Compane the COF of Exp(1/2) with the COF of the sam of two independent Esp(i) RV(s). Note that the expected value of Exp(h) is 2. The expected value of the sonn is also 2. Assuming He RVs model power, We have on average power of 2 Walks in boll cases. Movever, what is the probability that the power is > the expected value for when we have power distributed on Exp(1/2)? What is this prob for the sun of two independed Exp(i) RV(s)? Now consider In >2 Exp (1/2) independent RV(s). What is the expected value of the sum.? As N=D, what is the prob Ret sum takes a value > then the expected value? The example illustrates who some of all's Moy be useful in practice. The sum in our example can be Hought of ones the sum of powers necesived by multiple (n) autennas. An you can sea, we can keep Re arenajo received power la same While increasing the grab of othering power of the overage by using were antennas.

Probability and Random Processes

Sanjit Kaul



Statistics



We won't assume knowledge of the probabilistic model

We will soon use data to calculate statistics

Data comes from _____

Any statistic is function of data

 A statistic could, for example, be an estimate of the parameter of a PDF

Parameter Estimation



INDRAPRASTHA INSTITUTE of INFORMATION TECHNOLOGY **DELHI**



Creating the data

Total population m

• Sample size *n*

- Sample size is often much smaller than the total population
 - Often infeasible to measure all in total population



 Suppose we want to calculate the fraction of people in Delhi that are taller than 5 feet 6 inches.

- To calculate the exact fraction (probability p) we must measure the heights of all m people in Delhi
 - Total number of people taller than 5 feet 6 inches is

Note that p is unknown



- Suppose we want to calculate the fraction of people in Delhi that are taller than 5 feet 6 inches.
- To calculate the exact fraction (probability p) we must measure the heights of all m people in Delhi

Note that p is unknown

We want to estimate p using the sample of size n



• The *n* samples are chosen randomly

Are they independent samples?



• The *n* samples are chosen randomly

Are they independent samples?

 The height of a sample is a random variable of the _____ family.



• The *n* samples are chosen randomly

Are they independent samples?

- Think RVs that model the heights of the first two samples....
 - Are they independent RVs?

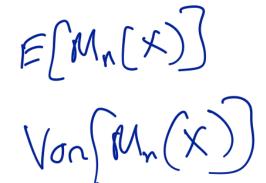


· We will assume samples are iid

Definition 7.1 Sample Mean

For iid random variables X_1, \ldots, X_n with PDF $f_X(x)$, the sample mean of X is the random variable

$$M_n(X) = \frac{X_1 + \dots + X_n}{n}.$$



Theorem 7.1

The sample mean $M_n(X)$ has expected value and variance

$$E[M_n(X)] = E[X], \quad Var[M_n(X)] = \frac{Var[X]}{n}.$$

Quiz 7.1

Let X be an exponential random variable with expected value 1. Let $M_n(X)$ denote the sample mean of n independent samples of X. How many samples n are needed to guarantee that the variance of the sample mean $M_n(X)$ is no more than 0.01?

$$Van(Mu(x)) = \frac{Van(x)}{n} \leq 0.01$$

$$N \geq \frac{Van(x)}{0.01}$$

Theorem 7.2 Markov Inequality

For a random variable X such that P[X < 0] = 0 and a constant c,

$$P\left[X \ge c^{2}\right] \le \frac{E\left[X\right]}{c^{2}}.$$

$$P\left[X \ge c^{2}\right] \le \int_{c^{2}} f_{x}(x) dx = \int_{c^{2}} \int_{c^{2}} c^{2} f_{x}(x) dx = \int_{c^{2}} \int_{c^{2}} f_$$