

INDIAN INSTITUTE OF TECHNOLOGY INDORE  
MA-204 NUMERICAL METHODS  
**Assignment -2- System of Linear equations**

1. Solve the following system of equations by Gauss-Jacobi method with initial approximation other than  $(1, 1, 1)$ :

$$\begin{aligned}8x + 2y - 2z &= 8 \\x - 8y + 3z &= -4 \\2x + y + 9z &= 12.\end{aligned}$$

2. Solve the following linear system by Jacobi method with  $X^{(0)} = (0, 0, 1)^T$ . Find out three iterations:

$$\begin{aligned}10x_1 + 3x_2 + x_3 &= 14 \\2x_1 - 10x_2 + 3x_3 &= -5 \\x_1 + 3x_2 + 10x_3 &= 14.\end{aligned}$$

3. Solve the system of equations

$$\begin{aligned}4x + y + 2z &= 4 \\3x + 5y + z &= 7 \\x + y + 3z &= 3\end{aligned}$$

by the Jacobi and Gauss-Seidel methods. In each case continue the iteration up to three steps starting with initial approximation  $x = 0, y = 0, z = 0$ .

4. Give an example of a matrix

- (a) which is Strictly-Row Diagonally Dominate but not Positive Definite.
- (b) which is Symmetric Positive Definite matrix but not Strictly-Row Diagonally Dominate.

5. Rewrite the following system of equations in two different ways so that the Jacobi iteration scheme converges:

$$\begin{aligned}3x_1 - 5x_2 + 47x_3 + 20x_4 &= 18 \\12x_1 + 16x_2 + 17x_3 + 50x_4 &= 25 \\17x_1 + 65x_2 - 13x_3 + 7x_4 &= 84 \\56x_1 + 23x_2 + 11x_3 - 19x_4 &= 36.\end{aligned}$$

6. Solve the following system of equations by Gauss-Seidel iterative method.

$$\begin{aligned}2x - y &= 7 \\-x + 2y - z &= 1 \\-y + 2z &= 1\end{aligned}$$

Use infinity norm in the following questions..

7. If  $A = \begin{pmatrix} \alpha/10 & \alpha/10 \\ 1 & 3/2 \end{pmatrix}$ , then show that  $\text{cond}(A)$  is minimum if  $\alpha = 12.5$  and the minimum is 11.
8. If  $A = \begin{pmatrix} 1 & \alpha \\ \alpha & 1 \end{pmatrix}$ ,  $\alpha \neq 1$ , then find  $\text{cond}(A)$ .
9. Verify that the system of equations:

$$\begin{aligned}400x_1 - 201x_2 &= 200 \\-800x_1 + 401x_2 &= -200\end{aligned}$$

is an ill-conditioned system.