Graphs: an introduction

Bijendra Nath Jain

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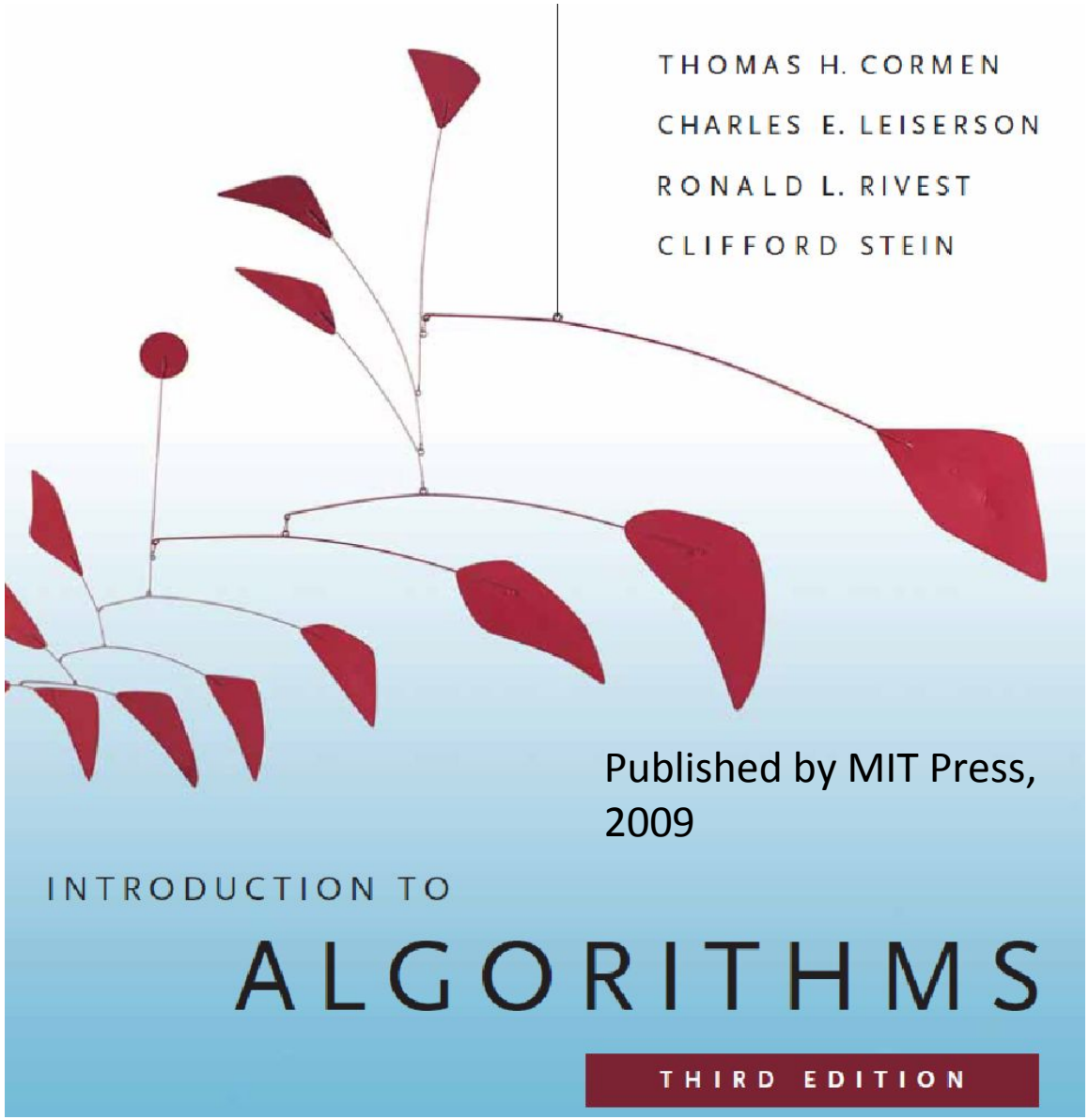
Outline of next 9 lectures

- Graphs:
  - Undirected graphs
  - Directed graphs
  - (Directed) acyclic graphs (or DAGs)
  - Sparse graphs
  - Weighted graphs
- Graph applications
- Representation of graphs:
  - Adjacency matrix
  - Linked lists
- Algorithms:
  - Traversal algorithms:
    - BFS
    - DFS
  - Topological sort
  - Minimum spanning trees
  - Dijkstra's Shortest path
    - One-to-one
    - One-to-many
    - Many-to-many

Text to be used for next 9 lectures



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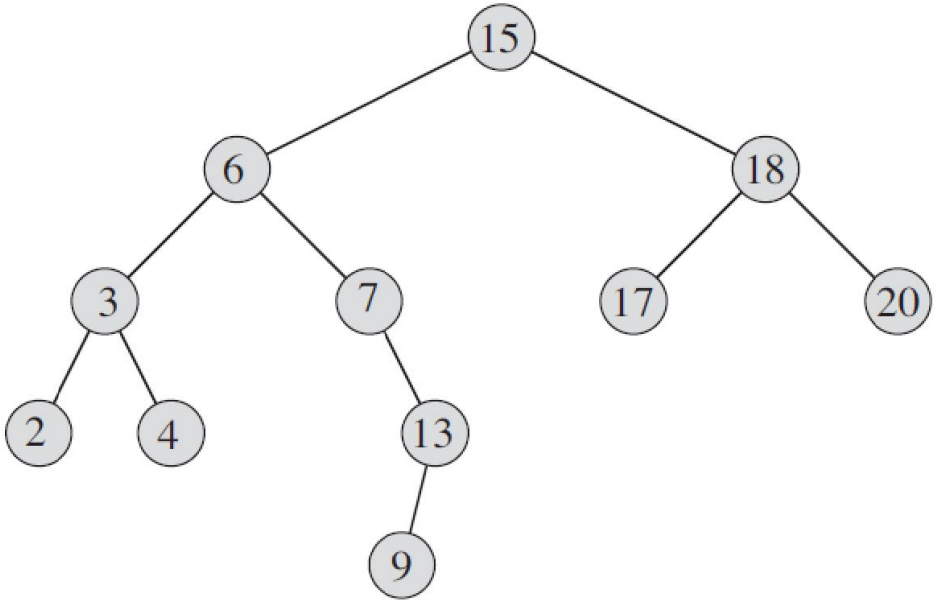
Some graph applications

- You are already familiar with trees, binary trees, and binary search trees
  - They are rooted
  - They are also graphs
- When looked upon as a graph, each edge is ‘directed’. That is, each edge identifies a pair of <parent node, child node>
- These graphs are special their representation is special, and algorithms are specific to search



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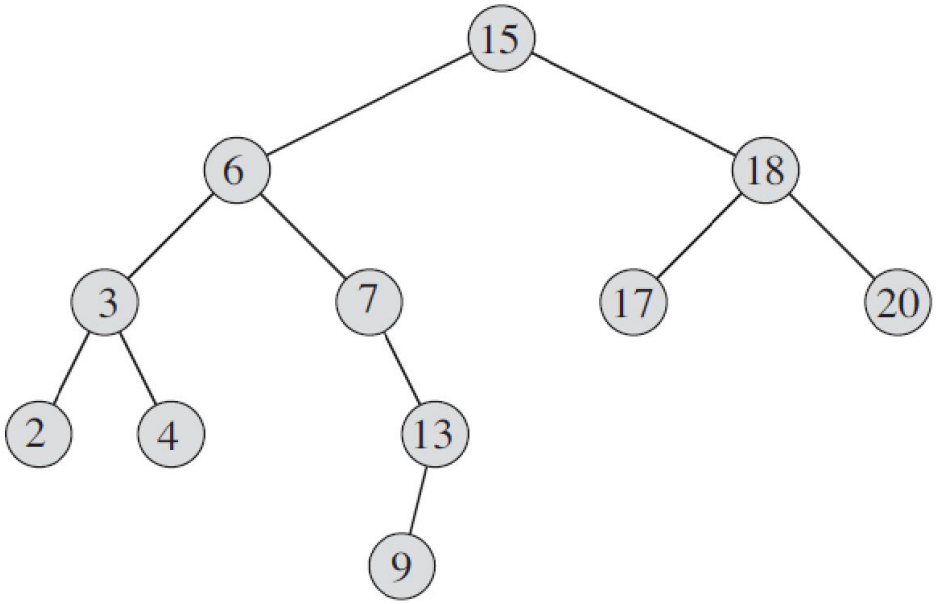
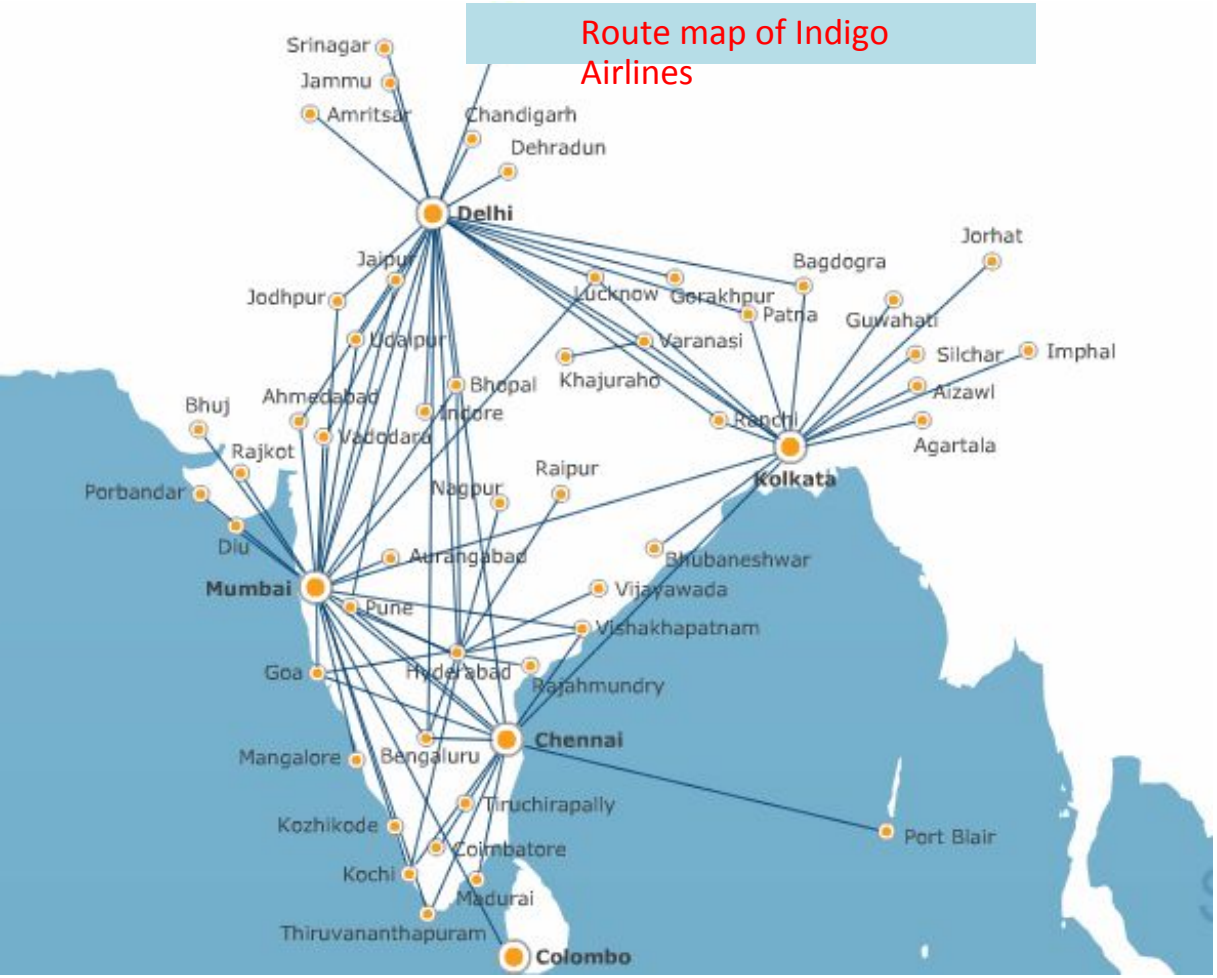


Some graph applications

- Maps:
  - Route map of an airline
  - Maps of highways in India
  - Internet: ERNet, in 1990



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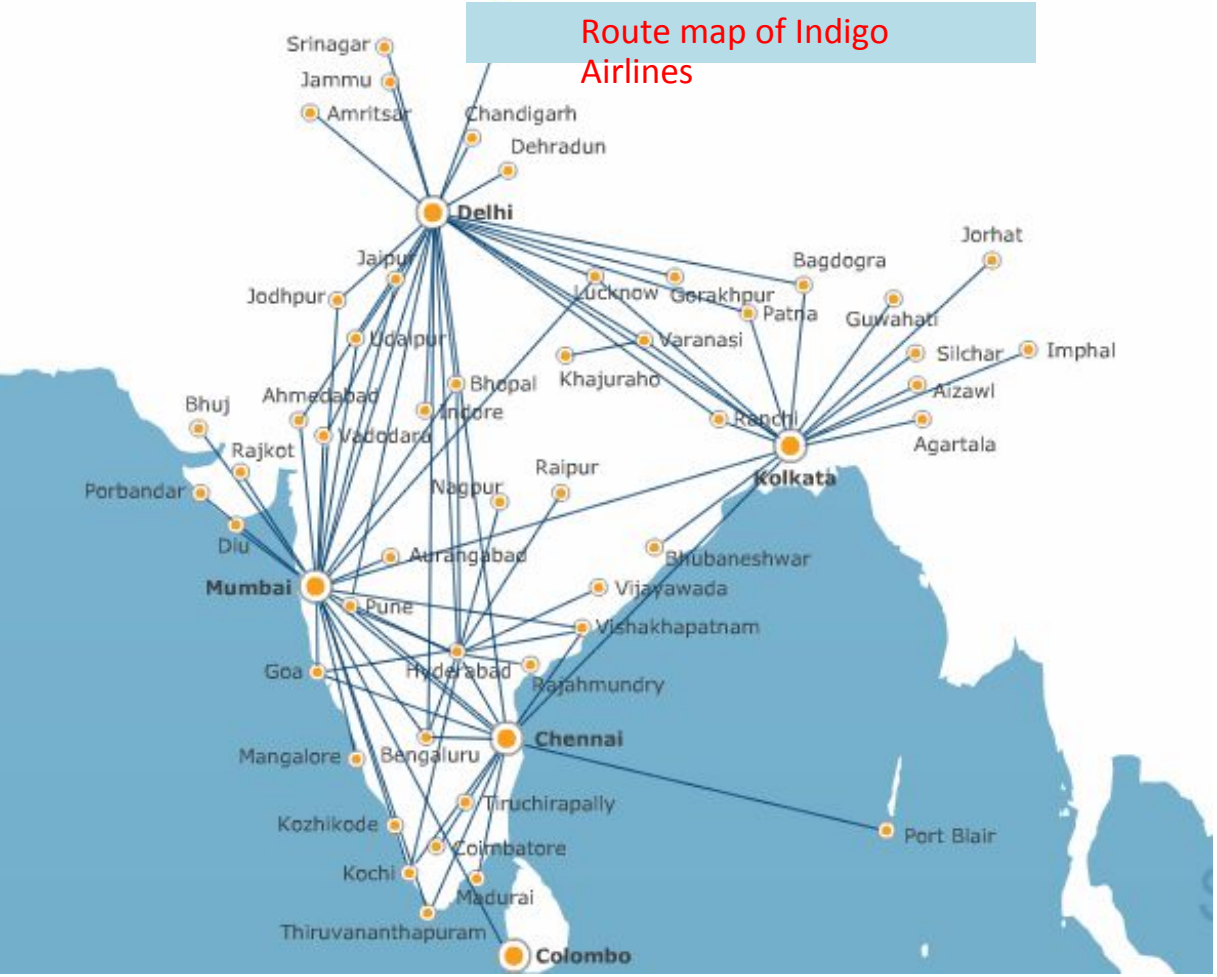

Reflect upon what problems would an airline attempt to solve

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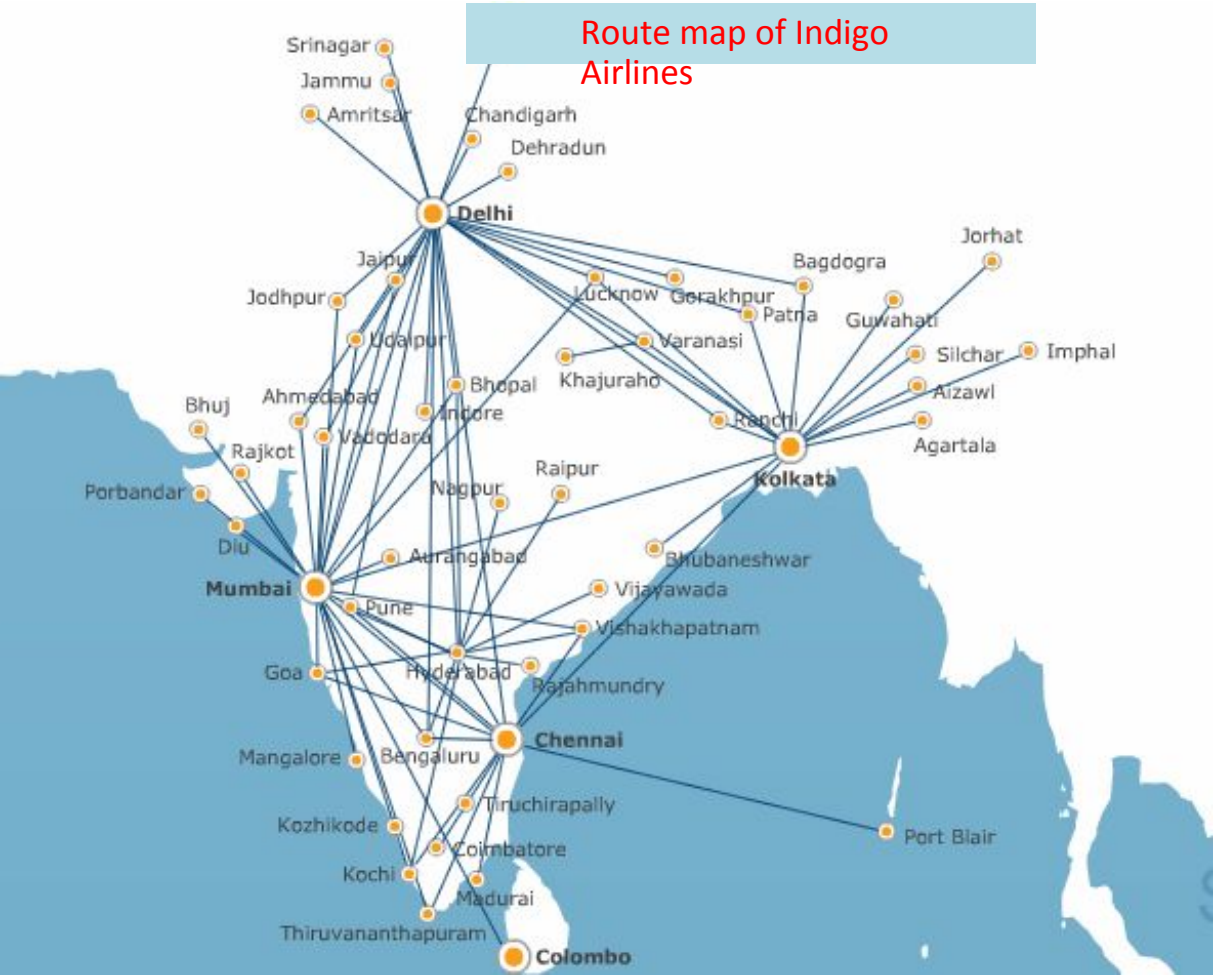

Reflect upon what problems would an airline attempt to solve

Describe at least one problem that an airline may wish to solve, but do so in some detail

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Reflect upon what problems would an highway planner attempt to solve

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Reflect upon what problems would an highway planner attempt to solve

Describe at least one problem that an a highway authority may wish to solve, but do so in some detail

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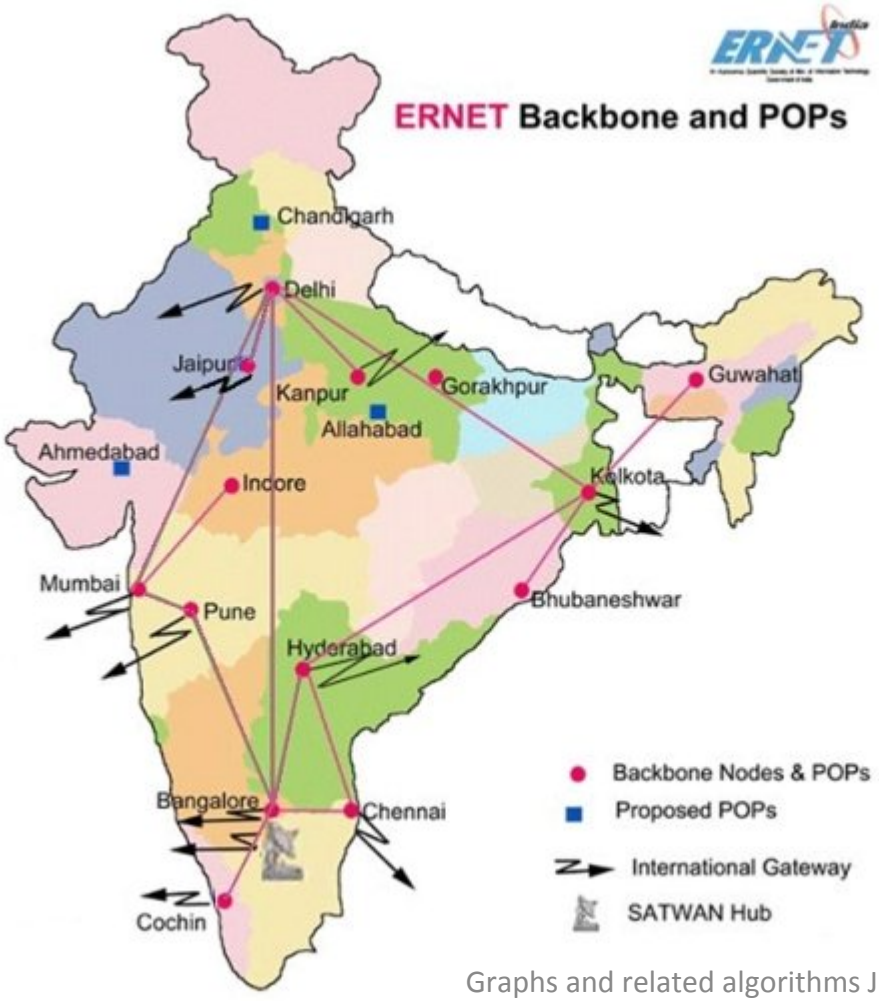

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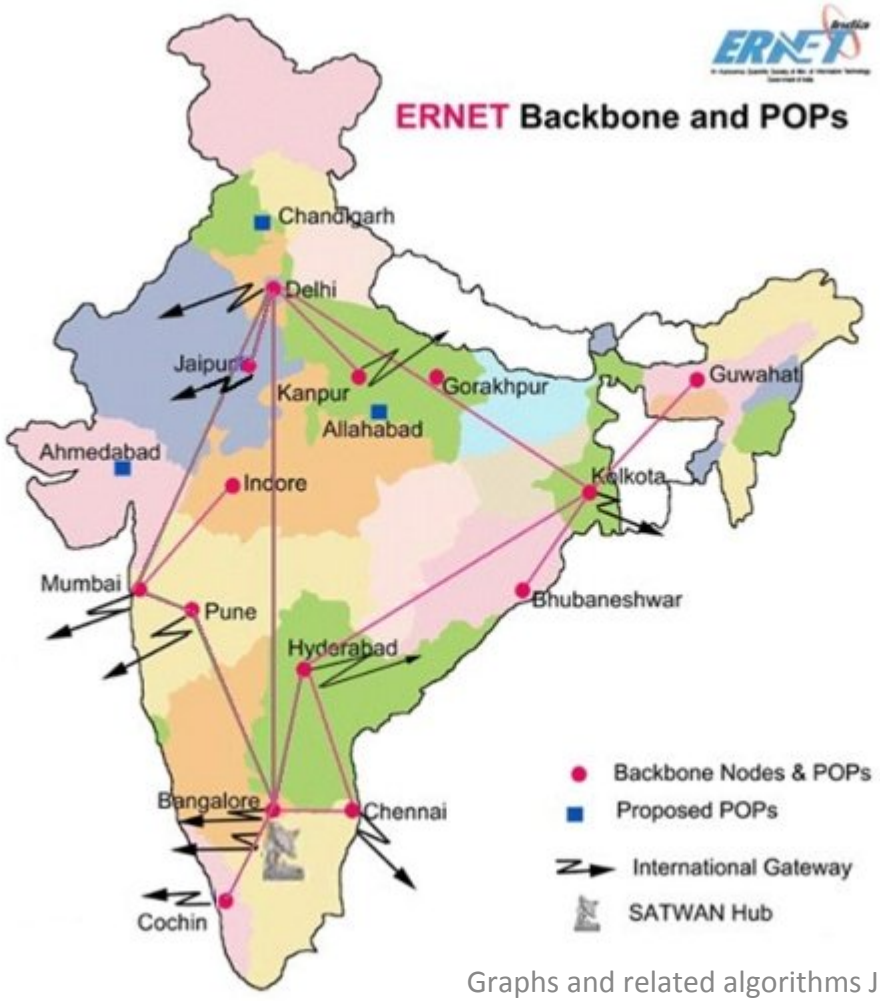

Reflect upon as to how network topology impacts throughput and latency in data networks

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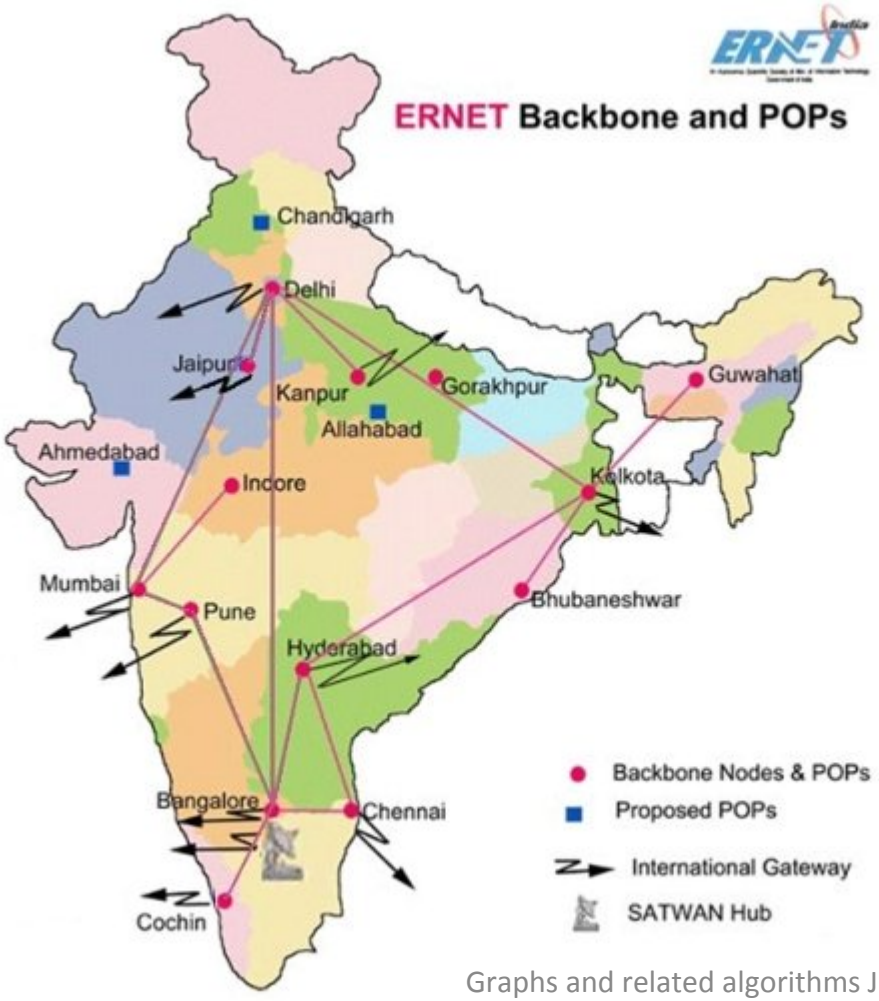
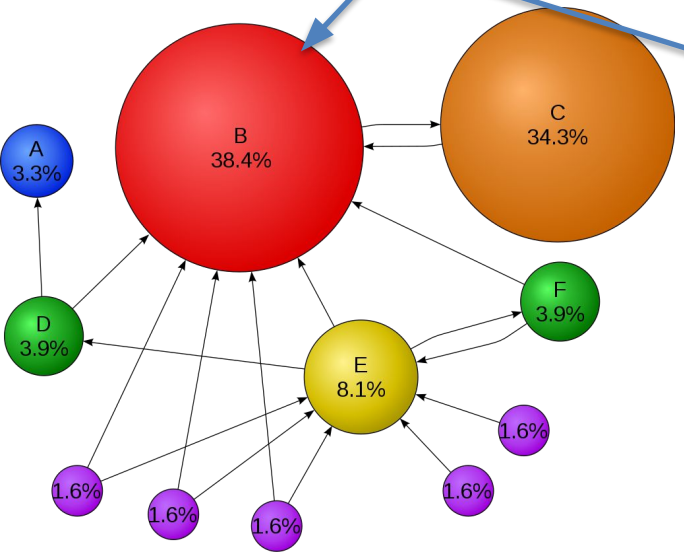


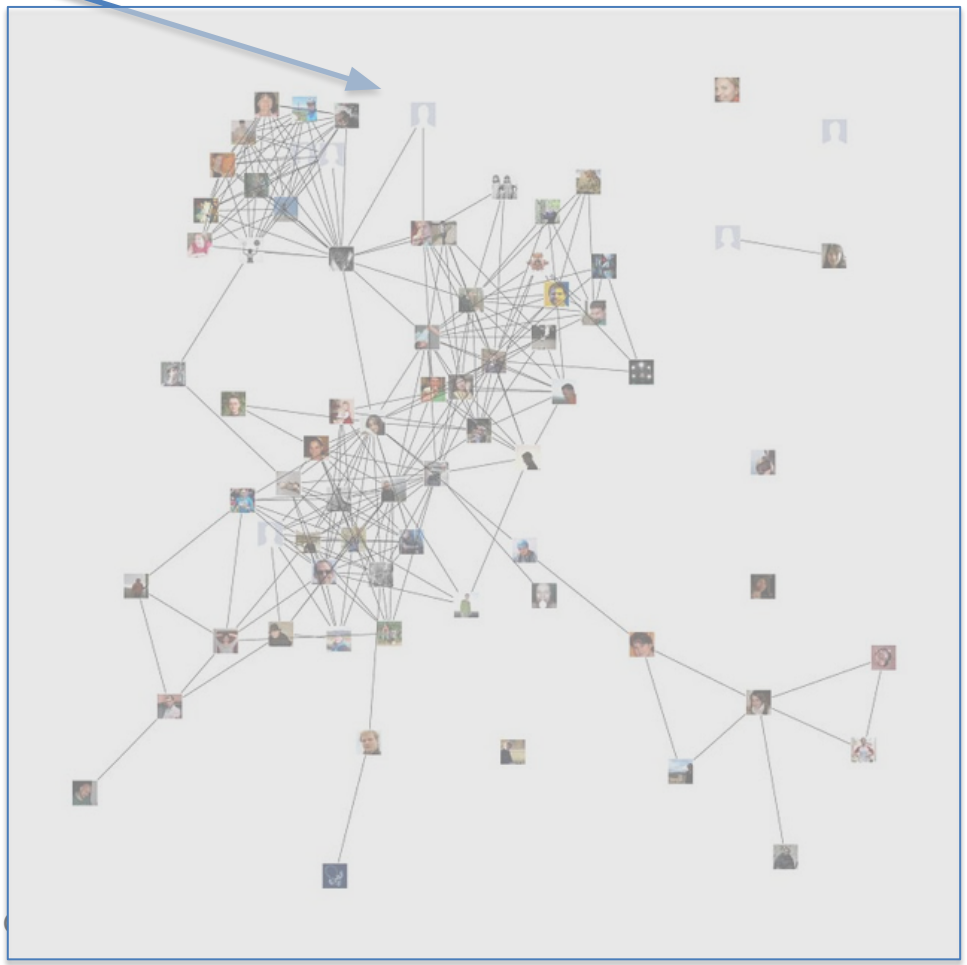
Reflect upon as to how network topology impacts throughput and latency in data networks

Describe at least one problem that an a network service provider may wish to solve, but do so in some detail

Some more graph applications

- More examples:
  - Web pages, and hyperlinks (see also https://en.wikipedia.org/wiki/PageRank)
  - Friends on social media
  - Etc.





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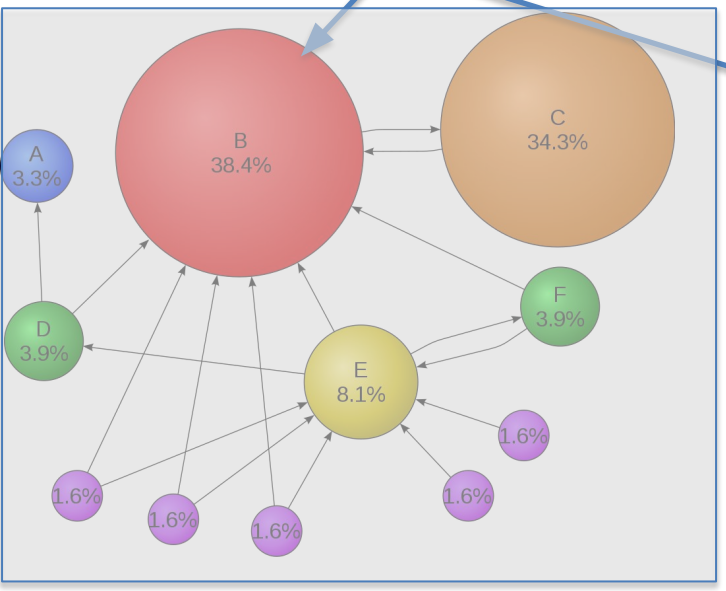
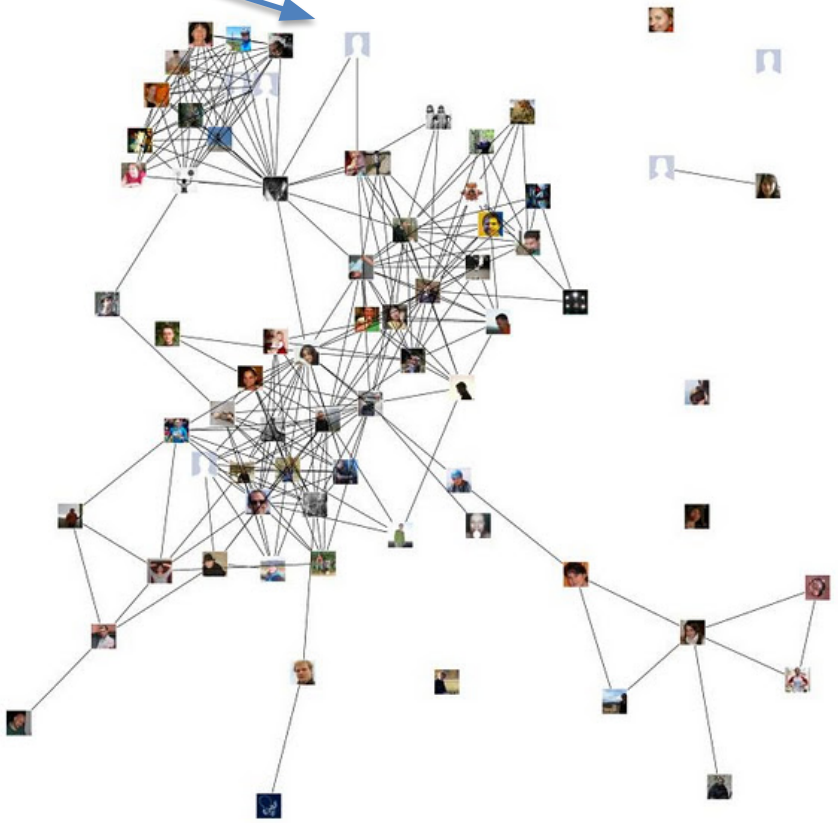




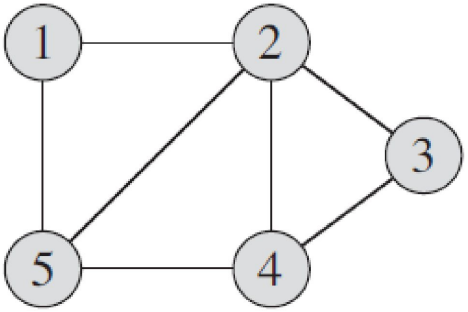
Some more graph applications

- Write a paragraph or two (with images) for 4 other graph applications, while identifying at least one problem that is IMP enough to be solved

What is a graph?

- A graph, G = (V, E), consists of:
  - Set of vertices, V
  - Set of edges, E, where edges are either ‘undirected’ or ‘directed’

- Undirected graph:



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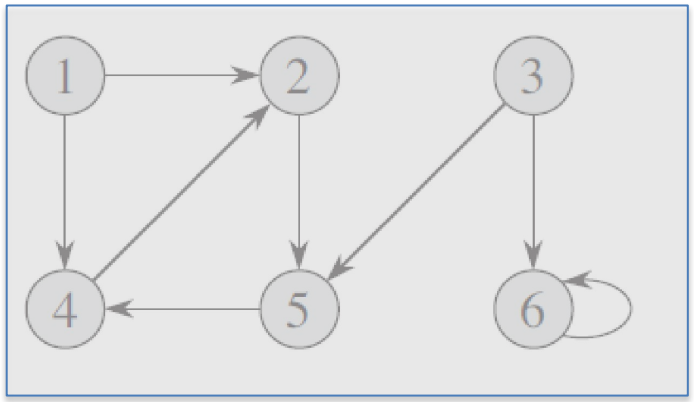

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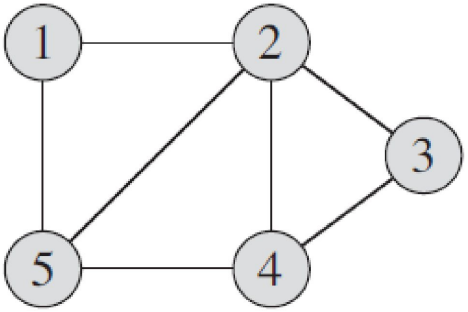

- G1 = (V1, E1), where
  V1 = {1, 2, 3, 4, 5},
  E1 = {(1, 2), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (4, 5)}
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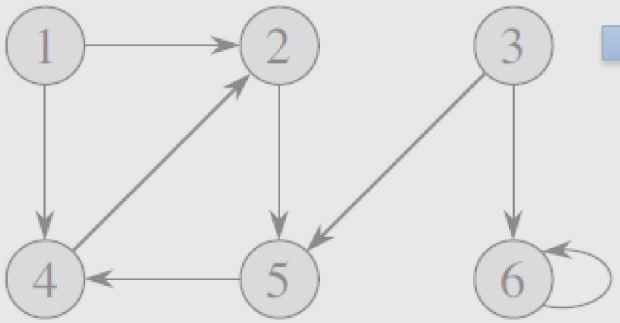

- G2 = (V2, E2), where
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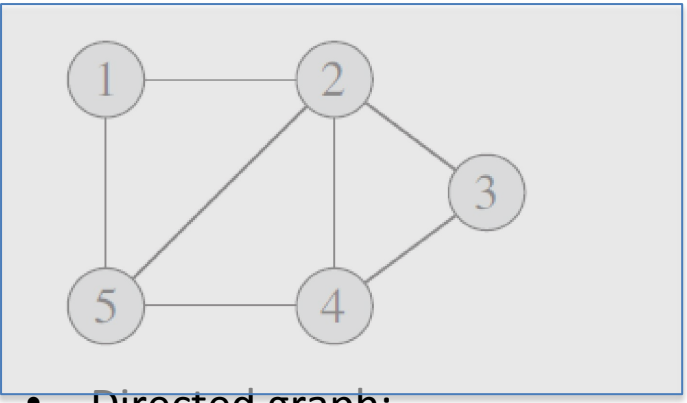

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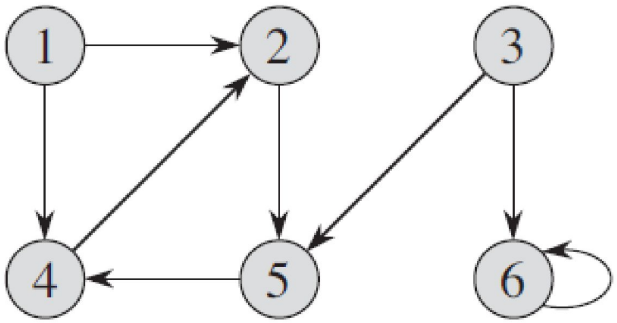

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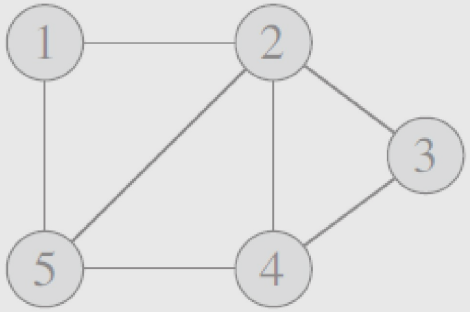

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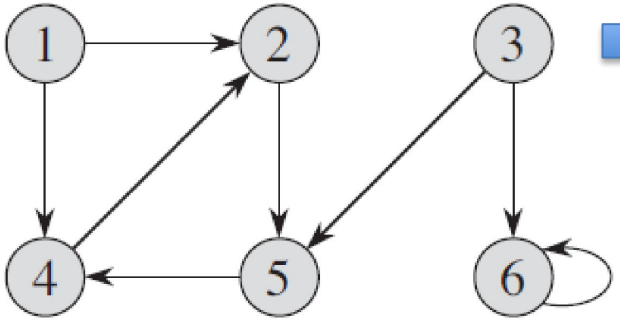

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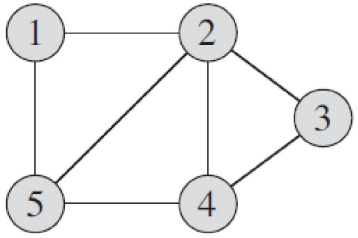
Some terminology: adjacency

- In an undirected graph:
  - Two vertices u and v are "adjacent" if (u,v) or (v, u) is an edge in E
  - Also edge e = (u,v) is incident to vertex u and to vertex v



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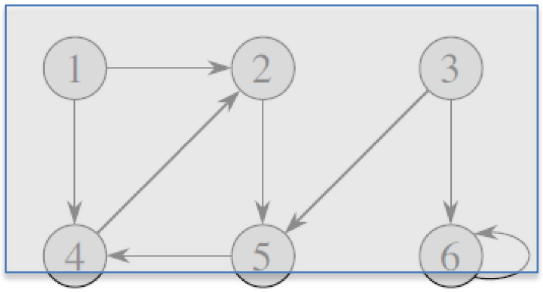

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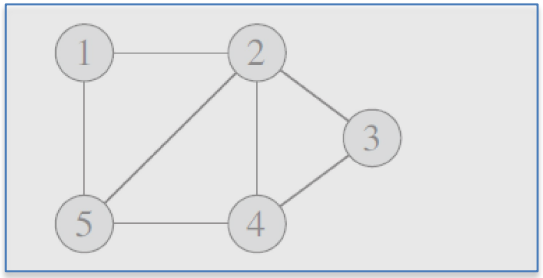

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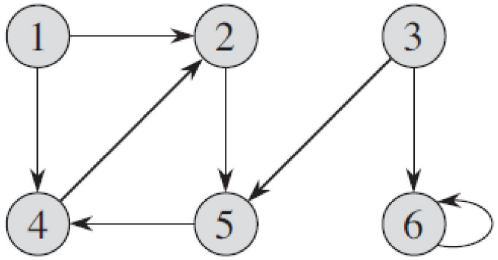
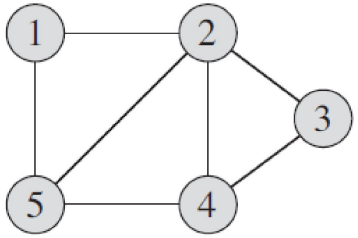
Note: the difference in "adjacent from" and "adjacent to"

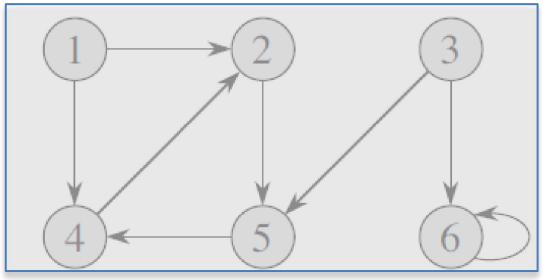


Some terminology: degree

- In an undirected graph the "degree" of vertex u is simply the number of edges that are incident **with** vertex u
  - a self-loop counts twice (both ends count)
  - The degree is denoted with deg(v)
  - If e=|E|, then always the case: $2e = \sum_{v \in V} \deg(v)$



- In a directed graph:
  - In-degree of vertex u is number of edges adjacent to vertex u, viz. terminate at vertex u
    - The In-degree is denoted as in-deg(u)
  - Out-degree of vertex u is number of edges adjacent from vertex u, viz. where vertex u is the initial vertex
    - The Out-degree is denoted as out-deg(u)



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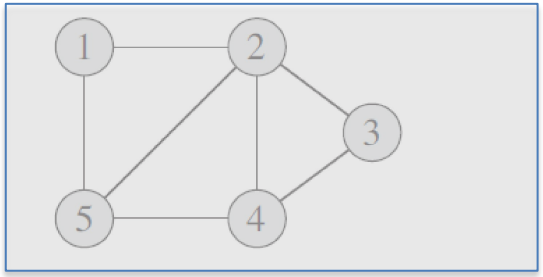

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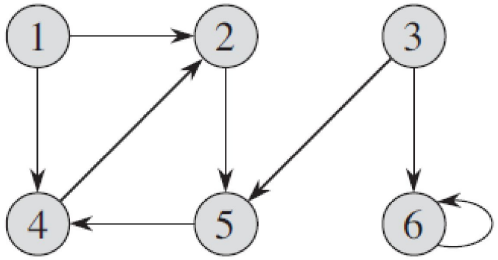
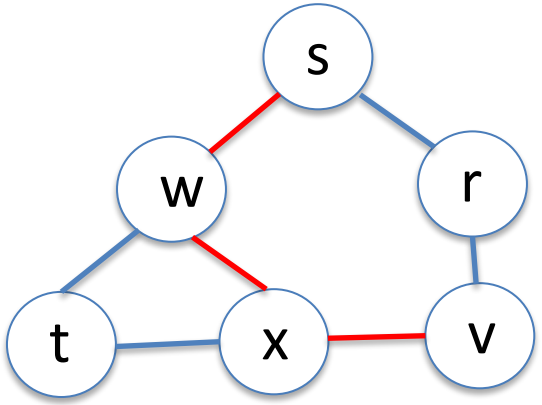


Some more terminology: paths in an undirected graph

Let undirected graph, G =(V, E)

- A path, p = (u_1, u_2, u_3, …, u_n) is said to be a simple path if:
  - Edges (u_i, u_{i+1}) are in E, and
  - Vertices u_1, u_2, u_3, …, u_n are all distinct



p = (s, w, x, v)

- A path, p = (u_1, u_2, u_3, …, u_n) is said to be a cycle if:
  - Edges (u_i, u_{i+1}) are in E, and
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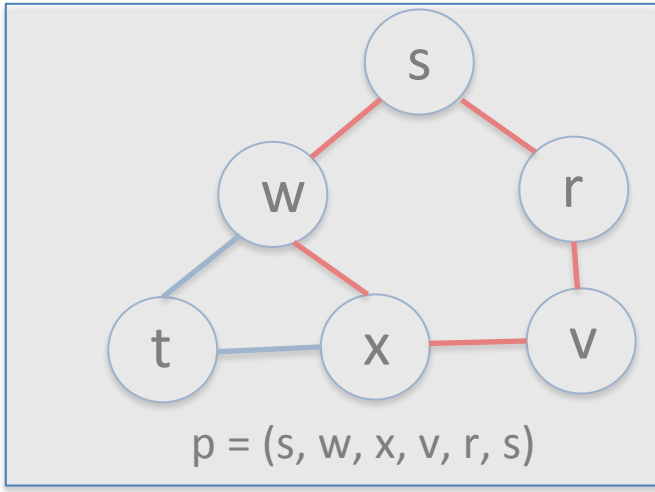
  - $u_1 = u_n$



p = (s, w, x, v, r, s)

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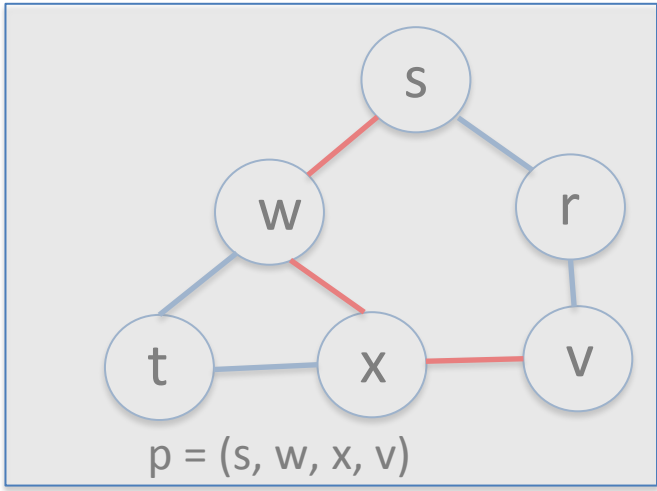

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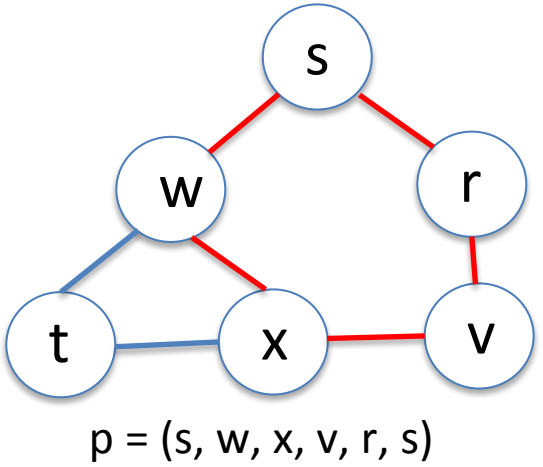

Some more terminology: paths in an directed graph

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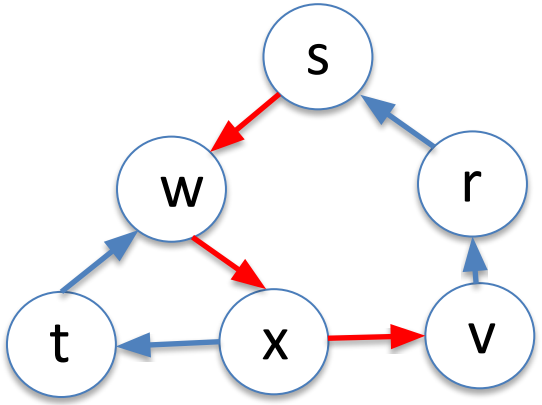
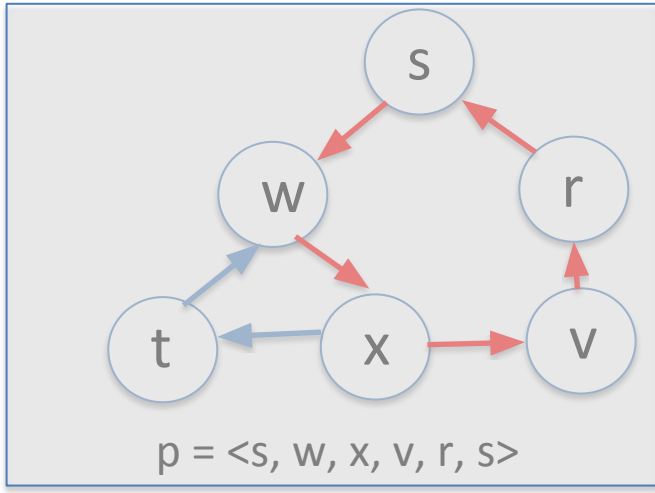
  - Edges <u_i, u_{i+1}> are in E, and
  - Vertices u_1, u_2, u_3, …, u_n are all distinct



p = <s, w, x, v>

- A path, p = <u_1, u_2, u_3, …, u_n> is said to be a cycle if:
  - Edges <u_i, u_{i+1}> are in E, and
  - Vertices u_2, u_3, …, u_n are all distinct, and
  - $u_1 = u_n$

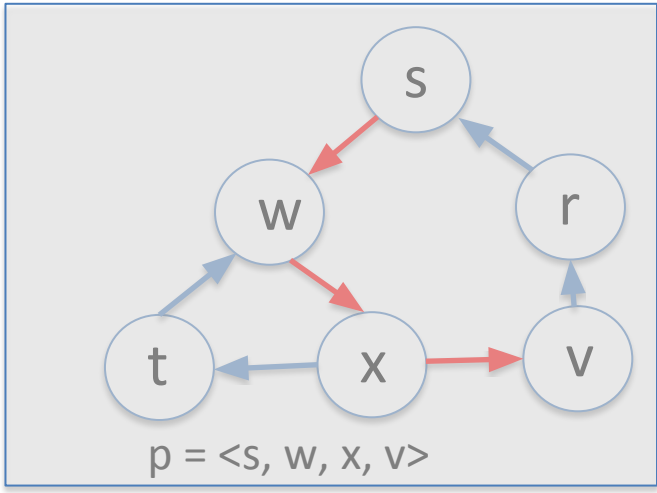


p = <s, w, x, v, r, s>

Some more terminology: paths in an directed graph

Let directed graph, G =(V, E)

- A path, p = <u_1, u_2, u_3, …, u_n> is said to be a simple path if:
  - Edges <u_i, u_{i+1}> are in E, and
  - Vertices u_1, u_2, u_3, …, u_n are all distinct



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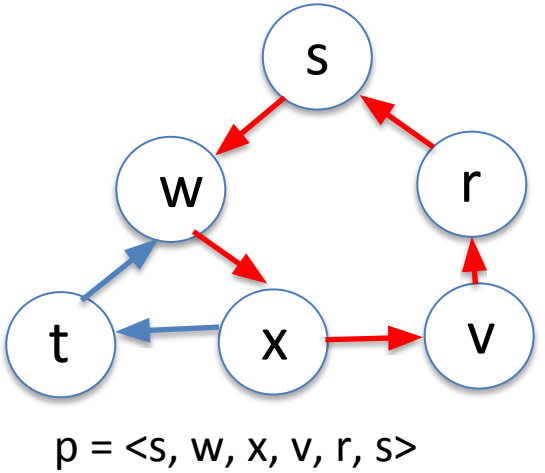
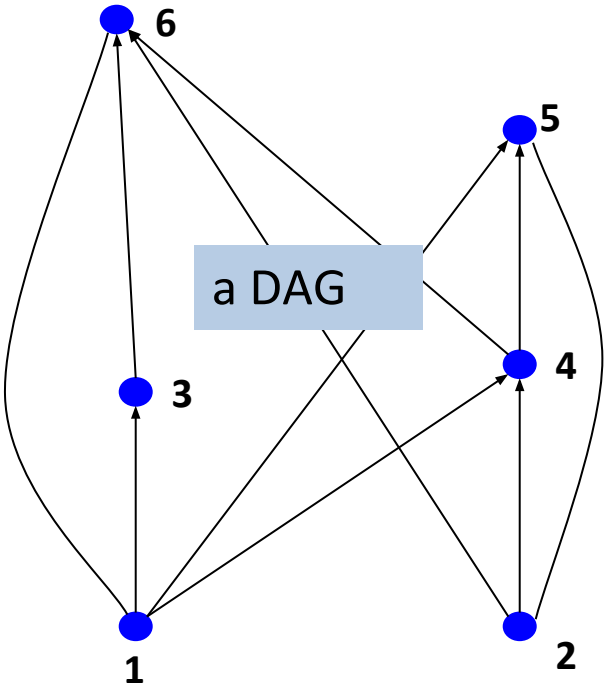
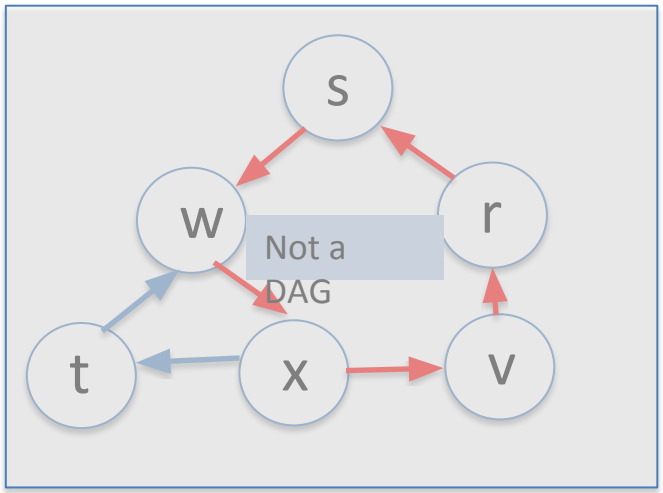


Some more terminology: directed acyclic graphs (DAGs)

Let directed graph, G =(V, E)

- Graph G is said to be a directed acyclic graph (or DAG) if:
  - There are no cycles in G





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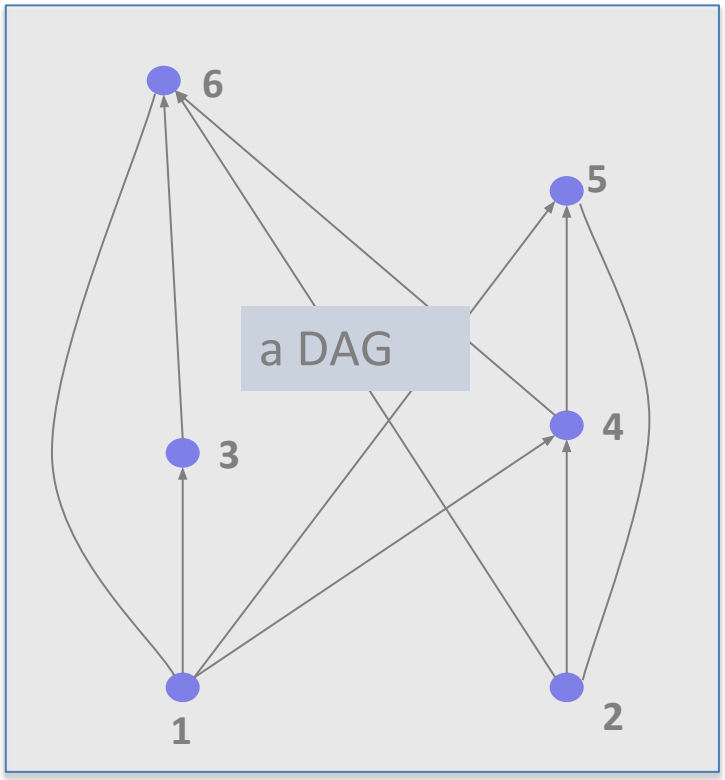
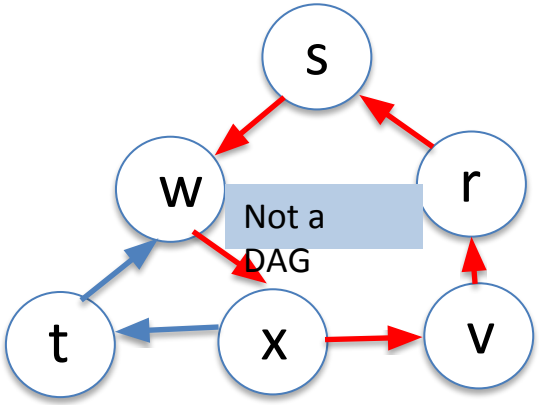
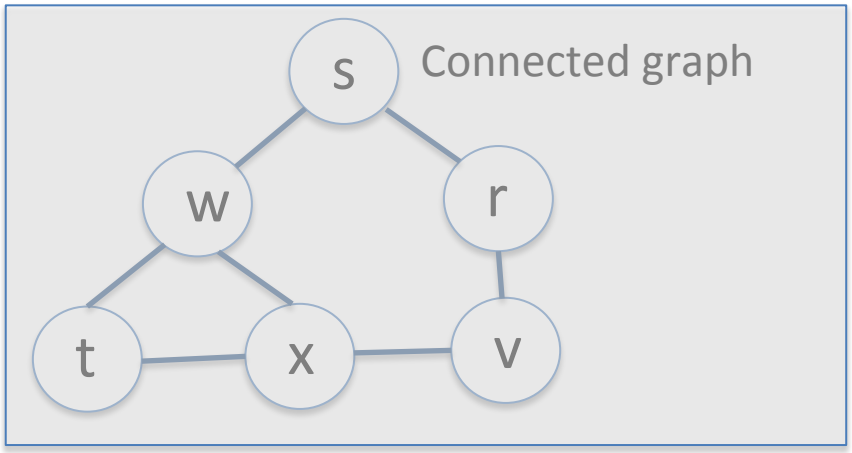


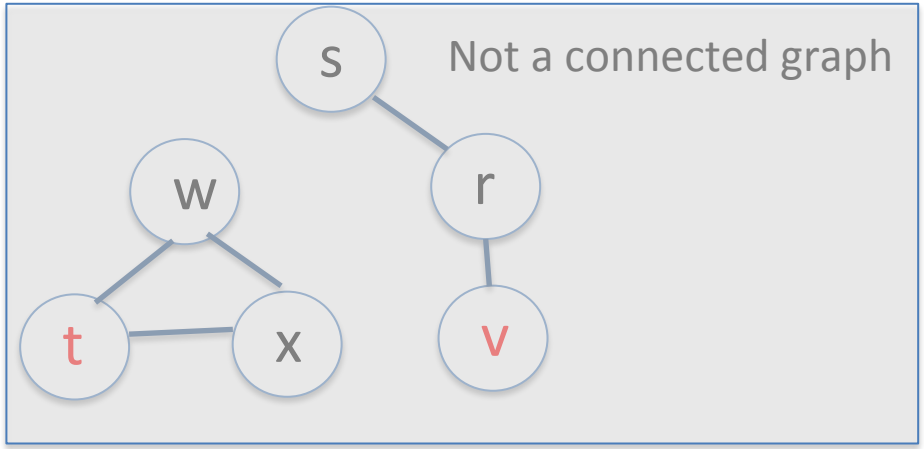


Some more terminology: connectedness of undirected graphs

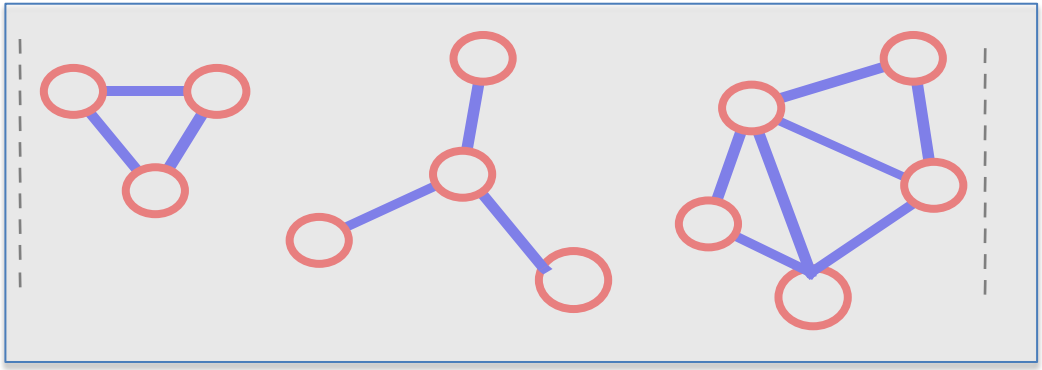
- Subgraph: subset of vertices, V_1 c V, and subset of edges. E_1 c E such that they form a graph

- A undirected graph, G =(V, E), is said to be connected if there is a path from u to v, for any and every given pair of vertices u and v, in V





- Connected components: maximal connected subgraph. E.g., the graph below has 3 connected components



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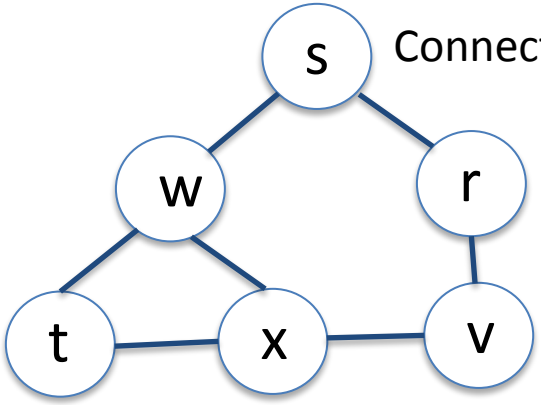
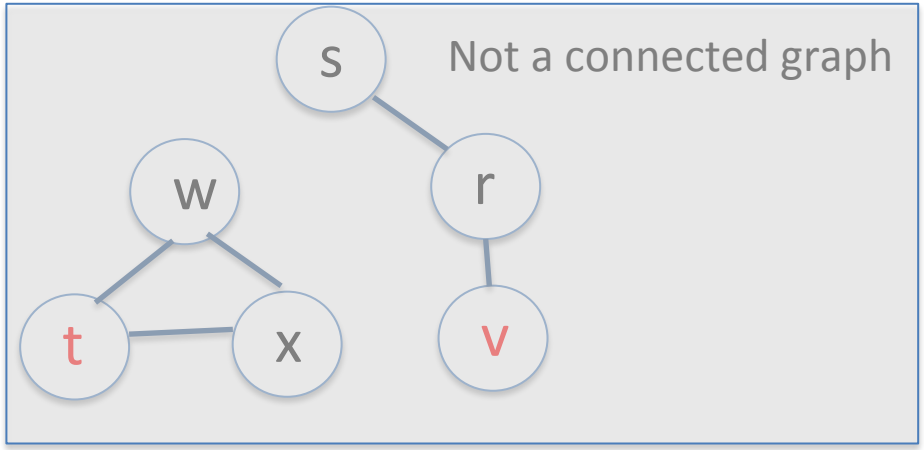
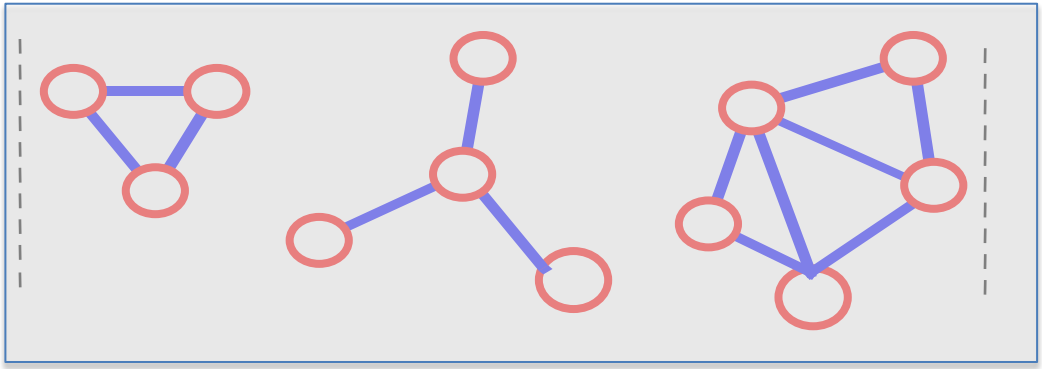


Connected graph



Not a connected graph

- Connected components: maximal connected subgraph. E.g., the graph below has 3 connected components



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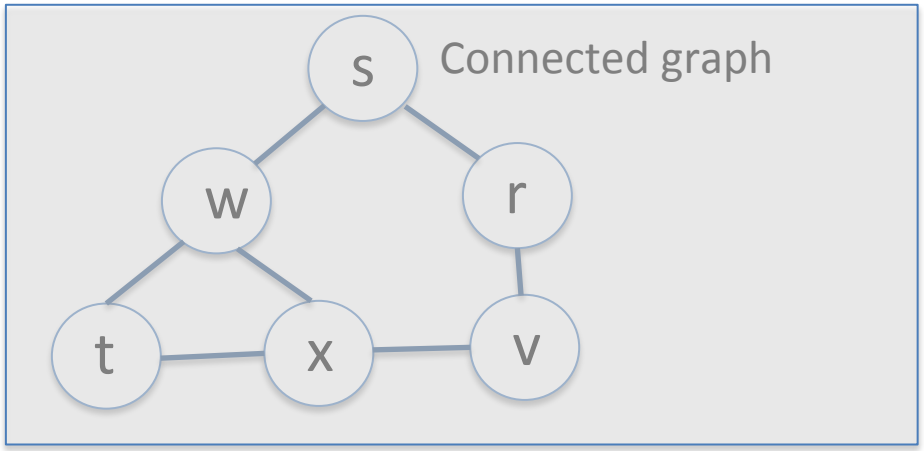

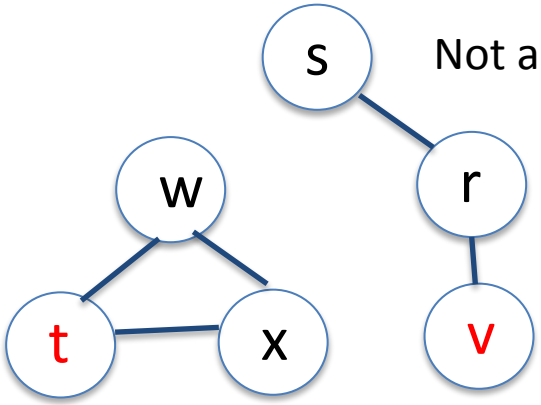

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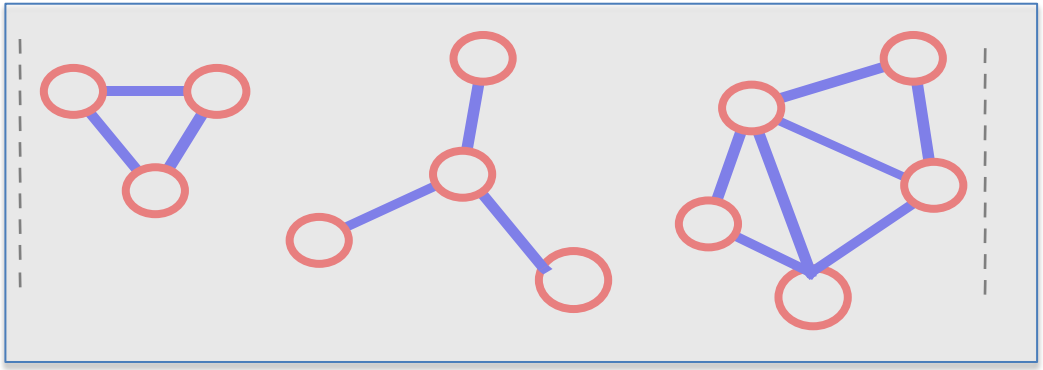

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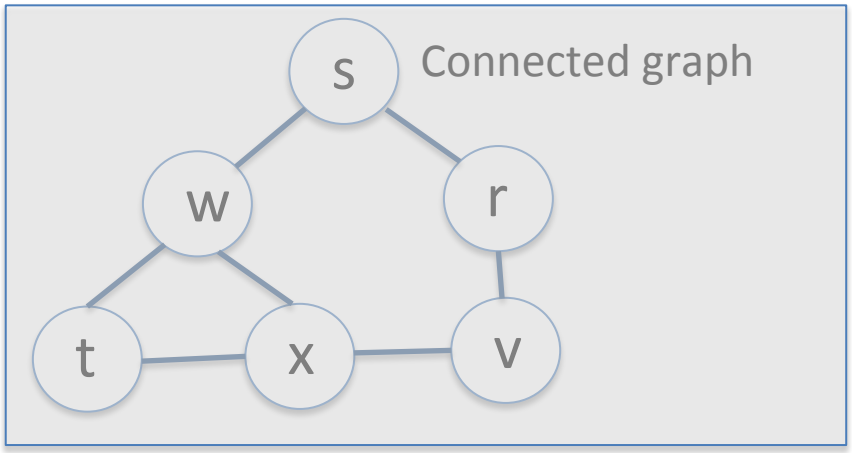

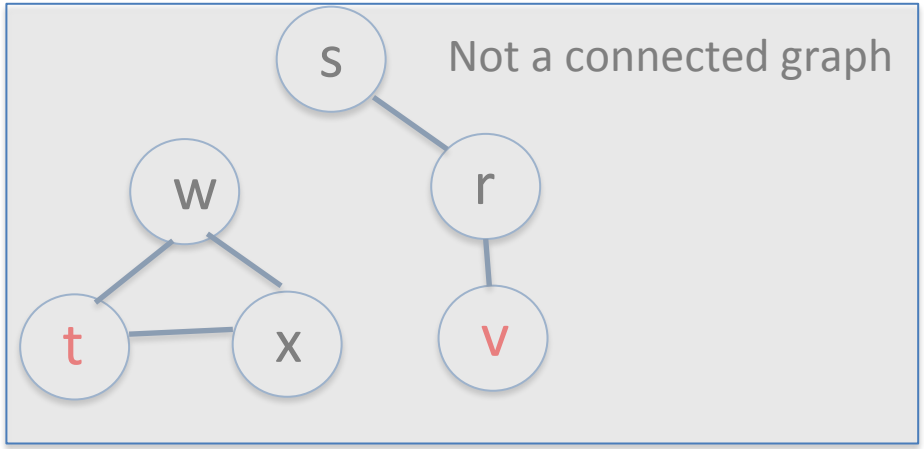

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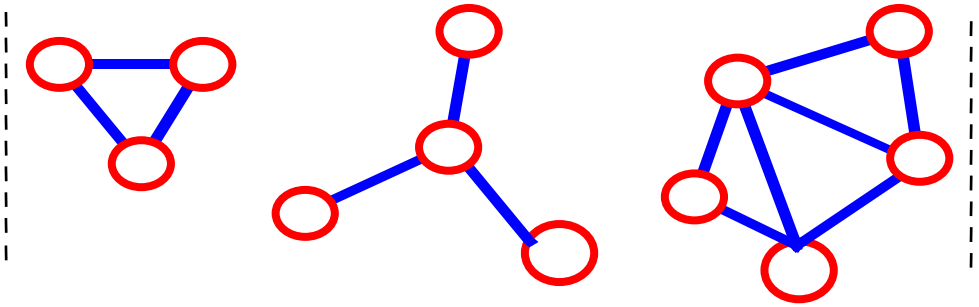
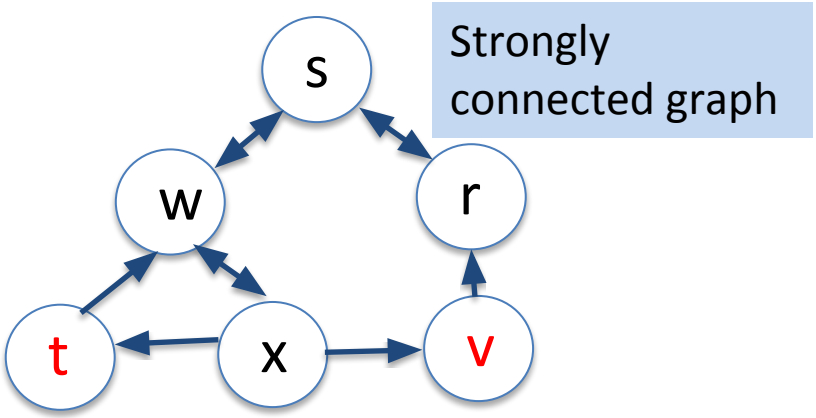
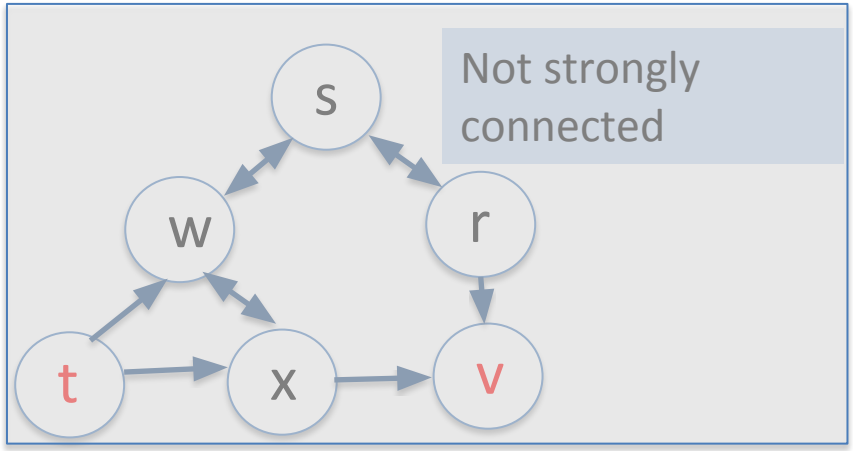


Some more terminology: connectedness of directed graphs

- A graph, G =(V, E), is strongly connected if there is a path from u to v, and from v to u, for any given pair of vertices u and v, in V





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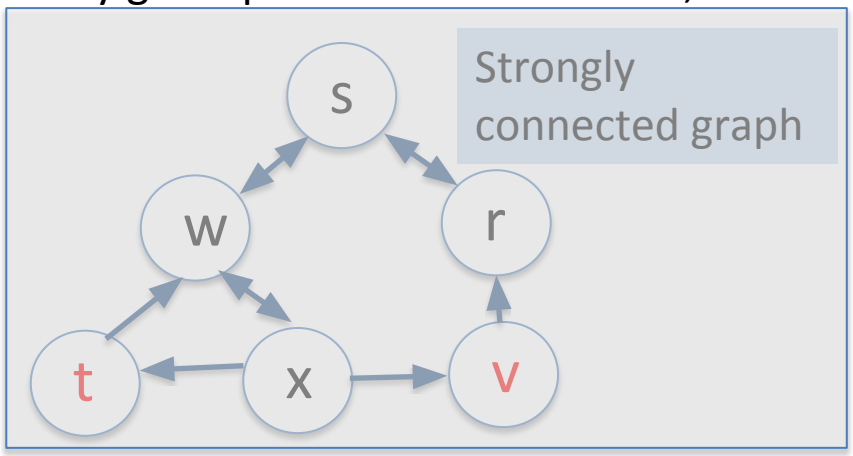

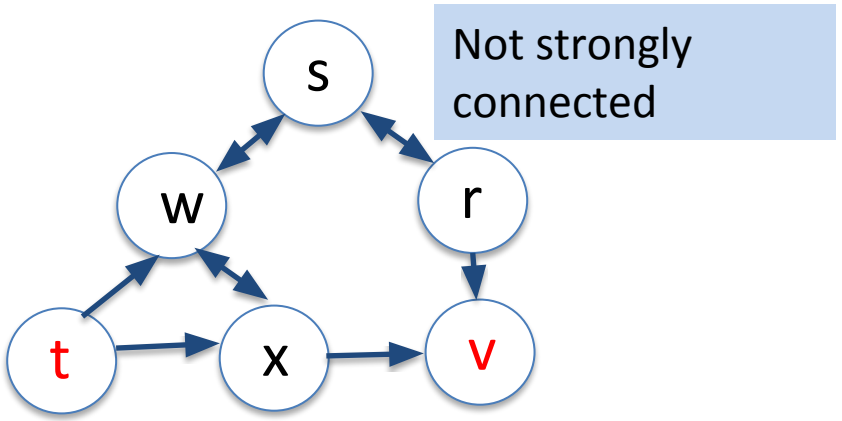

Representation of graphs using adjacency matrices

- A graph G = (V, E) may be represented by an n x n 'adjacency matrix', A = $\{a_{ij}\}$, where n = |V|
- For an 'undirected' graph, G = (V, E)

  $a_{ij} = a_{ji} = 1$, if there is an edge between verticies i and j

  $= 0$, otherwise
- For a 'directed' graph, G = (V, E)

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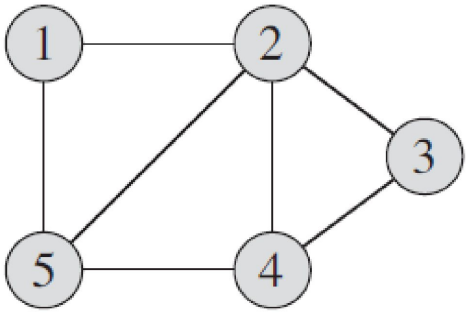

A =

					i=
0	1	0	0	1	1
1	0	1	1	1	2
0	1	0	1	0	3
0	1	1	0	1	4
1	1	0	1	0	5

j=

1	2	3	4	5

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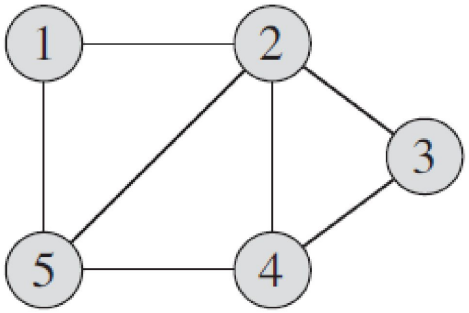

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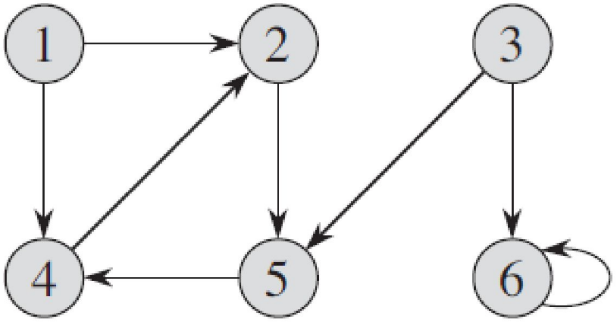

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0	0	0	0	0	1	6
1	2	3	4	5	6	

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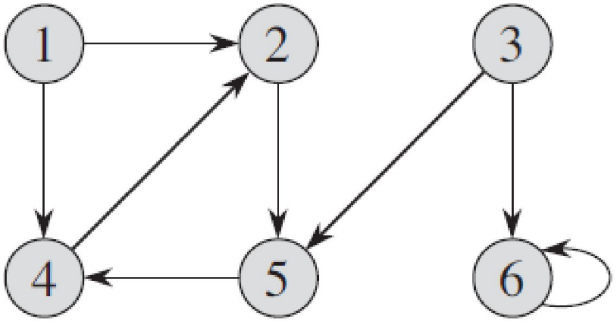

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1	2	3	4	5	6	

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- A graph G = (V, E) may instead be represented by an array of n 'adjacency lists' A_i
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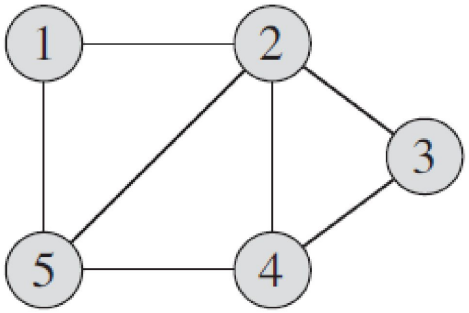

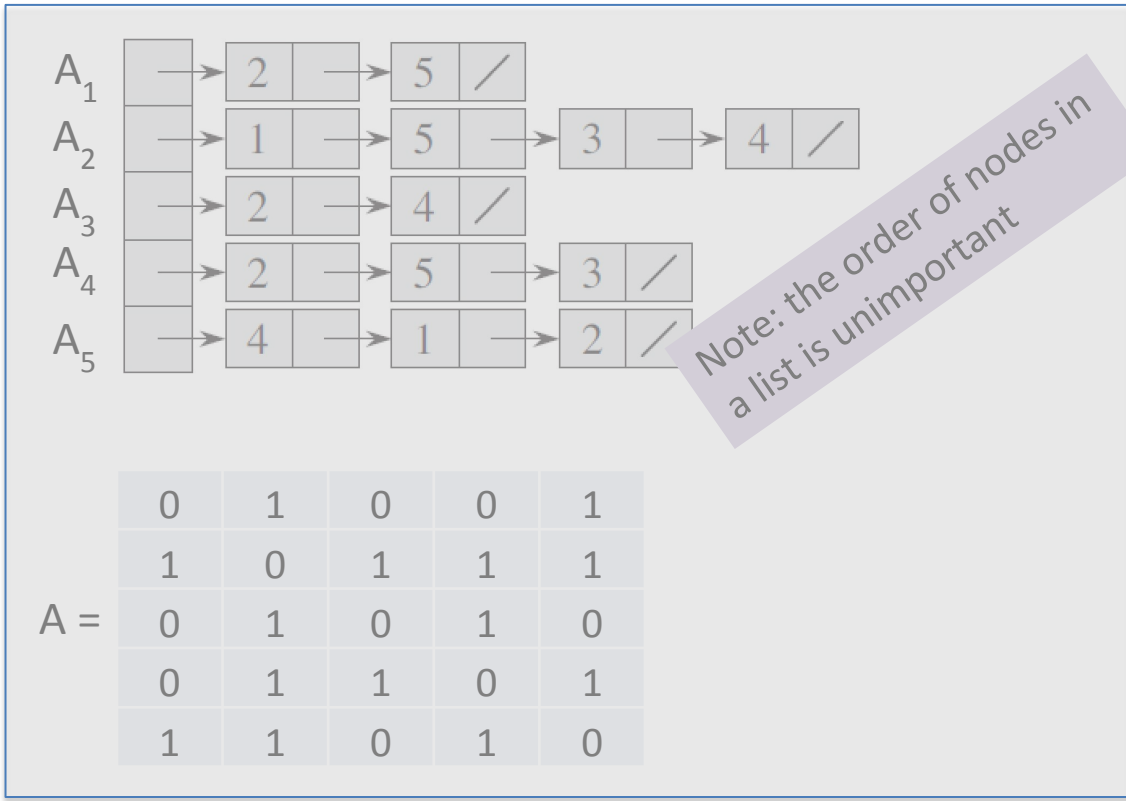

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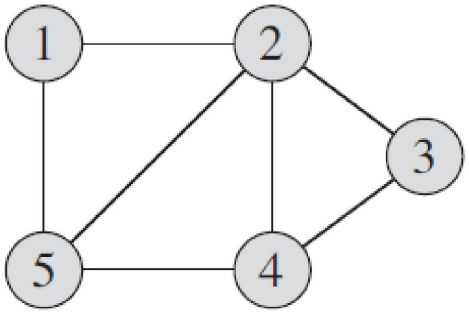

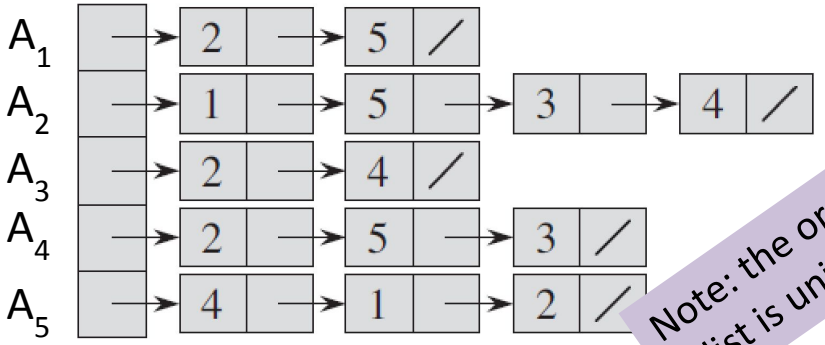

Note: the order of nodes in a list is unimportant

A =

0	1	0	0	1
1	0	1	1	1
0	1	0	1	0
0	1	1	0	1
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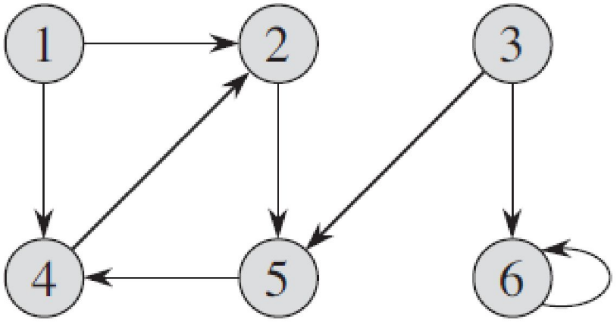

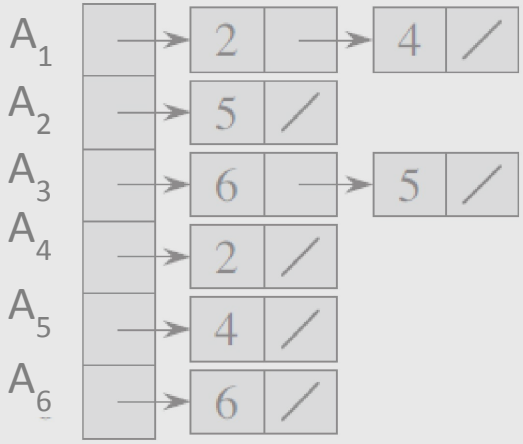

A =

0	1	0	1	0	0
0	0	0	0	1	0
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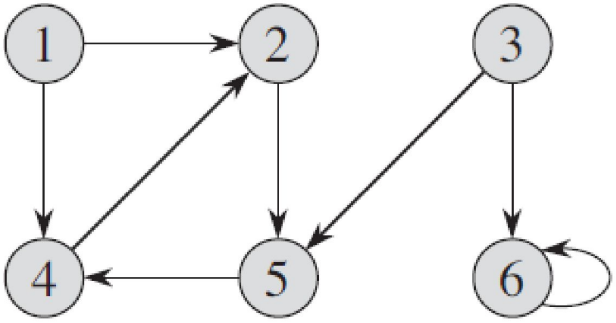

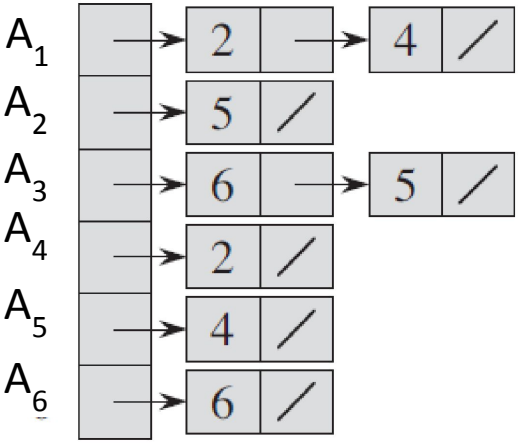

A =

0	1	0	1	0	0
0	0	0	0	1	0
0	0	0	0	1	1
0	1	0	0	0	0
0	0	0	1	0	0
0	0	0	0	0	1

Representation of graphs: adjacency matrices vs. adjacency lists

- Truly, this is a direct consequence of fact that an n x n matrix can be represented by a collection of n lists
- Memory requirements:
  - adjacency matrix: $O(|V|^2)$ bits
  - adjacency lists: $O(|V| + |E|)$ integers/addresses
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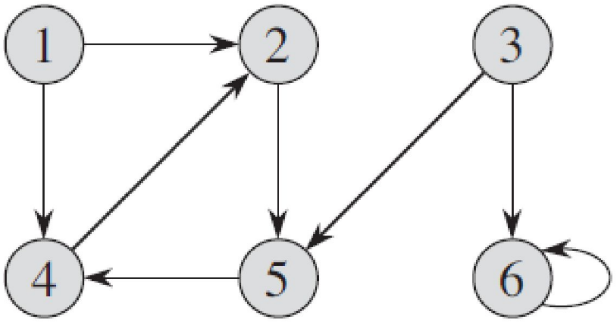
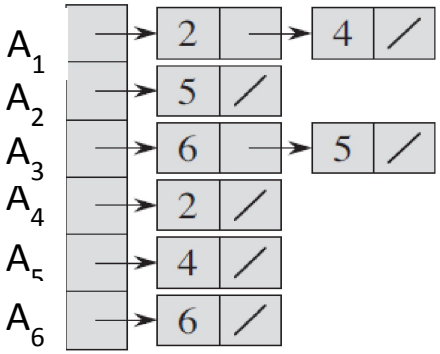
Note: time or space complexity related to graphs is invariably f(|V|, |E|)





A =

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0	0	0	0	1	0
0	0	0	0	1	1
0	1	0	0	0	0
0	0	0	1	0	0
0	0	0	0	0	1

Representation of graphs: adjacency matrices vs. adjacency lists

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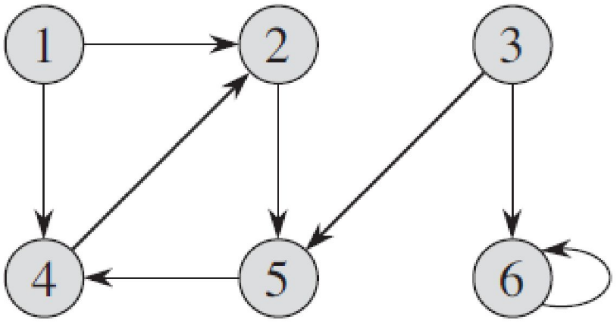

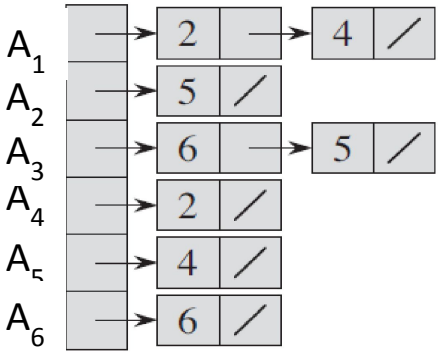

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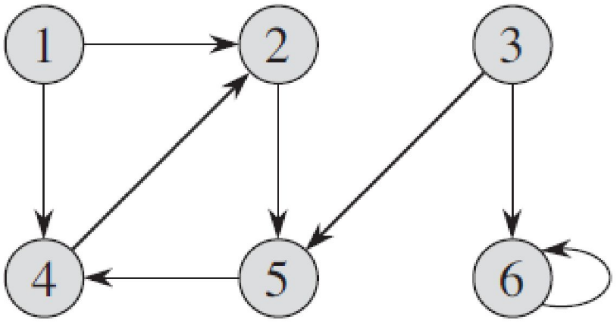

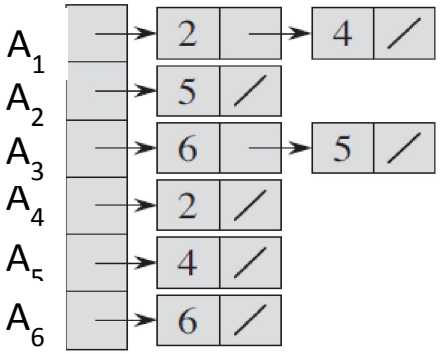

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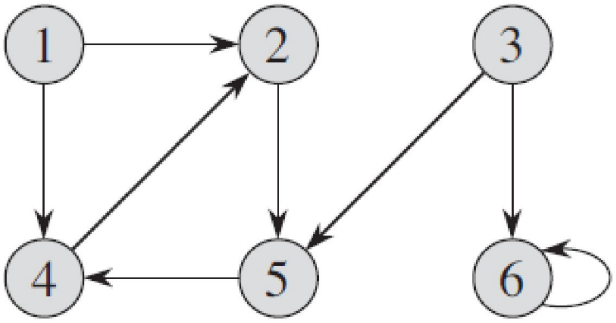

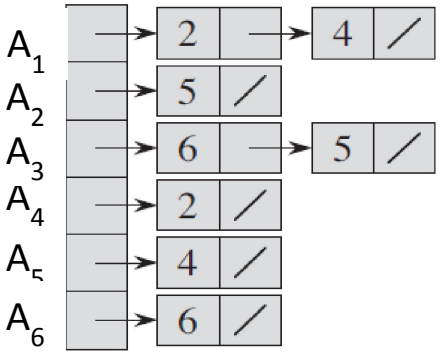

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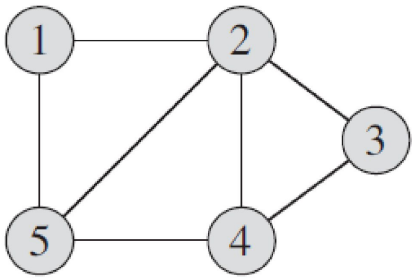
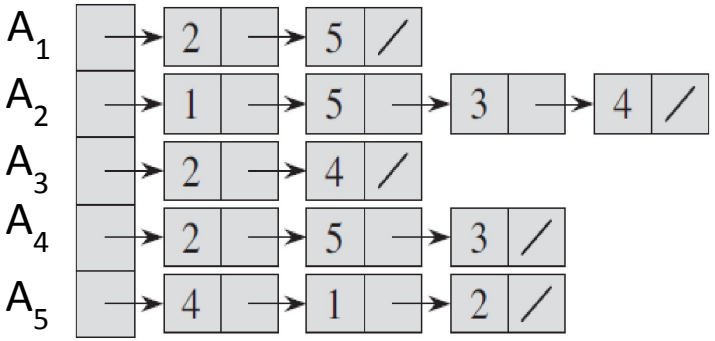
Representation of graphs: adjacency matrices vs. adjacency lists

- Here are some problems for which you must write algorithms assuming that the graph is represented using an adjacency matrix. Re-do this using adjacency lists
  - Count the number of edges in an undirected graph
  - Count the number of edges in a directed graph
  - What is the OUT-degree of each vertex in a directed graph
  - What is the IN-degree of each vertex in a directed graph





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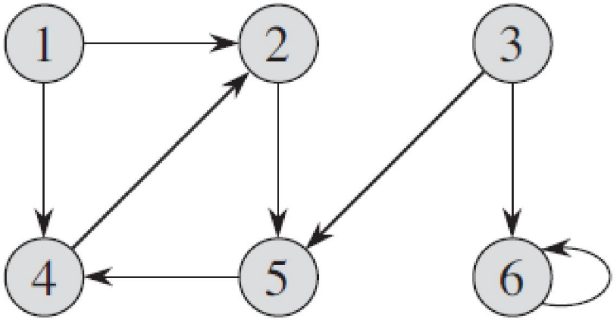

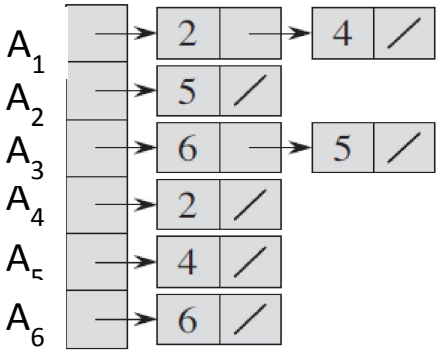

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0	0	0	1	0	0
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More on graphs

Sparse graphs

- Sparse graphs, where $|E| << |V|^2$
- Domestic network of American airlines:
  - $|V| = 218$ destinations, but only $|E| = 682$ routes $<< 23{,}653 = |V|^2$
- More efficient to work with adjacency list representation (218 lists, 682 list nodes)

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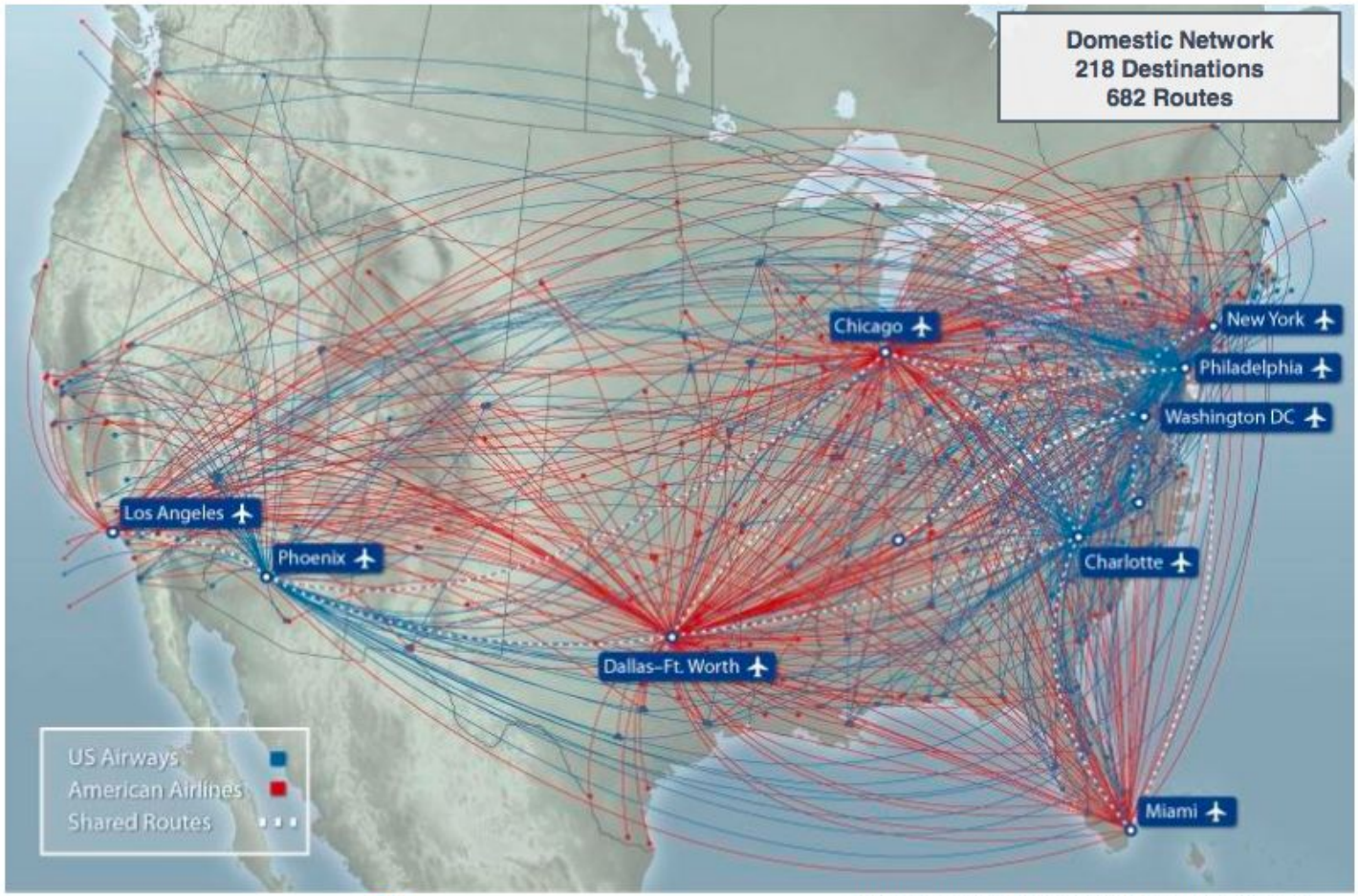


Diio 2013 published schedules as of January 25, 2013

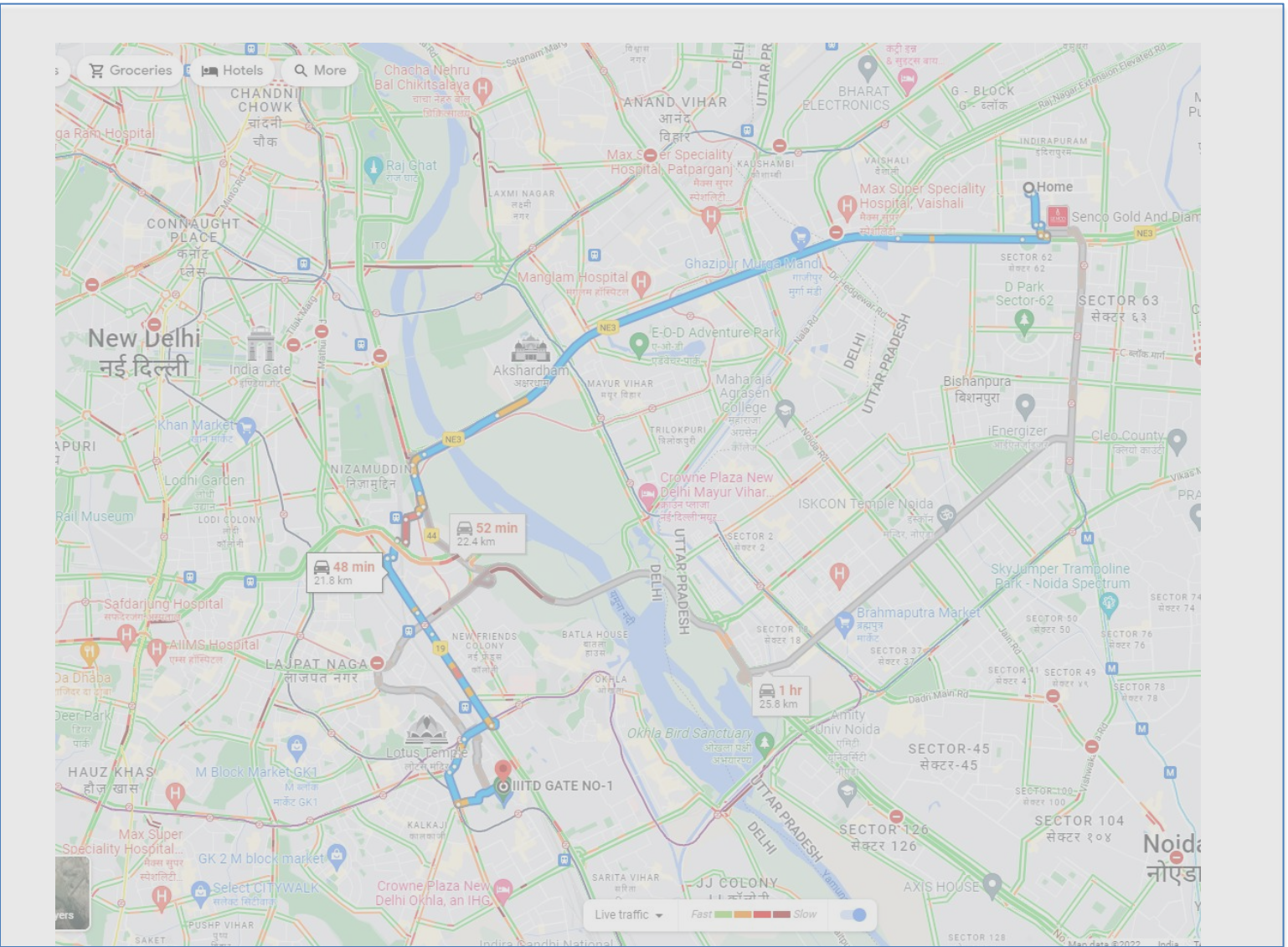
American Airlines

8

U·S AIRWAYS

Weighted graphs

- Graphs where a ”weight” is associated with every edge
- Weight may reflect ”distance”, ”delay”, ”cost”, ”congestion”, etc.
  - Helps evaluate ”shortest paths”



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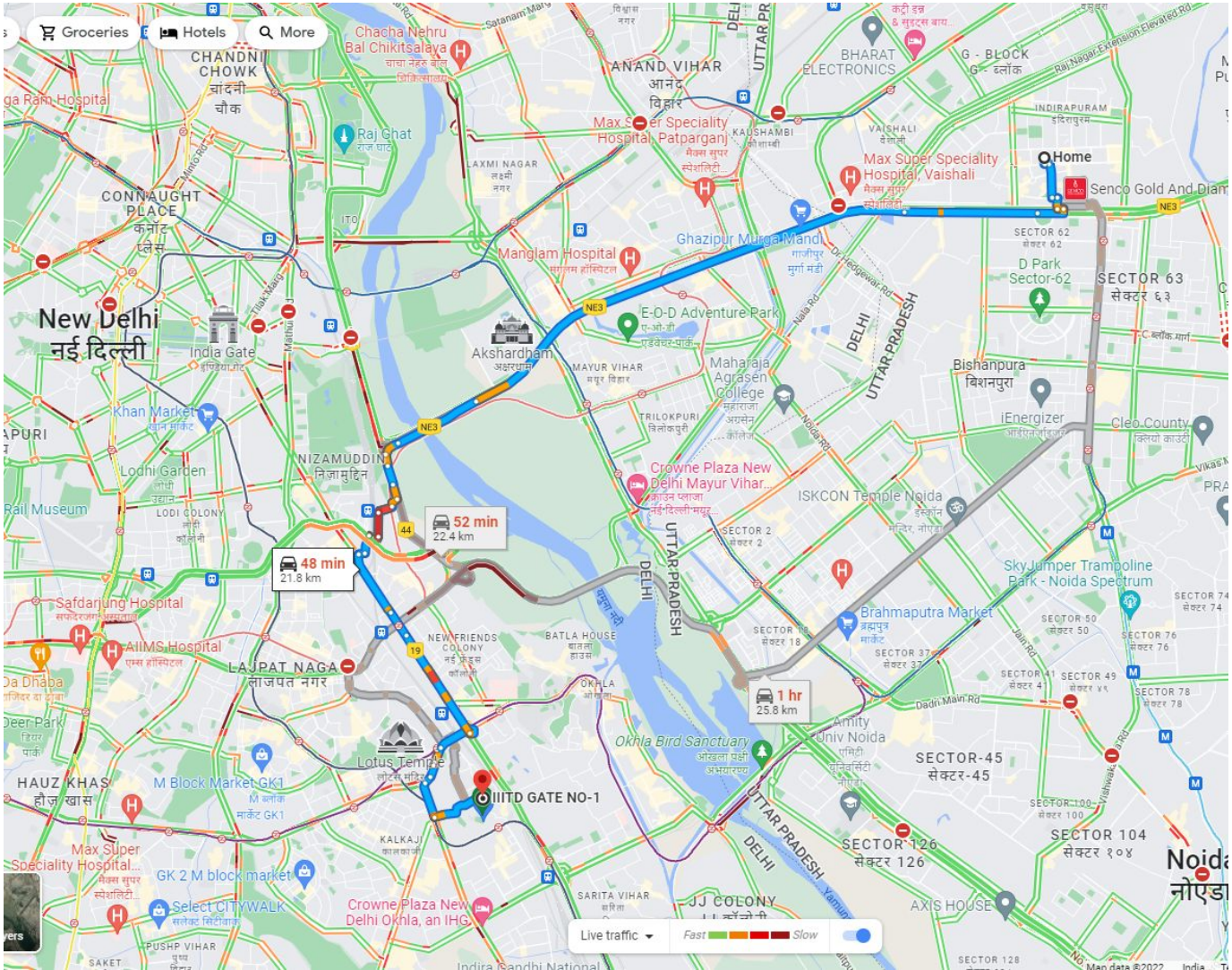

Weighted graphs, and their representation

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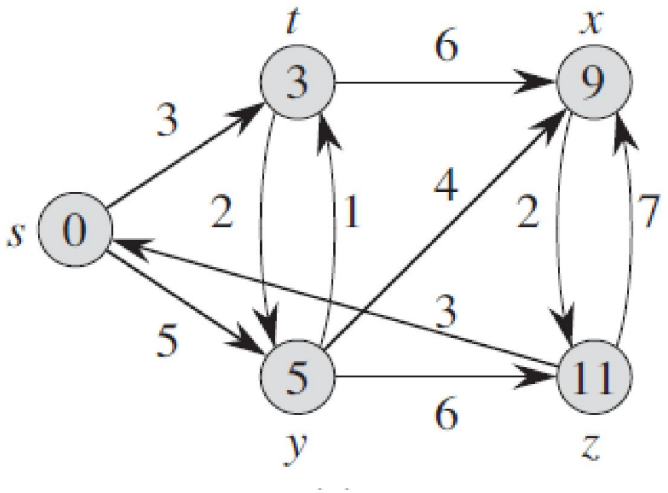


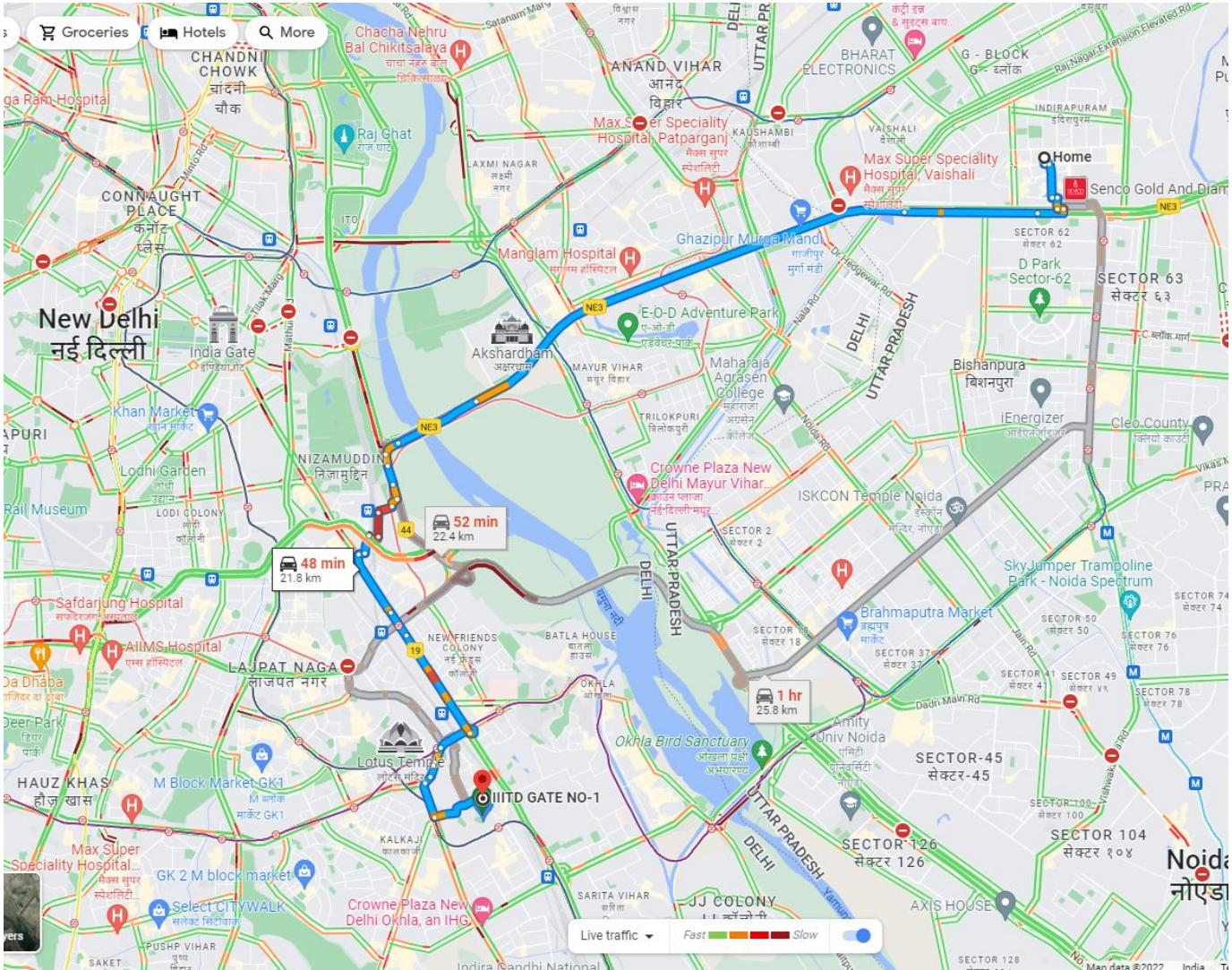
A =

	s	t	x	y	z
s	-	3	-	5	-
t	-	-	6	2	-
x	-	-	-	-	11
y	-	1	4	-	6
z	3	-	7	-	-

Some problems on graphs

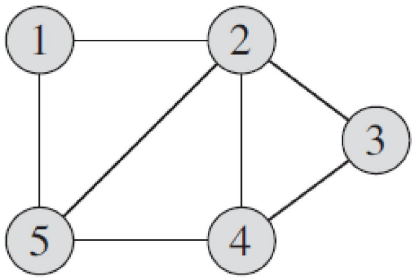
Shortest path algorithms

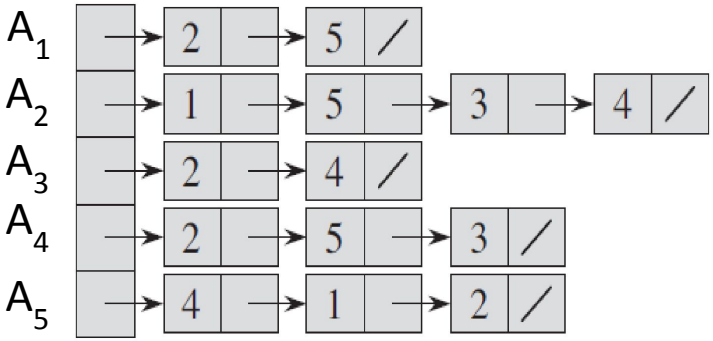
- One-to-one
- One-to-many
- Many-to-many



Traversal of graphs

- Viz. search for a vertex, or a path that satisfies a certain property, or is an “optimum”
  - Through an undirected graph or through a directed graph
- Different search techniques:
  - BFS
  - DFS
  - others





A =

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0	1	0	1	0
0	1	1	0	1
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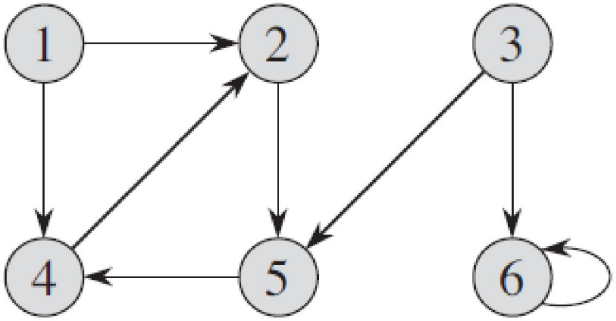
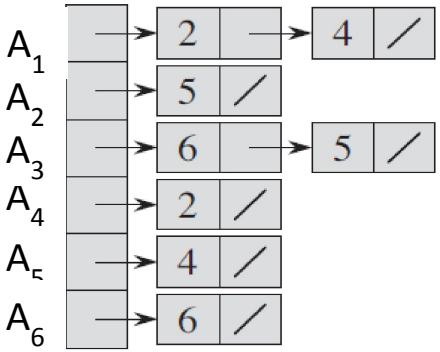




A =

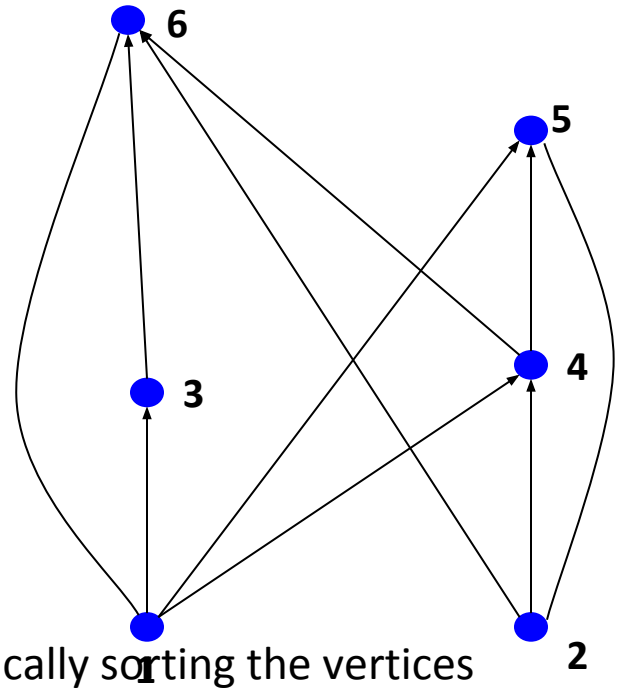
0	1	0	1	0	0
0	0	0	0	1	0
0	0	0	0	1	1
0	1	0	0	0	0
0	0	0	1	0	0
0	0	0	0	0	1

Topological sort

- Consider the following executable statements:

- S_1: a=0;
- S_2: b=1;
- S_3: c=a+1
- S_4: d=b+a;
- S_5: e=d+1;
- S_6: e=c+d;

Nodes = statements
Edges = precedence requirements

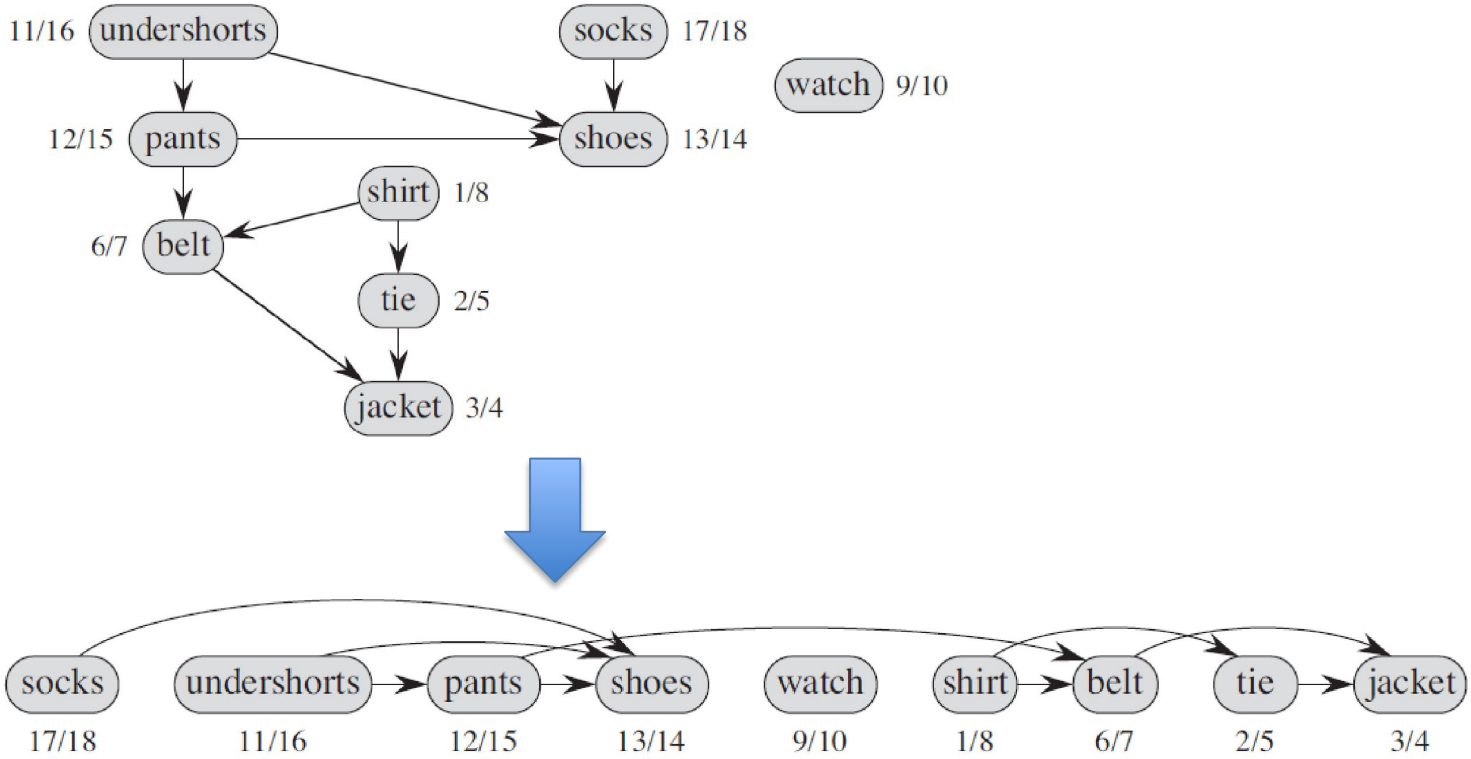


- Questions:

1. Which statements must execute before, for example, S_6?
2. Which statements can be executed in parallel?

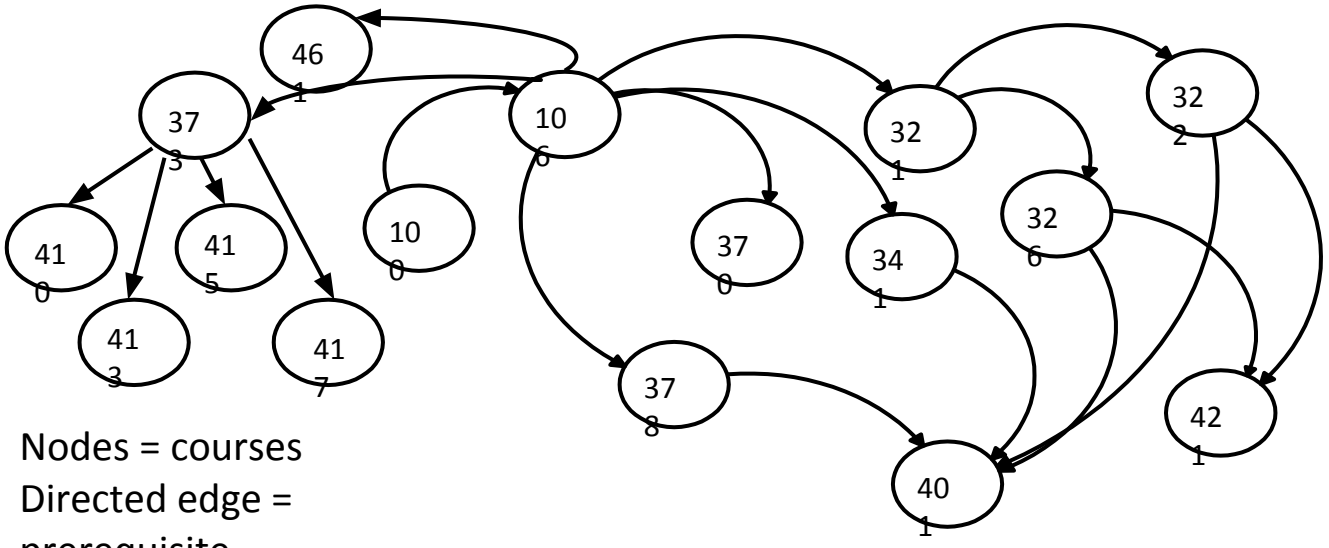
- Answers lie in studying the adjoining graph, G, and topologically sorting the vertices

Topological sort



Topological sort

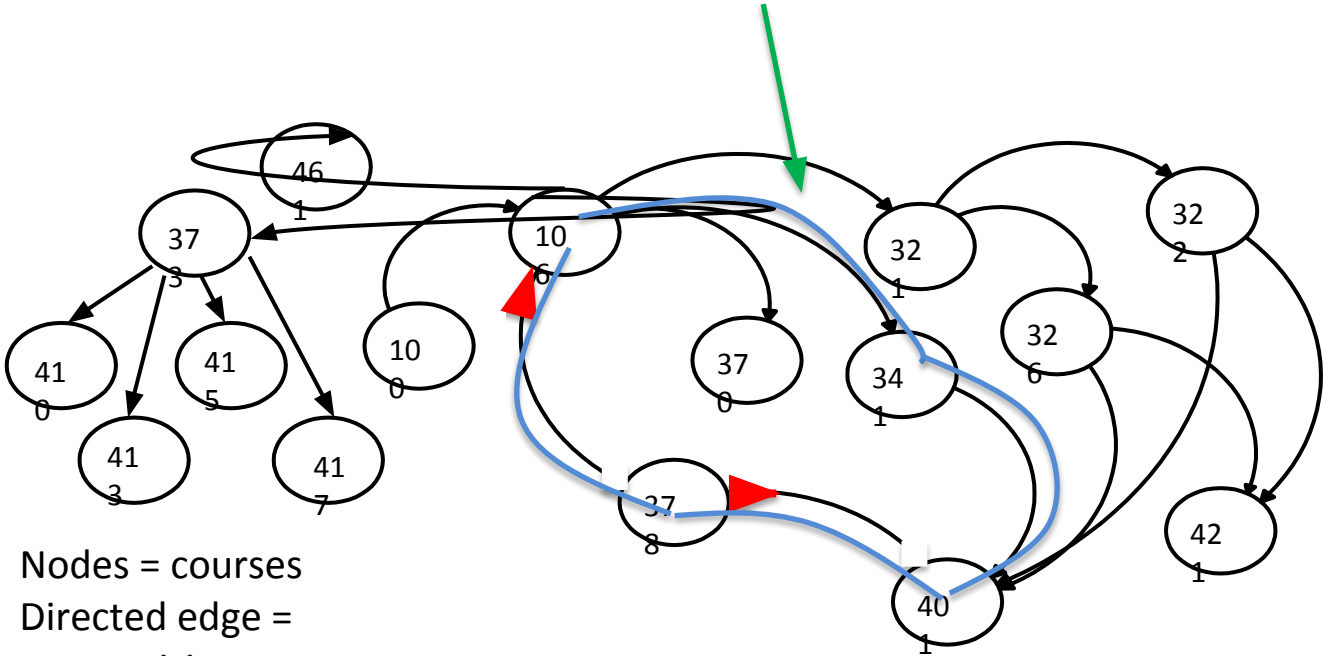
- In what possible sequence should one complete the courses so as to complete the program (consisting of these 16 courses)



Nodes = courses
Directed edge = prerequisite

Directed acyclic graphs,or DAGs

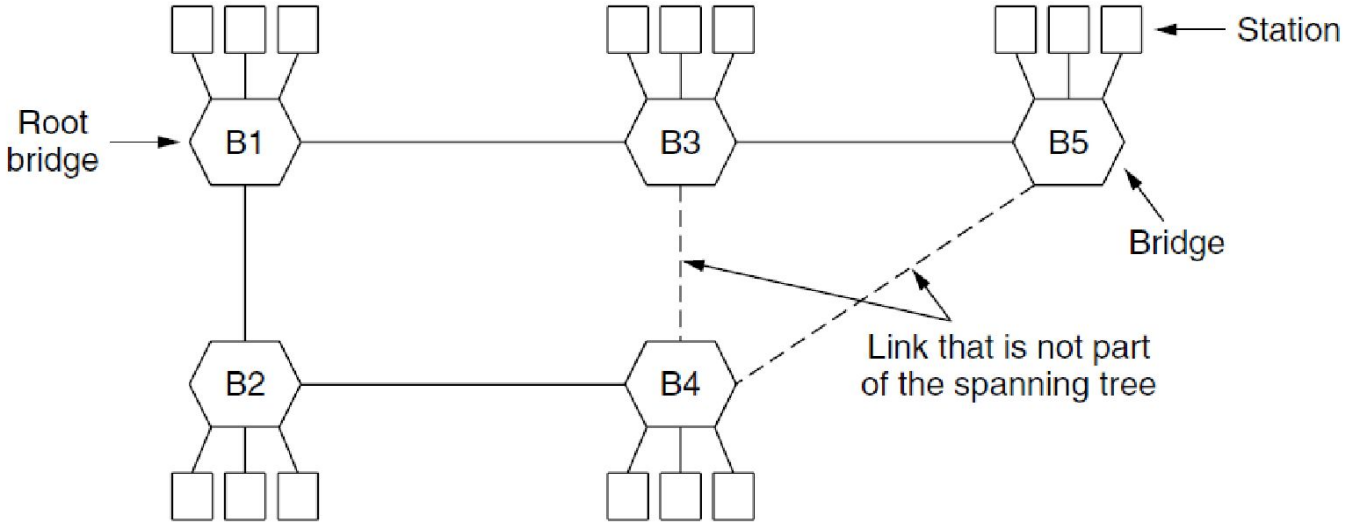
- Consider an 'directed' graph, G = (V, E), which has "cycles", as in diagram below showing pre-requisites
  - How does one complete the coursework
  - Vertices in a DAG (directed acyclic graph) only can be sorted topologically



Nodes = courses
Directed edge = prerequisite

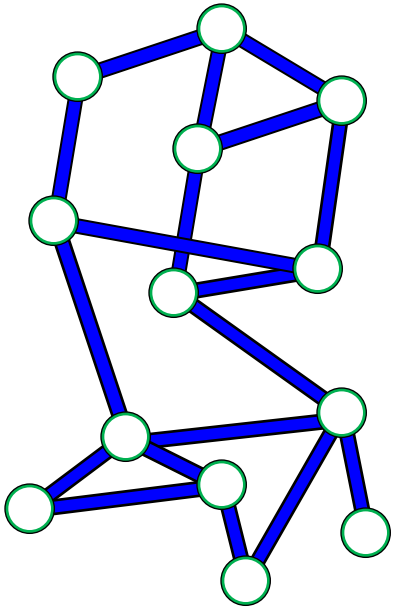
Other problems

- Minimum spanning tree
  - Spanning Tree algorithm applied to network of routers or bridges discovered by Radia Perlman in late 1970s/early 1980s

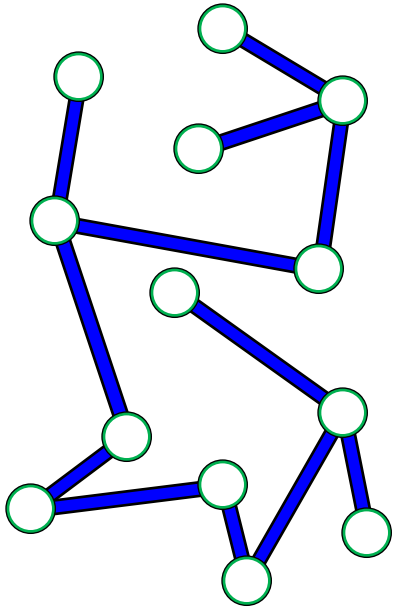


Other problems

- Minimum spanning tree
  - Another example
  - What can you say about fault-tolerance of a spanning tree



Graph G



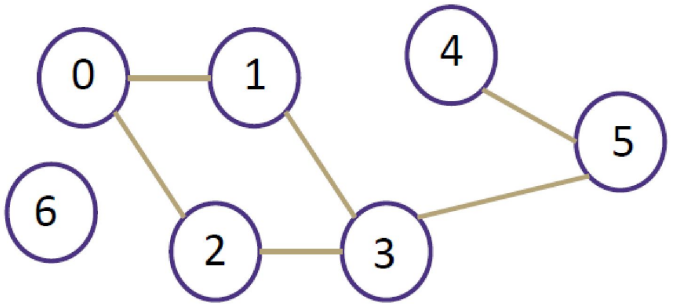
Spanning tree for Graph G

Q&A

Check these examples

- Courtesy https://courses.cs.washington.edu/courses/cse373/22sp/

Adjacency matrix or lists of directed/undirected graphs



	0	1	2	3	4	5	6
0	0	1	1	0	0	0	0
1	1	0	0	1	0	0	0
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3	0	1	1	0	0	1	0
4	0	0	0	0	0	1	0
5	0	0	0	1	1	0	0
6	0	0	0	0	0	0	0