$P[\text{Group 1} \mid \text{kicker kicks a field goal}] = 1/2$

$P[\text{Group 2} \qquad\qquad] = \frac{1}{3}$

$$K = (\underbrace{K \cap G_1}_{\text{kicker from } G_1}) \cup (\underbrace{K \cap G_2}_{\text{kicker from } G_i})$$

$$P[K] = P[K \cap G_1] + P[K \cap G_2]$$
$$= \underline{P[K|G_1]}\, P[G_1] + P[K|G_2]\, P[G_2]$$
$$= (1/2)\, P[G_1] + \frac{1}{3} P[G_2]$$
$$= \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)\left(2/3\right) =$$

K_i is the event that kicker i kicks a field goal.

$G_1^{(i)}$ is the event that the first kicker is from group i

$$K_1 = (K_1 \cap G_1^{(1)}) \cup (K_1 \cap G_1^{(2)})$$

$$P[K_1] = P[K_1 \cap G_1^{(1)}] + P[K_1 \cap G_1^{(2)}]$$
$$= P[K_1 | G_1^{(1)}]\, P[G_1^{(1)}] + P[K_1 \quad | G_1^{(2)}]\, P[G_1^{(2)}]$$
$$= (0.5)\left(3/9\right) + \left(\frac{1}{3}\right)\left(\frac{6}{9}\right)$$

$$P[K_2] = P[K_2 \cap (G_1^{(1)} \cap G_2^{(1)})] + P[K_2 \cap (G_1^{(1)} \cap G_2^{(2)})]$$
$$+ P[K_2 \cap (G_1^{(2)} \cap G_2^{(1)})] + P[K_2 \cap (G_1^{(2)} \cap G_2^{(2)})]$$

$$P[K_2 | G_1^{(1)} \cap G_2^{(1)}]\; P[G_1^{(1)} \cap G_2^{(1)}]$$
$$= \left(\frac{1}{2}\right) P[G_1^{(1)} \cap G_2^{(1)}]$$

$$P[G_1^{(1)} \cap G_2^{(1)}] = P[G_2^{(1)} | G_1^{(1)}]\, P[G_1^{(1)}]$$
$$= \left(\frac{2}{2+6}\right)\left(\frac{3}{9}\right) = \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)$$

$$P[G_1^{(1)}] = \frac{1}{3}$$

$$P[G_2^{(1)}] = ?$$

$$G_2^{(1)} = (G_2^{(1)} \cap G_1^{(1)}) \cup (G_2^{(1)} \cap G_1^{(2)})$$
$$P[G_2^{(1)}] = P[G_2^{(1)} | G_1^{(1)}]\, P[G_1^{(1)}] + P[G_2^{(1)} | G_1^{(2)}]\, P[G_1^{(2)}]$$
$$= \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{3}{8}\right)\left(\frac{2}{3}\right)$$
$$= \frac{1}{12} + \frac{1}{4} = \frac{4}{12} = \left(\frac{1}{3}\right)$$

Problem 1.9.5

There is a collection of field goal kickers, which can be divided into two groups 1 and 2. Group i has $3i$ kickers. On any kick, a kicker from group i will kick a field goal with probability $1/(i+1)$, independent of the outcome of any other kicks by that kicker or any other kicker.

(a) A kicker is selected at random from among all the kickers and attempts one field goal. Let K be the event that a field goal is kicked. Find $P[K]$.

(b) Two kickers are selected at random. For $j = 1, 2$, let K_j be the event that kicker j kicks a field goal. Find $P[K_1 \cap K_2]$. Are K_1 and K_2 independent events?

(c) A kicker is selected at random and attempts 10 field goals. Let M be the number of misses. Find $P[M = 5]$.