ECE113: Basic Electronics

WINTER 2021

S. S. Jamuar
B304, Academic Building
Email: sudhanshu@iiitd.ac.in

Office Hours:
Monday and Thursday (3:30 - 5pm)

Example

Switch S in Fig. 4(a) is arranged to disconnect the source and simultaneously short circuit the coil of having inductance of 2H and resistance 10 ohm. at time t = 0. If the initial current on coil I_0 = 20 A, Predict the current i after 0.2s has elapsed.

1. By Kirchhoff's voltage law,

$$\Sigma v = 0 = -v_L - v_R = -L\frac{di}{dt} - R_i$$

2. The homogeneous equation is

$$L\frac{di}{dt} + Ri = 0$$

3. Assuming an exponential solution, we write

$$i = A\,e^{st}$$

where s and A are to be determined.

4. Substituting into the homogeneous equation,

$$LsA\,e^{st} + RA\,e^{st} = (sL + R)A\,e^{st} = 0$$

If $sL + R = 0$, $s = -\frac{R}{L}$ and $i = A\,e^{-(R/L)t}$

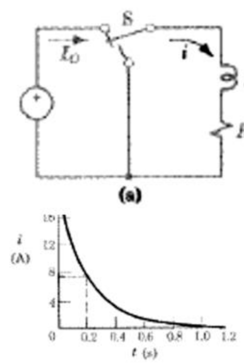


Fig. 4

Example

Switch S in Fig. 4(a) is arranged to disconnect the source and simultaneously short circuit the coil of having inductance of 2H and resistance 10 ohm. at time t = 0. If the initial current on coil I_0 = 20 A, Predict the current i after 0.2s has elapsed.

5. The energy stored in an inductance, $\frac{1}{2}Li^2$, cannot change instantaneously. Therefore, the current in the coil just after the switch is thrown must equal the current just before. At $t = 0^+$,

$$i = I_O = A\,e^0 = A \qquad \text{or} \qquad A = I_O = 20$$

Hence the solution is

$$i = I_O\,e^{-(R/L)t} = 20\,e^{-(10/2)t} = 20\,e^{-5t}\ \text{A} \qquad (4\text{-}10)$$

As shown in Fig. 4.4b, after 0.2 s the current is

$$i = 20\,e^{-5\times 0.2} = 20 \times 0.368 = 7.36\ \text{A}$$

The current decreases continually, but never becomes zero.

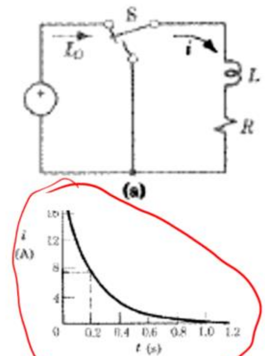
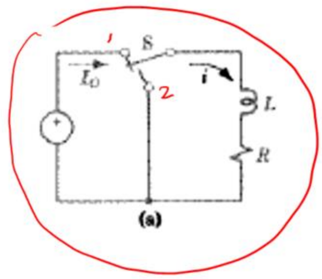
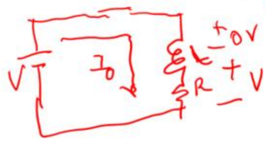


Fig. 4

Switch S in Fig. (a) is arranged to disconnect the source and simultaneously short circuit the coil of having inductance of 2H and resistance 10 ohm. at time t = 0. If the initial current on coil I_0 = 20 A, Predict the current i after 0.2s has elapsed.



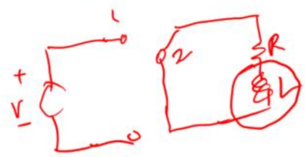
$I_0 = \frac{V}{R}$



$\boxed{E = \frac{1}{2} L I_0^2}$

I at t $(0-) = I_0$

I at t $(0+) = I_0$



$L\frac{di}{dt} + Ri = 0$

$i = Ae^{st}$

$LAse^{st} + RAe^{st} = 0$

$Ls + R = 0$

$s = -R/L$

$i(0+) = I_0 = A$

$i = I_0 e^{-t/(L/R)}$ Time const $= \frac{L}{R}$

General Procedure

General procedure to evaluate the natural response of an electrical circuit can be summarised as following:

- Write the governing equation using Kirchhoff's law.
- Reduce this to a homogeneous differential equation.
- Assume an exponential solution with undetermined constants.
- Determine the exponents from the homogeneous equation.
- Evaluate the coefficient from initial conditions.

Natural response of a RL circuit

Initial condition: $i(0) = I_0$

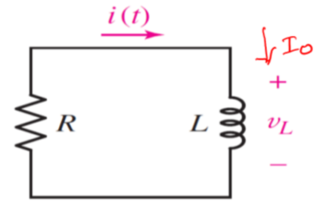
$Ri + v_L = Ri + L\frac{di}{dt} = 0$

$\frac{di}{dt} + \frac{R}{L}i = 0$

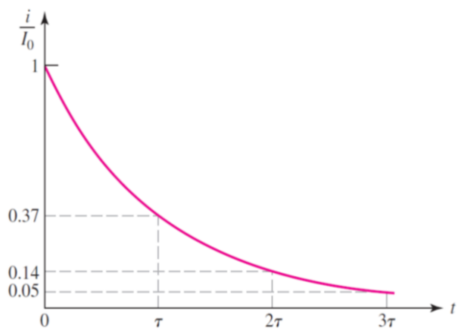
$i = I_0 e^{-\frac{R}{L}t}$

$i(t) = I_0 e^{-\frac{R}{L}t}$

- $p_R = i^2 R = I_0^2 R e^{-\frac{2Rt}{L}}$
- $w_R = \int_0^\infty p_R dt = \frac{1}{2} L I_0^2$
- Implies energy stored in the inductor is dissipated



Properties of the exponential response



$$\frac{i}{I_0} = e^{-\frac{R}{L}t}$$

At $t = \frac{L}{R}$

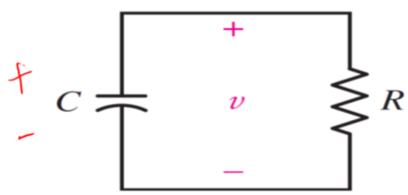
$$\frac{i}{I_0} = e^{-1} = 0.3679$$

$\frac{L}{R}$ is called the time constant and denoted as τ

That means

$$i = I_0 e^{-\frac{t}{\tau}}$$

Natural response of a RC circuit



$$C\frac{dv}{dt}+\frac{v}{R}=0$$

$$v=V_0e^{-t/RC}$$

Initial condition: $v(0)=V_0$

$$C\frac{dv}{dt}+\frac{v}{R}=0$$

Or

$$\frac{dv}{dt}+\frac{v}{RC}=0$$

$$v(t)=V_0e^{-\frac{t}{RC}}$$

Time constant $\tau=RC$

Problems

1. If the inductor of Fig. a has a current of i_L = 2 A at t = 0, find an expression for $i_L(t)$ valid for t >> 0, and its value at t = 200 μs.
2. For the circuit in Fig. b, find the voltage labeled v at t = 200 ms.
3. For the circuit of Fig. c, find the voltage labeled v at t = 200 μs.

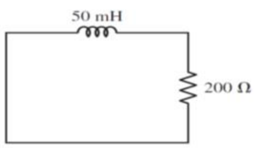


Fig. a

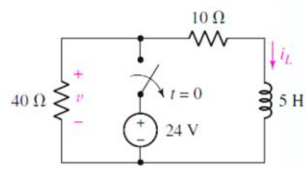


Fig. b

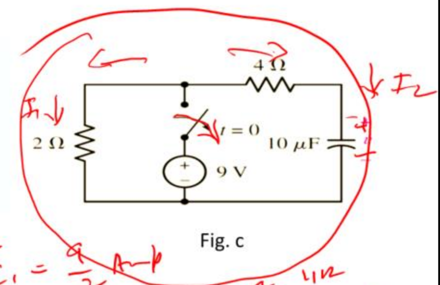
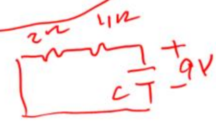


Fig. c

$t = 0^-$

$i_1 = \frac{9}{2}$ Amp

$I_2 = 9/4$ Amp



Unit step function

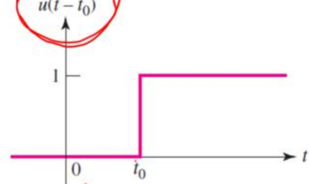
v V_0 R v

$u(t) = v$ V

$t<0 = 0$

$t>0 = v$

$t=0$ time

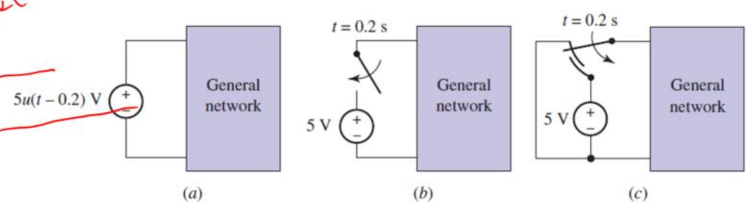


$$u(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

$u(t+t_0)$

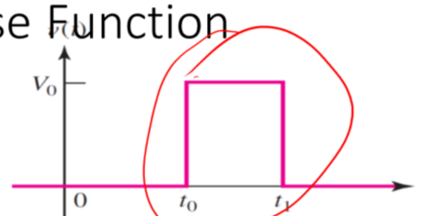
$t<t_0$ $u(t+t_0)$

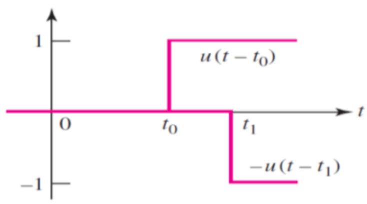
$-t_0$ t 0

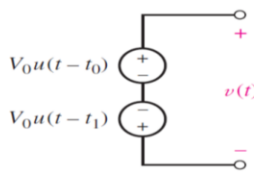


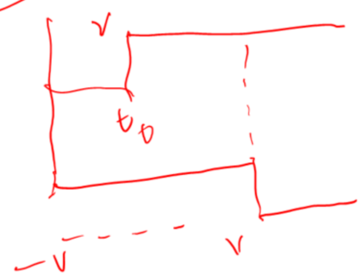
The Rectangular Pulse Function

$$v(t) = \begin{cases} 0 & t < t_0 \\ V_0 & t_0 < t < t_1 \\ 0 & t > t_1 \end{cases}$$







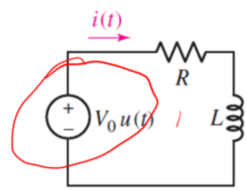


First order Circuits

- General form for first order, (linear) ordinary differential equation:

$$\frac{dx(t)}{dt} + ax(t) = f(t)$$

- x(t) is a circuit quantity (voltage, current), output quantity
- a is a constant, some function of the circuit elements
- $f(t)$ is a forcing function, usually the source voltage or current



- Combination of resistive elements and a capacitor or inductor
- 'First-order' refers to the order of the differential equation describing the circuit

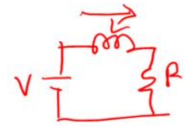
$$L\frac{di}{dt} + Ri = V_0 u(t)$$

x natural

x Forced

$x = x_{nat} + x_{for}$

First order Circuits



$\equiv I_f = \frac{V}{R}$

$I_{nat} = I_0 e^{-t/(L/R)}$

- Solution process:
  - Find the solution to the homogeneous equation
    - The solution is called the natural response (independent of the source applied)
    - A general solution has the form $x(t) = Ae^{-at}$
    - Often we write $x_n(t) = Ae^{-at}$
  - Look for a solution to the forced response
    - assume a forced response solution of the form $x_f(t)$
  - The complete solution is $x(t) = x_n(t) + x_f(t)$
  - Use initial conditions (i.e. x(0)) to determine constants

Complete response

$$L\frac{di}{dt} + Ri = V_0 u(t)$$

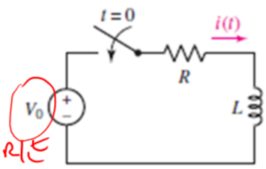
The solution consists of natural response plus forced response

Natural response i(t) = $\frac{V_0}{R} e^{-\frac{R}{L}t}$ for t = 0+

Forced response i(t) = $\frac{V_0}{R}$ for t > 0

Complete response

$$i = \frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t}$$



(a)

$i(t) = Ae^{-R/L\,t}$

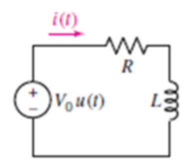
$i(t) = \frac{V_0}{R} + Ae^{-R/L\,t}$

$t=0 \quad i(0) = 0$

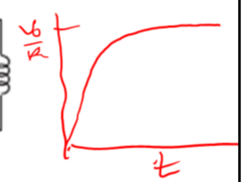
$A = -V_0/R$

$t = 0 \quad i = 0$

$t = \infty \quad i = \frac{V}{R}$



(b)



Complete response- General Approach

$$\frac{dx(t)}{dt} + Px(t) = Q$$

- For constant P and Q

$$x(t) = \frac{Q}{P} + Ae^{-Pt}$$

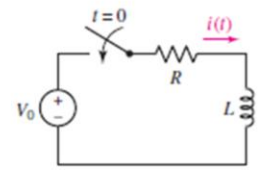
Forced response   Natural response

Intuitive understanding of responses

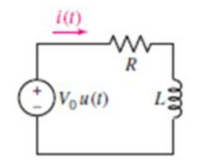
- Consider the forced RL circuit
- The circuit will eventually assume the forced response
- That means, After the natural response has died out, there can be no voltage across the inductor. Hence

$$i_f = \frac{V_0}{R}, \, and \qquad i = Ae^{-\frac{R}{L}t} \frac{V_0}{R}$$

- The current is zero prior to $t = 0$, and it cannot change value instantaneously since it is the current flowing through an inductor. Thus, the current is zero immediately after $t = 0$

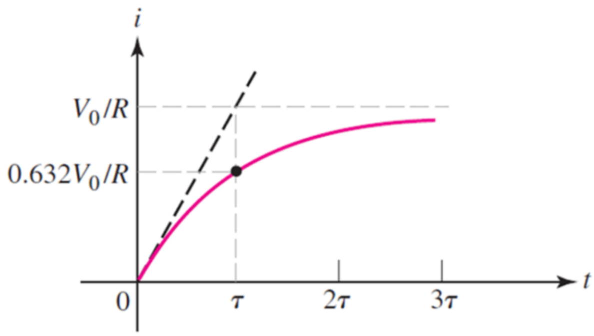


(a)



(b)

Output of unit step input



$$i = \frac{V_0}{R}(1 - e^{-Rt/L})$$

- Time constant = L/R

$V\,u(t)$

$t=0$ 3

12 mA

4R time const

Problems

1. Find $i(t)$ for $t=\infty, 3^-, 3^+, 100\mu s$ after application of source changes value in Fig. a.

$12(1-e^{-t/4R})$

2. Determine $i(t)$ for all values of time in the circuit in Fig. b.
3. Find the current response in a simple RL circuit when the forcing function is a rectangu… and duration t_0 in *Fig. c.*

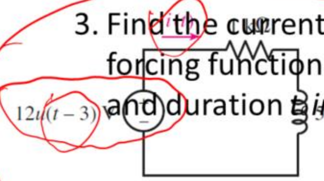
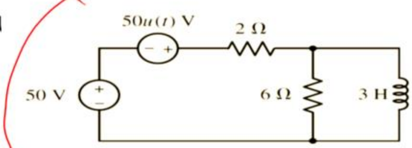


Fig. a



t_0

Fig. b

V_0

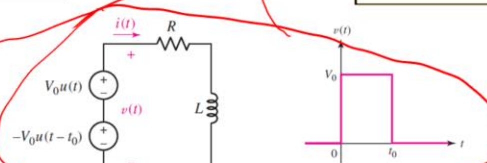


Fig. c

0

t_0 t

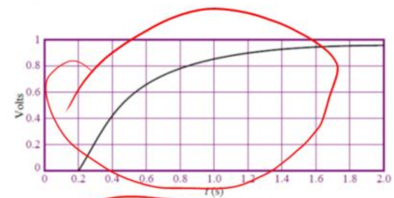
V_0

Transient Analysis

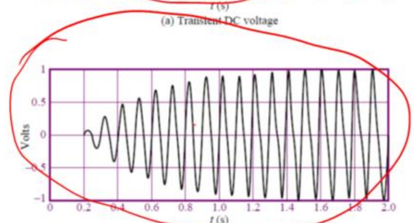
- To explore the solution of circuits that contain resistances, inductances, capacitances, voltage and current sources, and switches.
- The response of a circuit to the sudden application of a voltage or current is called transient response.
- The most common instance of a transient response in a circuit occurs when a switch is turned on or off—a rather common event in electrical circuits.
- We shall focus exclusively on the transient response of circuits in which a switch activates or deactivates a DC source.
- We shall restrict our analysis to first- and second-order

Transient Analysis

- The graphs of Figure 1 illustrate the result of the sudden appearance of a voltage across a hypothetical load [a DC voltage in Figure 1(a), an AC voltage in Figure 1(b)].
- The source voltage is turned on at time t = 0.2 s.
- The voltage waveforms of Figure 1 can be subdivided into three regions: a steady-state region, for $0 \le t \le 0.2$ s; a transient region for $0.2 \le t \le 2$ s (approximately); and a new steady-state region for $t > 2$ s, where the voltage reaches a steady DC or AC condition.
- The objective of transient analysis is to describe the behavior of a voltage or a current during the transition that takes place between two distinct steady-state conditions.



(a) Transient DC voltage



(b) Transient sinusoidal voltage

Figure 1