ECE113- Basic Electronics

Lecture week 5: Thevenin and Norton's Theorems

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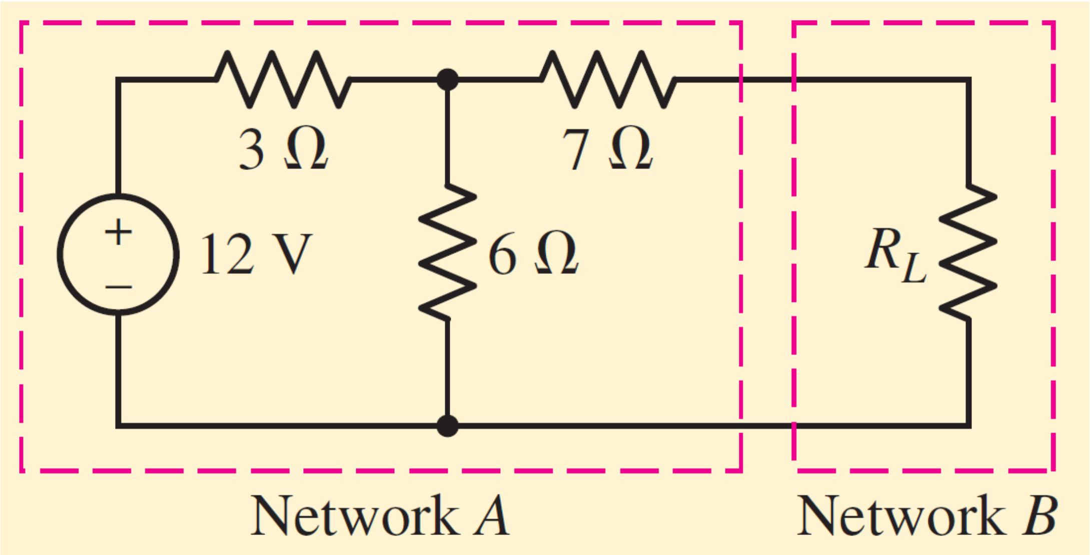
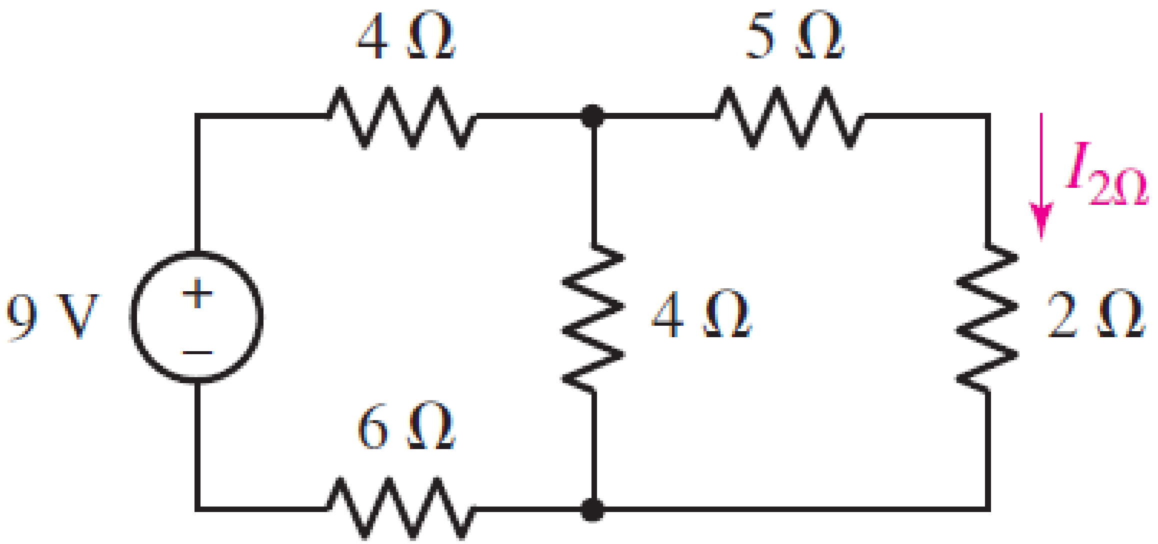
Example: Find Thevenin Equivalent



Ans: R_{TH} = 9 ohm, V_{TH} = 8V

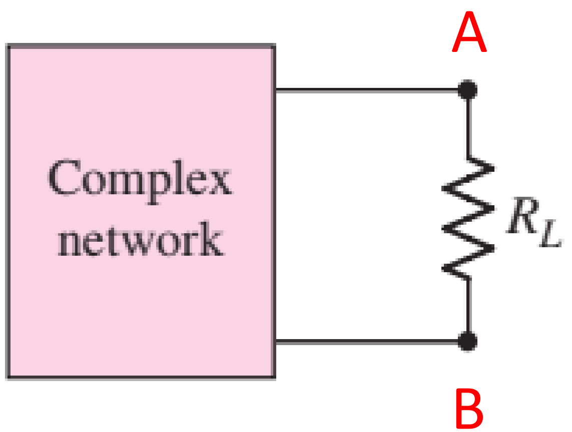
Example: Find Thevenin Equivalent



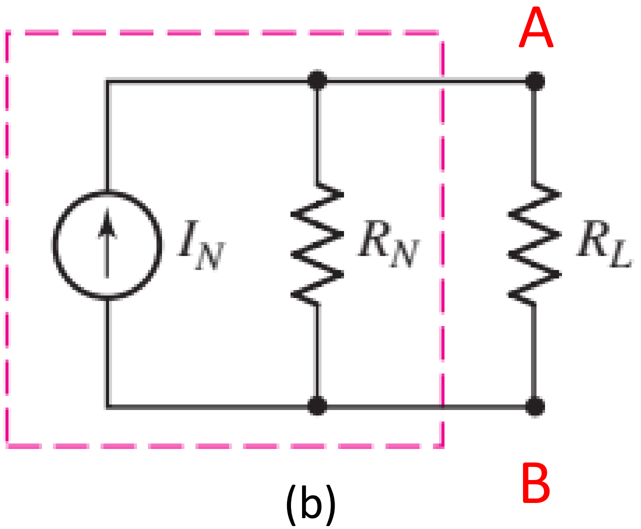
Ans: R_{TH} = 7.857 ohm, V_{TH} = 2.571 V, I_2 = 260.8 mA

Norton's Theorem

Any two terminal linear network containing energy sources and resistances (or impedances) can be replaced by an equivalent circuit consisting of a current source I_N in parallel with an resistance (or impedance) R_N, where <u>I_N *is the short circuit current between the terminals*</u> of the network and R_N is the resistance (or impedance) measured between the terminals with all the energy sources replaced by their internal resistance (or impedance).



(a)



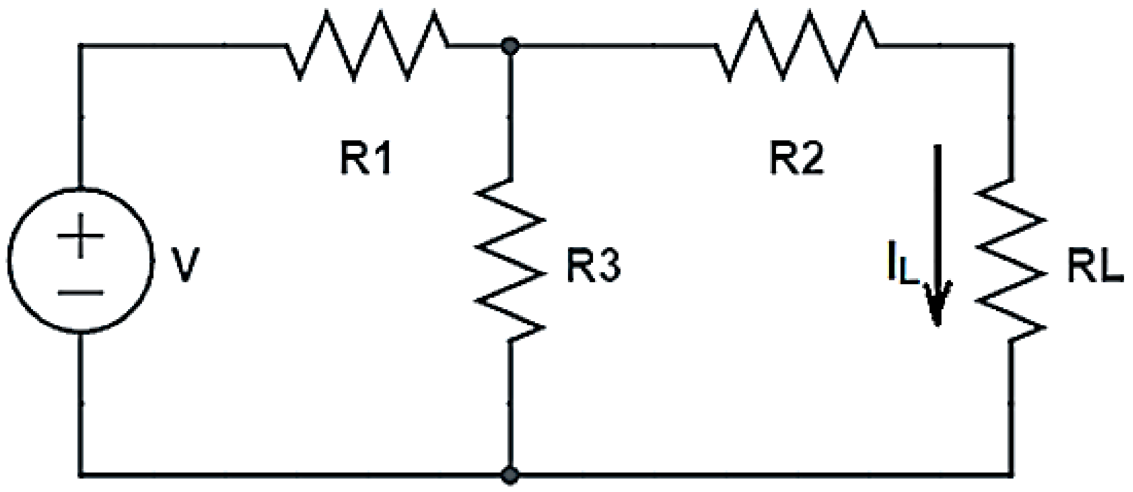
(b)

Norton's Theorem

1. **Given any linear circuit, rearrange it in the form of two networks, *A* and *B*, connected by two wires.** Network *A* is the network to be simplified; *B* will be left untouched. As before, if either network contains a dependent source, *its controlling variable must be in the same network*.
2. **Disconnect network *B*, and short the terminals of *A*.** Define a current i_{sc} as the current now flowing through the shorted terminals of network *A*.
3. **Turn off or "zero out" every independent source in network *A* to form an inactive network.** Leave dependent sources unchanged.
4. **Connect an independent current source with value i_{sc} in parallel with the inactive network.** Do not complete the circuit; leave the two terminals disconnected.
5. **Connect network *B* to the terminals of the new network *A*.** All currents and voltages in *B* will remain unchanged.

To determine Norton's equivalent circuit

To understand the concept of Norton's Theorem, let us consider a circuit as shown below.

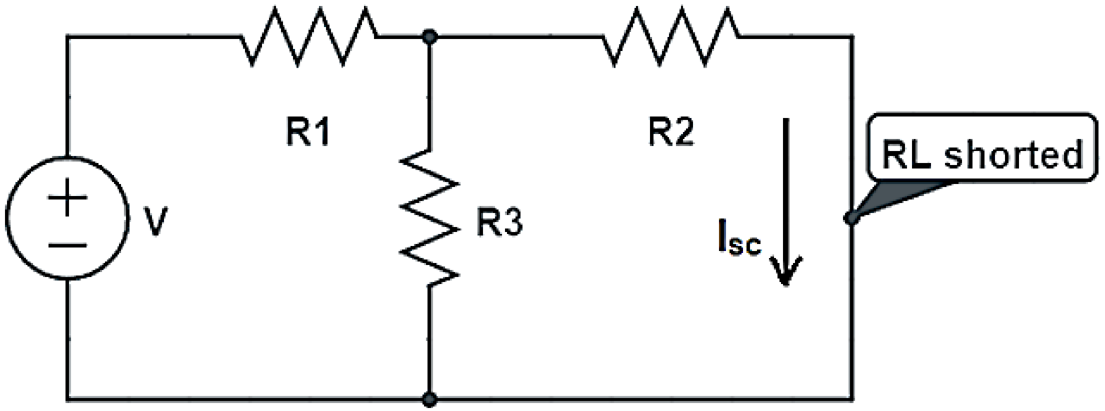


For the above circuit, we will try to find the Norton's equivalent circuit.

To determine Norton's equivalent circuit

First step, we will short circuit the load resistance R_L and find the short circuit current I_{SC}. Figure below depicts this step.

Let us now find short circuit current I_{SC} using conventional circuit analysis. The is calculated as shown below.



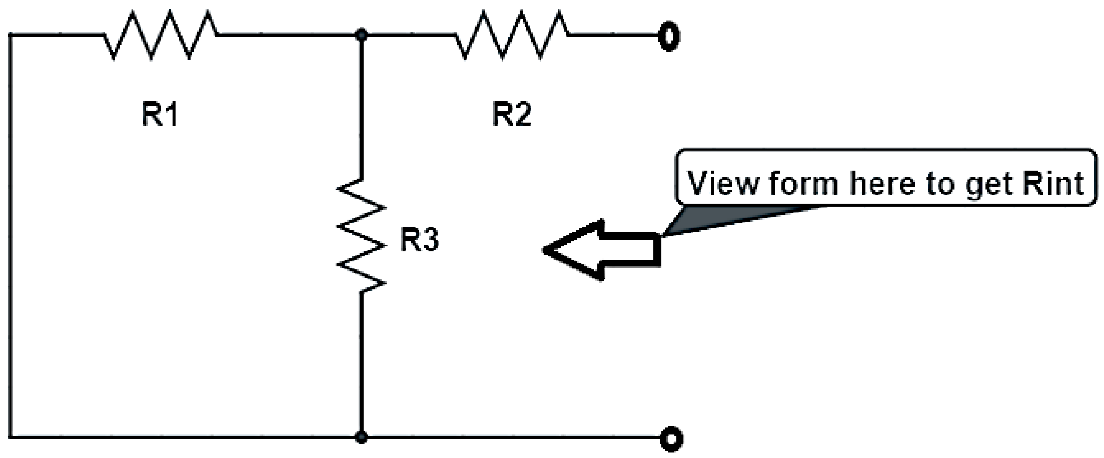
$$i = \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

$$\therefore\ Isc = \frac{iR_3}{(R_2 + R_3)}$$

This current I_{sc} is the magnitude or strength of current source of Norton's equivalent circuit

To determine Norton’s equivalent circuit

Second step, let us now find the resistance of the circuit. The main thing which should be taken care while calculating internal resistance is to replace current source by open circuit and voltage source by a short circuit. Also, keep the load terminals open and find the internal or equivalent resistance of circuit from open load terminal once you replaced all the sources. We will adopt this method.



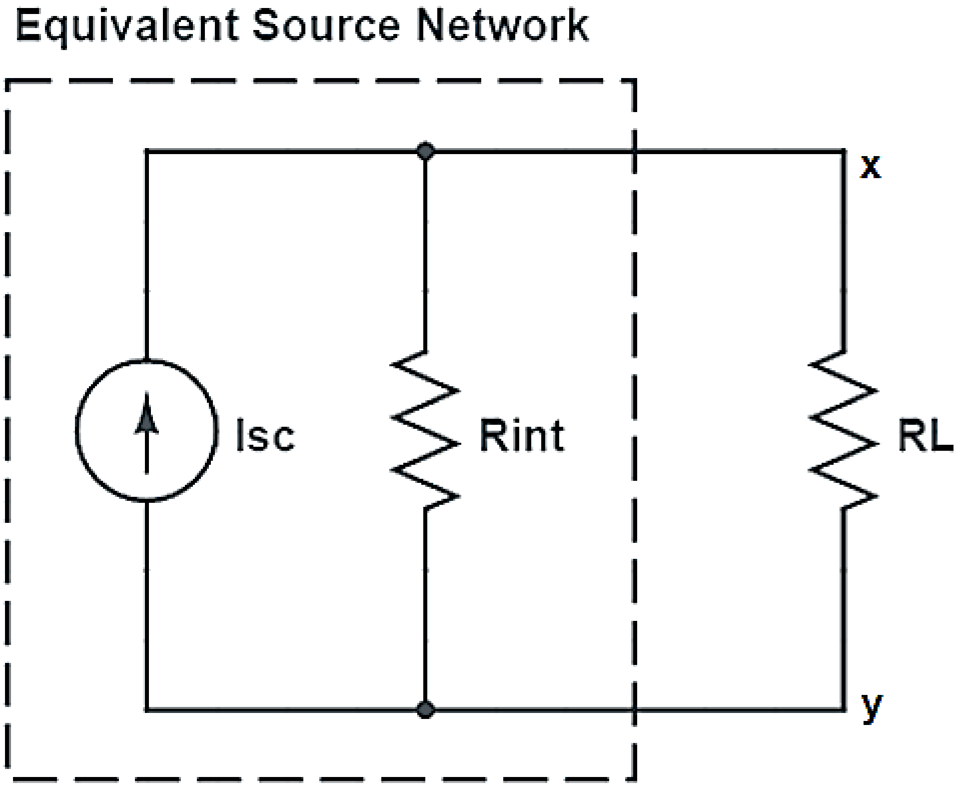
Let us call the resistance R_{int}.

$$R_{int} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

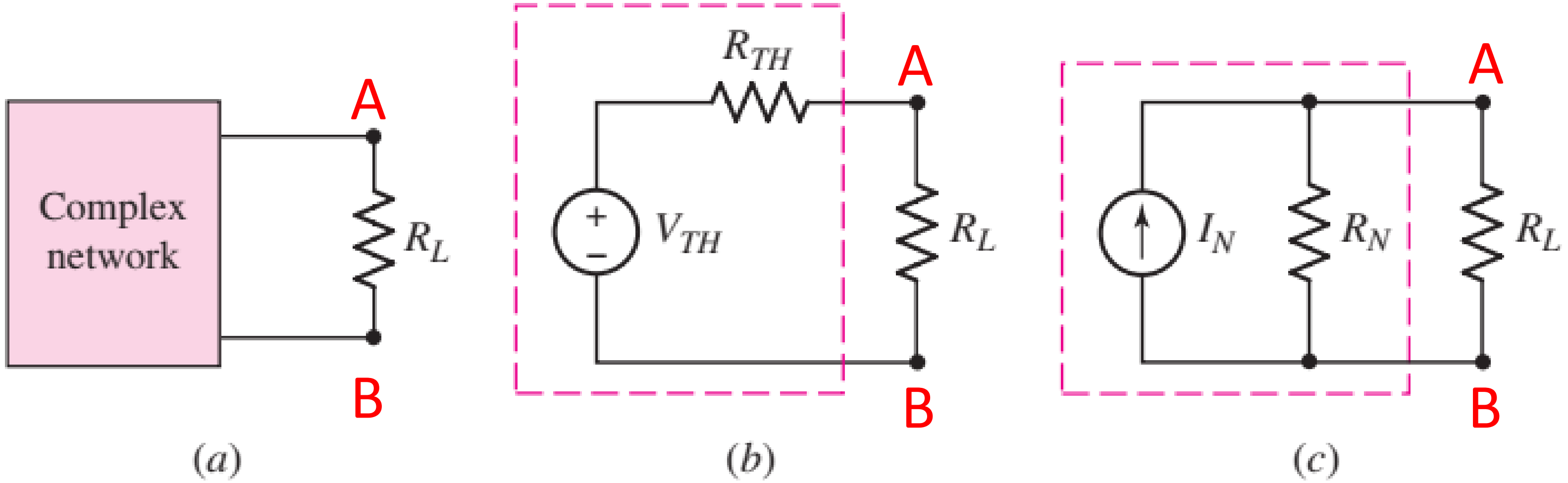
This R_{int} is the value of resistance which is to be connected in parallel with the current source I_{sc} calculated earlier. Well, it’s time to draw the Norton’s equivalent Circuit.

To determine Norton’s equivalent circuit

To draw Norton’s Equivalent Circuit, we connect current source I_{sc} in parallel with internal resistance R_{int}. This connection is called equivalent source network. The terminals x-y of equivalent source network is then wired to load resistance R_L to get the Norton’s equivalent circuit. This equivalent circuit is shown below.



Thevenin network ⟺ Norton network



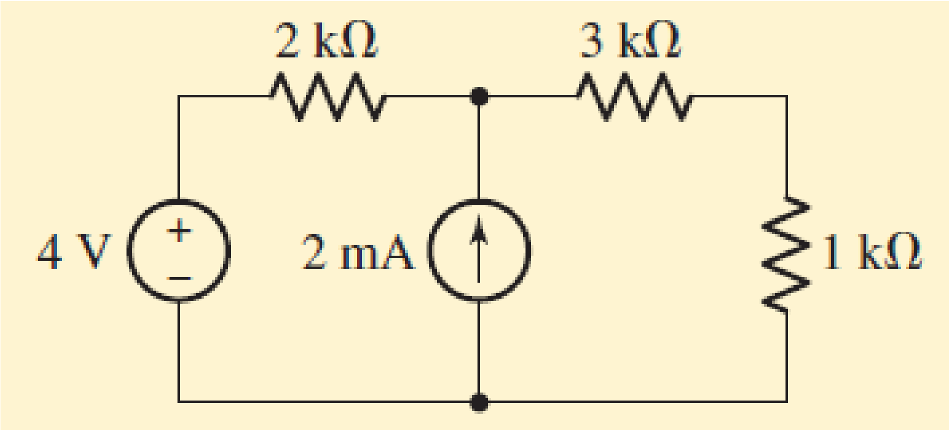
(a) A complex network including a load resistor R_L. (b) A Thévenin equivalent network connected to the load resistor R_L. (c) A Norton equivalent network connected to the load resistor R_L.

An equivalent current source of a Thevenin network is a Norton network

So, $I_N = \dfrac{V_{TH}}{R_{TH}}$ and $R_N = R_{TH}$

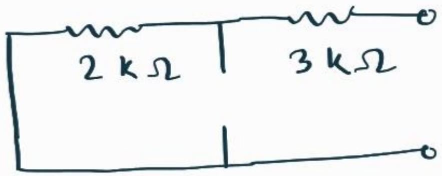
It is possible to obtain the Norton equivalent of a network by performing a source transformation on the Thévenin equivalent

Example: Find Norton Equivalent for load resistance 1 k ohm



Ans: 1.6 mA, 5 kOhm

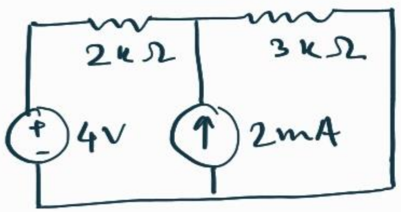
For R_N

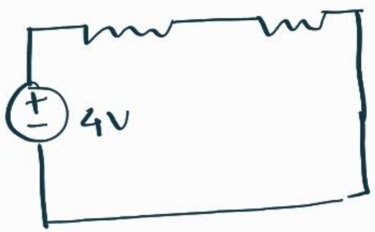


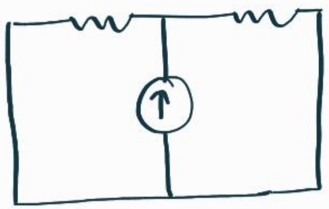
$\therefore R_N = 2+3 = 5\,k\Omega$

For I_N

Using superposition theorem,







$i_4 = \frac{4}{5000}$ A

$= 0.8$ mA

$i_2 = \frac{2\times2}{5}$ mA

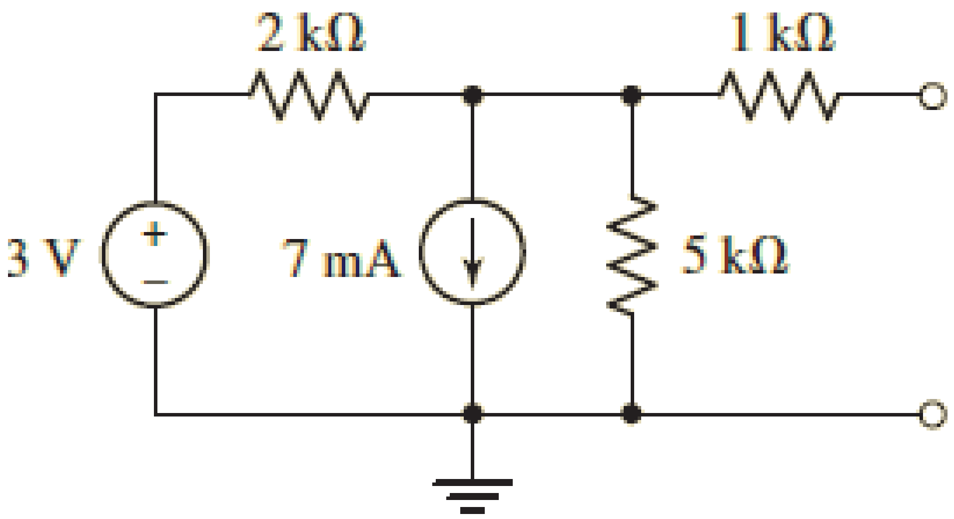
$= 0.8$ mA

$\therefore I_N = (0.8 + 0.8)$ mA

$= 0.16$ mA

$\therefore$ Norton's equivalent circuit

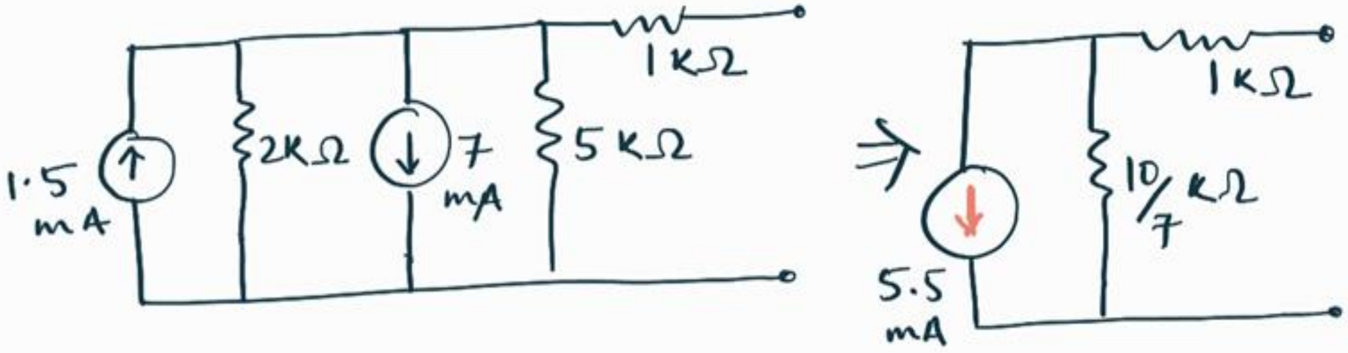
Example: Find Norton Equivalent and Thevenin Equivalent



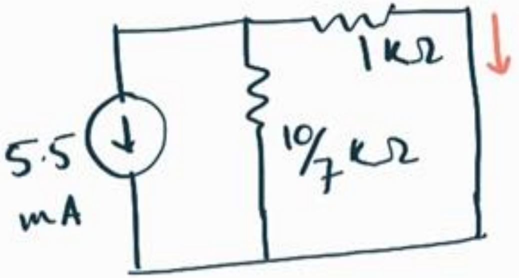
Ans: −3.235 mA, 2.429 k, −7.857 V.

Solution

Using source equivalence, we can convert the circuit as,



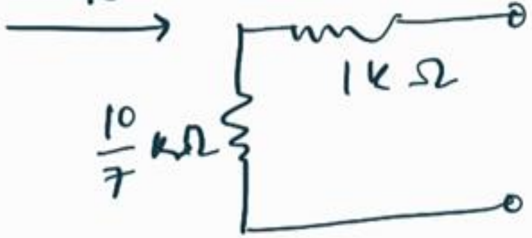
Therefore, for I_N



$$\therefore I_N = -\frac{\frac{10}{7} \times 5.5}{\left(\frac{10}{7}+1\right)} = -\frac{385}{119}$$

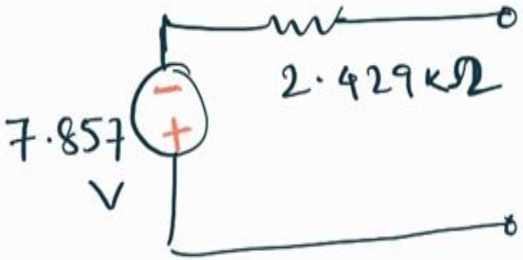
$$= -3.235\,mA$$

for R_N



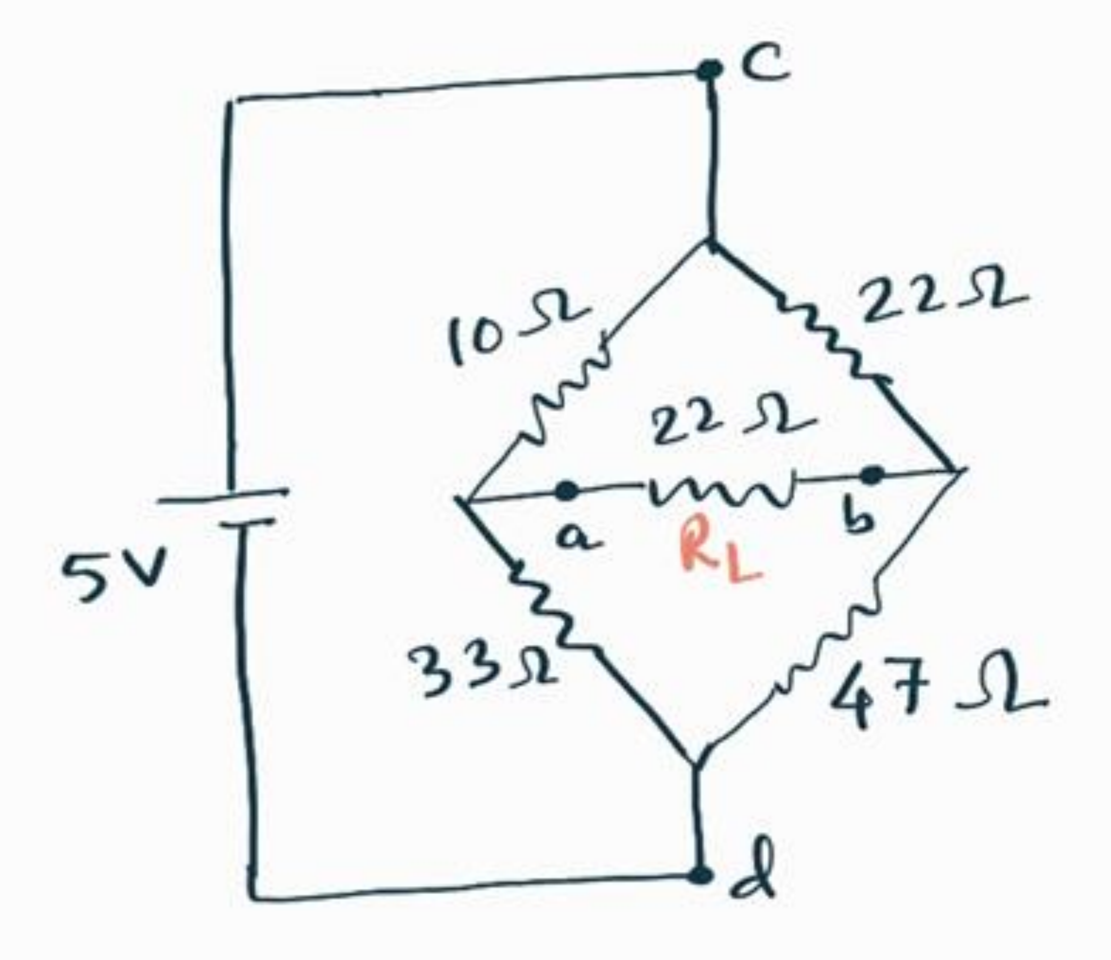
$$\therefore R_N = \frac{10}{7} + 1 = 2.429\,k\Omega$$

$$\therefore V_{TH} = I_N \times R_N = -3.235 \times 2.429 = -7.857\,V$$



Exercise 1

Apply Thevenin's theorem to find current through the resistor R_L in the following circuit

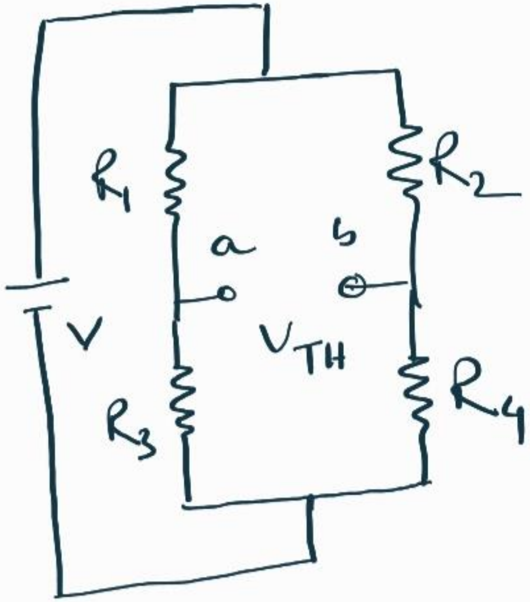


Ans: I_L = 9.62A

Exercise 1 solution

The Thevenin's voltage,

V_{TH} = voltage across R_3 − voltage across R_4



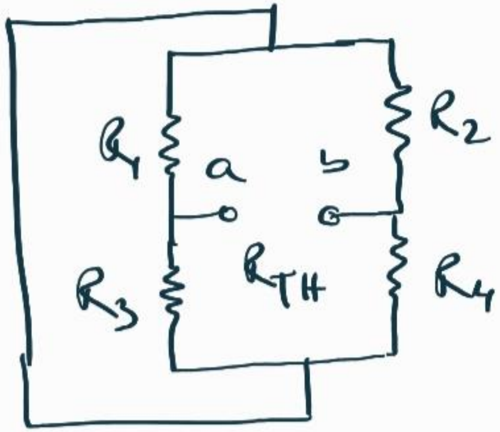
$$= \frac{V \cdot R_3}{R_1 + R_3} - \frac{V \cdot R_4}{R_2 + R_4}$$

$$= \left(\frac{5 \times 33}{10 + 33} - \frac{5 \times 47}{22 + 47}\right) V = 0.43 V$$

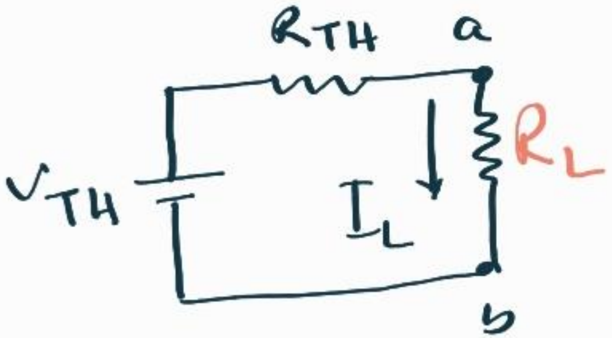
For, $R_{TH} = R_1 \| R_3 + R_2 \| R_4$

$$= \left(\frac{10 \times 33}{10 + 33} + \frac{22 \times 47}{22 + 47}\right) \Omega$$

$$= 22.69\ \Omega$$



Thevenin's equivalent

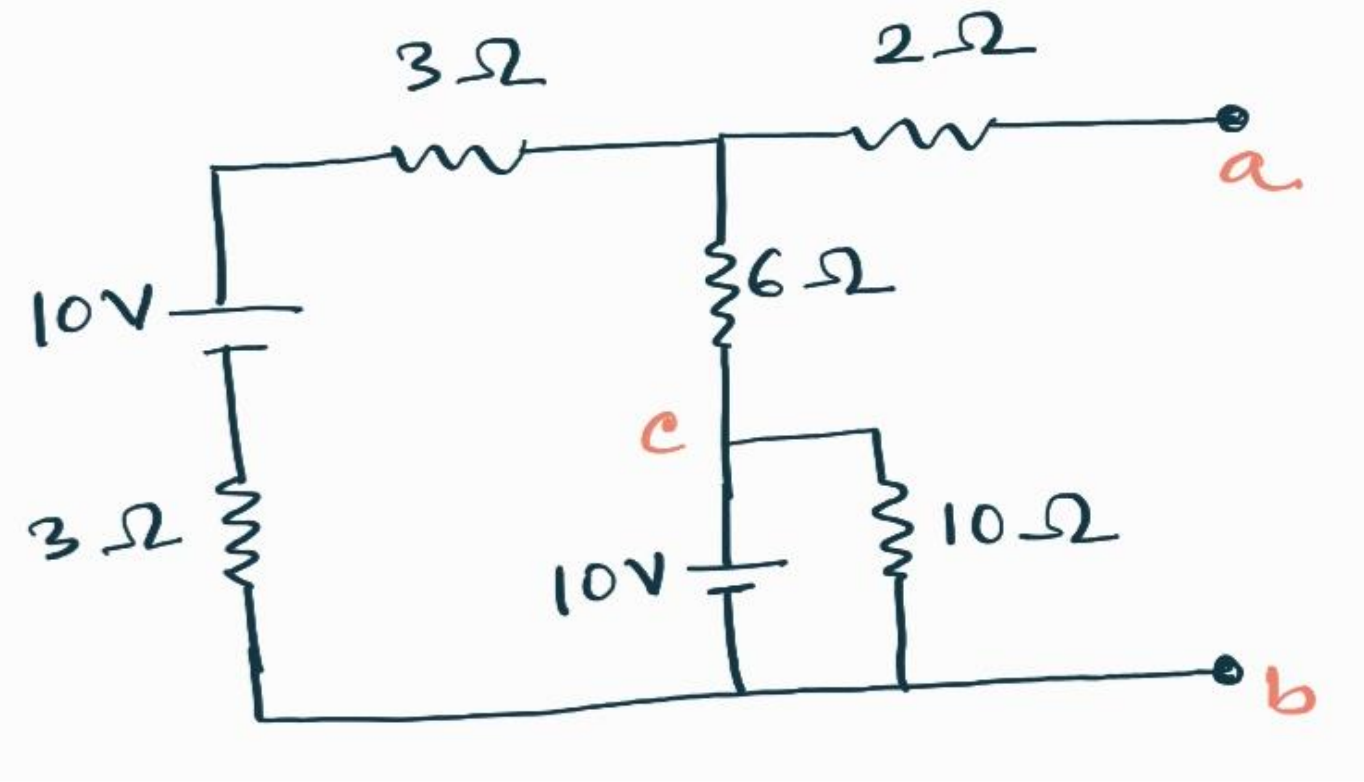


$$\therefore I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

$$= \frac{0.43}{22.69 + 22} = 9.62 A$$

Exercise 2

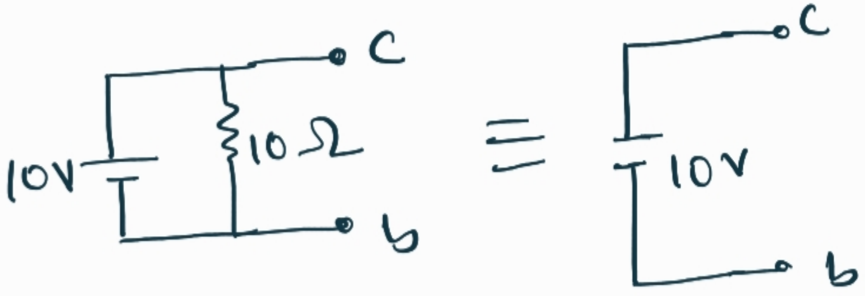
Find the Thevenin's and Norton's equivalent circuit between *a* and *b* of the following circuit



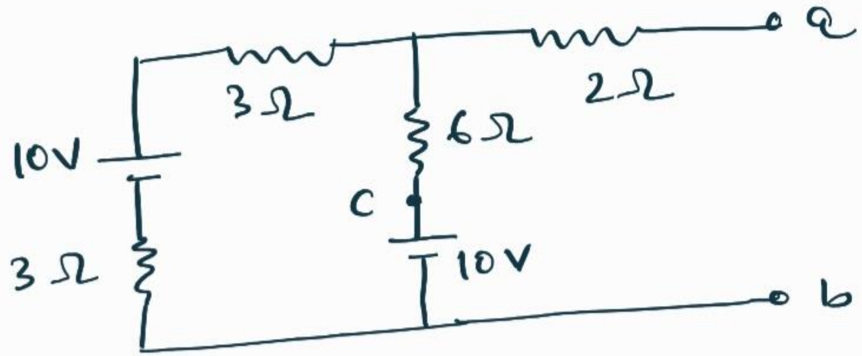
Ans: V_{TH} = 10 V, R_{TH} = 5 ohm, I_N = 2A

Exercise 2 solution

The 10V source with the parallel resistor 10Ω can be replaced by its equivalent by using Thevenin's theorem between the point c & b, as

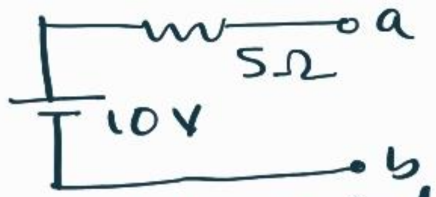


Therefore, the circuit can be redrawn as

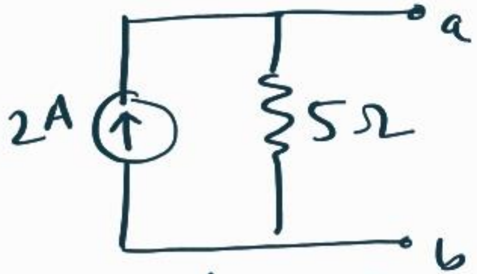


$\therefore R_{TH} = 2 + 6 \parallel (3+3) = 2 + 6 \parallel 6 = 5\,\Omega$

For V_{TH}, if we use KVL around the closed loop, current is zero in the loop. Since there is no voltage drop in any of the resistors, $V_{TH} = V_{ab} = 10V$



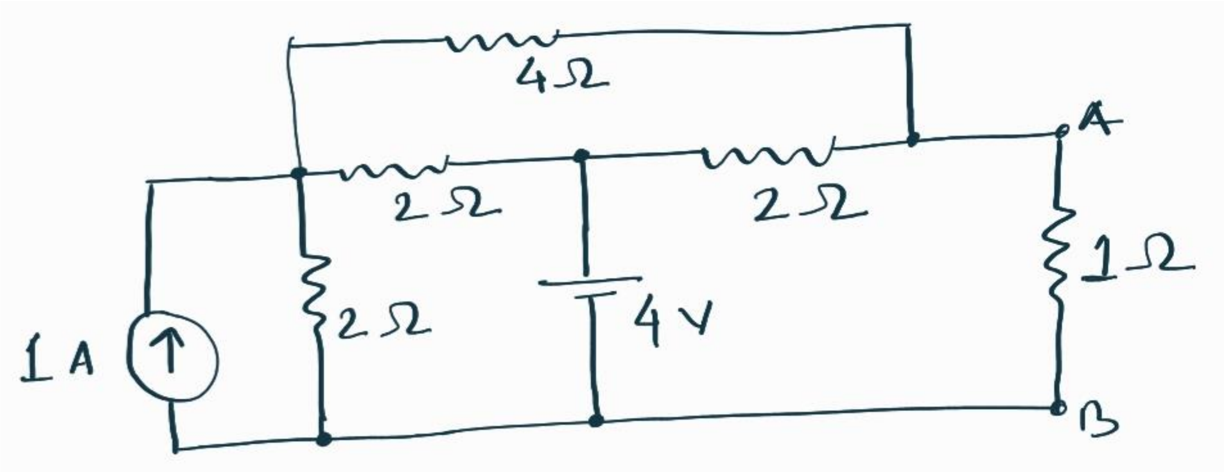
Thevenin's equivalent



Norton's equivalent

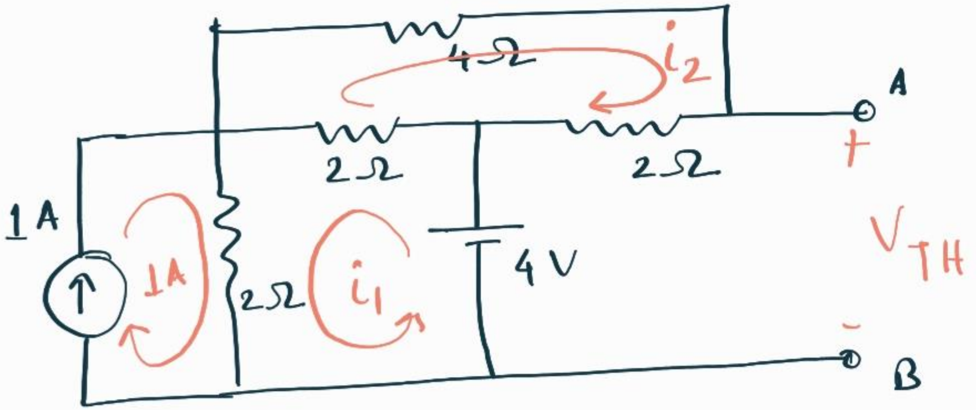
Exercise 3 and its solution

Calculate the power dissipation in the 1 ohm resistance in the following circuit



Ans: P = 2.34 W

For V_{TH},



Using KVL for loop i_1

$$2(i_1+i_2)+2(i_1+1)=4$$

$$\Rightarrow \quad 2i_1+i_2=1 \quad \text{——— (i)}$$

for loop i_2,

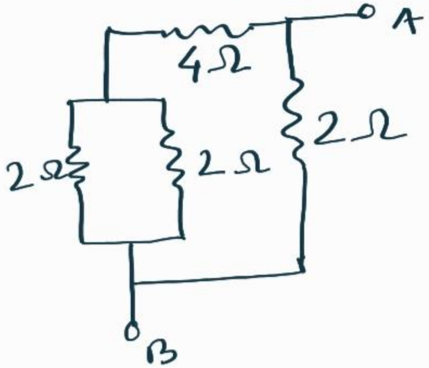
$$2(i_1+i_2)+4i_2+2i_2=0$$

$$\Rightarrow \quad i_1+4i_2=0 \quad \text{——— (ii)}$$

from (i) & (ii) $\longrightarrow$ $i_2 = -\frac{1}{7}$ A

So, $V_{TH} = 2i_2+4 = 4-2/7 = \frac{26}{7}$ V

Solution …

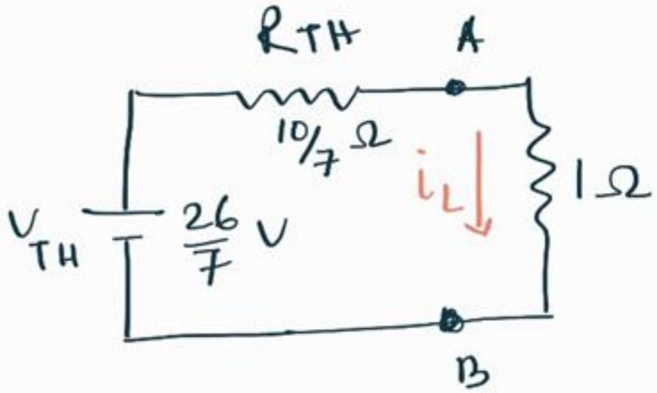


$$\Rightarrow \{(2 \| 2) + 4\} \| 2$$

$$= 5 \| 2 = \frac{10}{7}\,\Omega$$

$$\therefore R_{TH} = \frac{10}{7}\,\Omega$$

Then the equivalent circuit will be,



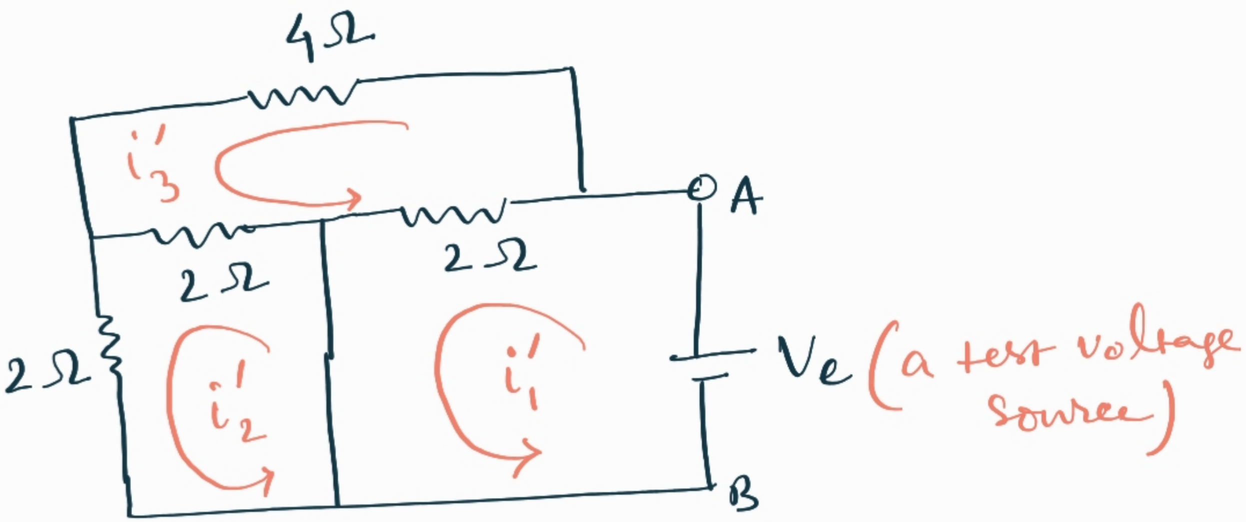
$$i_L = \frac{V_{TH}}{R_{TH} + 1}$$

$$= \frac{26/7}{17/7} = \frac{26}{17}\ \text{A}$$

Then the power dissipated in 1Ω resistance will be, $P = i_L^2 \times 1 = \left(\frac{26}{17}\right)^2 = 2.34\ \text{W}$

Another approach to find R_{TH} for this example

To find R_{TH}, we connect a test voltage source V_e accross A & B, and the circuit for R_{TH} is shown as,



Then using KVL,

for loop i_1, $2i_1' - 2i_3' = V_e$

$$\Rightarrow 2i_1' + 0 \cdot i_2' - 2i_3' = V_e \quad \text{— (III)}$$

for loop i_2, $2(i_2' - i_3') + 2i_2' = 0$

$$\Rightarrow 4i_2' - 2i_3' = 0$$

$$\Rightarrow 0 \cdot i_1' + 4i_2' - 2i_3' = 0 \quad \text{— (IV)}$$

for loop i_3, $4i_3' + 2(i_3' - i_2') + 2(i_3' - i_1') = 0$

$$\Rightarrow -2i_1' - 2i_2' + 8i_3' = 0 \quad \text{— (V)}$$

contd ...

Solving (iii), (iv), (v) for i_1, we get,

$$i_1' = \frac{\begin{vmatrix} V_e & 0 & -2 \\ 0 & 4 & -2 \\ 0 & -2 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & -2 \\ 0 & 4 & -2 \\ -2 & -2 & 8 \end{vmatrix}} = \frac{V_e(32-8) - 0 - 2(0-0)}{2(32-4) - 0 - 2(0+8)}$$

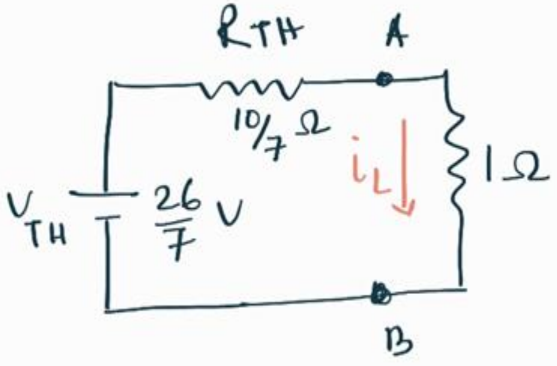
$$= \frac{28 V_e}{56 - 16}$$

$$= \frac{7 V_e}{10} \text{ A}$$

The current supplied by V_e is i_1', so that the effective resistance at A, B (i.e, the Thevenin's resistance R_{TH}) is

$$R_{TH} = \frac{V_e}{i_1'} = \frac{10}{7}\ \Omega$$

Then the equivalent circuit will be,



$$i_L = \frac{V_{TH}}{R_{TH} + 1}$$

$$= \frac{26/7}{17/7} = \frac{26}{17} \text{ A}$$

Then the power dissipated in 1Ω resistance will be,

$$P = i_L^2 \times 1 = \left(\frac{26}{17}\right)^2 = 2.34 \text{ W}$$

Thevenin equivalent when dependent source present:

- Find open circuit voltage
- Find Short circuit current
- From source equivalence find R_{th}

Reference:

- Engineering Circuit analysis, William H. Hayt Jr., Jack E. Kemmerly and Steven M. Durbin, 8th Edition, Tata McGraw Hill.
- http://www.electronics-tutorials.ws/dccircuits/dcp_8.html
- http://www.electronics-tutorials.ws/dccircuits/dcp_7.html