# K-Map (With Don't Care):

- In practice, in some applications the function is not specified for certain combinations of the variables.
- As an example, the four-bit bin for the decimal digits has six combinations that are not used and consequently are considered to be unspecified. ← output
- Functions that have unspecified outputs for some input combinations are called *incompletely specified functions* .
- In most applications, we simply don't care what value is assumed by the function for the unspecified minterms/maxterms. For this reason, it is customary to call the unspecified minterms/maxterms of a function as *don't-care conditions*
- These don't-care conditions can be used on a map to provide further simplification of the Boolean expression.

# K-Map (With Don't Care): SOP

don't care min terms
$m_{12}, m_{13}, m_{14}$ & $m_{15}$ o/p not sepecified

$$f(w,x,y,z) = \sum m(2,4,5,6,10) + D(12,13,14,15)$$

phi ← Greek

| $w,x$ \ $y,z$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ 0 | $m_1$ 0 | $m_3$ 0 | $m_2$ 1 |
| 01 | $m_4$ 1 | $m_5$ 1 | $m_7$ 0 | $m_6$ 1 |
| 11 | $m_{12}$ $\phi=1$ | $m_{13}$ $\phi=1$ | $m_{15}$ $\phi$ | $m_{14}$ $\phi=1$ |
| 10 | $m_8$ 0 | $m_9$ 0 | $m_{11}$ 0 | $m_{10}$ 1 |

3 PI
3 literals

$$f(w,x,y,z) = \bar{w}\cdot x\cdot\bar{y} + \bar{w}\cdot y\cdot\bar{z} + \bar{x}\cdot y\cdot\bar{z}$$

better for hardware

| $w,x$ \ $y,z$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ 0 | $m_1$ 0 | $m_3$ 0 | $m_2$ 1 |
| 01 | $m_4$ 1 | $m_5$ 1 | $m_7$ 0 | $m_6$ 1 |
| 11 | $m_{12}$ $\phi=1$ | $m_{13}$ $\phi=1$ | $m_{15}$ $\phi=0$ | $m_{14}$ $\phi=1$ |
| 10 | $m_8$ 0 | $m_9$ 0 | $m_{11}$ 0 | $m_{10}$ 1 |

$$f(w,x,y,z) = x\cdot\bar{y} + y\cdot\bar{z}$$

2 PI 2 literals

# K-Map (With Don't Care): POS

$$f(w,x,y,z) = \sum m(2,4,5,6,10) + D(12,13,14,15)$$

| $w,x$ \ $y,z$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ 0 | $m_1$ 0 | $m_3$ 0 | $m_2$ 1 |
| 01 | $m_4$ 1 | $m_5$ 1 | $m_7$ 0 | $m_6$ 1 |
| 11 | $m_{12}$ $\phi = 1$ | $m_{13}$ $\phi = 1$ | $m_{15}$ $\phi = 0$ | $m_{14}$ $\phi = 1$ |
| 10 | $m_8$ 0 | $m_9$ 0 | $m_{11}$ 0 | $m_{10}$ 1 |

$m_{15} = 1$ or 0

1111 → will never occur

| $w,x$ \ $y,z$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ 0 | $m_1$ 0 | $m_3$ 0 | $m_2$ 1 |
| 01 | $m_4$ 1 | $m_5$ 1 | $m_7$ 0 | $m_6$ 1 |
| 11 | $m_{12}$ $\phi = 1$ | $m_{13}$ $\phi = 1$ | $m_{15}$ $\phi = 0$ | $m_{14}$ $\phi = 1$ |
| 10 | $m_8$ 0 | $m_9$ 0 | $m_{11}$ 0 | $m_{10}$ 1 |

$$f(x,y,z) = (x+y)\cdot(w+\bar{y}+\bar{z})\cdot(x+\bar{y}+\bar{z})$$

$$f(x,y,z) = (x+y)\cdot(\bar{y}+\bar{z})$$

# K-Map (With Don't Care)

- Assigning the same values to the don't-cares for both SOP and POS implementations is not always a good choice.
- Sometimes it may be advantageous to give a particular don't-care the value 1 for SOP implementation and the value 0 for POS implementation, or vice versa.

$\phi \to m_{15} = 0$ $m_{15} = 1$

- In such cases the optimal SOP and POS expressions will represent different functions, but these functions will differ only for the valuations that correspond to these don't-cares.

# K-Map (7-segment display)

| $x_3x_2$ \ $x_1x_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 1 | 1 |
| 01 | 0 | 1 | 1 | 1 |
| 11 | $\phi$=1 | $\phi$=1 | $\phi$=1 | $\phi$=1 |
| 10 | 1 | 1 | $\phi$=1 | $\phi$=1 |

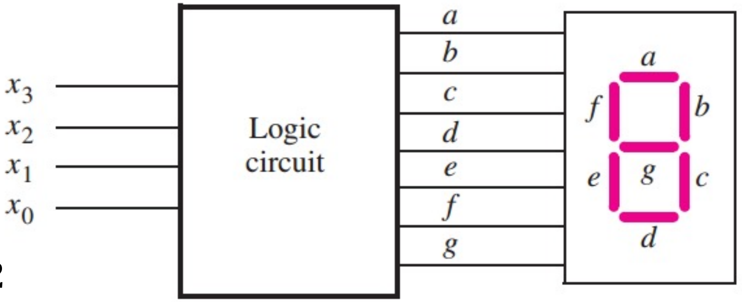
SOP

$$a = x_1 + x_3 + \overline{x_0}\,\overline{x_2} + x_0 x_2$$

all $\phi$s need not maintain ascribed value

| $x_3x_2$ \ $x_1x_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 1 | 1 |
| 01 | 0 | 1 | 1 | 1 |
| 11 | $\phi$=0 | $\phi$ | $\phi$ | $\phi$ |
| 10 | 1 | 1 | $\phi$ | $\phi$ |

POS




| | $x_3$ | $x_2$ | $x_1$ | $x_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 7 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 8 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |

$$a = (x_0 + x_1 + \overline{x_2}) \cdot (\overline{x_0} + x_1 + x_2 + x_3)$$

# K-Map (7-segment display)

$$f = x_3 + x_2\overline{x_1} + x_2\overline{x_0} + \overline{x_1}\,\overline{x_0}$$

| $x_3x_2$ \ $x_1x_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ 1 | $m_1$ 0 | $m_3$ 0 | $m_2$ 0 |
| 01 | $m_4$ 1 | $m_5$ 1 | $m_7$ 0 | $m_6$ 1 |
| 11 | $m_{12}$ $\phi=1$ | $m_{13}$ $\phi=1$ | $m_{15}$ $\phi=1$ | $m_{14}$ $\phi=1$ |
| 10 | $m_8$ 1 | $m_9$ 1 | $m_{11}$ $\phi=1$ | $m_{10}$ $\phi=1$ |

3rd dimension

advantage of Don't care inclusion ⊙ POS

$m_{15}$, $m_{11}$, $m_{10}$ → 1 SOP

$m_{12}$, $m_{13}$, $m_{14}$ → 1 for all

| $x_3x_2$ \ $x_1x_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ 1 | $m_1$ 0 | $m_3$ 0 | $m_2$ 0 |
| 01 | $m_4$ 1 | $m_5$ 1 | $m_7$ 0 | $m_6$ 1 |
| 11 | $m_{12}$ $\phi$ | $m_{13}$ $\phi$ | $m_{15}$ $\phi=0$ | $m_{14}$ $\phi$ |
| 10 | $m_8$ 1 | $m_9$ 1 | $m_{11}$ $\phi=0$ | $m_{10}$ $\phi=0$ |

$$f = (\overline{x_0} + \overline{x_1})\cdot(x_2 + \overline{x_1})\cdot(x_3 + x_2 + \overline{x_0})$$

| | $x_3$ | $x_2$ | $x_1$ | $x_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 7 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 8 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |

# K-Map (5-Variable):

| $x_4x_3x_2$ \ $x_1x_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 000 | $m_0$ | $m_1$ | $m_3$ | $m_2$ |
| 001 | $m_4$ | $m_5$ | $m_7$ | $m_6$ |
| 011 | $m_{12}$ | $m_{13}$ | $m_{14}$ | $m_{15}$ |
| 010 | $m_8$ | $m_9$ | $m_{11}$ | $m_{10}$ |
| 100 | $m_{16}$ | $m_{17}$ | $m_{19}$ | $m_{18}$ |
| 101 | $m_{20}$ | $m_{21}$ | $m_{23}$ | $m_{22}$ |
| 111 | $m_{28}$ | $m_{29}$ | $m_{31}$ | $m_{30}$ |
| 110 | $m_{24}$ | $m_{25}$ | $m_{27}$ | $m_{26}$ |

Barrier

1st one to recognize

BA → DC

$\bar{x}_4 \cdot \bar{x}_3 \cdot \bar{x}_2 \cdot \bar{x}_0$

$\bar{x}_3 \bar{x}_2 \bar{x}_1 x_0$

$\bar{x}_3 \bar{x}_2 \cdot \bar{x}_1 (\bar{x}_0 + x_0)$

for users' convenience

diff. of 16 are neighbours only $x_3$ changes

$x_4 \cdot \bar{x}_3 \bar{x}_2 \cdot \bar{x}_0$

$x_4 = 0$

| $x_3x_2$ \ $x_1x_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ 0 | $m_1$ 1 | $m_3$ 3 | $m_2$ 2 |
| 01 | $m_4$ 4 | $m_5$ 5 | $m_7$ 7 | $m_6$ 6 |
| 11 | $m_{12}$ 12 | $m_{13}$ 13 | $m_{15}$ 15 | $m_{14}$ 14 |
| 10 | $m_8$ 8 | $m_9$ 9 | $m_{11}$ 11 | $m_{10}$ 10 |

$f_1$

$x_4 = 1$

| $x_3x_2$ \ $x_1x_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_{16}$ 16 | $m_{17}$ 17 | $m_{19}$ 19 | $m_{18}$ 18 |
| 01 | $m_{20}$ 20 | $m_{21}$ 21 | $m_{23}$ 23 | $m_{22}$ 22 |
| 11 | $m_{28}$ 28 | $m_{29}$ 29 | $m_{31}$ 31 | $m_{30}$ 30 |
| 10 | $m_{24}$ 24 | $m_{25}$ 25 | $m_{27}$ 27 | $m_{26}$ 26 |

$f_2$ $f_1^*$

$f = \bar{x}_{n-1} \cdot f_1 + x_{n-1} f_1^*$

## Shannon's Theorem:

$$f(x_{n-1}, x_{n-2}, x_{n-3}, \ldots\ldots\ldots x_0) = \overline{x_{n-1}} \cdot f_1(x_{n-2}, x_{n-3}, \ldots\ldots\ldots x_0) + x_{n-1} \cdot f_1^*(x_{n-2}, x_{n-3}, \ldots\ldots\ldots x_0)$$

# K-Map (5-Variable)

- In 5-variable K-map, find PIs in each 4-variable section. Then, find overlapping PIs.
- Use these overlapping map without the fifth variable.

# K-Map (5-Variable)

- $f(A,B,C,D,E) = \sum m(0,1,2,4,5,6,10,13,14,18,21,22,24,26,29,30)$

**A = 0** (D = 1 for DE = 11, 10; C = 1 for BC = 01, 11; B = 1 for BC = 11, 10; E = 1 for DE = 01, 11)

| BC \ DE | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ 1 | $m_1$ 1 | $m_3$ 0 | $m_2$ 1 |
| 01 | $m_4$ 1 | $m_5$ 1 | $m_7$ 0 | $m_6$ 1 |
| 11 | $m_{12}$ 0 | $m_{13}$ 1 | $m_{15}$ 0 | $m_{14}$ 1 |
| 10 | $m_8$ 0 | $m_9$ 0 | $m_{11}$ 0 | $m_{10}$ 1 |

**A = 1** (D = 1 for DE = 11, 10; C = 1 for BC = 01, 11; B = 1 for BC = 11, 10; E = 1 for DE = 01, 11)

| BC \ DE | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_{16}$ 0 | $m_{17}$ 0 | $m_{19}$ 0 | $m_{18}$ 1 |
| 01 | $m_{20}$ 0 | $m_{21}$ 1 | $m_{23}$ 0 | $m_{22}$ 1 |
| 11 | $m_{28}$ 0 | $m_{29}$ 1 | $m_{31}$ 0 | $m_{30}$ 1 |
| 10 | $m_{24}$ 1 | $m_{25}$ 0 | $m_{27}$ 0 | $m_{26}$ 1 |

$\bar{D}\cdot\bar{E}$

C

$\bar{A}\cdot D\cdot\bar{E} + A\cdot D\cdot\bar{E}$

$= D\cdot\bar{E}$

$f = \bar{A}\cdot f_1 + A\cdot f_1^*$

$A\cdot D\cdot\bar{E}$   $A\cdot D\cdot\bar{E}$

$f_1(B,C,D,E) = \bar{B}\cdot\bar{D} + D\cdot\bar{E} + C\cdot\bar{D}\cdot E$

$f_1^* = D\cdot\bar{E} + C\cdot\bar{D}\cdot E + B\cdot\bar{C}\cdot\bar{E}$ exclusive

$f = D\cdot\bar{E} + C\cdot\bar{D}\cdot E + \bar{A}\cdot\bar{B}\cdot\bar{D} + A\cdot B\cdot\bar{C}\cdot\bar{E}$

# K-Map (5-Variable)

$$f(A,B,C,D,E) = \sum m(0,1,2,3,8,10,16,18,20,25)$$

**A = 0** (D = 1: DE columns 11, 10; C = 1: BC rows 01, 11; B = 1: BC rows 11, 10; E = 1: DE columns 01, 11)

| BC \ DE | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ 1 | $m_1$ 1 | $m_3$ 1 | $m_2$ 1 |
| 01 | $m_4$ 0 | $m_5$ 0 | $m_7$ 0 | $m_6$ 0 |
| 11 | $m_{12}$ 0 | $m_{13}$ 0 | $m_{15}$ 0 | $m_{14}$ 0 |
| 10 | $m_8$ 1 | $m_9$ 0 | $m_{11}$ 0 | $m_{10}$ 1 |

**A = 1** (D = 1: DE columns 11, 10; C = 1: BC rows 01, 11; B = 1: BC rows 11, 10; E = 1: DE columns 01, 11)

| BC \ DE | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_{16}$ 1 | $m_{17}$ 0 | $m_{19}$ 0 | $m_{18}$ 1 |
| 01 | $m_{20}$ 1 | $m_{21}$ 0 | $m_{23}$ 0 | $m_{22}$ 0 |
| 11 | $m_{28}$ 0 | $m_{29}$ 0 | $m_{31}$ 0 | $m_{30}$ 0 |
| 10 | $m_{24}$ 0 | $m_{25}$ 1 | $m_{27}$ 0 | $m_{26}$ 0 |

$m_0, m_2$

$m_{16}; m_{18}$

11001

$A \cdot B \cdot \bar{C} \cdot \bar{D} \cdot E$

$m_{25}$

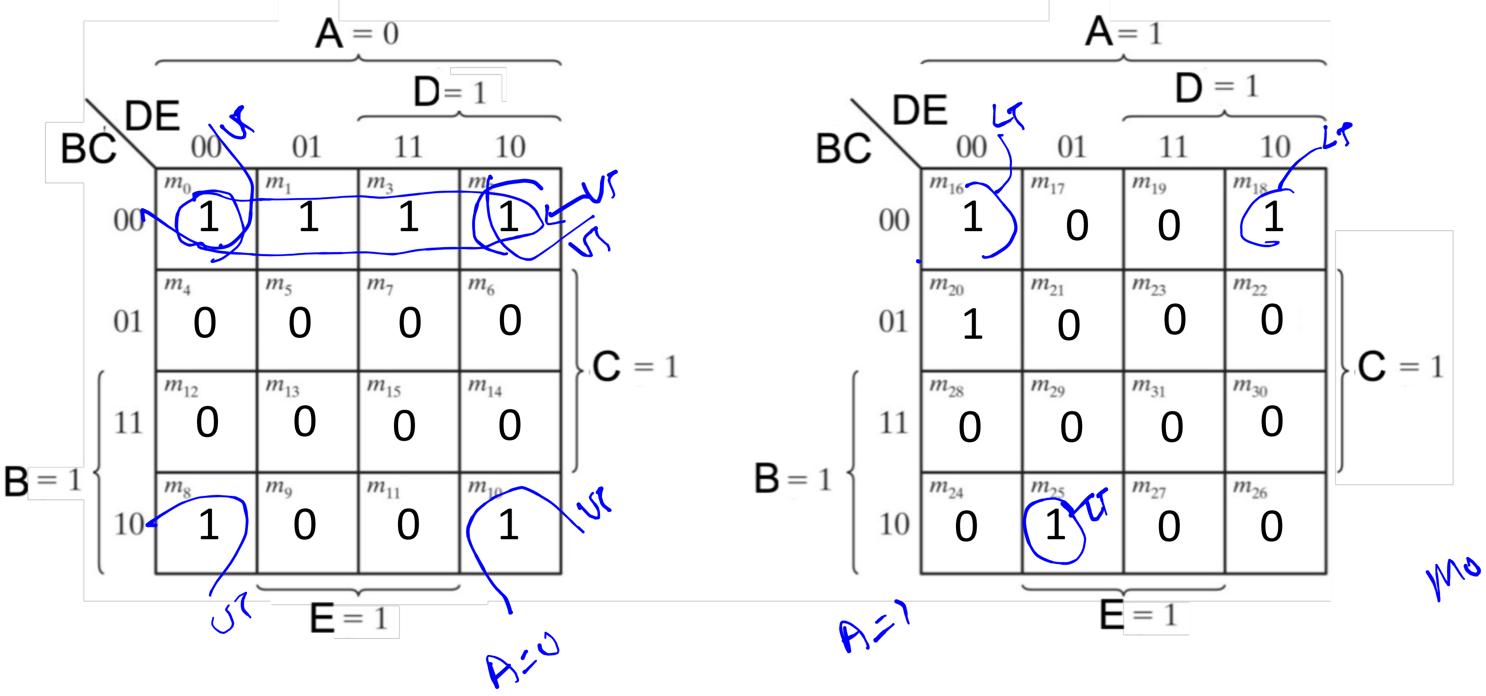
$$f_1(B,C,D,E) = \bar{B}\cdot\bar{C} + \bar{C}\cdot\bar{E}$$

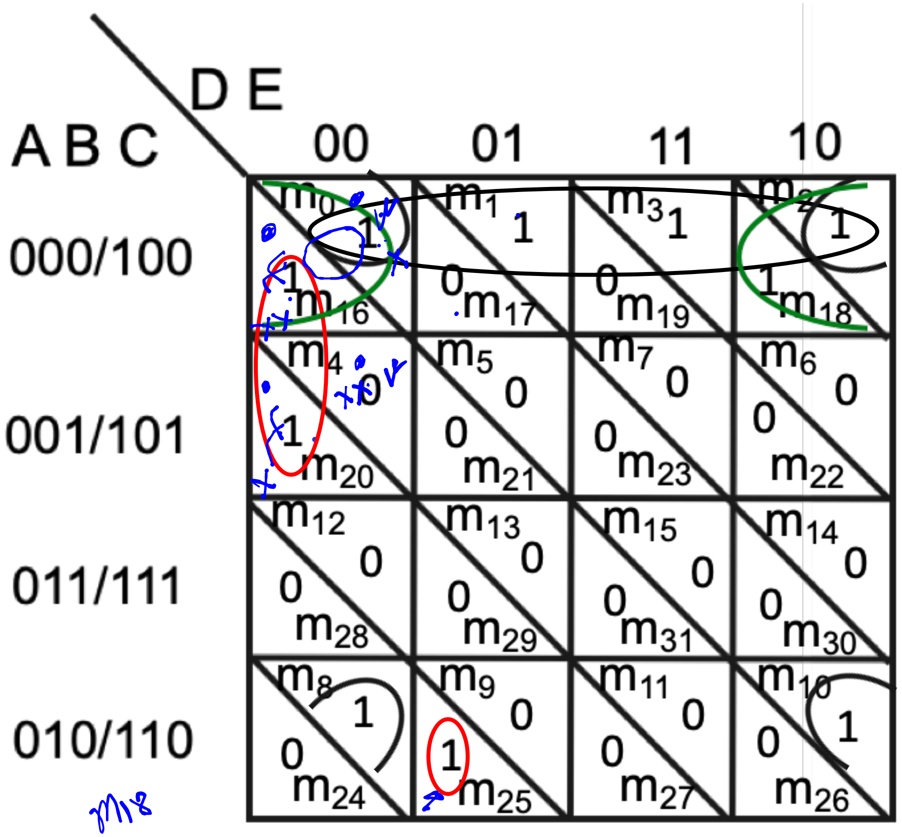
$$f_1^*(B,C,D,E) = \bar{B}\cdot\bar{D}\cdot\bar{E} + \bar{B}\cdot\bar{C}\cdot\bar{E} + B\cdot\bar{C}\cdot\bar{D}\cdot E$$

$$f(A,B,C,D,E) = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{C}\cdot\bar{E} + A\cdot\bar{B}\cdot\bar{D}\cdot\bar{E} + A\cdot B\cdot\bar{C}\cdot\bar{D}\cdot E + \bar{B}\cdot\bar{C}\cdot\bar{E}$$

# K-Map (5-Variable) --- An Alternate Style

$$f(A,B,C,D,E) = \sum m(0,1,2,3,8,10,16,18,20,25)$$




$m_0, m_{16}, m_2, m_{18}$

Green: Complete Square grouping

Black: Upper Triangle grouping

Red: Lower Triangle Grouping

$$f(A,B,C,D,E) = \bar{B}\cdot\bar{C}\cdot\bar{E} + \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{C}\cdot\bar{E} + A\cdot\bar{B}\cdot\bar{D}\cdot\bar{E} + A\cdot B\cdot\bar{C}\cdot\bar{D}\cdot E$$