The approximating polynomial in two form.

① Lagrange's formulation

② Newton's forward difference formula

① ⇒ In the Lagrange's form $P(x) = \sum_{i=1}^{K} L_i(x)\, y_i$

where $L_i = \dfrac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x\ldots}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1}\ldots}$

$\therefore \int_{x_0}^{x_K} f(x)\,dx = \int_{x_0}^{x_K} \sum_{i=0}^{k} L_i(x)\, y_i\, dx = \sum_{i=0}^{k} y_i \int_{x_{\ldots}}$

② ⇒ When the values of the function $y=f(x)$ are provi[...] equally spaced abscissas say $h = x_i - x_{i-1}$

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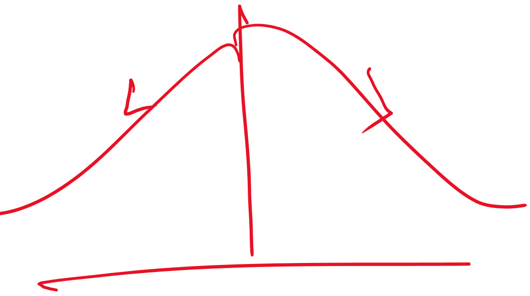
$P(p) = y_0 + p\,\Delta y_0 + \dfrac{p(p-1)}{2!}\Delta^2 y_0 + \cdots\cdots + P(\ldots$



$$\frac{- x_{i+1}) \cdots (x - x_n)}{) (x_i - x_{i+1}) \cdots (x_i - x_k)}$$

x_k

$L_i(x)\, dx$

o

ded at

$i = 1, 2, \cdots n$

$$\frac{(p-1) \cdots (p-k+1)}{k!} \Delta^k y_0$$

$$P(x) = y_0 + \frac{(x-x_0)}{h}\Delta y_0 + \frac{(x-x_0)(x-x_1)}{2!\,h^2}\Delta^2 y_0 + \cdots$$

$$\cdots + \frac{(x-x_0)(x-x_1}{k!\,h}$$

$+$

$$x = x_0 + ph \Rightarrow p = \frac{x-x_0}{h}$$

$x_1 = x_0 + h$

$x_2 = x_0 + 2h$

$$\therefore \int_{x_0}^{x_k} P(x)\,dx = \int_{x_0}^{x_k} f(x)\,dx = h\int_0^k P(p)\,dp$$

$$\text{Error} = E = \int_{x_0}^{x_k} R(x)\,dx \text{ . where}$$

$$R(x) = \frac{p(p-1)(p-2)\cdots(p-}{(k+1)!}$$

$$= \frac{(x-x_0)(x-x_1)\cdots(x}{(k+1)!}$$

$$E = \int_{x_0}^{x_k} R(x)\,dx$$

$$= \int_{x_0}^{x_k} \frac{(x-x_0)(x-x_1)\cdots(x-x_k)}{(k+1)} f^{k+1}(\xi)\,dx$$

$$= \frac{f^{k+1}(\xi)}{(k+1)!}\int_{x_0}^{x_k}(x-x_0)(x-x_1)\cdots(x-x_k)\,dx$$

$$h^{k+2}\,f^{k+1}(\xi)\int^{k} p(p-1)\cdots(p-k)\,dp$$

R_n

$$\frac{\ldots)\cdots(x-x_{k-1})}{h^k}\Delta^k y_0$$

$$x = x_0 + ph$$

$$\Rightarrow dx = h\,dp$$

$$x_n = x_0 + ph$$

$$x_n - x_0 = ph$$

$$\frac{x_n - x_0}{h} = \frac{ph}{h} \qquad p = n$$

$$= \frac{kh}{h} = k$$

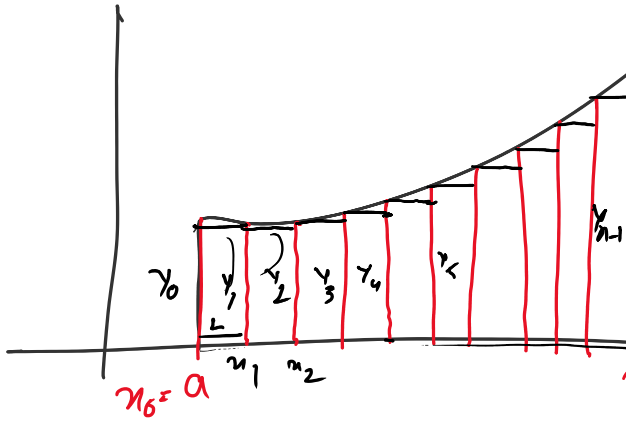
$$\underline{k)}$$

$$\underline{-x_k)}\, f^{k+1}(\xi)$$

$$\underline{a \le \xi \le b}$$

$$= \frac{h^{k+2} f^{k+1}(\xi)}{(k+1)!} \int_0^k p(p-1) \cdots (p-k)\, dp$$

Rectangular Rule



(x_0, x_1)

$$\int_{x_0}^{x_1} f(x)\,dx = y_0 \int_{x_0}^{x_1} dx$$

$$= y_0 (x_1 - x_0)$$

$$= h y_0$$

$\underline{x_0 - x_2}$ $\quad \int_{x_0}^{x_1} + \int_{x_1}^{x_2}$

$$\int_{x_0}^{x_n} f(x)\,dx = \int_{x_0}^{x_n} y(x)\,dx = \left[\int_{x_0}^{x_1} + \int_{x_1}^{x_2} + \cdots + \int_{x_{n-1}}^{x_n} \right]$$

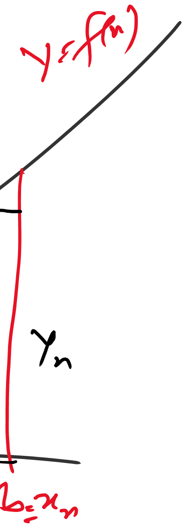
$$= h y_0 + h y_1 + h y_2 + \cdots + h y_{n-1}$$

$$\boxed{\int_{x_0}^{x_n} y(x)\,dx = h(y_0 + y_1 + \cdots + y_{n-1})}$$

This is known as Composite formula for n int

Monotonically increasing function

$$\int^b f(x)\,dx > h(y_0 + y_1 + \cdots + y_{n-1})$$



at $x = x_0$

$f(x)$ at x_0

$\underline{f(x_0)} = y_0$

$\int y(x)\, dx$

terval

function $\int_a^b f(x)\,dx > h\,(y_0 + y_1 + \dots + y_{n-1})$

Monotonically decreasing function

$$\int_a^b f(x)\,dx < h\,(y_0 + y_1 + \dots + y_{n-1})$$