Texas UCF

Kelly Criterion

What is Kelly Criterion?

- Kelly Criterion answers the question: “How much should I bet to grow my capital as fast as possible, without going broke, <u>IN THE LONG RUN?</u>

- Kelly is a formula for determining how much of your capital you should use, ***OVER A LONG PERIOD OF TIME,*** when making many repeated decisions that make or lose money. Can be applied to both fixed and variable probability/outcome events
  - Gambling/Casinos: Fixed probabilities with known outcomes
  - Investing/Trading: Non-fixed probabilities with unknown/uncertain outcomes. Here, we can use metrics like volatility and historical return estimates to apply Kelly despite not knowing our exact odds of “winning”

- Most useful for situations where you can’t afford to lose everything, e.x. Your investment portfolio. Rather than just maximizing expected value, it maximizes logarithmic/compounding wealth (more on this next slides)

Intuition behind Kelly (1/3)

- The problem with maximizing EV, though mathematically appealing, is you often go broke.
    - Imagine a game where I have 60% chance to double my money and 40% chance to lose it all. The EV-maximizing approach is to bet the entire bankroll every time since the EV is positive. But after a single loss, you'd have $0 left. <u>EV maximization doesn't consider survival</u>.
    - Tree diagram

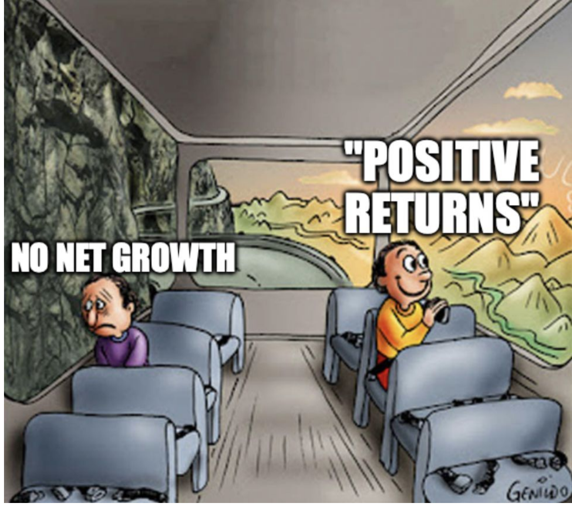
- Sometimes, we care more about growth rate than single outcomes (managing a portfolio example). So, Kelly focuses on repeated bet schemes. If you have to repeatedly make bets, the strategy that maximizes EV **is often NOT** the one that maximizes long-term wealth.

Intuition behind Kelly (2/3)

- From an intuitive standpoint, an "optimal" bet should always be a proportion of your bankroll (exponential growth), not a fixed or static amount. but:
  - Betting too little -> Not utilizing your bankroll as effectively as you could
  - Betting too much -> Risk going broke
- There must be some mathematical "sweet spot" where you maximize your long-run growth while minimizing your risk of ruin. To find this, we need to maximize the **geometric mean** of a distribution of payouts; oftentimes more accurate depictor of growth than arithmetic mean in the context of portfolios and bankrolls.
  - Imagine a scenario where in Year 1, your portfolio doubles in value (+100%). In Year 2, it halves in value (-50%). Your arithmetic average return is 25%, even though you broke even after 2 years. But your geometric return would be

    (**(1+1.0)***(1+-0.5)**)^(½) = 1, showing that your actual growth rate was 0%.

Intuition behind Kelly (3/3)



TLDR:

- Large numbers of repeated bets behave much differently than short-term single outcomes
- Maximizing EV approach ≠ Maximizing long-term wealth for repeated bets
- Arithmetic mean and growth rate are often poorly correlated; gain % when using arithmetic mean is weighted much more than loss % (earlier slide).
    - Also, arithmetic mean/% return doesn't reflect a portfolio compounding
    - So we use logarithmic returns to calculate total, compounded growth
    - Geometric mean is a much more accurate measure of long-term growth
- Kelly formula maximizes geometric/log growth rate.

Derivation of Kelly - Gambling Formula (1/2)

Let $\boldsymbol{f}$ be the fraction of your wealth bet per round. Let $\boldsymbol{b}$ be the payout odds; eg. if you get paid out 2x your original bet, then $b = 2$. Let $\boldsymbol{p}$ be the probability of winning, and $\boldsymbol{q} = 1 - p$ be the probability of losing. Let $\boldsymbol{W}$ be your wealth after one bet. Then,

$$W = (1 + fb) \text{ with probability } p;\ (1 - f) \text{ with probability } q$$

Expected Value[W] $= p\,(1 + fb) + q\,(1 - f)$

Since we care about long-term compounded wealth, we want to maximize the expected logarithm of W. This will maximize the products/compounding of many many W's (or bets).

$$\Rightarrow \mathrm{E}[\log W] = p \log(1 + fb) + q \log(1 - f)$$

We want to find the optimal fraction f that maximizes E[log W], so take the derivative with respect to f, and set it equal to zero (by the first derivative test for finding maxima):

$$0 = (p/(1 + fb))*b + (q/(1 - f))*-1$$

$$0 = (pb/(1 + fb)) - (q/(1 - f))$$

cont.

Derivation of Kelly - Gambling Formula (2/2)

We have $0 = (pb/(1 + fb)) - (q/(1 - f))$, and we need to solve for f (the optimal bet fraction).

$\Rightarrow pb/(1 + fb) = q/(1 - f)$

Cross multiply:

$\Rightarrow pb\,(1 - f) = q\,(1 + fb)$

$\Rightarrow pb - pbf = q + qfb$

Isolate f:

$\Rightarrow pb - q = qfb + pbf$

$\Rightarrow pb - q = f\,(qb + pb)$

$\Rightarrow f = (pb - q)/(qb + pb)$

$\Rightarrow f = (pb - q)/b\,(p + q)$

We know that $p + q = 1$ by definition of win and loss, so:

$\boldsymbol{f} = (pb - q)/b = \boldsymbol{p - (q/b)}$

Derivation of Kelly - Investing Formula

The derivation for the investing formula is similar, except this time we allow partial losses instead of losing the entire wager. ***W*** is defined the same - your expected wealth after one bet, or investment. Now, $E[W] = (1 + fg)$ with probability p and $(1 - fl)$ with probability q. ***g*** is the factor by which your wealth increases if you 'win', and ***l*** is the factor by which your wealth decreases if you 'lose'.

$E[\log W] = p \log(1 + fg) + q \log(1 - fl)$; take the derivative and set it equal to 0

$\Rightarrow 0 = (p/(1 + fg))*g + (q/(1 - fl))*(-l) \Rightarrow 0 = (pg(1 + fg)) - (ql/(1 - fl))$

$\Rightarrow (pg/(1 + fg)) = (ql/(1 - fl))$; cross multiply

$\Rightarrow pg\,(1 - fl) = ql\,(1 + fg) \Rightarrow pg - pgfl = ql + qlfg$; isolate f and factor it out

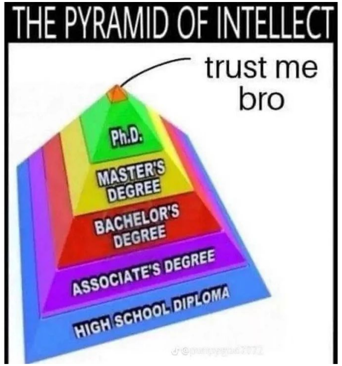
$\Rightarrow f(qlg + pgl) = pg - ql$ or $f = (pg - ql)/(qlg + pgl)$. Factor the denominator as $gl(p + q) = gl$

$\Rightarrow f = (pg - ql)/gl \Rightarrow f = pg/gl - ql/gl$

$\boldsymbol{\Rightarrow f = (p/l) - (q/g)}$

So, the optimal bet sizing increases when p (win) increases, and decreases when q (lose) increases. This makes intuitive sense

Side tangent - how are ***p***, ***q***, ***g***, ***l*** found in real life?



p and ***q*** - often estimated through backtesting/historical data.

- Simple example - if Apple stock has a positive daily return 60% of the time over the last years, estimate $p = 0.6$ and $q = 0.4$.

g and ***l*** - can be estimated using historical volatility data. One estimation formula for ***g***, ***l*** can be derived from the Expected Daily Move formula: EDM ≈ 0.8 * dailyVolatility ≈ 0.8 * annualizedVolatility/sqrt(252), and calculating log-adjusted returns (not arithmetic).

- Example - if Apple stock has an average positive Expected Daily Move of +1% from past volatility data/metrics, we might estimate $g = 0.01$, and estimate l accordingly.
- Other strategies exist - more in "Kelly for Portfolios" section

Example (not accurate data):

TSLA has historically been bullish. Suppose Tesla has an Expected Daily Move (EDM) of +6%, 70% of the time or EDM of -4%, 30% of the time. Then, the Kelly bet would be f = p/l - q/g = (0.7)/(0.04) - (0.3)/(0.06) = 17.5 - 5 = 12.5. However, this means committing 1250% of our capital. This is why traders often use fractional Kelly, since full Kelly can be too aggressive/unrealistic

Derivation of Kelly - Investing Formula

The derivation for the investing formula is similar, except this time we allow partial losses instead of losing the entire wager. Again, we want to optimize E[log *W*], with the extra addition that we can now lose parts of our bet - call this ***g***.

$E[W] = (1+fb)^p * (1-fa)^q$

$E[\log(W)] = p \log(1+fb) + q\log(1-fa)$

$dE/df = pb / (1+fb) - qa / (1-fa) = 0$

$pb / (1+fb) = qa / (1-fa)$

$pb (1-fa) = qa (1+fb) \rightarrow pb - f(pba) = qa + f(qab) \rightarrow f(qab) + f(pba) = pb - qa$

$f = (pb - qa) / (qab + pba) = (pb)/(qab + pba) - (qa) / (qab + pba) = p/(qa + pa) + q/(qb+pb)$

$= p/(a(p+q)) + q/(b(q+b)) = p/a + q/b$

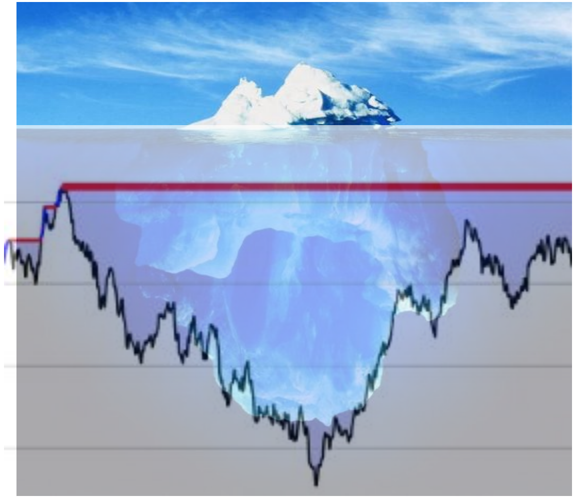
Games/Simulations

Weighted Coin Toss

- We can see Kelly in action by simulating a weighted coin toss game
- Game:
  - N timesteps
    - At each timestep, a weighted coin is flipped
      - The probability of the coin is known
    - You choose a bet-size % and bet on the tail being heads
    - Payout is compounded
  - The strategy involves ***choosing the bet size***
- We consider 3 strategies
  - All (always bet maximally) [Most EV]
  - Kelly (bet exactly based on kelly formula)
  - Partial Kelly (bet a fraction of kelly to be **conservative**)
  - 5x Kelly (bet more than kelly to be **aggressive**)

Weighted Coin Toss

- Based on these strategies, we gauge the following metrics
  - Average Value (EV)
  - Log Geometric Mean (What Kelly is maximizing)
  - Average Drawdown
    - The "lowest" point as a ratio to the previous highest point
  - Risk of Ruin
    - Probability of going bankrupt



Weighted Coin Toss

ALL

```
Average Value: 0.0
Log Geometric Mean: 0.0
Average Drawdown: 0.0
Risk of Ruin: 100.0%
```

KELLY

```
Average Value: 1.0388915000000123
Log Geometric Mean: 0.01821049743697178
Average Drawdown: 0.8040207381726617
Risk of Ruin: 0.0%
```

PARTIAL_KELLY

```
Average Value: 1.017276000000016
Log Geometric Mean: 0.01667326773056999
Average Drawdown: 0.8952133098878253
Risk of Ruin: 0.43%
```

5x KELLY

```
Average Value: 1.2121479999999984
Log Geometric Mean: -0.29468496528599675
Average Drawdown: 0.30778177428182196
Risk of Ruin: 0.575%
```

Weighted Coin Toss - Unintuitive Results

- Partial Kelly is expected to be more **conservative** than Kelly? Why more risk of ruin?
- In the simulation, if a strategy outputs to bet 0%, it also counts as “ruin”
    - Bets are discretized to the cent level, so partial kelly often stops betting entirely
- If we disable this, partial_kelly also has a Risk of Ruin of 0%.

PARTIAL_KELLY

```
Average Value: 1.017276000000016
Log Geometric Mean: 0.01667326773056999
Average Drawdown: 0.8952133098878253
Risk of Ruin: 0.43%
```

KELLY

```
Average Value: 1.038891500000123
Log Geometric Mean: 0.01821049743697178
Average Drawdown: 0.8040207381726617
Risk of Ruin: 0.0%
```

Blackjack Rules

- **Objective:** Achieve a hand total closer to 21 than the dealer without exceeding 21 (busting).
- **Values:** Cards 2–10 are face value, face cards count as 10, and aces count as 1 or 11.
- **Gameplay**: Players are dealt two cards and can repeatedly choose whether to "hit" (take another card) or "stand" (keep their hand).
    - If you hit and go over 21, you lose your bet
    - If you stand, then the dealer plays
- **Dealer Rules:** The dealer must hit until reaching at least 17; winning is determined by comparing totals.
- **Blackjack:** A blackjack is achieved from an ace and another card of value 10
- **Doubling Down & Splitting:** Players may double their bet for one extra card or split a pair into two hands adding an equivalent bet to the second hand.

Blackjack Betting

- Slightly more complex version of weighted coin toss
- Initial EV is always negative
  - Must make waiting bets (min bet) to get to circumstances where odds are favorable
  - Once u get to these favorable situations, need to bet more to outweigh the min bets
- Advantage players
  - Card count
  - Simple, Hi-Lo strategy
  - Complex, calculate exact remaining composition
- Composition: what cards remain in the shoe, given prior hands and current cards out

Blackjack - Strategies

Basic (No card counting)

- Assumes no count of cards
- Negative edge
- Optimal play given no additional info

Card Counting (Hi-Lo)

- Maintains a count of how many low vs. high cards have been seen
- Higher count means more high cards left in shoe
    - Higher advantages for blackjack (player edge), doubling down
- Indices/Deviations
    - Knowing the card count creates deviations from optimal strategy
        - Fab 4 (surrender)
        - Illustrious 18

Card Counting (Exact Composition)

-

Blackjack - Strategies

Strat 1: Basic (No card counting)	Strat 2: Card Counting (Hi-Lo)	Strat 3: Card Counting (Exact Composition)

- Assumes no count of cards
- Negative edge (-0.5%)
- Optimal play given no additional info
- Casinos hand cards with optimal strategies out, but you still lose money
- The only winning move in basic strategy is not to play
	- Maintains a count of how many low vs. high cards have been seen
	- Higher count means more high cards left in shoe
	    - Higher advantages for blackjack (player edge), doubling down
	- Indices/Deviations
	    - Knowing the card count creates deviations from optimal strategy
	        - Illustrious 18
	        - Fab 4 (surrender)
		- Creates more indices/deviations
		- Only relevant for computer/optimal play

Blackjack - Bet Sizing

- If we are playing the basic strategy without counting cards, what would kelly tell us to bet?
- If we are counting cards, how much should we bet?
  - Policy determines win percentage which determines Kelly
- How do we estimate win percentage
  - Payoff is not binary
    - Doubling down, blackjack 3:2, splitting, etc
  - Rougher estimations, each count provides us with some percentage edge
    - Estimate +1.1x on win, -1x on loss
  - Exact estimations, use monte carlo/dp simulation to get probability of winning

Blackjack - Estimating Win Probability

Problem:

- We know the composition of the cards left in the deck
- We want to find the probability that we (double lose, lose, tie, win, double win)

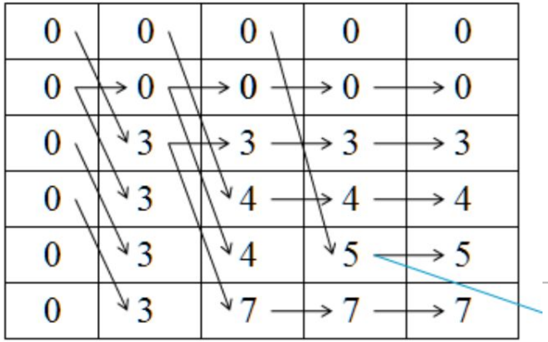
Possible Approach - Computational

- Step 1: Consider every possible hand and dealer face-up cards
  - This is ~ (20 * 10) since there are 20 possible sums we can start with and 10 sums of the dealer card
- Step 2: Solve for probability knowing the starting hand
  - Since the dealer is deterministic and the player plays before, we can solve for each player independently
    - We compute probabilities of ending up at all sums for both players, and combine this result afterwards to calculate the win probabilities
  - Exact DFS:
    - Too slow since each time we draw a card, the amount of possibilities multiples exponentially
  - Estimated Dynamic-Programming Approach

Blackjack - Estimating Win Probability

Estimated Dynamic-Programming Approach:

- Assume that the composition of the deck (probabilities that we draw each card) doesn't change throughout this turn. Also assume that our policy is deterministic throughout the turn.
- Then, our state space is massively reduced since we can represent any hand as the following
  - dp[sum of my cards (~20 possibilities)][do I have an ace (2 possible)][have I doubled (2 possible)]
- Transition: at any state, apply our policy and update the probabilities of the affected state using the composition
  - Example: If I'm at sum=10, then "hitting" would affect sums in the range [11, 21]
    - dp[15] += dp[10] * probability of drawing 5
- Deal with special cases with aces and doubling (which are nice because you can't undraw an ace, etc).



Blackjack - Estimating Win Probability

DP Results? (Code: code)

- Ran test of this approach with the standard blackjack strategy and full deck.
- DP outputs following probabilities
  - [DLose: 0.02287, Lose: 0.54901, Tie: 0.1103, Win: 0.3161, DWin: 0.00158]
- Ran simulation using the same strategy, with empirical results
  - [DLose 0.0524, Lose: 0.4377, Tie: 0.0836, Win: 0.3654, DWin: 0.0609]

General trends are the same, but there's definitely bugs in the dp. (Also ran out of time)

So, we decided to go with a more straightforward card counting heuristic method for estimating the win probability.

Blackjack - Simulation

- Calculate winning probability given some remaining shoe composition following some policy
  - Estimated based on .495 + scaling of current count
- Plug winning probability into Kelly to get bet sizing
- Find portfolio size that profits given some minimum bet
  - At what point will my kelly bet be large enough to offset my min bets
- Game Parameters
  - Hard 17
  - 3-2 Blackjack odds
  - 4 decks, 75% penetration (unrealistic)
  - $10 min bet, $25,000 max bet, $1,000 portfolio

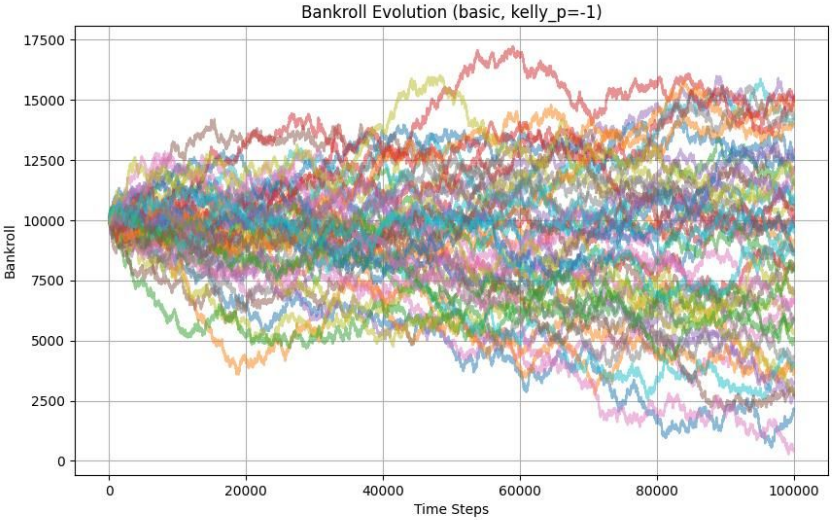
Blackjack - Simulation

- Policies
  - Basic strategy for play, min bet sizing
  - Hi-Lo strategy for play, card counting for bet sizing
  - ½ Kelly, 3x Kelly for bet sizing strategies
- Win estimation
  - Rough estimate scaling based on curr count
  - Exact probability via simulation/dp
- Simulation Parameters
  - 100,000 hands per trial
  - 50 trials per strategy
-

Blackjack - Simulation Results - Basic strategy

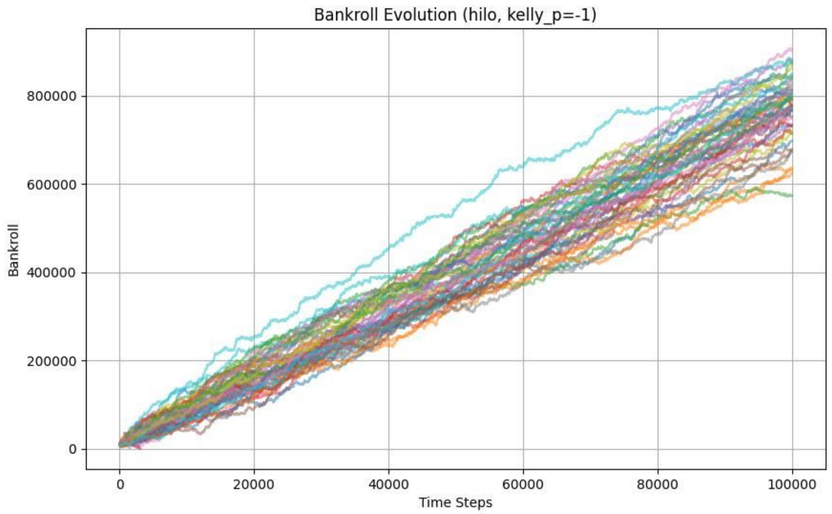
- Avg return per trial: -$1,258.50
- Only min bets
- Do NOT gamble

1-800-GAMBLER
HELP FOR PROBLEM GAMBLING

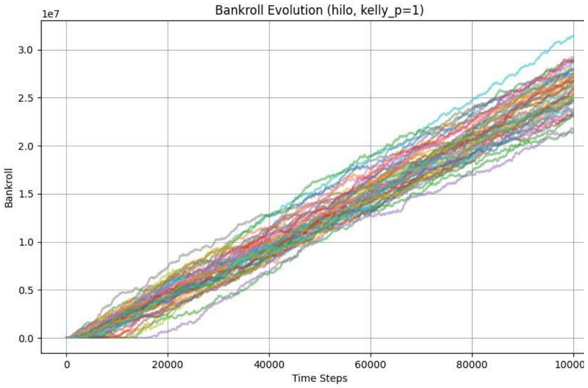


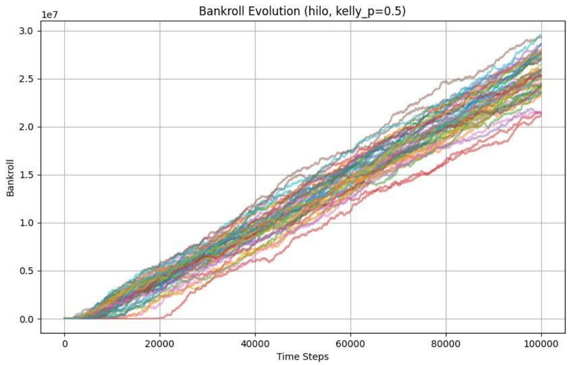
Blackjack - Simulation Results - HiLo Strategy, Constant Betting

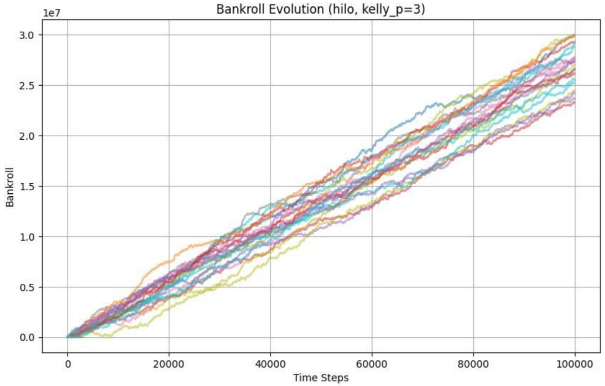
- Avg return per trial: $730,576.10
- Betting Size
  - Min bets when win-prob negative
  - Bets 100x min bet when win-prob positive
- Downsides?



Blackjack - Simulation Results - HiLo counting, Kelly betting







Full Kelly: $25,530,107.44

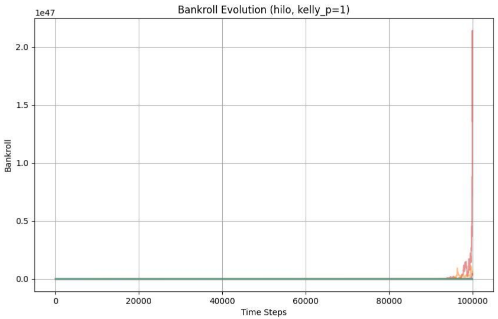
½ Kelly: $25,562,709.29

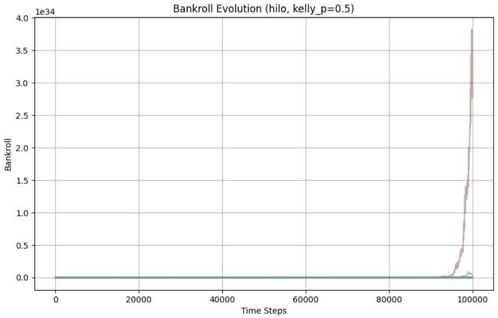
3x Kelly: $11,347,260.07

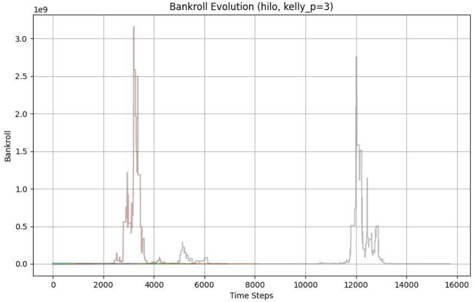
Findings corroborated by [***A Novel Approach of Option Portfolio Construction Using the Kelly Criterion***]()

Kelly tells us we grow exponentially? :(

Blackjack - Simulation Results - HiLo counting, Kelly betting, no max







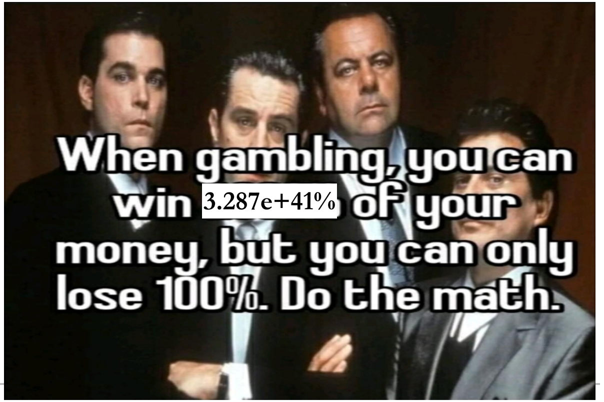
Full Kelly: $3.28779e+45

½ Kelly: $5.69789e+32

3x Kelly: -$9996.34

Previous slide was linear due to us imposing a max bet of $25,000

Uses rough estimate of p(win) using current card count



Game Conclusions/Future Work

Conclusions

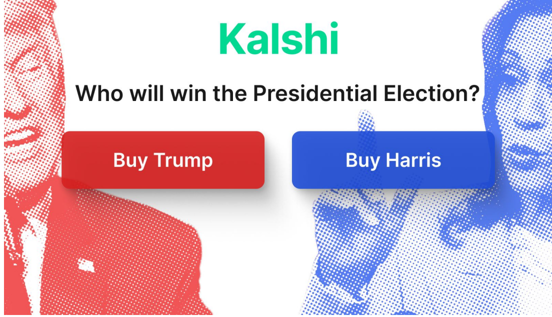
- Estimating probability of winning is difficult outside the basic games
    - Developing good estimators for win rate is a problem
    - Fractional Kelly can be very useful
- Don't be a degen and martingale when u lose
- Geometric growth >> linear growth

Future Work

- Fix and Optimize Dynamic Programming Estimate and Test w/ Kelly
- Find optimal strategy for blackjack when we know full composition
- RL agents for more complex games to estimate probability?

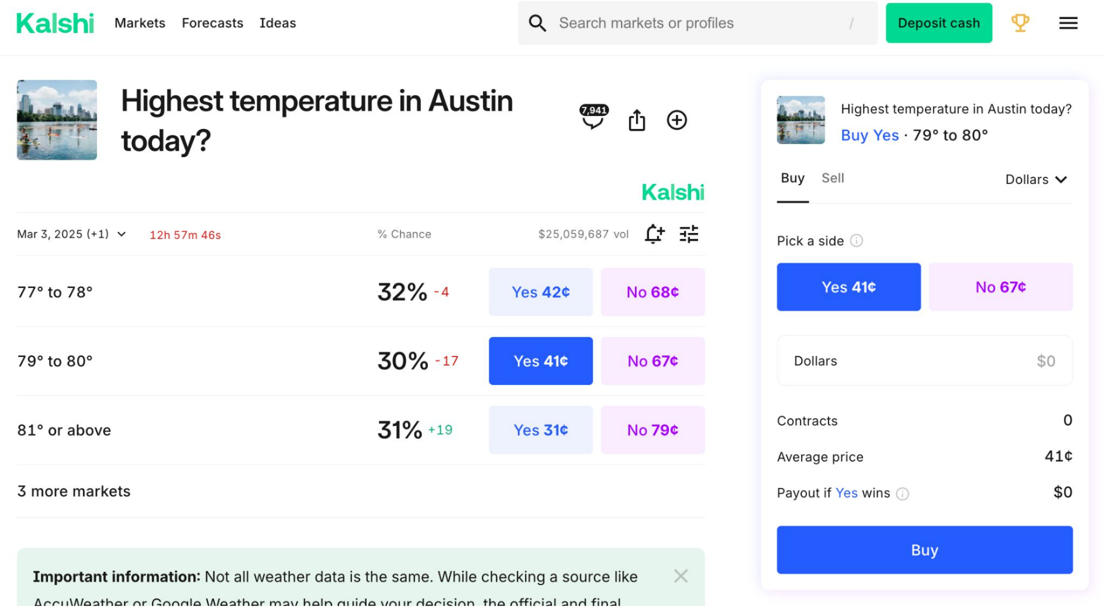
Kalshi

Market on Highest Temperature in Austin, TX



Strategy:

1. Retrieve Kalshi orderbook to find starting price at market open.
2. Check Visual Crossing API to get forecasted next-day high temperature in Austin.
3. Calculate Kelly to determine position size, and buy contracts up to $0.50.
4. Contracts maturity is 2 days after the market closes, so we don't compound every day.



Weather & Kalshi Data

Kalshi

- Kalshi lacks orderbook details to monitor minute by minute pricing which we needed for our Kelly calculation.
- Kalshi has a non-public API endpoint that contains approximate hour by hour pricing data and quantity of contracts available.
- It's important to note that Kalshi's API only had data up to November 1st, 2024

Weather

- Kalshi utilizes National Weather Service as the official weather report for Austin Bergstrom.
- We utilized Visual Crossing and used coordinates to get a forecasted high temp at Austin Bergstrom.
- Visual Crossing had an a 55% win rate compared to the actual high.

Code Snippet

```
# Calculate bet amount (Kelly %)
bet_amount = min(available_capital * 0.325, 1500)

# Check if our forecasted temperature falls within the correct range
forecast_correct = temp_in_range(temp, correct_range)

# Calculate outcome
if forecast_correct:
    # Win: Double the bet amount
    profit = bet_amount  # (We get our bet back plus the same amount)
    waiting_profits += profit  # Add profit to waiting queue
    day_result = "win"
    wins += 1
else:
    # Lose: Lose the entire bet
    profit = -bet_amount
    available_capital += profit  # Immediately reduce available capital
    portfolio_value += profit
    day_result = "lose"
    losses += 1
```

Kelly Calculation

Using Forecasted Weather Data compared with Actual Weather data, we deduced a 55% win rate using predicted max temperature data on Visual Crossing.

Restrictions:

- Not enough contracts at initial price of $0.30
- Price goes up as we buy more contracts

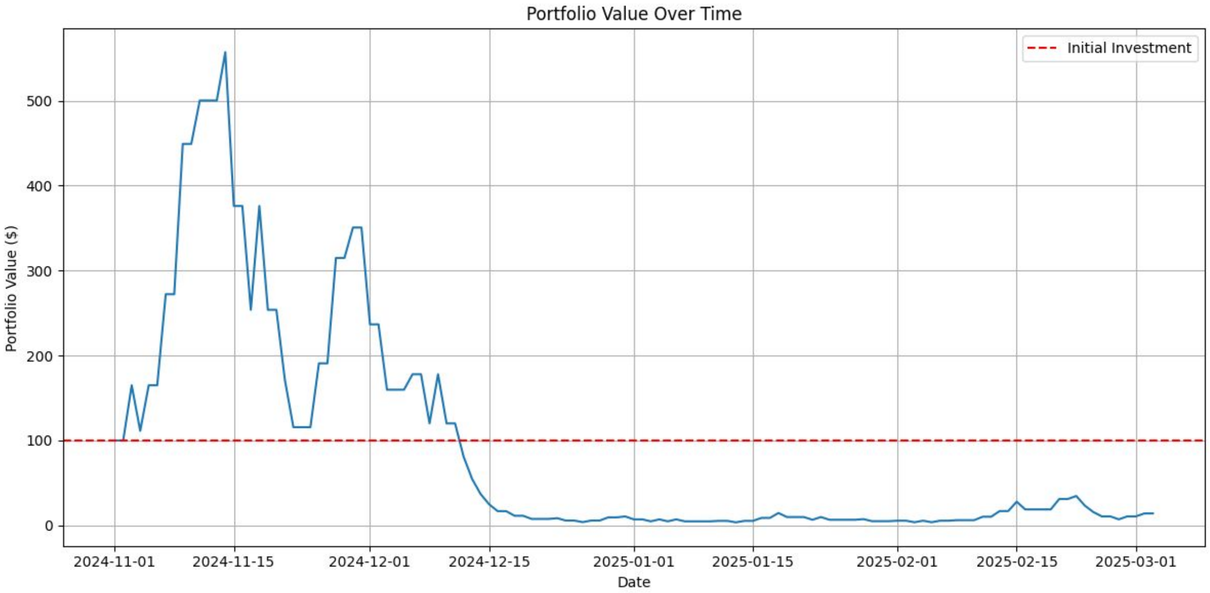
We lose all our money if we're wrong, ~double our money if we're right. ~2:1 odds

Kelly Formula:

Bet K% = 55%/1 - 45% / 2 = 32.5%

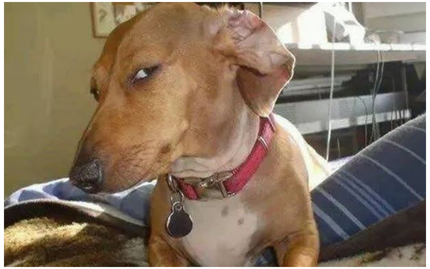
Actual Kelly: 32.5% - Bankruptcy

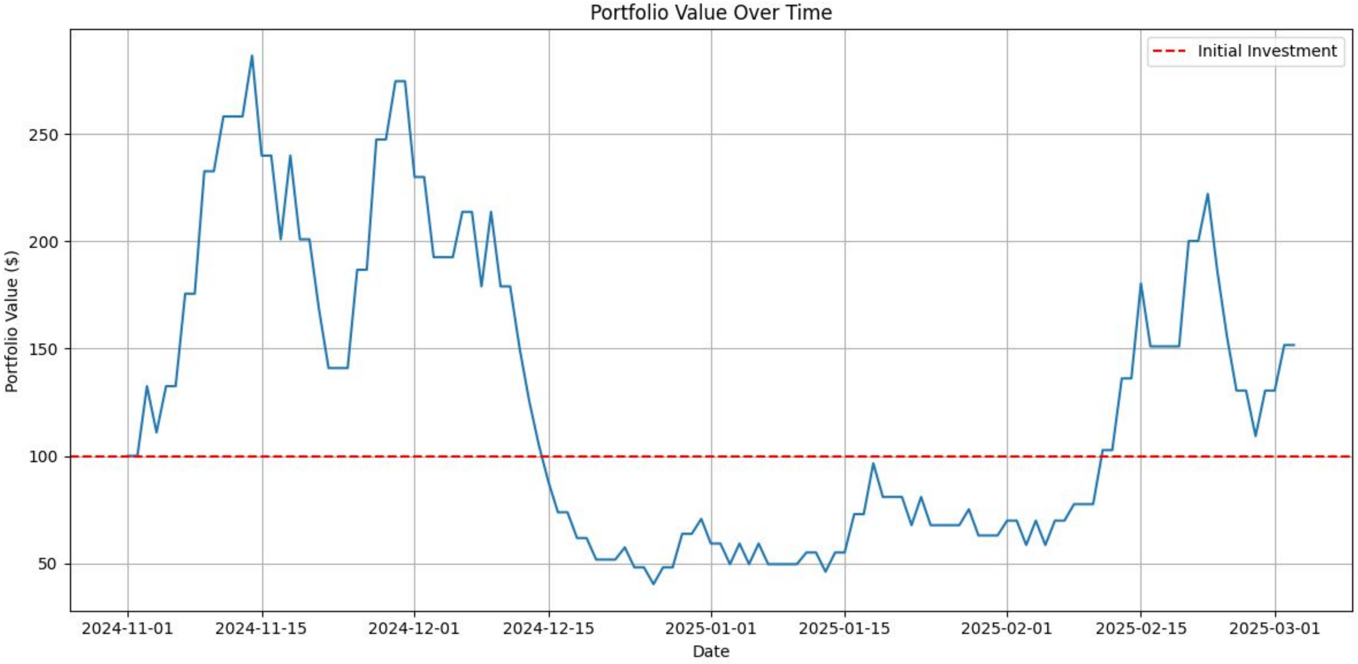
Backtesting Results:
Starting Portfolio: $100.00
Final Portfolio: $14.12
Total Return: -85.88%
Number of Trades: 120
Daily Sharpe Ratio: -0.0874
Annualized Sharpe Ratio: -1.6699
Maximum Drawdown: -99.35%



Fractional (Half) Kelly: 16.25%

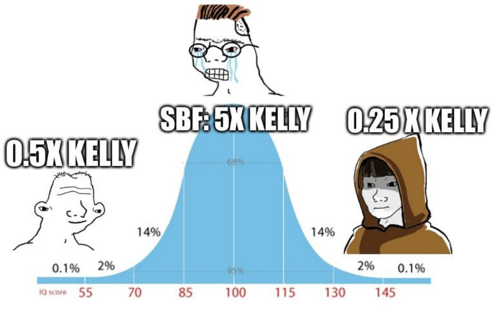
Backtesting Results:
Starting Portfolio: $100.00
Final Portfolio: $151.64
Total Return: 51.64%
Number of Trades: 120
Daily Sharpe Ratio: 0.0927
Annualized Sharpe Ratio: 1.7715
Maximum Drawdown: -85.95%

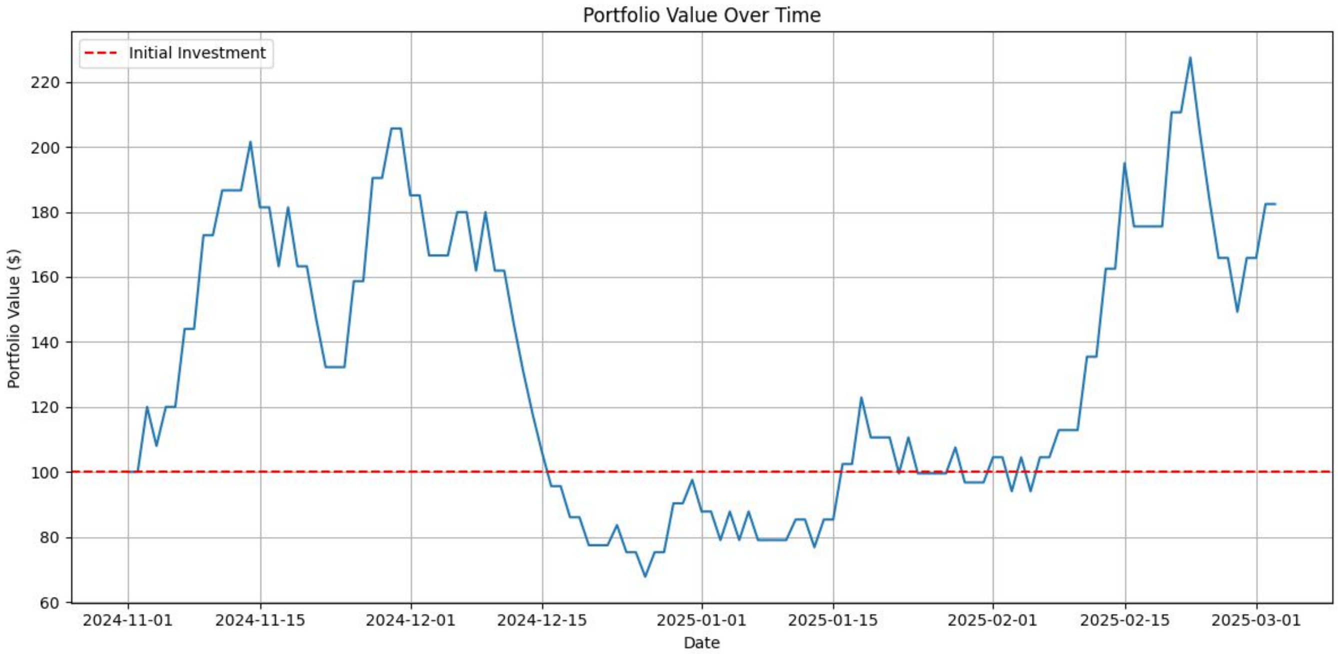




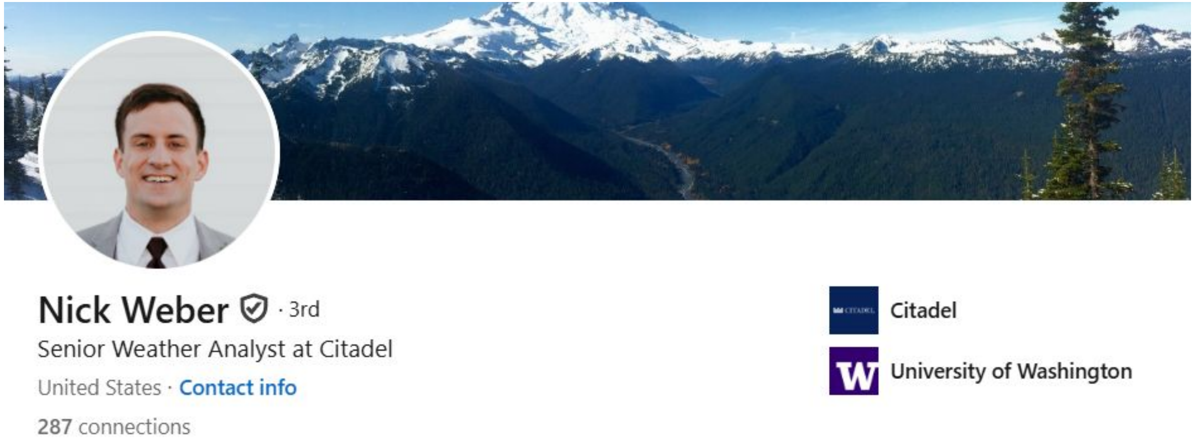
Quarter-ish Kelly: 10%

Backtesting Results:
Starting Portfolio: $100.00
Final Portfolio: $182.41
Total Return: 82.41%
Number of Trades: 120
Daily Sharpe Ratio: 0.0964
Annualized Sharpe Ratio: 1.8423
Maximum Drawdown: -67.06%





You could be this guy:



- We only had 4 months of data points to test on, and due to the volatility, our results could be inaccurate. A better way to test this in the future would be to manually paper trade this strategy for a few months.
- Fractional Kelly is Better, because full Kelly doesn't account for the risk.

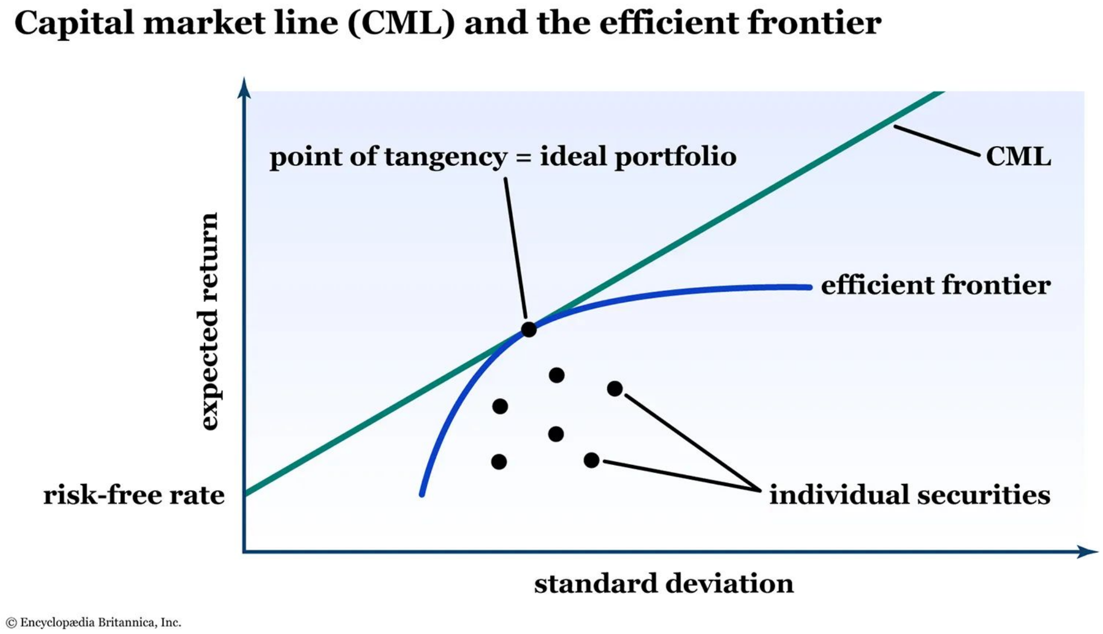
Portfolio Optimization

Markowitz Mean Variance Optimization

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \operatorname{Cov}(r_i, r_j)$$

Goal: maximize SHARPE



Markowitz

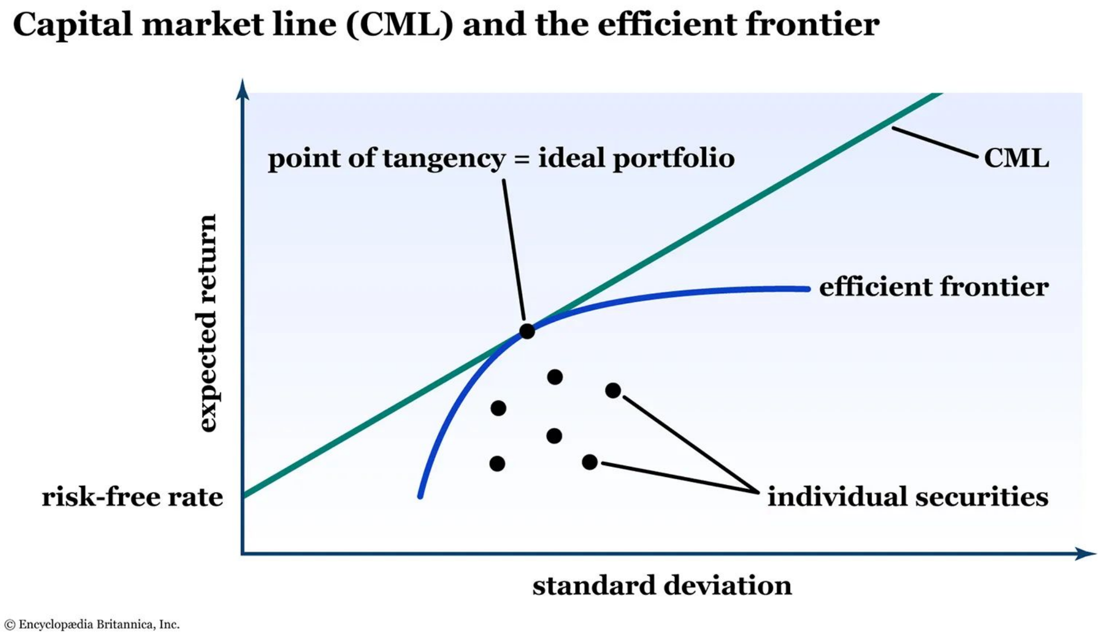
$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \, \mathrm{Cov}(r_i, r_j)$$

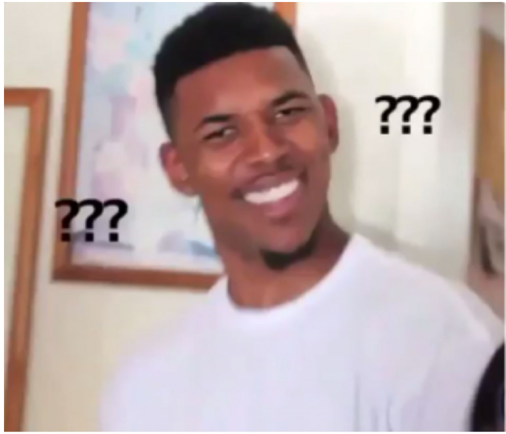
Goal: maximize SHARPE

= excess returns / unit of risk

= (E[Rp] - rf) / σ_p^2



Kelly Derivation



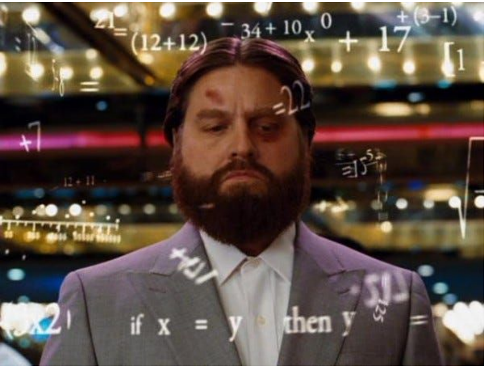
$$\boxed{W_{t+1} = W_t\,(1 + R_t)}$$

$$\ln(W_{t+1}) = \ln(W_t) + \ln(1 + R_t)$$

$$\max\ \mathbb{E}\left[\ln(W_{t+1})\right] = \max\ \mathbb{E}\left[\ln(1 + R_t)\right]$$

$$\boxed{\ln(1 + x) \approx x - \frac{x^2}{2}}$$

(taylor series expansion)



$$\mathbb{E}[\ln(1 + x)] \approx \mathbb{E}[x] - \tfrac{1}{2}\,\mathbb{E}[x^2].$$

$$\boxed{\mathbb{E}[\ln(1 + x)] \approx \mathbb{E}[x] - \tfrac{1}{2}\,\mathrm{Var}(x).}$$

Kelly Derivation

$$\boxed{\mathbb{E}[\ln(1+x)] \;\approx\; \mathbb{E}[x] \;-\; \tfrac{1}{2}\,\mathrm{Var}(x).}$$

where $\mathrm{Var}(R_p) = \sum_{i=1}^{n} w_i^2\,\mathrm{Var}(R_i) + 2\sum_{i<j} w_i\,w_j\,\mathrm{Cov}(R_i, R_j). = \boxed{F^T\,\Sigma\,F,}$

&

$$R_p = \boxed{r_f + F^T(R - r_f)}$$

$$\max_F \left[\;\underbrace{\mathbb{E}[R_p]}_{\text{linear term}} \;-\; \tfrac{1}{2}\;\underbrace{\mathrm{Var}(R_p)}_{\text{quadratic term}}\;\right] = \boxed{\max_F \left[r + F^T(M - R) - \tfrac{1}{2}\,F^T\Sigma F\right].}$$

where 0 ≤ f ≤ 1, and ∑F^t = 1

Markowitz vs Kelly

Markowitz	Kelly
- Goal: maximize SHARPE ratio (highest excess return per unit of risk) - Often leads to more diversified portfolios - Risk return tradeoff based off your constraints	- Goal: maximize long term log growth of wealth → CAN COME AT EXPENSE OF DIVERSIFICATION - Can often lead to riskier portfolio with higher returns - Fixed risk penalty (quadratic)

$$\text{Maximize } \frac{F^T(M - r_f)}{\sqrt{F^T \Sigma F}},$$

$$g(F) \approx r_f + F^T(M - r_f) - \frac{1}{2} F^T \Sigma F.$$

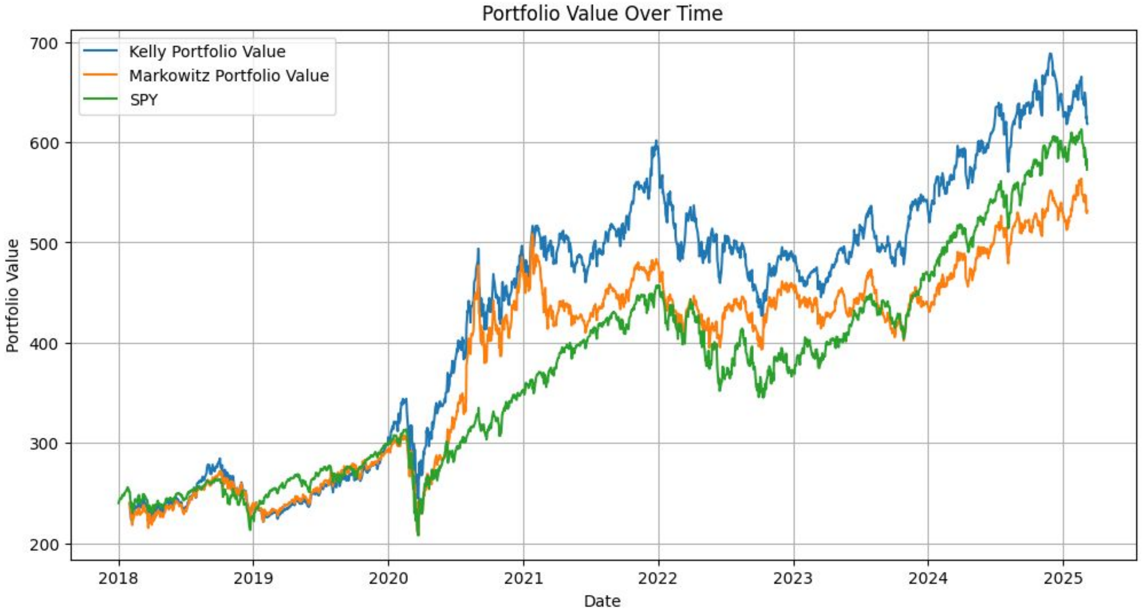
Portfolio Optimization Process

- Followed paper *Practical Implementation of the Kelly Criterion: Optimal Growth Rate, Number of Trades, and Rebalancing Frequency for Equity Portfolios*
- Process:
  - Calculated expected return and volatility for a rolling 1 year trailing window
  - Rebalanced portfolio based off new optimized trading dates for our new 1 year trailing window
  - For both markowitz and kelly, solved our same optimization problem
- Stocks we tested over:
  - Cracked S&P Basket (no nvda)
  - S&P Sector ETFs

Portfolio Optimization Results - Cracked S&P Stocks

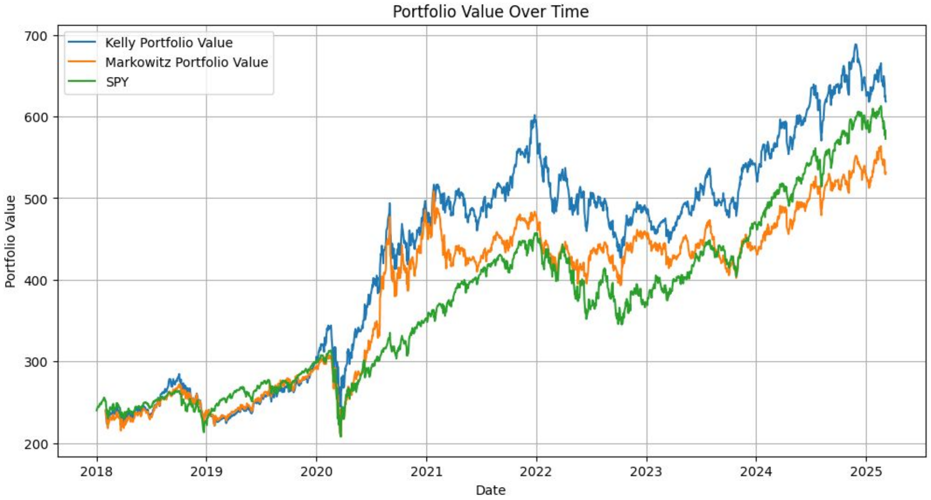
```python
stocks = [
    "AAPL",  # Apple.
    "MSFT",  # Microsoft
    "JNJ",   # Johnson & Johnson
    "PG",    # Procter & Gamble
    "JPM",   # JPMorgan Chase
    "VZ",    # Verizon
    "PFE",   # Pfizer Inc.
    "KO",    # Coca-Cola
    "DIS",   # Disney
    "PEP"    # PepsiCo
]
```

Me listening to an earnings call for a company that I own 1 share in



Portfolio Optimization Results - Cracked S&P Stocks - Portfolio Breakdown

Kelly Returns: 146%
Markowitz Returns: 115%
SPY Returns: 140%



Kelly

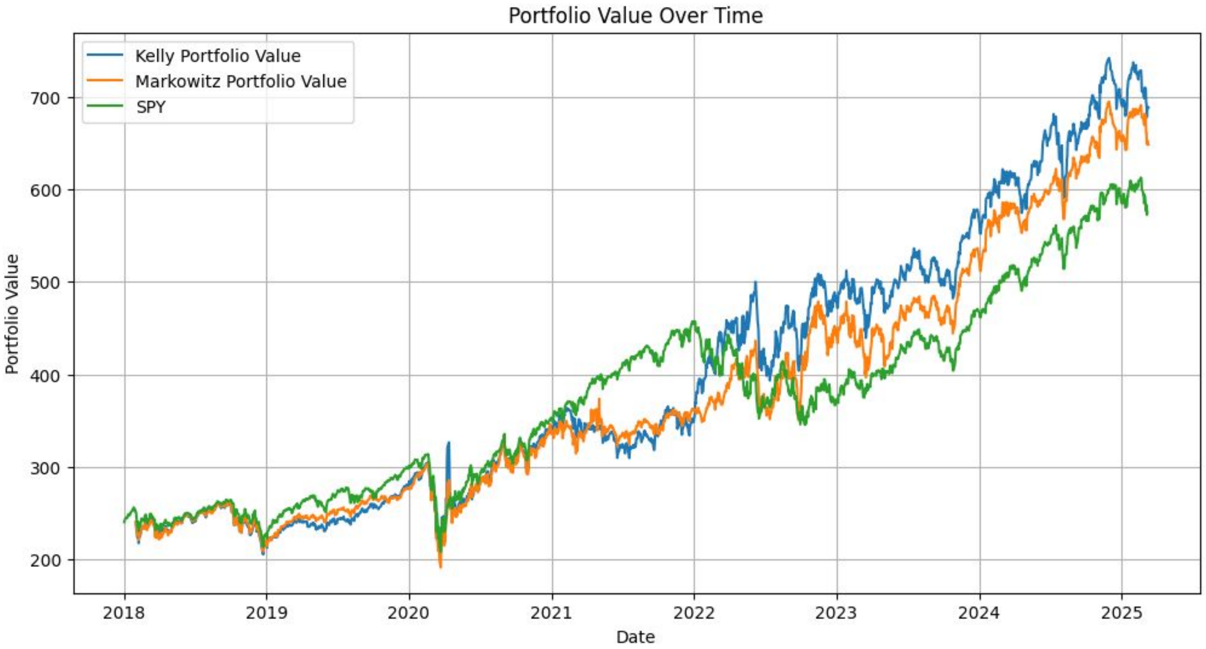
	Stock	Weight
0	AAPL	0.500
1	JPM	0.269
2	VZ	0.231
3	PFE	0.000
4	JNJ	0.000
5	DIS	0.000
6	MSFT	0.000
7	PEP	0.000
8	KO	0.000
9	PG	0.000

Markowitz

	Stock	Weight
0	KO	0.351
1	AAPL	0.280
2	VZ	0.206
3	JPM	0.163
4	JNJ	0.000
5	PEP	0.000
6	DIS	0.000
7	PFE	0.000
8	PG	0.000
9	MSFT	0.000

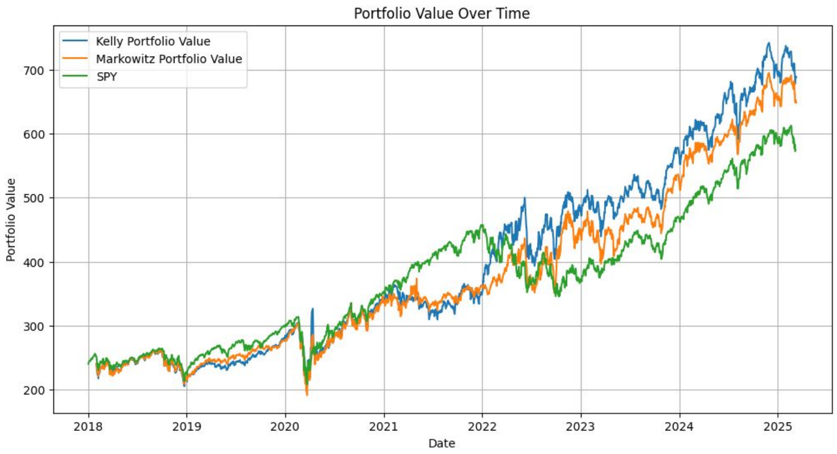
Portfolio Optimization Results - S&P Sector ETFs

```
stocks = [
    "XLY",  # Consumer Disc
    "XLP",  # Consumer Staples
    "XLE",  # Energy
    "XLF",  # Financials
    "XLV",  # Health Care
    "XLI",  # Industrials
    "XLB",  # Materials
    "XLK",  # Technology
    "XLU",  # Utilities]
]
```



Portfolio Optimization Results - S&P Sector ETFs - Portfolio Breakdown

Kelly = 187%
Mark = 170%
SPY = 140%



Kelly

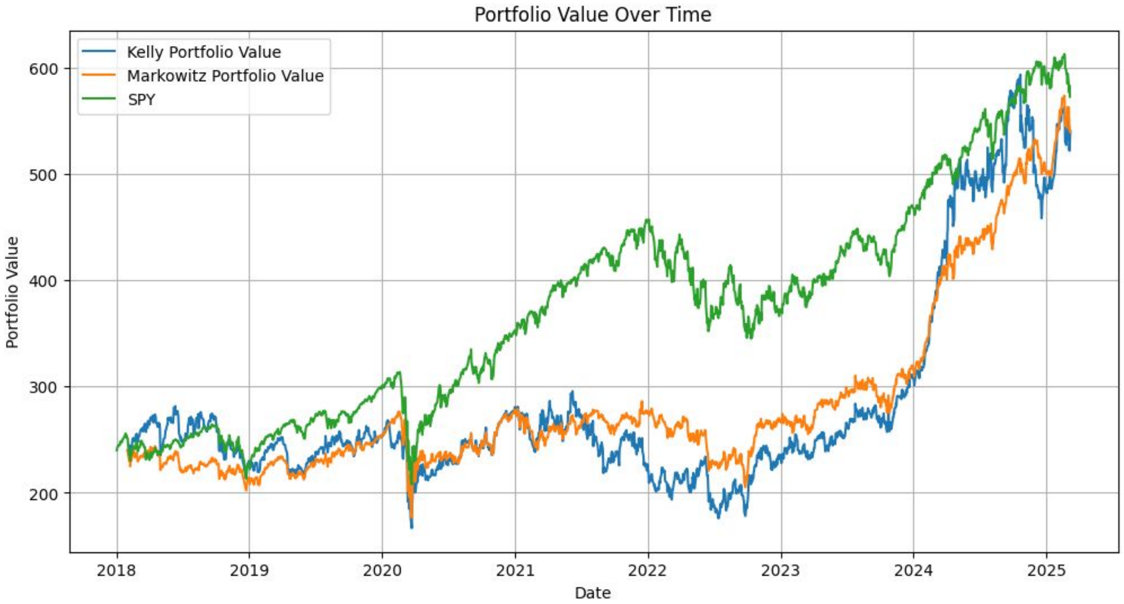
	Stock	Weight
0	XLU	0.75
1	XLF	0.25
2	XLB	0.00
3	XLE	0.00
4	XLK	0.00
5	XLI	0.00
6	XLV	0.00
7	XLP	0.00
8	XLY	0.00

Markowitz

	Stock	Weight
0	XLU	0.656
1	XLF	0.335
2	XLV	0.000
3	XLK	0.000
4	XLY	0.000
5	XLB	0.000
6	XLI	0.000
7	XLE	0.000
8	XLP	0.000

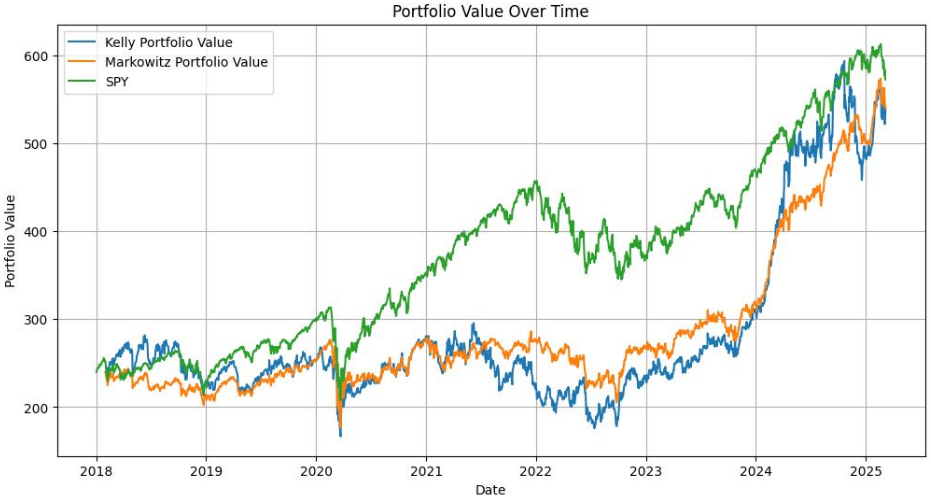
Portfolio Optimization Results - EuroStoxx50 Comps

```
stocks = [
    "NKE",  # Nike (for Adidas)
    "JPM",  # JPMorgan Chase (for Banca Intesa)
    "PFE",  # Pfizer (for Bayer)
    "WFC",  # Wells Fargo (for BBVA)
    "C",    # Citigroup (for BNP Paribas)
    "WMT",  # Walmart (for Carrefour)
    "VZ",   # Verizon (for Deutsche Telekom)
    "GM",   # General Motors (for Daimler)
    "BAC",  # Bank of America (for Deutsche Bank)
    "NEE",  # NextEra Energy (for Enel)
    "DUK",  # Duke Energy (for Engie)
    "XOM",  # ExxonMobil (for Eni)
    "PRU",  # Prudential Financial (for Generali)
    "EL",   # Estée Lauder (for L'Oreal)
    "RL",   # Ralph Lauren (for LVMH)
    "CSCO", # Cisco (for Nokia)
    "T",    # AT&T (for Orange)
    "GE",   # General Electric (for Philips)
    "LMT",  # Lockheed Martin (for Safran)
    "MRK",  # Merck (for Sanofi)
    "PNC",  # PNC Financial Services (for Santander)
    "ORCL", # Oracle (for SAP)
    "HON",  # Honeywell (for Schneider Electric)
    "RTX",  # Raytheon Technologies (for Siemens)
    "TMUS", # T-Mobile US (for Telefonica)
    "CVX",  # Chevron (for Total)
    "PG",   # Procter & Gamble (for Unilever)
]
```



Portfolio Optimization Results - EuroStoxx50 Comps - Portfolio Breakdown

Kelly = 126%
Mark = 121%
SPY = 140%



Kelly

	Stock	Weight
0	TMUS	0.5
1	T	0.5
14	VZ	0.0
25	GE	0.0
24	HON	0.0
23	CSCO	0.0
22	RL	0.0
21	CVX	0.0
20	PRU	0.0
19	XOM	0.0
18	DUK	0.0

Markowitz

	Stock	Weight
0	T	0.295
1	WMT	0.196
2	TMUS	0.195
3	RTX	0.141
4	CSCO	0.056
5	ORCL	0.047
6	GE	0.046
7	RL	0.016
8	NEE	0.008

Kelly Portfolio Optimization Conclusion

- Findings were corroborated by the paper:

  "These findings are consistent with the previous literature… which called this phenomenon **"portfolio condensation,"** and with Estrada [4] which reports that **portfolios built under the Kelly criterion are less diversified, have a higher expected return, and higher risk compared to those composed with the goal of maximizing risk-adjusted returns."**
- Avoid conflating long-term survival with decreased risk
  - Long-term survival is purely maxing out compounded growth
- "If the gamble is favorable or **the probability distribution of returns is known,** or can it be estimated correctly, **no other strategy can beat the Kelly criterion** in the long run if it is followed diligently"

Future Ideas + Conclusion

- Portfolio optimization: Optimize rebalancing period vs lookback period
  - Applications in options ?
  - Better basket of stocks to test over


Source: Trust me bro
POV: OUR PITCH
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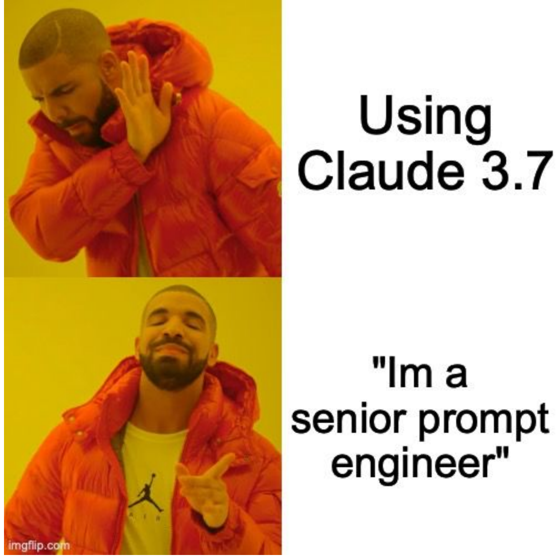
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Questions?