Definition

Let V be a vector space. Let S be an infinite subset of V. We say S is a *linearly independent* set if every finite subset of S is linearly independent.

Definition

Let V be a vector space. A set of vectors $\mathcal{B} \subset V$ is said to be a basis of V if

 \mathcal{B} is linearly independent.

(ii) \mathcal{B} spans V.

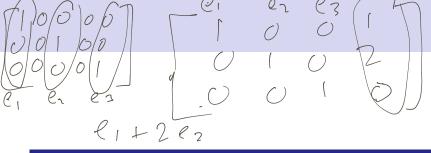
Whenever \mathcal{B} is a finite set, we say V is *finite dimensional*.

Example

The columns $\mathbf{e}_1, \dots, \mathbf{e}_n$ of the $n \times n$ identity matrix I_n form a basis of \mathbb{R}^n .

Theorem

Let A be an $m \times n$ matrix. The pivot columns of A form a basis for Col A.



Lemma

Let A be an $m \times n$ matrix in reduced echelon form, having k pivot columns, where $1 \le k \le m$. Then $\{e_1, \dots, e_k\}$ is a basis for Col A.

the pivot columns of A

 $e_{1}=(1,0.00)$ $b=(b_{1},...,b_{m})$ $e_{2}=(0,11,0.00)$ $=b_{1}e_{1}+b_{2}e_{2}-1-...+b_{m}e_{m}$ column of A which is not a pivot column. Then $b_i = 0$ for j = k+1, ...mNow b = b, l, l, l + . . + bmem => b= b, e, + . . . + bx ex E spar 3e, . . , ex3

Since & ei, ..., erz min of A Seli, ext col A.

=> (of A = Span Se1, ..., lp3. Lemma: If whis Sulspace of a vector space, and similar

Sive Wis doned Under Jahren inean Combination,

Claid., ---, ex an 1.j. (, l, + · · · + CR PR = 0. -) (C₁, (₂, ..., C_R, 0..., 0) = 0

Lemma

Let A be an $m \times n$ matrix and let A' be a matrix obtained by performing a row operation on A. Any linear dependence relation which holds between the columns of A also holds between the

The columns of A are linearly independent if and only if the columns of A' are linearly independent.

corresponding columns of A', and vice versa.

E is an elementary Support A= la...an Ean-Ean

Ciont - Chan = 0 C1, 621. -, ChEIR CIEA, + · · · + ChEan=0. i. The first statement follows. Since dementary matrices and in ventible, $C_{1}Ea_{1}+...+CnEa_{n}=0$ $=) E^{-1}(c_{1}Ea_{1}+...+cnE_{p})=0$ C) A, +. - · + C, A, = 0 is the visa! pant

Proposition

Let V be a vector space, and let S be a linearly independent subset of V. Any subset of S is linearly independent.

What is the contrapositive?

What about extending a linearly independent set to a bigger linearly independent set? How would we do this?

Proposition

independent.

Let $\{v_1, \ldots, v_n\}$ be a linearly independent set in a vector space V. If $w \notin Span(\{v_1, \ldots, v_n\})$ then the set $\{v_1, \ldots, v_n, w\}$ is linearly

Lemma

Let $S_1 \subset S_2 \subset V$. Then

 $\mathsf{Span}\, S_1 \subset \mathsf{Span}\, S_2.$

Lemma

Let W be a subspace of a vector space V. Let S be a subset of W. Then

Span $S \subset W$.