Karnaugh Map or K-Map

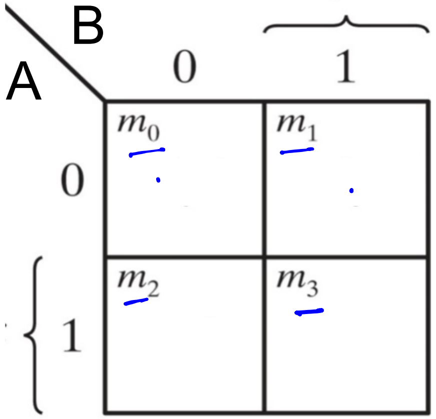
minimization → CSOP → SOP ↙ min operⁿ + min literals. Boolean Algebra

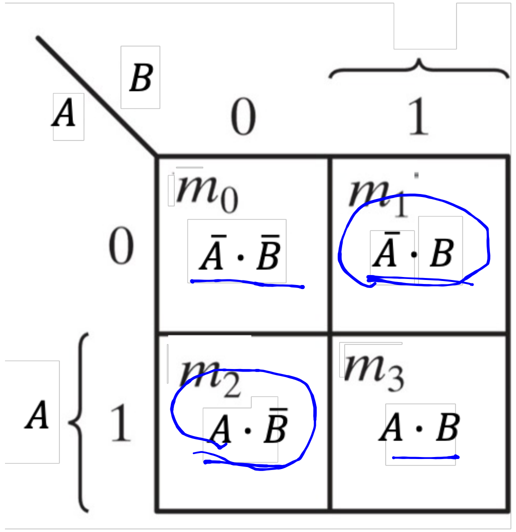
- Boolean Algebra: tedious and awkward because it lacks specific rules to predict each succeeding step
- The Karnaugh map (K-map) method provides a simple, straightforward procedure for minimizing Boolean functions.
- The K-map may be regarded as a pictorial form of a truth table.
- A K-map is a diagram made up of squares, with each square representing one minterm/maxterm of the variables of interest.

  SOP POS
- The simplified expressions produced by the map are always in one of the two standard forms: Sum Of Products (SOP) or Product Of Sums (POS).
- Simplest algebraic expression -> a minimum number of terms and with the smallest possible number of literals in each term.

Karnaugh Map (K-Map) with Minterms

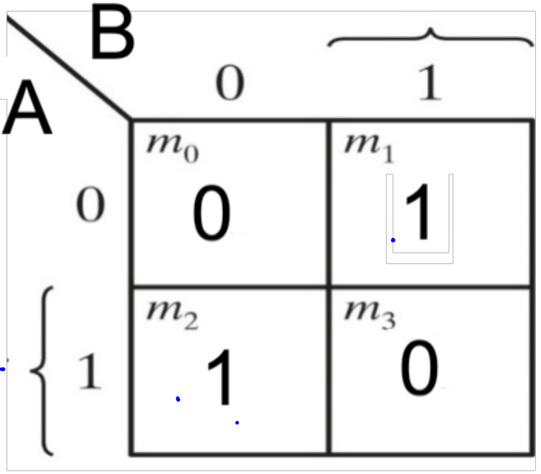
A	B	f
0	0	0
0	1	1
1	0	1
1	1	0





$f(A,B)=\sum m(1,2)$

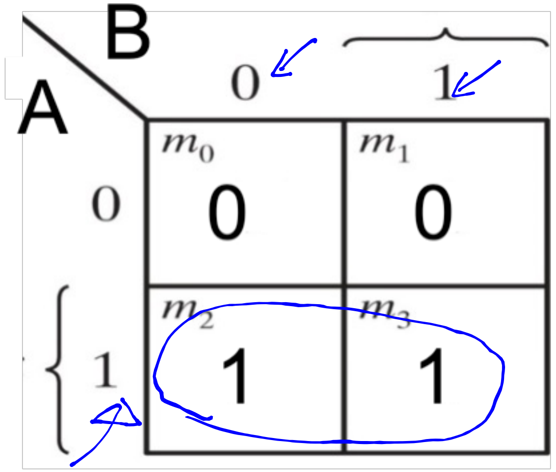
$f(A,B)=\bar{A}\cdot B+A\cdot\bar{B}$



$f(A,B)=\sum m(2,3)$

$f(A,B)=A\cdot\bar{B}+A\cdot B=A$

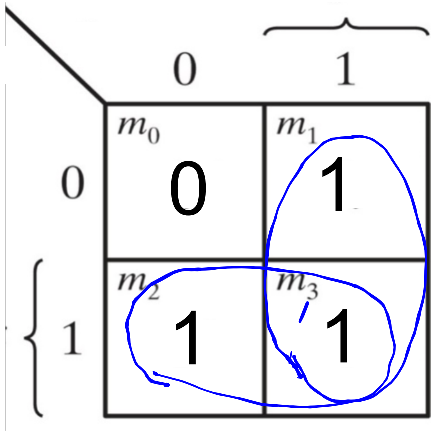
$A\cdot(B+\bar{B})=A$



$\rightarrow A$

two variables

$f(A,B)=A+B$

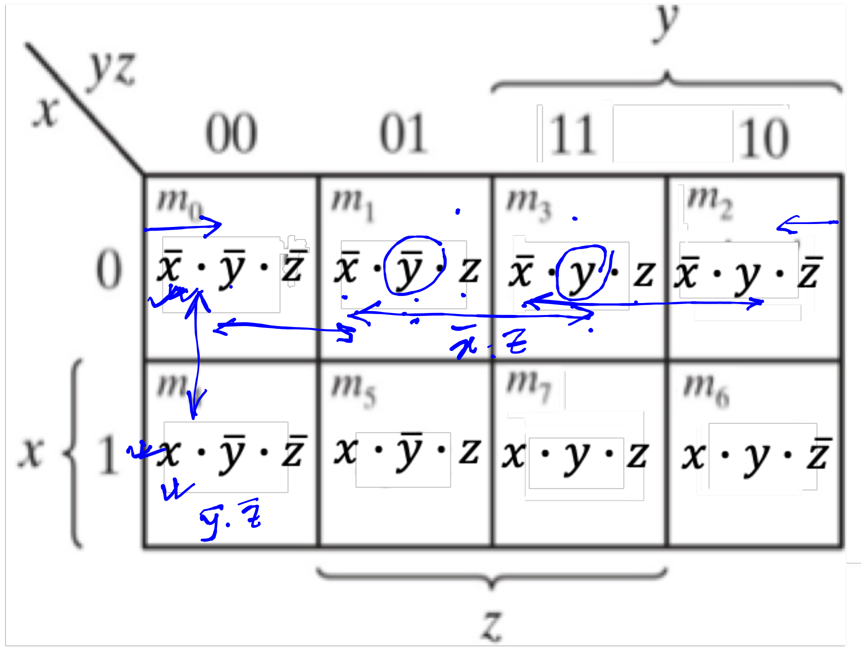


Karnaugh Map (Minterm Representation) for three variables:

8 squares.

0
1
2
3
4
5
6
7
~~8~~

000
001
010
011
⋮
111



?

Neighbours
↓
Combined

000
001
011
010

m_0 & m_2

K-map → cyclic

Any two minterms in adjacent squares (vertically or horizontally, but not diagonally, adjacent) that are ORed together will cause a removal of the complimented variables.

Karnaugh Map (3 Variable Map)

$x \backslash yz$	00	01	11	10
0	m_0 $\bar{x}\cdot\bar{y}\cdot\bar{z}$	m_1 $\bar{x}\cdot\bar{y}\cdot z$	m_3 $\bar{x}\cdot y\cdot z$	m_2 $\bar{x}\cdot y\cdot\bar{z}$
x 1	m_4 $x\cdot\bar{y}\cdot\bar{z}$	m_5 $x\cdot\bar{y}\cdot z$	m_7 $x\cdot y\cdot z$	m_6 $x\cdot y\cdot\bar{z}$

y spans columns 11 and 10; z spans columns 01 and 11.

Minterm Representation

$(x+y+\bar{z})\cdot(x+\bar{y}+\bar{z})$

$(x+\bar{z})\cdot(x+\bar{z})$

$+\,(x+\bar{z})\cdot\bar{y}\ \ldots\ldots$

$x \backslash yz$	00	01	11	10
0	M_0 $x+y+z$	M_1 $x+y+\bar{z}$	M_3 $x+\bar{y}+\bar{z}$	M_2 $x+\bar{y}+z$
x 1	M_4 $\bar{x}+y+z$	M_5 $\bar{x}+y+\bar{z}$	M_7 $\bar{x}+\bar{y}+\bar{z}$	M_6 $\bar{x}+\bar{y}+z$

y spans columns 11 and 10; z spans columns 01 and 11.

$(x+\bar{z})$

two changes.

Maxterm Representation

Any two minterms(maxterms) in adjacent squares (vertically or horizontally, but not diagonally, adjacent) that are ORed(ANDed) together will cause a removal of the complimented variables.

Karnaugh Map (4 Variable Map)

$\bar{A}\cdot\bar{C}$

AB \ CD	00	01	11	10
00	m_0 $\bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}$	m_1 $\bar{A}\cdot\bar{B}\cdot\bar{C}\cdot D$	m_3 $\bar{A}\cdot\bar{B}\cdot C\cdot D$	m_2 $\bar{A}\cdot\bar{B}\cdot C\cdot\bar{D}$
01	m_4 $\bar{A}\cdot B\cdot\bar{C}\cdot\bar{D}$	m_5 $\bar{A}\cdot B\cdot\bar{C}\cdot D$	m_7 $\bar{A}\cdot B\cdot C\cdot D$	m_6 $\bar{A}\cdot B\cdot C\cdot\bar{D}$
11	m_{12} $A\cdot B\cdot\bar{C}\cdot\bar{D}$	m_{13} $A\cdot B\cdot\bar{C}\cdot D$	m_{15} $A\cdot B\cdot C\cdot D$	m_{14} $A\cdot B\cdot C\cdot\bar{D}$
10	m_8 $A\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}$	m_9 $A\cdot\bar{B}\cdot\bar{C}\cdot D$	m_{11} $A\cdot\bar{B}\cdot C\cdot D$	m_{10} $A\cdot\bar{B}\cdot C\cdot\bar{D}$

$\bar{B}\bar{D}$

C

A

AB \ CD	00	01	11	10
00	M_0 $A+B+C+D$	M_1 $A+B+C+\bar{D}$	M_3 $A+B+\bar{C}+\bar{D}$	M_2 $A+B+\bar{C}+D$
01	M_4 $A+\bar{B}+C+D$	M_5 $A+\bar{B}+C+\bar{D}$	M_7 $A+\bar{B}+\bar{C}+\bar{D}$	M_6 $A+\bar{B}+\bar{C}+D$
11	M_{12} $\bar{A}+\bar{B}+C+D$	M_{13} $\bar{A}+\bar{B}+C+\bar{D}$	M_{15} $\bar{A}+\bar{B}+\bar{C}+\bar{D}$	M_{14} $\bar{A}+\bar{B}+\bar{C}+D$
10	M_8 $\bar{A}+B+C+D$	M_9 $\bar{A}+B+C+\bar{D}$	M_{11} $\bar{A}+B+\bar{C}+\bar{D}$	M_{10} $\bar{A}+B+\bar{C}+D$

Any two minterms/maxterms in adjacent squares (vertically or horizontally, but not diagonally, adjacent) that are ORed/ANDed together will cause a removal of the complimented variables.

K-Map (Example):

$$f(A,B,C) = \sum m(0,1,2,3,4,5)$$

$B = 1$ (columns 11, 10)

$A \backslash BC$	00	01	11	10
0	m_0 1	m_1 1	m_3 1	m_2 1
$A = 1$: 1	m_4 1	m_5 1	m_7 0	m_6 0

$C = 1$ (columns 01, 11)

$$f(A,B,C) = \sum m(0,1,2,3,7)$$

$B = 1$ (columns 11, 10)

$A \backslash BC$	00	01	11	10
0	m_0 1	m_1 1	m_3 1	m_2 1
$A = 1$: 1	m_4 0	m_5 0	m_7 1	m_6 0

$C = 1$ (columns 01, 11)

K-Map (Example):

$$f(A,B,C,D) = \sum m(0,4,8,10,11,12,13,15)$$

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

$$f(A,B,C,D) = \sum m(0,2,4,5,10,11,13,15)$$

AB \ CD	00	01	11	10
00	m_0 1	m_1 0	m_3 0	m_2 1
01	m_4 1	m_5 1	m_7 0	m_6 0
11	m_{12} 0	m_{13} 1	m_{15} 1	m_{14} 0
10	m_8 0	m_9 0	m_{11} 1	m_{10} 1

Truth Table

$$f(A,B,C,D) = \sum m(2,3,5,6,7,10,11,13,14)$$

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

K-Map

$x_2=0$ and $x_1=1$
$f=1$

$$f(x_3,x_2,x_1,x_0) = \overline{x_2}\cdot x_1 + x_3\cdot\overline{x_1}\cdot x_0$$

$x_3x_2 \backslash x_1x_0$	00	01	11	10
00	m_0 0	m_1 0	m_3 1	m_2 1
01	m_4 0	m_5 0	m_7 0	m_6 0
11	m_{12} 0	m_{13} 1	m_{15} 0	m_{14} 0
10	m_8 0	m_9 1	m_{11} 1	m_{10} 1

x_1

x_3

$$f(A,B,C,D) = \bar{C} + \bar{A}\cdot\bar{D} + \bar{B}\cdot\bar{D}$$

$AB \backslash CD$	00	01	11	10
00	m_0 1	m_1 1	m_3 0	m_2 1
01	m_4 1	m_5 1	m_7 0	m_6 1
11	m_{12} 1	m_{13} 1	m_{15} 0	m_{14} 0
10	m_8 1	m_9 1	m_{11} 0	m_{10} 1

K-Map (Maxterms)

$$f(A,B,C) = \prod(0,2,5,7)$$

$A \backslash BC$	00	01	11	10
0	M_0 0	M_1 1	M_3 1	M_2 0
1	M_4 1	M_5 0	M_7 0	M_6 1

$$f(x_3, x_2, x_1, x_0) = \prod M(2,3,5,6,7,10,11,13,14)$$

$x_3x_2 \backslash x_1x_0$	00	01	11	10
00	M_0 1	M_1 1	M_3 0	M_2 0
01	M_4 1	M_5 0	M_7 0	M_6 0
11	M_{12} 1	M_{13} 0	M_{15} 1	M_{14} 0
10	M_8 1	M_9 1	M_{11} 0	M_{10} 0

K-Map Definition:

- A **prime implicant** is a product term obtained by combining maximum possible number of adjacent squares in a K-map with the number of squares combined being a power of 2

AB \ CD	00	01	11	10
00	m_0 $\bar{A}\bar{B}\bar{C}\bar{D}$	m_1 $\bar{A}\bar{B}\bar{C}D$	m_3 $\bar{A}\bar{B}CD$	m_2 $\bar{A}\bar{B}C\bar{D}$
01	m_4 $\bar{A}B\bar{C}\bar{D}$	m_5 $\bar{A}B\bar{C}D$	m_7 $\bar{A}BCD$	m_6 $\bar{A}BC\bar{D}$
11	m_{12} $AB\bar{C}\bar{D}$	m_{13} $AB\bar{C}D$	m_{15} $ABCD$	m_{14} $ABC\bar{D}$
10	m_8 $A\bar{B}\bar{C}\bar{D}$	m_9 $A\bar{B}\bar{C}D$	m_{11} $A\bar{B}CD$	m_{10} $A\bar{B}C\bar{D}$

x \ yz	00	01	11	10
0	m_0 $\bar{x}\cdot\bar{y}\cdot\bar{z}$	m_1 $\bar{x}\cdot\bar{y}\cdot z$	m_3 $\bar{x}\cdot y\cdot z$	m_2 $\bar{x}\cdot y\cdot\bar{z}$
1	m_4 $x\cdot\bar{y}\cdot\bar{z}$	m_5 $x\cdot\bar{y}\cdot z$	m_7 $x\cdot y\cdot z$	m_6 $x\cdot y\cdot\bar{z}$

$2 - 2^1$

$2^2 - 4$

$2^3 - 8$

Prime Implicants: $\bar{x}\cdot y$, $y\cdot z$, $x\cdot z$, $x\cdot \bar{y}$

$$f(x,y,z) = \sum m(2,3,4,5,7)$$

x \ yz	00	01	11	10
0	m_0 0	m_1 0	m_3 1	m_2 1
1	m_4 1	m_5 1	m_7 1	m_6 0

$$f(A,B,C,D) = \sum m(0,2,3,5,7,8,9,10,11,13,15)$$

next slide

~~diagonally~~

AB \ CD	00	01	11	10
00	m_0 1	m_1 0	m_3 1	m_2 1
01	m_4 0	m_5 1	m_7 1	m_6 0
11	m_{12} 0	m_{13} 1	m_{15} 1	m_{14} 0
10	m_8 1	m_9 1	m_{11} 1	m_{10} 1

for MCx

PI: $B\cdot D$, $A\cdot D$, $A\cdot\bar{B}$, $\bar{B}\cdot\bar{D}$, $C\cdot D$, $\bar{B}C$

K-Map Definition:

Thanks Manas

- A prime implicant is an **essential prime implicant** if, it is the only prime implicant that uniquely covers one or more minterms.

EPI

Pair → 1

Quad → 2 variables

exclusive or unique cover

$$f(A,B,C,D)=\sum m(0,2,3,5,7,8,9,10,11,13,15)$$

Oct

AB \ CD	00	01	11	10
00	m_0 1	m_1 0	m_3 1	m_2 1
01	m_4 0	m_5 1	m_7 1	m_6 0
11	m_{12} 0	m_{13} 1	m_{15} 1	m_{14} 0
10	m_8 1	m_9 1	m_{11} 1	m_{10} 1

m_5+m_7 $m_{13}+m_{15}$

complete

$$f(x,y,z)=\sum m(2,3,4,5,7)$$

E.PI $B\cdot D$, $\bar{B}\cdot\bar{D}$

Quads

PI

NEPI → 2

$B\cdot D$; $\bar{B}\cdot\bar{D}$

x \ yz	00	01	11	10
0	m_0 0	m_1 0	m_3 1	m_2 1
1	m_4 1	m_5 1	m_7 1	m_6 0

$x\cdot\bar{y}$, $\bar{x}\cdot y$ EPI

same

EPI m_3 m_7 m_9 m_{11}

$$f(A,B,C,D)=B\cdot D+\bar{B}\cdot\bar{D}+C\cdot D+A\cdot D$$
$$f(A,B,C,D)=B\cdot D+\bar{B}\cdot\bar{D}+C\cdot D+A\cdot\bar{B}$$
$$f(A,B,C,D)=B\cdot D+\bar{B}\cdot\bar{D}+\bar{B}\cdot C+A\cdot D$$
$$f(A,B,C,D)=B\cdot D+\bar{B}\cdot\bar{D}+\bar{B}\cdot C+A\cdot\bar{B}$$

$$f(x,y,z)=x\cdot\bar{y}+\bar{x}\cdot y+y\cdot z$$
$$f(x,y,z)=x\cdot\bar{y}+\bar{x}\cdot y+x\cdot z$$

K-Map Based Minimization

- Find all prime implicants.
- Include all essential prime implicants in solution.
- Include minimum number of other prime implicants to cover all minterms/maxterms not covered by the essential prime implicants.
    - Minimize overlap between the prime implicants while including other prime implicants.

may not be unique

K-Map (HW)

- Find all prime implicants and the essential prime implicants
- $f(x, y, z) = \sum m(0,1,3,4,7)$
- $f(x, y, z) = \sum m(0,1,2,3,4,5,6,7)$
- $f(w, x, y, z) = \sum m(0,2,4,5,10,11,13,15)$
- $f(D, C, B, A) = \prod M(1, 2, 5, 6, 9, 10, 11, 13, 14)$
- $f(w, x, y, z) = \prod M(1, 2, 4, 5, 6, 7, 9, 10, 11, 12, 14)$
- $f(w, x, y, z) = \prod M(2, 3, 4, 5, 6, 10, 13)$

K-Map (HW)

Find minimized SOP

- $f(x, y, z) = \sum m(0,2,3,4,7)$
- $f(w, x, y, z) = \sum m(0,4,8,10,11,12,13,15)$
- $f(w, x, y, z) = \sum m(1,3,5,7,8,10,12,13,14)$

Find minimized POS

- $f(x, y, z) = \prod M(0,1,2,3,5,7)$
- $f(w, x, y, z) = \prod M(0,1,2,7,8,9,11,12,14,15)$
- $f(w, x, y, z) = \prod M(5,10,11,13,14,15)$

K-Map (With Don’t Care):

- In practice, in some applications the function is not specified for certain combinations of the variables.
- As an example, the four-bit binary code for the decimal digits has six combinations that are not used and consequently are considered to be unspecified.
- Functions that have unspecified outputs for some input combinations are called *incompletely specified functions* .
- In most applications, we simply don’t care what value is assumed by the function for the unspecified minterms/maxterms. For this reason, it is customary to call the unspecified minterms/maxterms of a function as *don’t-care conditions*
- These don’t-care conditions can be used on a map to provide further simplification of the Boolean expression.

K-Map (With Don't Care): SOP

$$f(w,x,y,z) = \sum m(2,4,5,6,10) + D(12,13,14,15)$$

$w,x \backslash y,z$	00	01	11	10
00	m_0 0	m_1 0	m_3 0	m_2 1
01	m_4 1	m_5 1	m_7 0	m_6 1
11	m_{12} ϕ	m_{13} ϕ	m_{15} ϕ	m_{14} ϕ
10	m_8 0	m_9 0	m_{11} 0	m_{10} 1

$$f(w,x,y,z) = \bar{w}\cdot x\cdot \bar{y} + \bar{w}\cdot y\cdot \bar{z} + \bar{x}\cdot y\cdot \bar{z}$$

$w,x \backslash y,z$	00	01	11	10
00	m_0 0	m_1 0	m_3 0	m_2 1
01	m_4 1	m_5 1	m_7 0	m_6 1
11	m_{12} ϕ	m_{13} ϕ	m_{15} ϕ	m_{14} ϕ
10	m_8 0	m_9 0	m_{11} 0	m_{10} 1

$$f(w,x,y,z) = x\cdot \bar{y} + y\cdot \bar{z}$$

K-Map (With Don't Care): POS

$$f(w,x,y,z) = \sum m(2,4,5,6,10) + D(12,13,14,15)$$

w,x \ y,z	00	01	11	10
00	m_0 0	m_1 0	m_3 0	m_2 1
01	m_4 1	m_5 1	m_7 0	m_6 1
11	m_{12} ϕ	m_{13} ϕ	m_{15} ϕ	m_{14} ϕ
10	m_8 0	m_9 0	m_{11} 0	m_{10} 1

$$f(x,y,z) = (x+y)\cdot(w+\bar{y}+\bar{z})\cdot(x+\bar{y}+\bar{z})$$

w,x \ y,z	00	01	11	10
00	m_0 0	m_1 0	m_3 0	m_2 1
01	m_4 1	m_5 1	m_7 0	m_6 1
11	m_{12} ϕ	m_{13} ϕ	m_{15} ϕ	m_{14} ϕ
10	m_8 0	m_9 0	m_{11} 0	m_{10} 1

$$f(x,y,z) = (x+y)\cdot(\bar{y}+\bar{z})$$

K-Map (With Don’t Care)

- Assigning the same values to the don’t-cares for both SOP and POS implementations is not always a good choice.
- Sometimes it may be advantageous to give a particular don’t-care the value 1 for SOP implementation and the value 0 for POS implementation, or vice versa.
- In such cases the optimal SOP and POS expressions will represent different functions, but these functions will differ only for the valuations that correspond to these don’t-cares.

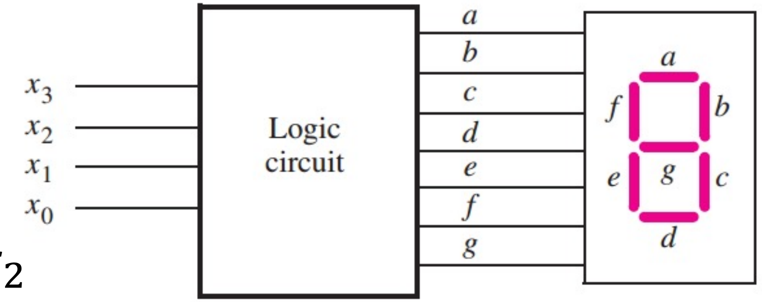
K-Map (7-segment display)

x_3x_2 \ x_1x_0	00	01	11	10
00	m_0 1	m_1 0	m_3 1	m_2 1
01	m_4 0	m_5 1	m_7 1	m_6 1
11	m_{12} ϕ	m_{13} ϕ	m_{15} ϕ	m_{14} ϕ
10	m_8 1	m_9 1	m_{11} ϕ	m_{10} ϕ

$$a = x_1 + x_3 + \overline{x_0}\,\overline{x_2} + x_0x_2$$

x_3x_2 \ x_1x_0	00	01	11	10
00	m_0 1	m_1 0	m_3 1	m_2 1
01	m_4 0	m_5 1	m_7 1	m_6 1
11	m_{12} ϕ	m_{13} ϕ	m_{15} ϕ	m_{14} ϕ
10	m_8 1	m_9 1	m_{11} ϕ	m_{10} ϕ

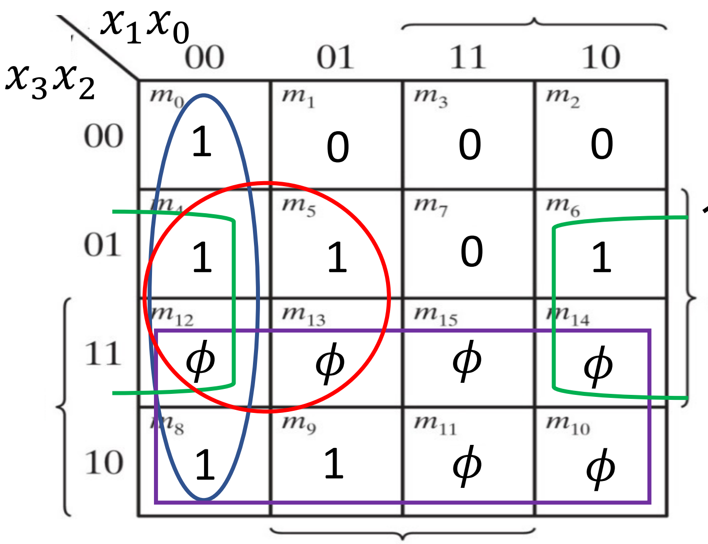
$$a = (x_0 + x_1 + \overline{x_2}) \cdot (\overline{x_0} + x_1 + x_2 + x_3)$$



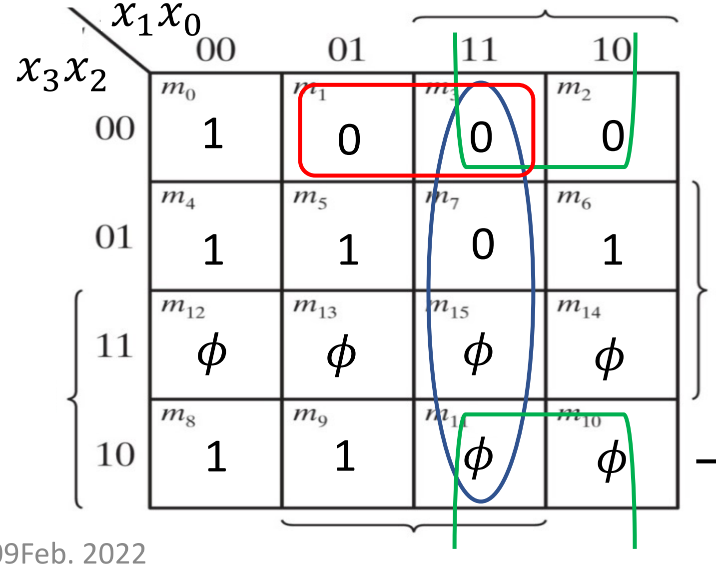
	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

K-Map (7-segment display)

$$f = x_3 + x_2\overline{x_1} + x_2\overline{x_0} + \overline{x_1}\,\overline{x_0}$$



	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1



$$f = (\overline{x_0} + \overline{x_1}) \cdot (x_2 + \overline{x_1}) \cdot (x_3 + x_2 + \overline{x_0})$$

K-Map (5-Variable):

AB \ CDE	000	001	011	010	110	111	101	100
00	m_0	m_1	m_3	m_2	m_6	m_7	m_5	m_4
01	m_8	m_9	m_{11}	m_{10}	m_{14}	m_{15}	m_{13}	m_{12}
11	m_{24}	m_{25}	m_{27}	m_{26}	m_{30}	m_{31}	m_{29}	m_{28}
10	m_{16}	m_{17}	m_{19}	m_{18}	m_{22}	m_{23}	m_{21}	m_{20}

$A = 0$

BC \ DE	00	01	11	10
00	m_0 0	m_1 1	m_3 3	m_2 2
01	m_4 4	m_5 5	m_7 7	m_6 6
11	m_{12} 12	m_{13} 13	m_{15} 15	m_{14} 14
10	m_8 8	m_9 9	m_{11} 11	m_{10} 10

$A = 1$

BC \ DE	00	01	11	10
00	m_{16} 16	m_{17} 17	m_{19} 19	m_{18} 18
01	m_{20} 20	m_{21} 21	m_{23} 23	m_{22} 22
11	m_{28} 28	m_{29} 29	m_{31} 31	m_{30} 30
10	m_{24} 24	m_{25} 25	m_{27} 27	m_{26} 26

Shannon's Theorem:

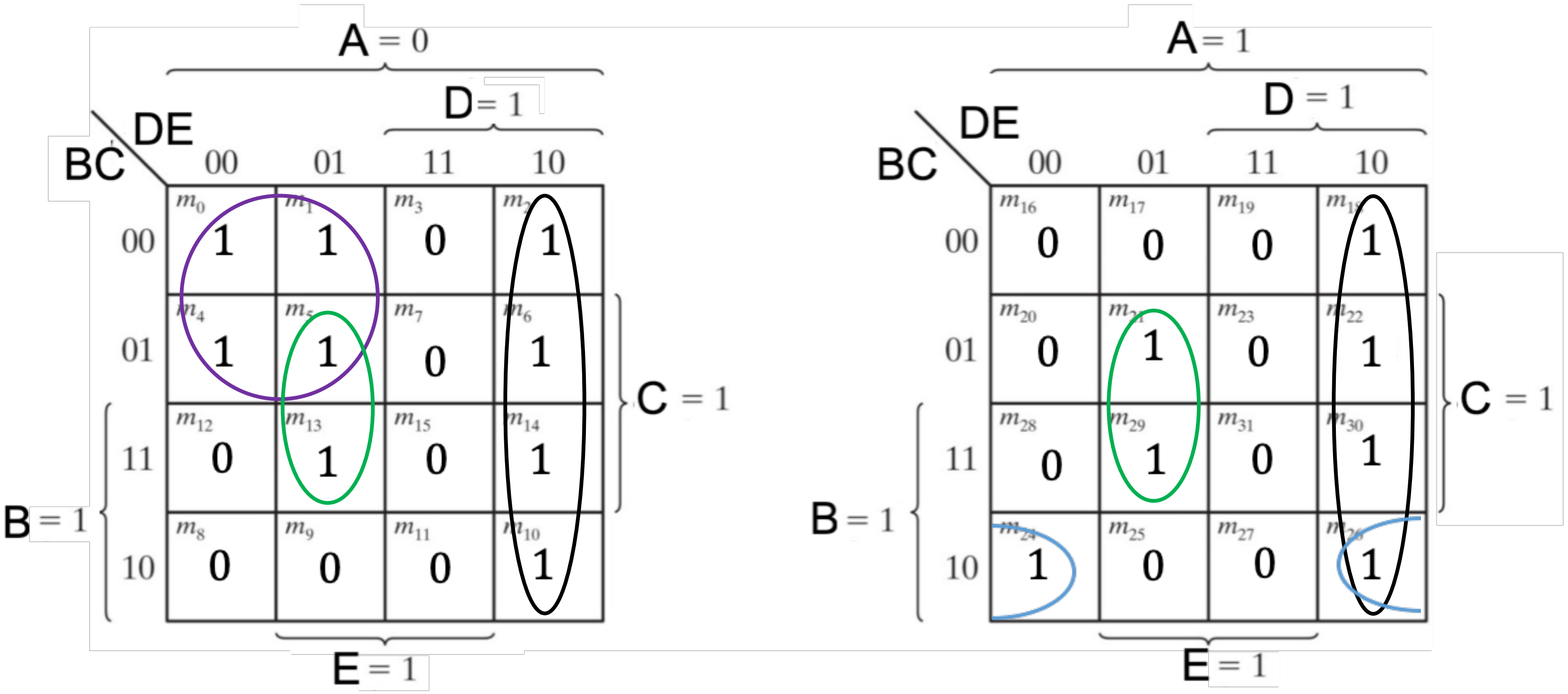
$$f(x_{n-1}, x_{n-2}, x_{n-3}, \dots\dots\dots x_0) = \overline{x_{n-1}} \cdot f_1(x_{n-2}, x_{n-3}, \dots\dots\dots x_0) + x_1 \cdot f_1^*(x_{n-2}, x_{n-3}, \dots\dots\dots x$$

K-Map (5-Variable)

- In 5-variable K-map, find PIs in each 4-variable section. Then, find overlapping PIs.
- Use these overlapping map without the fifth variable.

K-Map (5-Variable)

- $f(A, B, C, D, E) = \sum m(0,1,2,4,5,6,10,13,14,18,21,22,24,26,29,30)$



$$f_1(B, C, D, E) = \bar{B} \cdot \bar{D} + D \cdot \bar{E} + C \cdot \bar{D} \cdot E \qquad f_1^* = D \cdot \bar{E} + C \cdot \bar{D} \cdot E + B \cdot \bar{C} \cdot \bar{E}$$

$$f = \bar{A} \cdot \bar{B} \cdot \bar{D} + A \cdot B \cdot \bar{C} \cdot \bar{E} + D \cdot \bar{E} + C \cdot \bar{D} \cdot E$$

K-Map

$$f(A,B,C,D,E) = \sum m(0,1,2,3,8,10,16,18,20,25)$$

A = 0

BC \ DE	00	01	11	10
00	m_0 1	m_1 1	m_3 1	m_2 1
01	m_4 0	m_5 0	m_7 0	m_6 0
11	m_{12} 0	m_{13} 0	m_{15} 0	m_{14} 0
10	m_8 1	m_9 0	m_{11} 0	m_{10} 1

A = 1

BC \ DE	00	01	11	10
00	m_{16} 1	m_{17} 0	m_{19} 0	m_{18} 1
01	m_{20} 1	m_{21} 0	m_{23} 0	m_{22} 0
11	m_{28} 0	m_{29} 0	m_{31} 0	m_{30} 0
10	m_{24} 0	m_{25} 1	m_{27} 0	m_{26} 0

D = 1, C = 1, B = 1, E = 1

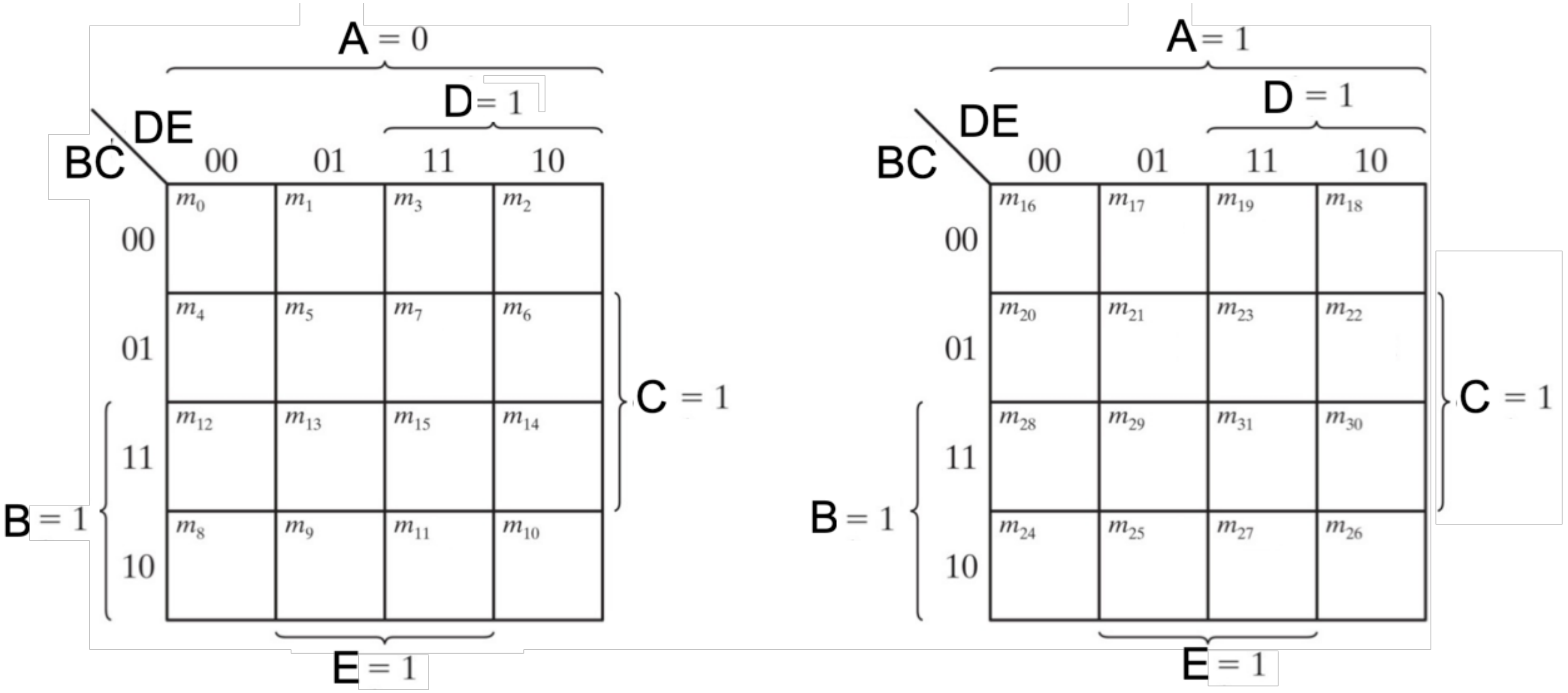
$$f_1(B,C,D,E) = \bar{B}\cdot\bar{C} + \bar{C}\cdot\bar{E}$$

$$f_1^*(B,C,D,E) = \bar{B}\cdot\bar{D}\cdot\bar{E} + \bar{B}\cdot\bar{C}\cdot\bar{E} + B\cdot\bar{C}\cdot\bar{D}\cdot E$$

$$f(A,B,C,D,E) = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{C}\cdot\bar{E} + A\cdot\bar{B}\cdot\bar{D}\cdot\bar{E} + A\cdot B\cdot\bar{C}\cdot\bar{D}\cdot E + \bar{B}\cdot\bar{C}\cdot\bar{E}$$

K-Map (5-Variable) --- Home Work

- $f(A,B,C,D,E) = \sum m(0,4,6,8,12,13,14,15,16,17,18,21,24,25,26,28,29,31)$



K-Map (HW):

- Find minimized SOP
- $f(x, y, z) = \sum m(0,2,3,4,7)$
- $f(x, y, z) = \sum m(0,1,2,3,5,7)$
- $f(w, x, y, z) = \sum m(0,4,8,10,11,12,13,15)$
- $f(w, x, y, z) = \sum m(1,3,5,7,8,10,12,13,14)$
- $f(w, x, y, z) = \sum m(0,1,2,7,8,9,11,12,14,15)$
- $f(w, x, y, z) = \sum m(5,10,11,13,14,15)$
- Consider the circuit which has 4-bit input and 1-bit output. The output of the circuit is high when input has odd number of 1’s. Draw the k-map and implement the circuit using only XOR gates.

K-Map (Homework)

- 4-bit to seven segment display converter.
- Binary to Gray code converter and vice-versa.
- BCD to excess-3 converter and vice-versa.
- 1’s complement adder and subtractor.
- 2’s complement adder and subtractor.
- Prime number detector for 4-bit input and 5-bit input for unsigned numbers.