0 1 2 3 4 5 6 7 8 9
9 8 7 6 5 4 3 2 1 0

overflow
added to LSD

decimal | Binary
LSB

$(r-1)$'s complement

r's complement

$r=10$
Decimal
9's compl.
736
263
999
$d_2 d_1 d_0$

$r=2$
Binary
11010
00101

1's compl.
4 bit
1's $x_3 x_2 x_1 x_0$
1 1 1 1
$- x_3 x_2 x_1 x_0$
$\bar{x}_3 \bar{x}_2 \bar{x}_1 \bar{x}_0$

$(r-1)$'s complement & add 1 → r's complement.

Signed Numbers (for subtraction):

- r's Complement Representation

In decimal 10's compliment is obtained by adding 1 to the 9's compliment of the number.

9's Compl. +1 −1

$176 - 048$

$176 + 951 - 999 + 1 - 1$

Add 1 to 9's compliment

$176 + 952 - 1000$

addition

carryout

remove

$1128 - 1000$

simpler than 9's complement

If there is a carry drop the carry

answer

128

4 digit

$0176 - 1348$

$0176 + 8652 - \boxed{10000}$

No carry out

$8828 - 10000$

$1171 + 1 = 1172$

No Carry add 1 to 9's compliment i.e., obtain the 10's compliment.

-1172

Same.

Signed Numbers (for subtraction):

easier than 1's complement

In binary, 2's compliment is obtained by adding 1 to the LSB of 1's compliment:

$1101 - 0110$

Add 1 to 1's compliment

larger

$1101 - 1110$

$1101 + 1010 - 10000$

$1101 + 0010 - 10000$

$10111 - 10000$

1's additional sum

additional sum

$\boxed{1111 - 10000}$

If there is a carry drop the carry

0111

No Carry obtain the 2's compliment.

$\boxed{-1}$

Signed Numbers

- 2's Complement Representation (shortcut)

$0 \quad \bar{b}_{n-1}\cdots\bar{b}_3\,\bar{b}_2\;1\;0$

- Given a number $B = b_{n-1}\, b_{n-2} \cdot\cdot\cdot\, b_1\, b_0$, its 2's complement, $K = k_{n-1}\, k_{n-2} \cdot\cdot\cdot\, k_1\, k_0$, can be found by examining the bits of B from right to left and taking the following action: copy all bits of B that are 0 and the first bit that is 1; then simply complement the rest of the bits

2's complement

$$1001 + 0001 \longleftarrow -- \quad 0110 \overset{}{\underset{\text{1's comp.}}{---\longrightarrow}} \boxed{1010}$$

$$10010 + 00001 \longleftarrow -- \boxed{01101} ---\longrightarrow \boxed{10011}$$

easy to take 2's complement

realize

Signed Numbers (for Base other than 2)

- r's Complement Representation: r's complement of any number *N*, can be formed by leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from r, and subtracting all higher significant digits from r-1.

Signed Numbers:

110.01 00110 acceptable drop 110.01 11001 1101 decimal 001.11 decimal point separator 432.11 decimal point fraction point 11100.11 decimal point tried failed

- In the previous definitions, it was assumed that the numbers did not have a radix point. If the original number *N* contains a radix point, the point should be removed temporarily in order to form the *r*'s or (*r* - 1)'s complement. The radix point is then restored to the complemented number in the same relative position.

1101+(-0101) -5 1101-0101 → 1101+1011-10000 16 11000

- The radix point is what all of us are familiar as 'decimal point' in Decimal number system. However, due to our preoccupation with decimal system and the comfort of our mind, we call the radix point as a 'decimal point' irrespective of the radix used.

0101 = 10000 - 1011

1101 1011 10000

- Compliment of a complimented number restores it back to the original number.

2's comp 110.01 001.11 easier Binary 110.01 multiply by 4 11001 00110 1's comp 001.10 421.56 ×100 42156 10^2

$$V(B) = -((b_{n-1}\times 2^{n-1}) - 1) + b_{n-2}\times 2^{n-2} + \cdots + b_1\times 2^1 + b_0 2^0$$

$$V(B) = -(b_{n-1}\times 2^{n-1}) + b_{n-2}\times 2^{n-2} + \cdots + b_1\times 2^1 + b_0\times 2^0$$

Signed Numbers

Note that all negative numbers have a '1' in the leftmost bit position (MSB); that is the way we distinguish them from positive numbers.

Decimal	Signed-1's Complement	Signed-2's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	1111	—	1000
−1	1110	1111	1001
−2	1101	1110	1010
−3	1100	1101	1011
−4	1011	1100	1100
−5	1010	1011	1101
−6	1001	1010	1110
−7	1000	1001	1111
−8	—	1000	—

11111101

00000101

position value

Left most has position value

indicator

makes no difference

carry

−8 to +7

1101 − 0000

1101

1101 + 1111

sign extension

Sample 8bit

Practice Problems (Negative numbers in Binary and Hexa decimal):

2's complement for binary and 16's complement for Hexadecimal

sign bit 6 bit

$$-31.25 = -(011111.01) \xrightarrow{\text{2's comp}} 1100000.11 = \underbrace{1110}_{14}\underbrace{0000}.\underbrace{1100}_{12} \dashrightarrow E0.C$$

comp. sign extension 16's comp.

$$-(\underbrace{0001}_{1}\underbrace{1111}_{15}.\underbrace{0100}) \dashrightarrow -1F.4 \xrightarrow{\text{16's comp.}} E0.C$$

-393.3125 do it

$$393.3125 = 0110001001.0101 = 000110001001.0101 \dashrightarrow 189.5$$

$$-393.3125 =$$

Signed Numbers: Addition/ Subtraction

Read & respond on Wednesday

Start here with your feedback

Decimal	Signed-1's Complement
+7	0111
+6	0110
+5	0101
+4	0100
+3	0011
+2	0010
+1	0001
+0	0000
−0	1111
−1	1110
−2	1101
−3	1100
−4	1011
−5	1010
−6	1001
−7	1000
−8	—

2	0010
+3	+ 0011
5	0101

5	0101
-6	+ 1001
-1	1110

1	0001
-7	+ 1000
-6	1001

5	0101
-3	+ 1100
2	10001
	+0001
	0010

-5	1010
-2	+ 1101
-7	10111
	+0001
	1000

Signed Numbers: Addition/ Subtraction

$x \oplus y = \bar{x}y + x\bar{y}$ expan

Decimal	Signed-1's Complement
+7	0111
+6	0110
+5	0101
+4	0100
+3	0011
+2	0010
+1	0001
+0	0000
−0	1111
−1	1110
−2	1101
−3	1100
−4	1011
−5	1010
−6	1001
−7	1000
−8	—

representation

MSB sign bit has a value

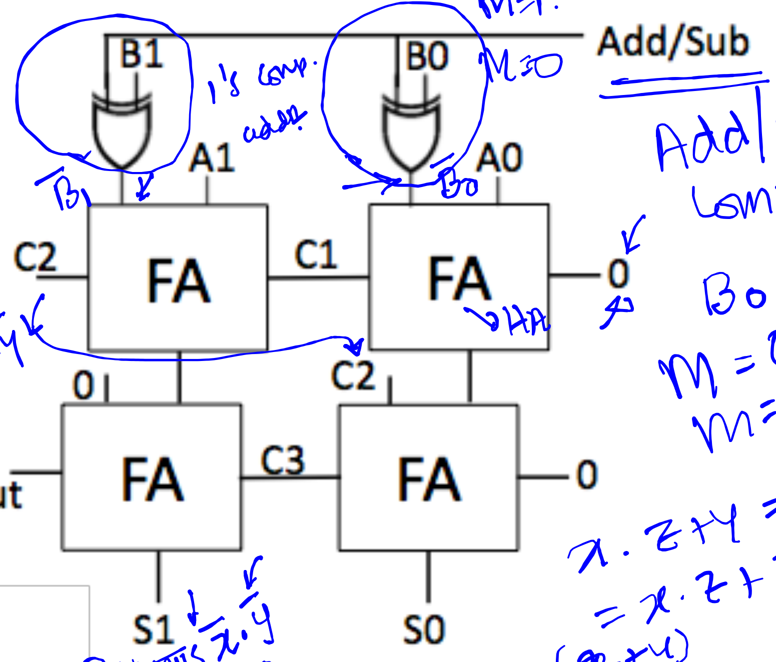
n-bit $-(2^{n-1}-1)$

1's compl. only for −ve number

- The addition of 1's complement numbers may or may not be simple.
- In some cases, a correction is needed, which amounts to an extra addition that must be performed. (2n full adders)
- Consequently, the time needed to add two 1's complement numbers may be twice as long as the time needed to add two unsigned numbers.

examples overflow → extra addition

resource intensive n-bit n FA



$M=1$ $M=0$ Add/Sub ← M

Add/sub command

1's comp. adder

Bo

$M=0 \rightarrow$ add

$M=1 \rightarrow$ sub.

$x \cdot z + y = (x \cdot z) + y$

$= x \cdot z + xy$

$x \cdot (z + y)$

20 mts. Concepts

10 mts. no questions accepted of slide 9's is not studied and responded $M=1$ = sub

+ve nos.

AND, OR, NOT

$\overline{x+y}$

$XOR = \bar{x}y + x\bar{y}$ 4 basic

$x \cdot \bar{y} + x$