Indian Institute of Technology Indore
MA203 Complex Analysis and Differential Equations-II
(Autumn Semester 2023)
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Tutorial Sheet 2

1. Form a PDE from $z = (x^2 + a)(y^2 + b)$ by eliminating arbitrary constants.

2. Eliminate the arbitrary constants a, b and c from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{1}$$

   and form the PDE.

3. Find PDEs by eliminating constants from the relation $z = ae^{-b^2 y} \cos bx$.

4. Form a PDE from $z = f(y/x)$ by eliminating arbitrary function.

5. Form a PDE by eliminating arbitrary function $z = x + y + f(x, y)$. Assume f' exists.

6. Solve the following for general solution

$$p + q = \cos x, \tag{2}$$

   where $p = \dfrac{\partial z}{\partial x}$, $q = \dfrac{\partial z}{\partial y}$.

7. Find the general solution of the following Lagrange's linear equation

$$(p - q)(x + y) = z. \tag{3}$$

8. Classify the following equations: (elliptic, hyperbolic and parabolic)

   1. $2\dfrac{\partial^2 u}{\partial x^2} + 4\dfrac{\partial^2 u}{\partial x \partial y} + 3\dfrac{\partial^2 u}{\partial y^2}$
   2. $\dfrac{\partial^2 u}{\partial t^2} + t\dfrac{\partial^2 u}{\partial x \partial t} + x\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} + 6u = 0.$
   3. $x^2\dfrac{\partial^2 u}{\partial t^2} + 3\dfrac{\partial^2 u}{\partial x \partial t} + x\dfrac{\partial^2 u}{\partial x^2} + 17\dfrac{\partial u}{\partial t} = 100u.$

9. Classify the following differential equation as to type in the second quadrant of $xy-$plane

$$\sqrt{y^2 + x^2 u_{xx}} + 2(x - y)u_{xy} + \sqrt{y^2 + x^2 u_{yy}} = 0. \tag{4}$$

10. Applying the method of separation of variables techniques, find the solution to the P.D.E

    1. $3u_x + 2u_y = 0.$
    2. $u_x = 2u_t + u$ where $u(x, 0) = 6e^{-3x}.$
    3. $\dfrac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$, given that $u = 0$ when $t = 0$ and $\dfrac{\partial u}{\partial t} = 0$ when $x = 0.$