'' in the equivalent circuit is $I'' = \dfrac{V_{Th}}{R_{Th} + R_L}$ ----------- (7)

Substituting for V_{Th} and R_{Th} from (5) and (6) in (7)

$$I'' = \frac{ER_2}{R_1 + R_2} \times \frac{1}{R_3 + \dfrac{R_1 R_2}{R_1 + R_2} + R_L} = \frac{ER_2}{(R_1 + R_2)\dfrac{R_3 R_1 + R_3 R_2 + R_1 R_L + R_2 R_L + R_1 R_2}{(R_1 + R_2)}}$$

or $$I'' = \frac{ER_2}{R_1 R_2 + R_1 R_3 + R_1 R_L + R_2 R_3 + R_2 R_L} \text{ ------------- (8)}$$

From equations (4) and (8), it is observed that $I' = I''$.

Hence Thevenin's theorem is verified.