V = vector space of all polynomials of degree less than or equal to 3.

$\mathbb{P}_3$

$\mathbb{P}_n$

$\operatorname{sec} B \rightarrow \mathbb{R}^n(t)$

$$a_0 + a_1 t + a_2 t^2 + a_3 t^3$$
$$b_0 + b_1 t + b_2 t^2 + b_3 t^3.$$

(a_0, a_1, a_2, a_3) $(0,0,0,0)$

$$\mathbb{P}_3 \overset{\text{bijection}}{\longleftrightarrow} \mathbb{R}^4$$

$$\mathbb{P}_n \longleftrightarrow \mathbb{R}^{n+1}$$

$0 = 0t^2$ $(\underline{a_0}, \dots, a_n)$

$\operatorname{span}\{t^2\}$
$\|$
$\{p(t) \in \mathbb{P}_n \mid p(t) = \underline{a}t^2\}$
$(0, 0, a, 0 \dots 0)$

$S := \{p(t) \in \mathbb{P}_n \mid p(0) = 0\}$
$(0, a_1, a_2, \dots, a_n)$

$p(t) = a_0 + a_1 t + \cdots + a_n t^n$

$p(0) = a_0$.

$\|$
$\operatorname{span}\{t, \dots, t^n\}$

$p(0) = 0$

$q(0) = 0$

$p+q(0) = 0.$

$cp(0) = 0.$

$p+q \in S$

$cp \in S.$

$(a_0, a_1, a_2, 0)$ $\quad$ $(a_0, a_1, 0, 0)$.

$\boxed{a_2 \neq 0}$ ✓ $\rightarrow$ $\boxed{a_1 \neq 0}$

$V = \mathbb{P}_3$.

$1 + t$

$1 - t$

1. all linear polynomials ✗
2. all constant polys. ✓
3. all quadratic polys. ✗

$p_1(t) = 1 + t^2$

$p_2(t) = 1 - t^2$

$a x^2 + b x + c$

a

2) $\leftarrow$ constant

not a quadratic poly.

$$\boxed{xy = k} \quad z = k \quad \underline{z \overset{f(x,y)}{=} -xy}$$

$(x_1 + x_2, y_1 + y_2, z_1 + z_2)$ $\quad 2x_1 = y_1 - z_1$

$2x_2 = y_2 - z_2$

$2(x_1 + x_2) = y_1 + y_2 - (z_1 + z_2)$

$S_1 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 ; \boxed{xy + z = 0} \right\}$

$(x, y, z) \in \mathbb{R}$

$S_2 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \boxed{2x = y - z} \right\}$ $x, y, z \in \mathbb{R}$

$S_3 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; x = 6t, \; y = 4t, \; t \in \mathbb{R} \right\}$

$t \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad t, z \in \mathbb{R}$

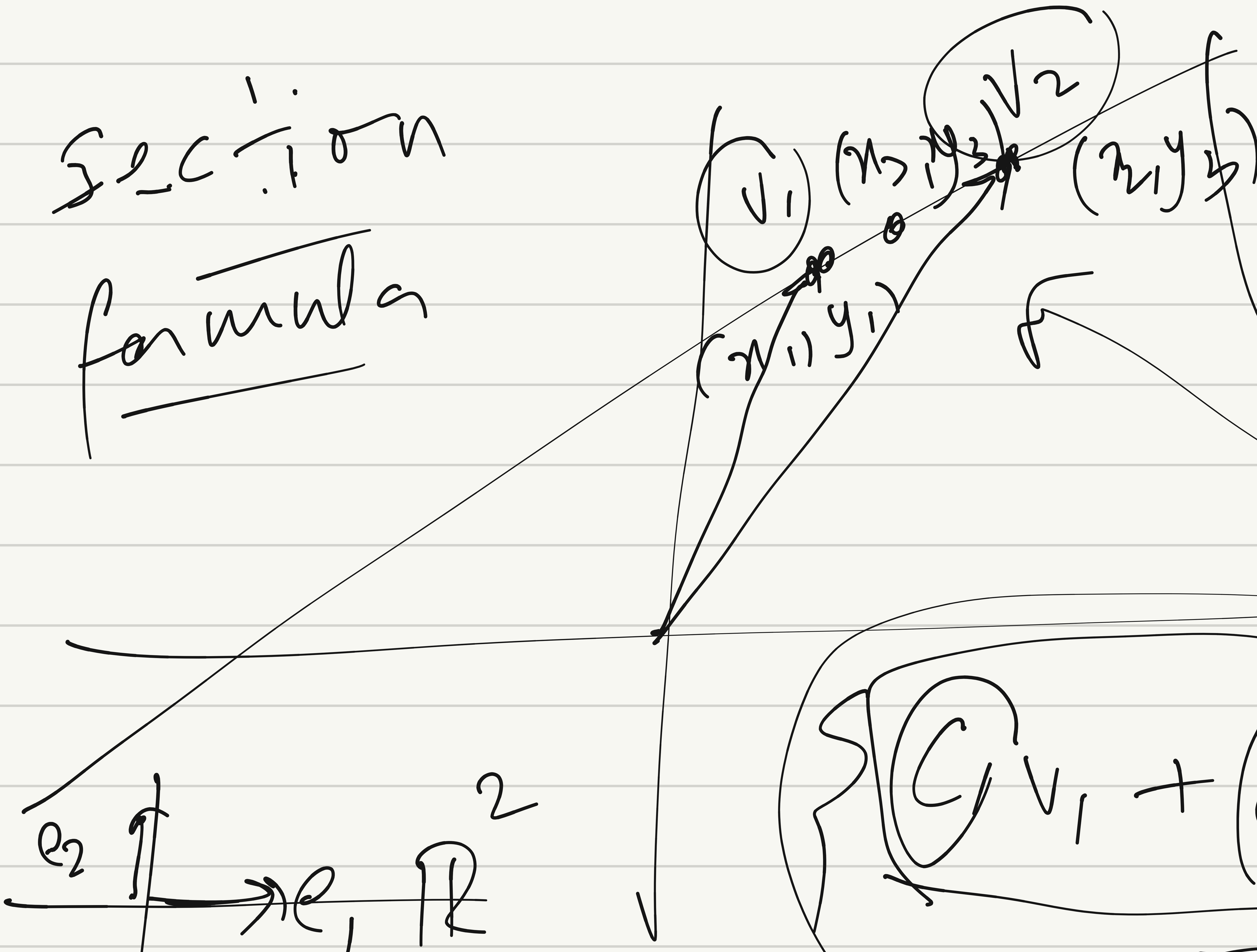
$\text{span}\left\{ \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$xy + z = 0$ $\quad (1, 0, 0)$

$z = 0$ $\quad (0, 1, 0)$

$1 + 0 = 0 \leftarrow (1, 1, 0)$

section formula



(x_1, y_1)

(x_2, y_2)

(x_3, y_3)

$\{c_1 v_1 + c_2 v_2 : c_1, c_2 \in \mathbb{R}, \; c_1 + c_2 = 1\}$

$x + y = 1$

(3) $e_1 \to e_2$

$e_2 \to e_1$

$x_1, \ldots, x_n \in \mathbb{R}$

l. i. over $\mathbb{Q}$, if

$q_1 x_1 + \cdots + q_n x_n = 0$

has no non-trivial rational solution

$e^{i\pi}, \pi, 1$ | $e, \pi, \pi - e, \pi e \leftarrow$

$\cos\pi + i\sin\pi = -1$ | $e, \pi, 1, i \leftarrow$ l. i.

$1 \cdot e - 1 \cdot \pi + 1 \cdot (\pi - e) + 0 \cdot \pi e$

$2c_1 = 0,\ 3c_2 = 0,\ 5c_3 = 0$

$\Rightarrow c_1 = c_2 = c_3 = 0$

$a, b, c \in \mathbb{R}^n$

$\uparrow$

l.i

$c_1(2a) + c_2(3b) + c_3(5c) = 0$

$2c_1 a + 3c_2 b + 5c_3 c = 0$

① $2a, 3b, 5c$.

② $2a-b, 2b-c, 2c-a$

③ $a+b, b+c, c+a$

④ $a-b, b-c, c-a$. $\leftarrow$

$$c_1(2a - b) + c_2(2b - c) + c_3(2c - a) = 0$$

$$\begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{bmatrix} \downarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0 & -1/2 \\ 0 & -1 & 2 \end{bmatrix}$$

$$(2c_1 - c_3)a + (-c+2c_2)b + c(-c_2 + 2c_3) = 0.$$

$$2c_1 - c_3 = 0$$
$$-c_1 + 2c_2 = 0$$
$$-c_2 + 2c_3 = 0$$