Probability and Random Processes

Continuous Random Variables

Sanjit Kaul

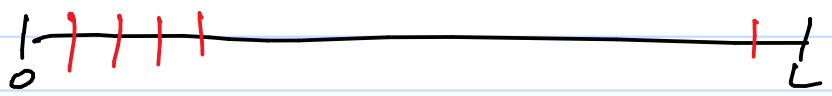
INDRAPRASTHA INSTITUTE *of*
INFORMATION TECHNOLOGY
DELHI

Continuous Random Variables

INDRAPRASTHA INSTITUTE *of*
INFORMATION TECHNOLOGY
DELHI

Intervals

- (x_1, x_2)
  - Includes neither x_1 nor x_2 but all real numbers in between
- $[x_1, x_2)$
- $(x_1, x_2]$
- $[x_1, x_2]$

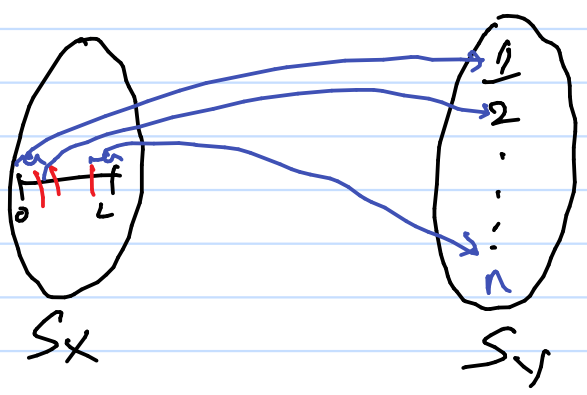


Pick randomly $x \in (0, L)$

$P[X = x] = ?$

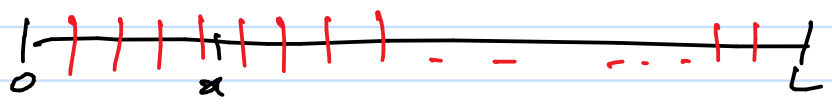
Note that X is a continuous RV since its range space $S_X = (0, L)$, which is continuum.

Create discrete RV Y by discretizing $(0, L)$ into n exclusive parts.



$$P[Y = y] = \begin{cases} \frac{1}{n} & y \in \{1, 2, \cdots, n\} \\ 0 & \text{otherwise} \end{cases}$$

$P[X = x] = ?$



When $\{X = x\}$ occurs, $\{Y = \cdots\}$ occurs

$$Y = \text{ceil}\left(x + \tfrac{L}{n}\right)$$

$$Y = \text{ceil}\left(\frac{x}{L/n}\right)$$

$$\{Y = \text{ceil}(x/(L/n))\} \supset \{X = x\}$$

$$P[\{Y = \text{ceil}(x/(L/n))\}] \geq P[X = x]$$

Taking the limit $n \to \infty$, we get $P[X = x] = 0$.

A Complete Description of a Continuous RV

A Very Narrow Carrom Board

- A very narrow board of length L can be approximated by a line
  - This is a terrible attempt on my part to emulate the example in RY

A Very Narrow Carrom Board

- A very narrow board of length L can be approximated by a line
  - This is a terrible attempt on my part to emulate the example in RY
- You shoot the striker (red)
- Your strike is powerful
- This ensures that the striker stops at any point along the thin board with equal probability
- The striker stops at a point in [0,L]

A Very Narrow Carrom Board

- Let X be the RV that describes the final location of the pointer
- We have $S_X = [0,L]$
- The range is a continuum. Thus X is a CRV
- What is $P[X = x]$ where $x \in S_x$?

A Very Narrow Carrom Board

- Let's think in discrete terms

- We split the board into n equal intervals
- Define the RV Y with $S_Y = \{1,2,\ldots,n\}$
- Y is a discrete uniform RV
- Note that in the discrete problem all values of X in interval k correspond to Y=k.

$$P[Y = k] = \boxed{}$$

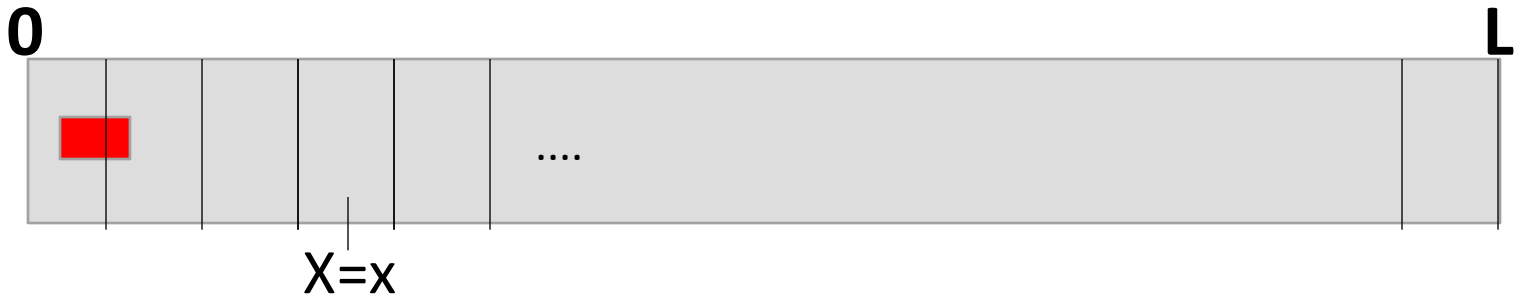
A Very Narrow Carrom Board

- Let's think in discrete terms

- We split the board into n equal intervals
- Define the RV Y with $S_Y = \{1,2,\ldots,n\}$
- Y is a discrete uniform RV
- Note that in the discrete problem all values of X in interval k correspond to Y=k.

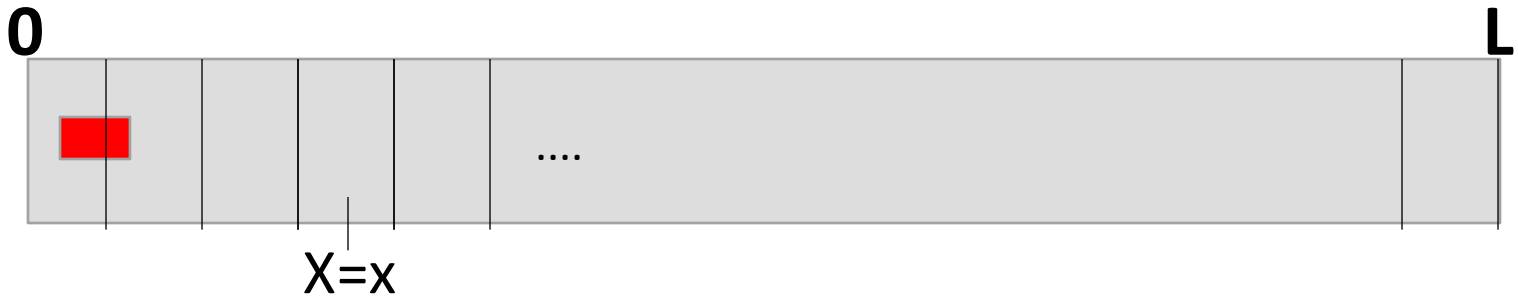
$$P[Y = k] = P[k - 1 \leq X < k]$$

A Very Narrow Carrom Board



If $X = x$ then $Y =$

A Very Narrow Carrom Board



If $X = x$ then $Y = \lceil x/(L/n) \rceil = \lceil nx/L \rceil$

- Also
- Therefore

$$P[X = x] \quad P[Y = \lceil nx/L \rceil] = 1/n$$

A Very Narrow Carrom Board

- Getting as close as possible to the continuous case

$$P[X = x] \leq \lim_{n \to \infty} 1/n = 0$$

- Therefore

$$P[X = x] = 0$$

- Every outcome {X=x} has a probability 0!
  - Note that this is not true for an interval on the board
- The PMF is not a useful way of describing a continuous RV
- What about the CDF?