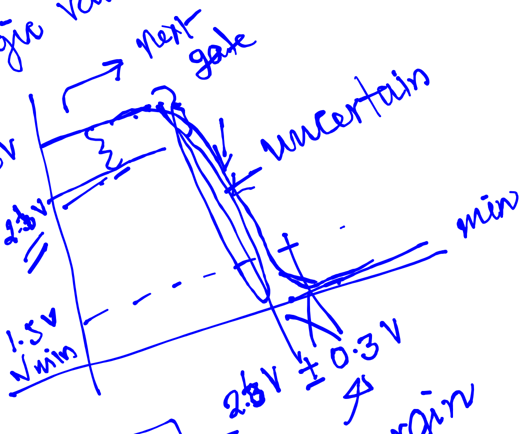
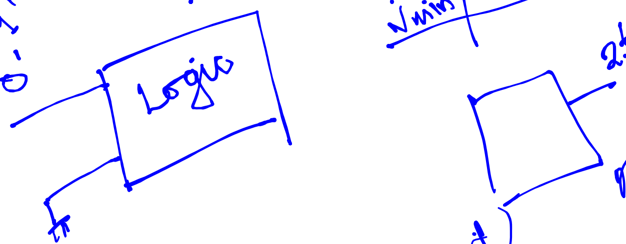
$$a\cdot b\cdot c\cdot d + a\cdot b = a\cdot b\cdot(1 + c\cdot d) = a\cdot b$$

Inverter
complements NOT logic gate

Use as logic value



0.3V



noise margin

mean → { 2.3V to 5V as '1'
1.8V to 0 as '0' }

② $f(a,b,c,d)$

$= a\cdot b\cdot c$

$= \underbrace{a\cdot b\cdot c\cdot d}+\underbrace{a\cdot b\cdot c\cdot \bar{d}}$

all the variables.

Before proceeding with details of canonical forms and representation let us respond to a couple of questions raised by students in the class, viz. proof for some of the Boolean Algebra theorems:

An exercise in using the Axioms/Postulates:

dual

Prove : $x \cdot 0 = 0, \quad x + 1 = 1$

Axiom 2 $\quad x + 0 = x, \qquad x.1 = x$

Axiom 3 $\quad x.y = y.x, \qquad x + y = y + x$

Axiom 4 $\quad \begin{cases} x \cdot (y + z) = x \cdot y + x \cdot z, \\ x + (y \cdot z) = (x + y) \cdot (x + z) \end{cases}$

Axiom 5 $\quad x \cdot \bar{x} = 0, \qquad x + \bar{x} = 1$

$$x \cdot 0 = 0$$

L.H.S. $= x \cdot 0 = x \cdot 0 + 0$

$$x \cdot 0 + 0 = x \cdot 0 + x \cdot \bar{x}$$

$$x \cdot 0 + x \cdot \bar{x} = x \cdot (0 + \bar{x})$$

$$x \cdot (0 + \bar{x}) = x \cdot \bar{x}$$

$x \cdot \bar{x} = 0 =$ R.H.S

$$x + 1 = 1$$

L.H.S. $= x + 1 = (x + 1) \cdot 1$

$$(x + 1) \cdot 1 = (x + 1) \cdot (x + \bar{x})$$

$\bar{x} = z$

$$(x + 1) \cdot (x + \bar{x}) = x + 1 \cdot \bar{x} = x + \bar{x}$$

$(x + \bar{x}) = 1 =$ R.H.S.

Axiom 4

An exercise in using the Axioms/Postulates:

Axiom 2 $\quad x + 0 = x, \qquad x.1 = x$

Axiom 3 $\quad x.y = y.x, \qquad x + y = y + x$

Axiom 4 $\quad \begin{cases} x \cdot (y+z) = x \cdot y + x \cdot z, \\ x + (y \cdot z) = (x+y) \cdot (x+z) \end{cases}$

Axiom 5 $\quad x \cdot \bar{x} = 0, \qquad \underline{x + \bar{x} = 1}$

Prove : $\underline{x \cdot x = x}$, $\underline{x + x = x}$

$$x \cdot x = x$$

L.H.S. $= x \cdot x = (x \cdot x) + 0$

$(x \cdot x) + \underline{(x \cdot \bar{x})} = x \cdot \underline{(x + \bar{x})}$

$x \cdot (x + \bar{x}) = x \cdot 1$

$x \cdot 1 = x =$ R.H.S

Dual

$$x + x = x$$

L.H.S. $= x + x = (x + x) \cdot 1$

$(x + x) \cdot (x + \bar{x}) = x + (x \cdot \bar{x})$

$x + (x \cdot \bar{x}) = x + 0$

$x + 0 = x =$ R.H.S.

Proof of De Morgan's Law:

Axiom 2 $\quad x + 0 = x, \qquad x.1 = x$

Axiom 3 $\quad x.y = y.x, \qquad x + y = y + x$

Axiom 4 $\quad x \cdot (y + z) = x \cdot y + x \cdot z,$
$\quad x + (y \cdot z) = (x + y) \cdot (x + z)$

Axiom 5 $\quad x \cdot \bar{x} = 0, \qquad x + \bar{x} = 1$

$A \cdot \bar{A} = 0$
$A + \bar{A} = 1$

Postulates accept

accepted are truth

Need No proof

condition iff $(x+y) \cdot \bar{x} \cdot \bar{y} = 0$
$(x+y) + \bar{x} \cdot \bar{y} = 1$

Basis

If these are true

Prove: $\overline{(x + y)} = \bar{x} \cdot \bar{y}$

We will show that $(i)\ (x + y) \cdot \bar{x} \cdot \bar{y} = 0$ *and* $(ii)\ x + y + \bar{x} \cdot \bar{y} = 1$

Theorems.

$3 \times 4 = 4 \times 3$

assumed complement

true

$y \cdot \bar{y} \cdot x$
$= (y \cdot \bar{y}) \cdot x$
$= 0 \cdot x = 0$ true

$(i)\ (x + y) \cdot (\bar{x} \cdot \bar{y}) = x \cdot \bar{x} \cdot \bar{y} + y \cdot \bar{x} \cdot \bar{y} = 0 + y \cdot \bar{y} \cdot \bar{x} = 0 + 0 = 0$

$A \cdot B = 0$
$A + B = 1$
$A = \bar{B}$
$B = \bar{A}$

$(ii)\ (x + y) + (\bar{x} \cdot \bar{y}) = (x + y + \bar{x}) \cdot (x + y + \bar{y}) = (1 + y) \cdot (1 + x) = 1 \cdot 1 = 1$

$A + B \cdot C = (A + B) \cdot (A + C)$

$(x+y) \cdot (\bar{x} \cdot \bar{y}) = 0$
$(x+y) + (\bar{x} \cdot \bar{y}) = 1$ then $\overline{x+y} = \bar{x} \cdot \bar{y}$

Boolean Algebra

Axiom 2 $x + 0 = x,$ $x.1 = x$

Axiom 3 $x.y = y.x,$ $x + y = y + x$

Axiom 4 $\left\{ \begin{array}{l} x \cdot (y+z) = x \cdot y + x \cdot z\,, \\ x + (y \cdot z) = (x+y) \cdot (x+z) \end{array} \right.$

Axiom 5 $x \cdot \bar{x} = 0,$ $x + \bar{x} = 1$

- Prove: $\overline{x \cdot y} = \bar{x} + \bar{y}$
- Prove : $x.y + x.\bar{y} = x$
- Prove : $x.(x+y) = x$

H.W.

numbers representation

Before proceeding with details of canonical forms and representation let us also investigate number systems which will be very useful in function representation as also some of the system to be discussed later.

Decimal and Binary Numbers

fingers - digits.

- The invention that we are most familiar with is decimal numbers, base 10, all the fingers and ~~thumbs~~. toes.
- Computers don't do well with decimal, at least not directly. relays base 10 → mechanical
- So when we deal with numbers in digital logic, it's much more likely that we'll use binary, base 2. 0,1 → base 2 binary numbers
- In decimal each position is called a digit.
- In binary each position is called a bit, four bits together is called a nibble and eight bits together is called a byte.

0 1 2 3 4 5 6 7 8 9

Decimal Numbers:

- Our number system is decimal, and position does count.
- Somebody, long, long ago, came up with the concept of "zero."
- This gave the system a placeholder so that a numeral in one position has a different value from a numeral in another position.
- So symbol and position both count. Since the base of the decimal number system is 10, the positions are powers of 10.

Decimal Numbers:

unit marker decimal pt

10^0 10^1 10^2

1 10 100 ....

.00001 $\frac{1}{100000}$

- Consider the number 32.063.
- The symbol "3" appears twice.
- In the position second to the left of the point, it has the value "thirty"; in the position third to the right of the point, it has the value "three thousandths."
- Each position has a unique multiplier value based on its position relative to the point. The position just to the left of the point has a multiplier value "1" or 10^0. The next position to the left has a multiplier value "10" or 10^1. Working to the right, the first position to the right of the point has a multiplier value "0.1" or 10^{-1}.
- And so on. We don't usually think of this, at least in this way, but it will be important when we work with binary numbers

Binary Numbers

- Binary numbers use only two symbols, the numerals 0 and 1.
- The 0 is still the position holder; the only "value" symbol is therefore 1.
- The base of this number system is 2, so all of the positions in the system are powers of 2.

  2^5 2^2 2^1 2^0 2^{-1} 2^{-2} 2^{-3} 2^{-4}

- Consider the binary number 110100.1011 → decimal equivalent
- Decimal equivalent of this binary number is 52.6875
- Let's see how we get it?

Binary Numbers

- We can "decode" the binary number 110100.1011 into decimal by writing it with powers of 2 in the right positions:

- The number 110100.1011

$$= 1\times2^5 + 1\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 0\times2^0 + 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3} + 1\times2^{-4} = 32 + 16 + 4 + 0.5 + 0.125 + 0.0625 = \underline{52.6875}$$

familiar

- **Positional number system:**

number systems

Binary 8 4 2 1
0 1 1 0 → decimal
Nibble → look in to position value

familiar 0×8 + 1×4 + 1×2 + 1×0

8 4 2 1 code
A B C D

DCBA code

0000
0001
0010
0011
0100
0101
0110
0111
1000
1001

Short coming

1010 1011 1100 1101 1110 1111 waste not used

don't care

$(67)_{10} \rightarrow 6{\times}10^1 + 7{\times}10^0 \quad \rightarrow Decimal$ Base 10

octal

$(67)_8 = 6{\times}8^1 + 7{\times}8^0 \quad \rightarrow \{(55)_{10}\} \rightarrow Octal$ Base 8

$(1011)_2 = 1{\times}2^3 + 0{\times}2^2 + 1{\times}2^1 + 1{\times}2^0 \rightarrow \{(10)_2\} \rightarrow Binary$ Base 2

16

$(6A)_{16} =? \rightarrow Hexadecimal$ Base 16

10 11 15

The digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Why hexadecimal

Number System Table:

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Post Algebra

2 valued Boolean Algebra

bit

language

numeral

Roman

5,1

Quaternary 0,1,2,3

Position digit

Multi valued logic

base 3, 4, 4

>100 yrs.

4.

0 to 9

X → 10

C → 100

10

10 fingers digit

8 symbols

1/4

1/8

1/32

base 7

Convenience

yes or no

00 64

22

16

8 byte

$8+7 = (15)_{10}$

systems.

comfortably realizable

$(836)_{10}$

→ 1000 0011 0110

Binary coded Decimal

0011 1101 1001 1100 . 0100

D 9 C

D9C

3D9C

coding

10 →

symbol →

0 1 2 3 4 5 6 7 8 9

8

0 1 2 3 4 5 6 7

Conversion from Decimal to Binary system:

57 to binary:

$\frac{57}{2} = 28 + 1$ 2^0

$\frac{28}{2} = 14 + 0$ 2^1

$\frac{14}{2} = 7 + 0$ 2^2

$\frac{7}{2} = 3 + 1$ 2^3

$\frac{3}{2} = 1 + 1$ 2^4

32 16 8 4 2 1

$57 = 111001$

0.625 to binary:

$\frac{0.125}{1/2}$

$0.625 \times 2 = 1.25$ 2^{-1}

$0.25 \times 2 = 0.5$ 2^{-2}

$0.5 \times 2 = 1.0$

0.101

$0.625 \times 2 \Rightarrow 1 + 0.25$

$0.25 \times 2 \rightarrow 0.5$

$0.5 \times 2 \rightarrow 1$

0.125

57.625 to binary:

Combine to do it

111001.101

divide → 86.75 ← repeated multiply

HW: Convert $(67)_{10}, (131)_{10}, (43.65625)_{10}, and (56.66)_{10}$ *to Binary*

Convert 10100101, 110.0011, 1000.0001 and 11001 to Decimal