

INDIAN INSTITUTE OF TECHNOLOGY INDORE

MA 203: Complex Analysis and Differential Equations-II

Autumn Semester

Tutorial -6 (Complex Analysis)

1. Solve (i) $e^z = -1$ (ii) $e^z = 1$ Ans: (i) $z = i(2k + 1)\pi, k \in \mathbb{Z}$.
2. Prove that $\sin z = 0$ iff $z = k\pi$, and $\cos z = 0$ iff $z = k\pi + \frac{\pi}{2}$, where $k \in \mathbb{Z}$.
3. Show that (i) $\sin(\bar{z})$ and (ii) $\cosh(\bar{z})$ are nowhere analytic.
4. Show that
 - (a) $\text{Log}(1 + i)^2 = 2\text{Log}(1 + i)$
 - (b) $\text{Log}(-1 + i)^2 \neq 2\text{Log}(-1 + i)$
 - (c) $\log(i^2) \neq 2\log(i)$
5. Find all roots of the equation $\log z = i\pi/2$. Ans: i
6. Find a parametric representation $\gamma : [a, b] \rightarrow \mathbb{C}$ for the following curves:
 - (a) The straight-line segment from 0 to $4 - 7i$ Ans: $\gamma(t) = (4 - 7i)t, 0 \leq t \leq 1$
 - (b) The upper half of $|z - 4 + 2i| = 3$ Ans: $\gamma(t) = 4 - 2i + 3(\cos t + i \sin t), 0 \leq t \leq \pi$
 - (c) $|z + 3 - i| = 5$, counter clockwise Ans: $\gamma(t) = -3 + i + 5(\cos t + i \sin t), 0 \leq t \leq 2\pi$
 - (d) $|z + 3 - i| = 5$, clockwise Ans: $-3 + i + 5(\cos(2\pi - t) + i \sin(2\pi - t)), 0 \leq t \leq 2\pi$
7. Give two different parametric representations of $y = \frac{1}{x}$ from $(1, 1)$ to $(4, \frac{1}{4})$.
8. Which properties such as smooth, piecewise smooth, closed, simple are applicable to the following curves
 - (a) $\gamma_1(t) = t + i|t|, t \in [-1, 1]$
 - (b) $\gamma_2(t) = |t| + it, t \in [-1, 1]$
 - (c) $\gamma_3(t) = |t^3| + it^3, t \in [-1, 1]$
9. Examine if the following curve is piecewise smooth for $t \in [-\pi, \pi]$

$$\gamma(t) = \begin{cases} t(1 + it \sin(1/t)), & t \neq 0 \\ 0, & t = 0. \end{cases}$$

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