

Tutorial Sheet

Tutorial: 2

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Practice Problems

1. Let $f : X \mapsto Y$ and $g : Y \mapsto Z$ be functions. Justify the following statements:
 - (i) If f and g are injective functions, then $g \circ f$ is also an injective function.
 - (ii) If f and g are surjective functions, then $g \circ f$ is also a surjective function.
2. Draw the Hasse diagram representing the partial ordering $\{(a, b) | a \text{ divides } b\}$ on the set $\{1, 2, 3, \dots, 15\}$.
3. Determine the maximal and minimal elements of the poset $(\{2, 3, 5, 6, 8, 9, 10, 12, 16, 18, 21\})$.
4. Determine whether the set of ordered pairs of non-negative integers, $(\mathbb{Z} \setminus \mathbb{Z}^-) \times \mathbb{Z} \setminus \mathbb{Z}^-$ is a well-ordered set.
5. Consider three integers, $a, b, c, \in \mathbb{Z}$. Show that if a is relatively prime to b but $a | bc$, then $a | c$. ↓ min element present always
6. Show that the set $\mathbb{Z}_n = \{0, 1, \dots, (n-1)\}$ for $n \geq 1$ is a group under addition modulo n .
7. Show that for each element a in a group G , there is a unique element b in G such that $a \cdot b = b \cdot a = e$, where e is the identity of the group.
8. Consider the group with the set $\{1, 7, 11, 13, 14\}$ under multiplication modulo 15. What is the order of the group? What is the order of element 13.
9. Solve the following recurrence relations along with the given initial conditions:
 - (i) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2, a_0 = 1, a_1 = 0$
 - (ii) $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2, a_0 = 6, a_1 = 8$
10. Consider the recurrence relation, $a_n = 2a_{n-1} + 2^n$. Find the solutions of the recurrence relation. Find the solution with $a_0 = 2$.