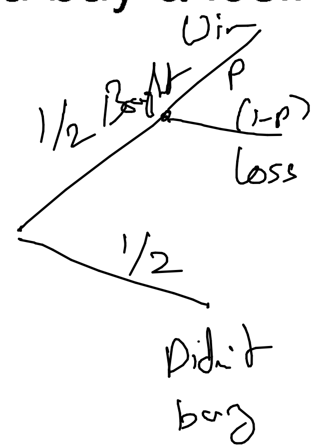
Problem 2.3.12

Suppose each day (starting on day 1) you buy one lottery ticket with probability $1/2$; otherwise, you buy no tickets. A ticket is a winner with probability p independent of the outcome of all other tickets. Let N_i be the event that on day i you do *not* buy a ticket. Let W_i be the event that on day i, you buy a winning ticket. Let L_i be the event that on day i you buy a losing ticket.

(a) What are $P[W_{33}]$, $P[L_{87}]$, and $P[N_{99}]$?



$P[W_{33}] = \left(\frac{1}{2}\right) p$

$P[L_{87}] = \left(\frac{1}{2}\right)(1-p)$

$P[N_{99}] = 1/2$

Suppose each day (starting on day 1) you buy one lottery ticket with probability $1/2$; otherwise, you buy no tickets. A ticket is a winner with probability p independent of the outcome of all other tickets. Let N_i be the event that on day i you do *not* buy a ticket. Let W_i be the event that on day i, you buy a winning ticket. Let L_i be the event that on day i you buy a losing ticket.

(a) What are $P[W_{33}]$, $P[L_{87}]$, and $P[N_{99}]$?

(b) Let K be the number of the day on which you buy your first lottery ticket. Find the PMF $P_K(k)$.

$$P_K(k) = \begin{cases} \left(\frac{1}{2}\right)^{k-1}\left(\frac{1}{2}\right) & k = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k$$

(c) Find the PMF of R, the number of losing lottery tickets you have purchased in m days.

(d) Let D be the number of the day on which you buy your jth losing ticket. What is $P_D(d)$? Hint: If you buy your jth losing ticket on day d, how many losers did you have after $d-1$ days?

$$P[L_i] = \left(\frac{1}{2}\right)(1-p), \quad i = 1, 2, \ldots, \overset{d}{m}.$$

$$P_R(r) = P[R=r] = \begin{cases} {}^{m}C_{r} \left(P[L_i]\right)^{r} \left(1 - P[L_i]\right)^{m-r} & r = 0, 1, \ldots, m, \\ 0 & \text{otherwise} \end{cases}$$

D is Pascal$(P[L_i], j)$

A student attends a lecture with probability 0.3. Every lecture attendance is taken with probability 0.7. A student's attendance is recorded when the student attends the lecture and the attendance is taken. Suppose there are a total of 10 lectures.

(a) What is the probability that the student attends 7 out of 10 lectures?

$${}^{10}C_7 \, (0.3)^7 \, (0.7)^3$$

A student attends a lecture with probability 0.3. Every lecture attendance is taken with probability 0.7. A student's attendance is recorded when the student attends the lecture and the attendance is taken. Suppose there are a total of 10 lectures.

(b) What is the probability that the student attends 7 out of 10 lectures and has an attendance of 4 out of 10?

P[Student attends 7 out of 10 & has an attendance of 4 out of 10]

= P[Student attends 4 when attendance is taken, Student attends 3 when attendance isn't taken, Student doesn't attend 3]

$$= {}^{10}C_4 \; {}^{6}C_3 \; {}^{3}C_3 \; \left((0.3)(0.7)\right)^4 \left((0.3)(0.3)\right)^3 (0.7)^3$$

A student attends a lecture with probability 0.3. Every lecture attendance is taken with probability 0.7. A student's attendance is recorded when the student attends the lecture and the attendance is taken. Suppose there are a total of 10 lectures.

(c) Suppose you are told that the student attended the 10th lecture and attendance was taken in the lecture. What is your updated belief about the occurrence of the event in part (b)?

P[Student attends 7 out of 10 & has an attendance of 6 out of (6) | Student attended 10th lecture in which attendance was taken]

A student attends a lecture with probability 0.3. Every lecture attendance is taken with probability 0.7. A student's attendance is recorded when the student attends the lecture and the attendance is taken. Suppose there are a total of 10 lectures.

(d) Suppose you are told that the student attended the 10th lecture. What is your updated belief about the occurrence of the event in part (b)?

P[Student attends 7 out of 10 & has an attendance of 4 out of 10 | Student attended 10th lecture] =

$\frac{1}{P[\text{Student attends } 10^{th}]}$ {P[Student attends 7 out of 10 & has an attendance of 4 out of 10 & Student attended 10th lecture in which attendance was taken]

+ P[. . . . . . 10th lecture in which attendance was not taken]}

Cumulative Distribution Function (CDF)

- **Def 2.11** The cumulative distribution function of a RV *X* is

$$F_X(x) = P[X \le x]$$

- **It is defined for all real *x***
- **Theorem 2.2 (part)**

For any DRV X with range $S_X = \{x_1, x_2, \ldots\}$ satisfying $x_1 \le x_2 \le x_3 \ldots$,

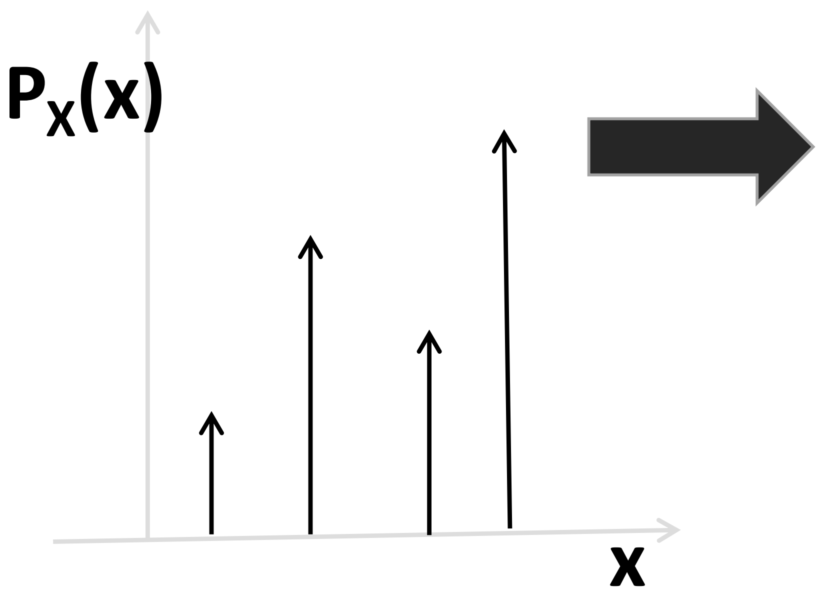
$\{X \le x\} = X \in (-\infty, x]$

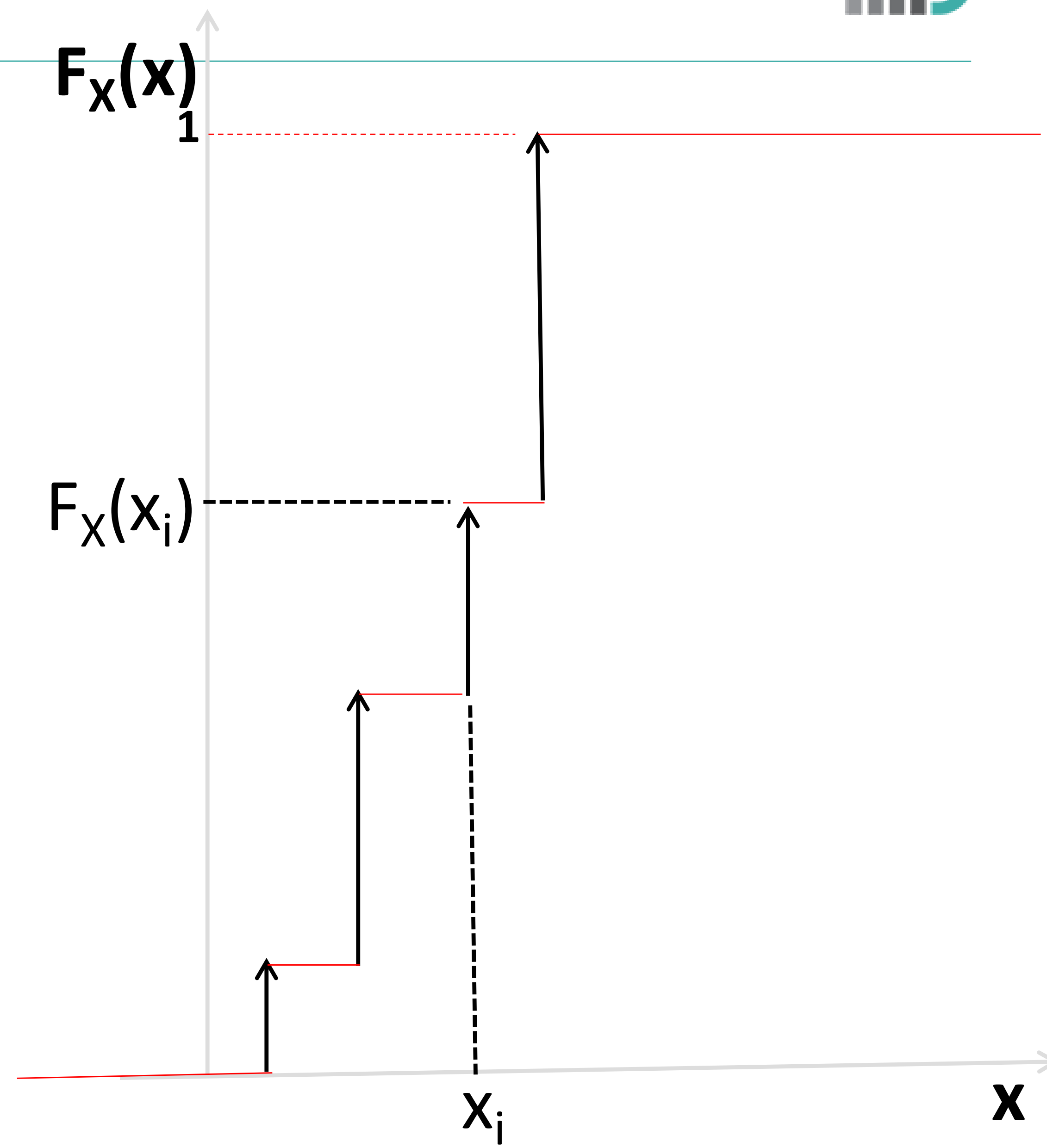
$$F_X(-\infty) = 0$$
$$F_X(\infty) = 1$$

For all $x' \ge x$, $\underbrace{F_X(x')}_{P[X \le x']} \ge \underbrace{F_X(x)}_{P[X \le x]}$

$\{X \le x'\} = X \in (-\infty, x']$

PMF -> CDF



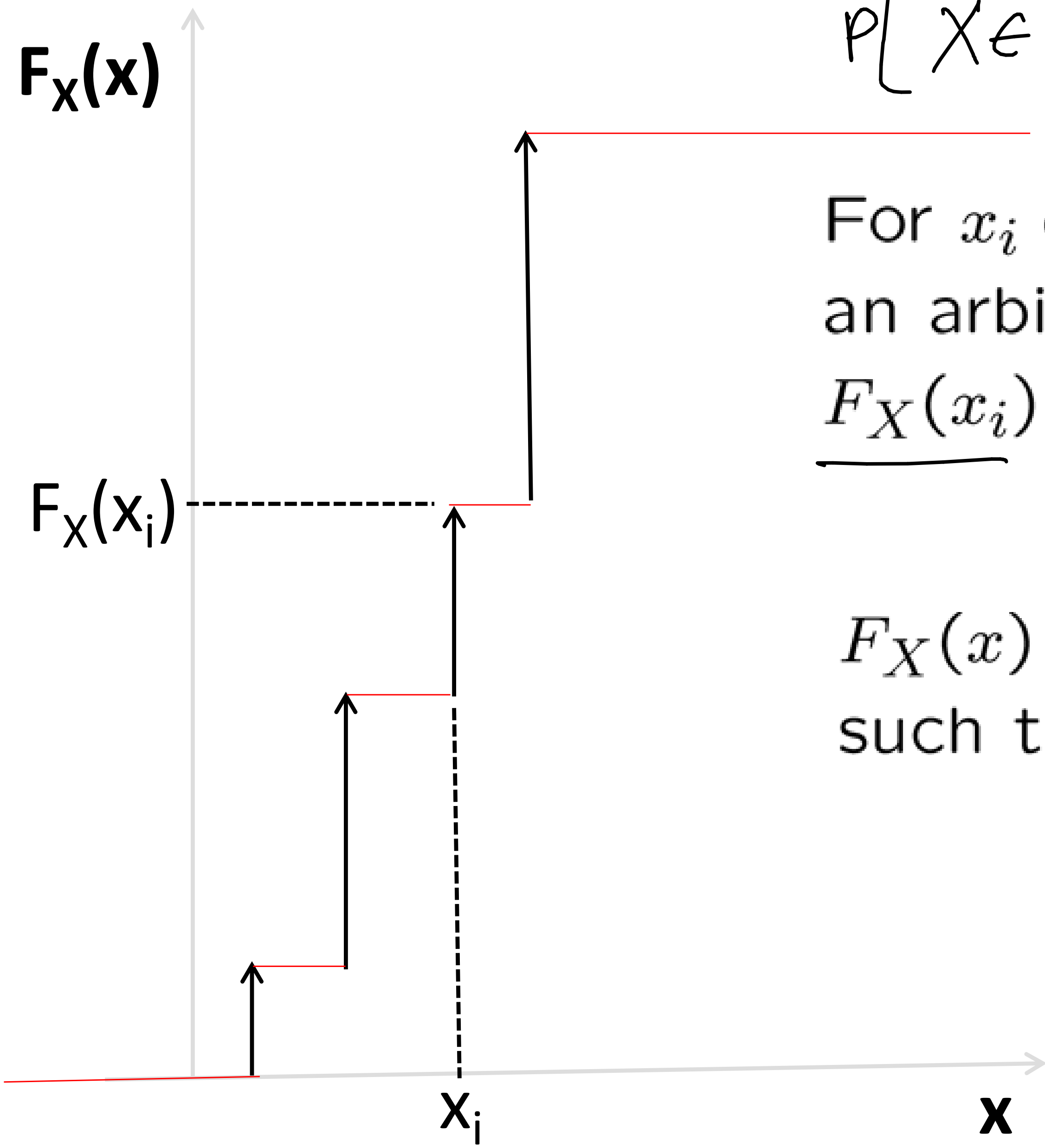


CDF

Starts at 0 and goes all the way to 1
Is non-decreasing
Jumps at points in the range of the RV
Jump at a point x equal to P[X=x]

Accumulate the probabilities in a PMF to get a CDF

CDF

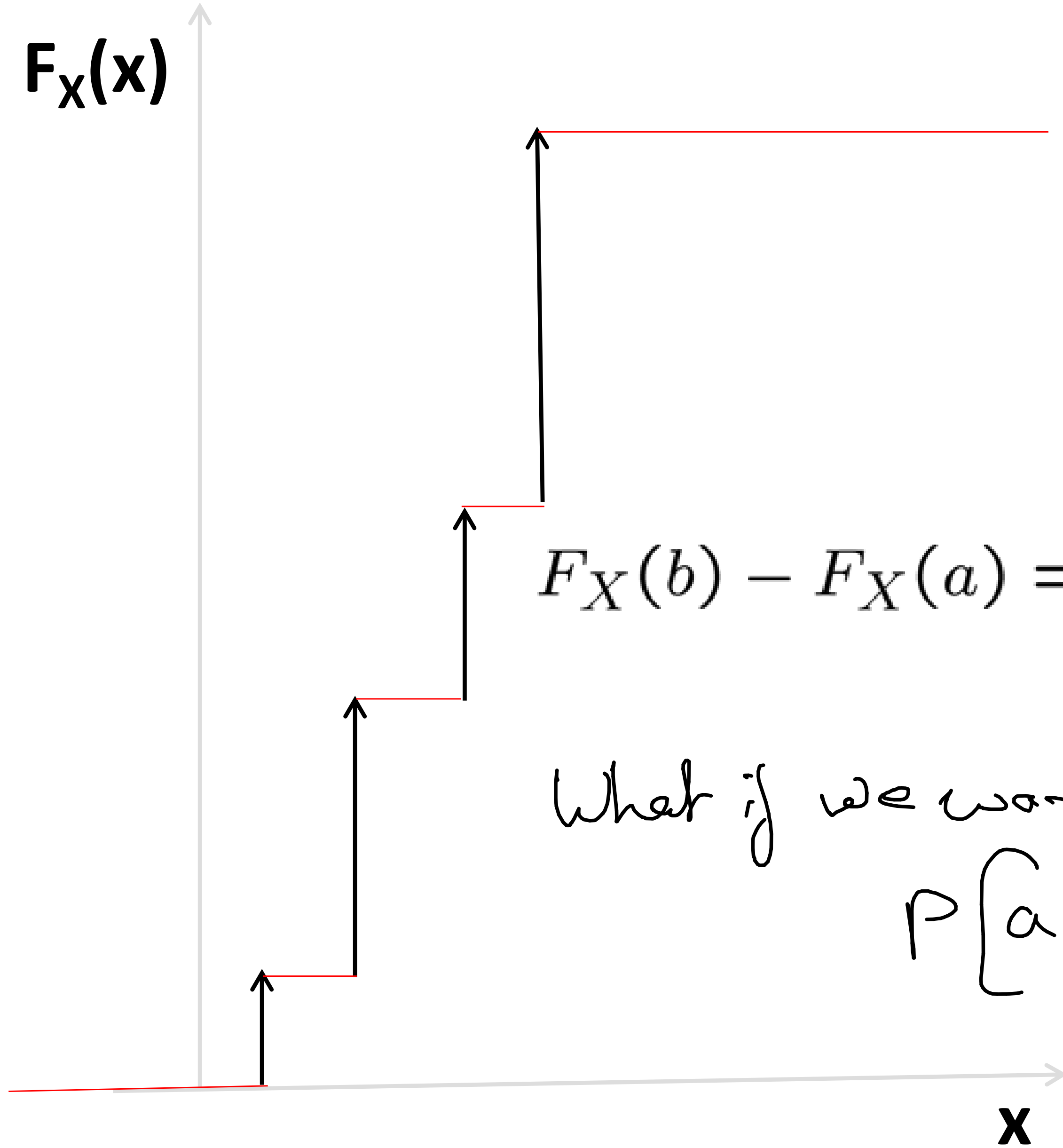


$P[X \in (x_i - \epsilon, x_i]]$
$= P[X \le x_i] -$
$P[X \le x_i - \epsilon]$

For $x_i \in S_X$ and ϵ,
an arbitrary small +ve no.,
$\underline{F_X(x_i)} - \underline{F_X(x_i - \epsilon)} = \underline{P_X(x_i)}$

$F_X(x) = F_X(x_i)$ for all x
such that $x_i \le x < x_{i+1}$.

CDF: Theorem 2.3



$$F_X(b) - F_X(a) = P[a < X \leq b]$$

What if we want

$P[a \leq X \leq b]$

CDF

Let $S_X = \{x_{min}, \ldots, x_{max}\}$.

$F_X(x_{max}) = ?$

$F_X(x_{min}) = ?$

For $x > x_{max}$, $F_X(x) = ?$

Example 2.24 Problem

In Example 2.11, let the probability that a circuit is rejected equal $p = 1/4$. The PMF of Y, the number of tests up to and including the first reject, is the geometric $(1/4)$ random variable with PMF

$$P_Y(y) = \begin{cases} (1/4)(3/4)^{y-1} & y = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases} \tag{2.40}$$

What is the CDF of Y?

$$P[Y \le y] = \sum_{k=1}^{y} \underbrace{P[Y = k]}_{(1/4)(3/4)^{k-1}}$$

Average of a Discrete Random Variable

- Mode (Def 2.12 – read from book)
- Median (Def 2.13 – read from book)
- Expected Value
  - When someone says average of a random variable, they usually mean the expected value of the RV!

Average of a Discrete Random Variable

- **Def 2.14** The expected value of X is

$$E[X] = \mu_X = \sum_{x \in S_X} x P_X(x).$$