$X \sim$ Binomial(n,p) RV

- Here the number of trials is fixed to n

- The Binomial RV counts the number of successful trials amongst the total of n trials

$$S_X = \{0, 1, 2, \ldots, n\}$$

- PMF? Simply calculate probabilities for all values in the range space...

$$P[X=x] = \begin{cases} {}^nC_k (1-p)^{n-k} p^k & k = 0, 1, 2, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

$P[X=k] = P[$k successes in n indep and id Bern trials$]$

$\{X=x\}$ = You obtain the k^{th} 1 at the x^{th} trial.

$x \geq k$

$\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$ . . . . $\underline{\quad}$
1 2 3 x

We have k 1(s) &
$x-k$ 0(s) &
the x^{th} outcome must be 1.

Any such sequence has a probability $p^k(1-p)^{x-k}$.

How many such sequences do I have?

$^{x-1}C_{k-1}$

$$P[X=x] = {}^{x-1}C_{k-1}\, p^k (1-p)^{x-k}$$

Binomial(n,p) RV

- You are not just interested in the first bit error
- You are interested in the number of bit errors in a packet of size n bits
- What is the probability that x out of n bits are in error, when each bit is in error with probability p, independent of other bits?
- **Def 2.7:** X is a binomial(n,p) random variable if the PMF of X has the form

$$P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x \in \{0, 1, \ldots\}, \\ 0 & \text{otherwise.} \end{cases}$$

Note that $n \geq 1$ and $0 < p < 1$.

Pascal(k,p) RV

- Example 2.15 and Definition 2.8
- Straightforward extension of Binomial
- Read from book!

Discrete Uniform(k,l) RV

- A fair coin has an equal probability of landing heads up and tails up
- The roll of a fair die has an equal probability of giving each of 1,2,3,4,5 and 6
- If 10 outcomes are possible and you have no reason to believe that any one outcome is more likely than another, you model the outcomes to be equiprobable

Discrete uniform*(k,l)* RV

- **Def 2.9** X is a discrete uniform*(k,l)* RV if the PMF of X has the form

$$P_X(x) = \begin{cases} 1/(l-k+1) & x = k, k+1, \ldots, l, \\ 0 & \text{otherwise.} \end{cases}$$

- Note that bit *k* and *l*, *k* < *l*, are integers by the definition of a **discrete** uniform RV.

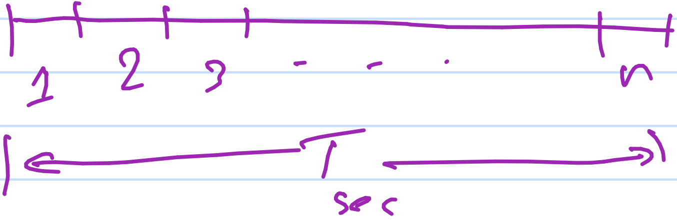
Discrete uniform*(k,l)* RV

- **Def 2.9** X is a discrete uniform*(k,l)* RV if the PMF of X has the form

$$P_X(x) = \begin{cases} 1/(l-k+1) & x = k, k+1, \ldots, l, \\ 0 & \text{otherwise.} \end{cases}$$

- Tossing a fair coin can be modeled using a discrete uniform*(100,101)* RV
  - Of course, typically we use discrete uniform*(0,1)*!
- For an experiment that requires rolling a fair die and noting the number
  - Use the discrete uniform(1,6) RV

The Binomial Picture



Poisson (α) Random Variable

- You stand at a railway ticket counter and note the number of customers that arrive at the counter every minute

- Turns out that in many such real world situations
  - Customers arriving at a restaurant
  - Packets arriving at a router
  - Students arriving to a class?

the **number of arrivals in a fixed time interval** can be modeled as a Poisson RV

Poisson (α) Random Variable

- The observation is a count and hence the range S_X is the set of non-negative integers
    - S_X is countably infinite
    - Poisson is indeed a discrete RV

Poisson (α) Random Variable

- **Def 2.10** X is a Poisson(α) RV if the PMF of X has the form

$$P_X(x) = \begin{cases} \alpha^x e^{-\alpha}/x! & x = 0, 1, 2, \ldots, \\ 0 & otherwise. \end{cases}$$

where the parameter $\alpha > 0$.

- If the average rate of arrivals is λ/second and time interval is of T seconds, $\alpha = \lambda T$.

Poisson (α) Random Variable

- **Def 2.10** X is a Poisson(α) RV if the PMF of X has the form

$$P_X(x) = \begin{cases} \alpha^x e^{-\alpha}/x! & x = 0, 1, 2, \ldots, \\ 0 & otherwise. \end{cases}$$

where the parameter $\alpha > 0$.

- If the average rate of arrivals is λ/second and time interval is of T seconds, α = λT.

- What explains the crazy looking expression?

Word Problem (Example 2.20)

- The number of database queries processed by a computer in any 10-second interval is a Poisson RV K with α=5 queries.

  - What is the probability that there will be no queries processed in a 10-second interval?

$$P_X(x) = \begin{cases} \alpha^x e^{-\alpha}/x! & x = 0, 1, 2, \ldots, \\ 0 & otherwise. \end{cases}$$

where the parameter $\alpha > 0$.

Word Problem (Example 2.20)

- The number of database queries processed by a computer in any 10-second interval is a Poisson RV K with α=5 queries.
  - What is the probability that at least two queries will be processed in a 2-second interval?

$$P_X(x) = \begin{cases} \alpha^x e^{-\alpha}/x! & x = 0, 1, 2, \ldots, \\ 0 & otherwise. \end{cases}$$

where the parameter $\alpha > 0$.

Quiz 2.3

Each time a modem transmits one bit, the receiving modem analyzes the signal that arrives and decides whether the transmitted bit is 0 or 1. It makes an error with probability p, independent of whether any other bit is received correctly.

(1) If the transmission continues until the receiving modem makes its first error, what is the PMF of X, the number of bits transmitted?

Quiz 2.3

Each time a modem transmits one bit, the receiving modem analyzes the signal that arrives and decides whether the transmitted bit is 0 or 1. It makes an error with probability p, independent of whether any other bit is received correctly.

(1) If the transmission continues until the receiving modem makes its first error, what is the PMF of X, the number of bits transmitted?

(2) If $p = 0.1$, what is the probability that $X = 10$? What is the probability that $X \geq 10$?

(3) If the modem transmits 100 bits, what is the PMF of Y, the number of errors?

(4) If $p = 0.01$ and the modem transmits 100 bits, what is the probability of $Y = 2$ errors at the receiver? What is the probability that $Y \leq 2$?

(5) If the transmission continues until the receiving modem makes three errors, what is the PMF of Z, the number of bits transmitted?

(6) If $p = 0.25$, what is the probability of $Z = 12$ bits transmitted?