Dual

$x \cdot y \triangleq x + y$

axioms

increase

$+$ / $-$

sign bit

0101

Subtract

negative

$\bar{b} \cdot a = 0$ ∴ Theorem

Definition

$x \to \bar{x}$

$7 \to 2, 3$ nr. complements

$\bar{a} = b$

Duality

$a \cdot \bar{b} = 0$

$a + \bar{b} = 1$

? $\overline{x + y} = \bar{x} \cdot \bar{y}$ ← fundamental

$x \cdot 1 = x$ symbol

$(x+y) \cdot \bar{x} \cdot \bar{y} = 0$

$x + y + \bar{x} \cdot \bar{y} = 1$

office hours.

5 – 6 Monday

Tuesday

Wednesday — 12–1

office hour.

+4

−4

sign bit 1 0

0 8 bit → sign

Let’s Move On: Logic Synthesis

realizing logic circuit

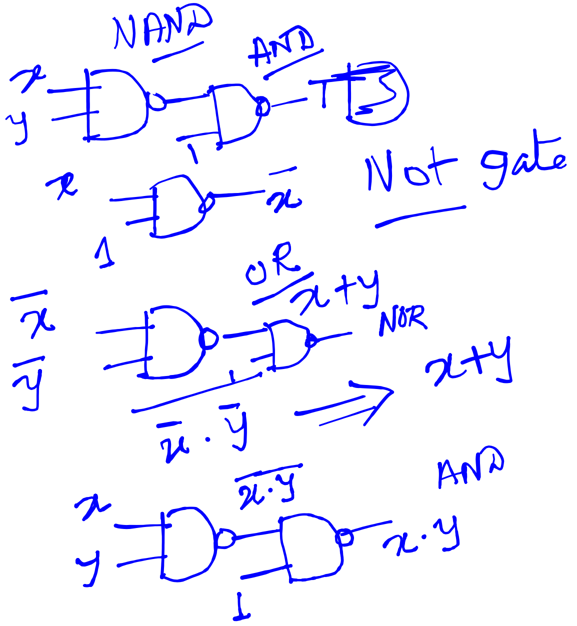
three gates
$\boxed{\text{Basic}}$ operations defined in

- Implement function using NOT, AND and OR gates
- Implement function using NAND gates
- Implement function using NOR gates

Universal Gates

NAND

manufacture



Definitions for Logic Synthesis:

- **Normal term:** Product or sum term in which no literal appears more than once, in any form.

when $x=1$ when $x=0$ $\bar{x}=1$ → literal 3 → 6 'state'

$$F_1(x, y, z) = (x + y) + \bar{x} \cdot \bar{z}$$

Normal

- **Minterm:** For a function of n variables, minterm is a normal product term with n literals.

Comp. normal

if any one 0 the term is 0

$$F_2(x, y, z) = \bar{x} \cdot \bar{y} \cdot z + \bar{x} \cdot y \cdot z + x \cdot \bar{y} \cdot \bar{z} + x \cdot \bar{y} \cdot z$$

- **Maxterm:** For a function of n variables, maxterm is a normal sum term with n literals.

maxterm

$$F(x, y, z) = (x + y + z) \cdot (x + \bar{y} + z) \cdot (\bar{x} + \bar{y} + z) \cdot (\bar{x} + \bar{y} + \bar{z})$$

even if one is 1 the term is 1

Truth Table

Minterms and Maxterms for Three Binary Variables

x	y	z	Minterms Term	Minterms Designation	Maxterms Term	Maxterms Designation
0	0	0	$\bar{x}\cdot\bar{y}\cdot\bar{z}$	m_0	$x+y+z$	M_0
0	0	1	$\bar{x}\cdot\bar{y}\cdot z$	m_1	$x+y+\bar{z}$	M_1
0	1	0	$\bar{x}\cdot y\cdot\bar{z}$	m_2	$x+\bar{y}+z$	M_2
0	1	1	$\bar{x}\cdot y\cdot z$	m_3	$x+\bar{y}+\bar{z}$	M_3
1	0	0	$x\cdot\bar{y}\cdot\bar{z}$	m_4	$\bar{x}+y+z$	M_4
1	0	1	$x\cdot\bar{y}\cdot z$	m_5	$\bar{x}+y+\bar{z}$	M_5
1	1	0	$x\cdot y\cdot\bar{z}$	m_6	$\bar{x}+\bar{y}+z$	M_6
1	1	1	$\boxed{x\cdot y\cdot z}$	m_7	$\bar{x}+\bar{y}+\bar{z}$	M_7

variable/literal
$x_1 \cdot x_2 \cdot x_3 \Rightarrow$
min term → expression
0 when any one of literals
1 when all are 1
DE
xyz
mt → minterm corr. to 0
min
lower case
minimum
left → right
given → 3
x.y.z →
m7
????
1234
upper case
Max

Logic Synthesis:

K-map

redrawn Table

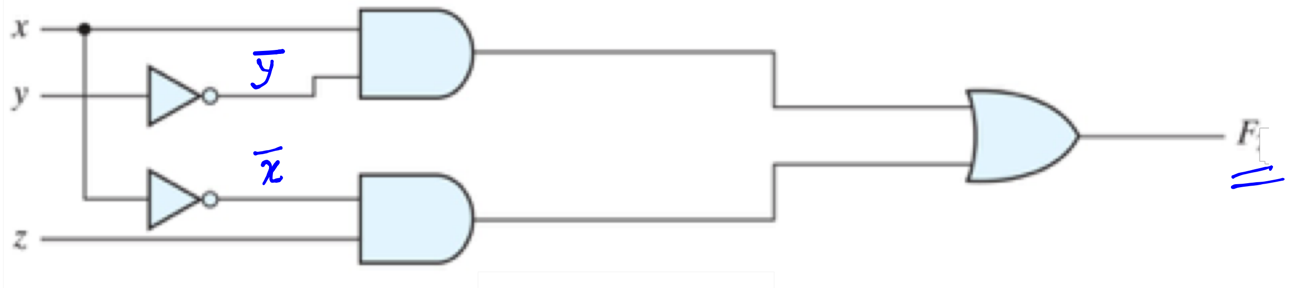
	x	y	z	F	
0	0	0	0	0	
1	0	0	1	1	$\bar{x} \cdot \bar{y} \cdot z$
2	0	1	0	0	
3	0	1	1	1	$\bar{x} \cdot y \cdot z$
4	1	0	0	1	$x \cdot \bar{y} \cdot \bar{z}$
5	1	0	1	1	$x \cdot \bar{y} \cdot z$
6	1	1	0	0	
7	1	1	1	0	

$F = (x+y+z)\cdot(x+\bar{y}+z)\cdot$
$(\bar{x}+\bar{y}+z)\cdot(\bar{x}+\bar{y}+\bar{z})$

max term exp.

$\bar{x}\cdot z(\bar{y}+y)$, $\bar{y}+y = 1$

$$F = \underbrace{\bar{x}\cdot\bar{y}\cdot z + \bar{x}\cdot y\cdot z}_{\bar{x}\cdot z} + \underbrace{x\cdot\bar{y}\cdot\bar{z} + x\cdot\bar{y}\cdot z}_{x\cdot\bar{y}}$$



000
001
010
100
011
110
101

Truth Table

DE	x	y	z	F	$\bar{F}$
0	0	0	0	0	1
1	0	0	1	1	0
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	1	0
5	1	0	1	1	0
6	1	1	0	0	1
7	1	1	1	0	1

product

Normal

$$F(x,y,z) = \underbrace{\bar{x}\cdot\bar{y}\cdot z + \bar{x}\cdot y\cdot z + x\cdot\bar{y}\cdot\bar{z} + x\cdot\bar{y}\cdot z}_{} = \underbrace{\bar{x}\cdot z + x\cdot\bar{y}}_{}$$

sum

minimized

Canonical Sum of Product or CSOP form

Sum of Product or SOP form

non-minimised

$$\boxed{F = m_1 + m_3 + m_4 + m_5}$$

representation

$$\bar{F} = \bar{x}\cdot\bar{y}\cdot\bar{z} + \bar{x}\cdot y\cdot\bar{z} + x\cdot y\cdot\bar{z} + x\cdot y\cdot z \rightarrow \boxed{\bar{x}\cdot\bar{z} + x\cdot y}$$

Max term

$$F = \bar{\bar{F}} = \underbrace{(x+y+z)\cdot(x+\bar{y}+z)\cdot(\bar{x}+\bar{y}+z)\cdot(\bar{\bar{x}}+\bar{y}+\bar{z})}_{}$$

sum

Canonical Product of Sum or CPOS form

$$= \underbrace{(x+z)\cdot(\bar{x}+\bar{y})}_{}$$

Normal

$$\boxed{F = M_0 \cdot M_2 \cdot M_6 \cdot M_7}$$

Product of Sum or POS form

Few More Definitions:

- **Sum of Product (SOP): SOP** expression is a set of logical product (AND) terms connected with logical sum (OR) operators.
- **Canonical SOP:** Each product term in SOP is a minterm
- **Product of Sum (POS): POS** expression is a set of logical sum (OR) terms connected with logical product (AND) operators.
- **Canonical POS:** Each sum term in POS is a maxterm

Representation for CSOP and CPOS forms:

Truth Table

DE	x	y	z	F	G
0	0	0	0	0	0
1	0	0	1	1	0
2	0	1	0	0	0
3	0	1	1	1	1
4	1	0	0	1	0
5	1	0	1	1	1
6	1	1	0	0	1
7	1	1	1	0	0

all the same

$$F(x,y,z) = \bar{x}\cdot\bar{y}\cdot z + \bar{x}\cdot y\cdot z + x\cdot\bar{y}\cdot\bar{z} + x\cdot\bar{y}\cdot z \qquad CSOP$$

result 4

$$F(x,y,z) = m_1 + m_3 + m_4 + m_5 \qquad CSOP$$

minterm rep. sop 2

$$F(x,y,z) = \sum m(1,3,4,5)$$

summation representation

statement sigma 3

$$F(x,y,z) = (x+y+z)\cdot(x+\bar{y}+z)\cdot(\bar{x}+\bar{y}+z)\cdot(\bar{x}+\bar{y}+\bar{z}) \qquad CPOS$$

$$F(x,y,z) = M_0\cdot M_2\cdot M_6\cdot M_7 \qquad CPOS$$

Pi product

$$F(x,y,z) = \prod M(0,2,6,7)$$

most commonly used convenient

$G = \bar{x}\cdot y\cdot z + x\bar{y}z + x\cdot y\cdot\bar{z}$

$\sum m(3,5,6)$

$\prod M(0,1,2,4,7)$