ECE113 – Basic Electronics

Lecture week 3: Series and parallel connected resistance and sources

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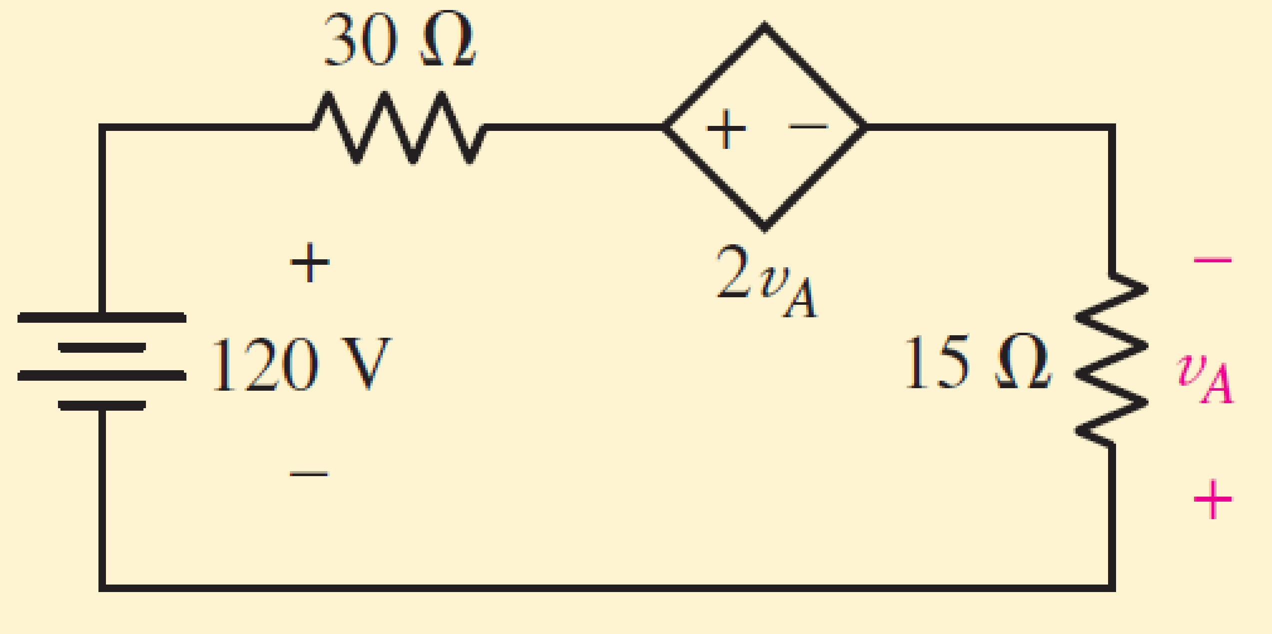
Single Loop Circuit (example 6)

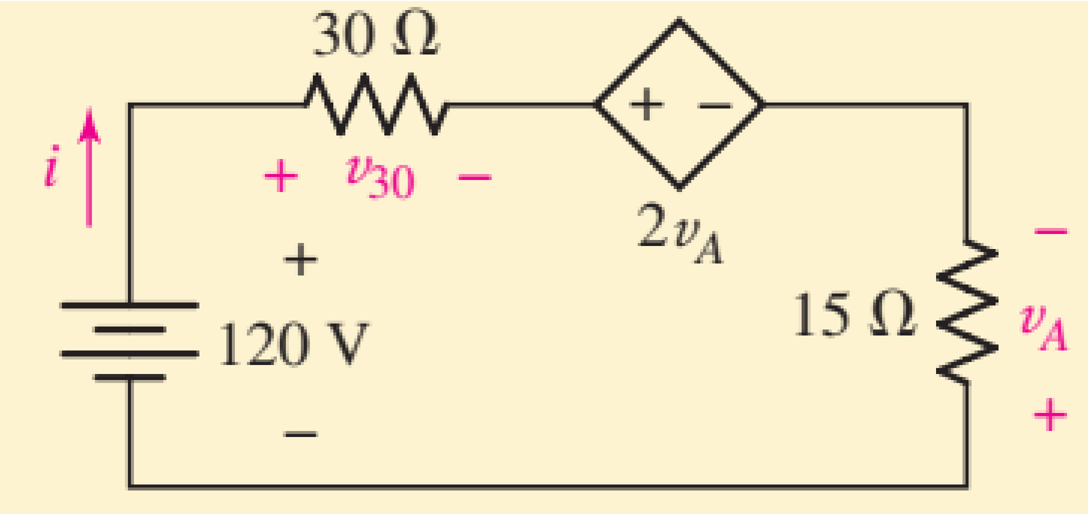
EXAMPLE 3.5

Compute the power absorbed in each element for the circuit shown in Fig. 3.13*a*.



$$
\begin{aligned}
p_{120\text{V}} &= (120)(-8) = -960 \text{ W} \\
p_{30\Omega} &= (8)^2(30) = 1920 \text{ W} \\
p_{\text{dep}} &= (2v_A)(8) = 2[(-15)(8)](8) \\
&= -1920 \text{ W} \\
p_{15\Omega} &= (8)^2(15) = 960 \text{ W}
\end{aligned}
$$

Example 6 solution



This circuit contains a dependent voltage source, the value of which remains unknown until we determine v_A. However, its algebraic value $2v_A$ can be used in the same fashion as if a numerical value were available. Thus, applying KVL around the loop:

$$-120 + v_{30} + 2v_A - v_A = 0 \qquad [7]$$

Using Ohm's law to introduce the known resistor values:

$$v_{30} = 30i \quad \text{and} \quad v_A = -15i$$

Note that the negative sign is required since i flows into the negative terminal of v_A.

Substituting into Eq. [7] yields

$$-120 + 30i - 30i + 15i = 0$$

and so we find that

$$i = 8 \text{ A}$$

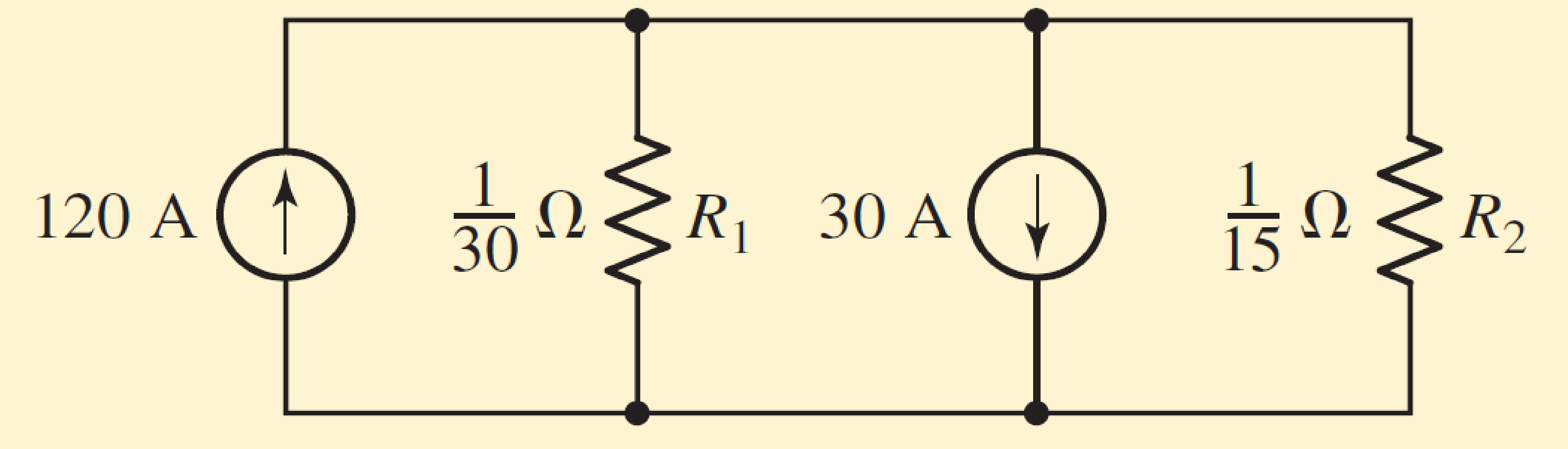
Computing the power *absorbed* by each element:

$$\begin{aligned} p_{120\text{V}} &= (120)(-8) &&= -960 \text{ W} \\ p_{30\Omega} &= (8)^2(30) &&= 1920 \text{ W} \\ p_{\text{dep}} &= (2v_A)(8) &&= 2[(-15)(8)](8) \\ & &&= -1920 \text{ W} \\ p_{15\Omega} &= (8)^2(15) &&= 960 \text{ W} \end{aligned}$$

Single node pair circuit (example 7)

Text book example 3.6 (Hayt and Kemmerly)

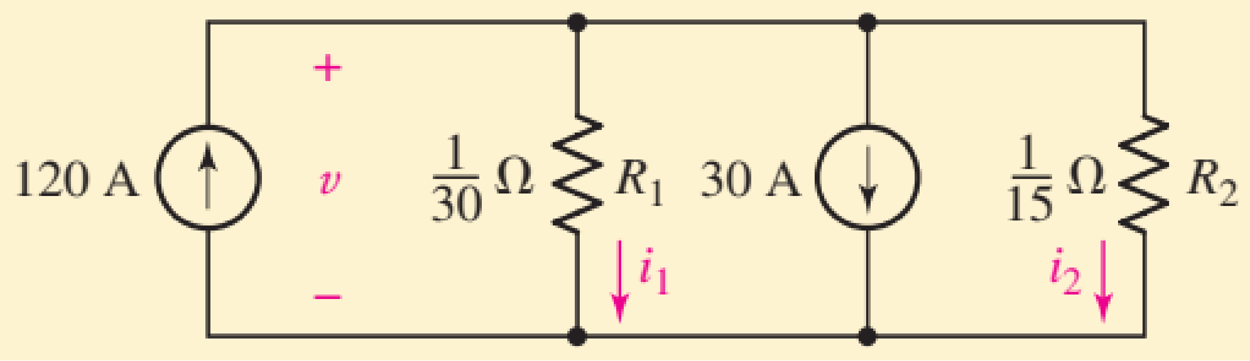
Find the voltage, current, and power associated with each element in the circuit



$$p_{R1} = 30(2)^2 = 120 \text{ W} \quad \text{and} \quad p_{R2} = 15(2)^2 = 60 \text{ W}$$

$$p_{120\text{A}} = 120(-2) = -240 \text{ W} \quad \text{and} \quad p_{30\text{A}} = 30(2) = 60 \text{ W}$$

Example 7 solution



Determining either current i_1 or i_2 will enable us to obtain a value for v. Thus, our next step is to apply KCL to either of the two nodes in the circuit. Equating the algebraic sum of the currents leaving the upper node to zero:

$$-120 + i_1 + 30 + i_2 = 0$$

Writing both currents in terms of the voltage v using Ohm's law

$$i_1 = 30v \quad \text{and} \quad i_2 = 15v$$

we obtain

$$-120 + 30v + 30 + 15v = 0$$

Solving this equation for v results in

$$v = 2\text{ V}$$

and invoking Ohm's law then gives

$$i_1 = 60\text{ A} \quad \text{and} \quad i_2 = 30\text{ A}$$

The absorbed power in each element can now be computed. In the two resistors,

$$p_{R1} = 30(2)^2 = 120\text{ W} \quad \text{and} \quad p_{R2} = 15(2)^2 = 60\text{ W}$$

and for the two sources,

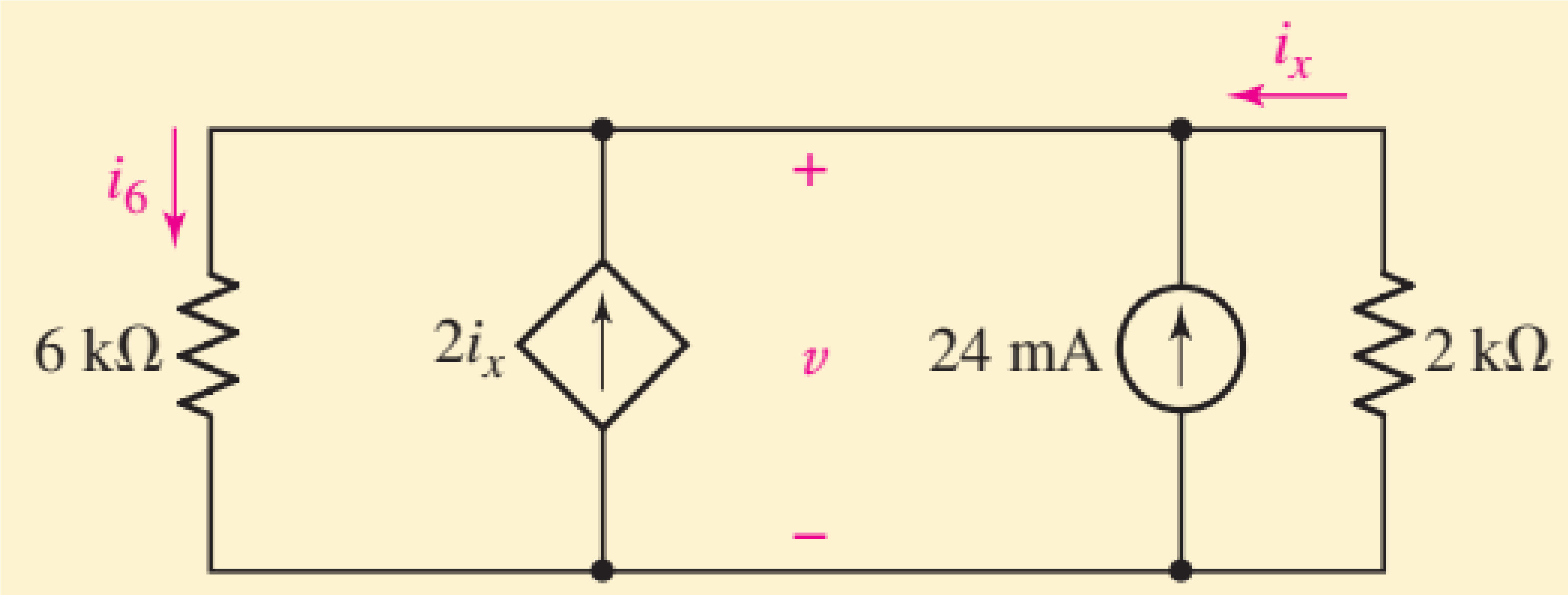
$$p_{120\text{A}} = 120(-2) = -240\text{ W} \quad \text{and} \quad p_{30\text{A}} = 30(2) = 60\text{ W}$$

Since the 120 A source absorbs negative 240 W, it is actually *supplying* power to the other elements in the circuit. In a similar fashion, we find that the 30 A source is actually *absorbing* power rather than *supplying* it.

Single node pair circuit (example 8)

Textbook example 3.7

Determine the value of v and the power supplied by the independent current source



Ans: 14.4 V

Example 8 solution

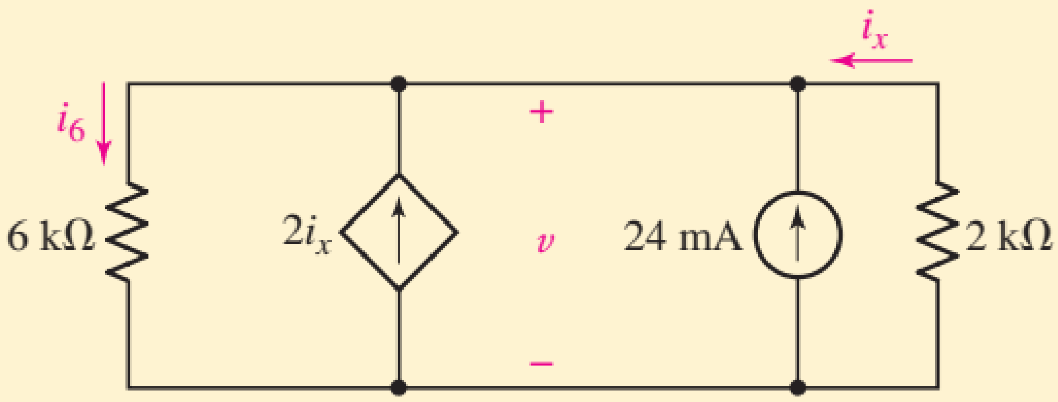


FIGURE 3.17 A voltage v and a current i_6 are assigned in a single-node-pair circuit containing a dependent source.

By KCL, the sum of the currents leaving the upper node must be zero, so that

$$i_6 - 2i_x - 0.024 - i_x = 0$$

Again, note that the value of the dependent source ($2i_x$) is treated the same as any other current would be, even though its exact value is not known until the circuit has been analyzed.

We next apply Ohm's law to each resistor:

$$i_6 = \frac{v}{6000} \quad \text{and} \quad i_x = \frac{-v}{2000}$$

Therefore,

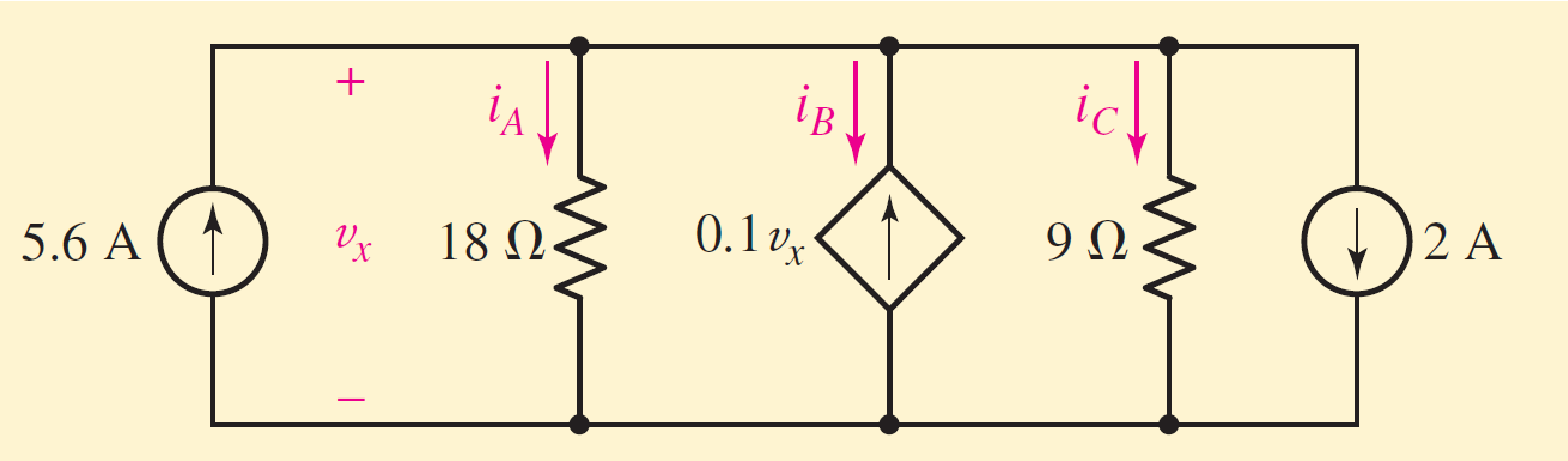
$$\frac{v}{6000} - 2\left(\frac{-v}{2000}\right) - 0.024 - \left(\frac{-v}{2000}\right) = 0$$

and so $v = (600)(0.024) = 14.4$ V.

Any other information we may want to find for this circuit is now easily obtained, usually in a single step. For example, the power supplied by the independent source is $p_{24} = 14.4(0.024) = 0.3456$ W (345.6 mW).

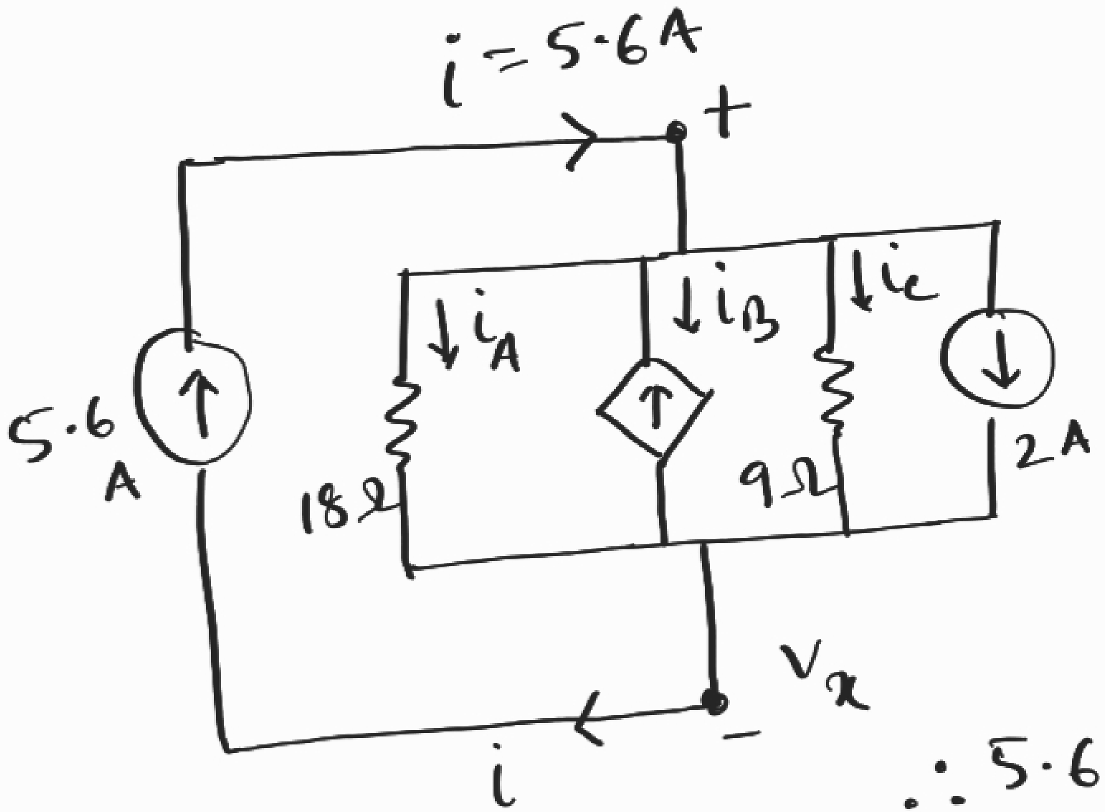
Single node pair circuit (Example 9)

Practice 3.8 find i_A, i_B, i_c



Ans: 3 A; −5.4 A; 6 A

Example 9 solution



$$i = i_A + i_B + i_C + 2$$

$$i_A = \frac{V_x}{18}$$

$$i_B = -0.1V_x$$

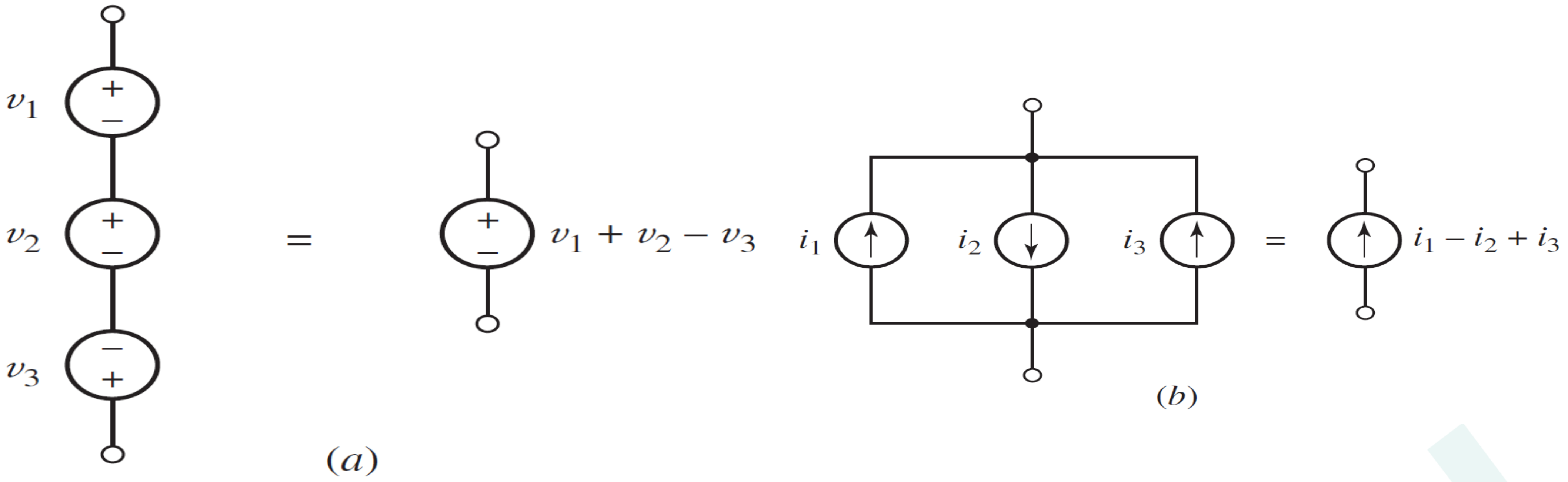
$$i_C = \frac{V_x}{9}$$

$$\therefore 5.6 = \frac{V_x}{18} - 0.1V_x + \frac{V_x}{9} + 2$$

$$\Rightarrow V_x = 54\,\Omega$$

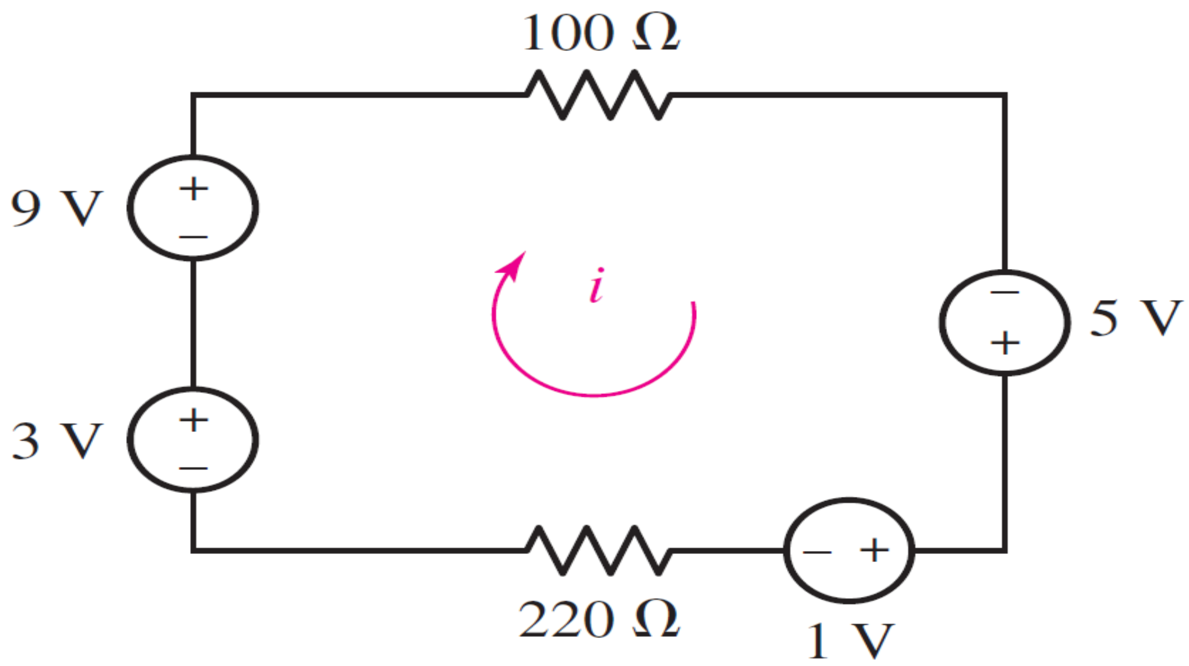
$$\therefore i_A = 3A \; ; \; i_B = -5.4A \; ; \; i_C = 6A$$

Series and parallel connected sources



Example 10

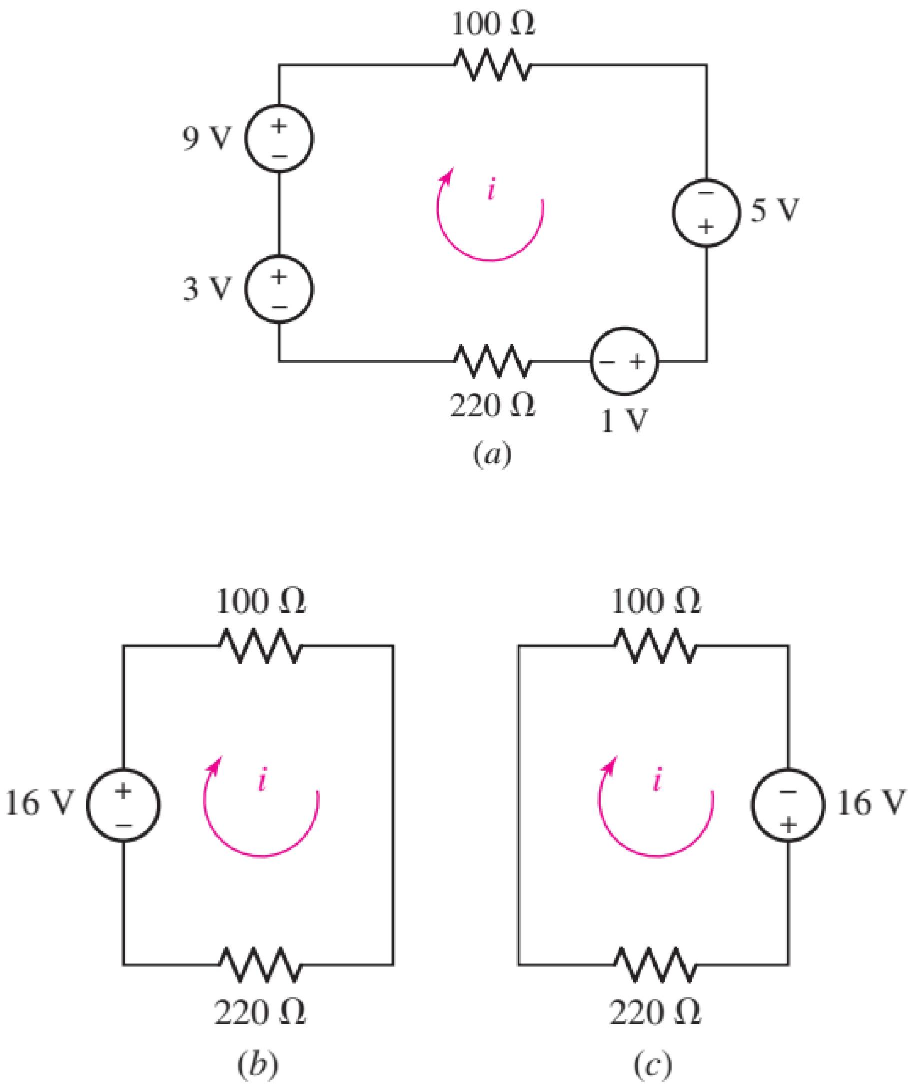
Determine the current i in the circuit by first combining the sources into a single equivalent voltage source.



(a)

Ans: 50 mA

Example 10 solution



To be able to combine the voltage sources, they must be in series. Since the same current (i) flows through each, this condition is satisfied.

Starting from the bottom left-hand corner and proceeding clockwise,

$$-3 - 9 - 5 + 1 = -16 \text{ V}$$

so we may replace the four voltage sources with a single 16 V source having its negative reference as shown in Fig. 3.20*b*.

KVL combined with Ohm's law then yields

$$-16 + 100i + 220i = 0$$

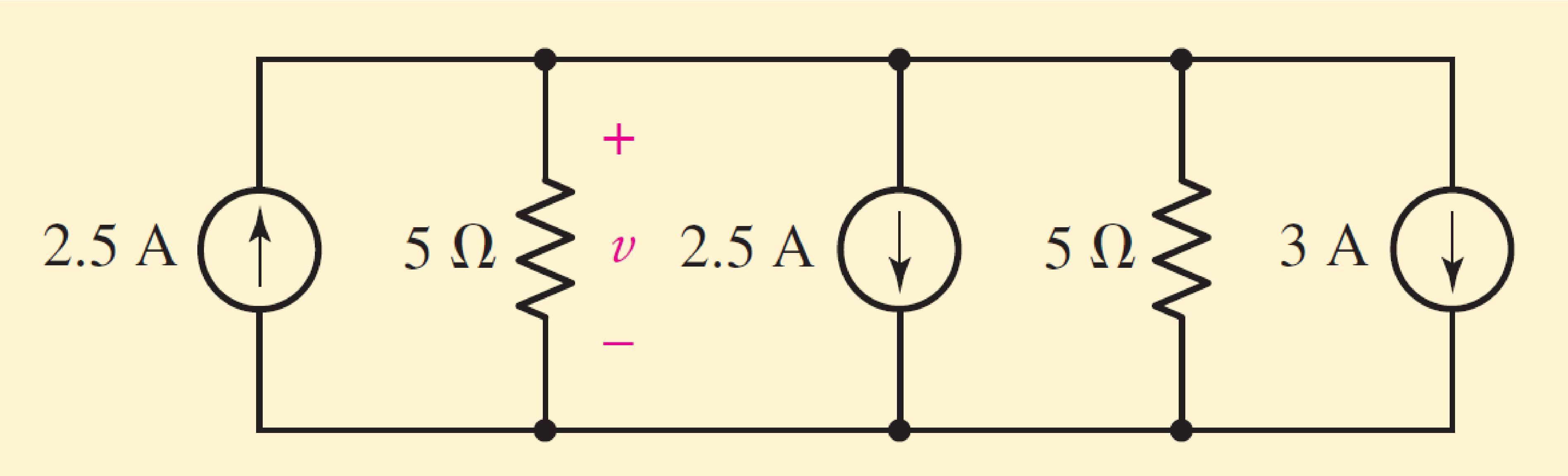
or

$$i = \frac{16}{320} = 50 \text{ mA}$$

We should note that the circuit in Fig. 3.20*c* is also equivalent, a fact easily verified by computing i.

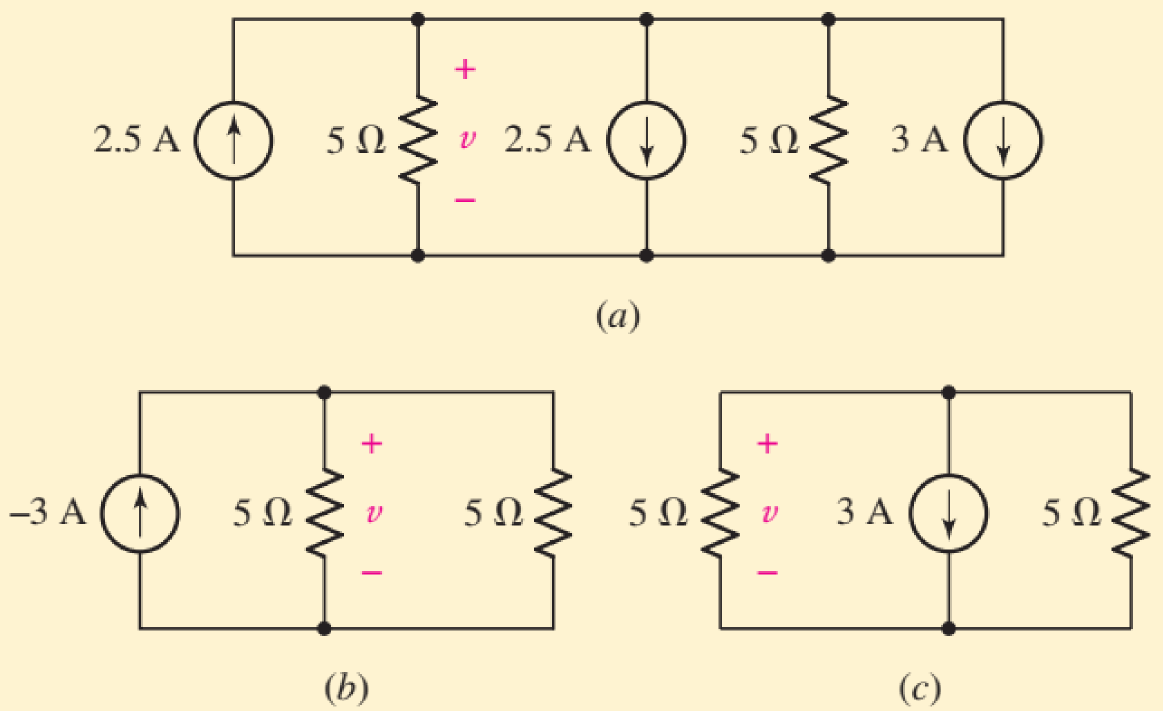
Example 11

Determine the voltage v in the circuit by first combining the sources into a single equivalent current source.



Ans: 7.5V

Example 11 solution



The sources may be combined if the same voltage appears across each one, which we can easily verify is the case. Thus, we create a new source, arrow pointing upward into the top node, by adding the currents that flow into that node:

$$2.5 - 2.5 - 3 = -3 \text{ A}$$

One equivalent circuit is shown in Fig. 3.22*b*.

KCL then allows us to write

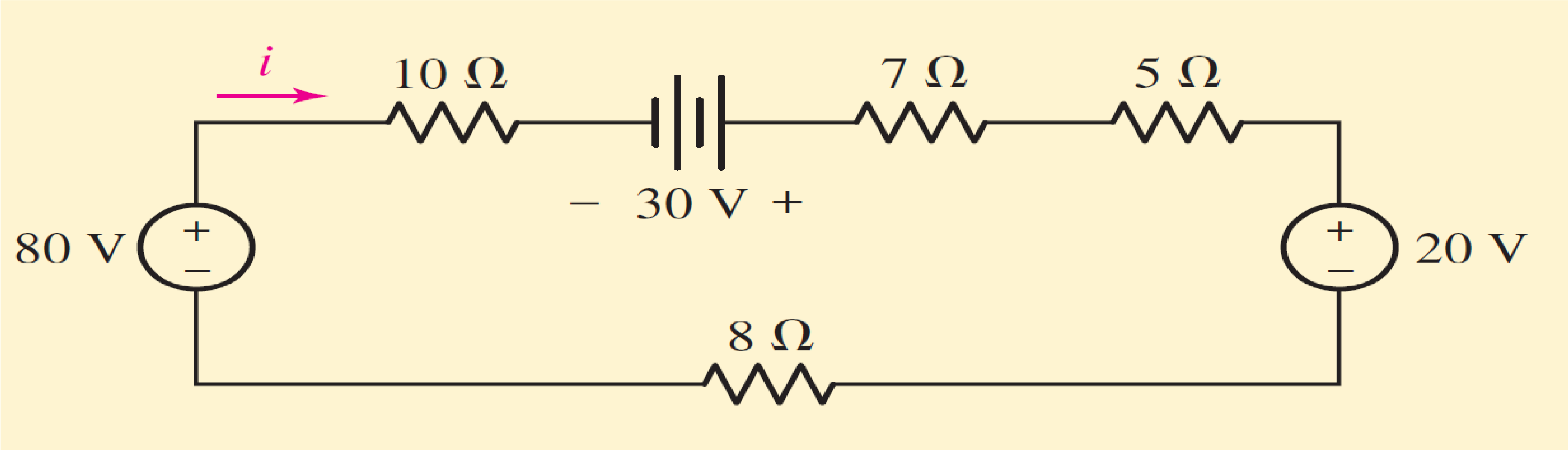
$$-3 + \frac{v}{5} + \frac{v}{5} = 0$$

Solving, we find $v = 7.5$ V.

Another equivalent circuit is shown in Fig. 3.22*c*.

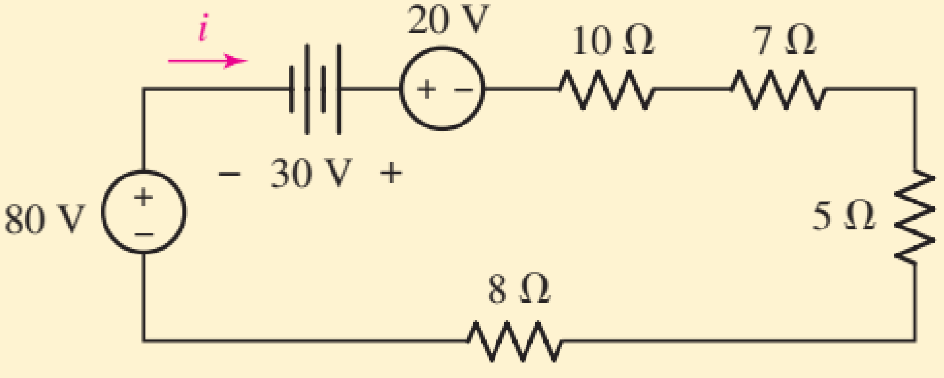
Example 12

Use resistance and source combinations to determine the current i in the Fig. and the power delivered by the 80 V source.

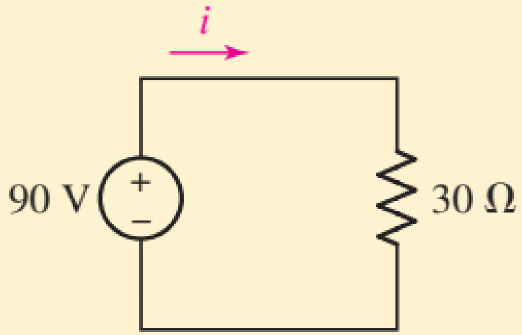


Ans: 3A, 240W

Example 12 solution



(b)



(c)

FIGURE 3.27 (*a*) A series circuit with several sources and resistors. (*b*) The elements are rearranged for the sake of clarity. (*c*) A simpler equivalent.

We first interchange the element positions in the circuit, being careful to preserve the proper sense of the sources, as shown in Fig. 3.27*b*. The next step is to then combine the three voltage sources into an equivalent 90 V source, and the four resistors into an equivalent 30 Ω resistance, as in Fig. 3.27*c*. Thus, instead of writing

$$-80 + 10i - 30 + 7i + 5i + 20 + 8i = 0$$

we have simply

$$-90 + 30i = 0$$

and so we find that

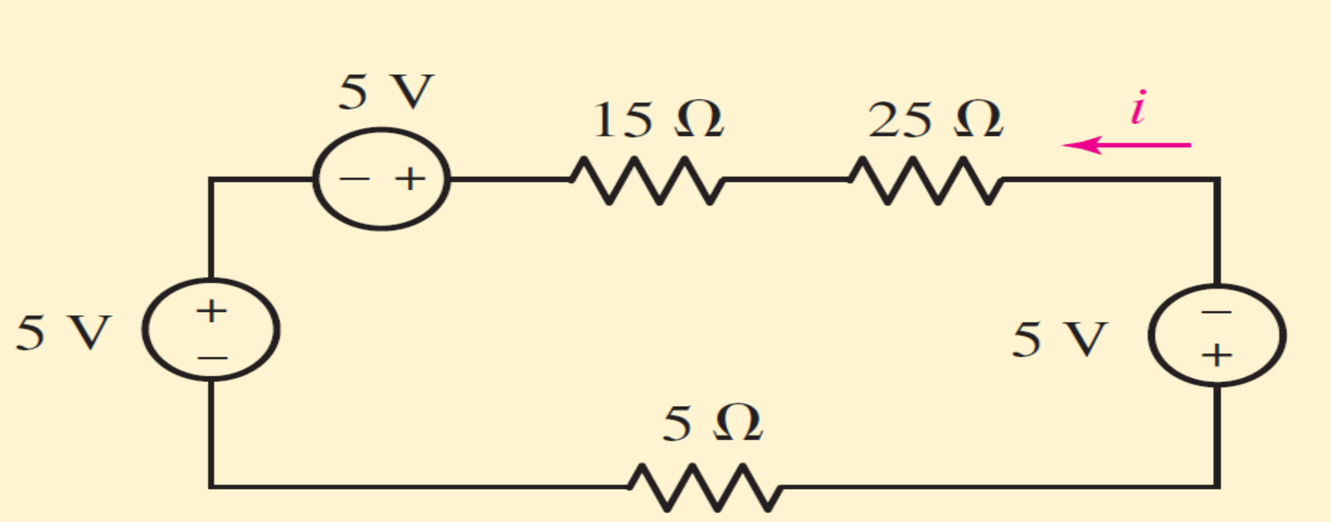
$$i = 3 \text{ A}$$

In order to calculate the power delivered to the circuit by the 80 V source appearing in the given circuit, it is necessary to return to Fig. 3.27*a* with the knowledge that the current is 3 A. The desired power is then 80 V × 3 A = 240 W.

It is interesting to note that no element of the original circuit remains in the equivalent circuit.

Example 13 and its solution

Determine i in the circuit



$V_{eq} \Rightarrow 5+5+5 \ V = 15 \ V$

$R_{eq} \Rightarrow 15+25+5 \ \Omega = 45 \ \Omega$

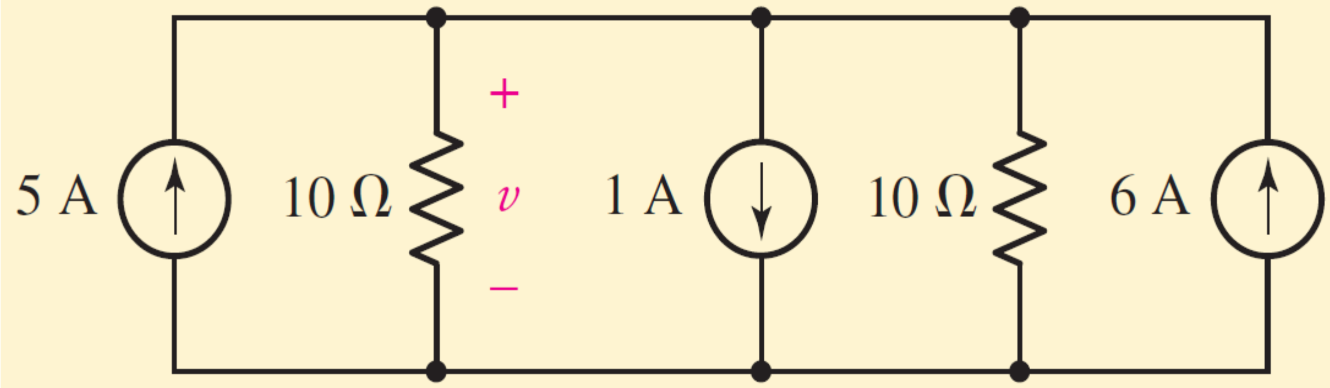
$i = -\frac{15}{45} = -333 \ mA$

'-ve' → for the direction of the current.

Ans: -333 mA

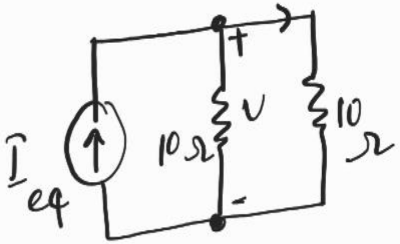
Example 14 and its solution

Determine v in the circuit by first combining the three current sources, and then the two 10 Ω resistors.



Ans: 50V

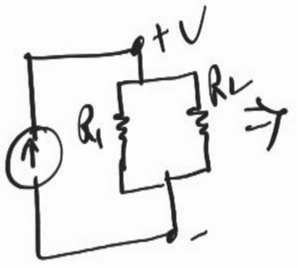
3.10 Determine the voltage v in the circuit of Fig. 3.23 after first replacing the three sources with a single equivalent source.



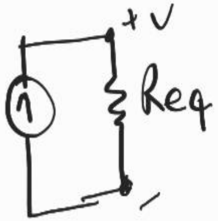
$I_{eq} = 5 - 1 + 6 = 10\,A$

$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = 5\,\Omega$

$\Downarrow$

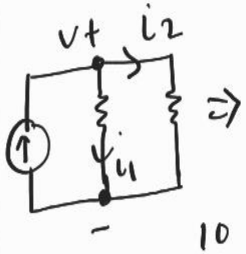


$\Rightarrow$



$\therefore v = 50V$,

Another approach



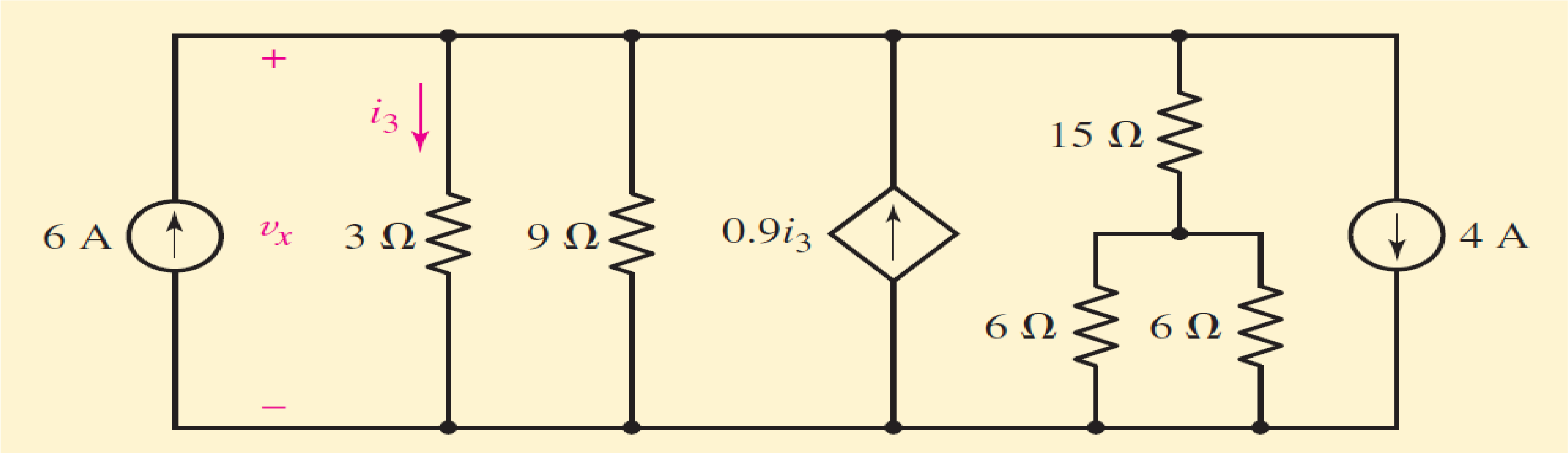
$\Rightarrow i_{eq} = i_1 + i_2$

$10 = \frac{v}{10} + \frac{v}{10}$

$\therefore v = 50V$

Example 15

Calculate the power and voltage of the dependent source



Example 15 solution

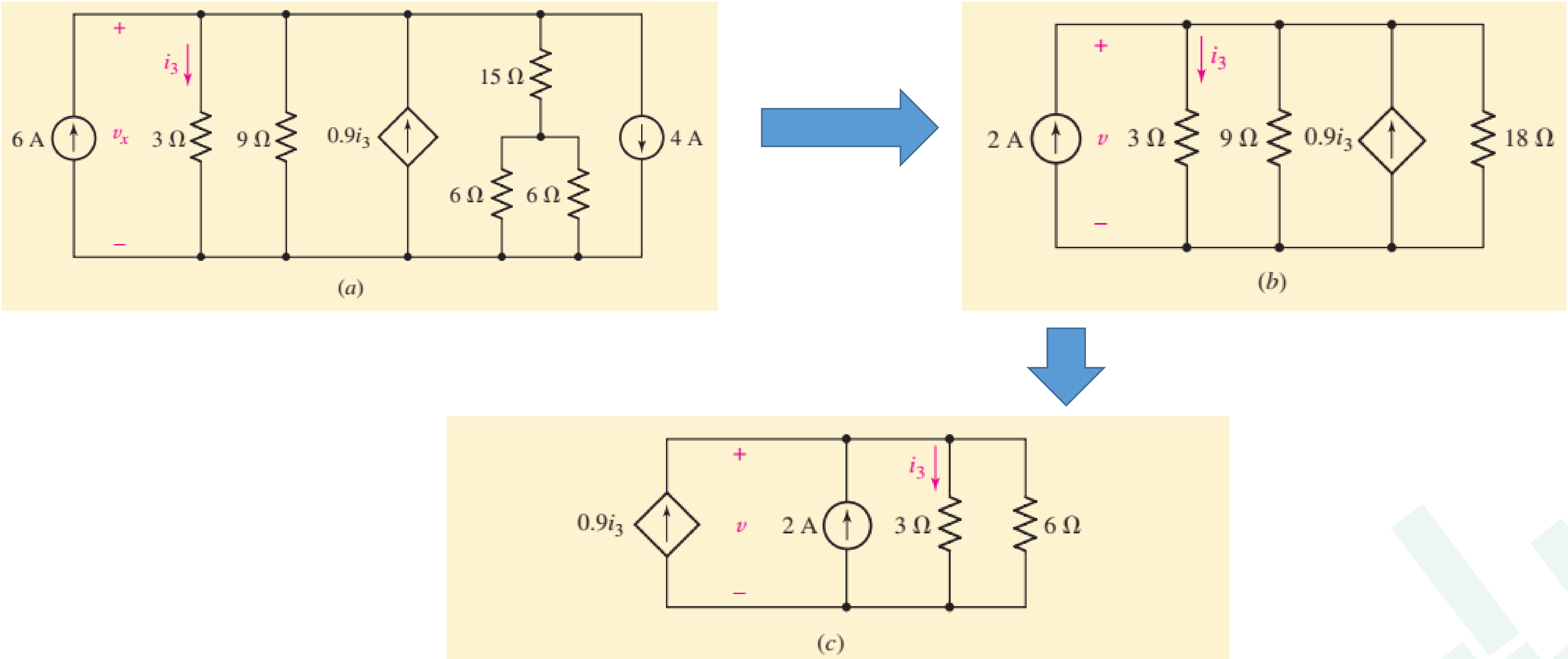
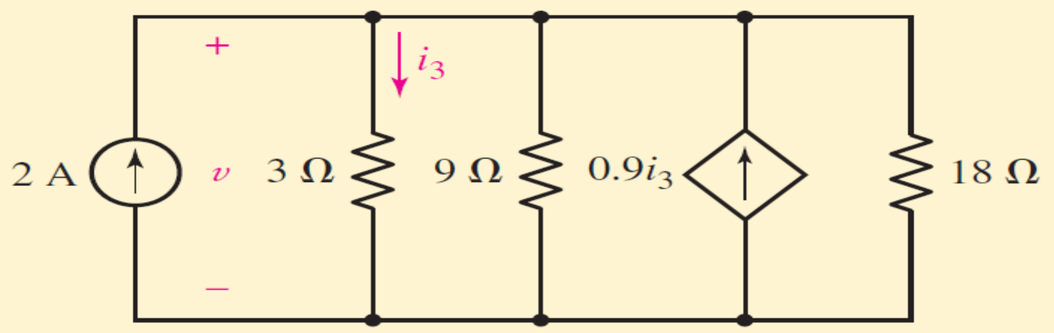
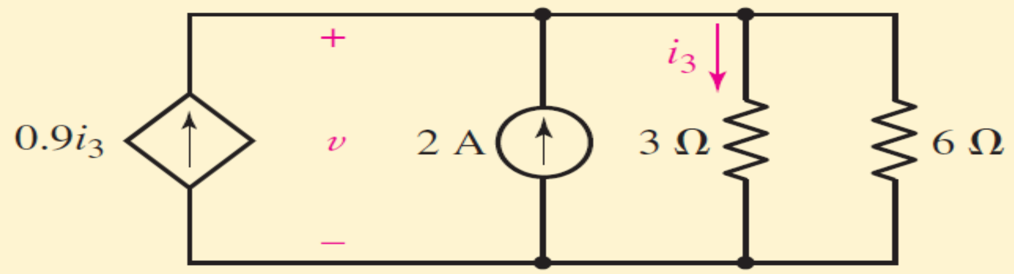


FIGURE 3.31 (*a*) A multinode circuit. (*b*) The two independent current sources are combined into a 2 A source, and the 15 Ω resistor in series with the two parallel 6 Ω resistors are replaced with a single 18 Ω resistor. (*c*) A simplified equivalent circuit.

Contd…

Example 15 solution





Ans: 30 W

Applying KCL at the top node of Fig. 3.31*c*, we have

$$-0.9i_3 - 2 + i_3 + \frac{v}{6} = 0$$

Employing Ohm's law,

$$v = 3i_3$$

which allows us to compute

$$i_3 = \frac{10}{3}\text{ A}$$

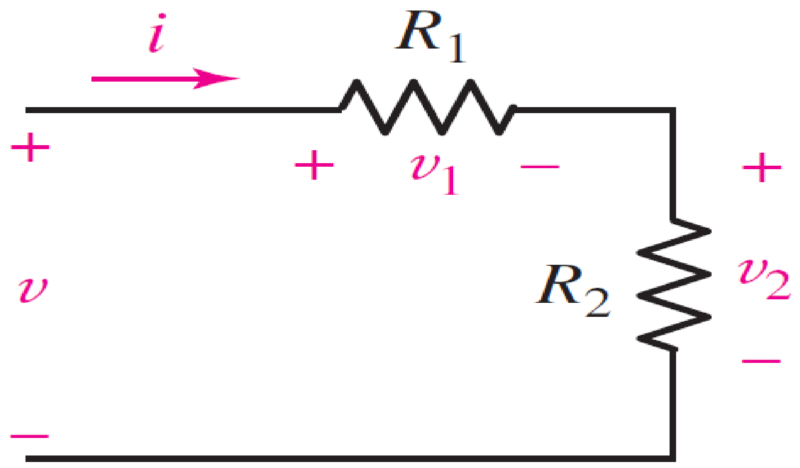
Thus, the voltage across the dependent source (which is the same as the voltage across the 3 Ω resistor) is

$$v = 3i_3 = 10\text{ V}$$

The dependent source therefore furnishes $v \times 0.9i_3 = 10(0.9)(10/3) = 30$ W to the remainder of the circuit.

Voltage and current division

Voltage Division



$$v_2 = \frac{R_2}{R_1 + R_2} v$$

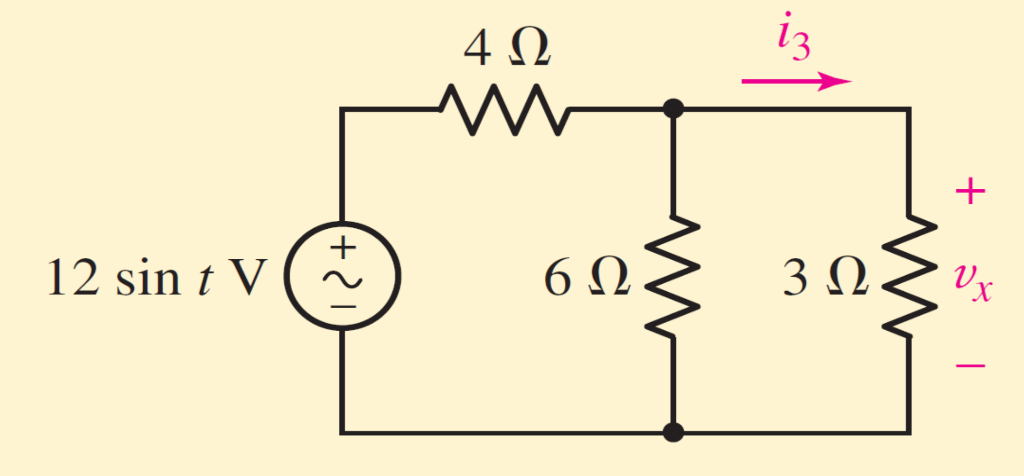
$$v_1 = \frac{R_1}{R_1 + R_2} v$$

For N number of resistors in series,
Voltage across kth resistor

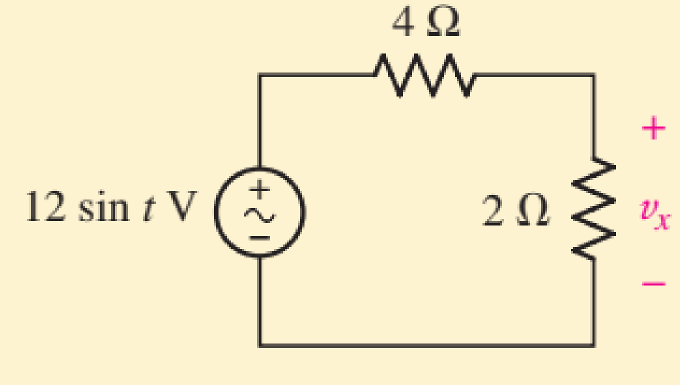
$$v_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} v$$

Voltage and current division; Example 16 and its solution

Example 3.13: Determine v_x in the circuit



Ans: 4 sin t



(b)

We first combine the 6 Ω and 3 Ω resistors, replacing them with $(6)(3)/(6+3) = 2\ \Omega$.

Since v_x appears across the parallel combination, our simplification has not lost this quantity. However, further simplification of the circuit by replacing the series combination of the 4 Ω resistor with our new 2 Ω resistor would.

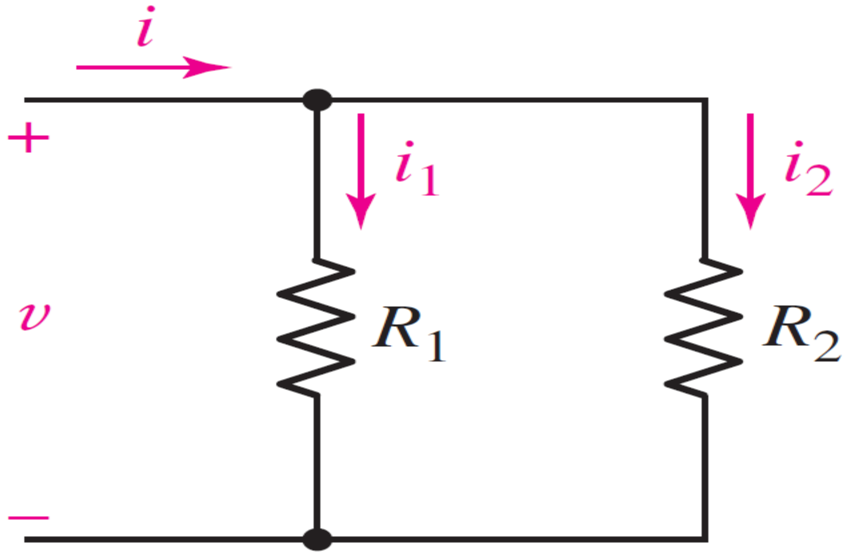
Thus, we proceed by simply applying voltage division to the circuit in Fig. 3.35*b*:

$$v_x = (12 \sin t)\frac{2}{4+2} = 4 \sin t \qquad \text{volts}$$

Voltage and current division

Current division

$$i_1 = i\frac{R_2}{R_1 + R_2} \qquad i_2 = i\frac{R_1}{R_1 + R_2}$$



For N parallel resistors, current through the kth resistor is

$$i_k = i\frac{\dfrac{1}{R_k}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_N}}$$

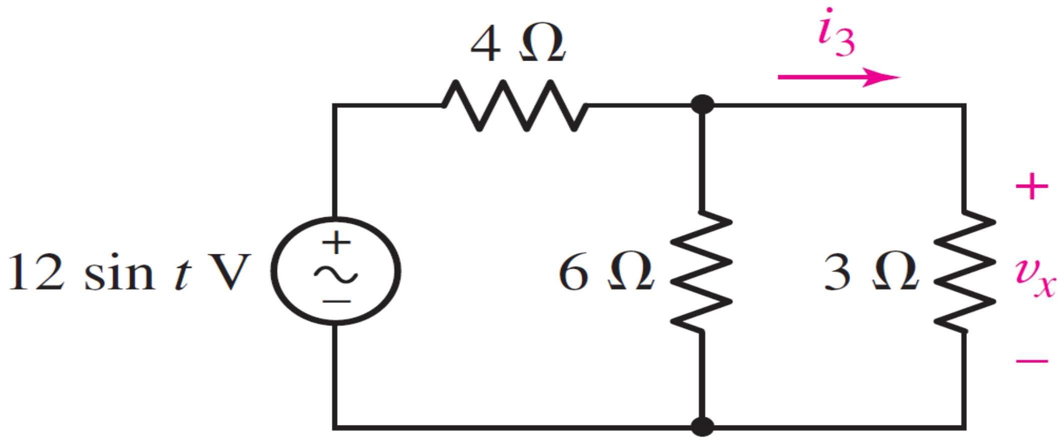
Written in terms of conductances,

$$i_k = i\frac{G_k}{G_1 + G_2 + \cdots + G_N}$$

Without parallel resistors, there is no way that current division can be applied. Remember, parallel resistors must be branches between the same pair of nodes

Voltage and current division Example 17 and its solution

Example 3.14 (same as previous circuit) find i_3



The total current flowing into the 3 Ω–6 Ω combination is

$$i(t) = \frac{12 \sin t}{4 + 3\|6} = \frac{12 \sin t}{4 + 2} = 2 \sin t \qquad \text{A}$$

and thus the desired current is given by current division:

$$i_3(t) = (2 \sin t)\left(\frac{6}{6+3}\right) = \frac{4}{3} \sin t \qquad \text{A}$$

Ans: $\frac{4}{3}$sin t

Nodal Analysis

- Nodal analysis and Mesh analysis— allow to investigate circuits with a consistent, methodical approach
- Nodal analysis is based on KCL
- Mesh analysis is based on KVL (will be discussed in the next lecture)
- A node must be chosen as reference and voltages at other nodes must be assigned with respect to the reference
- Hence, for N number of nodes, N-1 equations can be formed. If number of unknowns are more than N-1, then the problem will be unsolvable

Nodal analysis

Procedure:

1. Choose a node as reference. All voltages shall be measured with respect to this reference (also termed as local ground)
2. Label all the remaining node voltages with respect to the reference
3. Write KCL for all the nodes except the reference
4. Solve equations to find node voltages
5. Calculate branch voltages and currents as asked in questions

For constructing KCL equations, you may choose (obviously its not universal rule, the choice is yours

$$\sum \text{currents entering the node from current sources} = \sum \text{currents leaving the node through resistors}$$

Nodal analysis

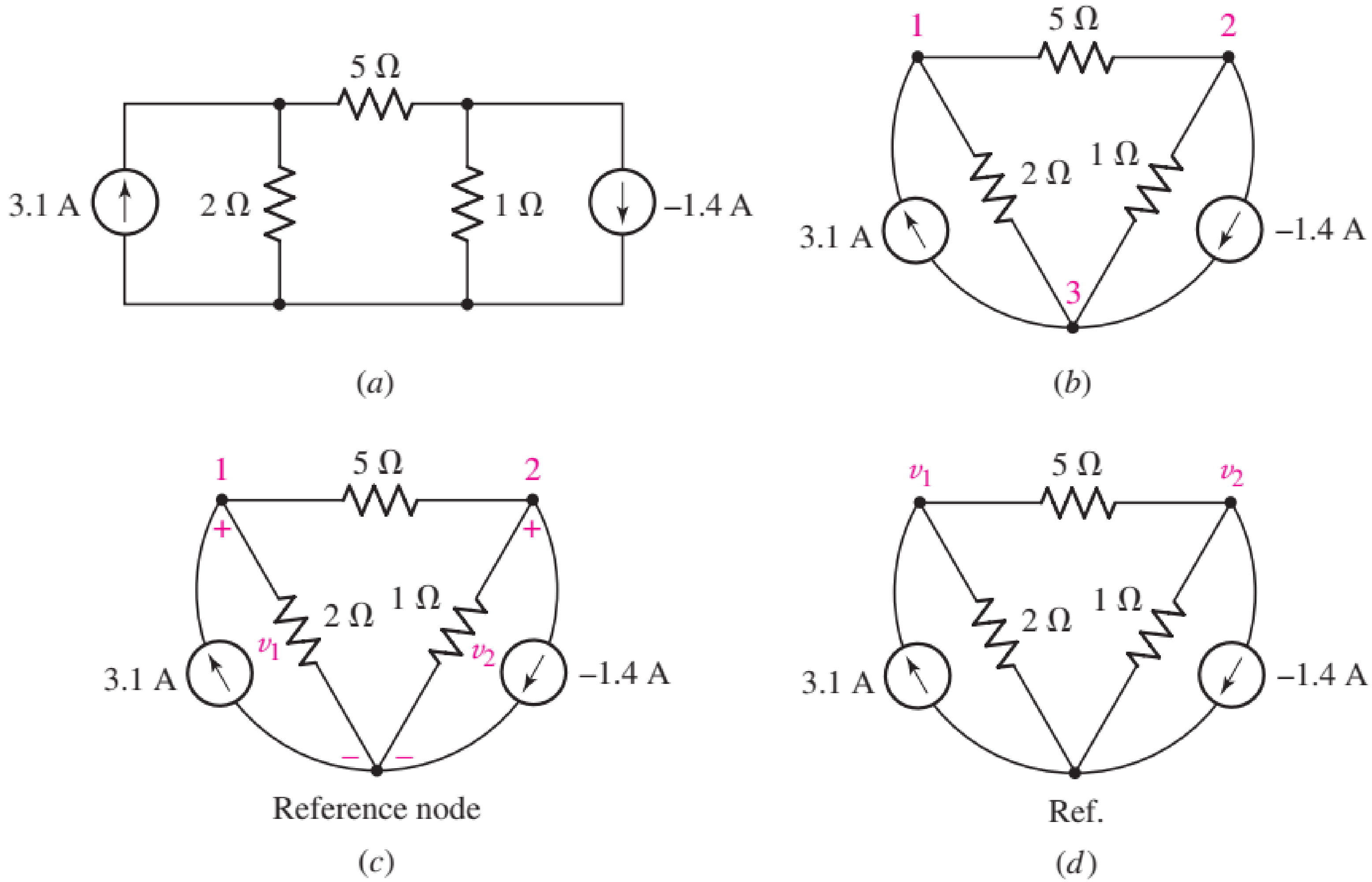


FIGURE 4.1 (*a*) A simple three-node circuit. (*b*) Circuit redrawn to emphasize nodes. (*c*) Reference node selected and voltages assigned. (*d*) Shorthand voltage references. If desired, an appropriate ground symbol may be substituted for "Ref."

Our goal will be to determine the voltage across each element

We now apply KCL to nodes 1 and 2. We do this by equating the total current leaving the node through the several resistors to the total source current entering the node. Thus,

$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1 \qquad [1]$$

or

$$0.7v_1 - 0.2v_2 = 3.1 \qquad [2]$$

At node 2 we obtain

$$\frac{v_2}{1} + \frac{v_2 - v_1}{5} = -(-1.4) \qquad [3]$$

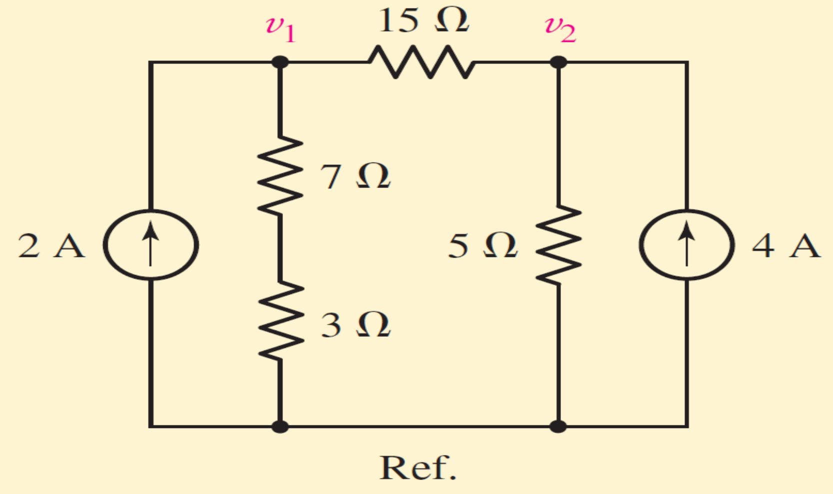
or

$$-0.2v_1 + 1.2v_2 = 1.4 \qquad [4]$$

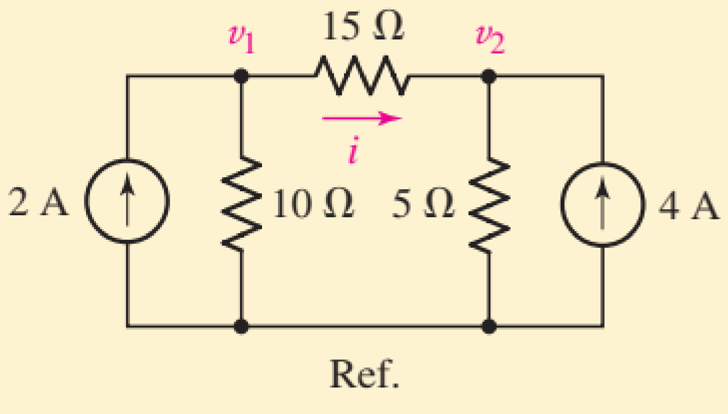
Equations [2] and [4] are the desired two equations in two unknowns, and they may be solved easily. The results are $v_1 = 5$ V and $v_2 = 2$ V.

Nodal Analysis Example 18 and its solution

Example 4.1: Determine the current flowing left to right through the 15 ohm resistor of Fig



Ans: 0



(*b*)

Nodal analysis will directly yield numerical values for the nodal voltages v_1 and v_2, and the desired current is given by $i = (v_1 - v_2)/15$.

Before launching into nodal analysis, however, we first note that no details regarding either the 7 Ω resistor or the 3 Ω resistor are of interest. Thus, we may replace their series combination with a 10 Ω resistor as in Fig. 4.2*b*. The result is a reduction in the number of equations to solve.

Writing an appropriate KCL equation for node 1,

$$2 = \frac{v_1}{10} + \frac{v_1 - v_2}{15} \qquad [5]$$

and for node 2,

$$4 = \frac{v_2}{5} + \frac{v_2 - v_1}{15} \qquad [6]$$

Rearranging, we obtain

$$5v_1 - 2v_2 = 60$$

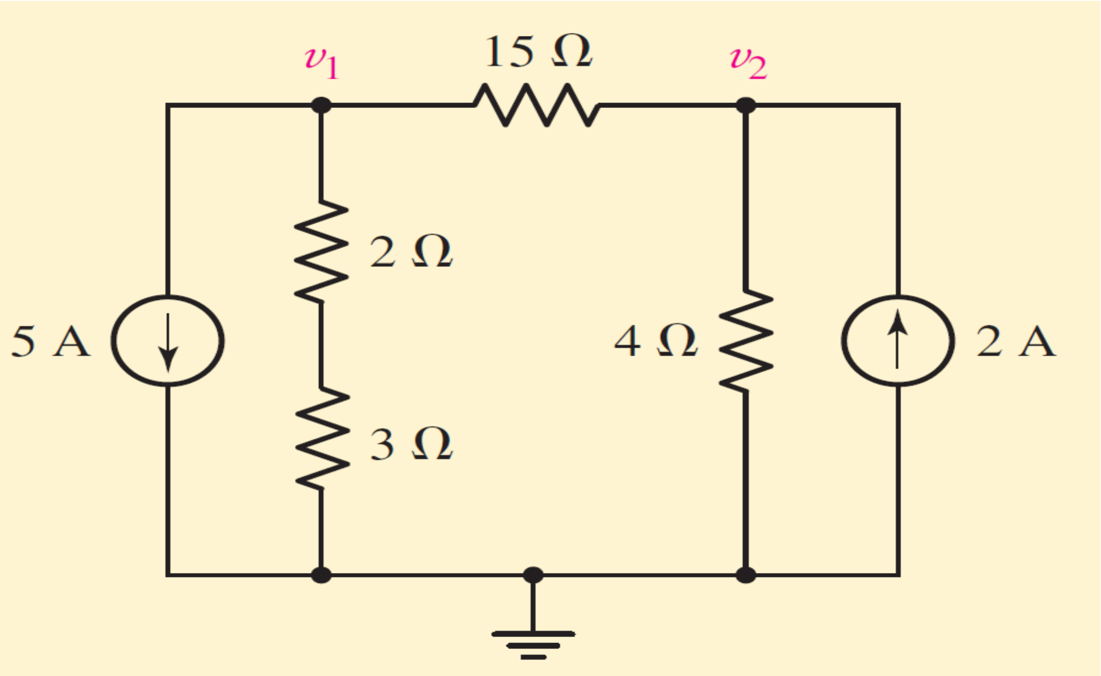
and

$$-v_1 + 4v_2 = 60$$

Solving, we find that $v_1 = 20$ V and $v_2 = 20$ V so that $v_1 - v_2 = 0$. In other words, ***zero current*** is flowing through the 15 Ω resistor in this circuit!

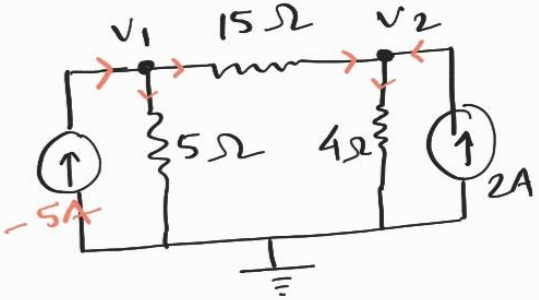
Nodal Analysis; Example 19 and its solution

Practice Problem 4.1: determine the nodal voltages v1 and v2.



Ans: v1 = −145/8 V, v2 = 5/2 V.

An equivalent circuit



at node v_1

$$-5 = \frac{v_1}{5} + \frac{v_1 - v_2}{15}$$

$$\Rightarrow 4v_1 - v_2 = 75 \quad \text{— (1)}$$

at Node v_2

$$2 + \frac{v_1 - v_2}{15} = \frac{v_2}{4}$$

$$\Rightarrow -4v_1 + 19v_2 = 120 \quad \text{— (2)}$$

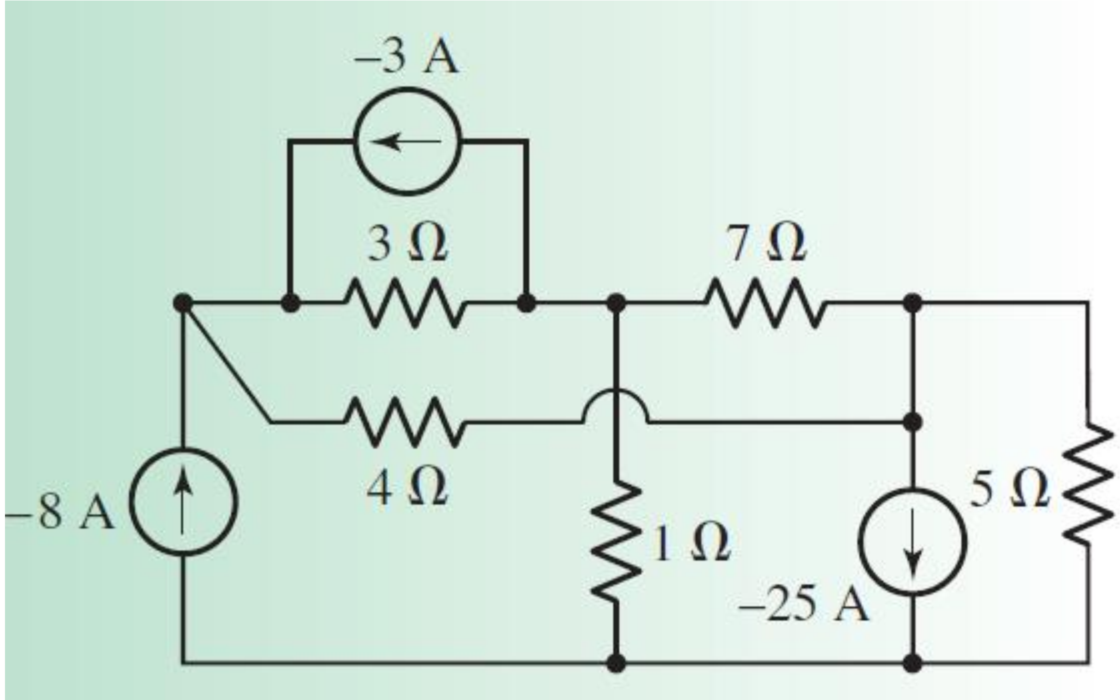
From eqn (1) & (2)

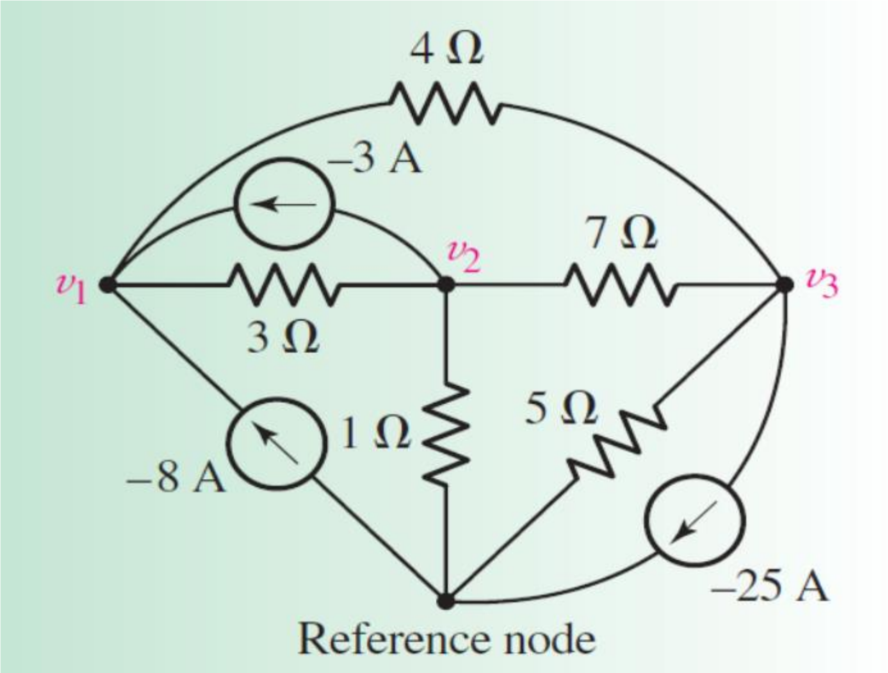
$$18 v_2 = 45 \quad \therefore v_2 = \frac{5}{2}$$

and $v_1 = -\frac{145}{8}$

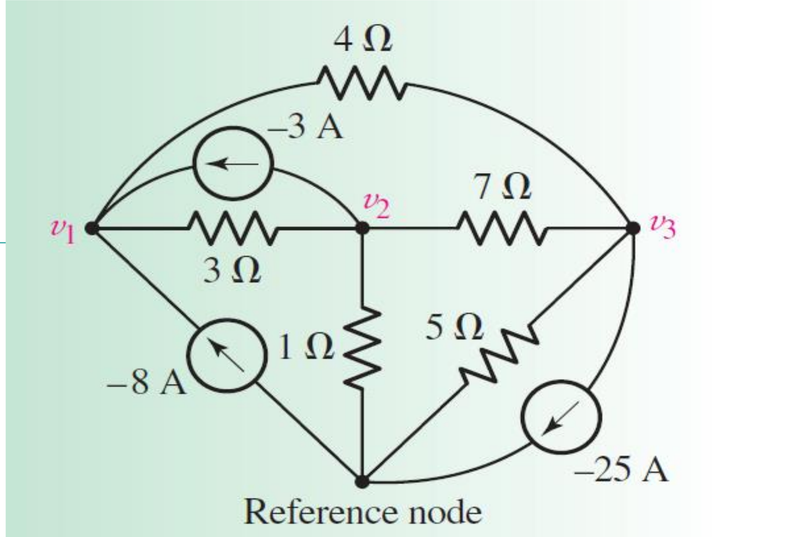
Nodal Analysis

Example 4.2: Determine the nodal voltages for the circuit





Nodal Analysis



Ans: 5.412 V, 7.736 V, 46.32 V

We begin by writing a KCL equation for node 1:

$$-8 - 3 = \frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4}$$

or

$$0.5833v_1 - 0.3333v_2 - 0.25v_3 = -11$$

At node 2:

$$-(-3) = \frac{v_2 - v_1}{3} + \frac{v_2}{1} + \frac{v_2 - v_3}{7}$$

or

$$-0.3333v_1 + 1.4762v_2 - 0.1429v_3 = 3$$

And, at node 3:

$$-(-25) = \frac{v_3}{5} + \frac{v_3 - v_2}{7} + \frac{v_3 - v_1}{4}$$

or, more simply,

$$-0.25v_1 - 0.1429v_2 + 0.5929v_3 = 25$$

$$v_1 = \frac{\begin{vmatrix} -11 & -0.3333 & -0.2500 \\ 3 & 1.4762 & -0.1429 \\ 25 & -0.1429 & 0.5929 \end{vmatrix}}{\begin{vmatrix} 0.5833 & -0.3333 & -0.2500 \\ -0.3333 & 1.4762 & -0.1429 \\ -0.2500 & -0.1429 & 0.5929 \end{vmatrix}} = \frac{1.714}{0.3167} = 5.412 \text{ V}$$

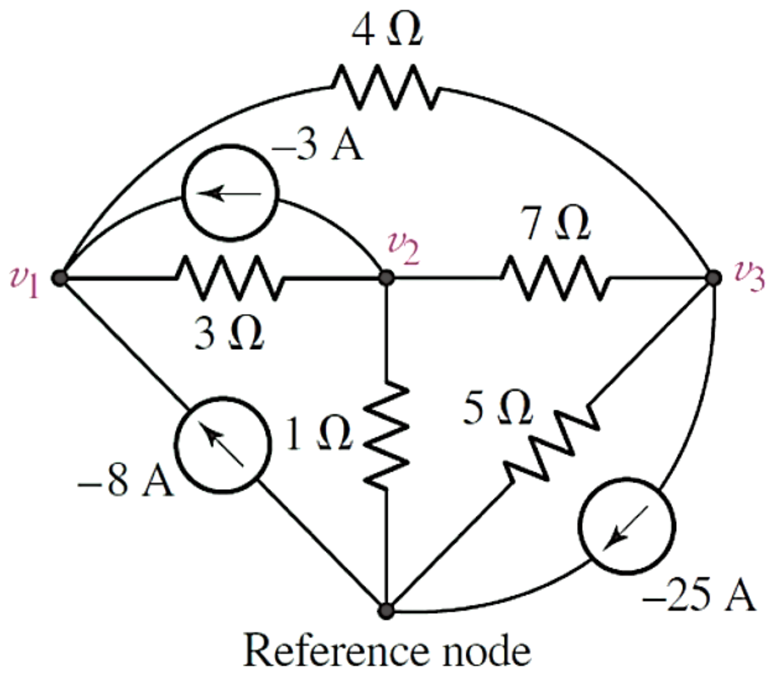
Similarly,

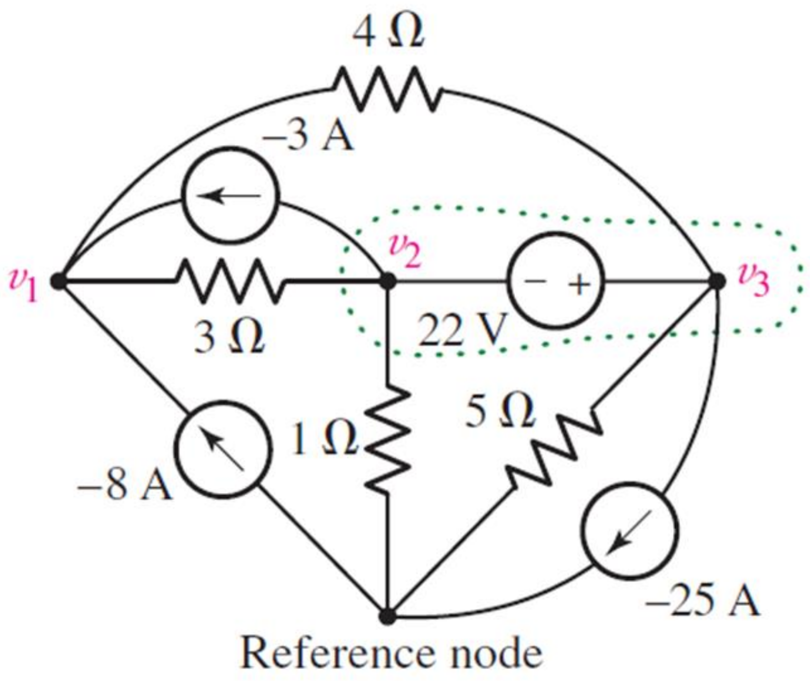
$$v_2 = \frac{\begin{vmatrix} 0.5833 & -11 & -0.2500 \\ -0.3333 & 3 & -0.1429 \\ -0.2500 & 25 & 0.5929 \end{vmatrix}}{0.3167} = \frac{2.450}{0.3167} = 7.736 \text{ V}$$

and

$$v_3 = \frac{\begin{vmatrix} 0.5833 & -0.3333 & -11 \\ -0.3333 & 1.4762 & 3 \\ -0.2500 & -0.1429 & 25 \end{vmatrix}}{0.3167} = \frac{14.67}{0.3167} = 46.32 \text{ V}$$

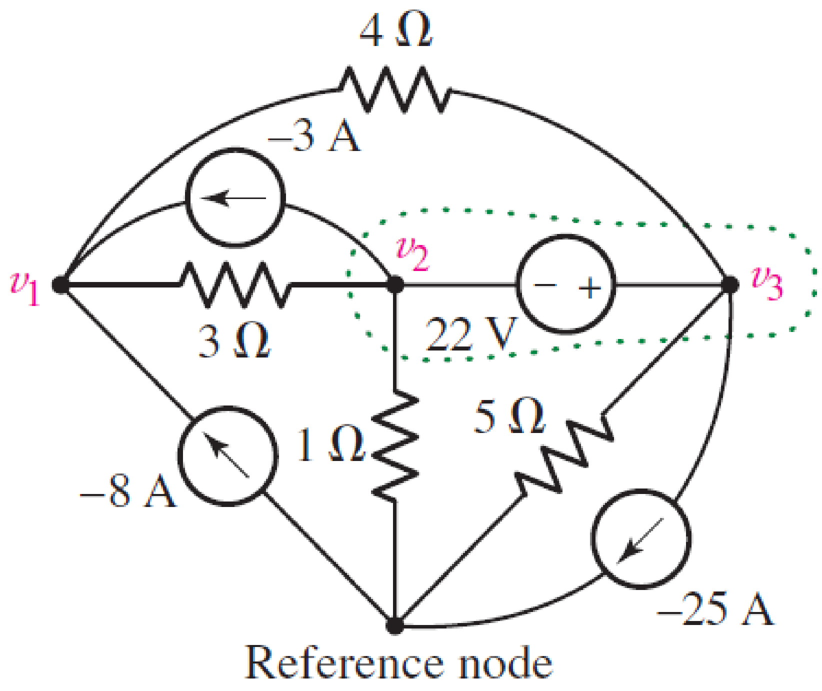
Super-node





When a voltage source is present between two nonreference nodes, a super node may be used to avoid introducing an unknown variable for the current through the voltage source. Instead of applying KCL to each of the two nodes of the voltage source element, KCL is applied to an imaginary node consisting of both the nodes together. **This imaginary node is called a super node**.

Super-node



- Used to handle the Independent Voltage sources in Nodal analysis
- In presence of a voltage source, it is difficult to form KCL equations
- The easier method is to treat node 2, node 3, and the voltage source together as a super-node and apply KCL
- This is okay because if the total current leaving node 2 is zero and the total current leaving node 3 is zero and no extra current is supplied by the voltage source. Then the total current leaving the combination of the two nodes is zero.

Keep in mind that:

A supernode is formed when a voltage source is connected between two nonreference nodes and any elements connected in parallel with it.

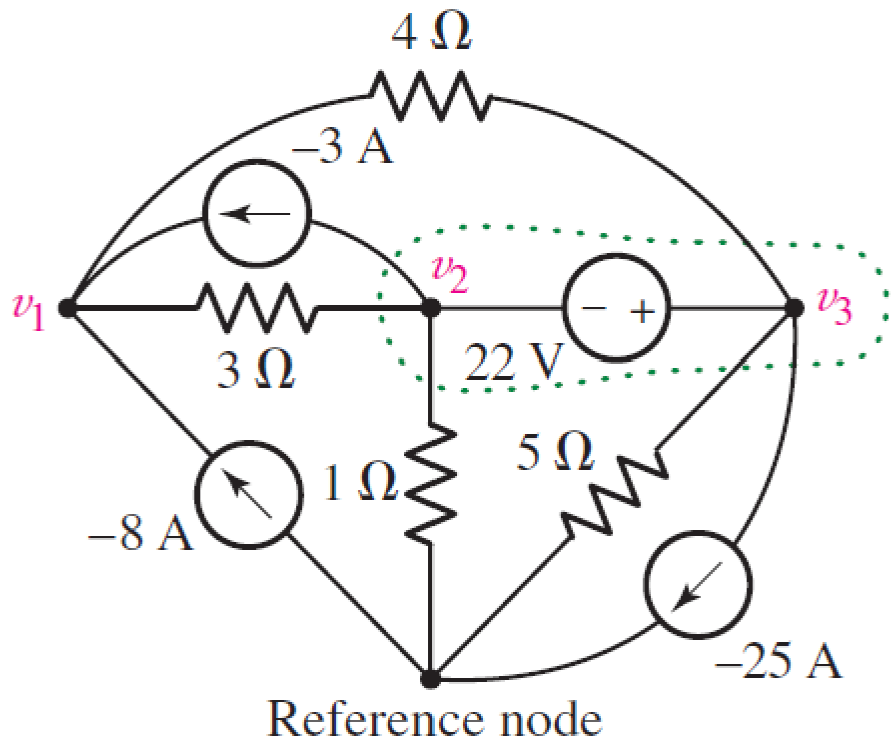
Procedure for Supernode Analysis

The procedure for supernode analysis won't be different with the procedure of nodal analysis we have learned before such as:

1) Identify all the nodes in the circuit including the supernode.
2) Set a node as a reference node. It usually acts as a ground so just add a ground symbol to it.
3) Assign node voltage to other nodes (v_1, v_2, v_3,etc).
4) Remove the voltage source from the circuit first. Because, a supernode has no voltage of its own.
5) Write the KCL supernode equations (currents entering a supernode are equal to the currents leaving the supernode). We have to simultaneously write the nodal equation of two nodes taken as Supernode.
6) Use the KVL equation for the loop where a voltage source exists if you need to find the relationship of two nodes where a voltage source exists. (To complete the missing equation).
7) Write all the KCL equations you can find (make the sum of the leaving current from a branch equal to zero).
8) Use substitution, elimination, Cramer's rule, etc.

Example 4.5

Determine the value of the unknown node voltage v1



Ans: 1.071 V

The KCL equation at node 1 is unchanged from Example 4.2:

$$-8-3=\frac{v_1-v_2}{3}+\frac{v_1-v_3}{4}$$

or

$$0.5833v_1-0.3333v_2-0.2500v_3=-11 \qquad [17]$$

Next we consider the 2-3 supernode. Two current sources are connected, and four resistors. Thus,

$$3+25=\frac{v_2-v_1}{3}+\frac{v_3-v_1}{4}+\frac{v_3}{5}+\frac{v_2}{1}$$

or

$$-0.5833v_1+1.3333v_2+0.45v_3=28 \qquad [18]$$

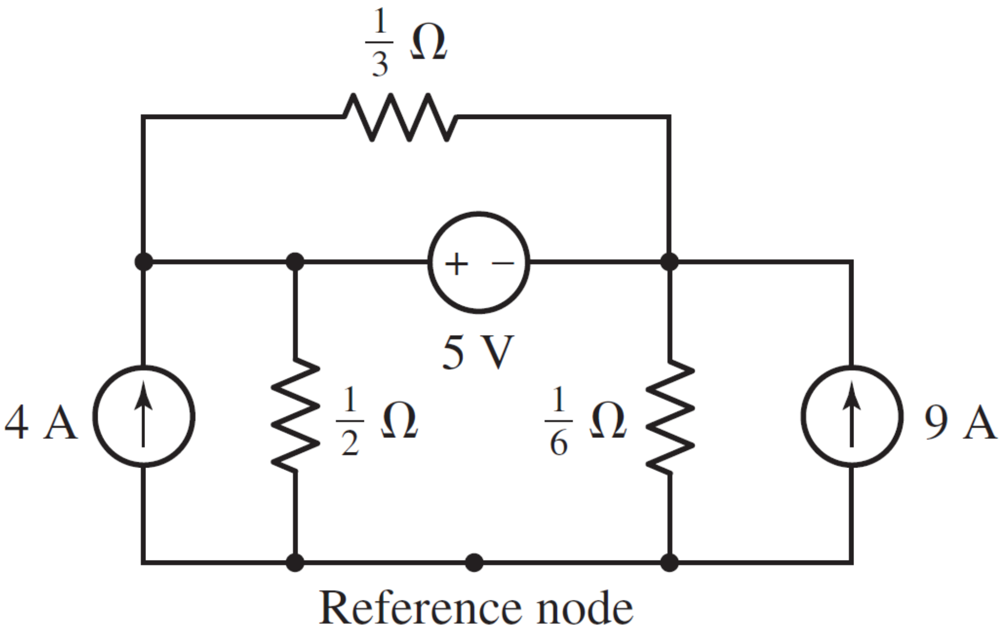
Since we have three unknowns, we need one additional equation, and it must utilize the fact that there is a 22 V voltage source between nodes 2 and 3:

$$v_2-v_3=-22 \qquad [19]$$

Solving Eqs. [17] to [19], the solution for v_1 is 1.071 V.

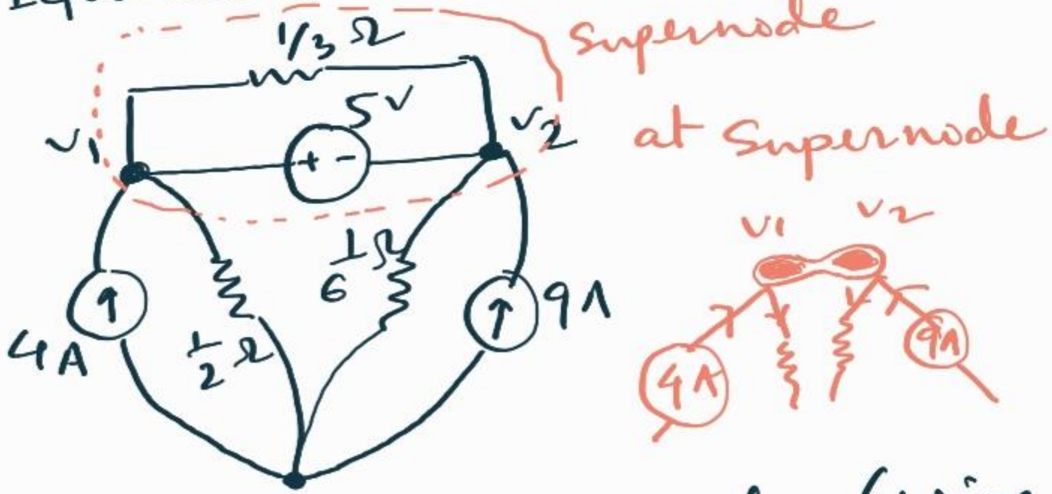
Practice 4.4

For the circuit of Fig., compute the voltage across each current source.



Ans: 5.375 V, 375 mV

Equivalent circuit



Then at the supernode (using KCL)

$$-4 + \frac{v_1}{\frac{1}{2}} + \frac{v_2}{1/6} - 9 = 0$$

$$\therefore\ 2v_1 + 6v_2 = 13 \quad \text{— (1)}$$

and also using KVL,

$$v_1 - v_2 = 5\text{ V} \quad \text{— (2)}$$

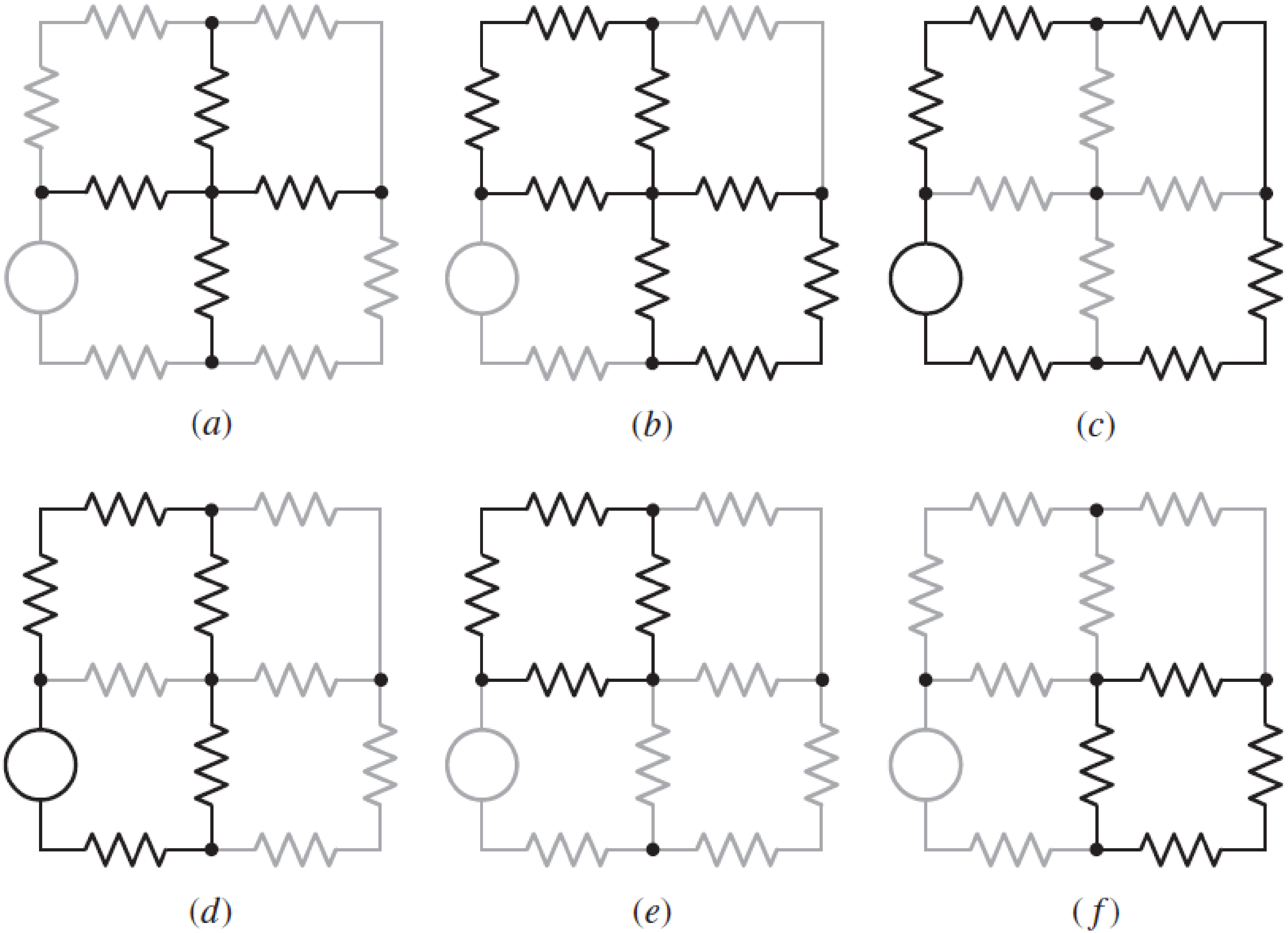
$$\Rightarrow v_1 = 5.375\text{V};\quad v_2 = 0.375\text{ V}$$

Mesh analysis

- Mesh is a loop that does not contain any loop inside
- Approach based on KVL
- What KCL is to node analysis, KVL is to Mesh analysis
- Applicable to planar circuit
- If it is possible to draw the diagram of a circuit on a plane surface in such a way that no branch passes over or under any other branch, then that circuit is said to be a planar circuit.
- If our circuit contains M meshes, then we expect to have M mesh currents and therefore will be required to write M independent equations

Planar circuits are the circuits that can be drawn on a plane surface with no wires crossing each other. **Non-Planar circuits** are the circuits that can be drawn on a plane surface with some wires crossing each other

Mesh analysis



(a) The set of branches identified by the heavy lines is neither a path nor a loop. (b) The set of branches here is not a path, since it can be traversed only by passing through the central node twice. (c) This path is a loop but not a mesh, since it encloses other loops. (d) This path is also a loop but not a mesh. (e, f) Each of these paths is both a loop and a mesh.

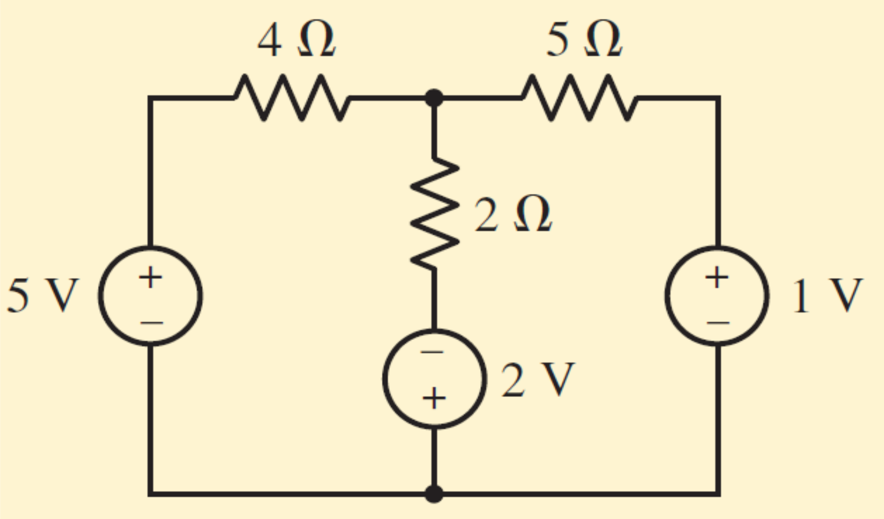
Mesh analysis

Procedure

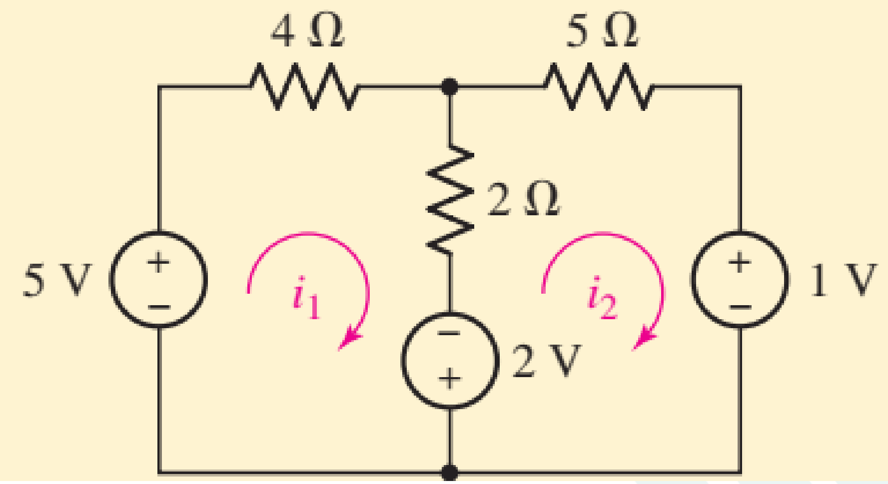
1. Determine if the circuit is a planar circuit. If not, perform nodal analysis instead.
2. Count the number of meshes (M). Redraw the circuit if necessary.
3. Label each of the M mesh currents. Generally, defining all mesh currents to flow clockwise results in a simpler analysis.
4. Write a KVL equation around each mesh. If a current source lies on the periphery of a mesh, no KVL equation is needed and the mesh current is determined by inspection.
5. Express any additional unknowns such as voltages or currents other than mesh currents in terms of appropriate mesh currents. This situation can occur if current sources or dependent sources appear in our circuit.
6. Organize the equations. Group terms according to mesh currents.
7. Solve the system of equations for the mesh currents (there will be M of them).

Example 4.7

Determine the power supplied by the 2 V source



Ans: 2.474 W.



We first define two clockwise mesh currents as shown in Fig. 4.17*b*.

Beginning at the bottom left node of mesh 1, we write the following KVL equation as we proceed clockwise through the branches:

$$-5 + 4i_1 + 2(i_1 - i_2) - 2 = 0$$

Doing the same for mesh 2, we write

$$+2 + 2(i_2 - i_1) + 5i_2 + 1 = 0$$

Rearranging and grouping terms,

$$6i_1 - 2i_2 = 7$$

and

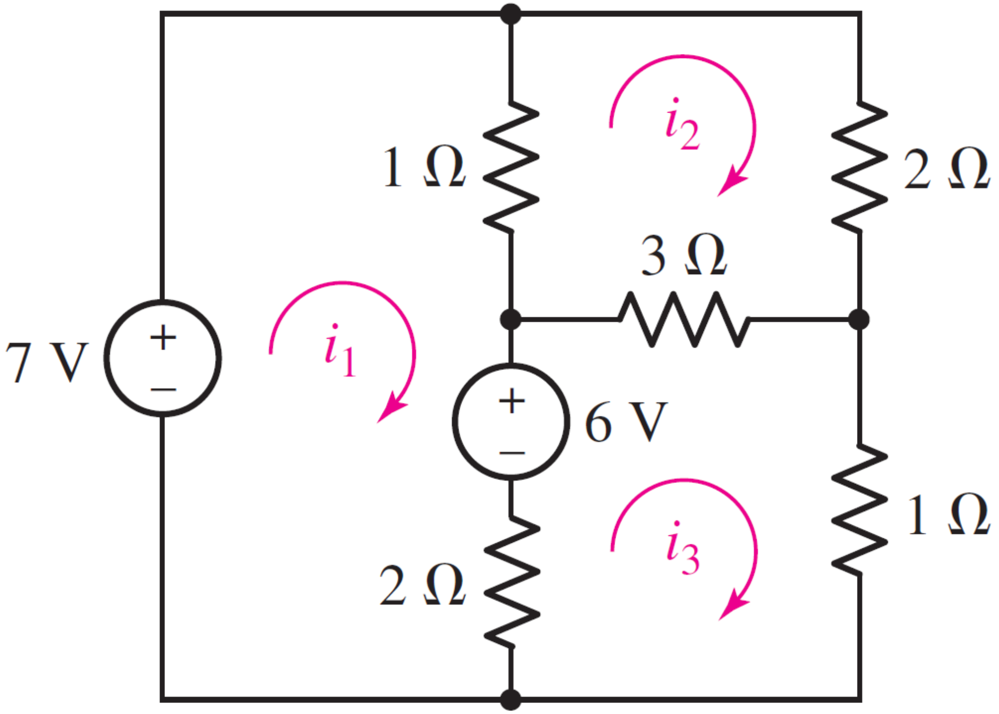
$$-2i_1 + 7i_2 = -3$$

Solving, $i_1 = \frac{43}{38} = 1.132$ A and $i_2 = -\frac{2}{19} = -0.1053$ A.

The current flowing out of the positive reference terminal of the 2 V source is $i_1 - i_2$. Thus, the 2 V source supplies $(2)(1.237) = 2.474$ W.

Example 4.8

Use mesh analysis to determine the three mesh currents in the circuit



Ans: $i1$ = 3 A, $i2$ = 2 A, and $i3$ = 3 A

The three required mesh currents are assigned as indicated in Fig. 4.19, and we methodically apply KVL about each mesh:

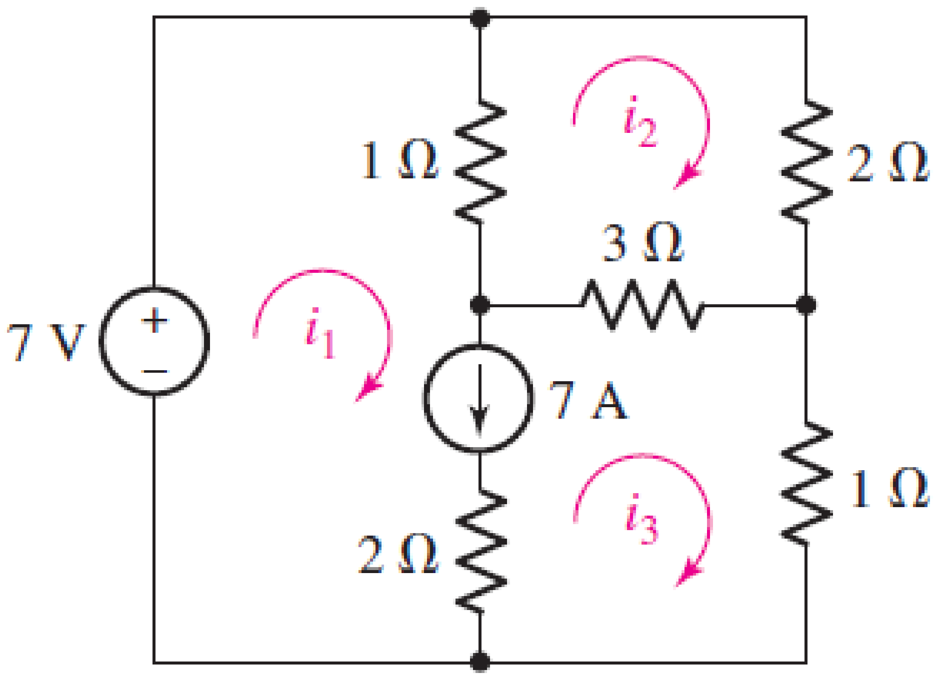
$$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$$
$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$
$$2(i_3 - i_1) - 6 + 3(i_3 - i_2) + 1i_3 = 0$$

Simplifying,

$$3i_1 - i_2 - 2i_3 = 1$$
$$-i_1 + 6i_2 - 3i_3 = 0$$
$$-2i_1 - 3i_2 + 6i_3 = 6$$

and solving, we obtain $i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 3$ A.

Supermesh



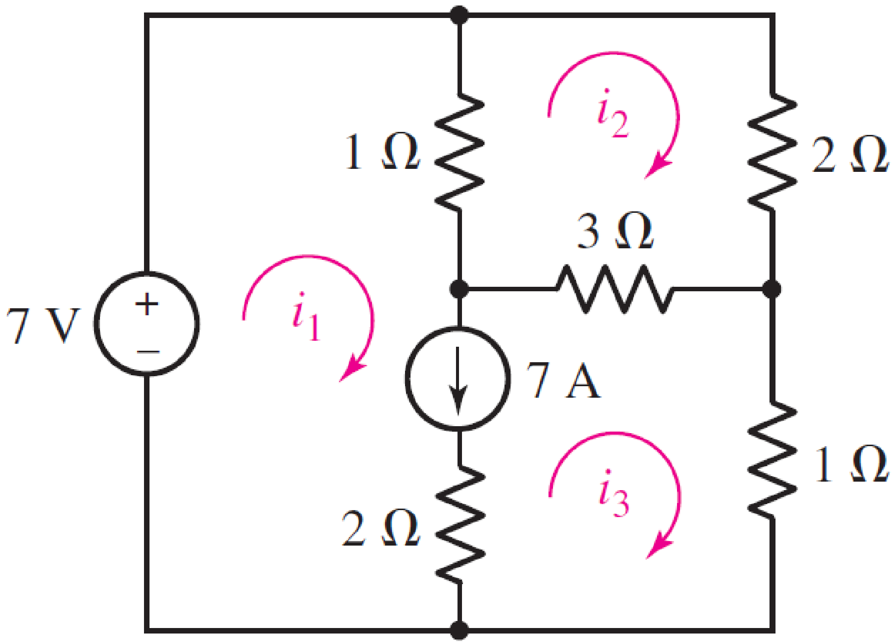
- Used to handle the Independent Current sources in Mesh analysis
- "Super-mesh" is created from two meshes that have a current source as a common element; the current source is in the interior of the super-mesh.
- The number of meshes reduces by 1 for each current source present

A supermesh is formed when two adjacent meshes share a common current source and none of these (adjacent) meshes contain a current source at the outer loop.

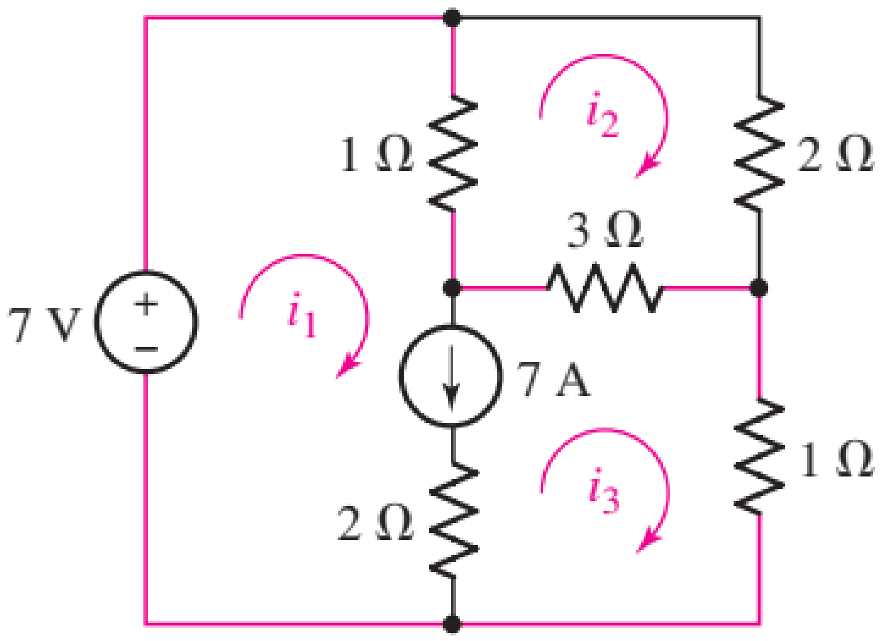
The circuit is first treated as if the current source is not there, like voltage source in a supernode

Example 4.11

Determine the three mesh currents in Fig.



Ans: i1 = 9 A, i2 = 2.5 A, and i3 = 2 A



(*b*)

FIGURE 4.24 (*a*) A three-mesh circuit with an independent current source. (*b*) A supermesh is defined by the colored line.

We note that a 7 A independent current source is in the common boundary of two meshes, which leads us to create a supermesh whose interior is that of meshes 1 and 3 as shown in Fig. 4.24*b*. Applying KVL about this loop,

$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$$

or

$$i_1 - 4i_2 + 4i_3 = 7 \qquad [32]$$

and around mesh 2,

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

or

$$-i_1 + 6i_2 - 3i_3 = 0 \qquad [33]$$

Finally, the independent source current is related to the mesh currents,

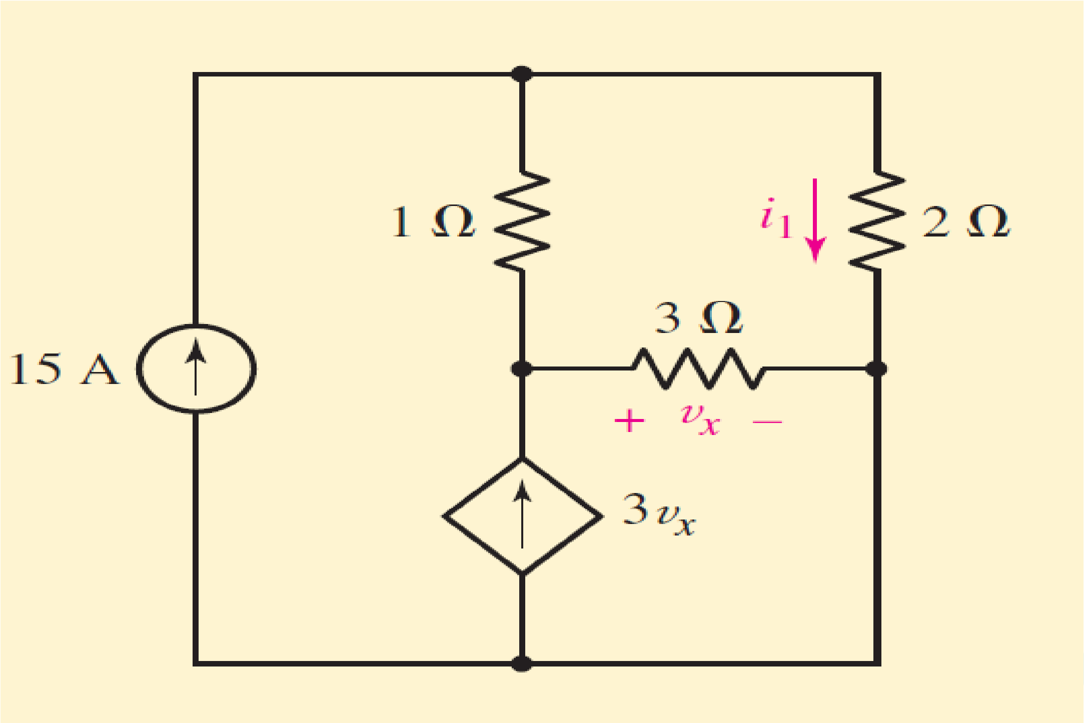
$$i_1 - i_3 = 7 \qquad [34]$$

Solving Eqs. [32] through [34], we find $i_1 = 9$ A, $i_2 = 2.5$ A, and $i_3 = 2$ A.

Nodal analysis for dependent source

Ex. 4.4

Determine the power supplied by the dependent source



Ans: 55.1 W

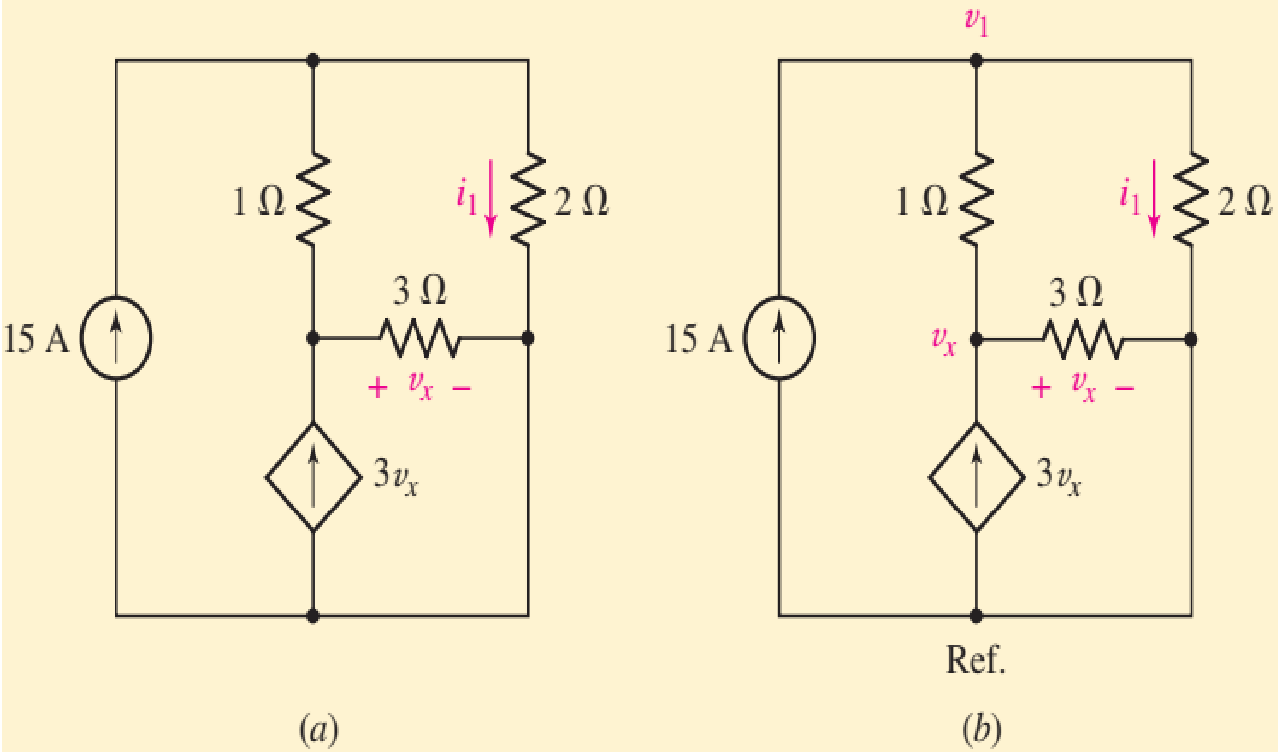


FIGURE 4.7 (*a*) A four-node circuit containing a dependent current source. (*b*) Circuit labeled for nodal analysis.

We select the bottom node as our reference and label the nodal voltages as shown in Fig. 4.7*b*. We have labeled the nodal voltage v_x explicitly for clarity. Note that our choice of reference node is important in this case; it led to the quantity v_x being a nodal voltage.

Our KCL equation for node 1 is

$$15 = \frac{v_1 - v_x}{1} + \frac{v_1}{2} \qquad [15]$$

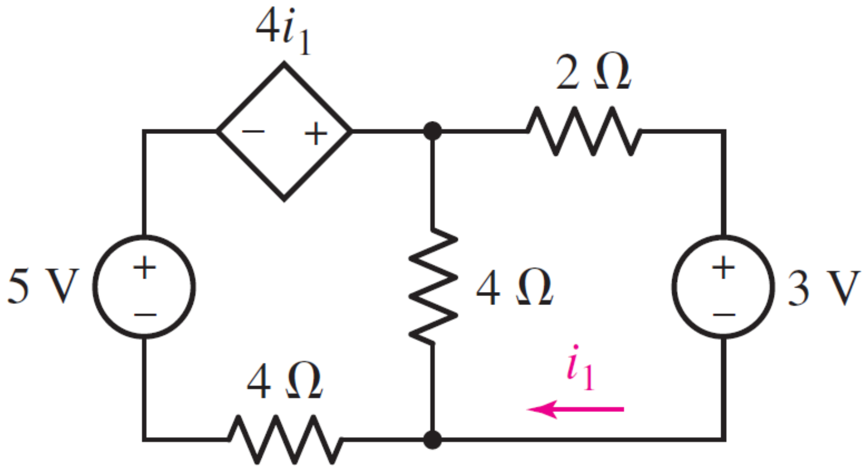
and for node *x* is

$$3v_x = \frac{v_x - v_1}{1} + \frac{v_2}{3} \qquad [16]$$

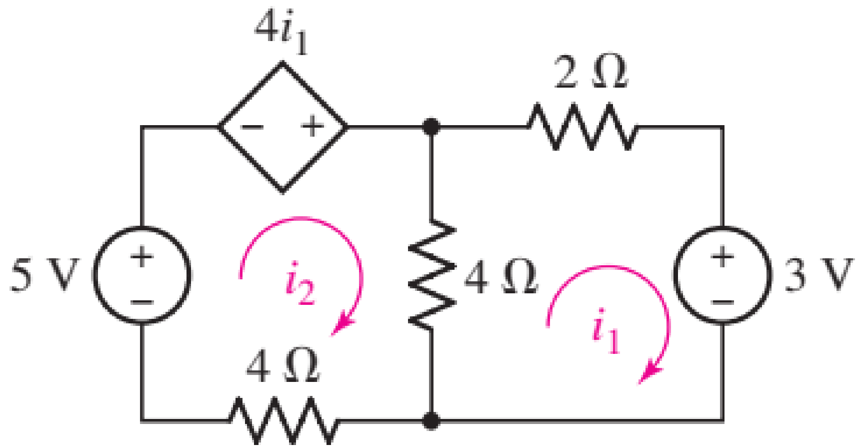
Grouping terms and solving, we find that $v_1 = \frac{50}{7}$ V and $v_x = -\frac{30}{7}$ V. Thus, the dependent source in this circuit generates $(3v_x)(v_x) = 55.1$ W.

Mesh analysis with dependent source

Ex 4.9; Determine the current i1 in the circuit



Ans: −250 mA



The current i_1 is actually a mesh current, so rather than redefine it we label the rightmost mesh current i_1 and define a clockwise mesh current i_2 for the left mesh, as shown in Fig. 4.21*b*.

For the left mesh, KVL yields

$$-5 - 4i_1 + 4(i_2 - i_1) + 4i_2 = 0 \qquad [27]$$

and for the right mesh we find

$$4(i_1 - i_2) + 2i_1 + 3 = 0 \qquad [28]$$

Grouping terms, these equations may be written more compactly as

$$-8i_1 + 8i_2 = 5$$

and

$$6i_1 - 4i_2 = -3$$

Solving, $i_2 = 375$ mA, so $i_1 = -250$ mA.

What method should be used?

- If node voltages are required, nodal analysis is preferred
- If mesh current is required, mesh analysis may be used
- For circuits with dependent source, apply a method according to the controlling quantity