ECE 113- Basic Electronics

Lecture week 7: Capacitor, Inductor, RL and RC circuits

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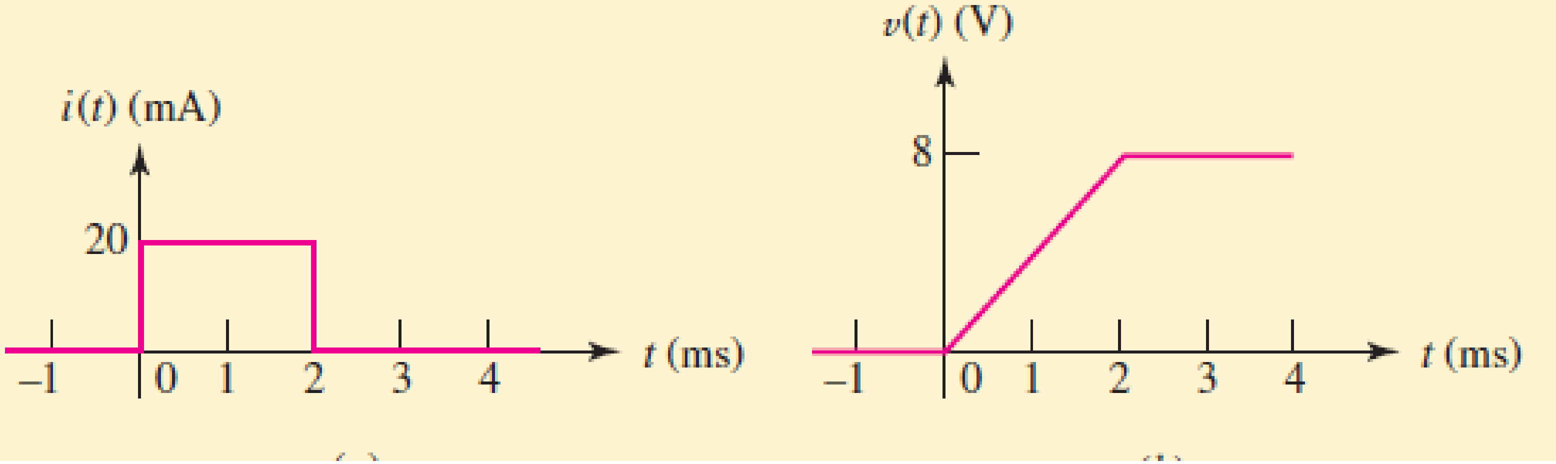
Current-voltage relationship in a capacitor

Current-voltage relationship

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0)}$$

Find the capacitor voltage that is associated with the current shown graphically in Fig. 7.5*a*. The value of the capacitance is 5 μF.



- Power:

$$p = vi = vC\frac{dv}{dt}$$

Stored energy can be calculated by integrating power considering zero initial voltage:

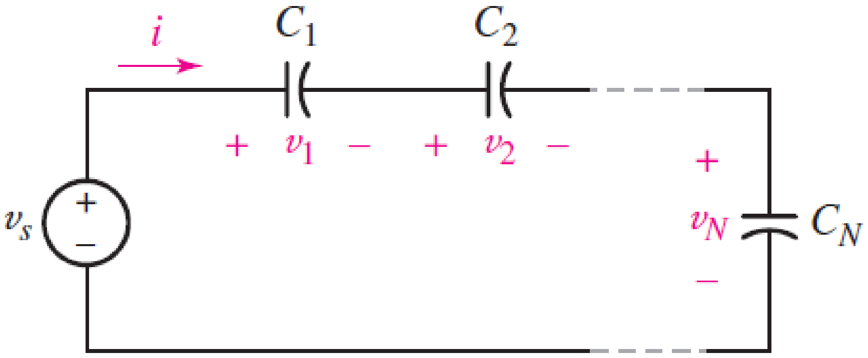
$$w_C(t) = \frac{1}{2}Cv(t)^2$$

Applicability of KCL and KVL

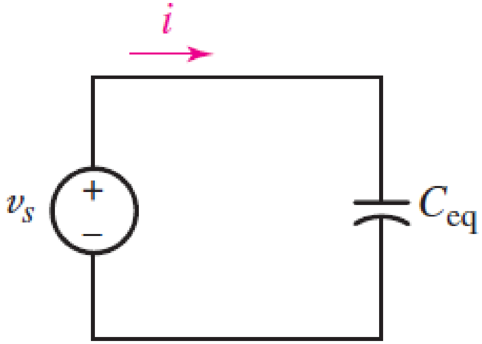
- L and C are linear elements
- Kirchhoff's laws can be applied (are not restricted to only resistance)

Capacitors in series

Same current flows through all capacitor, so same amount of charge will accumulate in each capacitor



(a)



(b)

- To see the series formula, consider the individual voltages across each capacitor

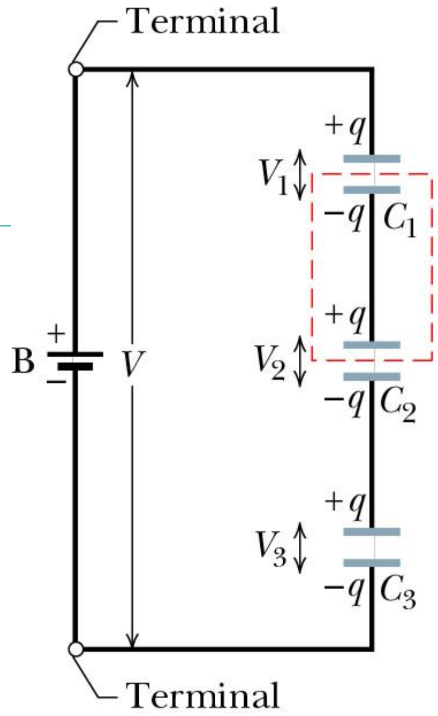
$$V_1 = \frac{q}{C_1},\ V_2 = \frac{q}{C_2},\ V_3 = \frac{q}{C_3}$$

- The sum of these voltages is the total voltage of the battery, V

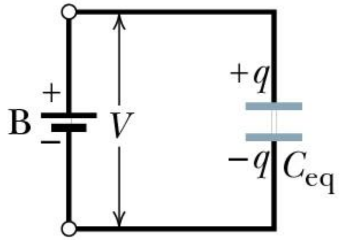
$$V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

- Since $V/q = 1/C_{eq}$, we have

$$\frac{V}{q} = \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



(a)

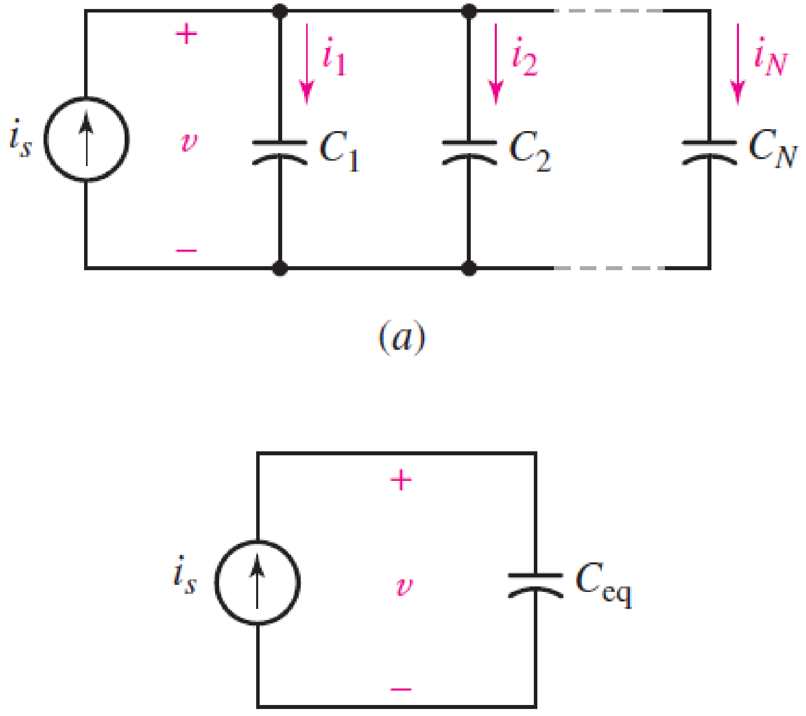


(b)

Capacitors in series: $\displaystyle \frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$

Capacitors in Parallel

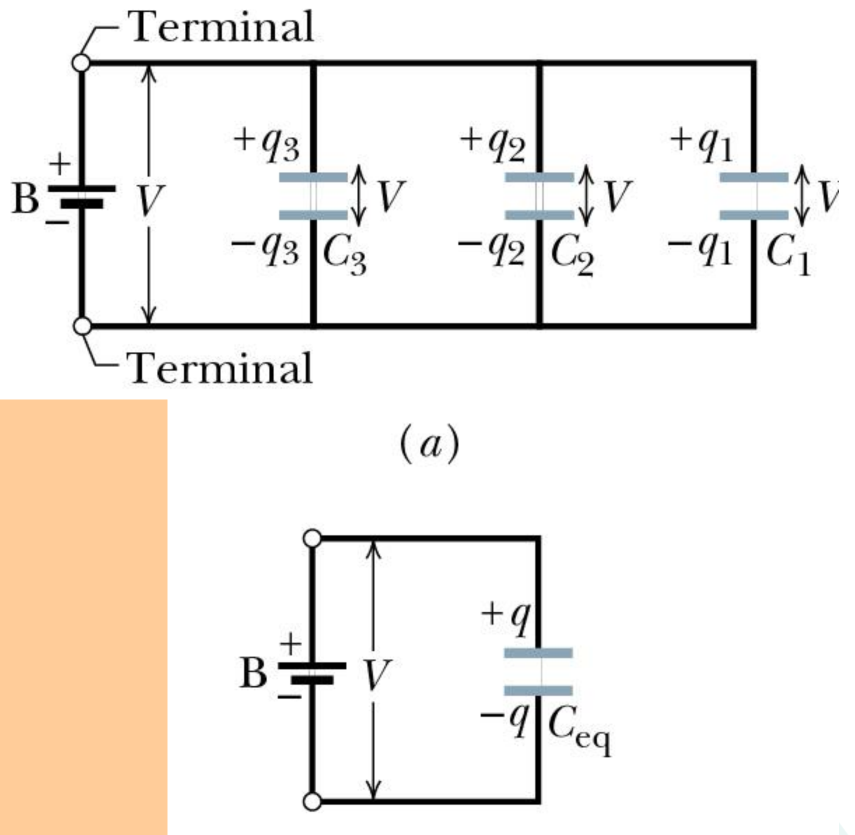
Current is divided among the parallel branches. So, different amount of charge will accumulate in different capacitor but the voltage is same.



(a)

(b)

$$V=\frac{q_1}{C_1},\quad V=\frac{q_2}{C_2},\quad V=\frac{q_3}{C_3}$$



(a)

(b)

$$V=\frac{q_1}{C_1},\quad V=\frac{q_2}{C_2},\quad V=\frac{q_3}{C_3}$$

$$\Rightarrow$$

$$q=q_1+q_2+q_3$$

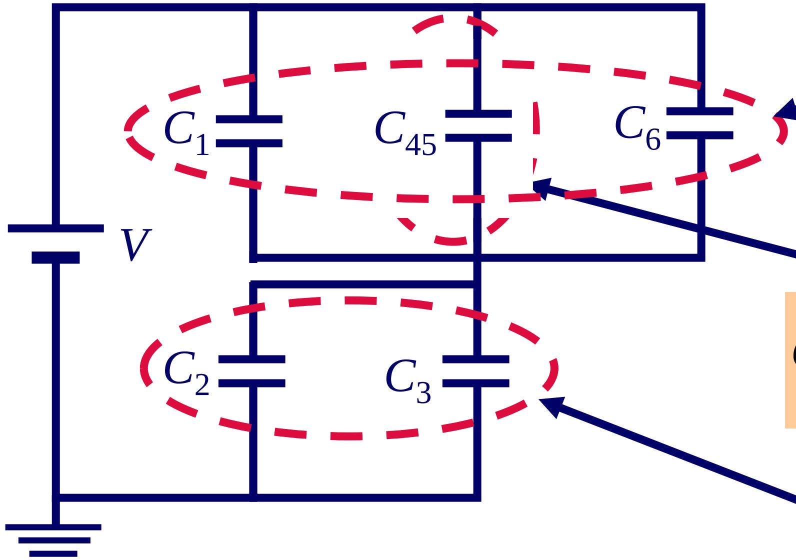
$$=C_1V+C_2V+C_3V=V(C_1+C_2+C_3)$$

$$\Rightarrow$$

$$C_{eq}V=V(C_1+C_2+C_3)\Rightarrow C_{eq}=C_1+C_2+C_3$$

Capacitors in parallel: $C_{eq}=\sum_{j=1}^{n}C_j$

Example Capacitor Circuit



parallel

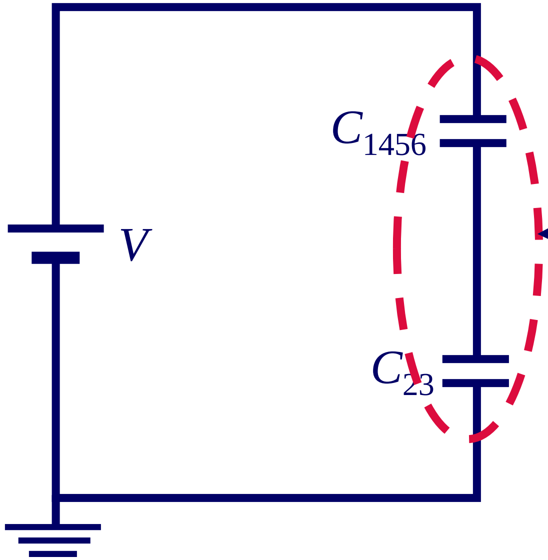
$$C_{1456} = C_1 + C_{45} + C_6$$

series

$$C_{45} = \frac{C_4 C_5}{C_4 + C_5}$$

parallel

$$C_{23} = C_2 + C_3$$



$$C_{45} = \frac{C_4 C_5}{C_4 + C_5}$$

$$C_{1456} = C_1 + C_{45} + C_6$$

$$C_{23} = C_2 + C_3$$

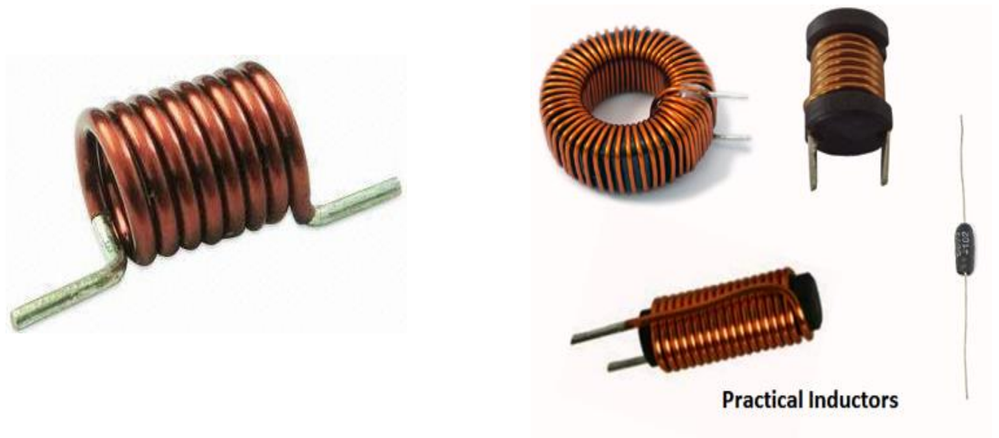
$$C_{123456} = \frac{C_{1456} C_{23}}{C_{1456} + C_{23}}$$

Complete solution

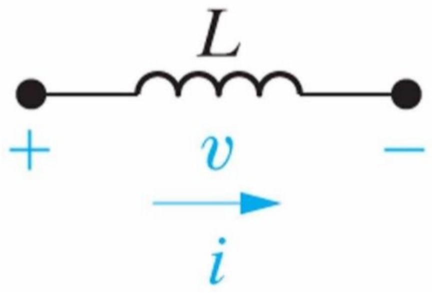
$$C_{123456} = \frac{\left(C_1 + \frac{C_4 C_5}{C_4 + C_5} + C_6\right)(C_2 + C_3)}{C_1 + \frac{C_4 C_5}{C_4 + C_5} + C_6 + C_2 + C_3}$$

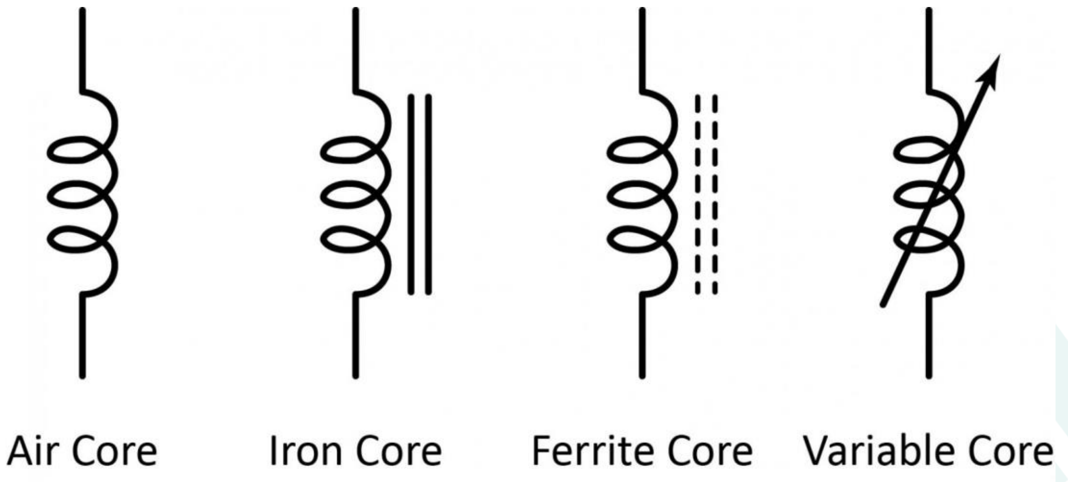
Inductor

The behavior of **inductors** is based on magnetic fields where the source of magnetic field is current. If the current is varying with time, <u>the magnetic field is varying with time which induces a voltage in the conductor.</u>



Inductance is symbolized by “L”





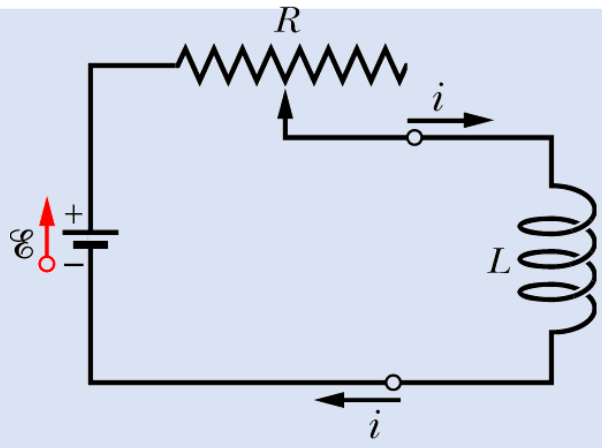
Inductor Symbols

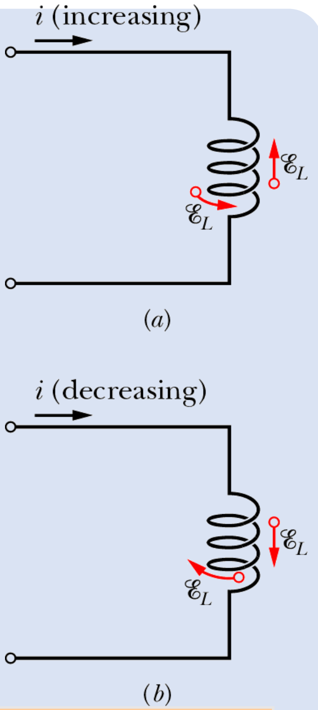
Induction and Inductance

Magnetic Induction

- Faraday's Law
  - "A changing magnetic flux induces an electromotive force (emf)" $\varepsilon = -\dfrac{\Delta\Phi_B}{\Delta t}$
- Lenz's Law
  - "the magnetic field produced by an induced current always opposes any changes in the magnetic flux"

- When we try to run a current through a coil of wire, the *changing* current induces a "back-EMF" that opposes the current.
- That is because the changing current creates a changing magnetic field, and the increasing magnetic flux through the coils of wire induce an opposing EMF.
- We seek a description of this that depends only on the geometry of the coils (i.e., independent of the current through the coil).
- We call this the inductance (c.f. capacitance). It describes the proportionality between the current through a coil and the magnetic flux induced in it.





$$L = \frac{N\Phi_B}{i}$$

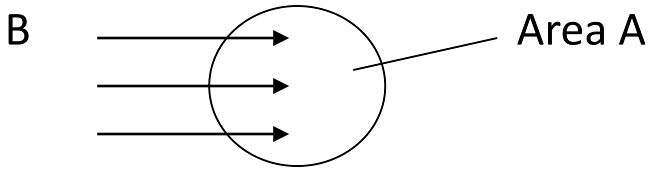
Inductance

Inductance units: henry (H), 1 H = 1 T-m^2/A

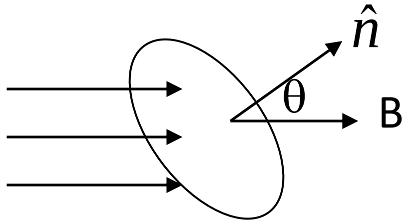
Magnetic flux

<u>A reminder in how to find the Magnetic flux across an area</u>

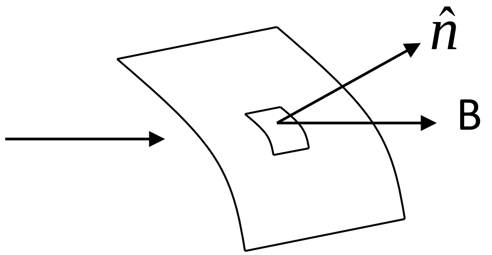
- $\phi_m = BA$



- $\phi_m = \vec{B}\cdot\hat{n}dA$
  $= \vec{B}\cdot d\vec{A}$
  $= B\cos\theta dA$



- $\phi_m = \int \vec{B}\cdot\hat{n}dA$



- We need a way to calculate the *amount of magnetic field* that passes through a loop.
- Similar to the definition of electric flux, we define a magnetic flux $\Phi_B = \int \vec{B}\cdot d\vec{A}$
- Magnetic flux is a scalar.
- In uniform magnetic field, the magnetic flux can be expressed as $\Phi_B = BA\cos\theta$
- SI unit is the weber (Wb):

  1 weber = 1 Wb = 1 T m^2

Self-inductance

- Induced emf acts like a battery pushing current in the opposite direction. This phenomenon is called ***self-inductance*** (or inductance)
  - The current changing through a coil induces a current in the *same* coil.
  - The induced current opposes the original applied current, from Lenz's Law.
  - The circuit element that uses self-inductance is called an ***inductor***
  - Energy is stored in the magnetic field of an inductor.
  - There is an energy density associated with the magnetic field.

From now, own "inductance" means self-inductance

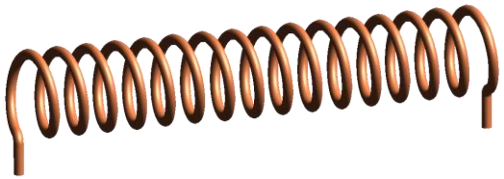
- Since for any inductor $L = \frac{N\Phi_B}{i}$ then $iL = N\Phi_B$, $L\frac{di}{dt} = N\frac{d\Phi_B}{dt}$
- But Faraday's Law says $\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$

The self-induced EMF is opposite to the direction of change of current

•Mutual induction -- An emf is induced in a coil as a result of a changing magnetic flux produced by a second coil.

Inductance of a Solenoid

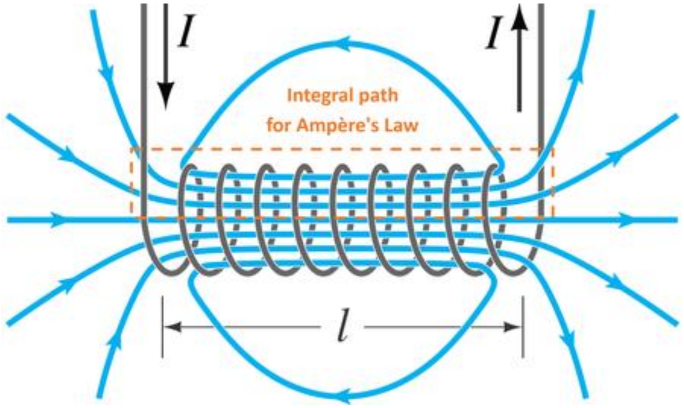
- Consider a solenoid. Applying Ampere's circuital law to the solenoid, the magnetic field inside a solenoid is



$$Bl = \mu_0 iN \Rightarrow B = \mu_0 in$$

number of turns per unit length $n = N/l$.

- The magnetic flux through the solenoid is then $\Phi_B = \int B \cdot dA = \mu_0 inA$



- The inductance of the solenoid is then: $L = \frac{N\Phi_B}{i} = \frac{N\mu_0 inA}{i} = nl\mu_0 nA = \mu_0 n^2 lA$

- Note that this depends only on the geometry. Since $N = nl$, this can also be written $L = \frac{\mu_0 N^2 A}{l}$ Compare with capacitance of a capacitor $C = \frac{\varepsilon_0 A}{l}$

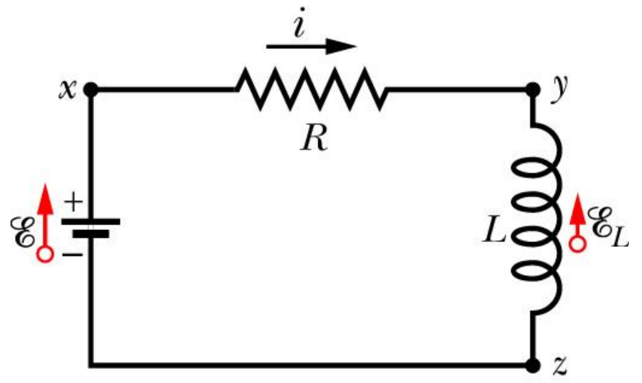
Ampere's circuital law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

Can also write $\mu_0 = 4\pi \times 10^{-7}$ H/m = 1.257 μH/m

Compare with $\varepsilon_0 = 8.85$ pF/m

Energy Stored in Magnetic Field



- By Kirchoff's Loop Rule, we have $\mathcal{E} = iR + L\dfrac{di}{dt}$

- We can find the power in the circuit by multiplying by *i*.

$$\mathcal{E}i = i^2R + Li\frac{di}{dt}$$

power stored in magnetic field

power provided by battery

power dissipated in resistor

- Power is rate that work is done, i.e. $P = \dfrac{dU_B}{dt} = Li\dfrac{di}{dt}$

- So $dU_B = Li\,di$, or after integration $U_B = \dfrac{1}{2}Li^2$ Energy in magnetic field

Recall for electrical energy in a capacitor: $U_E = \dfrac{q^2}{2C} = \dfrac{1}{2}CV^2$

Magnetic energy density, u_B is $u_B = \dfrac{U_B}{V} = \dfrac{B^2}{2\mu_0}$

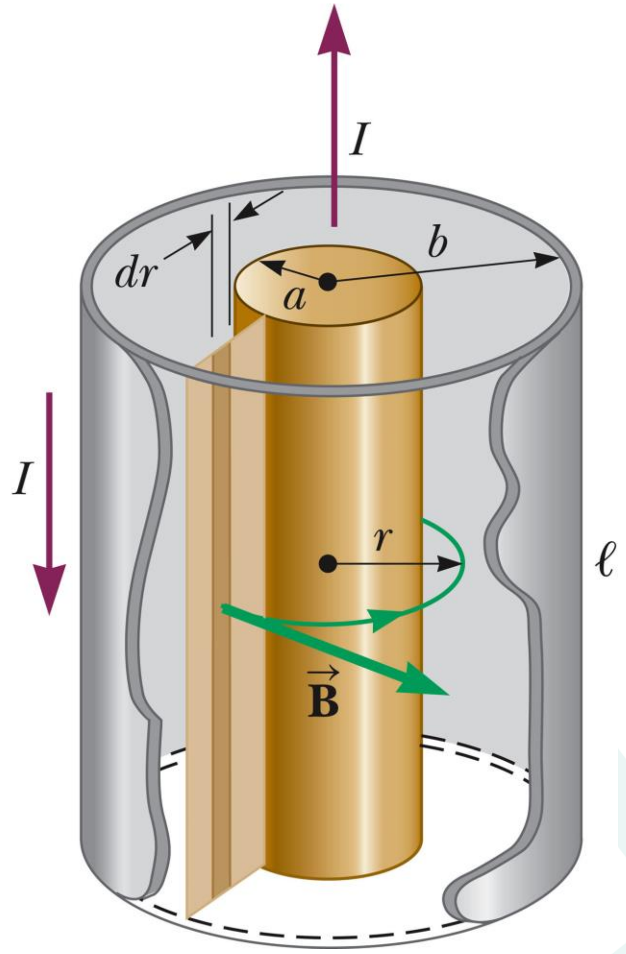
Inductance of a Coaxial Cable

- Calculate L of a length ℓ for the cable
- The total flux is

$$\Phi_B = \int B\, dA = \int_a^b \frac{\mu_o I}{2\pi r} \ell\, dr$$

$$= \frac{\mu_o I \ell}{2\pi} \ln\left(\frac{b}{a}\right)$$

- Therefore, L is $L = \frac{\Phi_B}{I} = \frac{\mu_o \ell}{2\pi} \ln\left(\frac{b}{a}\right)$



Inductors with magnetic core

- Most practical inductors are made by wrapping a wire coil around a magnetic material
- The magnetic material increases the inductance
  - Magnetic material has greater permeability than free space

$$\mu_o \to \mu, \; \mu > \mu_o$$

- Most inductors contain a material that produces a larger L in a smaller package

Material	relative permeability μ/μ0
Iron (99.95% pure Fe annealed in H)	200000
Ferritic stainless steel (annealed)	1000 – 1800
Ferrite	350 - 500
Ferrite (cobalt nickel zinc)	40 – 125
Carbon steel	100
Austenitic stainless steel	1.003 – 1.05
Neodymium magnet	1.05
Platinum	1.000265
Aluminum	1.000022
Concrete (dry)	1
Vacuum	1
Hydrogen	1.0000000
Water	0.999992
Bismuth	0.999834

Inductor

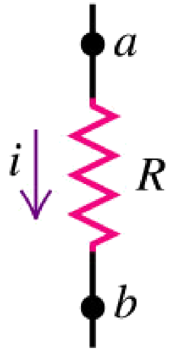
Voltage-current relationship

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt' + i(t_0)}$$

Power $p = vi = Li\,\frac{di}{dt}$

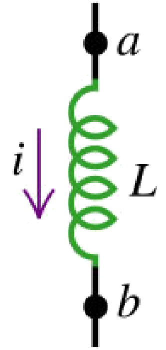
Energy (zero initial current):

$$w_L(t) = \frac{1}{2}Li(t)^2$$



$V_{ab} = iR$

(a) Resistor with current i flowing from a to b: potential drops from a to b



$V_{ab} = L\frac{di}{dt}$

(b) Inductor with current i flowing from a to b:

- If $di/dt > 0$: potential drops from a to b
- If $di/dt < 0$: potential increases from a to b
- If i is constant ($di/dt = 0$): no potential difference

Potential difference across a resistor depends on the current

Potential difference across an inductor depends on the **rate of change** of the current

Inductor

Given the waveform of the current in a 3 H inductor as shown in Fig. 7.12*a*, determine the inductor voltage and sketch it.

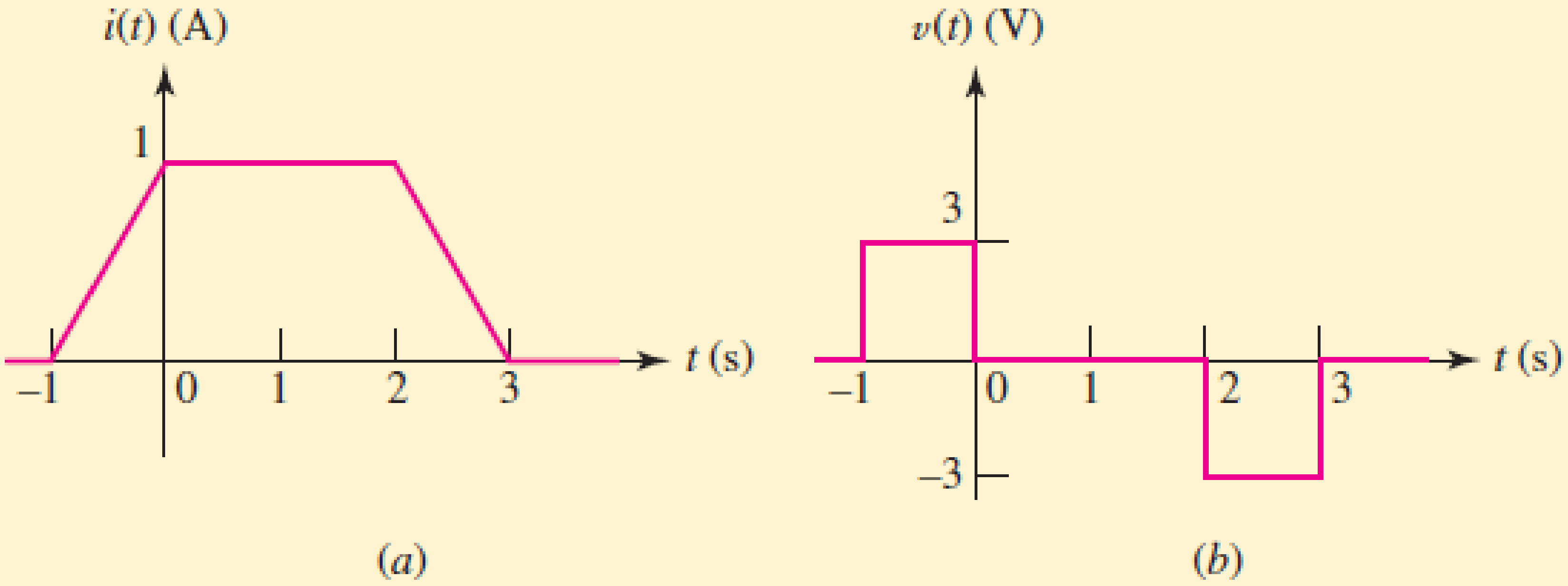
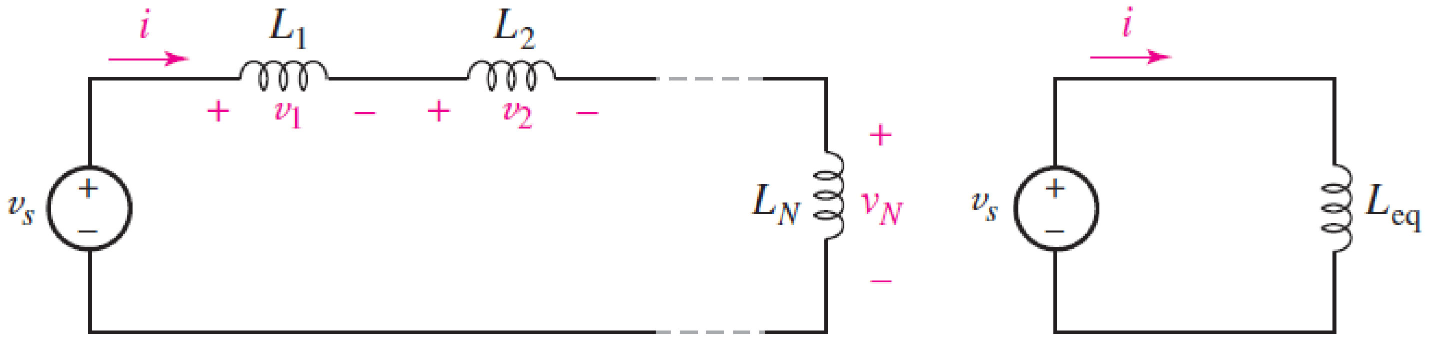


FIGURE 7.12 (*a*) The current waveform in a 3 H inductor. (*b*) The corresponding voltage waveform, $v = 3\, di/dt$.

Inductors in series



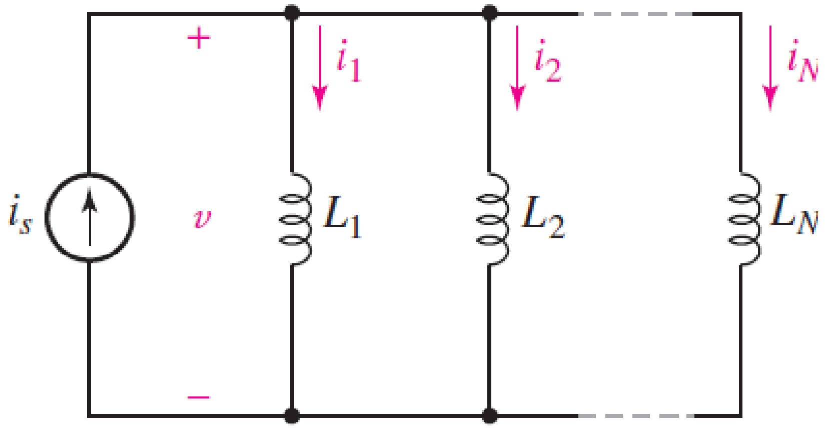
(a) (b)

$$v_1 = L_1\frac{di}{dt}, \qquad v_2 = L_2\frac{di}{dt}, \qquad \text{and} \qquad v_3 = L_3\frac{di}{dt}.$$

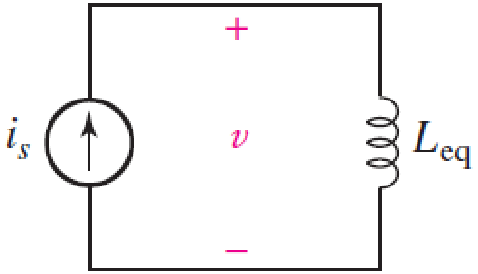
$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di}{dt},$$

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_n.$$

Inductors in parallel



(a)



(b)

$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v\,d\tau + i_1(t_0),$$

$$i_2 = \frac{1}{L_2}\int_{t_0}^{t} v\,d\tau + i_2(t_0),$$

$$i_3 = \frac{1}{L_3}\int_{t_0}^{t} v\,d\tau + i_3(t_0).$$

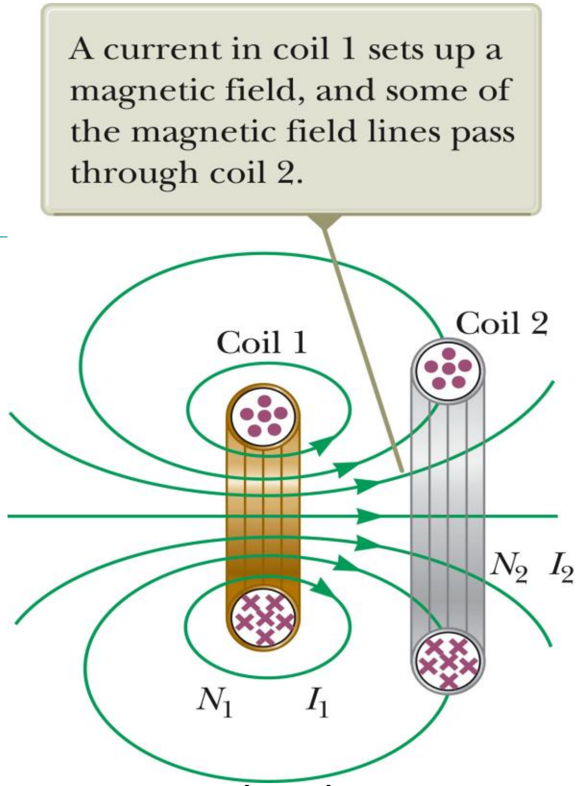
$$i = i_1 + i_2 + i_3.$$

$$i = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,d\tau + i(t_0).$$

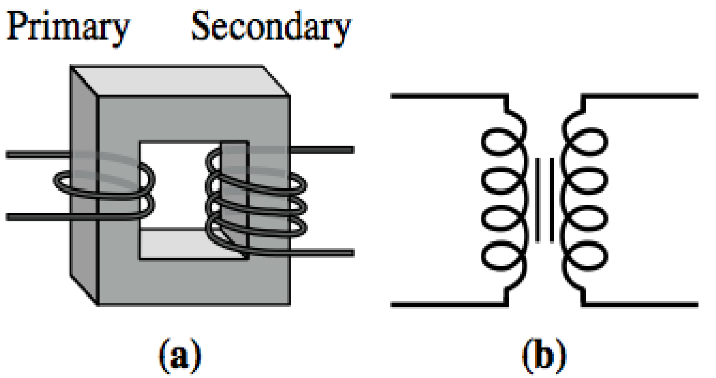
$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} v\,d\tau + i_1(t_0) + i_2(t_0) + i_3(t_0).$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Mutual Inductance

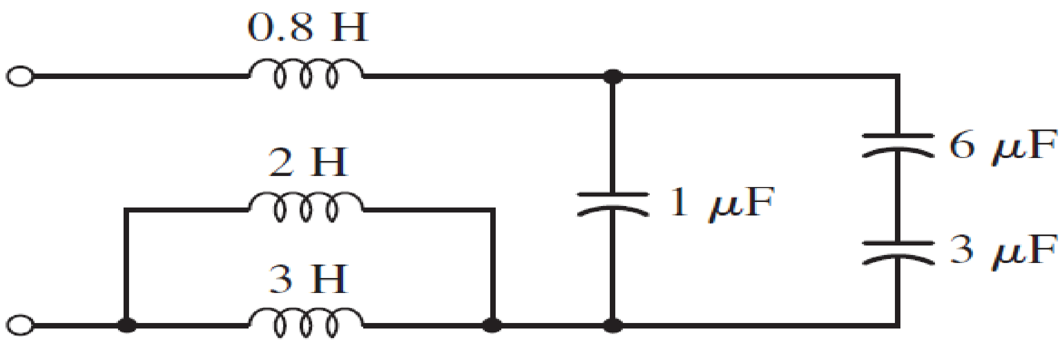


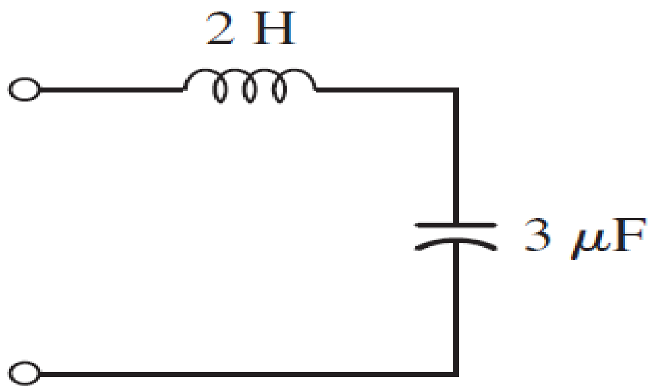
- The magnetic flux through the area enclosed by a circuit often varies with time because of time-varying currents in nearby circuits.
- This process is known as *mutual induction* because it depends on the interaction of two circuits.
  - The current in coil 1 sets up a magnetic field. Some of the magnetic field lines pass through coil 2.
  - Coil 1 has a current I_1 and N_1 turns. Coil 2 has N_2 turns.
- The **mutual inductance** M_{12} of coil 2 with respect to coil 1 is $M_{12} \equiv \dfrac{N_2 \Phi_{12}}{I_1}$
- Mutual inductance depends on the geometry of both circuits and on their orientation with respect to each other.
- If current I_1 varies with time, the emf induced by coil 1 in coil 2 is $\varepsilon_2 = -N_2 \dfrac{d\Phi_{12}}{dt} = -M_{12} \dfrac{dI_1}{dt}$
- If the current is in coil 2, there is a mutual inductance M_{21}.
- If current 2 varies with time, the emf induced by coil 2 in coil 1 is $\varepsilon_1 = -M_{21} \dfrac{dI_2}{dt}$
- In mutual induction, the emf induced in one coil is always proportional to the rate at which the current in the other coil is changing. The mutual inductance in one coil is equal to the mutual inductance in the other coil, i.e. $M_{12} = M_{21} = M$; The induced emf's can be expressed as $\varepsilon_1 = -M \dfrac{dI_2}{dt}$ and $\varepsilon_2 = -M \dfrac{dI_1}{dt}$



Example 7.8

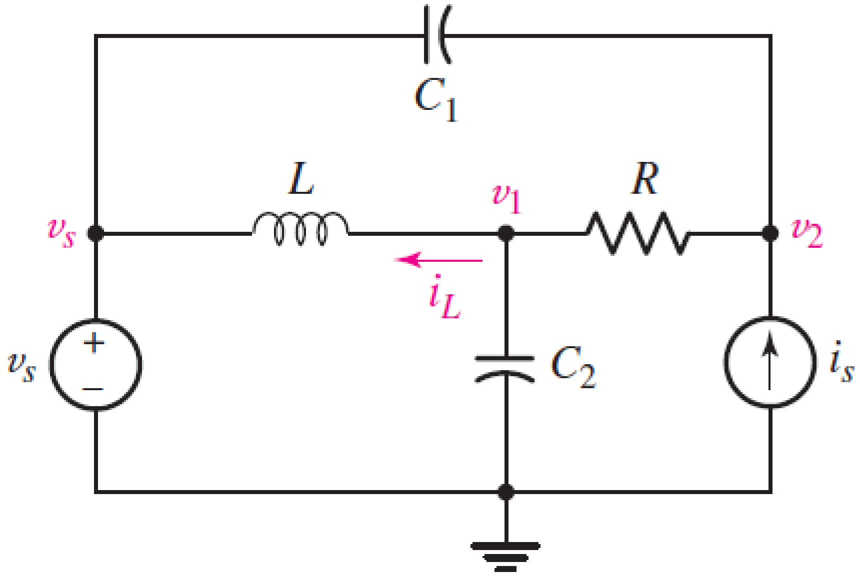
- Simplify the network



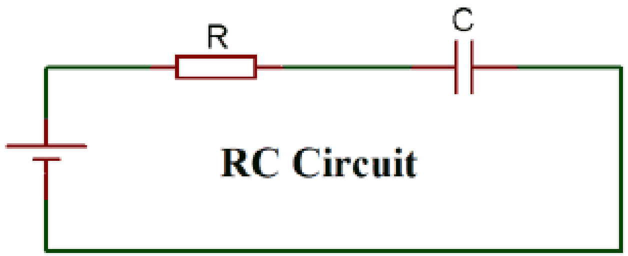


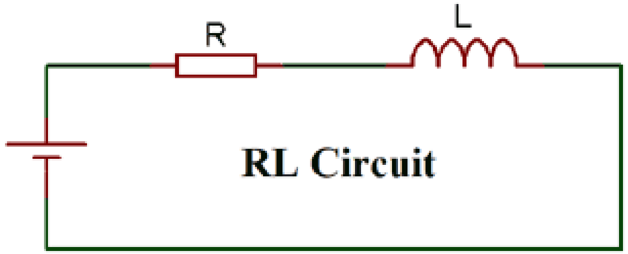
Nodal equations for RLC circuit- Example

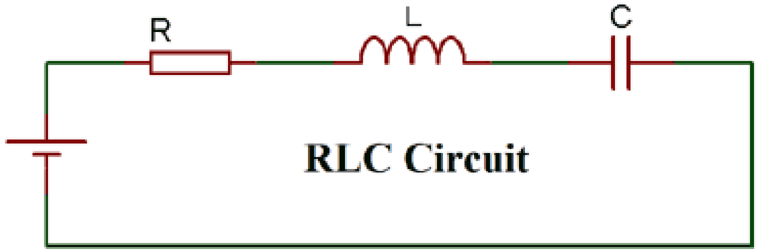
Example 7.9: Write appropriate nodal equations for the circuit



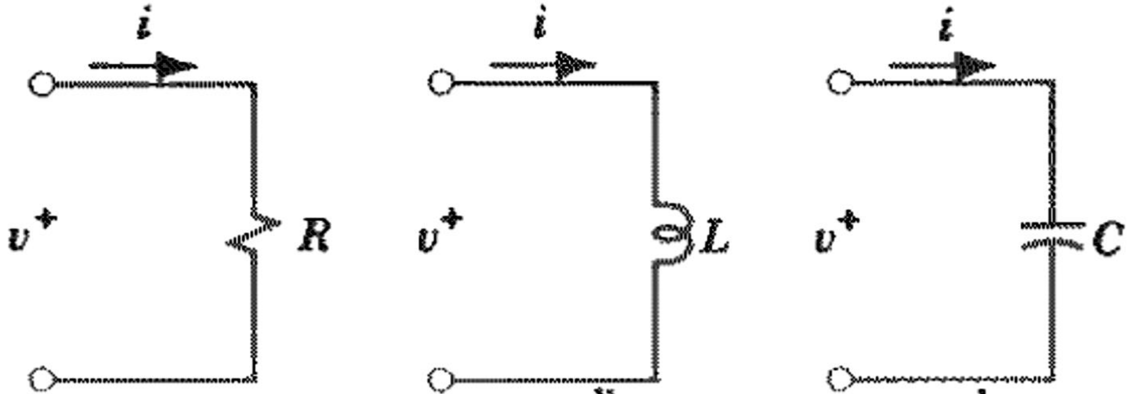
RC, RL, RLC circuits examples







Circuit Elements



Resistance	Inductance	Capacitance
$v = Ri$	$v = L\dfrac{di}{dt}$	$i = C\dfrac{dv}{dt}$
$i = \dfrac{v}{R} = Gv$	$i = \dfrac{1}{L}\int_{-\infty}^{t} v\,dt$	$v = \dfrac{1}{C}\int_{-\infty}^{t} i\,dt$

Response of a circuit

- In predicting the behavior of electrical circuit with L and C, we must take into account two different sources of energy:
    - External energy source known as forced response and
    - Internal energy source known as natural response
- External energy source in an electrical circuit may be a DC voltage, step input, pulse input, sinusoidal input etc.
- Forced response can be maintained indefinitely by continuously applying the input signal, where as natural response tends to die out.
- The natural response will reveal the characteristics of the system.
- After the natural response has become negligibly small, conditions are said to have reached the steady state.
- For L and C, we will express the circuit's behavior using differential equation. This is obtained by applying KCL and KVL and the equations are are homogeneous linear differential equations.
- Solution of this type of equation is voltage or current as a function of time (in electrical circuit context)
- The solution is also called as **response** of a circuit
- In few lectures, we will study the behavior of RC, RL and RLC for non sinusoidal inputs.

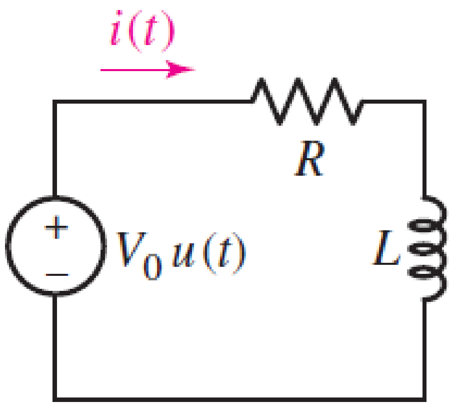
First Order Circuits

- A first order circuit is characterized by a first order differential equation.
- There are two types of first order circuits:
  - Resistive capacitive, called RC
  - Resistive inductive, called RL

Combination of resistive elements and a capacitor or inductor

- There are also two ways to excite the circuits:
  - Initial conditions
  - Independent sources

Key Words:
Transient Response of Circuits, Time constant



A driven RL circuit

$$L\frac{di}{dt} + Ri = V_0 u(t)$$

First order Circuits

- General form for first order, (linear) ordinary differential equation:

$$\frac{dx(t)}{dt} + ax(t) = f(t)$$

- x(t) is a circuit quantity (voltage, current)
- a is a constant, some function of the circuit elements
- $f(t)$ is a forcing function, usually the source voltage or current

- **Solution process**:
  - Find the solution to the homogeneous equation
    - The solution is called the natural response (independent of the source applied)
    - A general solution has the form $x(t) = Ae^{-at}$
    - Often we write $x_n(t) = Ae^{-at}$
  - Look for a solution to the forced response
    - assume a forced response solution of the form $x_f(t)$
  - The complete solution is $x(t) = x_n(t) + x_f(t)$
  - Use initial conditions (i.e. x(0)) to determine constants

Complete response - General Approach

$$\frac{dx(t)}{dt} + Px(t) = Q$$

- For constant P and Q

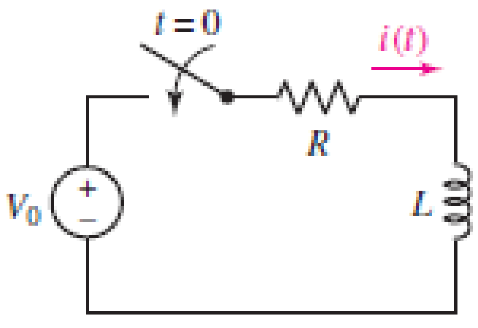
$$x(t) = \frac{Q}{P} + Ae^{-Pt}$$

Forced response

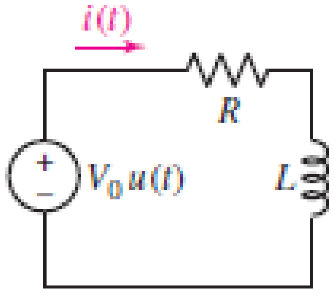
Natural response

At t → ∞, forced response is basically the <u>steady state response</u>

Intuitive understanding of responses



(a)



(b)

$$L\frac{di}{dt} + Ri = V_0 u(t)$$

Soln. $\Rightarrow i = \frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t}$

- Consider the previous forced RL circuit
- The circuit will eventually assume the forced response
- That means, After the natural response has died out, there can be no voltage across the inductor. Hence,

$$i_f = \frac{V_0}{R}$$

$$i = Ae^{-\frac{R}{L}t} + \frac{V_0}{R}$$

- The current is zero prior to t = 0, and it cannot change value instantaneously since it is the current flowing through an inductor. Thus, the current is zero immediately after t = 0

$$0 = A + \frac{V_0}{R}$$

$$i = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Transient and steady state response

- The **complete response** can be written as:

$$v = v_n + v_f$$

- Where the nature response is v_n and forced response is v_f
- **natural response** is v_n $\quad v_n \sim e^{-t/\tau}$
- And the **forced response** is: $v_f = v - v_n$
- Note that the eventual response of the circuit is to reach $\boldsymbol{V_s}$ after the natural response decays to zero.

Another Perspective of response – transient and steady state response

- Another way to look at the response is to break it up into the transient response and the steady state response:

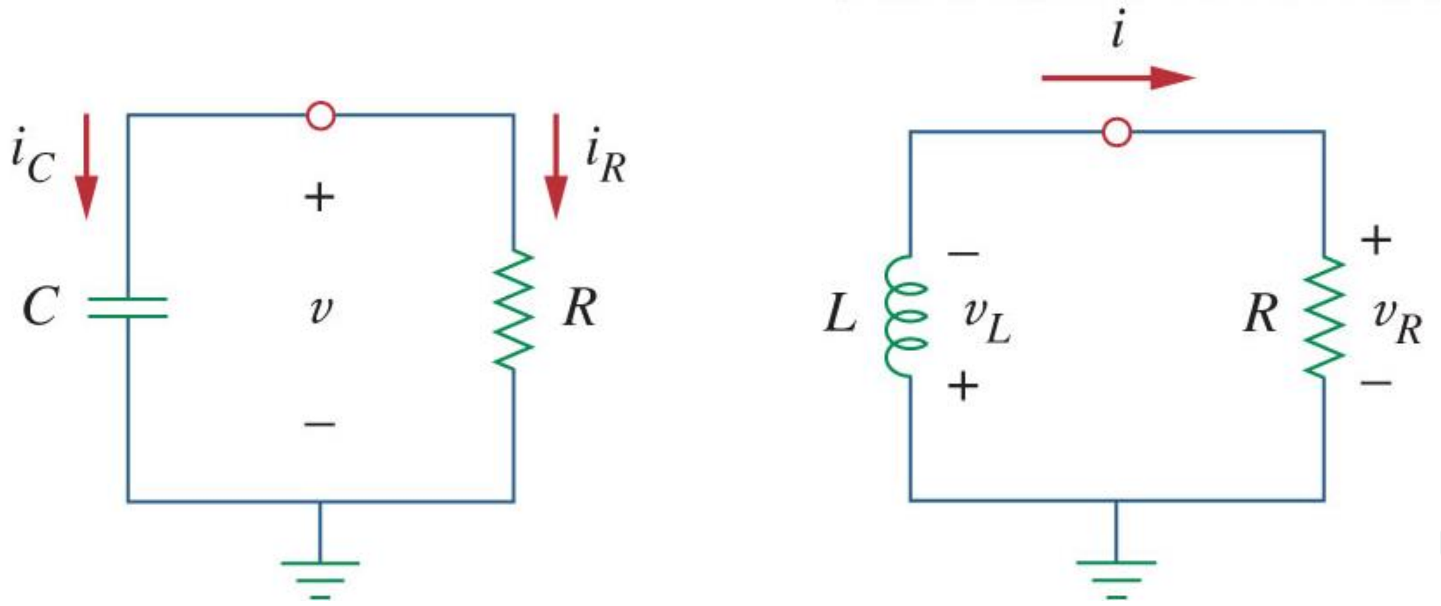
$$v = v_t + v_s$$

- Where the **transient response** is: $v_t = v - v_s$
- And the **steady state response** is: $v_s = V_s$

Therefore, transient response is equivalent to the natural response, as t → ∞, forced response is basically the steady state response

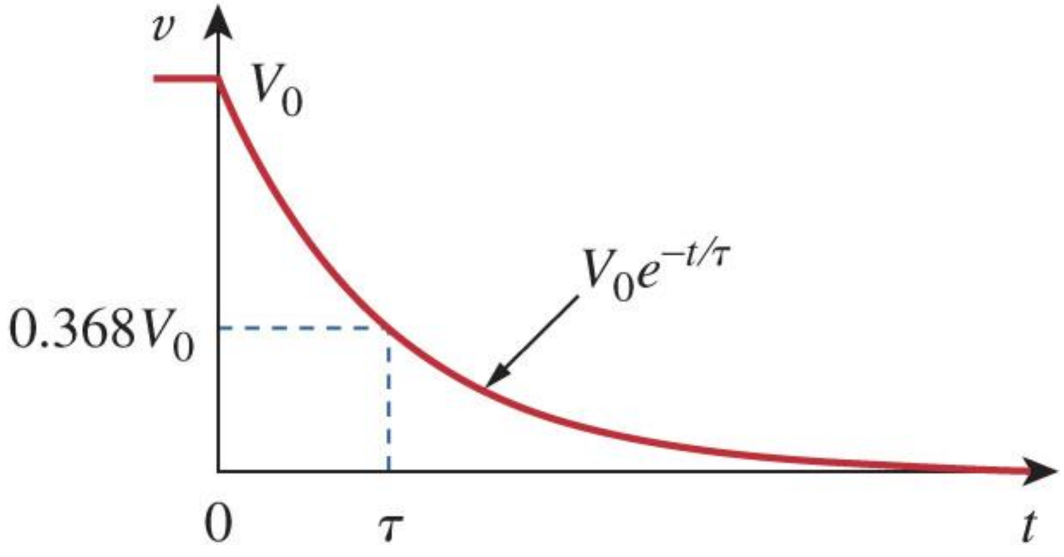
Source Free Circuit (natural response/discharging)

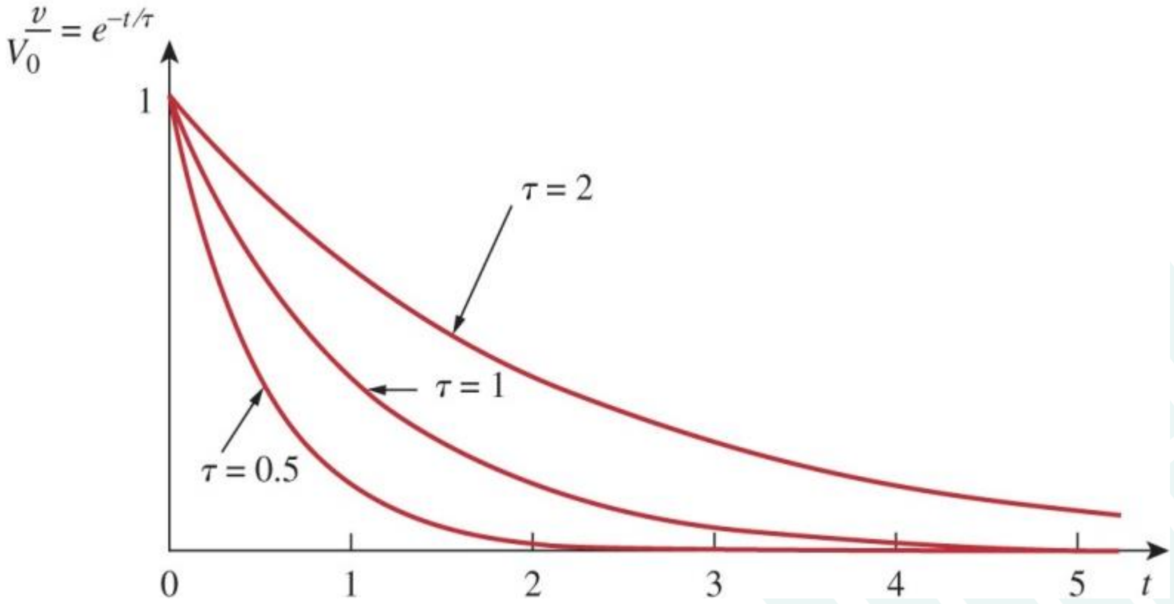
- A source free RC/RL circuit occurs when its dc source is suddenly disconnected.
- The **energy stored in the capacitor/inductor is released to the resistors**.
- Consider a series combination of a resistor and a initially charged capacitor or inductor as shown below:
- we will start with an initial condition (like fully charged capacitor or the steady state current of inductor circuit)



Natural/transient Response; Time Constant

- The result shows that the voltage response of the circuit is an exponential decay of the initial voltage.
- Since this is the response of the circuit without any external applied voltage or current, the response is called the natural response.
- **The speed at which the voltage decays (or increases during charging) can be characterized by how long it takes the voltage to drop (or gain during charging) to *1/e* of the initial voltage.**
- **This is called the time constant and is represented by τ.**
- After five time constants the voltage on the capacitor is less than one percent.
- After five time constants a capacitor is considered to be either fully discharged or charged
- A circuit with a small time constant has a fast response and vice versa.



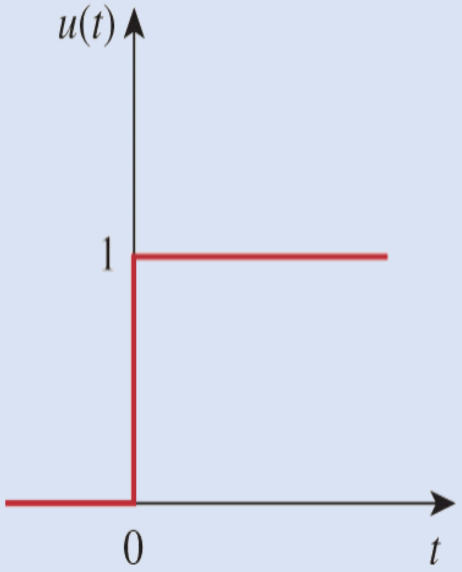


Singularity Functions

- Before we consider the response of a circuit to an external voltage, we need to cover some important mathematical functions.
- Singularity functions serve as good approximations to switching on or off a voltage.
- The three most common singularity functions are the <u>unit step</u>, <u>unit impulse</u>, and <u>unit ramp</u>.

The Unit Step

- A step function is one that maintains a constant value before a certain time and then changes to another constant afterwards.
- The prototypical form is zero before *t=0* and one afterwards.
- See the graph for an illustration.



- Mathematically, the unit step is expressed as:

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

- The switching time may be shifted to $t=t_0$ by:

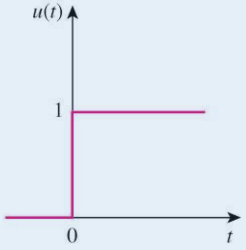
$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

- Note that this results in a *delay* in the switch.
- The unit step function is written as *u(t)*

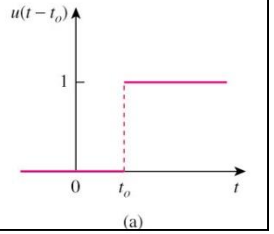
The Unit Step and Equivalent Circuit

- The unit step function $u(t)$ is 0 for negative values of t and 1 for positive values of t.

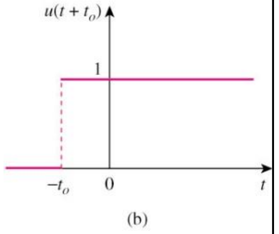
$$u(t)=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$$



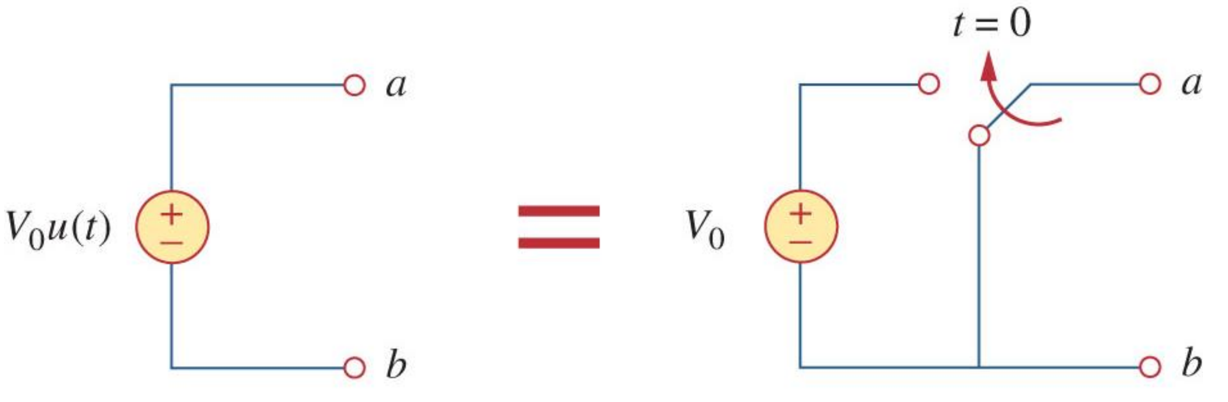
$$u(t-t_o)=\begin{cases}0, & t<t_o\\ 1, & t>t_o\end{cases}$$



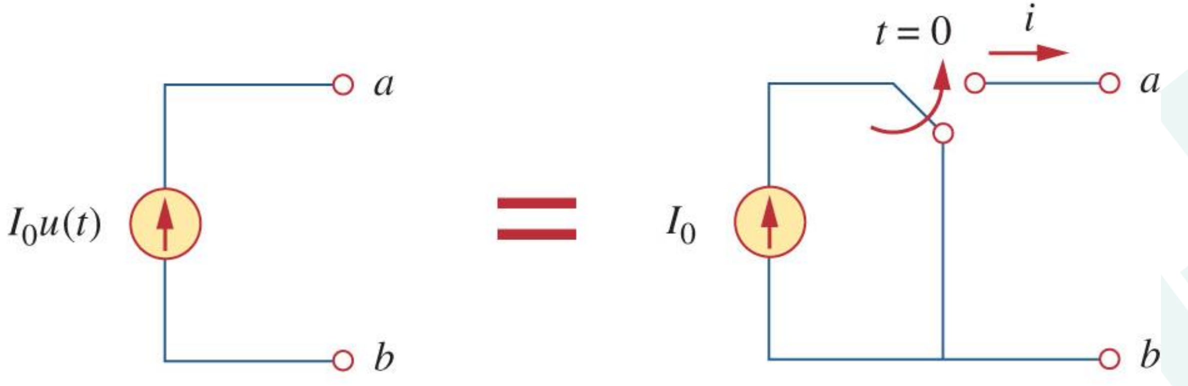
$$u(t+t_o)=\begin{cases}0, & t<-t_o\\ 1, & t>-t_o\end{cases}$$



- The unit step function has an equivalent circuit to represent when it is used to switch on a source.
- The equivalent circuits for a voltage and current source are shown.



(a) (b)



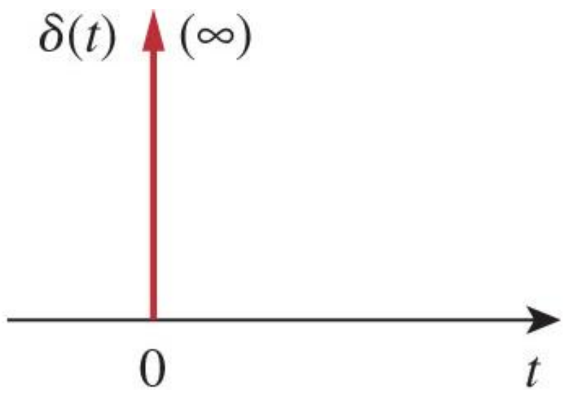
(a) (b)

The Unit Impulse and Unit Ramp Function

- **The derivative of the unit step function is the unit impulse function.**
- This is expressed as:

$$\delta(t)=\begin{cases} 0 & t<0 \\ \text{Undefined} & t=0 \\ 0 & t>0 \end{cases}$$

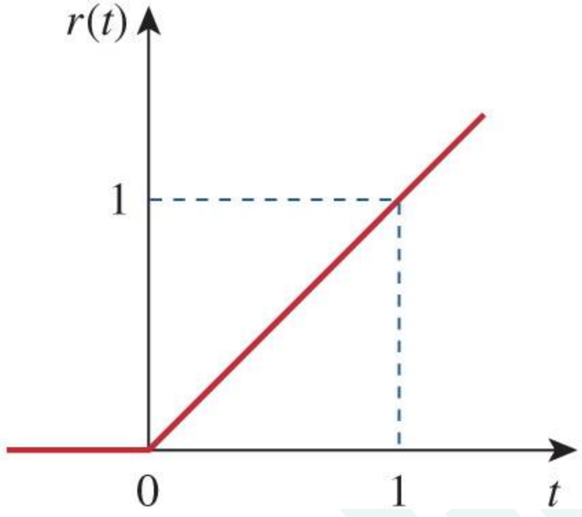
- Voltages of this form can occur during switching operations.



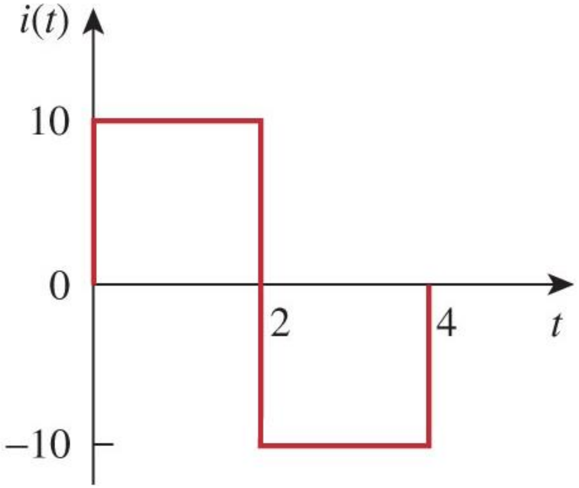
- **Integration of the unit step function results in the unit ramp function**:

$$r(t)=\begin{cases} 0, & t\le 0 \\ t, & t\ge 0 \end{cases}$$

- Much like the other functions, the onset of the ramp may be adjusted.



Example of a ramp function



$$i(t) = \begin{cases} 0 & t < 0 \\ 10 & 0 < t < 2 \\ -10 & 2 < t < 4 \end{cases}$$

$$i(t) = 10[u(t) - u(t-2)] - 10[u(t-2) - u(t-4)]$$

$$i(t) = \mathbf{10[u(t) - 2u(t-2) + u(t-4)]\,A}$$

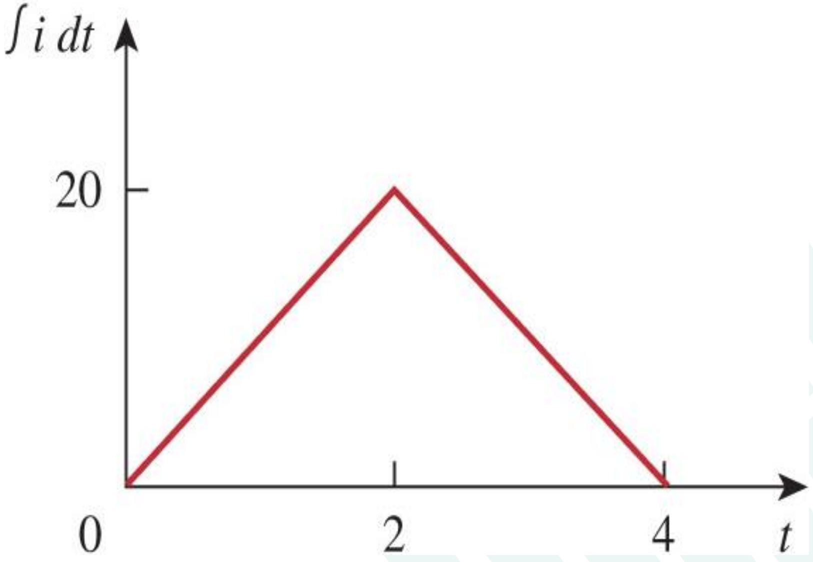
Let $I = \int_{-\infty}^{t} i\,dt$.

For $t < 0$, $\quad I = 0$.

For $0 < t < 2$, $\quad I = \int_0^t 10\,dt = 10t$

For $2 < t < 4$, $\quad I = \int_0^2 10\,dt - 10\int_2^t dt = 20 - 10t\big|_2^t = 40 - 10t$

For $t > 4$, $\quad I = 20 - 10t\big|_2^4 = 0$

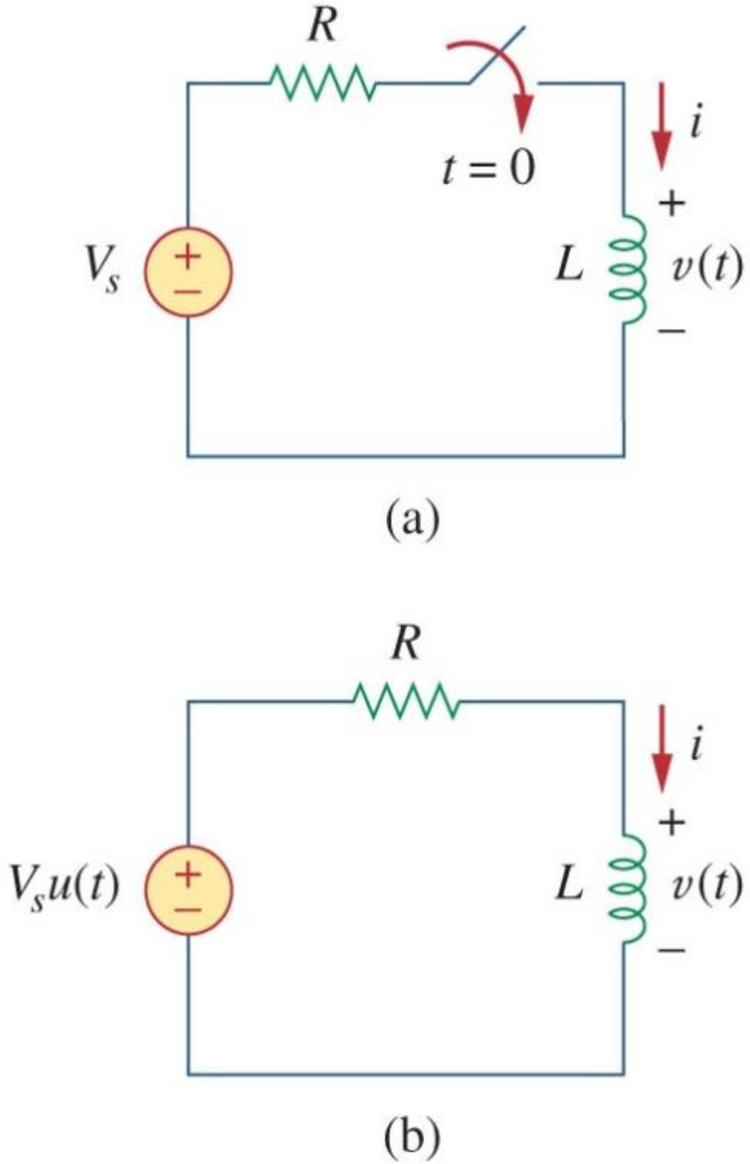


Step Response of series RL/RC Circuit (charging)

- When a <u>DC source is suddenly applied to a RC/RL circuit, the source can be modeled as a **step function**</u>.
- The circuit response is known as the *step response*.
- Let's consider the circuit shown here.
- We will use the <u>transient</u> and <u>steady state response</u> approach.
- We know that the transient response will be an exponential:

$$i_t = Ae^{-t/\tau}$$

- We can find the voltage on the inductor/capacitor/resistor as a function of time.



Series RL circuit (charging/growth of current/forced response)

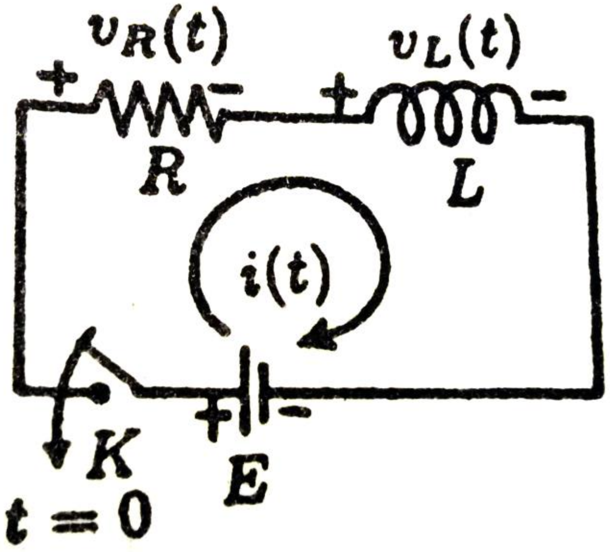


Fig.14.2. A series *RL* circuit

Suppose, the key K is closed at time $t = 0$

Let ***i(t)*** is the current in the circuit at any time 't' after closing the switch K.

So, $$L\frac{di}{dt} + Ri = E \quad \Rightarrow \frac{di}{E - Ri} = \frac{dt}{L}$$

Which upon integration gives, $$\Rightarrow -\frac{1}{R}\ln(E - Ri) = \frac{t}{L} + C_1$$

C_1 : integration const.

At, $t = 0$, $i = 0$. Hence, $$C_1 = -\frac{1}{R}\ln E$$

Therefore, $$\ln\frac{E - Ri}{E} = \frac{-Rt}{L} \quad \Rightarrow 1 - \frac{Ri}{E} = e^{\frac{-R}{L}t} \Rightarrow \boxed{i = \frac{E}{R}(1 - e^{\frac{-R}{L}t})}$$

Variation of current with time

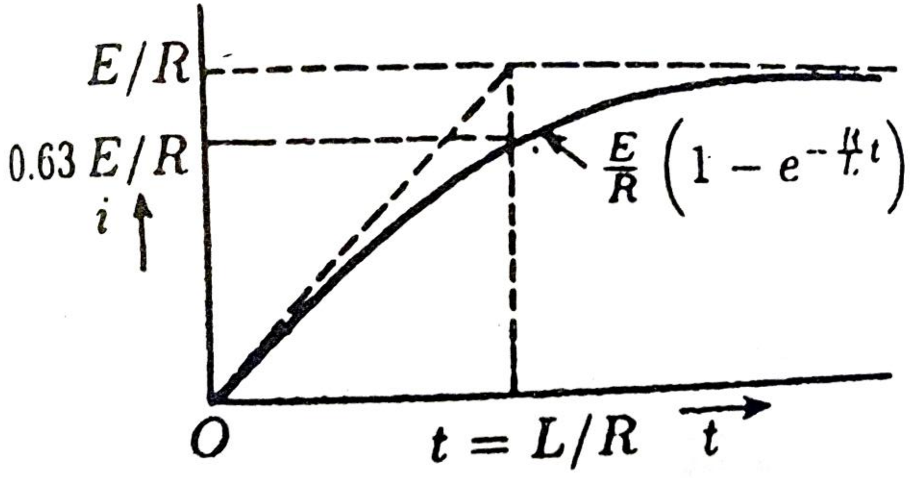
Steady state current (i_s) is obtained for $t \rightarrow \infty$ $$\Rightarrow i_s = \frac{E}{R}$$

The transient current (i_t) is therefore, $$\Rightarrow i_t = i - i_s = -\frac{E}{R}e^{-\frac{R}{L}t}$$

At $t \rightarrow \infty$; $i_t \rightarrow 0$

Contd..

The variation of i with t, i.e., the growth of the current in RL circuit



The quantity L/R has a dimension of time and is called the time-constant of the circuit (denoted by τ)

$$\text{At } t = \tau = L/R, \quad \Rightarrow i = i_s\left(1-\frac{1}{e}\right) = 0.63 i_s$$

- i.e. the time taken for the current to reach 0.63 or 63% of the final value is the time constant for this charging circuit.
- It measures the rapidity with which the final state is approached.
- The greater the L and smaller the R, the larger the time constant, τ, i.e. the longer the time taken to attain the steady state.

The tangent to the eq. of i at $t = 0$ intersects the straight line $i = E/R$ at $t = \tau = L/R$ (see the fig). Thus the time constant can also be defined as the time in which the steady state would be reached if the current was allowed to increase at the initial rate.

Contd..

The time dependence of the voltage drop across the resistance R is given by

$$v_R(t) = Ri(t) = E(1 - e^{-t/\tau})$$

where $\tau = L/R$

The voltage drop across the inductance L is

$$v_L(t) = E - v_R(t) = Ee^{-t/\tau}$$

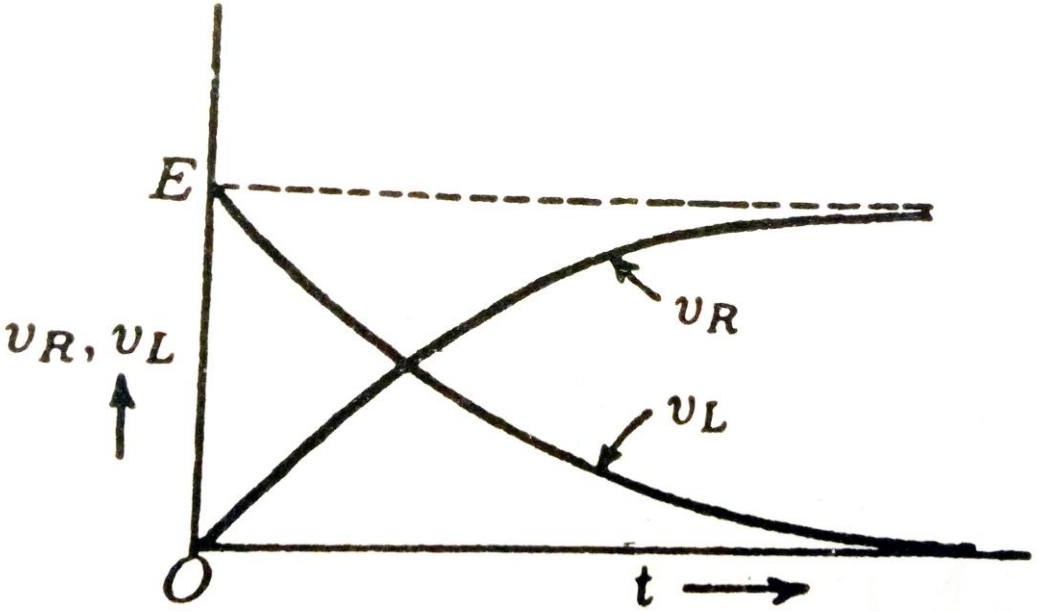


Fig.14.4. Time dependence of v_R and v_L

Discharging/decay of current/natural response

Here, the switch K_2 is initially open and the switch K_1 is initially closed for a sufficient long time so that the steady state has been reached in the series RL circuit. The steady state current is $i_s = E/R$

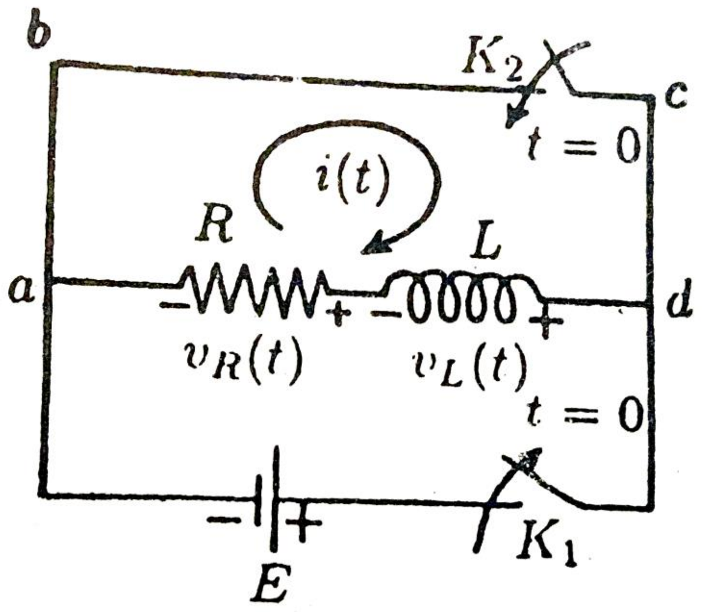


Fig.14.5. Decay of current in a series RL circuit

After the steady state current, at time $t = 0$ the switch K_1 is opened and simultaneously K_2 is closed. In the closed path *abcd,* the driving emf is zero. So, it's the natural response. *Using KVL,*

$$L\frac{di}{dt} + Ri = 0 \qquad \Rightarrow \frac{di}{i} = -\frac{R}{L}dt$$

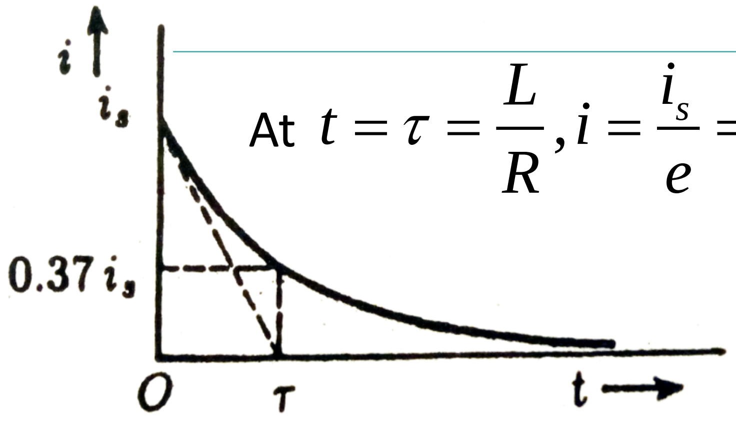
Integration gives, $\Rightarrow \ln i = -\frac{R}{L}t + C_2$ $\qquad C_2$: integration const.

At, $t = 0$, $i = i_s = E/R$. So, $C_2 = \ln i_s = \ln\frac{E}{R}$

$$\therefore \ln\left(\frac{i}{i_s}\right) = -\frac{R}{L}t \Rightarrow \boxed{i = i_s e^{-\frac{R}{L}t} = \frac{E}{R}e^{-\frac{R}{L}t}}$$

Decay of the current in series RL circuit after sudden removal of the emf.

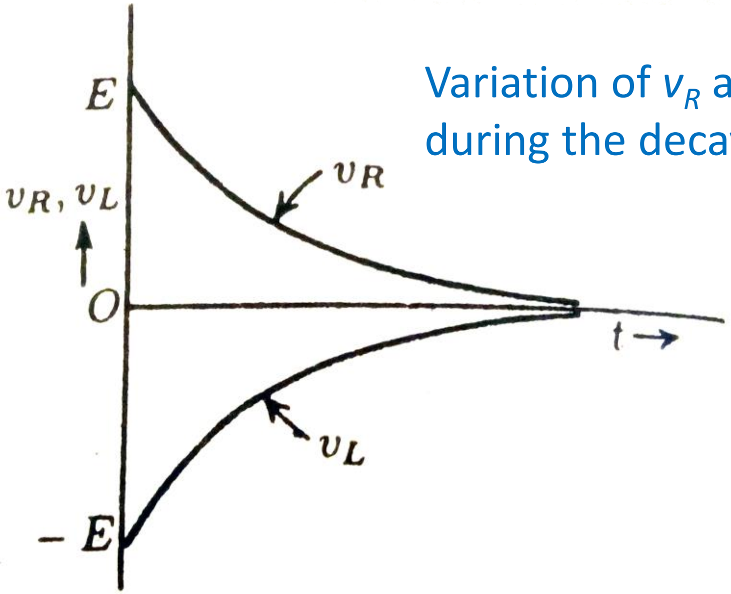
Contd..



At $t = \tau = \dfrac{L}{R}, i = \dfrac{i_s}{e} = 0.37 i_s$

Fig.14.6. Decay of current in *RL* curcuit

- The time constant L/R is the time taken by the current to decay to 0.37 or 37% of its initial value. Again, it is the rapidity of the decay of the current. The smaller the L and greater the R, the quicker the decay of the current.
- The tangent to the eq. of i at $t = 0$ intersects the time axis $t = \tau = L/R$ (see the fig). Thus time constant can also be viewed as the time in which the current would decay to zero, if it was allowed to decrease at the initial rate, i.e., the rate at $t=0$.



Variation of v_R and v_L with t during the decay of current

The time dependence of the voltage drop across the resistance R is given by

$$v_R(t) = Ri(t) = Ee^{-\frac{R}{L}t}$$

The voltage drop across the inductance L is $v_L(t) = -v_R(t) = -Ee^{-\frac{R}{L}t}$

-ve sign in v_L shows that the polarity of v_L is opposite to that assumed in the analysis

Question: If at $t=0$, only switch K_1 is opened and K_2 is not simultaneously closed, then what will happen?

Series RC circuit (charging/growth of charge/forced response)

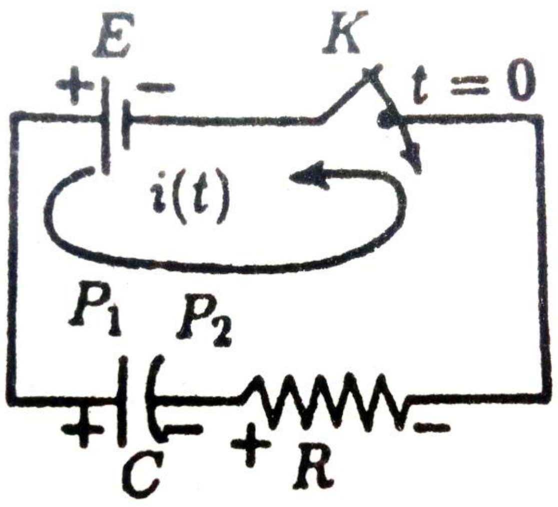


Fig.14.8. Charging of a capacitor through a resistance

The current ***i(t)*** that flows in the circuit is the charging current.

At any instant of time 't' after closing the key K, the voltage across the capacitor is $v_C(t)$ and across the resistance is $v_R(t)$. Then from KVL,

$$v_C(t) + v_R(t) = E$$

We have, $v_C(t) = \dfrac{q}{C}$ and $v_R(t) = Ri = R\dfrac{dq}{dt}$ as, $i = \dfrac{dq}{dt}$ and $q = Cv_C$

$$\therefore \frac{q}{C} + R\frac{dq}{dt} = E \Rightarrow \frac{dq}{E - q/C} = \frac{dt}{R}$$

Integrating we have, $-C\ln\left(E - \dfrac{q}{C}\right) = \dfrac{t}{R} + C_3$ C_3 : integration const.

At $t = 0$, $q = 0$, therefore, $C_3 = -C\ln E$ $\therefore \ln\left(\dfrac{E - q/C}{E}\right) = -\dfrac{t}{CR} \Rightarrow \boxed{q = EC\left(1 - e^{-t/(CR)}\right)}$

Therefore, at $t = \infty$, $q = q_0 = EC$ is the final charge of the capacitor

Contd..

Growth of the capacitor charge

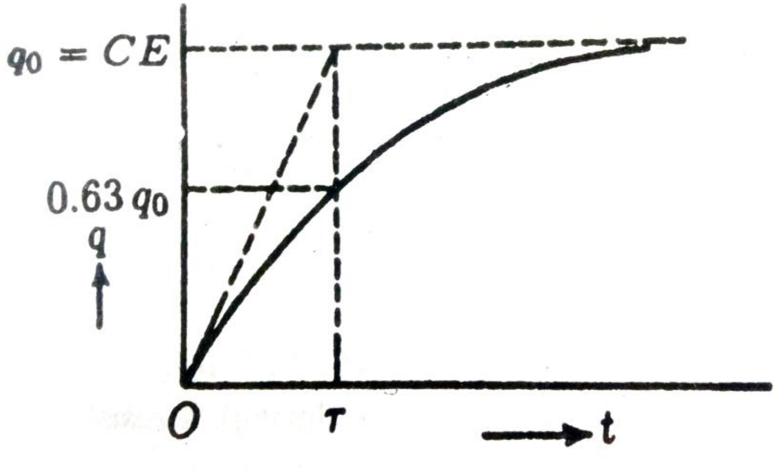


Fig.14.9. Variation of capacitor charge with time during charging

$t = \tau = CR$ = time constant of the RC circuit

At $t = \tau = CR$, $\Rightarrow q = q_0\left(1 - \frac{1}{e}\right) = 0.63 q_0$

The tangent to the growth curve at $t = 0$ intersects the line $q = q_0 = CE$ at $t = \tau = CR$ (see the fig). Thus the time constant can also be defined as the time in which the capacitor would be fully charged if it was allowed to charge at the initial rate.

The charging current,

$$i = \frac{dq}{dt} = \frac{q_0}{CR} e^{-\left(\frac{t}{CR}\right)} = i_0 e^{-\left(\frac{t}{CR}\right)}$$

where $i_0 = q_0/CR$ is the initial current at $t = 0$.

Clearly, with increasing time, the current i decays exponentially with the time constant $\tau = CR$. The $v_R(t)$ and $v_C(t)$ are,

$$v_R(t) = Ri = \frac{q_0}{C} e^{-t/CR} = E e^{-t/CR} \qquad v_L(t) = E - v_R(t) = E(1 - e^{-t/CR})$$

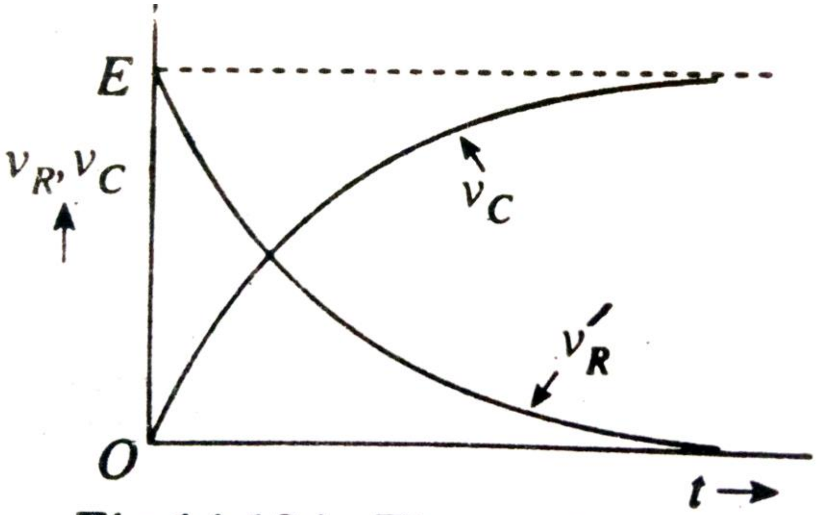


Fig.14.10A. Time variation of v_R and v_C

Discharging (natural response) of the Series RC circuit

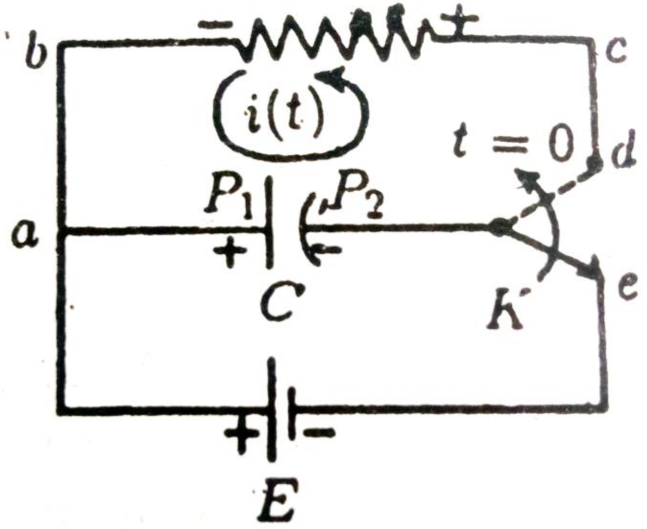


Fig.14.10. Discharging of a capacitor through a resistance

- In the circuit, switch K is initially at position *e* so that the capacitor is fully charged i.e. $q_0 = CE$ and switch is then moved quickly to position *d* at time $t = 0$
- The current corresponding to this is called discharging current and during discharging, the electrostatic energy of the capacitor is dissipated as the Joule heat in the resistance R.
- If q(t) is the charge at time t and as we have $i = dq/dt$, so, in the closed path *abcd,* KVL will give

$$\frac{q}{C} + R\frac{dq}{dt} = 0 \Rightarrow \frac{dq}{q} = -\frac{dt}{CR}$$

Integrating we have, $\ln q = -\frac{t}{CR} + C_4$ C_4 : integration const.

At $t = 0$, $q = q_0$, therefore, $C_4 = \ln q_0$ $\Rightarrow \ln\left(\frac{q}{q_0}\right) = -\frac{t}{CR} \Rightarrow q = q_0 e^{-t/CR}$

Charge on the capacitor decays exponentially with time.

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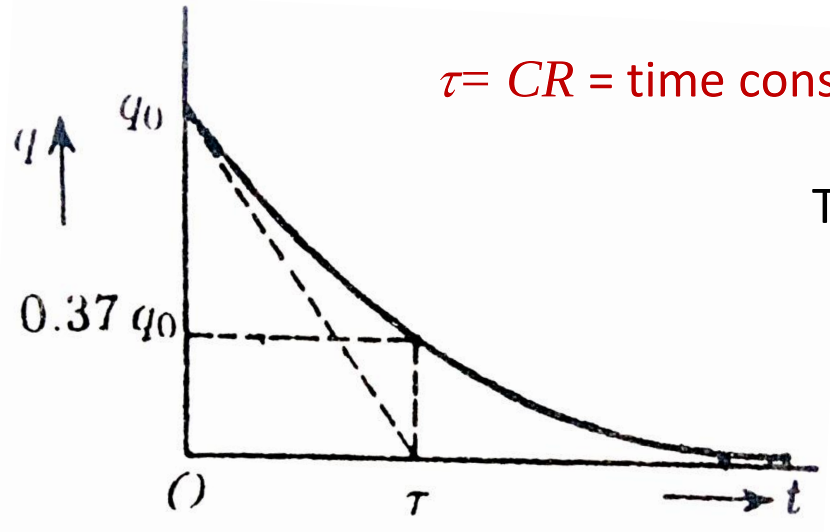


Fig.14.11. Decay of capacitor charge with time

τ = CR = time constant of the RC circuit

At $t = \tau = CR, \Rightarrow q = {q_0}/{e} = 0.37q_0$

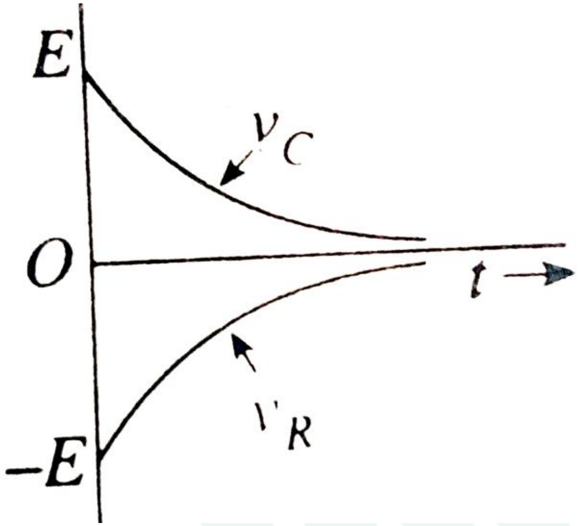
Therefore, at $t = \tau = CR$, capacitor charge falls to 37% of its initial value

The discharging current, $i = \dfrac{dq}{dt} = -\dfrac{q_0}{CR} e^{-\left(\frac{t}{CR}\right)} = -i_0 e^{-\left(\frac{t}{CR}\right)}$

The '– ve' current indicates the current $i(t)$ flows in the direction opposite to that was assumed in the analysis and shown in the circuit. The maximum value of the current $i(t)$ is q_0/CR occurring at $t = 0$.

The $v_R(t)$ and $v_C(t)$ during discharging are,

$$v_R(t) = Ri = -\frac{q_0}{C} e^{-t/CR} = -Ee^{-t/CR} \qquad v_C(t) = \frac{q}{C} = \frac{q_0}{C} e^{-t/CR} = Ee^{-t/CR}$$



'– ve' sign in $v_R(t)$ signifies that the polarity of the voltage drop across R is opposite to that assumed in the analysis

Observation:

If a signal voltage v_i is applied to a series RC circuit, then E in the KVL will be v_i and thus we can write $v_C + v_R = v_i$

Then the voltage across the capacitor C is, $$v_C = \frac{q}{C} = \frac{1}{C}\int i\,dt$$ Since, $i = \dfrac{dq}{dt}$

$$\Rightarrow v_C = \frac{1}{CR}\int v_R dt \Rightarrow v_C = \frac{1}{CR}\int (v_i - v_C)dt$$ Since, $v_R = iR$

If $\mathbf{v_i >> v_C}$; Then $v_C = (1/CR)\int v_i dt$ i.e. the capacitor voltage is proportional to the integral of the input voltage. The circuit is then referred to as an ***Integrating circuit*** or an ***integrator***. The condition $v_i >> v_C$ is when the time constant CR of the circuit is very large so that the capacitor charges up slowly.

Again, the voltage across the resistance R is, $$v_R = iR = R\frac{dq}{dt} = CR\frac{dv_C}{dt} = CR\frac{d}{dt}(v_i - v_R)$$

If $\mathbf{v_i >> v_R}$; Then $v_R = CR\dfrac{dv_i}{dt}$ i.e. the voltage is across R proportional to the time derivative of the input voltage. The circuit is then referred to as an ***differentiating circuit*** or an ***differentiator***. The condition $v_i >> v_R$ is achieved when the time constant CR of the circuit is very small so that the capacitor is quickly charged.