Discrete Random Variables

Chapter 2 in the book by RY

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Random Variables

- Our Experiment contained
  - Procedure
  - Observation
  - Model

- Sample space
  - The set of all possible outcomes

- Examples
  - {Excellent, Good, Fair, Poor}
  - {Heads, Tails}
  - {A+, A, A-, B, B-, C, C-, D, F}

Random Variables

- Our Experiment contained
  - Procedure
  - Observation
  - Model
- Under the model we assigned probabilities to outcomes in the sample space S

Random Variables

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- Under the model we assigned probabilities to outcomes in the sample space S

- Now we will assign numbers to outcomes in S
  - How should we?

Random Variables

- Now we will assign numbers to outcomes in S
  - How should we?
- We have a sample space
- In addition, define a range space
  - Range space is a set of numbers
- An experiment must result in exactly one number in the range space

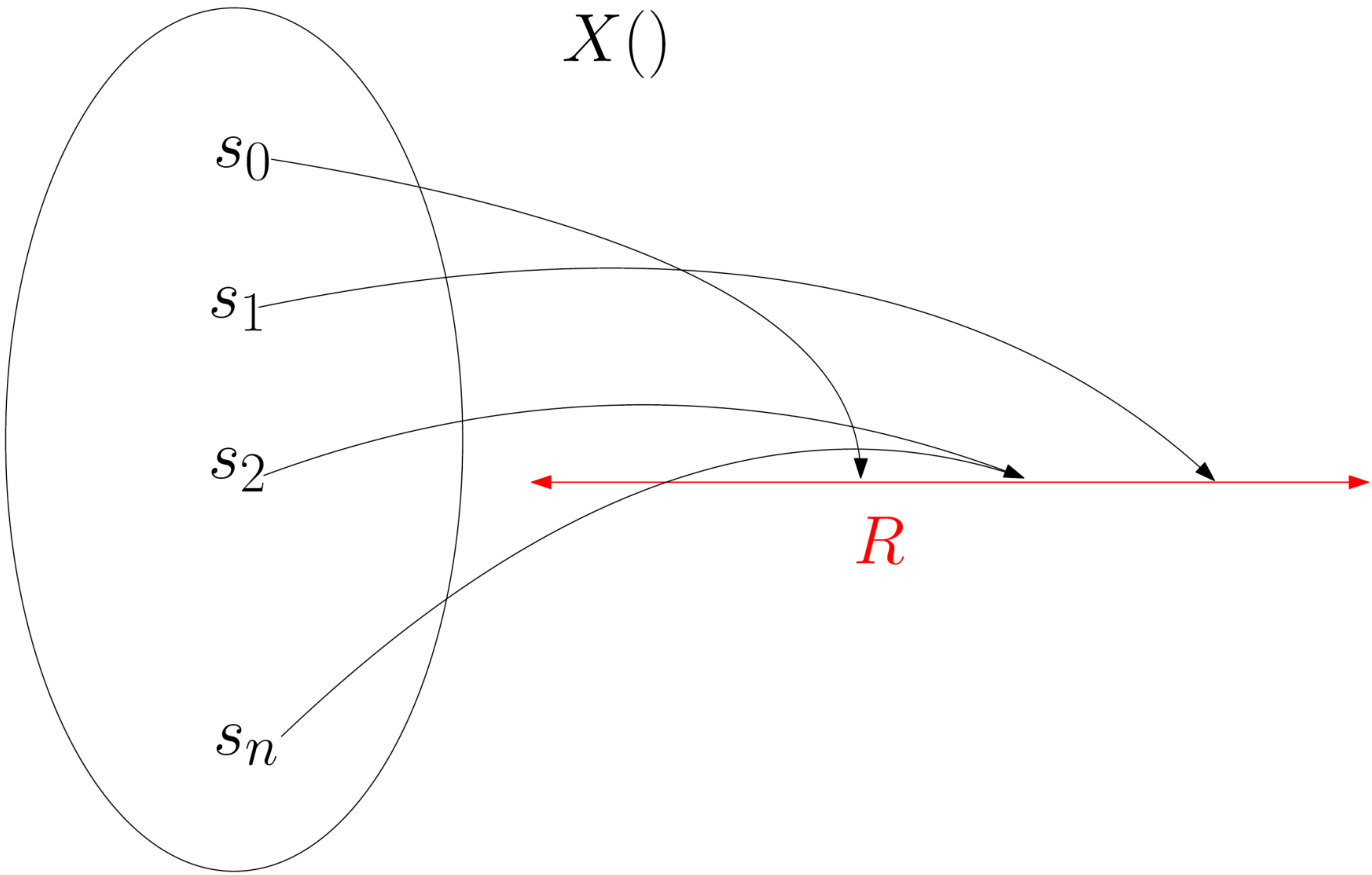
Random Variable

- An experiment must result in exactly one number in the range space

Random Variable

- An experiment must result in exactly one number in the range space
- A Random Variable is **a function** and maps outcomes in the sample space to numbers in a range space

Random Variable



Random Variable

- Coin Tossing Experiment
  - Procedure: Toss a coin once
  - Observation: Heads (1) or Tails (0)
    - Absence/Presence; Pass/Fail; Success/Failure; Accept/Reject; Rich/Poor; Male/Female; Infected/Healthy
    - You are happy to make Boolean observations! That is good enough for your experiment
  - Model: P[0] = 0.5, P[1] = 0.5
  - Our Random Variable, say X(.), takes values of 0 and 1

Random Variable

- **Some Notation**

Random variable X has a range $S_X = \{0, 1\}$.

Range of X is the set of possible values of X

Random variable Y has a range $S_Y \ldots$

Random Variable

- The Sample and Range space may be the same
  - You are asked to count the number of buses that pass in 10 minutes.
  - The number of buses belongs to the set of non-negative integers (Range space and Sample space)

Random Variable

- Random variable maybe a function of the observations in the sample space
  - Toss a coin five times
  - Number of heads belongs to the set {0,1,2,3,4,5} and is a random variable.

- The range space is … $S_X = \{0, 1, 2, 3, 4, 5\}$

- When an experiment is performed the RV … X takes a value in ~~the~~ … S_X

Experiment and the Random Variable

- The RV may be a function of another RV
  - Suppose you earn Rs 100 if the number of heads is even and have to pay Rs 100 if the number of heads is odd
  - Let RV G denote your gain at the end of the experiment of 5 coin tosses

$$G = f(X) = \begin{cases} 100 & \text{if } X = 0, 2, 4, \\ -100 & \text{if } X = 1, 3, 5. \end{cases}$$

S_G S_X

Def 2.1: Random Variable

- **Random Variable**: A RV **consists of** an experiment with a probability measure P[.] defined on a sample space S and a **function** that assigns a real number to each outcome in the sample space of the experiment
  - There is an underlying experiment
  - The function defined is just a mapping from outcomes to numbers and is in no way related to a specific experiment

Def 2.1: Random Variable

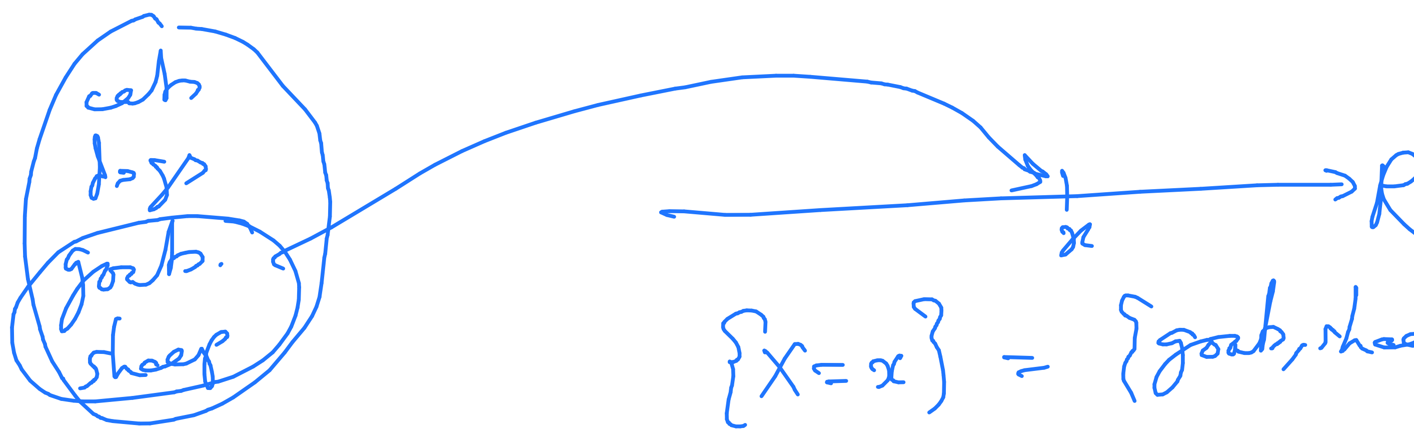
- Notation
  - RV is denoted by a capital letter. The value it assumes is denoted by the corresponding small letter
  - $X = x$, X is the RV and x is the value
  - Writing $X = z$ is correct too. Here z is the value
  - $X(.)$ denotes the function corresponding to the RV X

Never Forget the Experiment!

The event

$$\{X = x\} = \boxed{\{s \in S \mid X(s) = x\}}$$

What is the set of outcomes corresponding to the event X=x?



$$\{X = x\} = \{\text{goats}, \text{sheep}\}$$

Discrete Random Variable

- X is a **discrete** random variable if its range S_X is a countable set

$$S_X = \{x_1, x_2, \ldots\}$$

- $\{X = x_i\}$ and $\{X = x_j\}$, where $x_i \neq x_j$ are disjoint!
- Note that there is no constraint on the values x_1, x_2,…

Discrete Random Variable

- The range must be a countable set
  - Finite, Countably infinite

- Range cannot be the interval (0,1)
- It can be {0.01, 0.05, 0.08, 0.1,…}
- It can be {0.001, 0.5, 0.08, 0.9999}
- It can be the set of integers

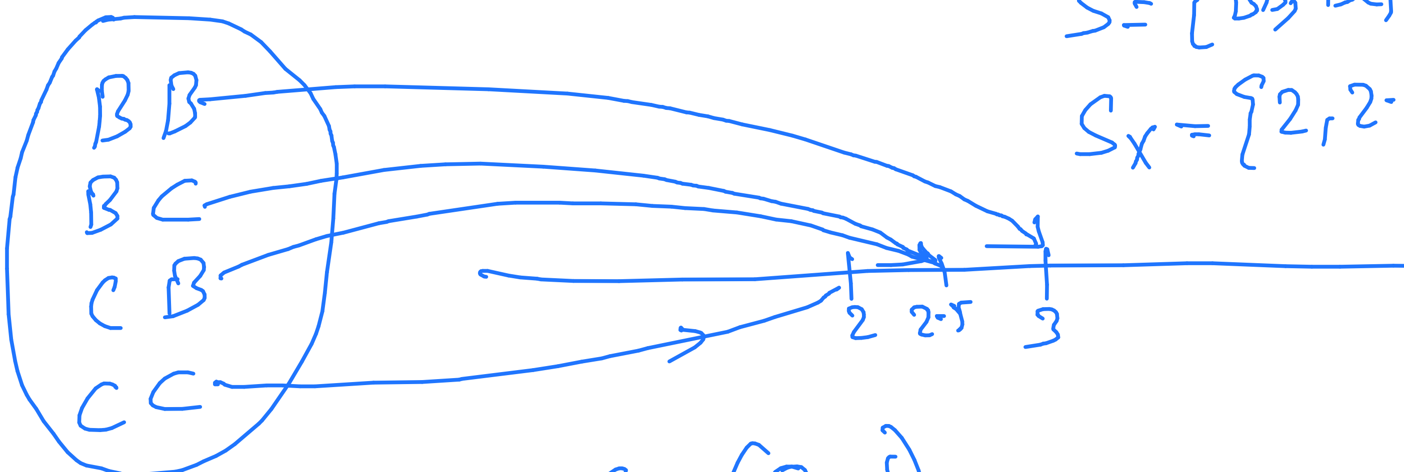
- X(.) maps outcomes in sample space S to values in the range S_X

Quiz 2.1

A student takes two courses. In each course, the student will earn a B with probability 0.6 or a C with probability 0.4, independent of the other course. To calculate a grade point average (GPA), a B is worth 3 points and a C is worth 2 points. The student's GPA is the sum of the GPA for each course divided by 2. Make a table of the sample space of the experiment and the corresponding values of the student's GPA, G.

$S = \{BB, BC, CB, CC\}$

$S_X = \{2, 2.5, 3\}$



$S = (0, 1)$

Probability Mass Function

- **Def 2.4** The probability mass function (PMF) of a discrete random variable X is

$$P_X(x) = P[X = x]$$

Probability of the event X=x

Notation!

Probability Mass Function

- **Def 2.4** The probability mass function (PMF) of a discrete random variable X is

$$P_X(x) = P[X = x]$$

- Note that the PMF is defined for all *x*, not just the *x* that have a mapping to one or more outcomes
  - **The domain of P_X is the set of real numbers**

Probability Mass Function

- There is nothing sacrosanct about x. Just a convention to use a capital letter for a RV and corresponding small for the value

$$P_X(u) = P[X = u]$$

Fickle Coin

- You toss a fickle coin once. The sample space S={heads, tails, standing}.
- The outcomes heads, tails and standing occur with probabilities 0.7, 0.2 and 0.1 respectively.
- In our model, if tails is observed, then the RV $X = 0$. For the outcome heads, $X = 1$. For the outcome standing $X=2$.

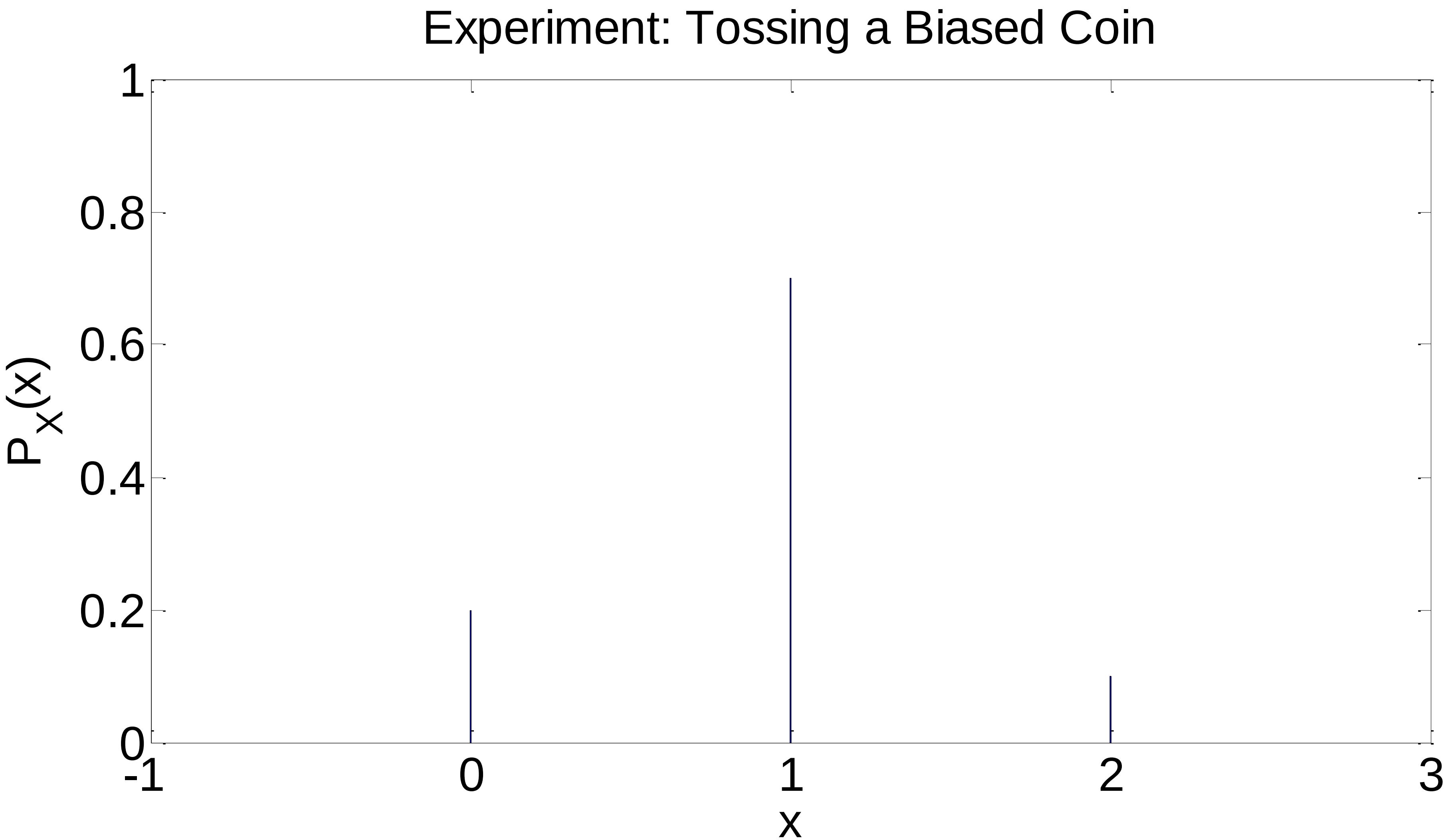
Fickle Coin

- P[X=0] = 0.7, P[X=1] = 0.2, P[X=2] = 0.1

- The PMF is

$$P_X(x) = \begin{cases} 0.2 & x = 0, \\ 0.7 & x = 1, \\ 0.1 & x = 2, \\ 0 & otherwise. \end{cases}$$

PMF: Tossing a Biased Coin…



PMF

- Is $P_X(\pi)$ defined for the experiment? What is its value?
- **The PMF contains all the information about the RV X**