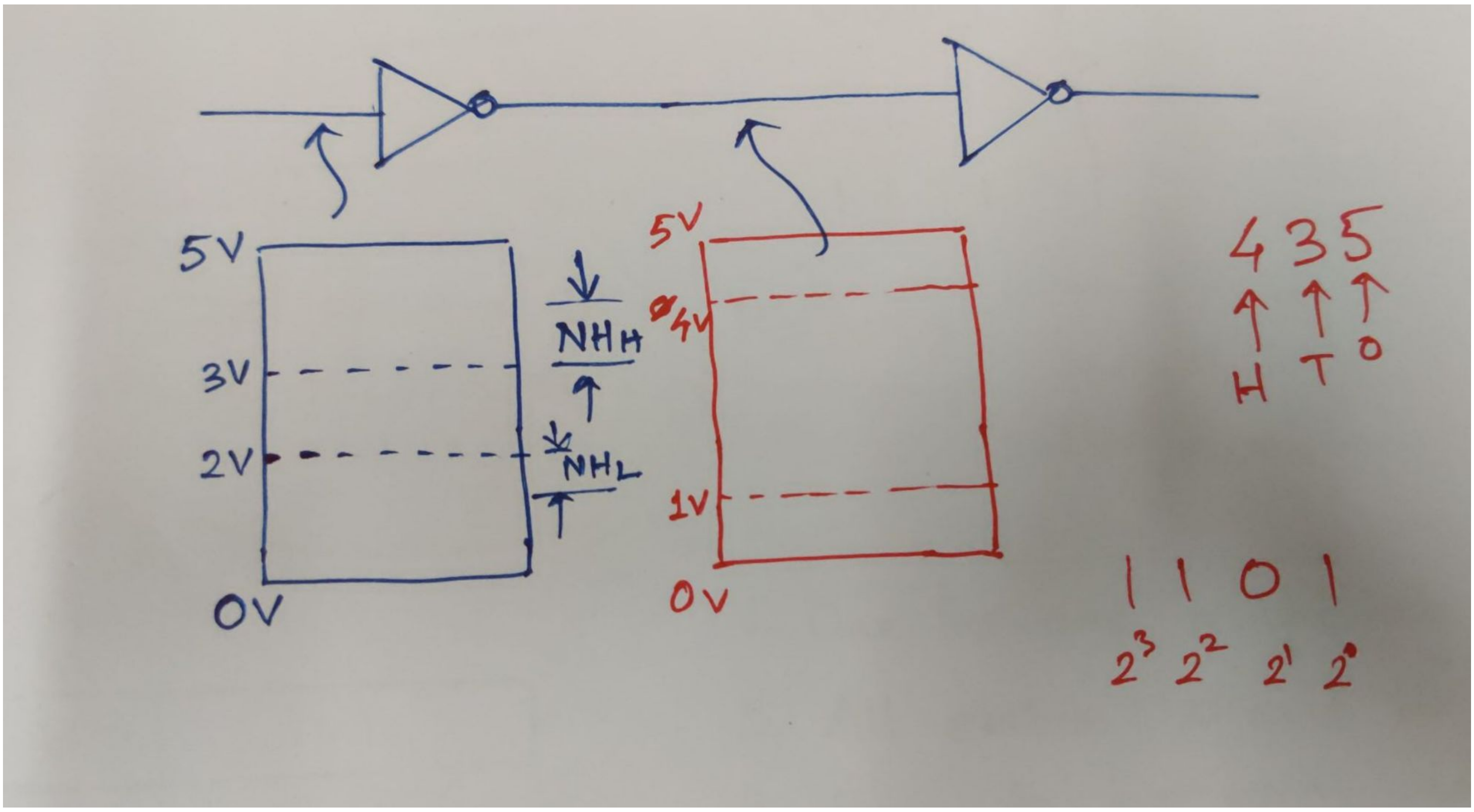

5V
3V
2V
0V
NH H
NH L
5V
4V
1V
0V
4 3 5
H T O
1 1 0 1
2^3 2^2 2^1 2^0

Negative Integers.

4 bit.

0000
⋮
1111

Set of +ve & -ve numbers

S

Mapping

Set of +ve numbers.

1. One-to-one mapping
2. All entries should be mapped.
3. Easy to perform addition/subtraction etc..

sign bit	$\lvert u \rvert$

$$\text{Sign bit} = \begin{cases} 1, & u < 0 \\ 0, & u > 0 \end{cases}$$

$5 \rightarrow 0101$

$-5 \rightarrow 1101$

1's complement

$3 \rightarrow 0011$

$-3 \rightarrow 1100$

x	$\bar{x} = 1 - x$
1	0
0	1

$15 - 3$

$= 12$

$= 1100$

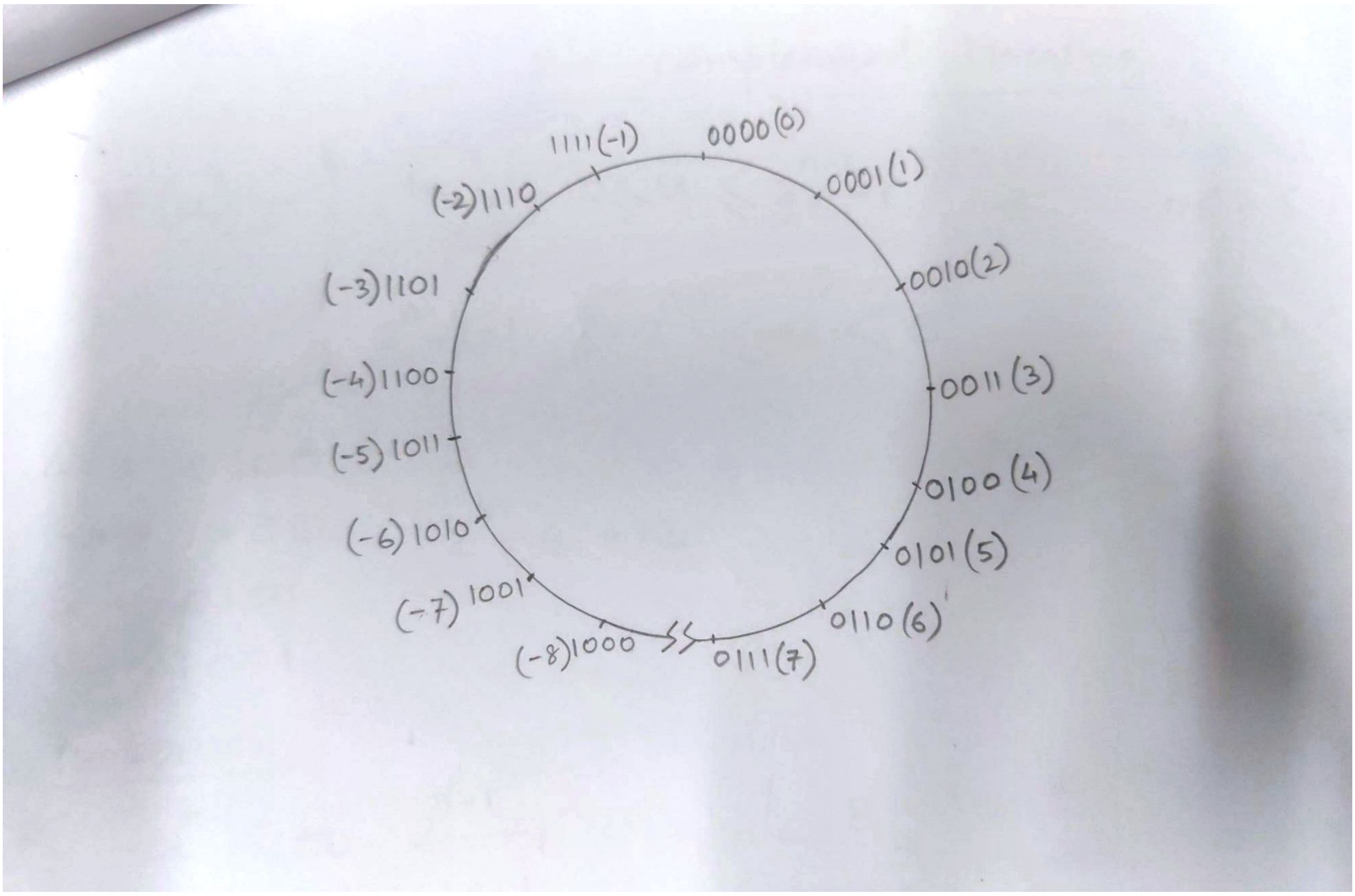
$$\boxed{2^n - 1 - |u|}$$

$$F(u) = \begin{cases} u, & u > 0 \\ \sim(|u|) \text{ or } \left(2^n - 1 - |u|\right), & u < 0 \end{cases}$$

-7 to 8

$-3 + 7 = 4 \rightarrow 0100$

$8 + 7 = 15 \rightarrow 1111$


0000 (0)
0001 (1)
0010 (2)
0011 (3)
0100 (4)
0101 (5)
0110 (6)
0111 (7)
(-8) 1000
(-7) 1001
(-6) 1010
(-5) 1011
(-4) 1100
(-3) 1101
(-2) 1110
1111 (-1)

2's complement Notation

$$F(u) = \begin{cases} u, & 0 \leq u \leq 2^{n-1} - 1 \\ 2^n - |u|, & -2^{n-1} \leq u < 0 \end{cases}$$

$0 \leq u \leq 7$

$-8 \leq u < 0$

$n = 4$

$4 \leftarrow 0100$

$-4 \leftarrow 2^4 - 4 = 12 = 1100$

1101

$2^3 \; 2^2 \quad 2^0$

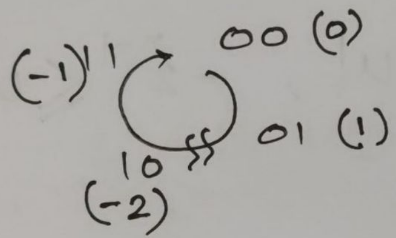
Properties.

* -2^{n-1} to $2^{n-1} - 1$
* Unique representation of 0 (0000)
* MSB is equal to sign bit.

Number Circle

$\boxed{5 + (-2)}$

2 bit



$M + N$

$5 \quad -2$

$2^n - N$

$2^4 - 2 = 14$

Computing 2's complement

* $2^n - u$

$$= \underbrace{2^n - 1 - u}_{\text{1's complement.}} + 1$$

$$\boxed{2 \times (-3)} =$$

$0010 \quad 1101$

$= 26 \bmod 2^4 = 10$

$= 1010$

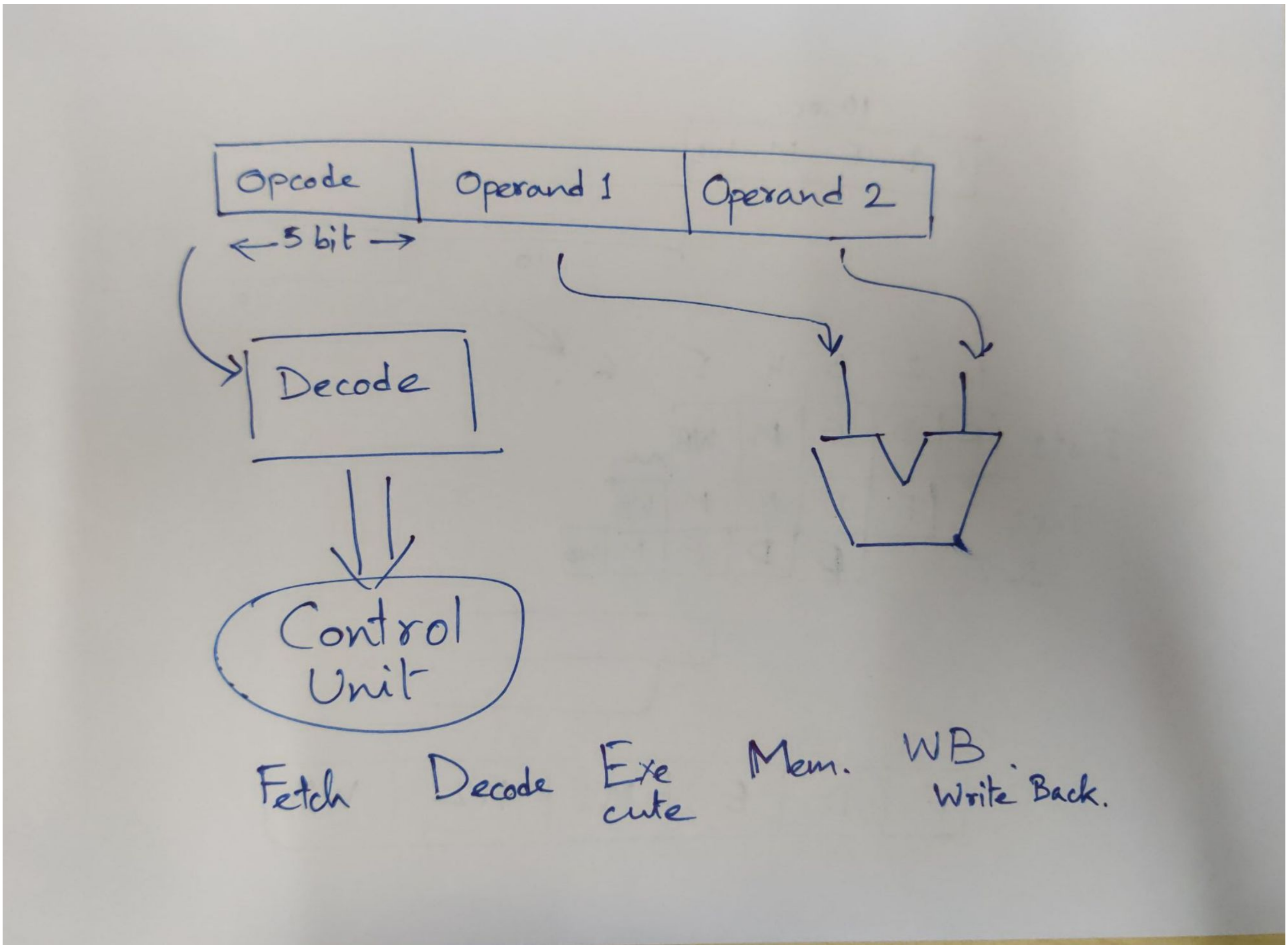
$a \equiv b$

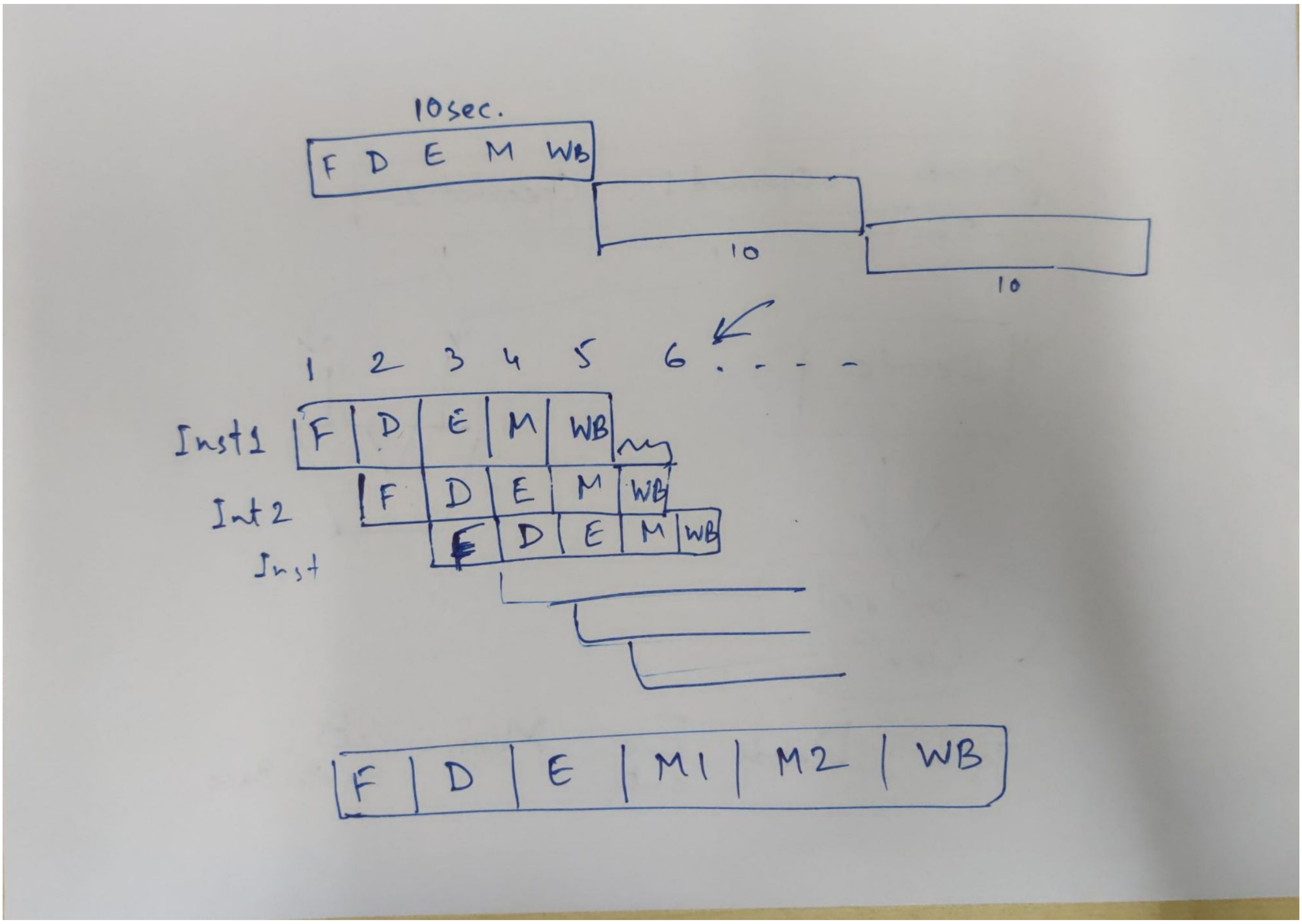
$$\boxed{a = b \bmod 2^n}$$

$a = 3$

$b = 19 \bmod 16$

$= 3$


Opcode
Operand 1
Operand 2
5 bit
Decode
Control Unit
Fetch
Decode
Exe cute
Mem.
WB
Write Back.


10sec.
F D E M WB
10
10
1 2 3 4 5 6 . - - -
Inst1 F D E M WB
Int 2 F D E M WB
Inst F D E M WB
F D E M1 M2 WB

Sign Extension

Convert a n bit number to a
m - bit number (2's complement)

$$\underset{8}{m} > \underset{4}{n}.$$

+ve
Add $(m-n)$ 0's in the MSB positions.

0100 $\longrightarrow$ 00000100

-ve :

$$F(u) = 2^{n} - |u|$$

$$F'(u) = 2^{m} - |u|$$

$$2^m - u - (2^n - u)$$

$$F_4(-4) = 2^4 - 4$$
$$= 1100$$

$$= 2^m - 2^n$$

$$= 2^n + 2^{n+1} + 2^{n+2} \cdots\cdots + 2^{m-1}$$

$$= \underbrace{11111111}_{m-n}\ \underbrace{0000\cdots0}_{n.}$$

$$F_8(-4) = 11110000 + 1100$$
$$= 1111\underline{1100}$$

To convert a -ve number
Add $(m-n)$ 1's in the MSB positions.

* In both cases extend the sign bit by $m-n$ positions.

Overflow theorem:

$u+v$

$u \cdot v < 0$, there will never be an overflow.

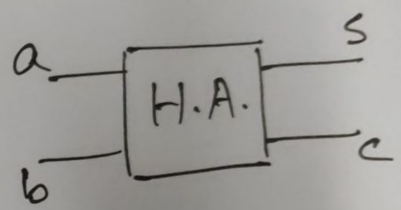
$u \cdot v > 0$, an overflow is possible.

u & v are of same sign.

→ the sign of the result is different from u & v then there is overflow.

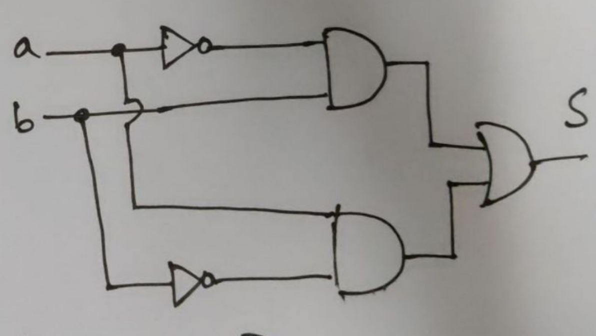
Truth Table of Half adder

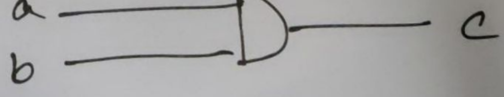
a	b	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



$$S = \bar{a}b + a\bar{b}$$

$$C = a.b.$$





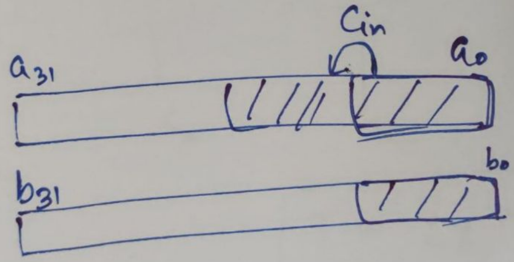
FA truth table

a	b	Cin	S	Cout
0	0	0	0	0
0	1	0	1	0
1	0	0	1	0
1	1	0	0	1
0	0	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

$$S = a \oplus b \oplus C_{in}$$

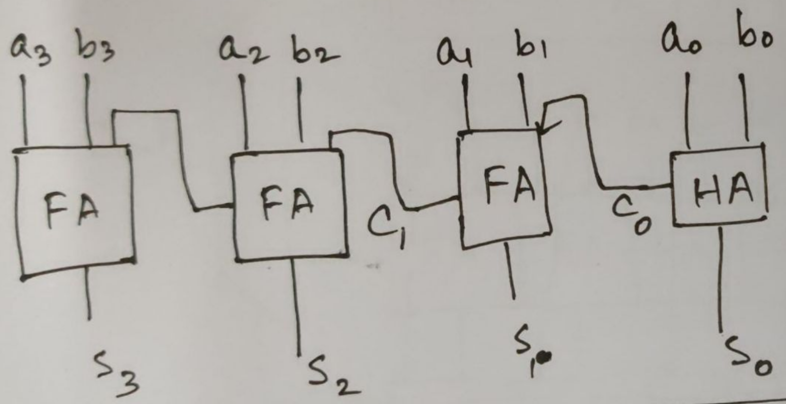
$$= a \cdot \bar{b}\, \bar{C}_{in} + \bar{a} \cdot b \cdot \bar{C}_{in} + \bar{a}\, \bar{b}\, C_{in} + a \cdot b\, C_{in}$$

$$C_{out} = a \cdot b + a \cdot C_{in} + b \cdot C_{in}$$

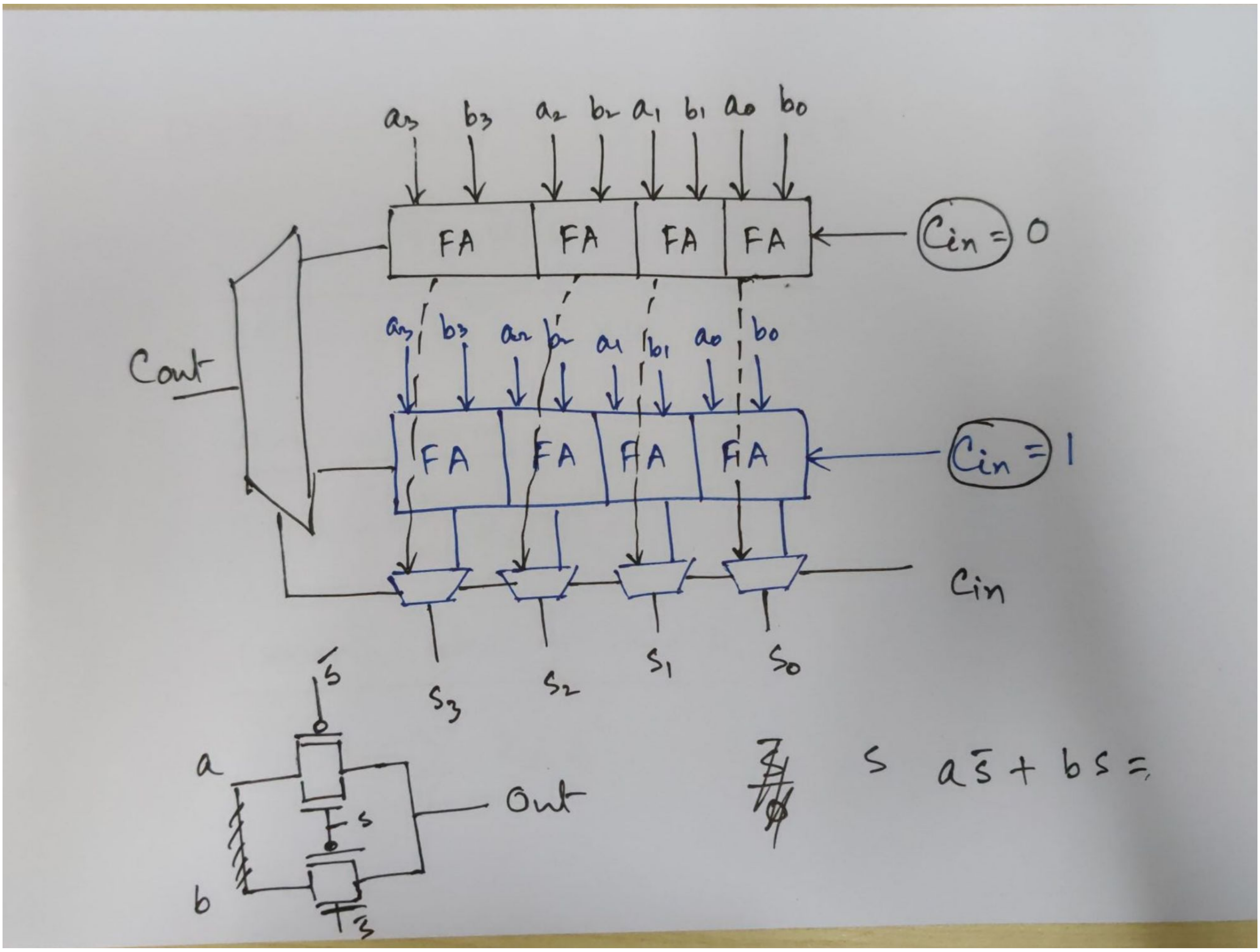


A : $a_3\ a_2\ a_1\ a_0$

B : $b_3\ b_2\ b_1\ b_0$



$$\boxed{T = t_h + (n-1)t_f}$$


a_3 b_3 a_2 b_2 a_1 b_1 a_0 b_0
FA FA FA FA
$C_{in} = 0$
a_3 b_3 a_2 b_2 a_1 b_1 a_0 b_0
FA FA FA FA
$C_{in} = 1$
Cout
Cin
s_3 s_2 s_1 s_0
$\bar{s}$
a
b
s
Out
$a\bar{s} + bs =$

$$0.375 = 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$$

$$\Rightarrow 0.011$$

$$3.29 = 3 \times 10^{0} + 2 \times 10^{-1} + 9 \times 10^{-2}$$

Generic form

$$A = \sum_{i=-n}^{n} x_i \, 10^{i}$$

Generic form for base 2

$$A = \sum_{i=-n}^{n} x_i \, 2^{i}$$