Huffman Coding and Red-Black Tree

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Huffman Coding

Huffman Coding

- Are used to compress information.
  - Like WinZip (although WinZip doesn't use the Huffman algorithm).
  - JPEGs do use Huffman as part of their compression process.

- **Basic idea:** Instead of storing characters in a file as 8-bit ASCII value, store the more frequently occurring characters using fewer bits and less frequently occurring characters using more bits
  - On average this should decrease the filesize (usually by 1/2).

Huffman Coding (Cont.)

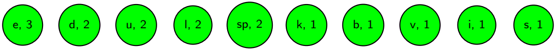
An Example: Consider the string,

"duke blue devils"

- Do a frequency count of the characters:

e	d	u	l	space	k	b	v	i	s
3	2	2	2	2	1	1	1	1	1

- Next we use a Greedy algorithm to build up a Huffman Tree.
  - Start with nodes for each character.



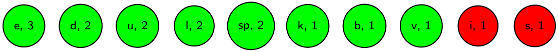
Huffman Coding (Cont.)

- Pick the nodes with the smallest frequency and combine them together to form a new node.
  - The selection of these nodes is the Greedy part.

- Remove the two selected nodes are from the set and replace it with a combined node.

- Continue until only 1 node left in the set.

Huffman Coding (Cont.)



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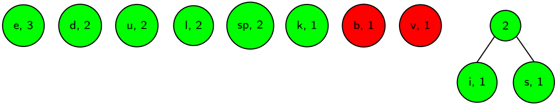

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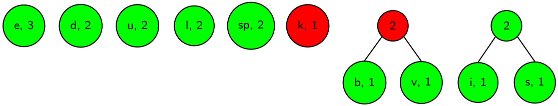

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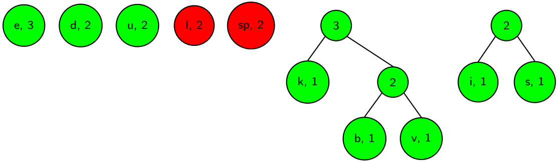

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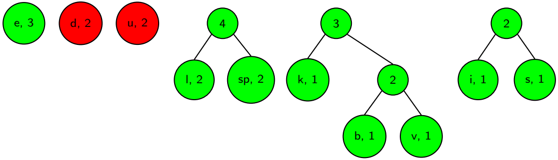

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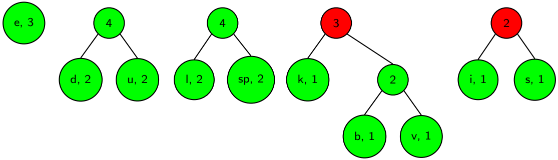

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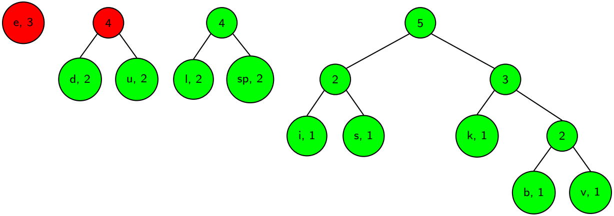

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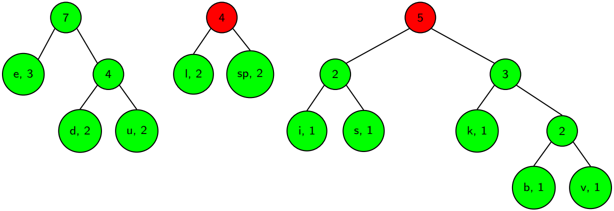

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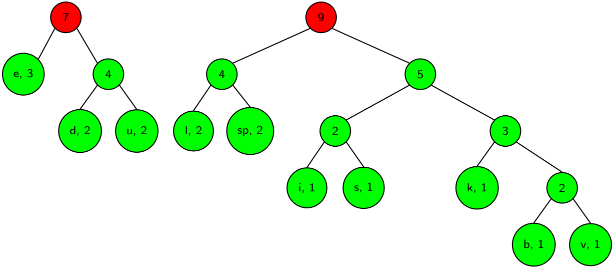

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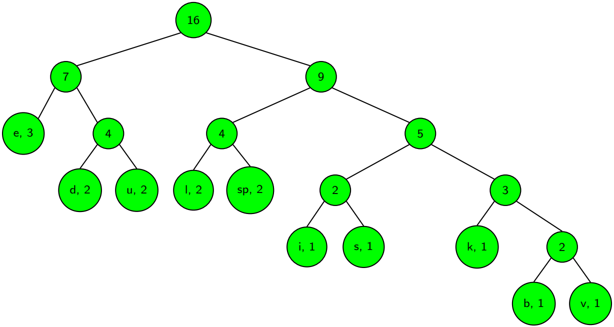


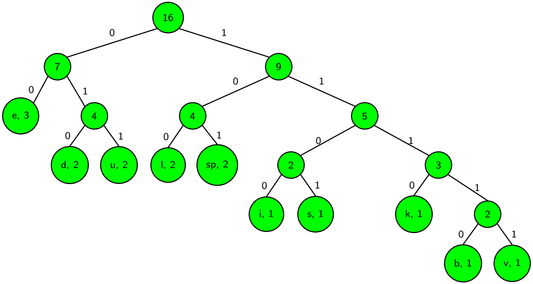
Huffman Coding: Assign Codes

- **Assign codes to the tree:**
  - 0: For left child.
  - 1: For Right child.

- A traversal of the tree from root to leaf gives the Huffman code for that particular leaf character.

- **Note:** No code is the prefix of another code.

Huffman Coding: Code Assignment



e	d	u	l	space	i	s	k	b	v
00	010	011	100	101	1100	1101	1110	11110	11111

Huffman Coding: Compression

- Use these codes to encode the string.
- Encoding of "duke blue devils":

010 011 1110 00 101 11110 100 011 00 101 010 00 11111 1100 100 1101

- Grouped then into bytes:

01001111 10001011 11101000 11001010 10001111 11100100 1101*xxxx*

- $\therefore$ it takes 7 bytes of space.
- In contrast, the uncompressed string takes 16 characters $\times$ 1 byte/char = 16 bytes.

Huffman Decoding: Uncompression

- Reading the compressed file bit by bit.
  - Start at the root of the tree.
  - If a 0 is read, head left.
  - If a 1 is read, head right.
  - When a leaf is reached decode that character and start over again at the root of the tree

- $\therefore$ Huffman table information needs to be saved as a header in the compressed file.
  - Doesnt add a significant amount of size to the file for large files (which are the ones you want to compress anyway).
  - Or we could use a fixed universal set of codes/freqencies.

Homework

- Implement Huffman encoding and decoding in C.
- Using it create a compression software that takes as input a text file and outputs a compressed file with the Huffman encoding table in the header of the file.
- Also create a uncompression software that will take the compressed file created above and output the original text file.

Red-Black Tree (RBT)

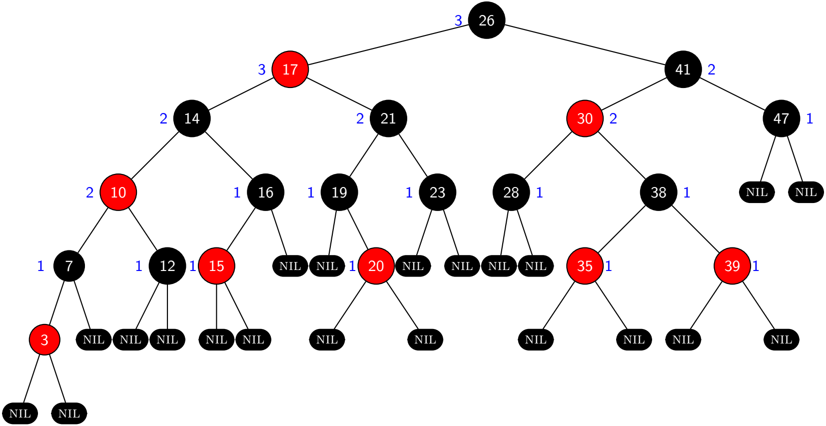
Properties

- Another example of balanced tree.
- Like AVL tree, ensures basic dynamic-set operations take $\mathcal{O}(\log n)$ in the worst case.
- It is a BST satisfying the following properties.
  1. Every node is either **red** or **black**.
  2. Root is **black**.
  3. Every leaf is **black**.

     Note: Only the null pointers are considered as leaf nodes. All other nodes are considered as internal nodes.
  4. If a node is **red** the both its children are **black**.
  5. For each node, all paths from the node to descendant leaves contain the same number of black nodes.

An Example Tree

Black-height: # black node on any path from, but not including, a node x down to a leaf. It is denoted by $bh(x)$.



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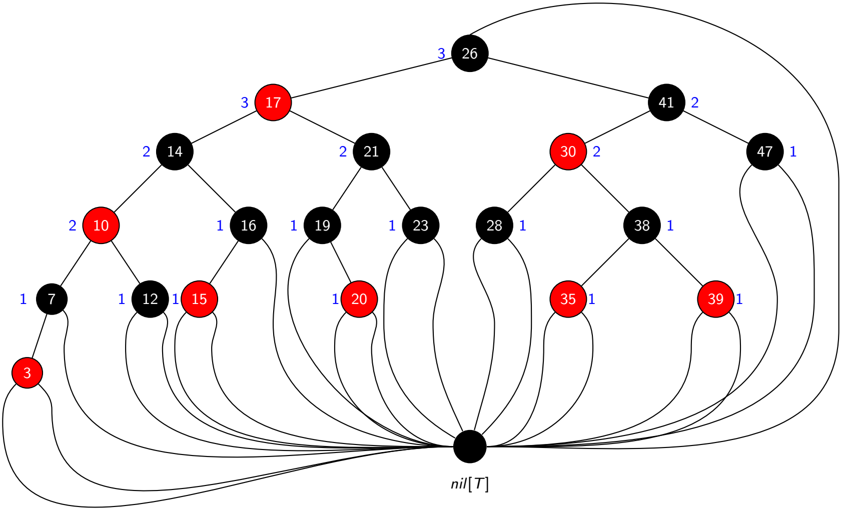

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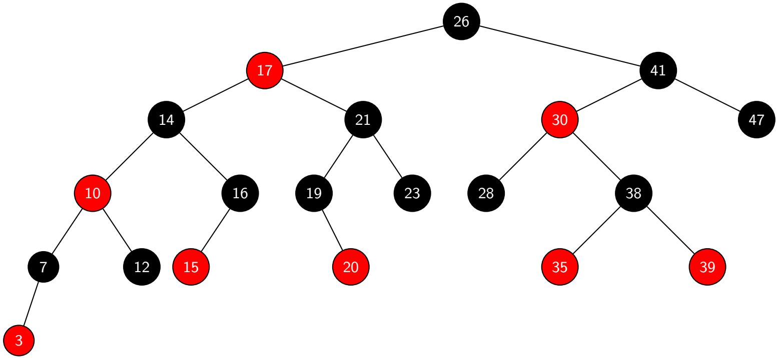

RBT Node in C

```
typedef struct RBTNode {
  int nKey;
  int nColour;              // 0 - Red and 1 - black
  struct RBTNode *pParent;
  struct RBTNode *pLeft;
  struct RBTNode *pRight;
} RBTNode;
```

Sentinel $nil[T]$**:**

- It is a RBTNode.
- Color = Black, i.e., nColour = 1.
- nKey, pParent, pLeft, pRight can be set to arbitrary values.

Lemma

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A RBT with n internal nodes has height at most $2\lg(n+1)$.

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A RBT with n internal nodes has height at most $2\lg(n+1)$.

Claim: A subtree rooted at any node x contains at least $2^{bh(x)} - 1$ internal nodes.

Lemma

Proof of the claim: The proof is by induction on the height of x.

Base Case ($bh(x) = 0$): Then x must be a leaf ($nil[T]$) node $\Rightarrow$ # of internal nodes is at least $2^0 - 1 = 0$.

Inductive Step: Height of the children of x is either $bh(x)$ or $bh(x) - 1$.

Then by the inductive hypothesis a sub-tree rooted at x contains at least

$$(2^{bh(x)-1} - 1) + (2^{bh(x)-1} - 1) + 1 \ = \ 2^{bh(x)} - 1$$

internal nodes.

Lemma

Proof of the Lemma: Let h be the height of the tree.

By Property 4, at least half of the nodes on any simple path from the root to a leaf, not including the root, must be black
$\Rightarrow \; bh(x) \geq h/2$.

$\therefore \; n \; \geq 2^{h/2} - 1 \quad \Rightarrow \quad h \; \leq \; \lg(n+1)$.

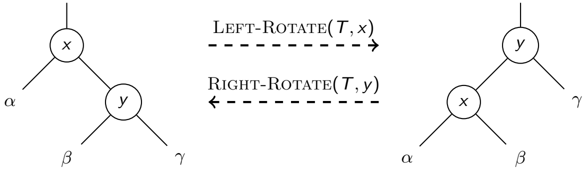
Rotations in a RBT

Rotations

- Insert and Delete operations may violate the red-black property.
- Can be restored by
  - changing the colours of some of the nodes and
  - also change the pointer structure.
- The pointer structure is changed via the following two types of rotation.
  - Left rotation.
  - Right rotation.

Left and Right Rotations

Left-Rotate: $right[x] \neq nil[T]$ and $p[root] = nil[T]$.
Right-Rotate: $left[x] \neq nil[T]$ and $p[root] = nil[T]$.



Complexity: $\mathcal{O}(1)$.

LEFT-ROTATE(T, x)

```
BEGIN
  y ← right[x]            // Set y
  right[x] ← left[y]      // Turn y's left sub-tree into x's right sub-tree

  if left[y] ≠ nil[t]
    then p[left[y]] ← x

  p[y] ← p[x]             // Link x's parent to y

  if p[x] = nil[T]
    then root[T] ← y
  else if x = left[p[x]]
    then left[p[x]] ← y
  else
    right[p[x]] ← y

  left[y] ← x
  p[x] ← y
END
```

LEFT-ROTATE(T, x)

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y ← right[x]                  // Set y
right[x] ← left[y]            // Turn y's left sub-tree into x's right sub-tree

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if p[x] = nil[T]
   then root[T] ← y
else if x = left[p[x]]
   then left[p[x]] ← y
else
   right[p[x]] ← y

left[y] ← x
p[x] ← y
```

END

Homework: Write the algorithm for the RIGHT-ROTATE.

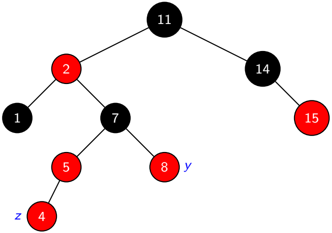
RBT: Insertion

Insertion

- Insert the z into the RBT T as if it was a BST.
- Colour the node z as red.

  Note: Colouring z as red does not effect the black height!
- Call an auxiliary procedure RB-INSERT-FIXUP to recolour nodes and perform rotations.

RB-INSERT-FIXUP(T, z)



Recall: Properties of a RBT:

1. Every node is either red or black.
2. Root is black.
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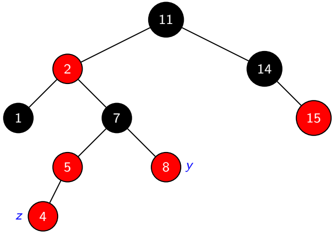

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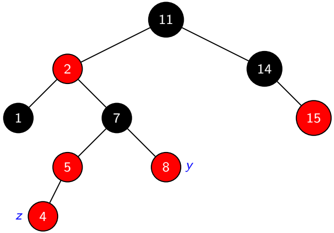

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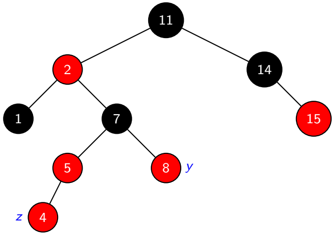

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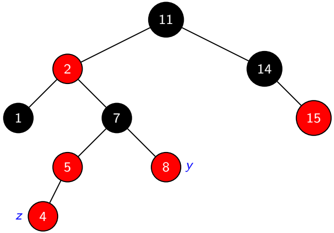

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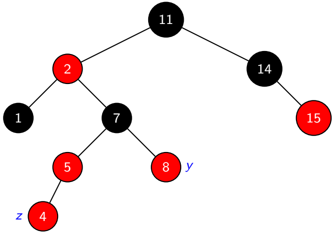

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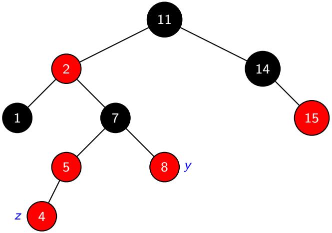

The following cases may arise:

- **Case 1:** z's uncle is red.

1. **while** $color[p[z]] = \text{RED}$

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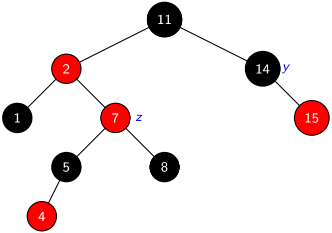

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- **Case 1:** z's uncle is red.

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while color[p[z]] = RED
    if p[z] = left[p[pz]]
        y ← right[p[p[z]]]
        if colour[y] = RED
            colour[p[z]] ← BLACK
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            colour[p[p[z]]] ← RED
            z ← p[p[z]]
```

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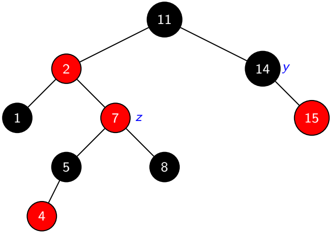

The following cases may arise:

- **Case 1:** z's uncle is red.
- **Case 2:** z's uncle is black and z is a right child.

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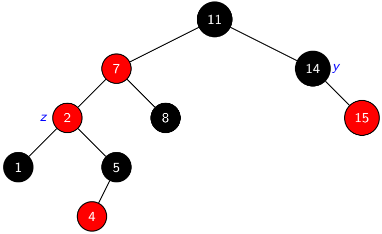

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            z ← p[p[z]]
        else if z = right[p[z]]
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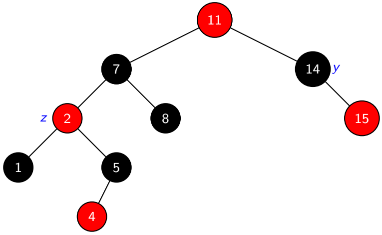
            LEFT-ROTATE(T, z)
```

RB-INSERT-FIXUP(T, z)



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- **Case 1:** z's uncle is red.
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- **Case 3:** z's uncle is black and z is a left child.
- **Note:**
  - Properties 2 is violated.
  - Properties 4 is violated for the node 11.

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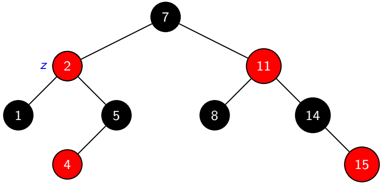

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            RIGHT-ROTATE(T, p[p[z]])
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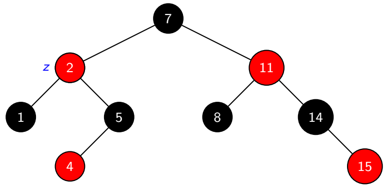

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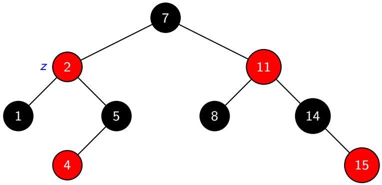
1. **while** $color[p[z]] = \text{RED}$
2. **if** $p[z] = left[p[pz]]$
3. $y \leftarrow right[p[p[z]]]$
4. **if** $colour[y] = \text{RED}$
5. $colour[p[z]] \leftarrow \text{BLACK}$
6. $colour[y] \leftarrow \text{BLACK}$
7. $colour[p[p[z]]] \leftarrow \text{RED}$
8. $z \leftarrow p[p[z]]$
9. **else if** $z = right[p[z]]$
10. $z \leftarrow p[z]$
11. LEFT-ROTATE(T, z)
12. $colour[p[z]] \leftarrow \text{BLACK}$
13. $colour[p[z]] \leftarrow \text{RED}$
14. RIGHT-ROTATE($T, p[p[z]]$)
15. **else** (same as Line 3 to 14 with "right" and "left" interchanged)

RB-INSERT-FIXUP(T, z)



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2      if p[z] = left[p[pz]]
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4          if colour[y] = RED
5              colour[p[z]] ← BLACK
6              colour[y] ← BLACK
7              colour[p[p[z]]] ← RED
8              z ← p[p[z]]
9          else if z = right[p[z]]
10             z ← p[z]
11             LEFT-ROTATE(T, z)
12             colour[p[z]] ← BLACK
13             colour[p[z]] ← RED
14             RIGHT-ROTATE(T, p[p[z]])
15     else (same as Line 3 to 14 with
       "right" and "left" interchanged)
16 colour[root[T]] ← BLACK
```

Thank You for your kind attention!

Books and Other Materials Consulted

1. Huffman Coding part taken from the following website.

2. Red-Black Tree part taken from Chapter 13 of the *Introduction to Algorithms* book by Thomas H Cormen, Charles E Leiserson, Ronald L Rivest, Clifford Stein.

Questions!!