ECE 113- Basic Electronics

Lecture week 10: Continuation of AC response in circuits (under steady state condition)

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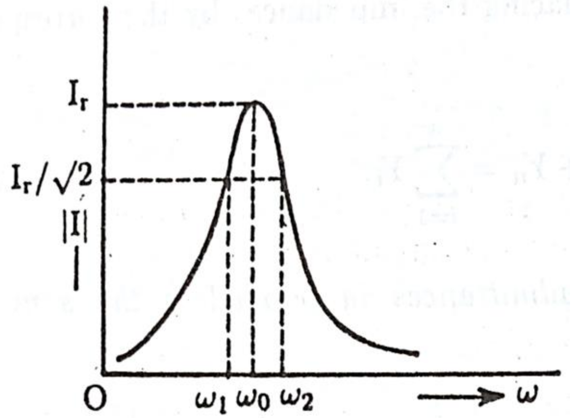
Bandwidth frequencies of a series RLC circuit



Current response in series RLC circuit

Let ω_x is the angular frequency at which current is $I = I_r/\sqrt{2}$

$$\frac{I_r}{\sqrt{2}} = \frac{V}{\sqrt{2}R} = \frac{V}{\left[R^2 + \left(\omega_x L - \frac{1}{\omega_x C}\right)^2\right]^{1/2}}$$

$$\text{or,}\quad 2R^2 = R^2 + \left(\omega_x L - \frac{1}{\omega_x C}\right)^2$$

$$\text{or,}\quad \omega_x L - \frac{1}{\omega_x C} = \pm R$$

$$\text{or,}\quad \omega_x^2 LC \mp \omega_x RC - 1 = 0.$$

Only positive roots of this eq. are acceptable

Since at frequency ω_x, current is

$$I = \frac{V}{|Z|} = \frac{V}{\left[R^2+\left(w_x L - \frac{1}{w_x C}\right)^2\right]^{1/2}}$$

The roots are

$$\omega_1 = \frac{\sqrt{R^2C^2 + 4LC} - RC}{2LC}$$

$$\text{and}\quad \omega_2 = \frac{\sqrt{R^2C^2 + 4LC} + RC}{2LC}$$

$$\omega_2 - \omega_1 = \frac{R}{L} = \frac{\omega_0 R}{\omega_0 L} = \frac{\omega_0}{Q}$$

$$\text{or,}\quad Q = \frac{\omega_0}{\omega_2 - \omega_1}.$$

This eq. relates the Q factor with bandwidth
→ Higher the Q, smaller the bandwidth

At frequencies ω_1 and ω_2 the power dissipation is $(I_r/\sqrt{2})^2 R$ → half the power dissipated at resonance. So ω_1 and ω_2 are called **half power frequencies**

Capacitor voltage in series RLC circuit

The voltage across capacitor C is $V_C = -\frac{j}{\omega C}\frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$ where V is the applied voltage

The magnitude of V_C is

$$|V_C| = \frac{V}{\omega C\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{[\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2]^{1/2}} \quad \text{(a)}$$

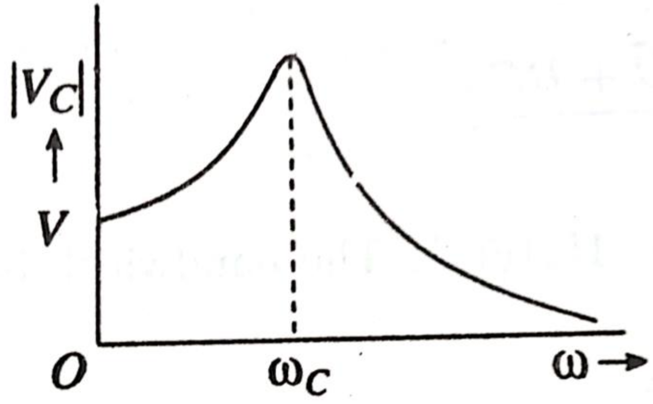


Fig.15.17A. Variation of $|V_C|$ with ω

$|V_C|$ is maximum at $\omega = \omega_C$ when the denominator on the r.h.s. of the eq. (a) is minimum. Therefore, for maximum $|V_C|$, we have

$$\frac{d}{d\omega}[\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2] = 0$$

or, $2\omega R^2 C^2 + 4\omega LC(\omega^2 LC - 1) = 0$

or, $2LC(\omega^2 LC - 1) + R^2 C^2 = 0$

or, $\omega = \omega_C = \left[\frac{1}{LC}\left(1 - \frac{R^2 C}{2L}\right)\right]^{1/2}$

or, $\omega_C = \omega_0\left[1 - \frac{1}{2Q^2}\right]^{1/2}$ (b)

From eq. (b), $\boldsymbol{\omega_C < \omega_0}$ where ω_0 is the resonant frequency.

→ This is expected, as current peaks at ω_0 and the capacitor voltage is a product of current and $1/\omega C$ which increases with decreasing frequency. **When Q is large $\omega_C \simeq \omega_0$**

→ ω_C is sometimes called capacitor charge resonance or voltage resonance, but current resonance is true resonance as current and voltage in the circuit is in phase.

Sinusoidal voltage applied to a parallel RLC circuit

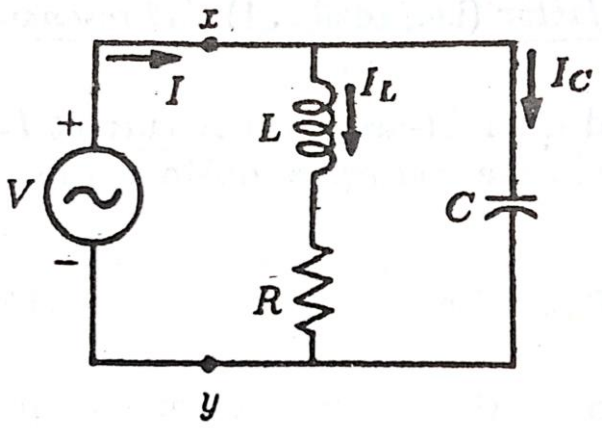


Fig.15.19. A parallel resonant circuit

Let I_L and I_C are phasors for current through the inductor and the capacitor, respectively, and I is the current from the source.

→ I_L and I_C are almost 180° out of phase. I_C leads source voltage V by 90° while I_L lags source voltage V by 90°.

By KCL, the source current I is a phasor sum of I_C and I_L, and in resonance V and I are in phase.

We have, $$I = IC + IL \qquad (1)$$

But, $I = V/Z$; where Z is the impedance offered by the circuit at the terminal x, y in the circuit.

Also, $$I_L = \frac{V}{R + j\omega L} \quad \text{and} \quad I_c = \frac{V}{1/(j\omega C)} = j\omega CV. \qquad (2)$$

Therefore, from (1) and (2),

$$\frac{V}{Z} = \frac{V}{R + j\omega L} + j\omega CV$$

$$\text{or,} \quad \frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$$\frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

[Y is the admittance]

$$\text{or,} \quad Y = \frac{1}{Z} = \frac{R + j(\omega C R^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2} \qquad (3)$$

In resonance, *I* and *V* will be in phase when the imaginary part of Y is zero, i.e.,

$$\omega CR^2 + \omega^3 L^2 C - \omega L = 0$$

$$\text{or,} \quad \omega^2 L^2 C = L - CR^2$$

$$\text{or,} \quad \omega = \omega_p = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$\omega_P \rightarrow$ parallel resonance angular frequency

Corresponding parallel resonance frequency, $$f_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

The impedance of the circuit is purely resistive at resonance

Admittance in resonance → $$Y_p = \frac{R}{R^2 + \omega_p^2 L^2} = \frac{R}{L/C} = \frac{CR}{L}$$

and the impedance in resonance → $$Z_p = \frac{1}{Y_p} = \frac{L}{CR}$$

If R is small → Z_P is large.

For R → 0, $Z_P \rightarrow \infty$

Also when R → 0 ; $\omega_P \rightarrow \frac{1}{\sqrt{LC}}$ the series resonance angular frequency