These Equalities are Valid for Both Continuous and Discrete RV(s)

Theorem 3.5

For any random variable X,

(a) $E[X - \mu_X] = \boxed{}$

(b) $E[aX + b] = \boxed{}$

(c) $\mathrm{Var}[X] = \boxed{}$

(d) $\mathrm{Var}[aX + b] = \boxed{}$

$E[(X - \mu_X)^2]$

$g(X) = (X - \mu_X)^2$

N students in a RDS section

In any minute, the probability that a student is on the phone is $p = 0.2$.

Let X be the no. of minutes before no student is on the phone.

$$P[X=x] = \begin{cases} \left(1-(1-p)^N\right)^{x-1} (1-p)^N, & x \in \{1,2,\dots\} \\ 0 & \text{otherwise} \end{cases}$$

Note that in any minute the success event is the event that no student is on the phone during the minute. Therefore any minute's outcome can be modeled using a Bernoulli RV, say Z, where $Z=1$ if no student is on the phone. Otherwise $Z=0$. The PMF of Z is:

$$P[Z=z] = \begin{cases} (1-p)^N & z=1 \\ 1-(1-p)^N & z=0 \\ 0 & \text{otherwise} \end{cases}$$

The RV X is simply geometric$\left((1-p)^N\right)$.

Families of Continuous RV(s)

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Uniform(a,b)

Definition 3.5 Uniform Random Variable

X is a uniform (a, b) random variable if the PDF of X is

$$f_X(x) = \begin{cases} 1/(b-a) & a \le x < b, \\ 0 & \textit{otherwise}, \end{cases}$$

where the two parameters are $b > a$.

- CLAIM: The probability of an outcome being in any interval of a given size Δ is the same
  - How can we conclude this?

VVVS Problem

- What is E[X]?
  - L/2
- What is $E[X^2]$?
  - $L^2/3$
- What is Var[X]?
  - $L^2/3 - L^2/4 = L^2/12$

$$E[X^n] = \int_{-\infty}^{\infty} x^n f_X(x)\, dx$$

$$= \int_a^b x^n \frac{1}{b-a}\, dx$$

Discrete and Continuous Uniform RV(s)

Theorem 3.7

Let X be a uniform (a, b) random variable, where a and b are both integers. Let $K = \lceil X \rceil$. Then K is a ______ random variable.

- You want to describe the RV K

$$S_K = \{a, a+1, \dots, b\}$$

For $k \in S_K$, $P[K = k] = P[X \in (k-1, k]]$

$$= \int_{k-1}^{k} \frac{1}{b-a}\, dx = \frac{1}{b-a}$$

Discrete and Continuous Uniform RV(s)

Theorem 3.7

Let X be a uniform (a, b) random variable, where a and b are both integers. Let $K = \lceil X \rceil$. Then K is a ______ random variable.

- You want to describe the RV K
- K is a discrete RV. It is described completely by its PMF P[K = k]

a k b

Given the ceil function, all x that are > k-1 and <= k correspond to the outcome {K=k}

$$P[K = k] = P[k - 1 < x \leq k] = \frac{1}{b-a}$$

Discrete and Continuous Uniform RV(s)

$$P[K = k] = P[k - 1 < x \leq k] = \frac{1}{b-a}$$

- The above is true for k = a+1, a+2,…, b
- P[K=k] = 0 otherwise.
- Clearly K is a discrete uniform RV

Exponential(λ)

Definition 3.6 Exponential Random Variable

X *is an* exponential (λ) random variable *if the PDF of* X *is*

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & \textit{otherwise}, \end{cases}$$

where the parameter $\lambda > 0$.

$P[X > 0] = ?$

$$= \int_0^\infty f_X(x)\,dx$$

$$= 1$$

$P\left[X \leq \frac{1}{\lambda}\right]$

$$= \int_0^{1/\lambda} \lambda e^{-\lambda x}\,dx$$

$$= 1 - \frac{1}{e}$$

$E[X]$

$$= \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$= \int_0^\infty x \lambda e^{-\lambda x}\,dx$$

Exponential(λ)

Definition 3.6 Exponential Random Variable

X *is an* exponential (λ) random variable *if the PDF of* X *is*

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & \textit{otherwise}, \end{cases}$$

where the parameter $\lambda > 0$.

- The inter-arrival time between two packet arrivals at a server may be modeled as an exp RV
- Received power may also be modeled as an exp RV

Exponential(λ)

Definition 3.6 Exponential Random Variable

X *is an* exponential (λ) random variable *if the PDF of* X *is*

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & \textit{otherwise}, \end{cases}$$

where the parameter $\lambda > 0$.

- What is the average inter-arrival time?