ECE 113- Basic Electronics

Lecture week 6: Maximum Power Transfer Theorem, Y- Δ transformation, Capacitor, Inductor

Dr. Ram Krishna Ghosh, Assistant Professor

Office: B601, Research and Development Block

Email: rkghosh@iiitd.ac.in

INDRAPRASTHA INSTITUTE *of*
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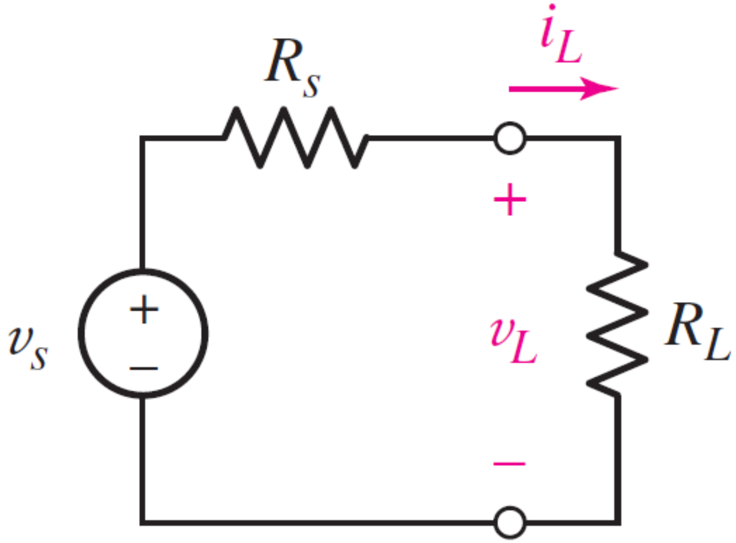
Maximum Power Transfer Theorem

- An independent voltage source in series with a resistance R_s, or an independent current source in parallel with a resistance R_s, delivers maximum power to a load resistance R_L such that $R_s = R_L$.

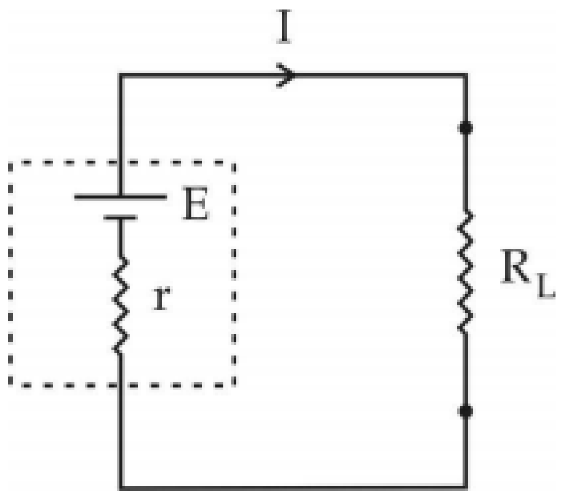
Alternatively

- A network delivers maximum power to a load resistance R_L when R_L is equal to a the Thévenin equivalent resistance of the network.



Proof of Maximum power transfer theorem

Consider a network with a source of emf E and internal resistance r connected to a load resistance R_L. The current I in the circuit is



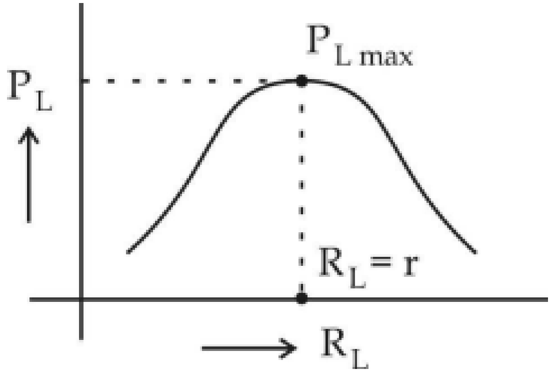
$$I = \frac{E}{R_L + r} \text{ ----------- (1)}$$

The power delivered to load resistance R_L is $P_L = I^2 R_L$ or $P_L = \left(\frac{E}{R_L + r}\right)^2 R_L$

(from equation (1))

$$P_L = \frac{E^2}{(R_L + r)^2} R_L \text{ ---------- (2)}$$

The variation of P_L with R_L is as shown.



P_L is found to be maximum for a particular value of R_L when

P_L is maximum, $\frac{dP_L}{dR_L} = 0$

[$\because$ No variation of P_L with R_L at $P_{L\,max}$]

Proof of Maximum power transfer theorem

i.e., $\frac{d}{dR_L}\left(\frac{E^2R_L}{(R_L+r)^2}\right)=0$ or $\frac{d}{dR_L}\left[E^2R_L(R_L+r)^{-2}\right]=0$

Differentiating $E^2\left[R_L(-2)(R_L+r)^{-3}+(R_L+r)^{-2}\right]=0$ or $\frac{-2R_L}{(R_L+r)^3}+\frac{1}{(R_L+r)^2}=0$ or

$$\frac{2R_L}{(R_L+r)^3}=\frac{1}{(R_L+r)^2}$$

Thus, $\frac{2R_L}{R_L+r}=1 \quad \Rightarrow \quad 2R_L=R_L+r$ or $\boxed{R_L=r}$

Thus the power delivered to the load resistance is maximum when the load resistance is equal to the internal resistance of the source.

To show that the maximum power transfer efficiency of a circuit is 50%:

The power across the load $P_L = I^2 R_L = \frac{E^2}{(R_L + r)^2} R_L$ ----------- (1)

From the maximum power transfer theorem, P_L is maximum when $R_L = r$. Putting this condition in equation (1),

$$P_{L\,\max} = \frac{E^2}{(2R_L)^2} R_L \Rightarrow P_{L\,\max} = \frac{E^2}{4R_L} \quad \text{------------- (2)}$$

The power that is taken from the voltage source is (or power generated by the source),

$$P = I^2 (R_L + r) = \frac{E^2}{(R_L + r)^2}(R_L + r) \quad \text{or} \quad P = \frac{E^2}{R_L + r}. \quad \text{When } R_L = r, \quad P = \frac{E^2}{2R_L} \quad \text{-------- (3)}$$

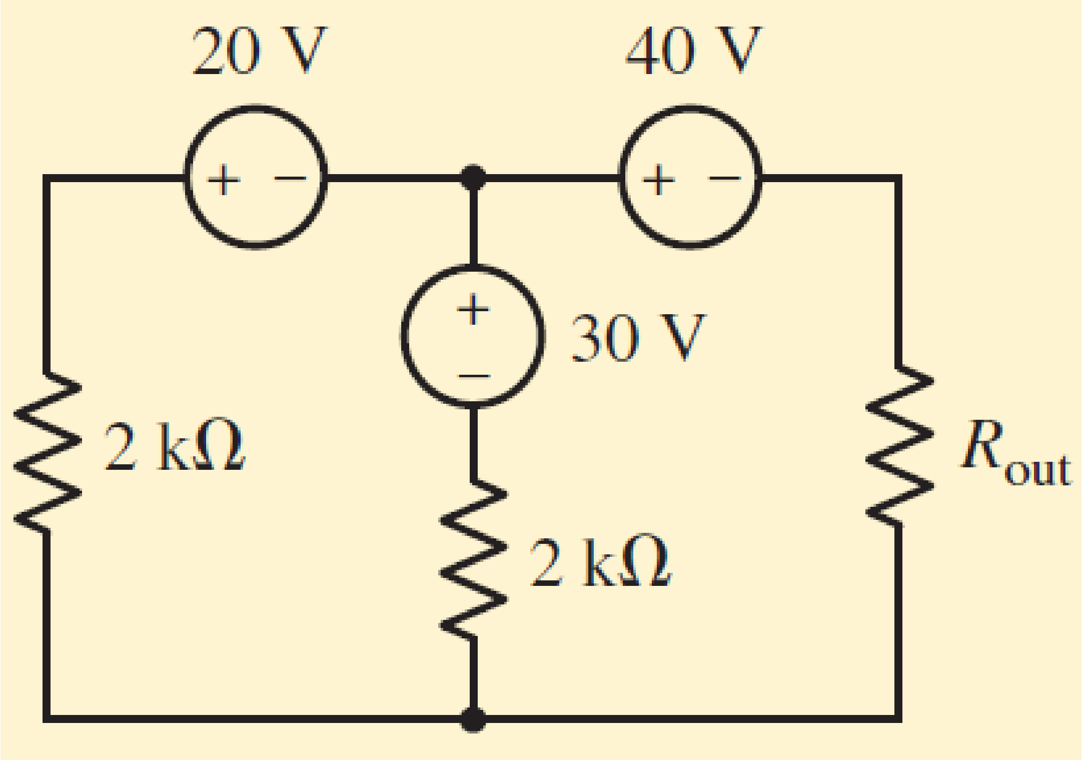
Dividing equation (2) by (3)

$$\frac{P_{L\,\max}}{P} = \frac{E^2}{4R_L} \times \frac{2R_L}{E^2} = \frac{1}{2} \quad \text{or} \quad P_{L\max} = \frac{P}{2}$$

Thus the maximum power delivered to the load is only half the power generated by the source or the maximum power transfer efficiency is 50%. The remaining 50% power is lost across the internal resistance of the source.

Practice

(a) If R_{out} =3 k ohm, find the power delivered to it.
(b) What is the maximum power that can be delivered to any R_{out}?

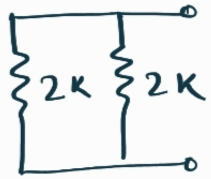


Ans: 230 mW; 306 mW

solution

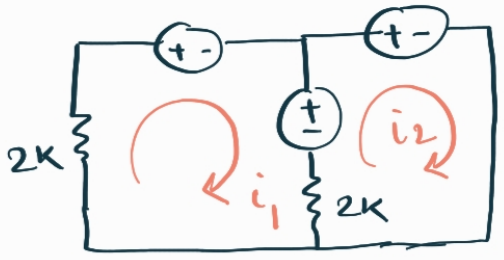
1) Find Norton's equivalent circuit and then proceed.

For R_N



$\Rightarrow R_N = 1\,K\Omega$

For I_N



For loop i_1

$$2i_1 + 20 + 30 + 2(i_1 - i_2) = 0$$

$$\Rightarrow 2i_1 - i_2 = -25 \quad \text{--- (i)}$$

For loop i_2

$$2(i_2 - i_1) - 30 + 40 = 0$$

$$\Rightarrow i_1 - i_2 = 5 \quad \text{--- (ii)}$$

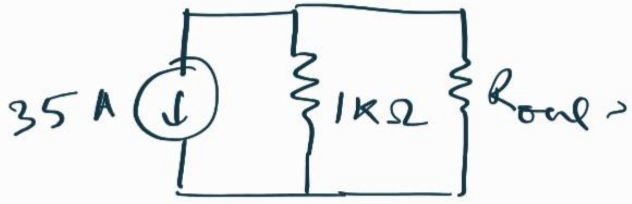
From (i) & (ii)

$$2i_1 - i_2 = -25$$

$$\underline{\underset{-}{2i_1} \underset{+}{-} 2i_2 = \underset{-}{10}}$$

$$i_2 = -35\ A$$

So, the Norton's equivalent circuit is,



(a) if $R_{load} = 3\,K\Omega$,

Then Power delivered to it will be,

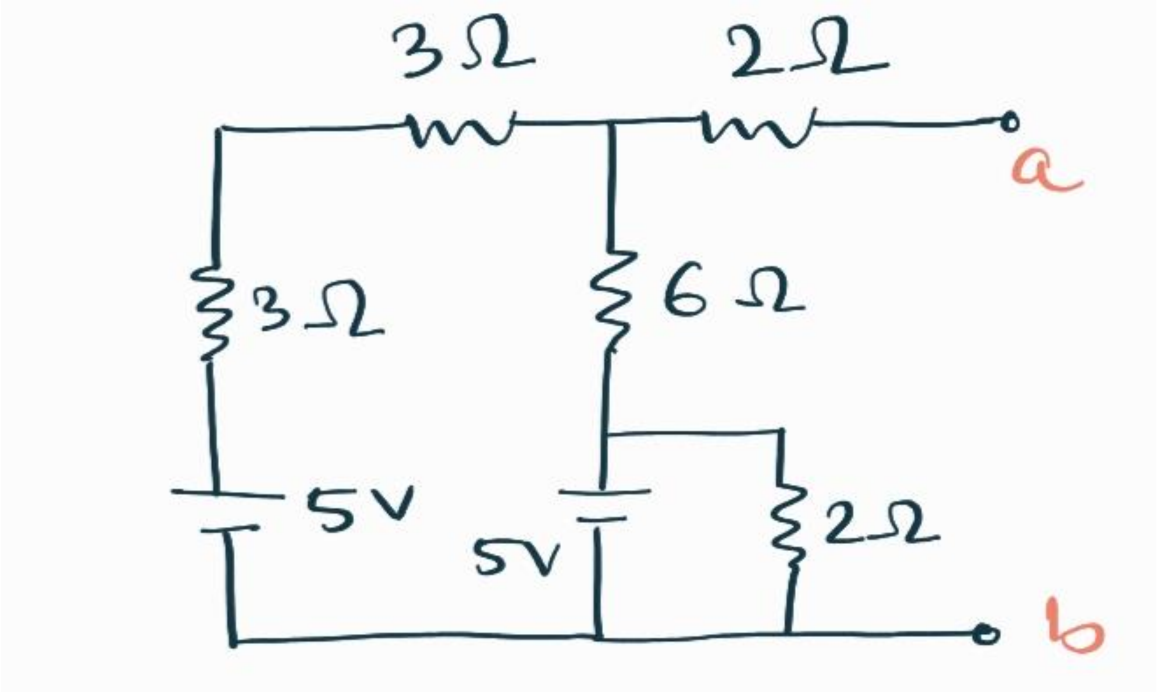
$$P = \left(\frac{35}{4}\right)^2 \times 3 = 230\ mW$$

(b) $P = P_{max}$ when $R_{load} = R_N$

$$\therefore P_{max} = \left(\frac{35}{2}\right)^2 \times 1 = 306\ mW$$

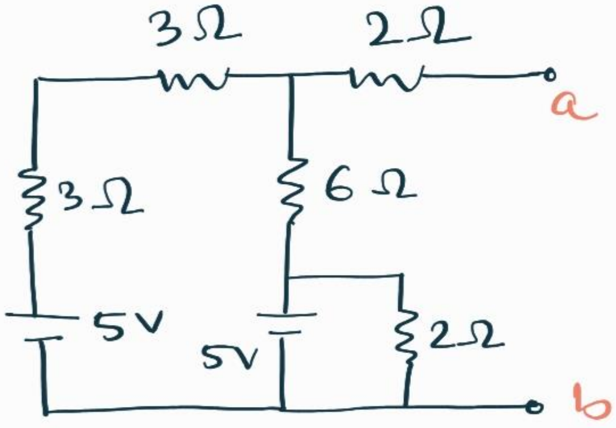
Exercise

Determine the resistance to be connected across a, b to dissipate maximum power in the following circuit and calculate the power

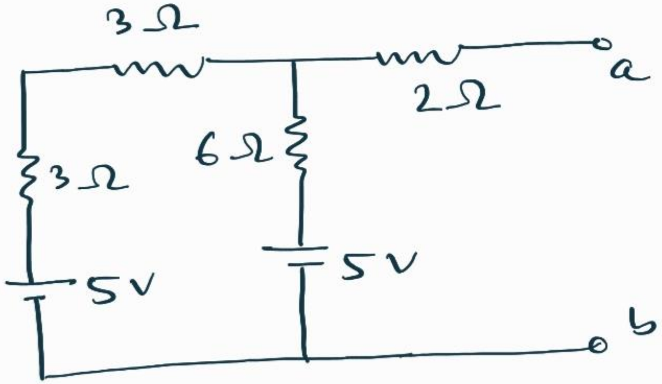


Ans: R_L = 5 ohm, P = 1.25 W

Solution



The effective circuit is,



So, $R_{TH} = 2 + \dfrac{6 \times (3+3)}{6 + (3+3)} = 5\,\Omega$

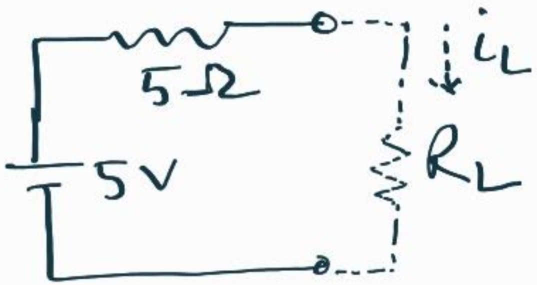
for V_{TH}, the current in closed loop is zero.

So, $V_{TH} = V_{ab} = 5\,\Omega$

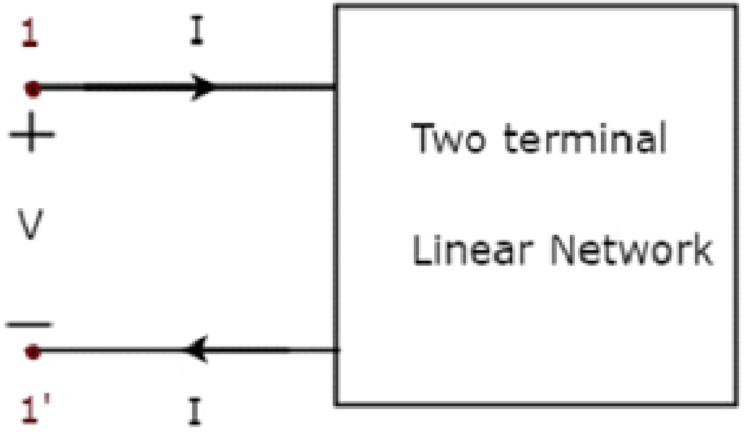
By maximum power transfer theorem, the resistance R_L connected across a, b will dissipate maximum power when $R_L = R_{TH}$.

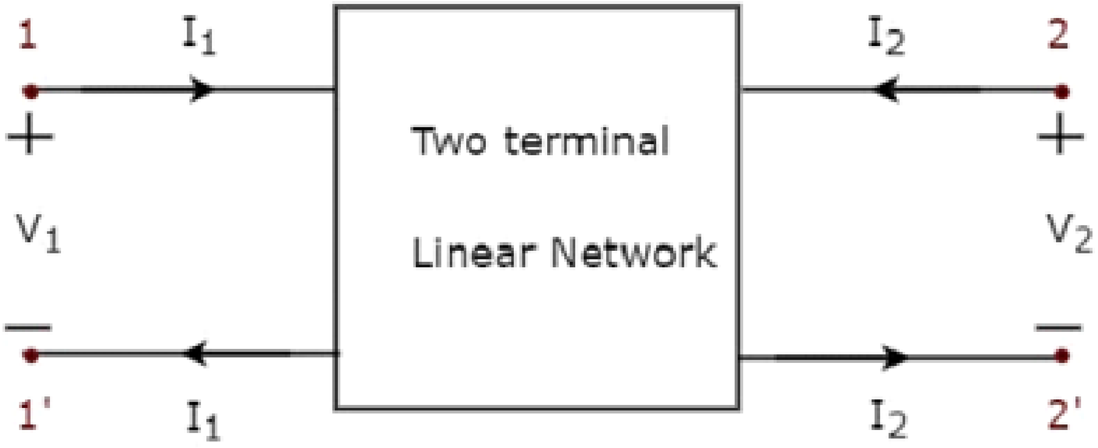
So, $i_L = \dfrac{5}{5+5} = 0.5\,A$; $P_{max} = i_L^2 R_L = (0.5)^2 \times 5$
$= 1.25\,W$

Then the Thevenin's equivalent circuit is



Y - Δ or Δ -Y Transformations

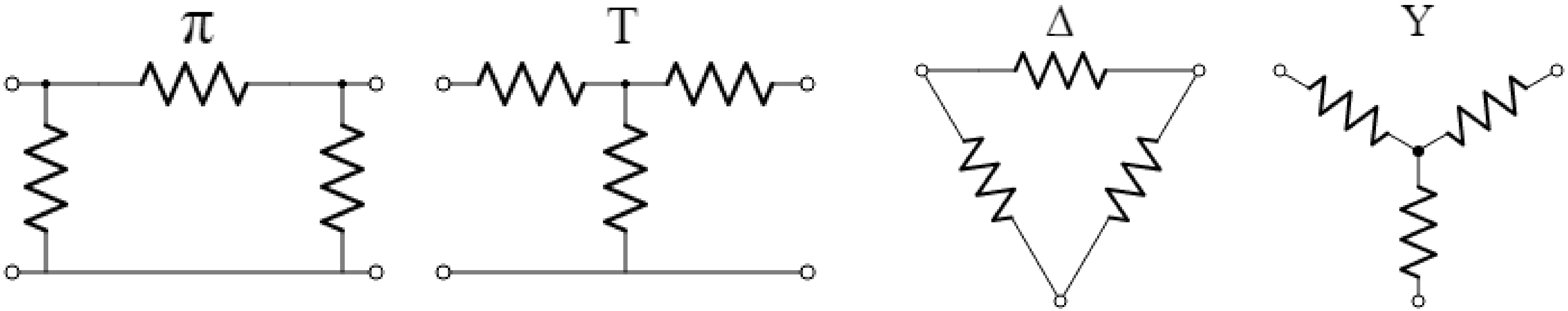




One-port circuit: When any voltage source is connected across the terminals, the current entering through any one of the two terminals, equals the current leaving the other terminal. For example, resistance, inductance and capacitance acts as a one-port.

Two-port circuit: a two-port is a circuit having <u>two pairs of terminals</u>. Each pair behaves as a one-port; current entering in one terminal must be equal to the current living the other terminal.

Y - Δ or Δ -Y Transformations

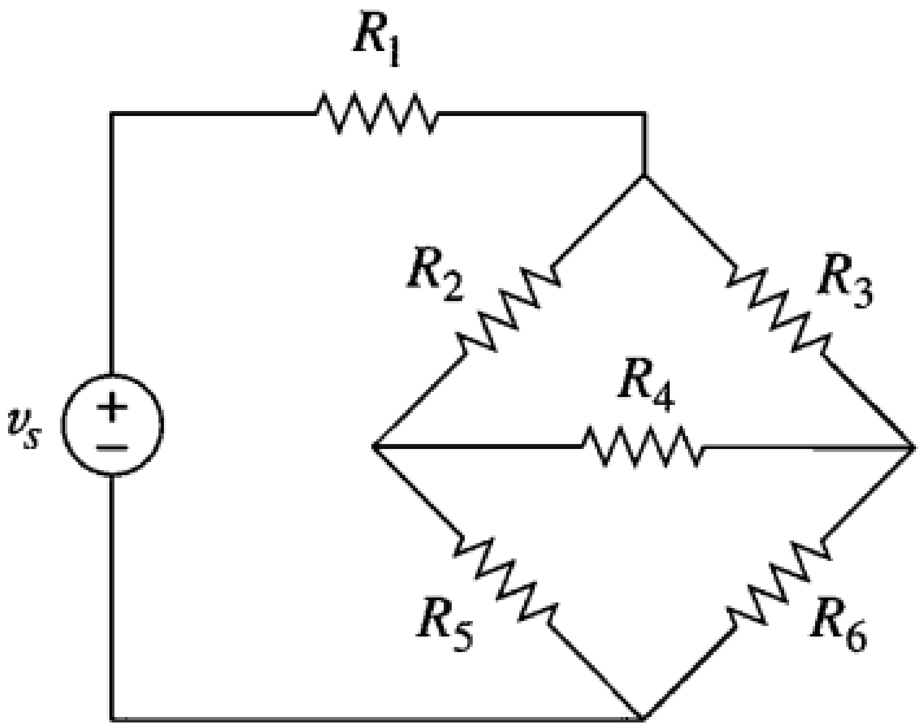


The most important subclass of two-port networks is the one in which the minus reference terminals of the input and output ports are at the same. This circuit configuration is readily possible to consider the ‘***π or Δ*** ’– network also as a three-terminal network as in the fig.

Another frequently encountered circuit configuration that shown in the fig. is approximately referred to as a three-terminal ***T or Y*** connected circuit as well as two-port circuit.

Y - Δ or Δ -Y Transformations

How to Combine $R_1 - R_6$?

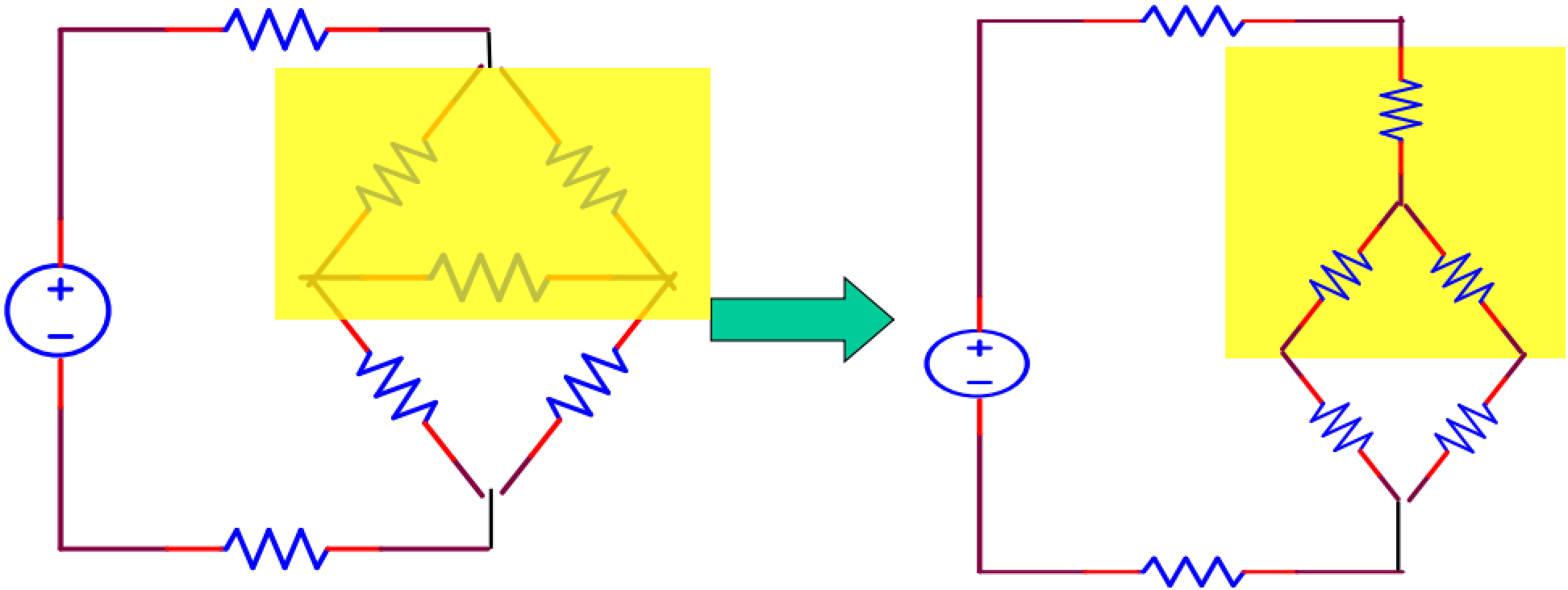


- Solution: Use three terminal equivalent networks:
  a) Wye (Y) or Tee (T)
  b) Delta (Δ) or Pi (π)

- Useful in:
  a) Three phase networks
  b) Electrical filters
  c) Matching networks

Y - Δ or Δ -Y Transformations

How to Apply Transformation?



Y - Δ or Δ -Y Transformations

Δ-TO-Y AND Y-TO-Δ

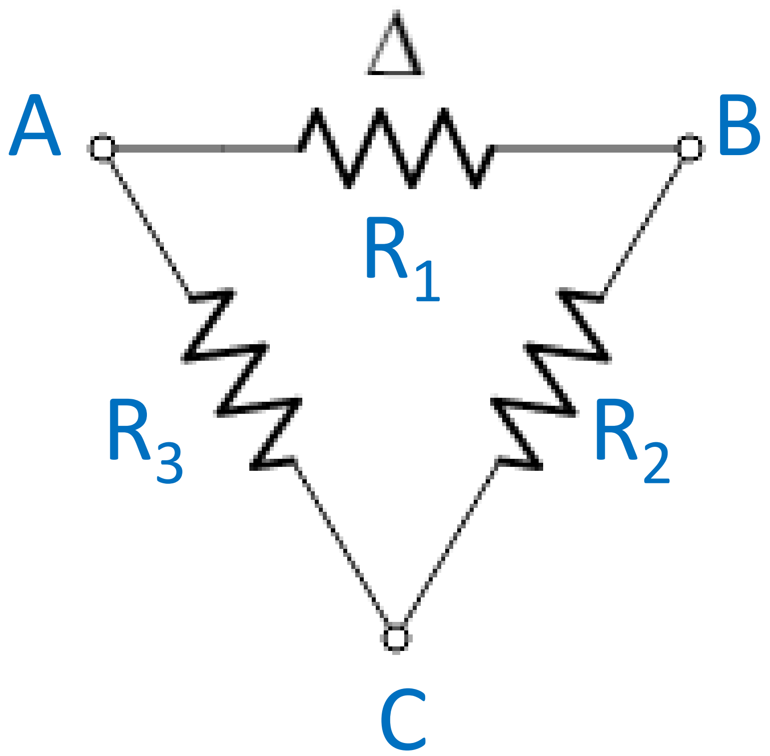
Wye-Delta (and vice versa) Transformation

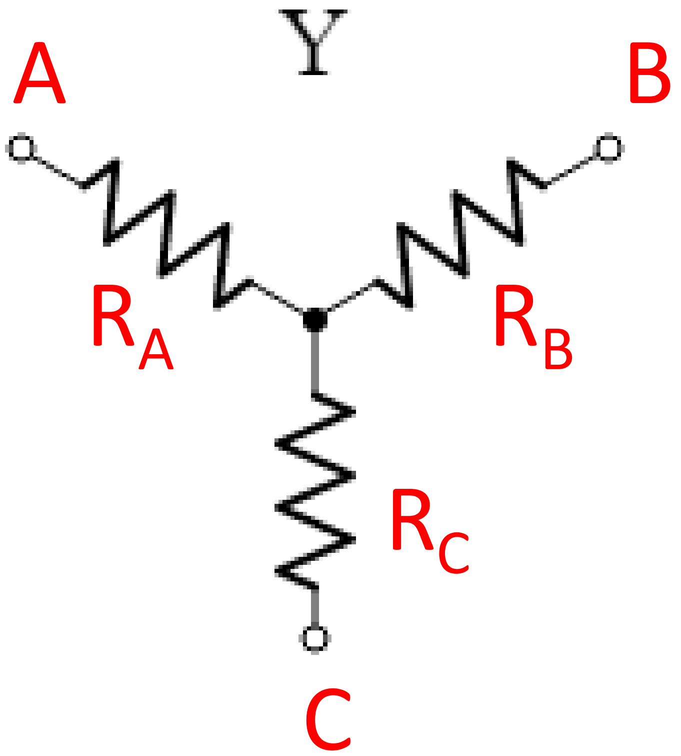
- The Y-Δ transform, also written wye-delta and also known by many other names, is a mathematical technique to simplify the analysis of an electrical network.

- The aim is to analyse the circuit when the resistors are neither in parallel nor in series.

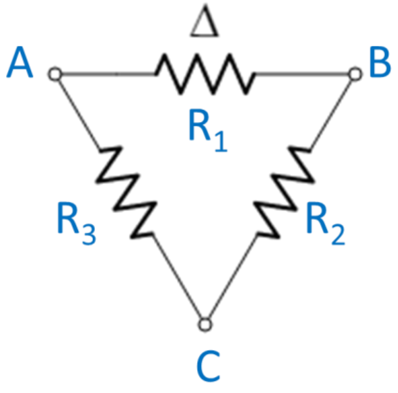
Y - Δ or Δ -Y Transformations

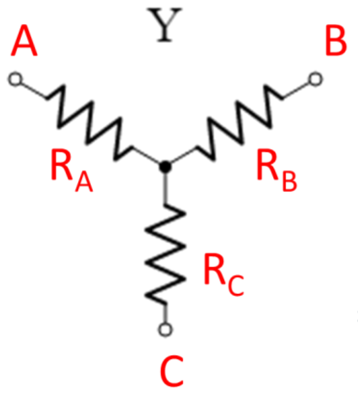
Consider the following resistances of the Δ and Y





Y - Δ or Δ -Y Transformations





For Δ, the resistance between the points A and B will be

$$R_{AB} = R_1 \parallel (R_2 + R_3)$$

$$\Rightarrow R_{AB} = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \quad \text{-----} (a)$$

For Y, if we measure the resistance value between points A and B, we will get

$$R_{AB} = R_A + R_B \quad \text{---} (b)$$

Since the two systems are identical, resistance measured between two terminals in both systems must be equal

$$R_A + R_B = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \quad \text{—} (i)$$

(for A and B)

$$R_B + R_C = \frac{R_2(R_3+R_1)}{R_1+R_2+R_3} \quad \text{—} (ii)$$

(for B and C)

$$R_C + R_A = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} \quad \text{—} (iii)$$

(for C and A)

Y - Δ or Δ -Y Transformations

Adding equations (I), (II) and (III) we get,

$$2(R_A + R_B + R_C) = \frac{2(R_1R_2 + R_2R_3 + R_3R_1)}{R_1 + R_2 + R_3}$$

$$\Rightarrow R_A + R_B + R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1 + R_2 + R_3} \quad \text{—— (IV)}$$

If the Δ connected system has same resistance ***R*** at its three sides then equivalent star resistance **r** will be,

$$r = \frac{R.R}{R+R+R} = \frac{R}{3}$$

Subtracting equations (I), (II) and (III) from equation (IV) we get

$$R_A = \frac{R_3R_1}{R_1 + R_2 + R_3} \quad \text{—— (V)}$$

$$R_B = \frac{R_1R_2}{R_1 + R_2 + R_3} \quad \text{—— (VI)}$$

$$R_C = \frac{R_2R_3}{R_1 + R_2 + R_3} \quad \text{—— (VII)}$$

Δ -Y Transformations

Y - Δ or Δ -Y Transformations

For **Y – Δ transformation** we just multiply equations (v), (VI) and (VI), (VII) and (VII), (V) that is by doing (v) × (VI) + (VI) × (VII) + (VII) × (V) we get,

$$R_A R_B + R_B R_C + R_C R_A$$

$$= \frac{R_1 R_2^2 R_3 + R_1 R_2 R_3^2 + R_1^2 R_2 R_3}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1 R_2 R_3 (R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3} \quad \text{—— (VIII)}$$

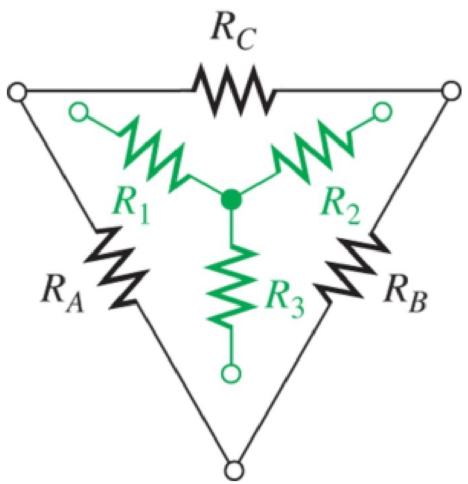
Now dividing equation (VIII) by equations (V), (VI) and equations (VII) separately we get,

$$\boxed{\begin{aligned} R_3 &= \frac{R_A R_B + R_B R_C + R_C R_A}{R_A} \\ R_1 &= \frac{R_A R_B + R_B R_C + R_C R_A}{R_B} \\ R_2 &= \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} \end{aligned}}$$

Y - Δ Transformations

A diagram helping to remember Y - Δ or Δ -Y Transformations

Δ-Y Conversion



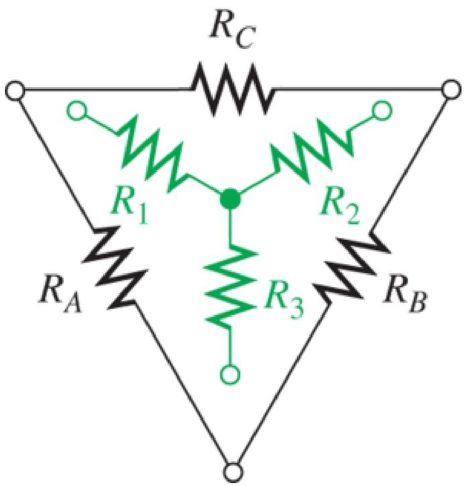
$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_Y = \frac{\textit{Product of two adjacent R in } \Delta}{\textit{Sum of all three R in } \Delta}$$

Y-Δ Conversion



$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

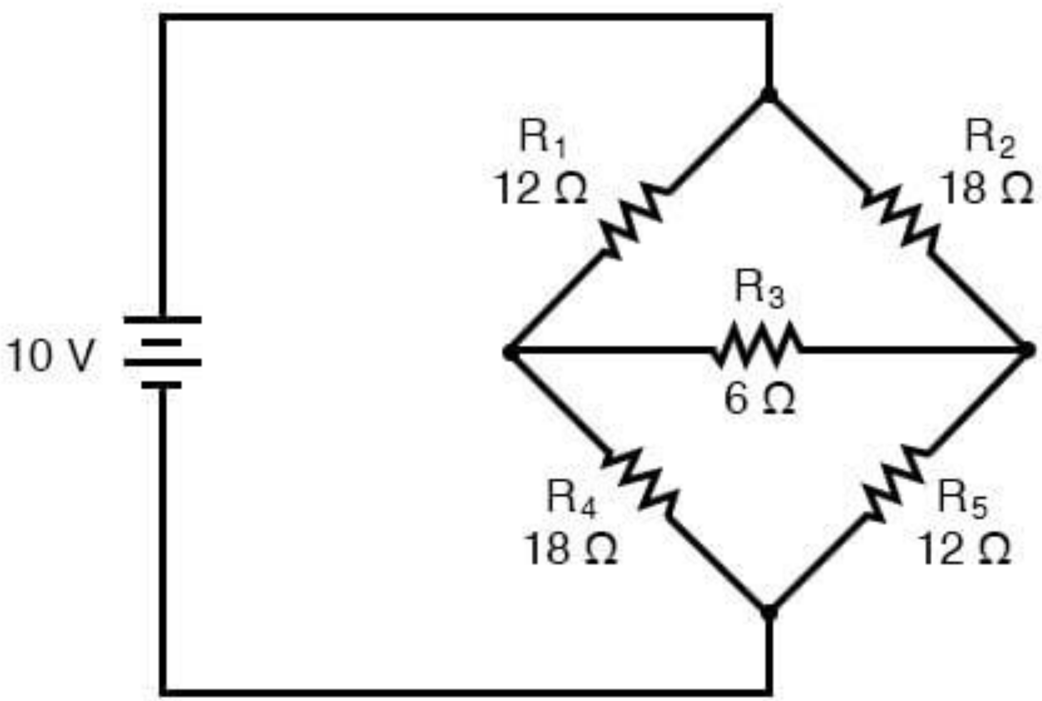
$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

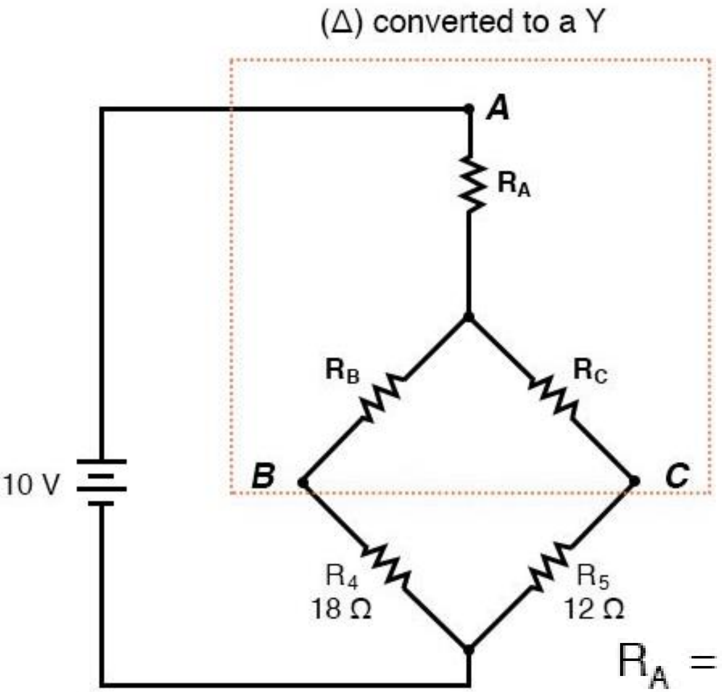
$$R_\Delta = \frac{\textit{Sum of all double products of R in Y}}{\textit{R of Y arm opposite to the desired } \Delta \textit{ arm}}$$

Exercise

Find the voltage drop across each resistances



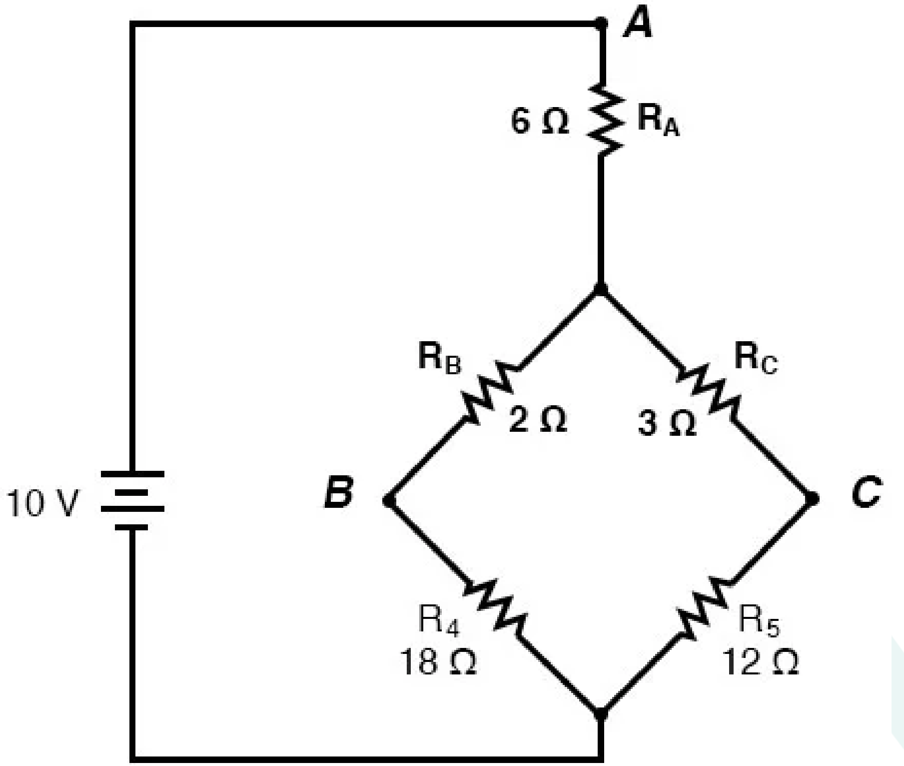
Exercise solution page1



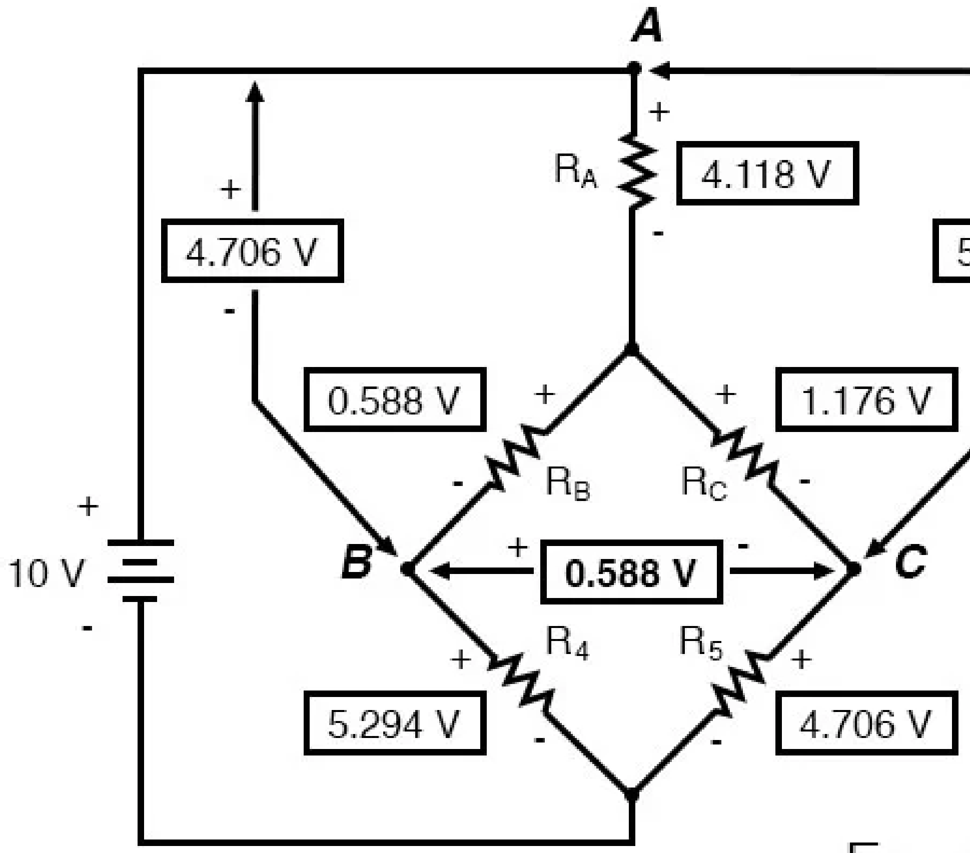
$$R_A = \frac{(12\ \Omega)(18\ \Omega)}{(12\ \Omega) + (18\ \Omega) + (6\ \Omega)} = \frac{216}{36} = \mathbf{6\ \Omega}$$

$$R_B = \frac{(12\ \Omega)(6\ \Omega)}{(12\ \Omega) + (18\ \Omega) + (6\ \Omega)} = \frac{72}{36} = \mathbf{2\ \Omega}$$

$$R_C = \frac{(18\ \Omega)(6\ \Omega)}{(12\ \Omega) + (18\ \Omega) + (6\ \Omega)} = \frac{108}{36} = \mathbf{3\ \Omega}$$



Exercise solution page2

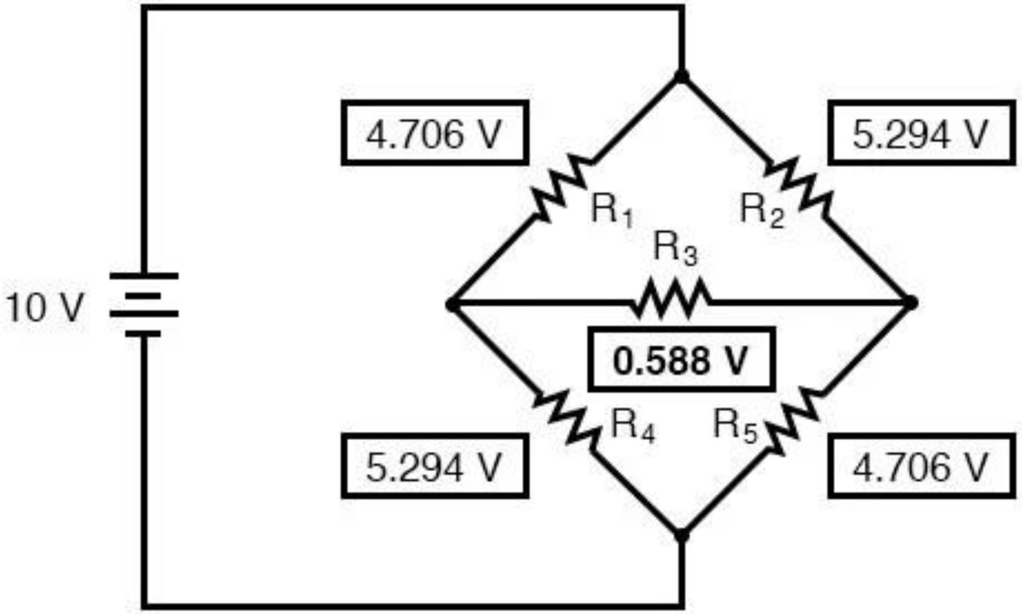


$E_{A\text{-}B} = 4.706\ V$

$E_{A\text{-}C} = 5.294\ V$

$E_{B\text{-}C} = 588.24\ mV$

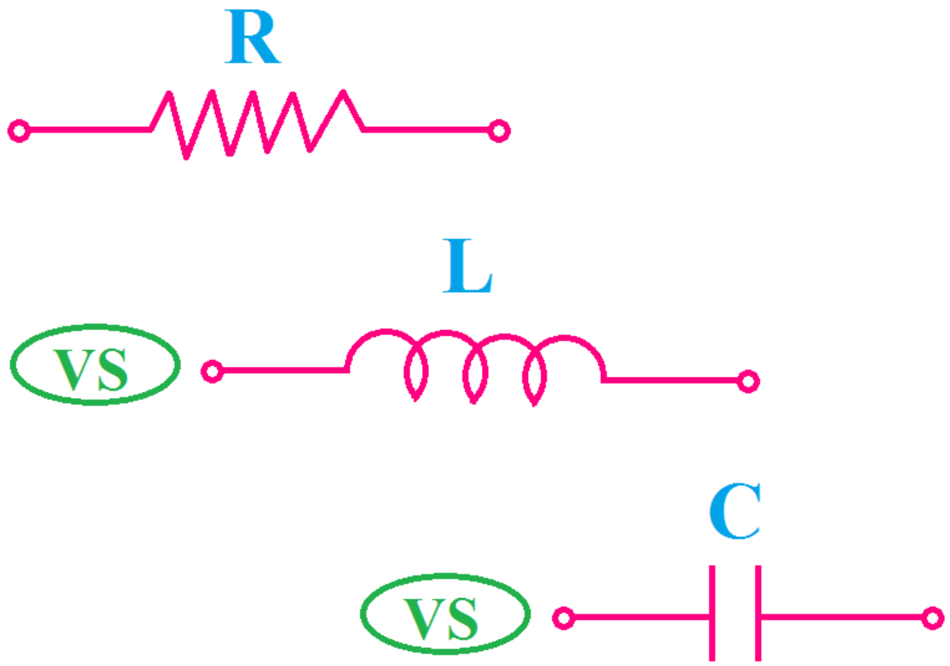
Now that we know these voltages, we can transfer them to the same points A, B, and C in the original bridge circuit:



Capacitors and Inductors

Capacitors and Inductors

Capacitors
and
Inductors



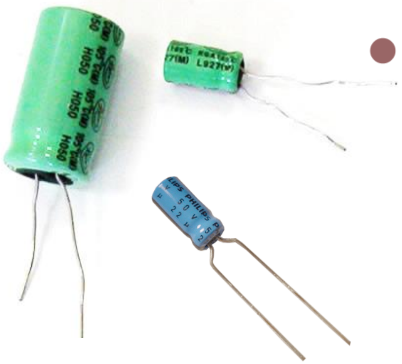
- These elements store and deliver finite amount of energy
- Passive linear circuit elements
- The current-voltage relationships for these new elements are time dependent
- Ideal Capacitors and Inductors can neither generate nor dissipate energy

Capacitors

- A capacitor is a device which is used to store electrical charge (a surprisingly useful thing to do in circuits!).

- Effectively, any capacitor consists of a pair of conducting plates separated by an insulator. The insulator is called a dielectric and is often air, paper or oil.

- Unit : Farad (1 C/V)



- Capacitor

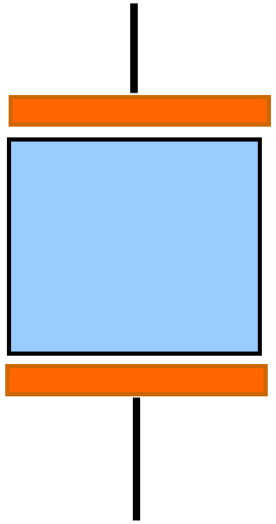
  = conductor (metal plate)

  \+

  insulator (dielectric)

  \+

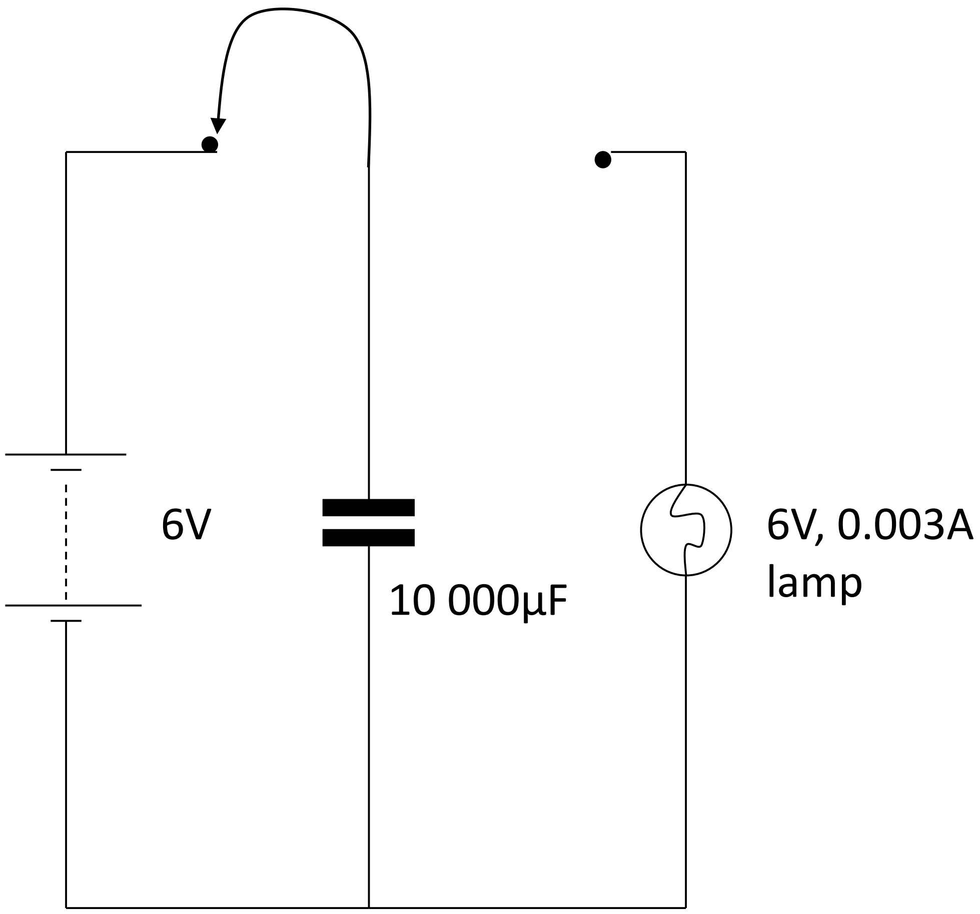
  conductor (metal plate)



Ceramic Capacitor Symbol

Variable Capacitor Symbol

Action of a capacitor



Set up the circuit.

Connect the flying lead of the capacitor to the battery.

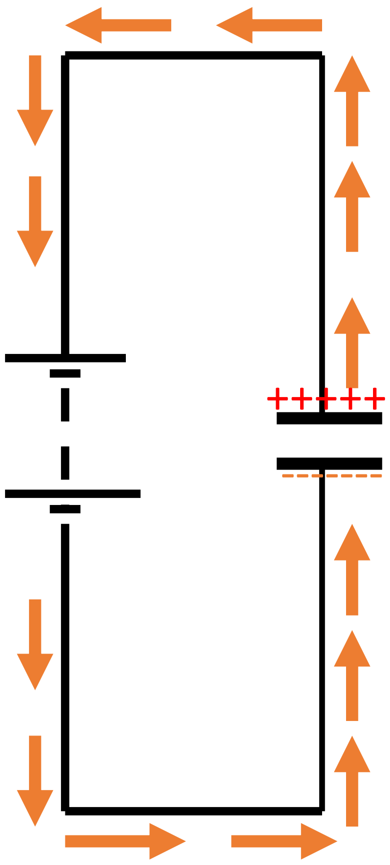
Connect it to the lamp.

What do you observe.

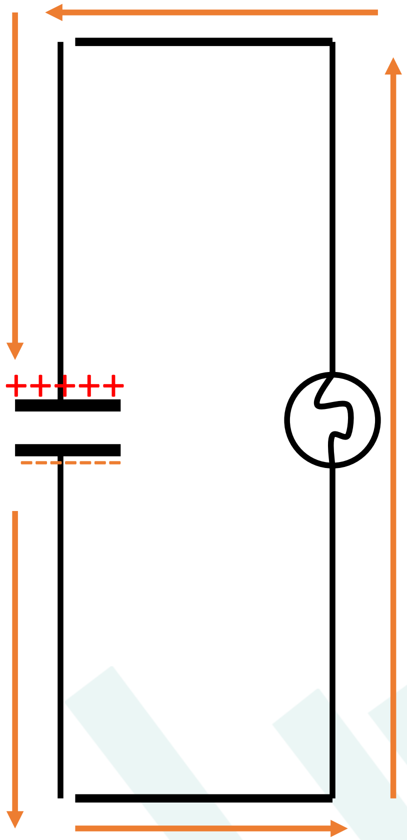
Try putting a 100Ω resistor in series with the lamp. What effect does it have?

What is happening

- When the capacitor is connected to the battery, a momentary current flows.
- Electrons gather on the plate attached to the negative terminal of the battery. At the same time electrons are drawn from the positive plate of the capacitor

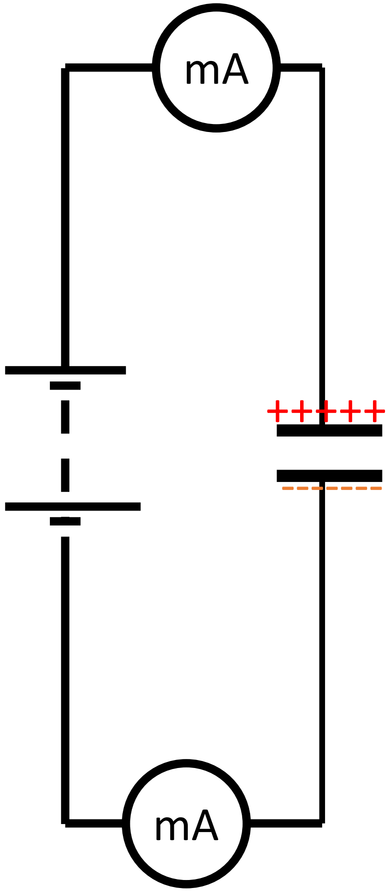


- When the capacitor is connected to the lamp, the charge has the opportunity to rebalance and a current flows lighting the lamp.
- This continues until the capacitor is completely discharged.

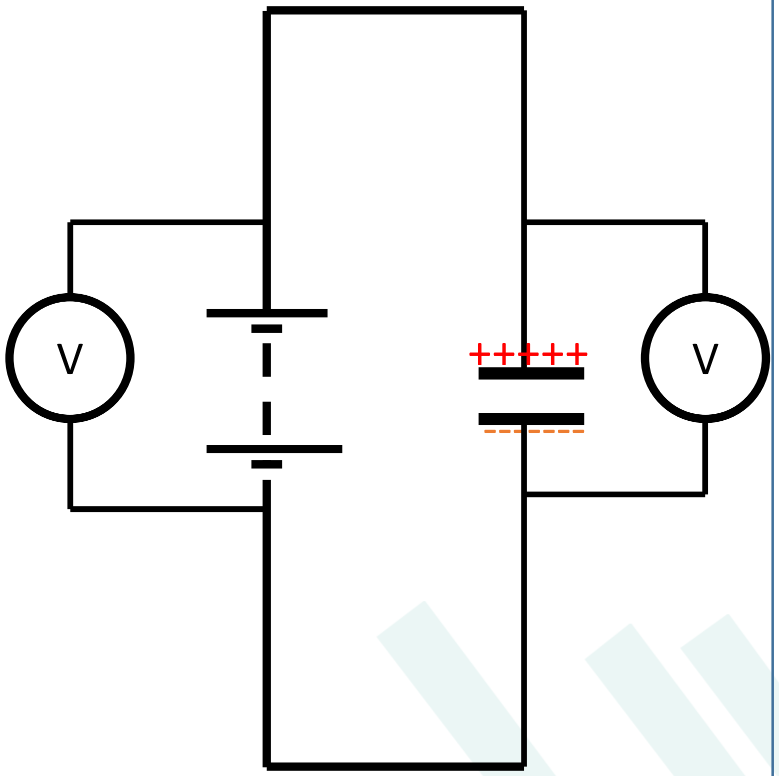


While the capacitor is charging

- Although the current falls as the capacitor is charging the current at any instant in both of the meters is the same, showing that the charge stored on the negative plate is equal in quantity with the charge stored on the positive plate.



- When the capacitor is fully charged the potential difference measured across the capacitor is equal and opposite to the potential difference across the battery, so there can be no further current flow.

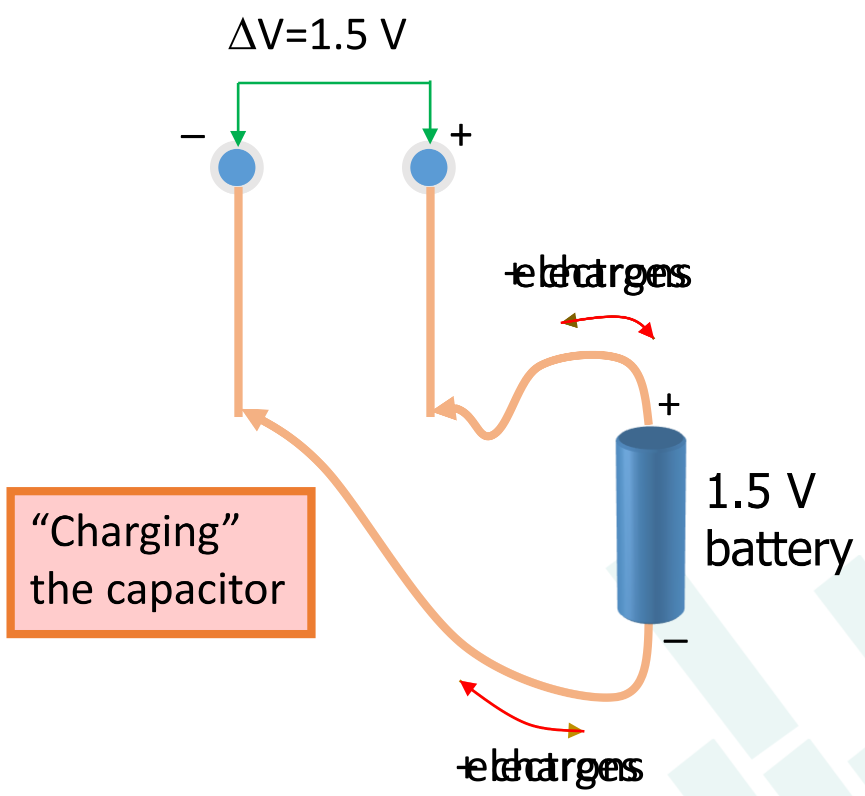


Capacitance

What is Capacitance (C)?

- From the word “capacity,” it describes how much charge an arrangement of conductors can hold for a given voltage applied.

- Charges will flow until the right conductor’s potential is the same as the + side of the battery, and the left conductor’s potential is the same as the – side of the battery.



- How much charge is needed to produce an electric field whose potential difference is 1.5 V?
- Depends on capacitance:

$$q = CV$$

definition of capacitance

Capacitance Depends on Geometry

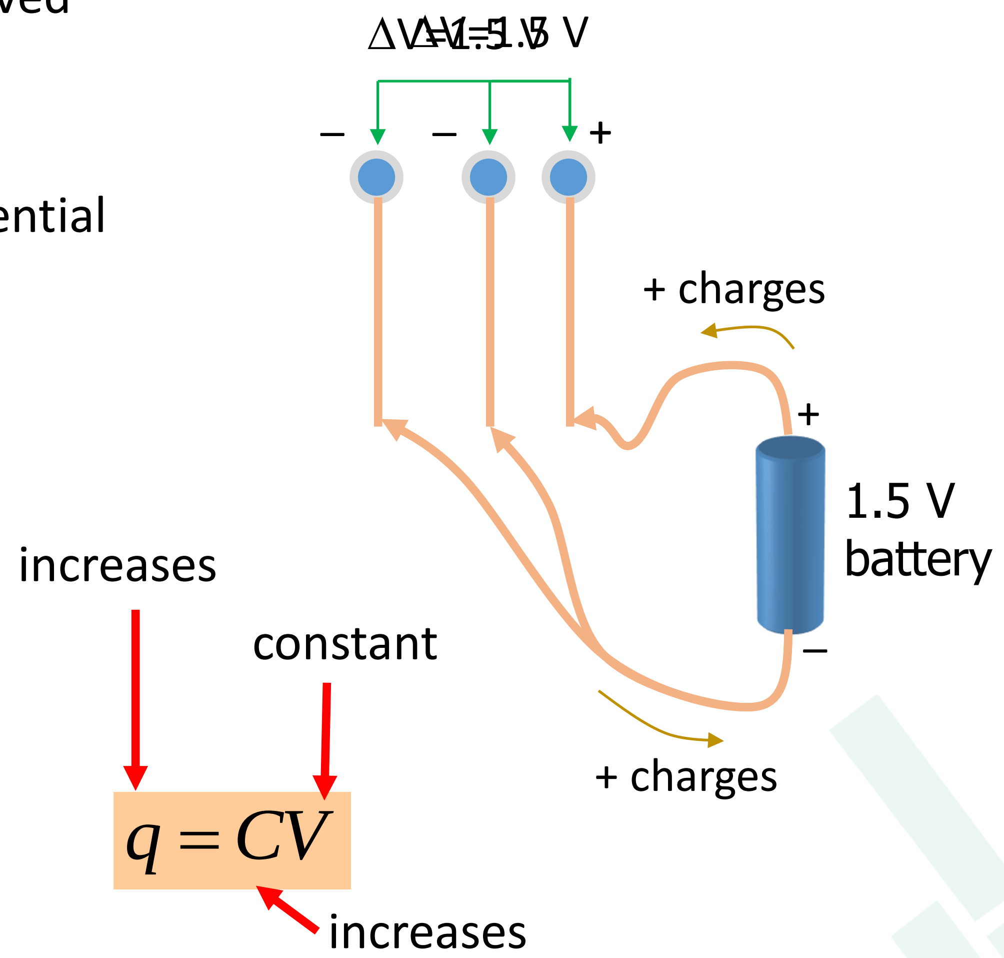
- What happens when the two conductors are moved closer together?

- They are still connected to the battery, so the potential difference cannot change.

- But recall that $V = -\int \vec{E} \cdot d\vec{s}$

- Since the distance between them decreases, the E field has to increase.

- Charges have to flow to make that happen, so now these two conductors can hold more charge. I.e. the capacitance increases.

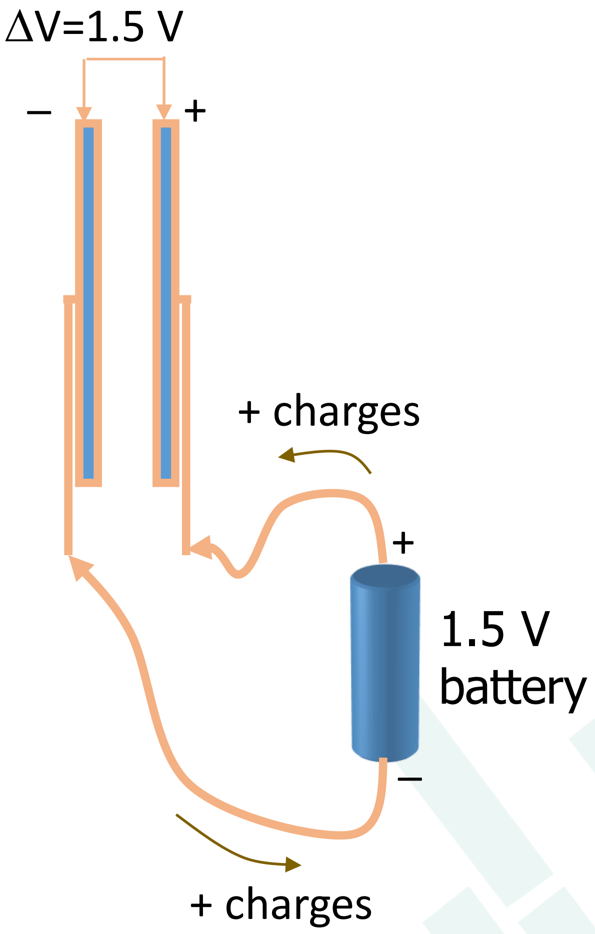


Capacitance Depends on Geometry

- What happens if we replace the small conducting spheres with large conducting plates?
- The plates can hold a lot more charge, so the capacitance goes way up.

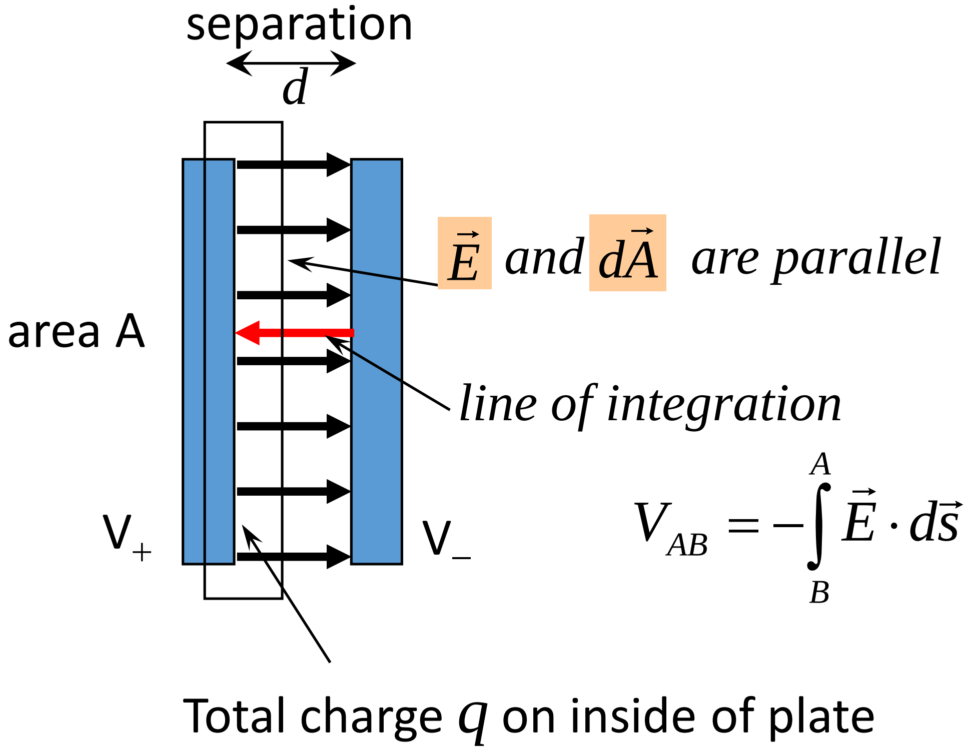
- Here is a capacitor that you can use in an electronic circuit.
- We will discuss several ways in which capacitors are useful.
- But first, let's look in more detail at what capacitance is.

Circular plates



Capacitance for Parallel Plates

- Parallel plates make a great example for calculating capacitance, because
  - The E field is constant, so easy to calculate.
  - The geometry is simple, only the area and plate separation are important.
- To calculate capacitance, we first need to determine the $\boldsymbol{E}$ field between the plates. We use Gauss' Law, with one end of our gaussian surface closed inside one plate, and the other closed in the region between the plates (neglect fringing at ends):



$$V_{AB} = -\int_B^A \vec{E}\cdot d\vec{s}$$

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = q \quad \text{so} \quad q = \varepsilon_0 EA$$

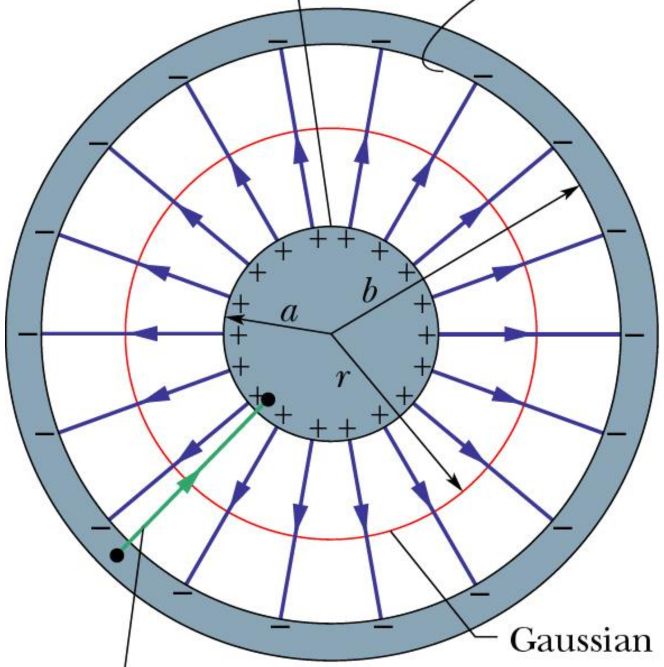
- Need to find potential difference $V = V_+ - V_- = -\int \vec{E}\cdot d\vec{s}$
- Since E=*constant*, we have $V = Ed$, so the capacitance is

$$C = q/V = \frac{\varepsilon_0 EA}{Ed} = \frac{\varepsilon_0 A}{d}$$

Gauss's Law:
The net electric flux through any hypothetical closed surface is equal to $1/\varepsilon_0$ times the net electric charge within that closed surface

Capacitance for Other Configurations (Cylindrical)

- Cylindrical capacitor
  - The E field falls off as $1/r$.
  - The geometry is fairly simple, but the V integration is slightly more difficult.
- To calculate capacitance, we first need to determine the $\boldsymbol{E}$ field between the plates. We use Gauss' Law, with a cylindrical gaussian surface closed in the region between the plates (neglect fringing at ends):



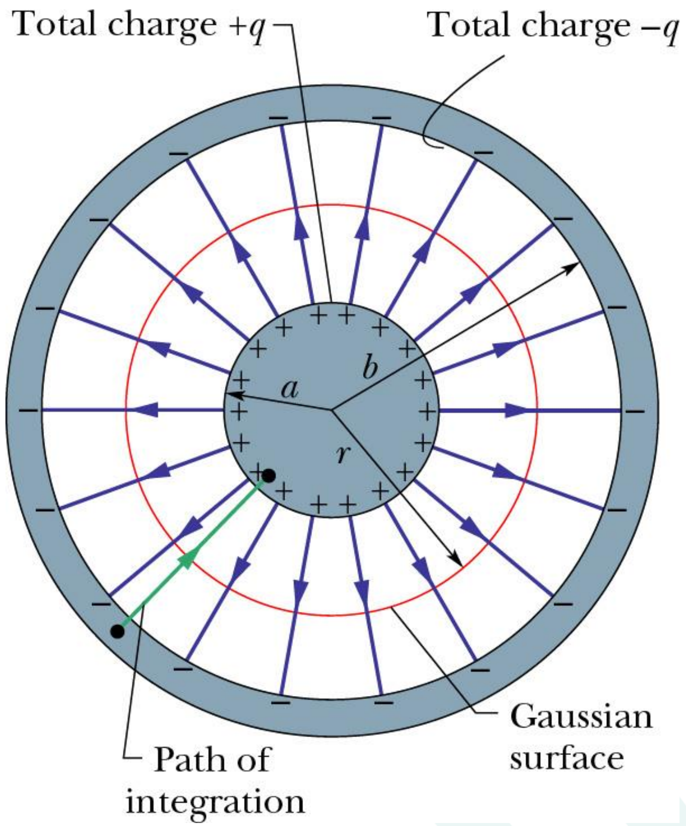
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$ So $$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$$ or $$E = q/(2\pi\varepsilon_0 rL)$$

- Need to find potential difference $$V = V_+ - V_- = -\int \vec{E} \cdot d\vec{s}$$

- Since $E \sim 1/r$, we have $$V = \frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$, so the capacitance is $$C = q/V = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

Capacitance for Other Configurations (Spherical)

- Spherical capacitor
  - The E field falls off as $1/r^2$.
  - The geometry is fairly simple, and the V integration is similar to the cylindrical case.
- To calculate capacitance, we first need to determine the ***E*** field between the spheres. We use Gauss' Law, with a spherical gaussian surface closed in the region between the spheres:



$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$ So $$q = \varepsilon_0 E A = \varepsilon_0 E (4\pi r^2)$$ or $$E = q/(4\pi\varepsilon_0 r^2)$$

- Need to find potential difference $$V = V_+ - V_- = -\int \vec{E} \cdot d\vec{s}$$

- Since $E \sim 1/r^2$, we have $$V = \frac{q}{4\pi\varepsilon_0} \int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$, so the capacitance is $$C = q/V = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Capacitance Summary

- Parallel Plate Capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

- Cylindrical (nested cylinder) Capacitor

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

- Spherical (nested sphere) Capacitor

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

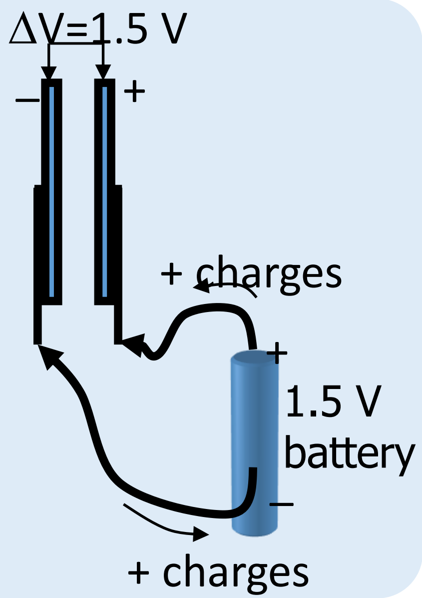
- Capacitance for isolated Sphere

$$C = 4\pi\varepsilon_0 R$$

Capacitors Store Energy

- When charges flow from the battery, energy stored in the battery is lost. Where does it go?
- An arrangement of charge is associated with potential energy. One way to look at it is that the charge arrangement stores the energy. Recall the definition of electric potential $U = qV$
- For a distribution of charge on a capacitor, a small element dq will store potential energy $dU = V\,dq$ .Thus, the energy stored by charging a capacitor from charge 0 to q is

$$U = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C} = \tfrac{1}{2}CV^2$$



- Another way to think about the stored energy is to consider it to be stored in the electric field itself.
- The total energy in a parallel plate capacitor is $U = \tfrac{1}{2}CV^2 = \dfrac{\varepsilon_0 A}{2d}V^2$
- The volume of space filled by the electric field in the capacitor is $vol = Ad$, so the *energy density* is

$$u = \frac{U}{vol} = \frac{\varepsilon_0 A}{2dAd}V^2 = \tfrac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$ but $$V = -\int \vec{E}\cdot d\vec{s} = Ed$$ for a parallel plate capacitor,

so $$u = \tfrac{1}{2}\varepsilon_0 E^2$$

Energy stored in electric field

Dielectrics in capacitors

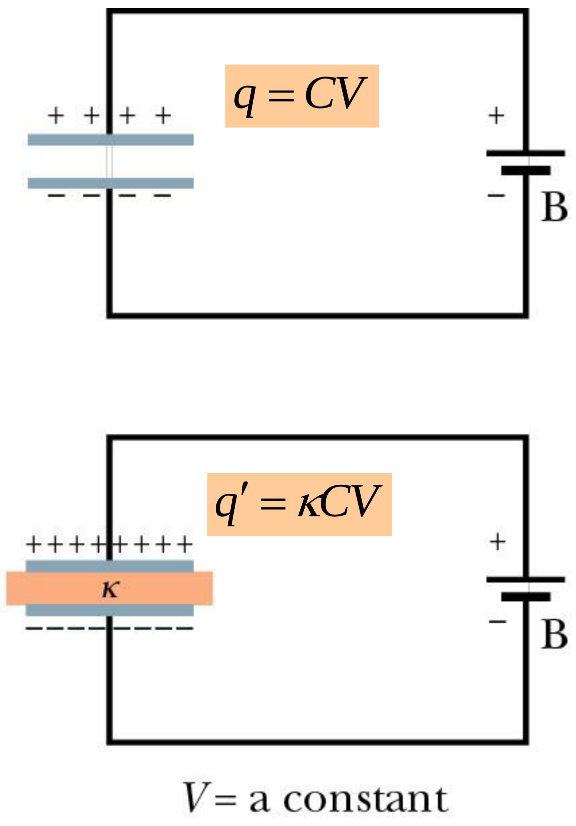
- You may have wondered why we write ε_0 (permittivity of free space), with a little zero subscript. It turns out that other materials (water, paper, plastic, even air) have different permittivities $\varepsilon = \kappa\varepsilon_0$. The κ is called the *dielectric constant*, and is a unitless number. For air, κ = 1.00054 (so ε for air is for our purposes the same as for “free space.”)
- In all of our equations where you see ε_0, you can substitute $\kappa\varepsilon_0$ when considering some other materials (called dielectrics).
- The nice thing about this is that we can increase the capacitance of a parallel plate capacitor by filling the space with a dielectric:

$$C' = \frac{\kappa\varepsilon_0 A}{d} = \kappa C$$

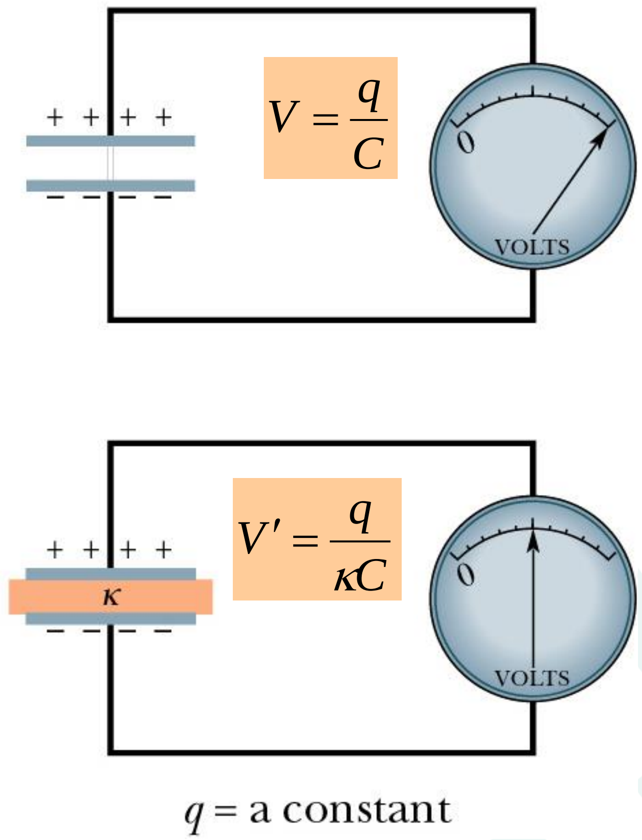
Material	Dielectric Constant κ
Air	1.00054
Polystyrene	2.6
Paper	3.5
Transformer Oil	4.5
Pyrex	4.7
Ruby Mica	5.4
Porcelain	6.5
Silicon	12
Germanium	16
Ethanol	25
Water (20º C)	80.4
Water (50º C)	78.5
Titania Ceramic	130
Strontium Titanate	310

What Happens When You Insert a Dielectric?

- With battery attached, V=const, so more charge flows to the capacitor

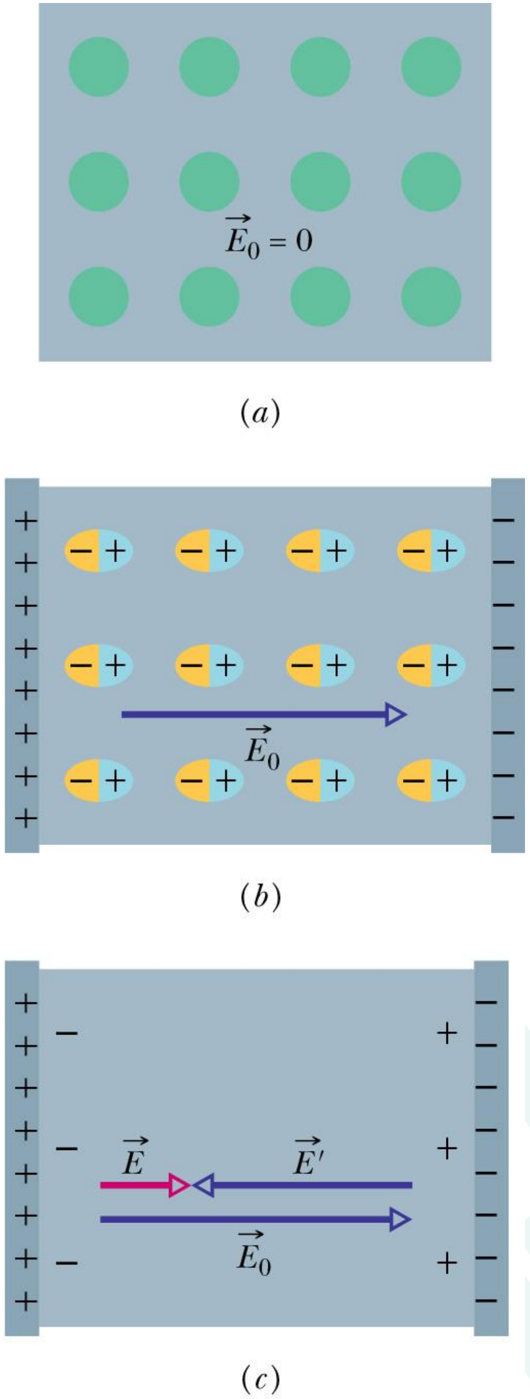


- With battery disconnected, q=const, so voltage (for given q) drops.



What Does the Dielectric Do?

- A dielectric material is made of atoms/molecules.
- Polar dielectrics already have a dipole moment (like the water molecule).
- Non-polar dielectrics are not naturally polar, but actually stretch in an electric field, to become polar.
- The molecules of the dielectric align with the applied electric field in a manner to oppose the electric field.
- This reduces the electric field, so that the net electric field is less than it was for a given charge on the plates.
- This lowers the potential (case b of the previous slide).
- If the plates are attached to a battery (case a of the previous slide), more charge has to flow onto the plates.



What Changes?

A parallel plate capacitor is connected to a battery of voltage V. If the plate separation is decreased, which of the following increase?

A. II, III and IV.
B. I, IV, V and VI.
C. I, II and III.
D. All except II.
E. All increase.

I.	Capacitance of capacitor
II.	Voltage across capacitor
III.	Charge on capacitor
IV.	Energy stored on capacitor
V.	Electric field magnitude between plates
VI.	Energy density of E field

$$q = CV$$

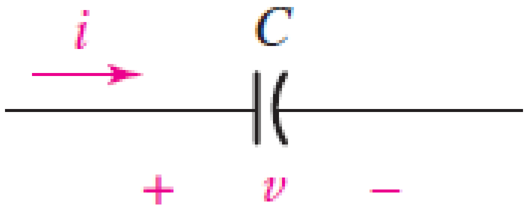
$$C = \frac{\varepsilon_0 A}{d}$$

$$U = \tfrac{1}{2} C V^2$$

$$u = \tfrac{1}{2} \varepsilon_0 E^2$$

Capacitor

Capacitance: $$C = \frac{dQ}{dv} \text{ or } Q = Cv$$



Current through a capacitor: $$i = C\frac{dv}{dt}$$

Zero current flowing through a capacitor when applied DC voltage, but a non-zero current for applied AC voltage

Determine the current i flowing through the capacitor of Fig. 7.1 for the two voltage waveforms of Fig. 7.3 if $C = 2$ F.