# ECE 113: Basic Electronics

Lecture 2: Voltage and current sources, resistor, Kirchoff’s laws

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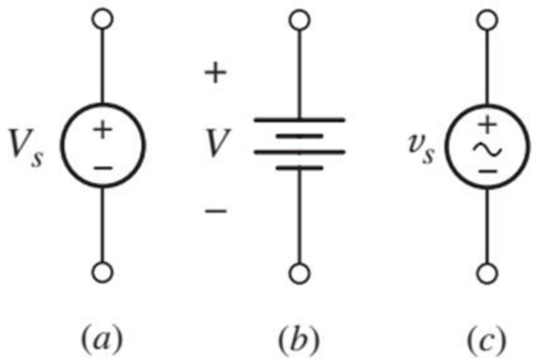
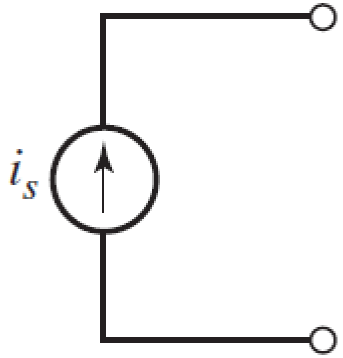
# Independent and dependent source

Independent source



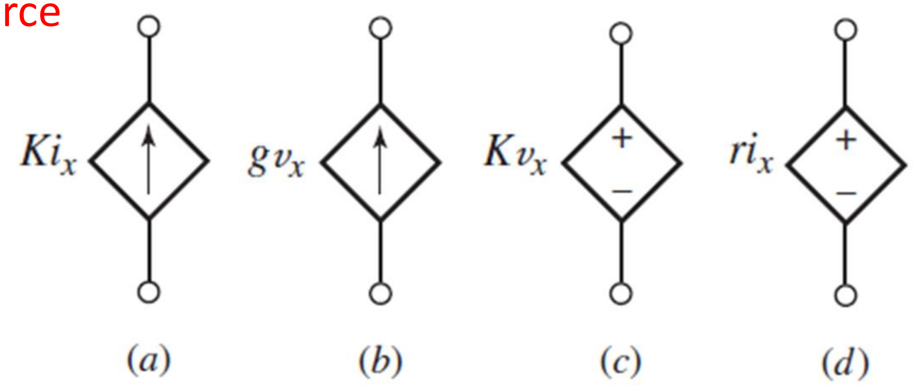

■ **FIGURE 2.16** (*a*) DC voltage source symbol; (*b*) battery symbol; (*c*) ac voltage source symbol.

Terminal voltage is completely independent of the current




Current is completely independent of the terminal voltage

Dependent source




■ **FIGURE 2.18** The four different types of dependent sources: (*a*) current-controlled current source; (*b*) voltage-controlled current source; (*c*) voltage-controlled voltage source; (*d*) current-controlled voltage source.

K is a dimensionless scaling const. g is a scaling factor with units of A/V and r is a scaling factor with units of V/A

- These are also ideal sources
- Dependent on current or voltage at some other area of the circuit
- Again independent of terminal voltage/current

# Networks and Circuits

**Circuit:** A circuit may be defined as a complete path for electric current flow.

**Branch:** A group of circuit elements, usually in series, and having two terminals is called a branch of a circuit.

**Network:** A network is a combination of circuit elements or branches interconnected in some ways.

**Mesh or Loop:** A loop is any closed path formed by a number of branches in a network. A mesh is the simplest possible closed path.

**Node or junction:** A node or junction is simply a common point where two or more circuit components meets.

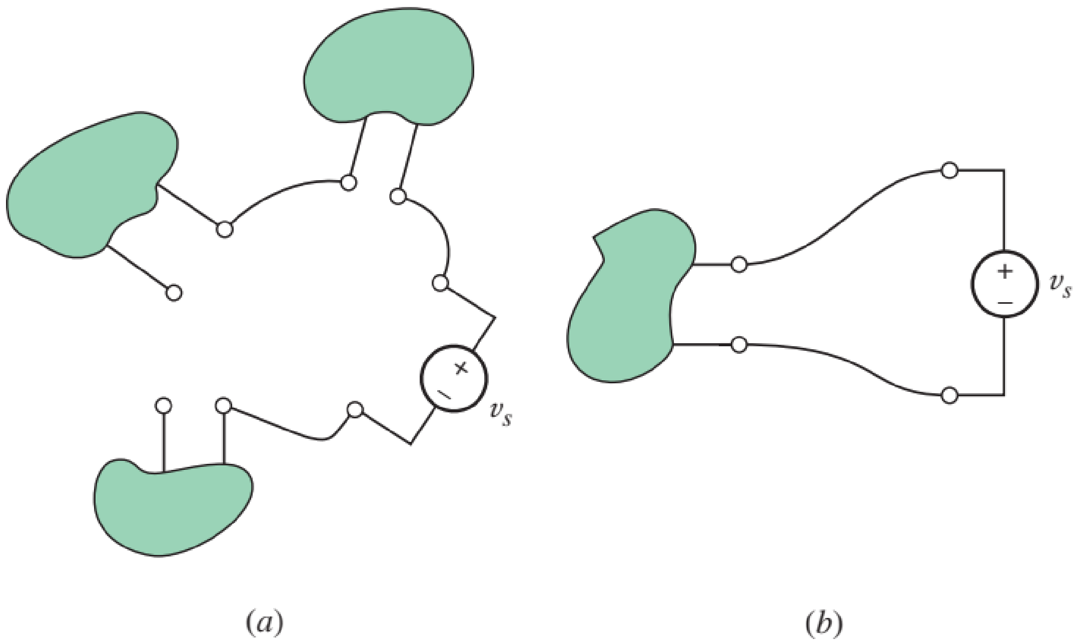
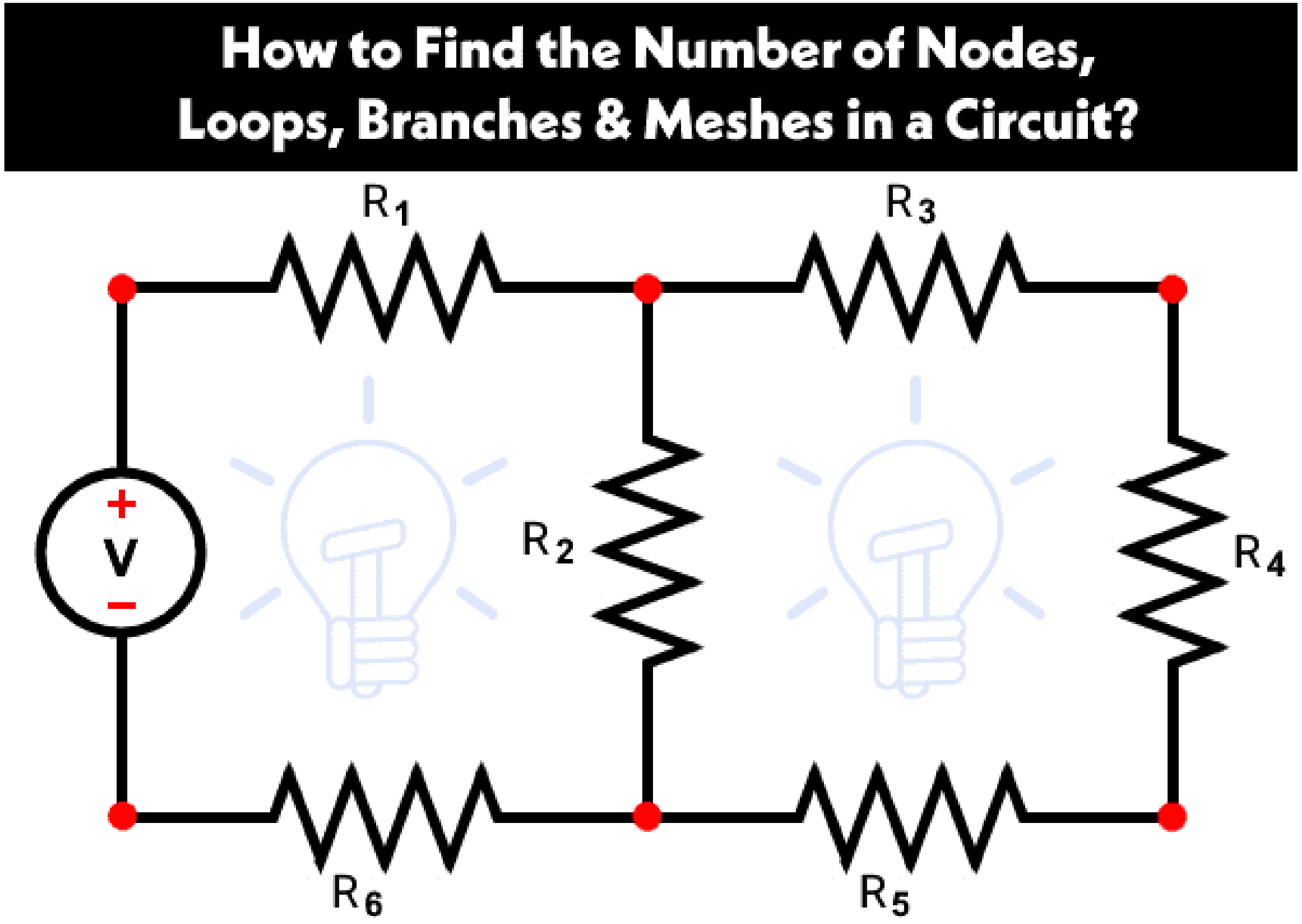
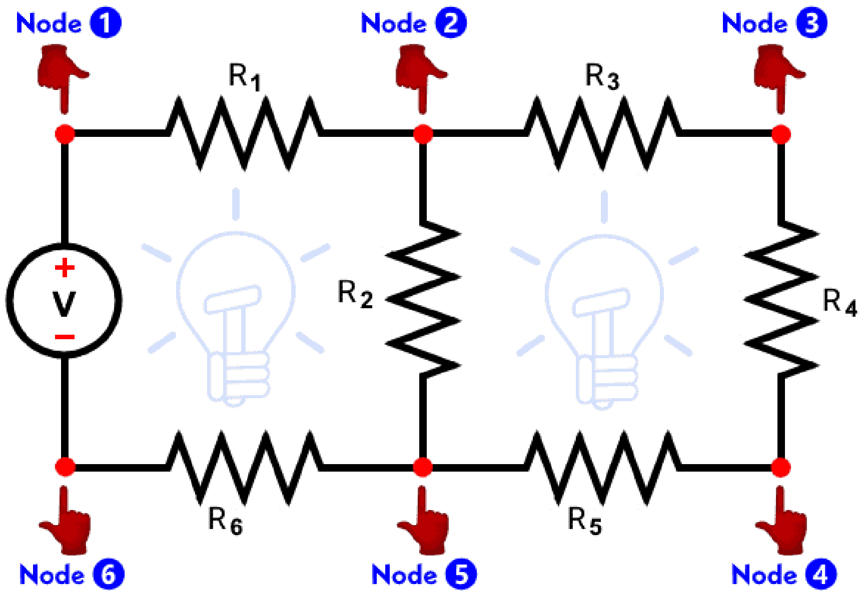



**FIGURE 2.21** (*a*) A network that is not a circuit. (*b*) A network that is a circuit.

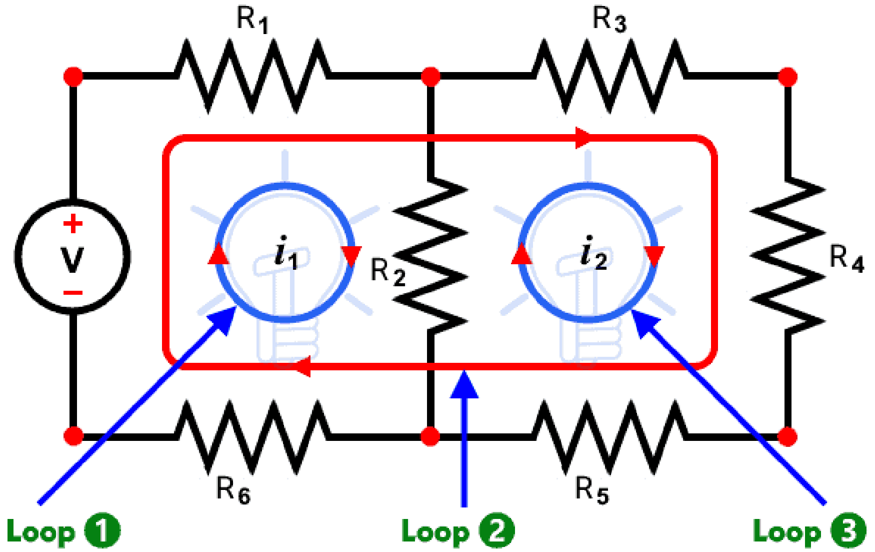
# Networks and Circuits



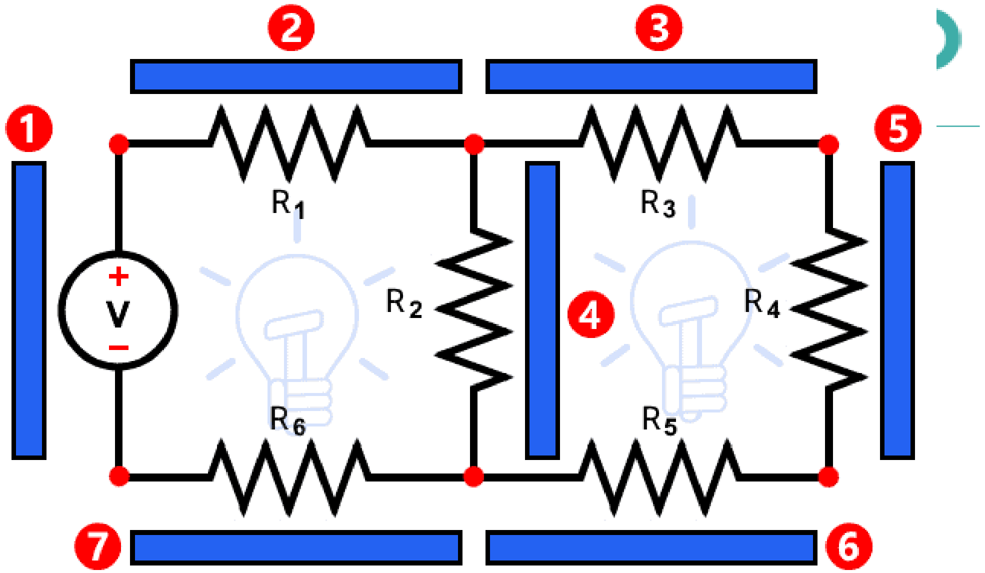
## Finding Nodes in a Circuit



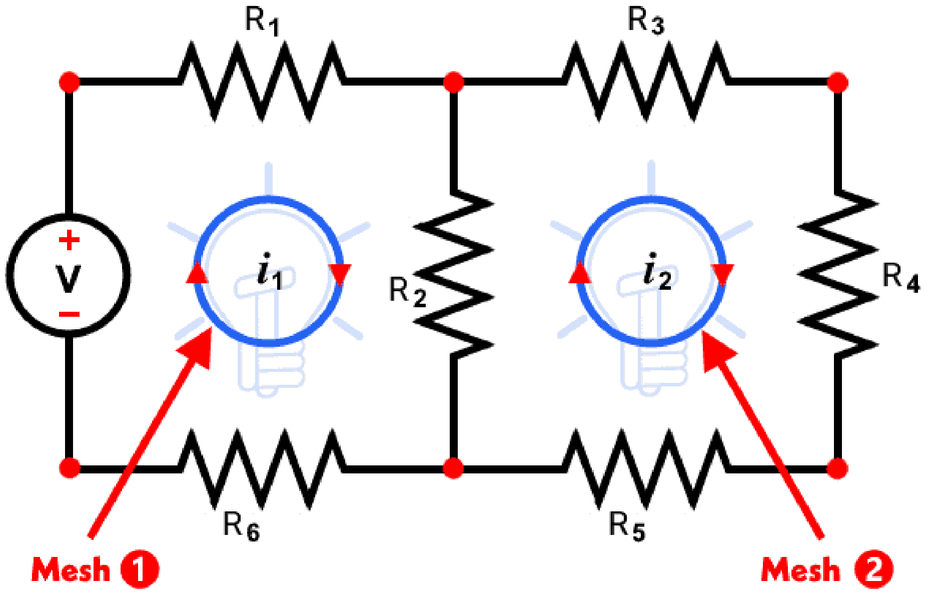

## Finding Loops in a Circuit



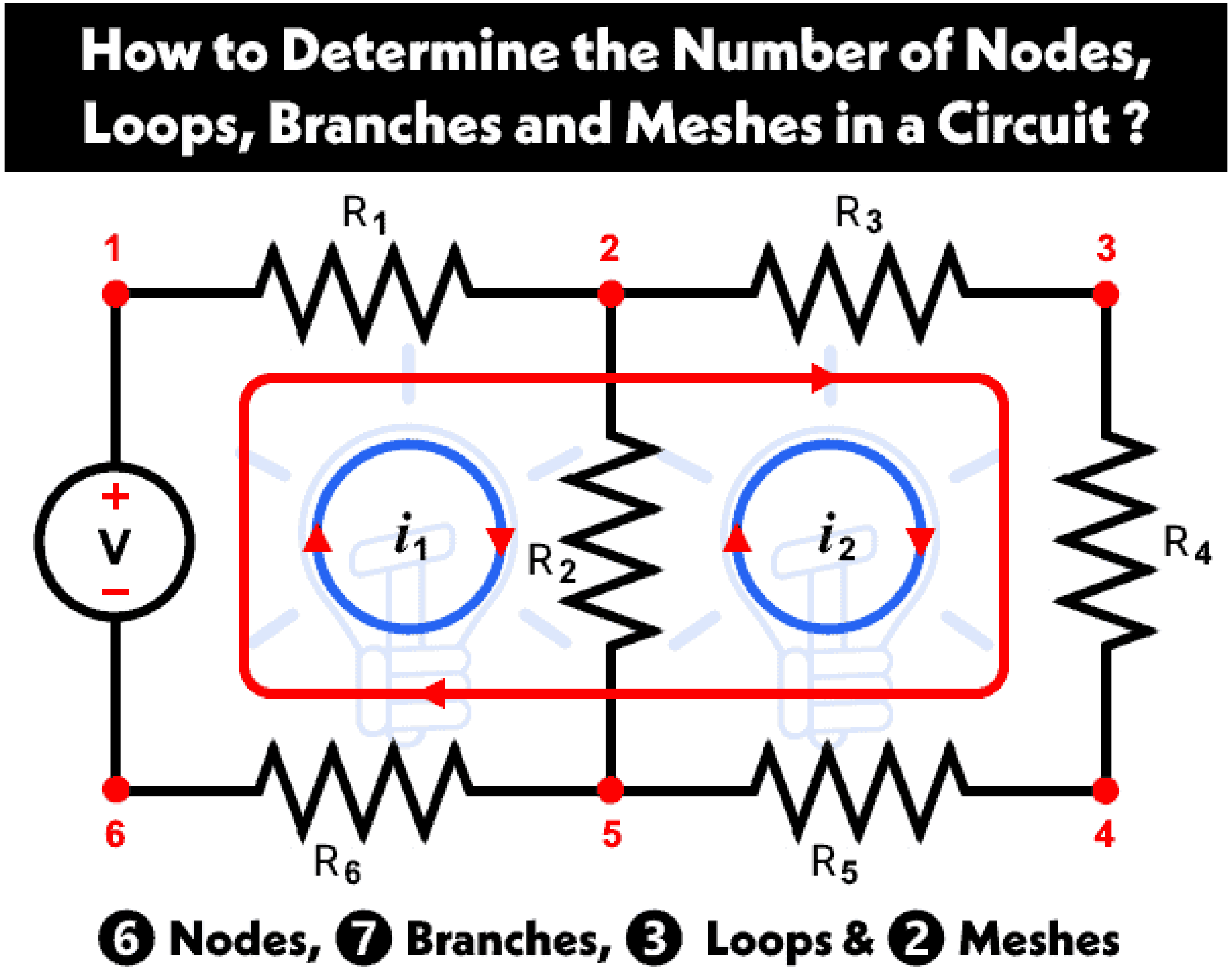

## Finding Branches in a Circuit



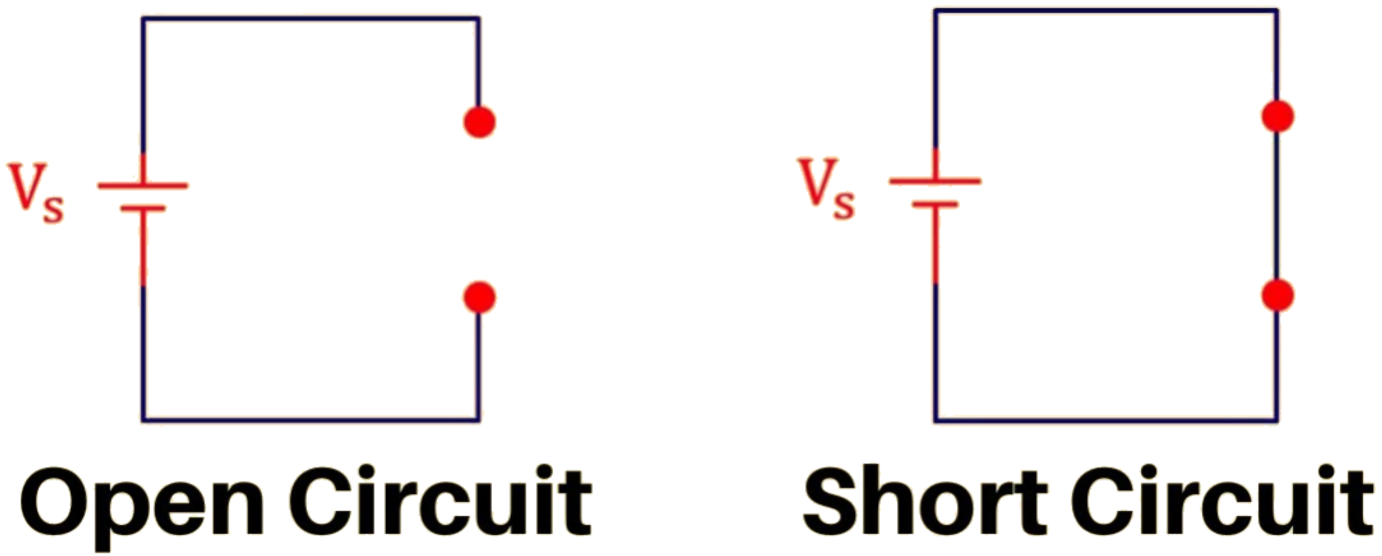

## Finding Meshes in a Circuit



# Networks and Circuits



# Networks and Circuits

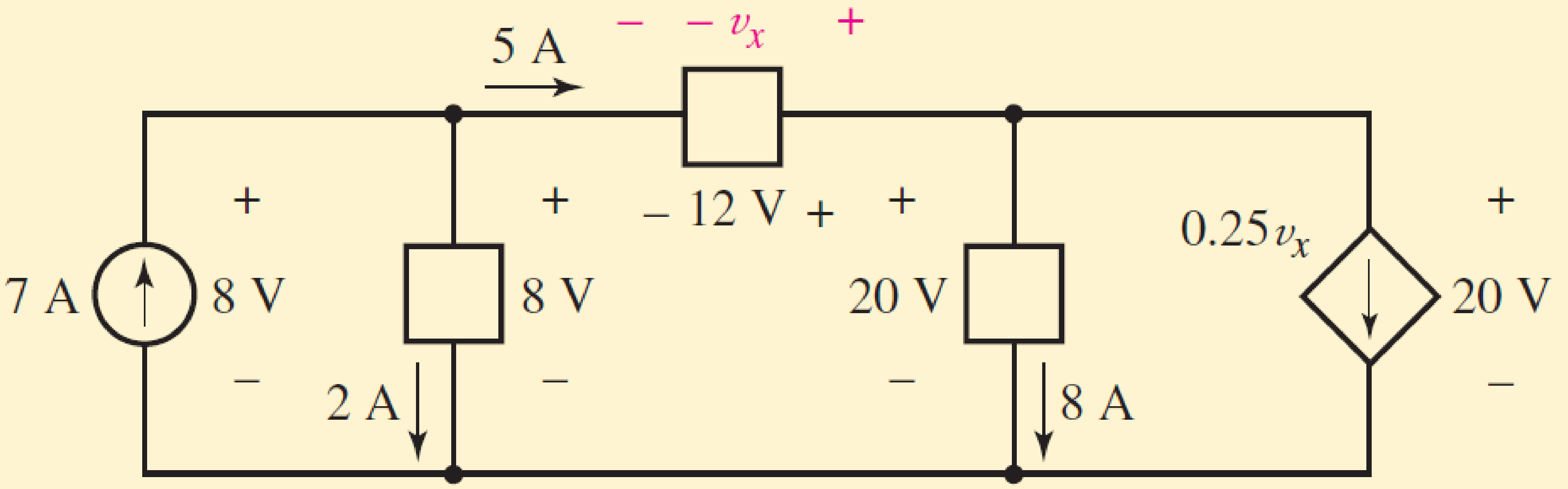



**<u>Open circuit:</u>** If the load connected between any two terminals of any network is made infinite, i.e., the conducting path between the terminals is made open then the terminals are said to be open circuited.

**<u>Short circuit:</u>** If any two terminals of a network are connected by a wire of almost zero resistance then the terminals are said to be short circuited

# Example problem

Find the power *absorbed* by each element in the circuit in Fig.




Ans: (left to right) −56 W; 16 W; −60 W; 160 W; −60 W.

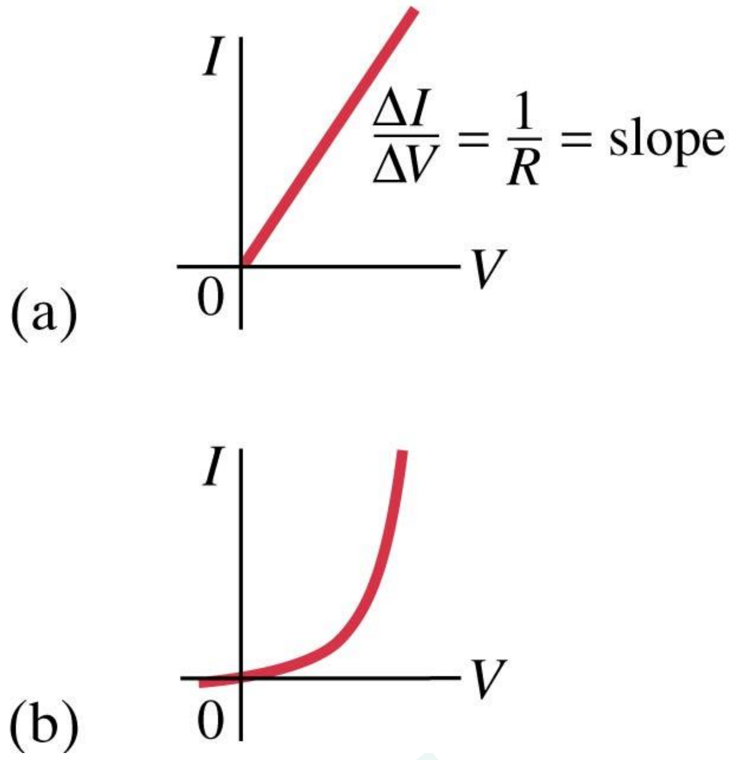
# Ohm's law

- Ohm's law states that the current, ***I*** flowing in a circuit, is directly proportional to the applied voltage ***V*** at constant temperature.

$$I \,\alpha\, V$$

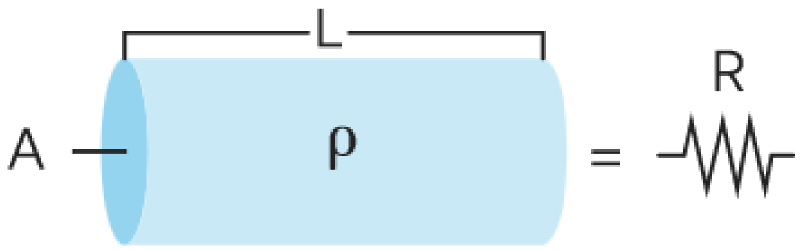
$$I = \frac{V}{R}$$

$$or \quad V = I.R \qquad or \quad R = \frac{V}{I}$$




In many conductors, the resistance is independent of the voltage; this relationship is called Ohm's law. Materials that do not follow Ohm's law are called nonohmic.

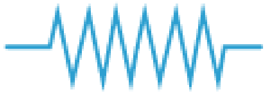
# Resistivity

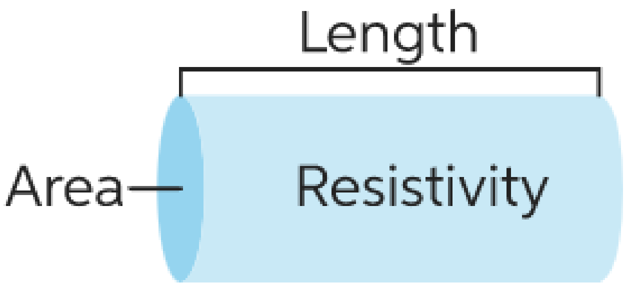



L = length
A = area
ρ = resistivity

$$R = \rho \frac{L}{A}$$

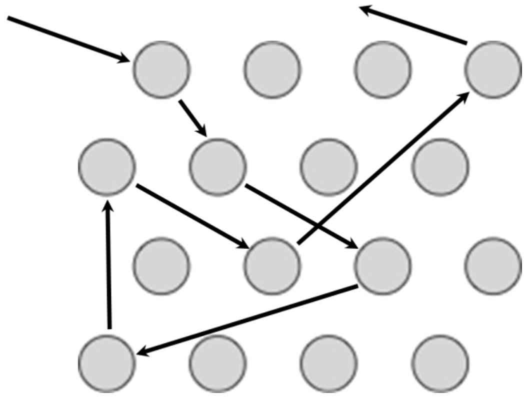
**Resistance Vs Resistivity**





# Drude model (a classical approach of classical conductivity)

**The basic assumptions of the Drude model**



1. between collisions the interaction of a given electron

independent electron approximation
free electron approximation

2. collisions are instantaneous events
Drude considered electron scattering off the impenetrable ion cores

the specific mechanism of the electron scattering is not considered below

3. an electron experiences a collision with a probability per unit time $1/\tau$
$dt/\tau$ – probability to undergo a collision within small time $dt$
randomly picked electron travels for a time $\tau$ before the next collision
$\boldsymbol{\tau}$ is known as the relaxation time, the collision time, or the mean free time
$\boldsymbol{\tau}$ is independent of an electron position and velocity

4. after each collision an electron emerges with a velocity that is randomly directed and with a speed appropriate to the local temperature

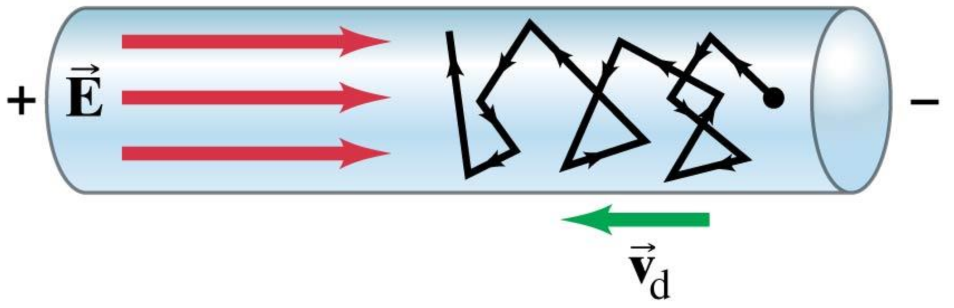
# Drude theory: electrical conductivity

If we apply an electric field, the equation of motion is

$$m_e \frac{d\mathbf{v}}{dt} = -e\mathcal{E}$$

integration gives

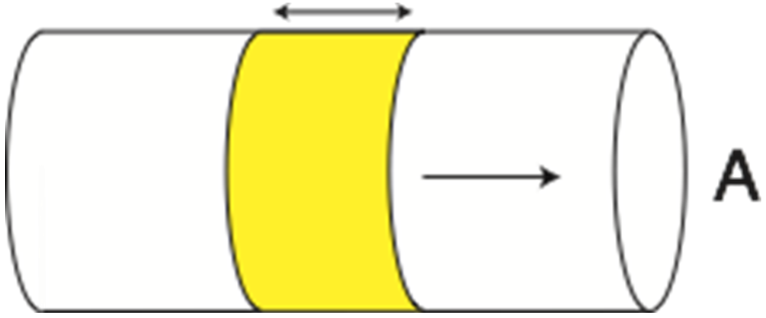
$$\mathbf{v}(t) = \frac{-e\mathcal{E}t}{m_e}$$




and if $\boldsymbol{\tau}$ is the average time between collisions then the average drift speed is

$$\overline{\mathbf{v}} = \frac{-e\mathcal{E}\tau}{m_e}$$

# Drude theory: electrical conductivity




number of electrons passing in unit time

$$n|\bar{\mathbf{v}}|A$$

current of negatively charged electrons

$$-en|\bar{\mathbf{v}}|A$$

current density

$$\mathbf{j} = n\bar{\mathbf{v}}(-e)$$

and with $\bar{\mathbf{v}} = \frac{-e\mathcal{E}\tau}{m_e}$ we get

Ohm's law

$$\mathbf{j} = \frac{ne^2\tau}{m_e}\mathcal{E}$$

# Drude theory: electrical conductivity

Ohm's law

$$\mathbf{j} = \frac{ne^2\tau}{m_e}\boldsymbol{\mathcal{E}}$$

$$\mathbf{j} = \sigma\boldsymbol{\mathcal{E}} = \frac{\boldsymbol{\mathcal{E}}}{\rho}$$

and we can define the conductivity

$$\sigma = \frac{ne^2\tau}{m_e} = n\mu e$$

and the resistivity

$$\rho = \frac{m_e}{ne^2\tau}$$

and the mobility

$$\mu = \frac{e\tau}{m_e}$$

$$|\mathbf{v}| = \mu|\boldsymbol{\mathcal{E}}|$$

# Resistor

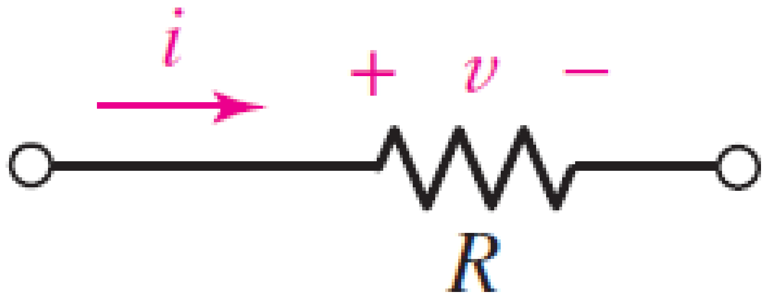
- Resistance: A measure of difficulty to flow electrons through a circuit element
- Can be measured using Ohm's Law

$$V \propto I$$

or

$$V = RI$$

Unit of resistance: Ohm (Ω)



# Resistor

- Relation with power:

$$P = VI = I^2R = \frac{V^2}{R}$$

**EXAMPLE 2.3**

The 560 Ω resistor shown in Fig. 2.24*b* is connected to a circuit which causes a current of 42.4 mA to flow through it. Calculate the voltage across the resistor and the power it is dissipating.

Ans: 23.7 V, 1.003/5/7 W

# Resistor

- Resistivity: Resistance of per unit length per unit area of an uniform conductor

$$R = \frac{\rho L}{A}$$

- Conductance:

$$G = \frac{1}{R} = \frac{I}{V}$$

Power relation with conductance:

$$P = \frac{I^2}{G} = V^2 G$$

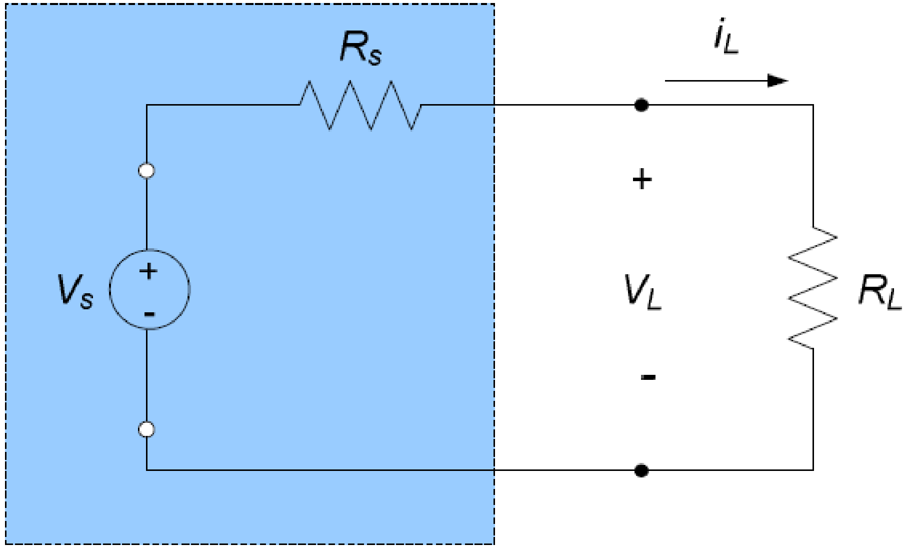
# Example

**EXAMPLE 2.4**

A dc power link is to be made between two islands separated by a distance of 24 miles. The operating voltage is 500 kV and the system capacity is 600 MW. Calculate the maximum dc current flow, and estimate the resistivity of the cable, assuming a diameter of 2.5 cm and a solid (not stranded) wire.

Ans: 1200 A, R = 417 Ohm,  area = 4.9 cm^2, length = 3.9*10^6 cm, rho = 520 mu Ohm. cm

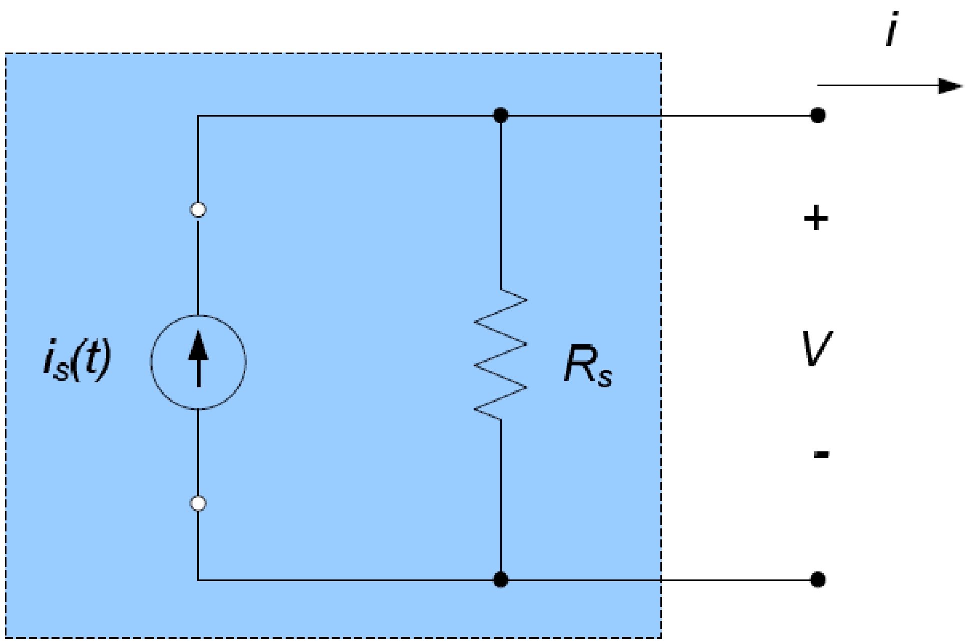
# Non-ideal Voltage Source




- Modelled as an ideal voltage source and internal resistance
- Implication:
  - When a load draws large current, supply voltage decreases and the converse
- Does not provide infinite power
- Note: Ideal voltage source means a source with zero internal resistance

# Non-ideal current source

- Modelled as an ideal current source and internal resistance in parallel
- Implication:
  - Higher terminal voltage results in lower supply current
- Note: Ideal current source means a source with infinite internal resistance

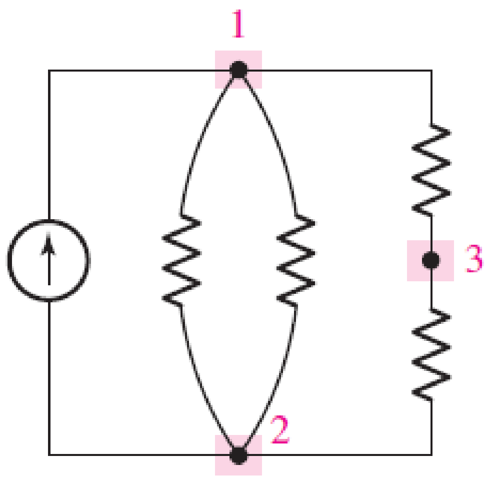


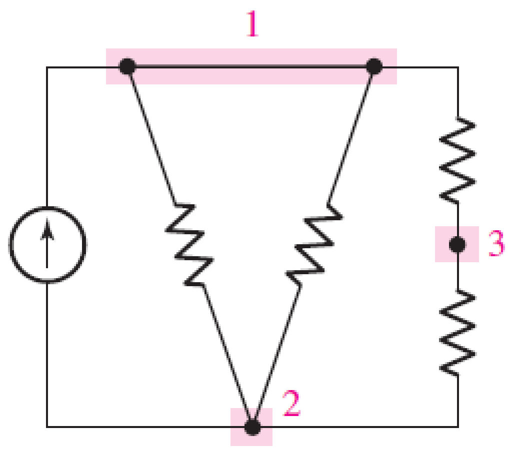
# Nodes and loops

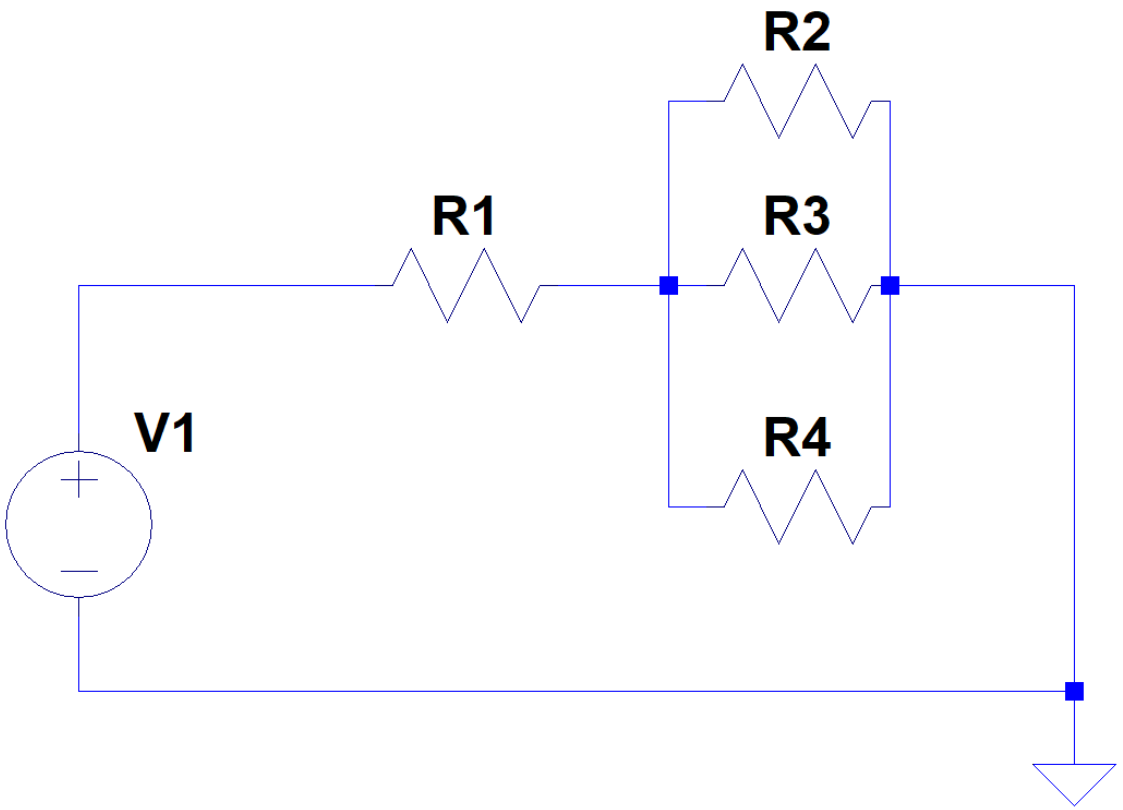
- **<u>Node:</u>** A point at which two or more circuit elements have a common connection

- **<u>Loop:</u>** If a set of nodes and elements are traversed in such a way that *no node is encountered more than once, then the set of nodes and elements are called a <u>path</u>*. If the start and end point of a path is the same node, then it is called a loop.

# Node

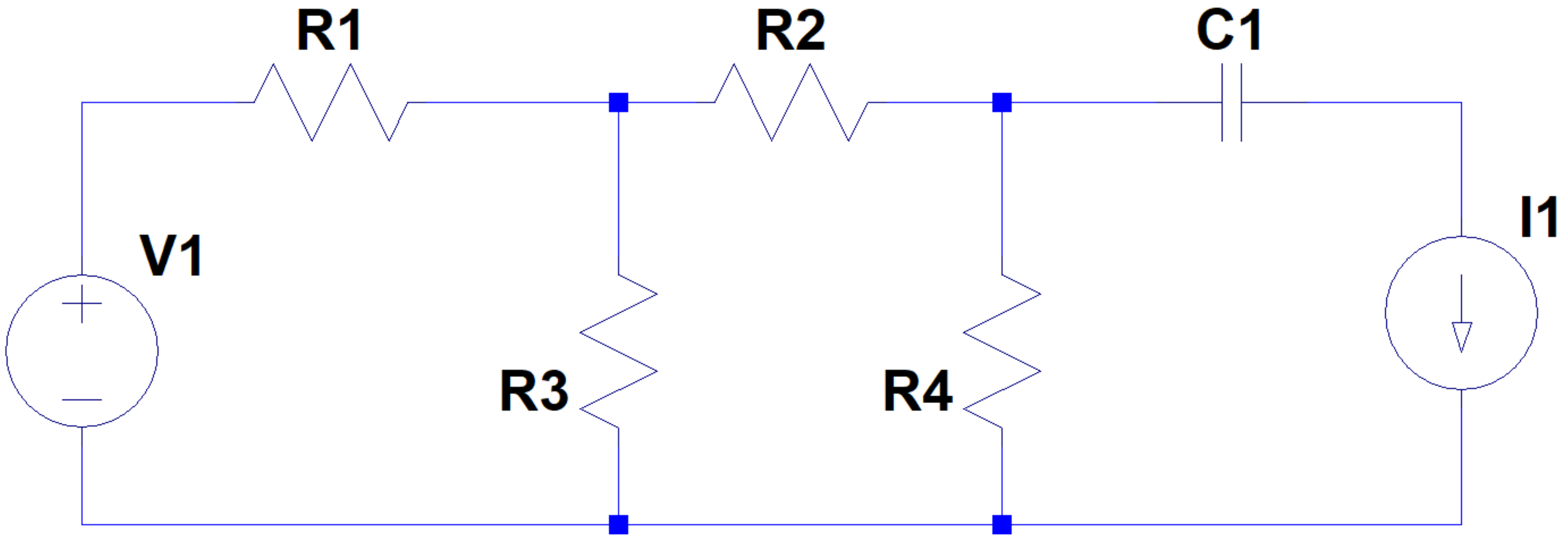



How many nodes are there in the above circuit?

# Loop



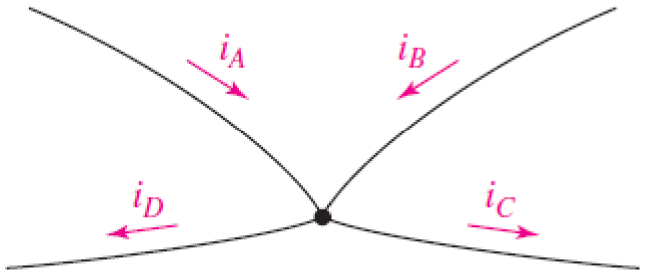
# Kirchhoff’s Laws

- Proposed by Gustav Kirchhoff [keerkh-hawf]
- Kirchhoff’s Current Law (KCL): tells us the relationship of currents at a node
- Kirchhoff’s Voltage Law (KVL): Tells us the relationship of voltages in a loop

# Kirchhoff’s Current Law

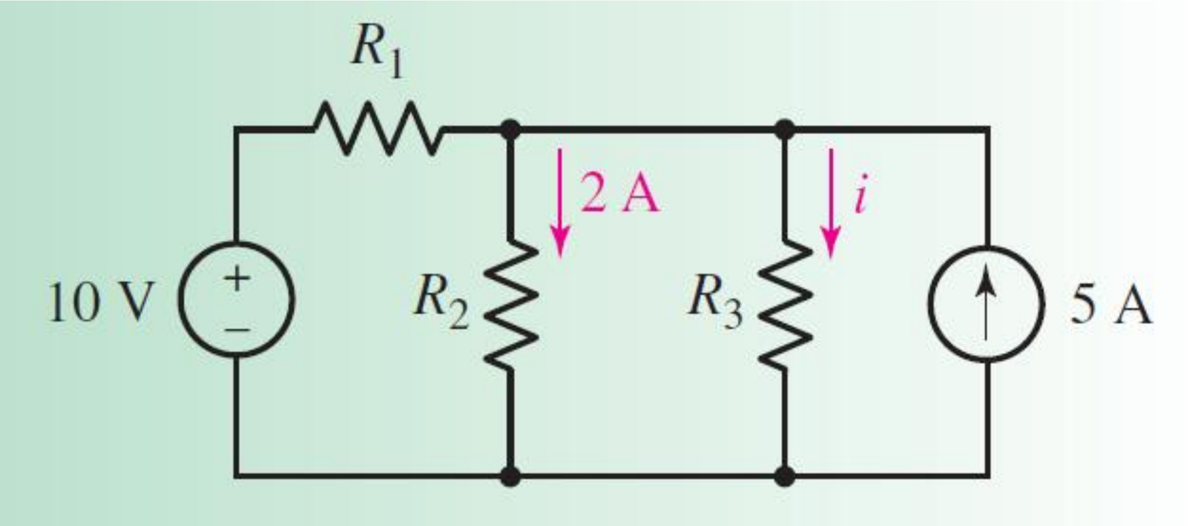
- The algebraic sum of the currents entering any node is zero
- Implication: current entering a node is equal to the current exiting the node
- Conservation of charge implies KCL

$$\sum_{n=1}^{N} i_n = 0$$




- **Current entering +ve, exiting -ve**

# Example 1




For the circuit in the Fig, compute the current through resistor R3 if it is known that the voltage source supplies a current of 3 A.

Ans: 6 A

▶ ***Identify the goal of the problem.***

The current through resistor $R_3$, labeled as $i$ on the circuit diagram.

▶ ***Collect the known information.***

The node at the top of $R_3$ is connected to four branches.

Two of these currents are clearly labeled: 2 A flows out of the node into $R_2$, and 5 A flows into the node from the current source. We are told the current out of the 10 V source is 3 A.

▶ ***Devise a plan.***

If we label the current through $R_1$ (Fig. 3.3*b*), we may write a KCL equation at the top node of resistors $R_2$ and $R_3$.

▶ ***Construct an appropriate set of equations.***

Summing the currents flowing into the node:

$$i_{R_1} - 2 - i + 5 = 0$$

The currents flowing into this node are shown in the expanded diagram of Fig. 3.3*c* for clarity.

▶ ***Determine if additional information is required.***

We have one equation but two unknowns, which means we need to obtain an additional equation. At this point, the fact that we know the 10 V source is supplying 3 A comes in handy: KCL shows us that this is also the current $i_{R_1}$.

▶ ***Attempt a solution.***

Substituting, we find that $i = 3 - 2 + 5 = 6$ A.

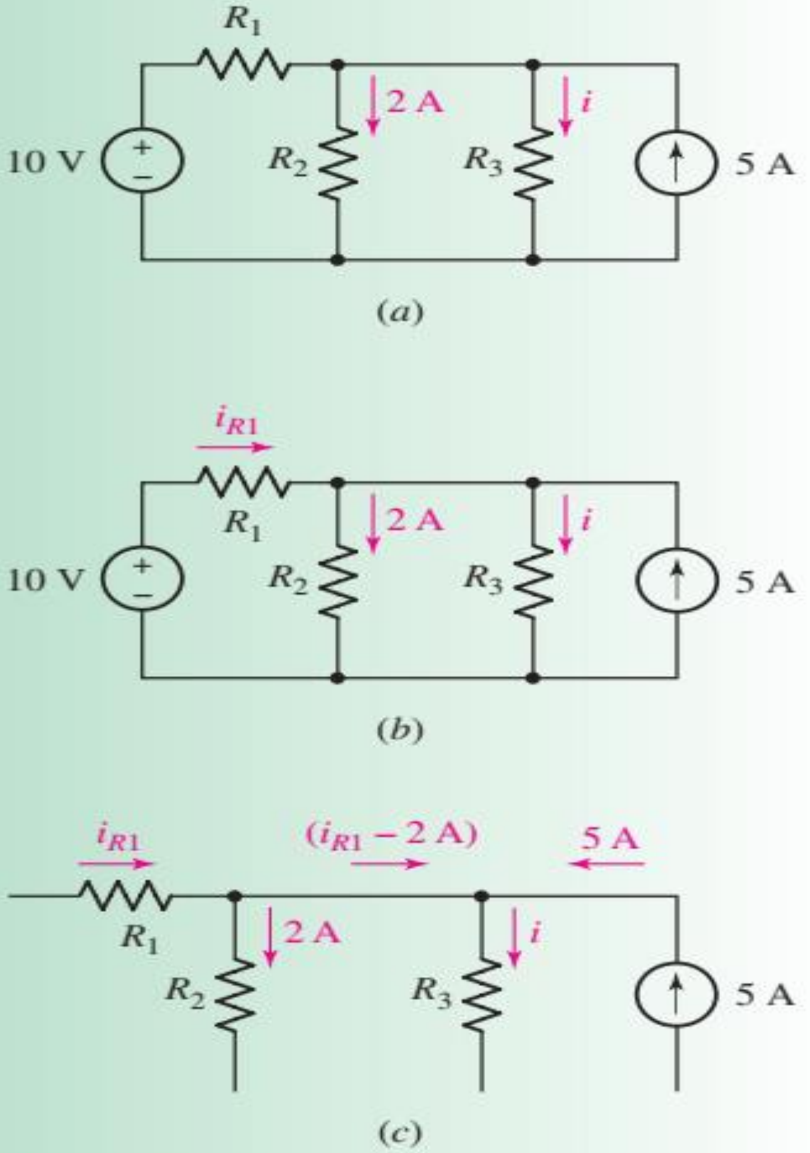
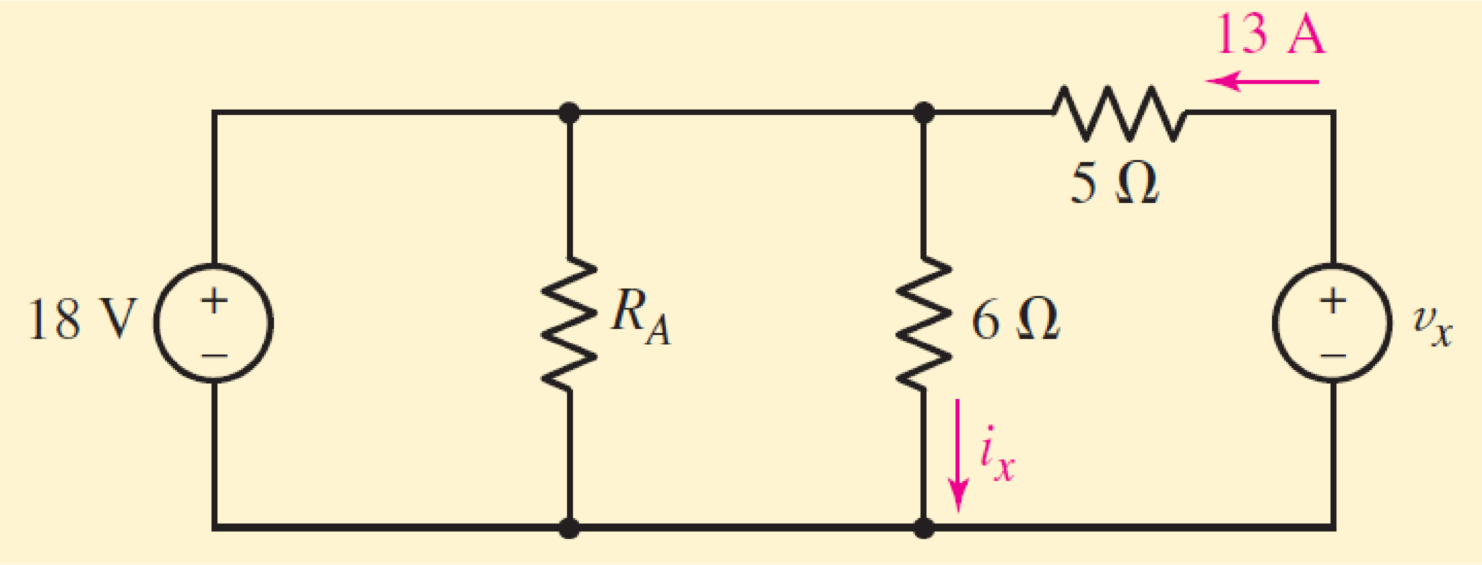



**FIGURE 3.3** (*a*) Simple circuit for which the current through resistor $R_3$ is desired. (*b*) The current through resistor $R_1$ is labeled so that a KCL equation can be written. (*c*) The currents into the top node of $R_3$ are redrawn for clarity.

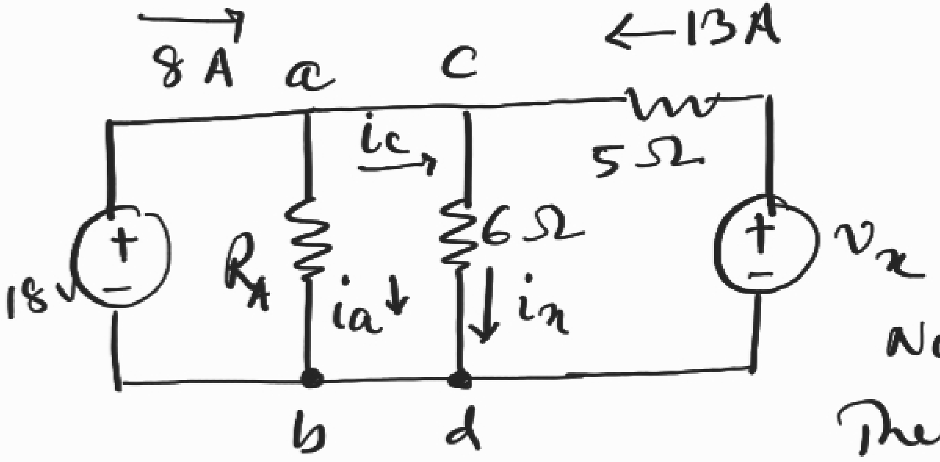
# Example 2

If $i_x$ = 3 A and the 18 V source delivers 8 A of current, what is the value of $R_A$?




Ans: 1 ohm

# Example 2 solution




given $i_x = 3A$

voltage at ab = 18 V

Now if we use KCL at a

Then we can write

$$8 = i_a + i_c$$

$$\therefore i_c = 8 - i_a$$

Now current at c;

$$i_c + 13 = i_x$$

$$\Rightarrow 8 - i_a + 13 = 3 \Rightarrow i_a = 18$$

Using Ohm's law, $V_{ab} = R_A \cdot i_a \quad \therefore R_A = \frac{18V}{18A} = 1\Omega$

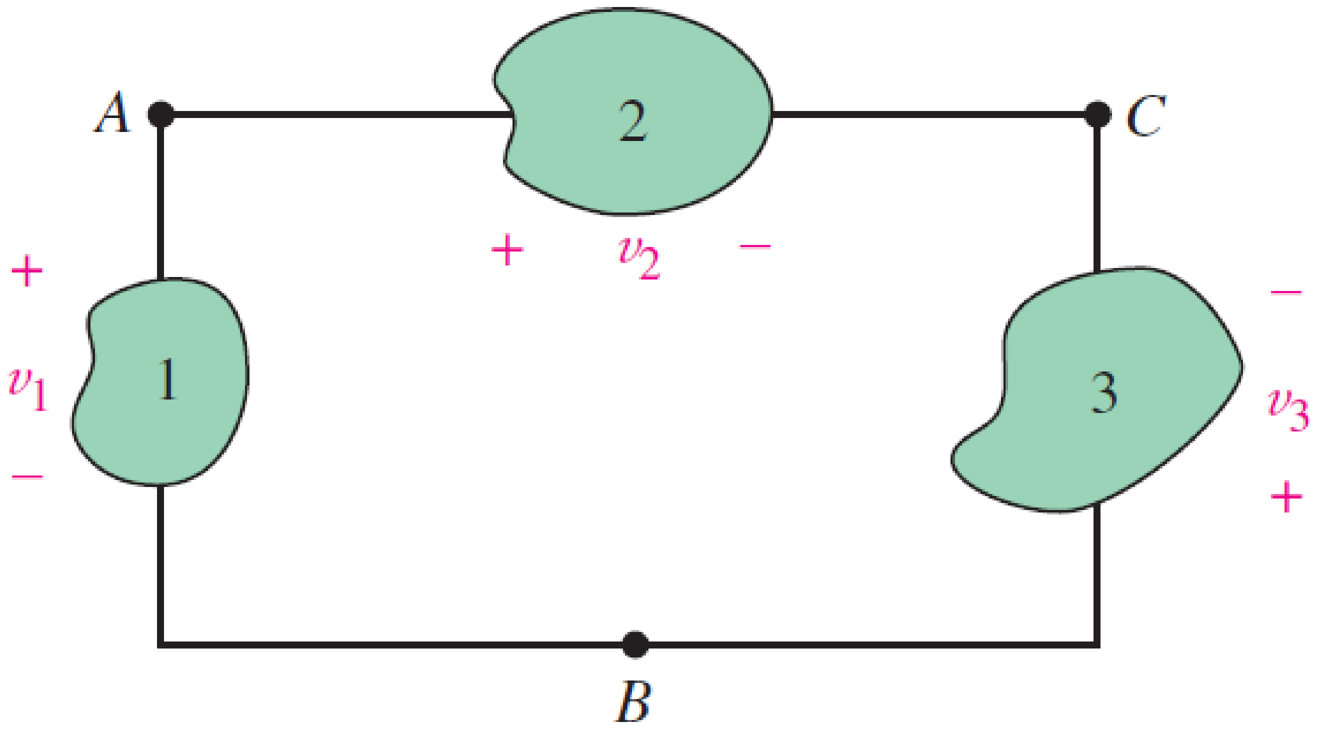
# Kirchhoff's Voltage Law

- The algebraic sum of the voltages around any closed path is zero

$$\sum_{n=1}^{N} v_n = 0$$

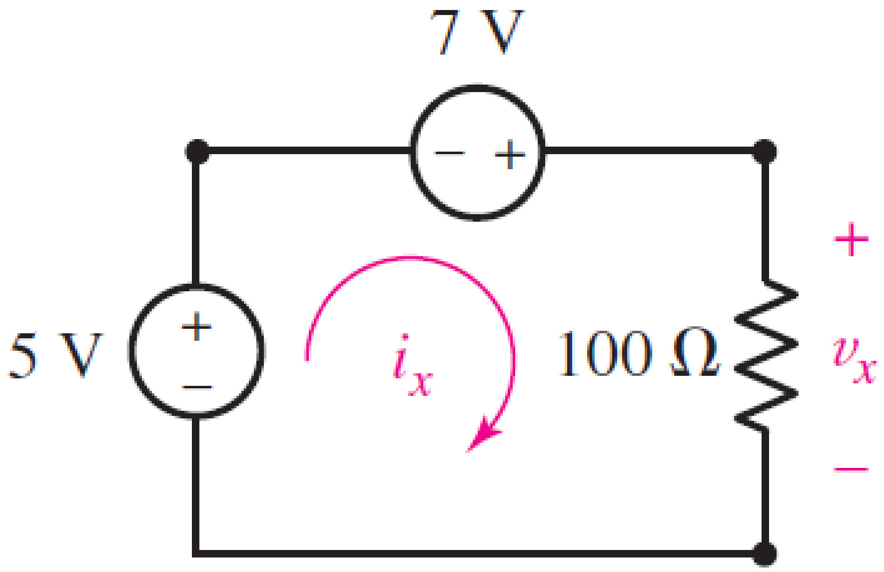
- Convention: sign should be –ve if a –ve side of an element is encountered

# Kirchhoff's Voltage Law



# Example 3

In the circuit, find $v_x$ and $i_x$.




Ans: 12V ,120 mA

# Example 3 solution

We know the voltage across two of the three elements in the circuit. Thus, KVL can be applied immediately to obtain $v_x$.

Beginning with the bottom node of the 5 V source, we apply KVL clockwise around the loop:

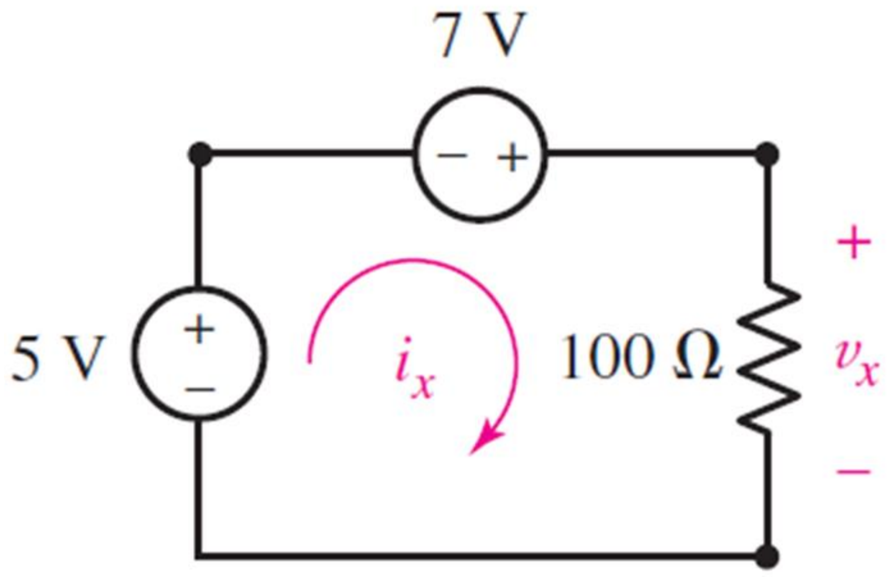
$$-5 - 7 + v_x = 0$$

so $v_x = 12$ V.

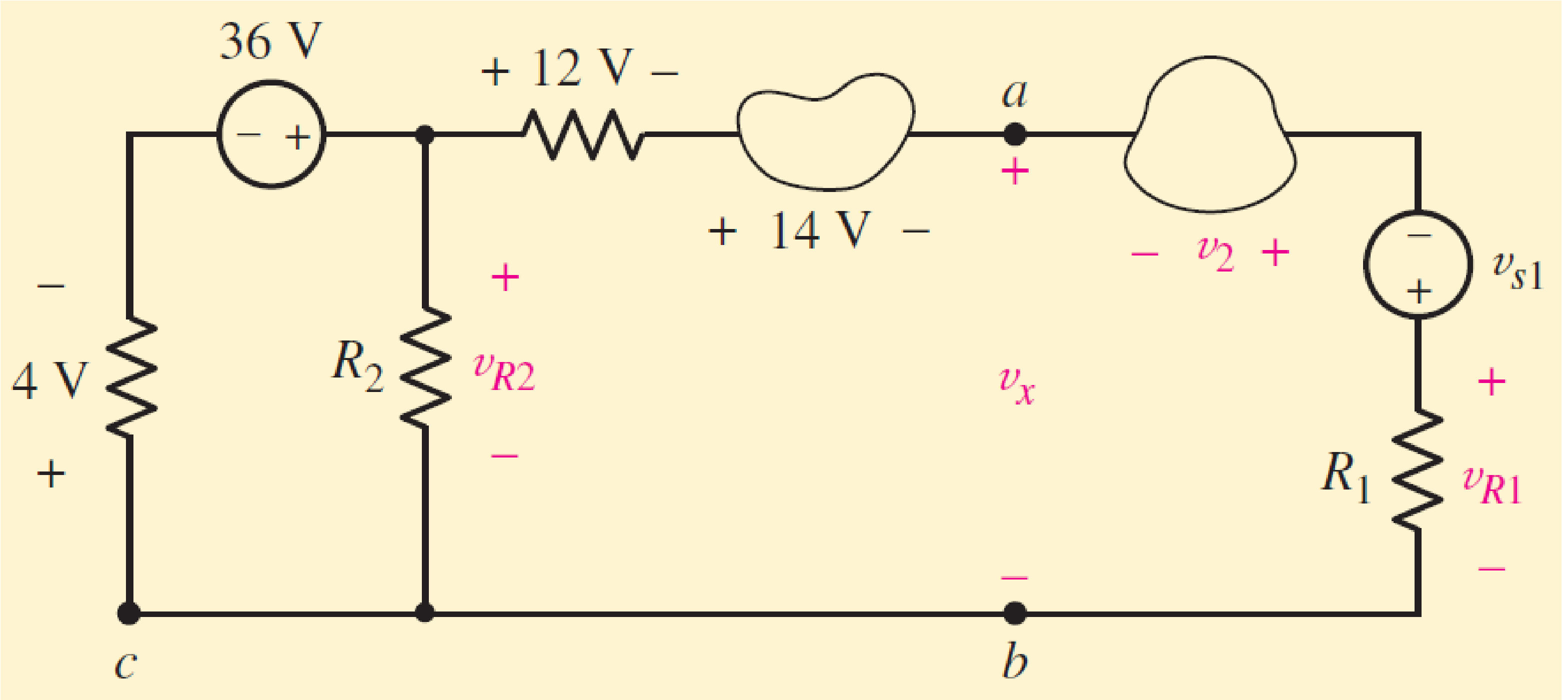
KCL applies to this circuit, but only tells us that the same current ($i_x$) flows through all three elements. We now know the voltage across the 100 Ω resistor, however.

Invoking Ohm's law,

$$i_x = \frac{v_x}{100} = \frac{12}{100} \text{ A} = 120 \text{ mA}$$



# Example 4




Find $v_{R2}$ (the voltage across R2) and the voltage labeled $v_x$.

Ans: 32 V, 6 V

# Example 4 solution

The best approach for finding $v_{R2}$ is to look for a loop to which we can apply KVL. There are several options, but the leftmost loop offers a straightforward route, as two of the voltages are clearly specified. Thus, we find $v_{R2}$ by writing a KVL equation around the loop on the left, starting at point $c$:

$$4 - 36 + v_{R2} = 0$$

which leads to $v_{R2} = 32$ V.

To find $v_x$, we might think of this as the (algebraic) sum of the voltages across the three elements on the right. However, since we do not have values for these quantities, such an approach would not lead to a numerical answer. Instead, we apply KVL beginning at point $c$, moving up and across the top to $a$, through $v_x$ to $b$, and through the conducting lead to the starting point:

$$+4 - 36 + 12 + 14 + v_x = 0$$

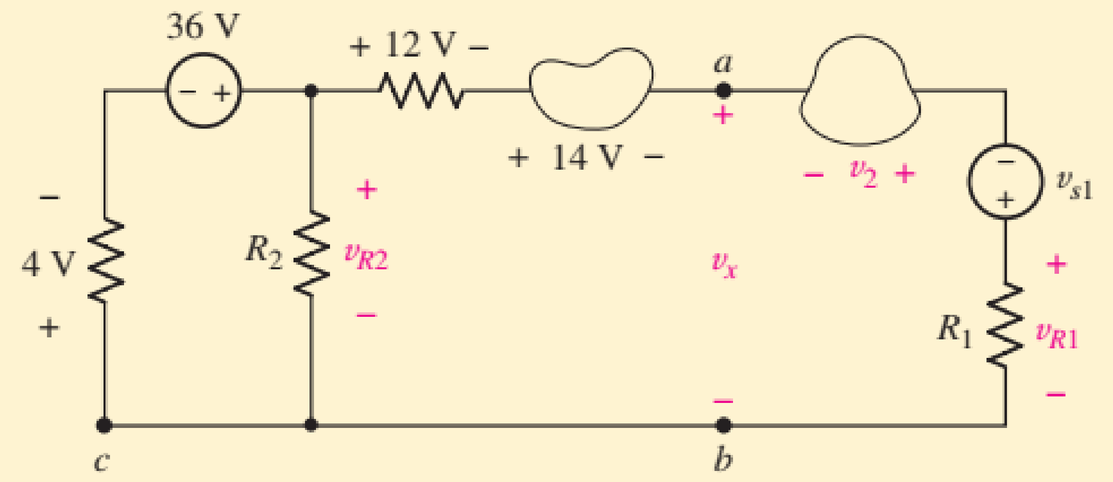
so that

$$v_x = 6 \text{ V}$$

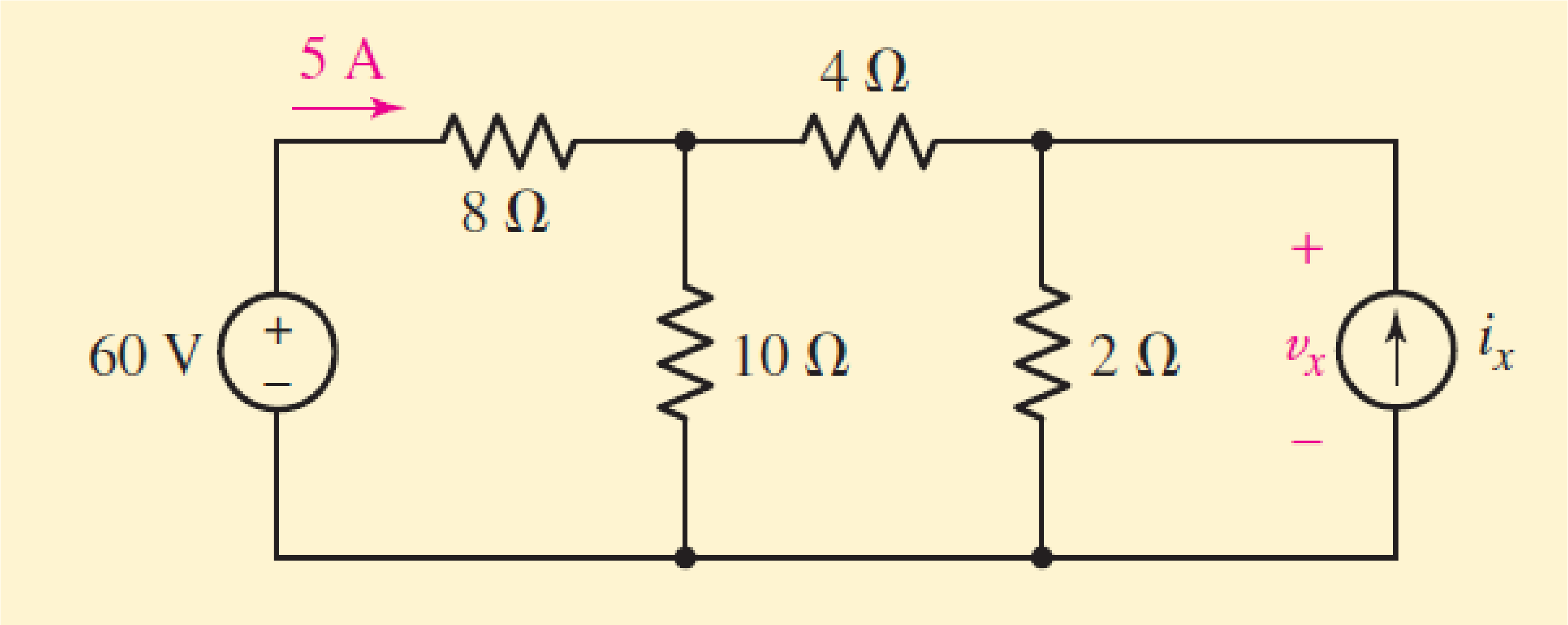
***An alternative approach:*** Knowing $v_{R2}$, we might have taken the shortcut through $R_2$:

$$-32 + 12 + 14 + v_x = 0$$

yielding $v_x = 6$ V once again.



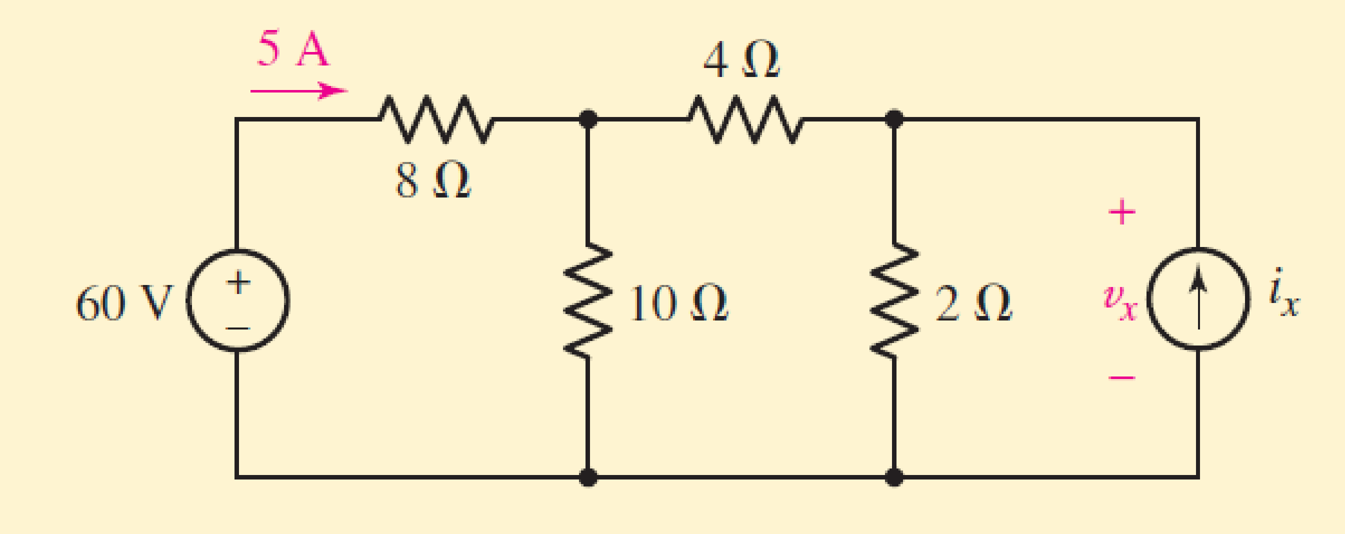
# Example 5

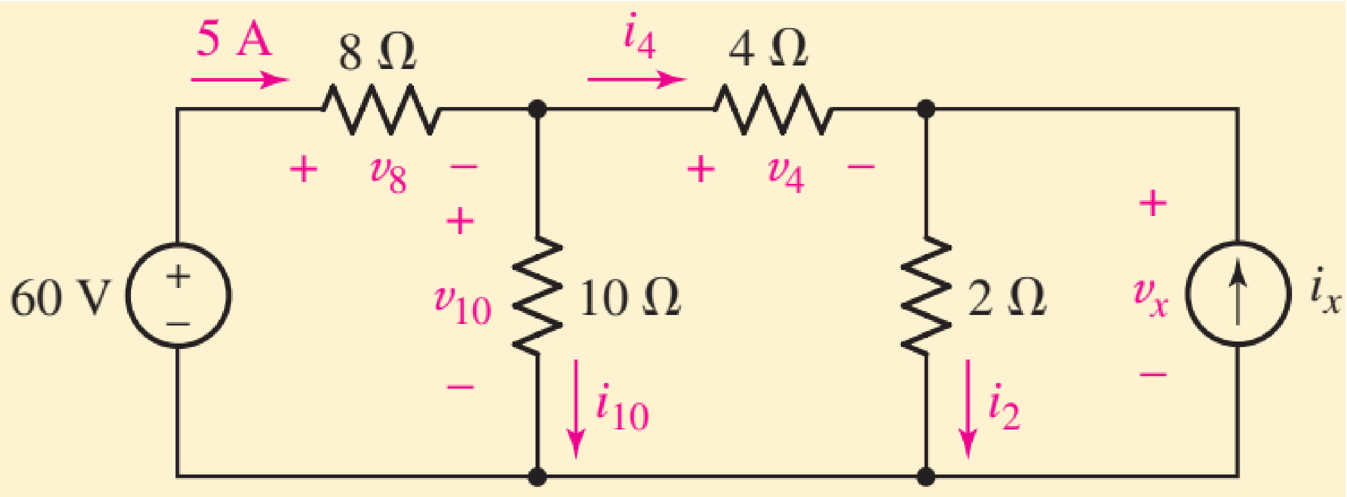



Determine $v_x$

Ans: 8V

# Example 5 solution




We begin by labeling voltages and currents on the rest of the elements in the circuit (Fig. 3.10*b*). Note that $v_x$ appears across the 2 Ω resistor and the source $i_x$ as well.

If we can obtain the current through the 2 Ω resistor, Ohm's law will yield $v_x$. Writing the appropriate KCL equation, we see that

$$i_2 = i_4 + i_x$$

Unfortunately, we do not have values for any of these three quantities. Our solution has (temporarily) stalled.

Since we were given the current flowing from the 60 V source, perhaps we should consider starting from that side of the circuit. Instead of finding $v_x$ using $i_2$, it might be possible to find $v_x$ directly using KVL. We can write the following KVL equations:

$$-60 + v_8 + v_{10} = 0$$

and

$$-v_{10} + v_4 + v_x = 0 \qquad [5]$$

This is progress: we now have two equations in four unknowns, an improvement over one equation in which *all* terms were unknown. In fact, we know that $v_8 = 40$ V through Ohm's law, as we were told that 5 A flows through the 8 Ω resistor. Thus, $v_{10} = 0 + 60 - 40 = 20$ V, so Eq. [5] reduces to

$$v_x = 20 - v_4$$

If we can determine $v_4$, the problem is solved.

The best route to finding a numerical value for the voltage $v_4$ in this case is to employ Ohm's law, which requires a value for $i_4$. From KCL, we see that

$$i_4 = 5 - i_{10} = 5 - \frac{v_{10}}{10} = 5 - \frac{20}{10} = 3$$

so that $v_4 = (4)(3) = 12$ V and hence $v_x = 20 - 12 = 8$ V.

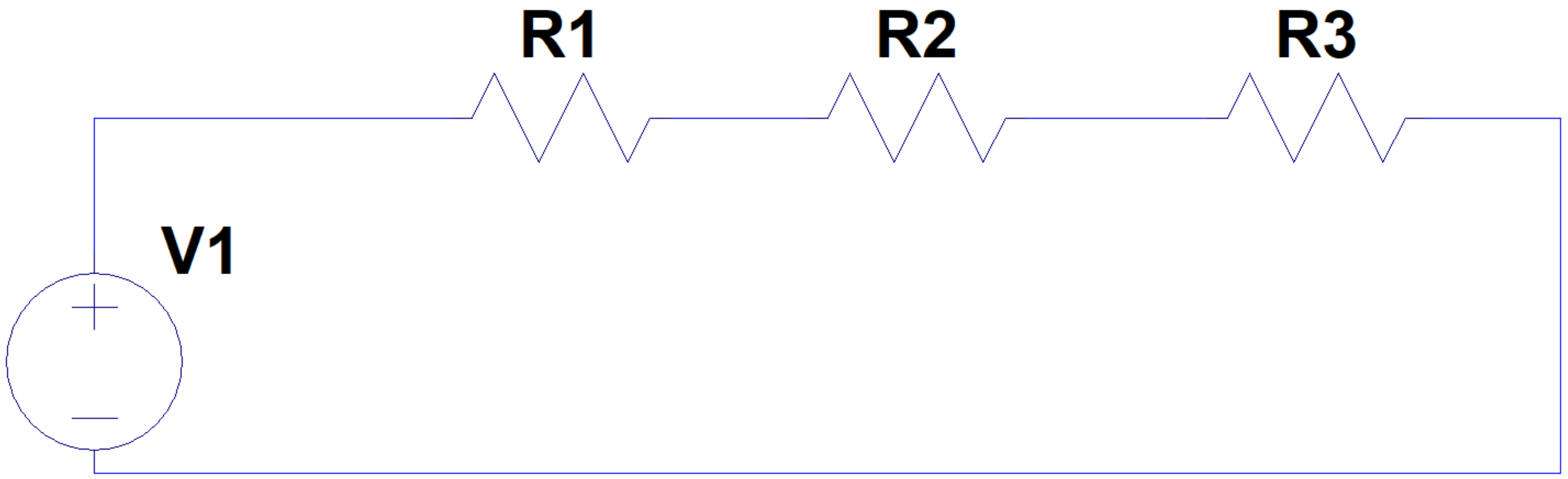
# Reference

- Hayt and Kemmerly, Chapter 3

# Series and parallel circuits

Series circuit

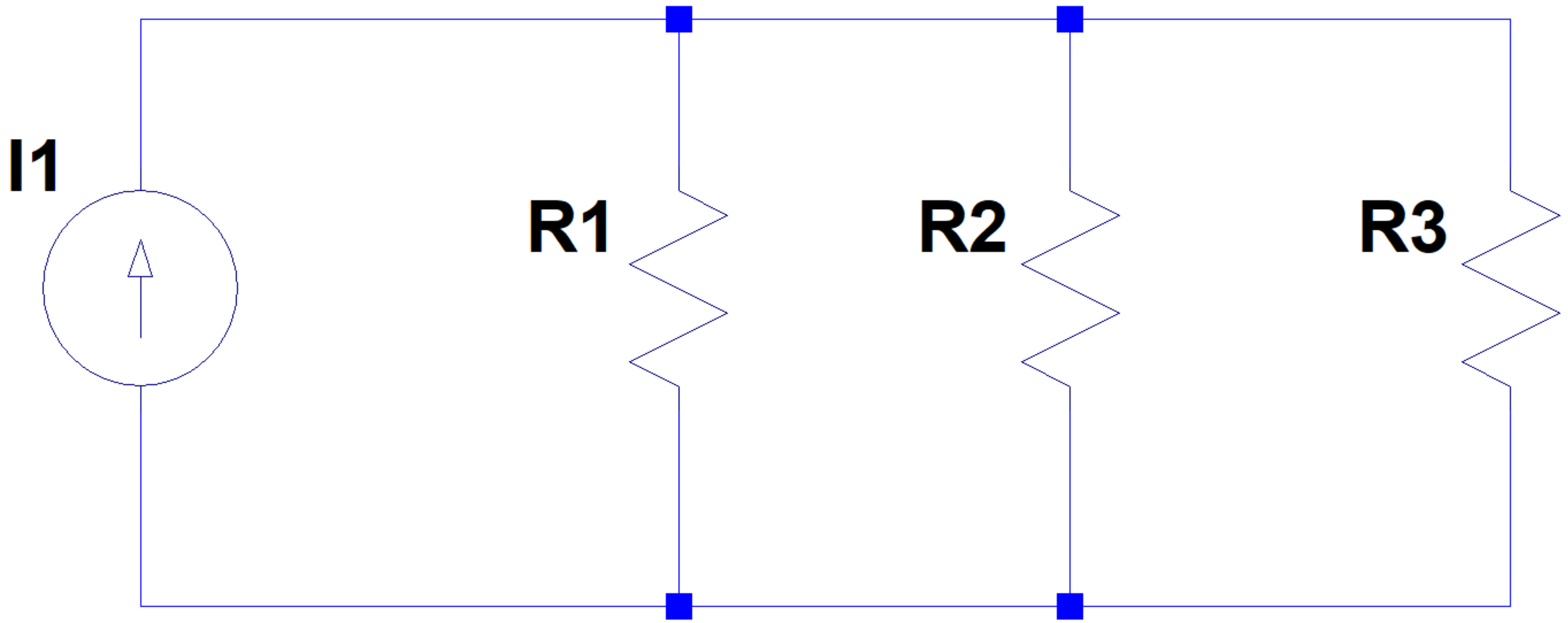
$$v_1 = (R_1 + R_2 + R_3)i$$



# Series and parallel circuits

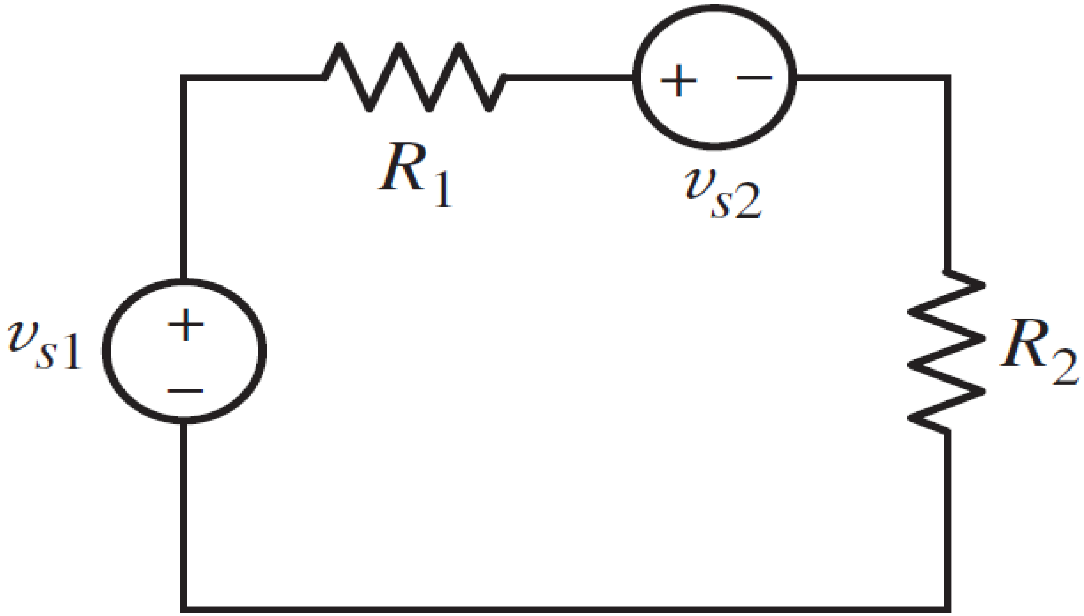
Parallel circuit

$$i_1 = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$$



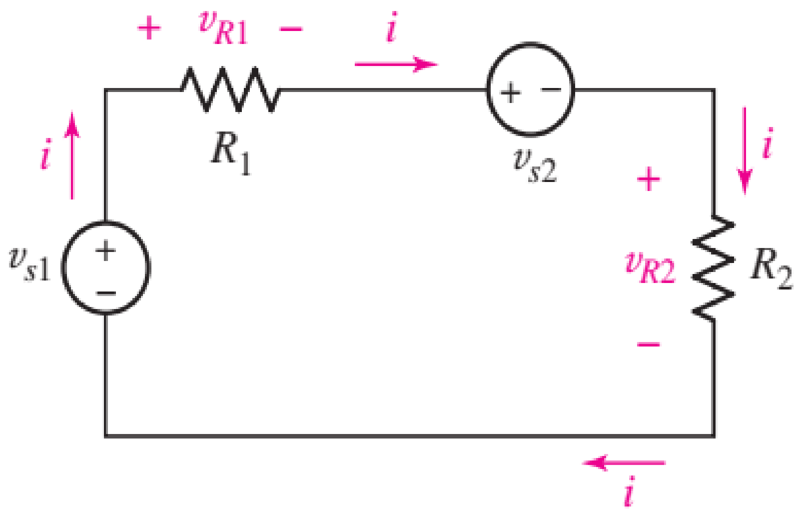
# Single loop circuit

We seek the current *through* each element, the voltage *across* each element, and the power *absorbed* by each element




Current through the circuit

$$i = \frac{v_{s1} - v_{s2}}{R_1 + R_2}$$




$$-v_{s1} + v_{R1} + v_{s2} + v_{R2} = 0$$

We then apply Ohm's law to the resistive elements:

$$v_{R1} = R_1 i \quad \text{and} \quad v_{R2} = R_2 i$$

Substituting into Eq. [6] yields

$$-v_{s1} + R_1 i + v_{s2} + R_2 i = 0$$

Since $i$ is the only unknown, we find that

$$i = \frac{v_{s1} - v_{s2}}{R_1 + R_2}$$

The voltage or power associated with any element may now be obtained by applying $v = Ri$, $p = vi$, or $p = i^2 R$.