INDIAN INSTITUTE OF TECHNOLOGY INDORE

MA 203: Complex Analysis and Differential Equations-II

Autumn Semester 2023

Tutorial −2 (Complex Analysis)

1. Explain the geometrical meaning of the relations given in problems:
   (a) $|z - z_0| < R$, $|z - z_0| > R$, $|z - z_0| = R$.
   (b) $|z - 2| + |z + 2| = 5$.
   Ans: An ellipse with foci at the points $z = \pm 2$ and major semi-axis $5/2$.
   (c) $|z - z_1| = |z - z_2|$.
   (d) $\mathrm{Re} z \geq m$
   (e) $\mathrm{Re} z + \mathrm{Im} z < 1$.
   (f) $\mathrm{Arg}\ z = \theta_0$.
   (g) $-\pi < \mathrm{Arg}\ z < \pi$.
   (h) $\frac{\pi}{6} < \mathrm{Arg}\ z < \frac{\pi}{4}$.
   (i) $1 < |z| < 3$.
2. What are the boundary point(s) of a deleted neighborhood of a point $z_0 \in \mathbb{C}$?
3. Is the subset $D = \{z : 0 \leq \mathrm{Arg}(z) \leq \pi/4, z \neq 0\}$ open? Neither open nor closed
4. Is the subset $D = \{z : |z| < 1\} \cup \{z : |z + 2| \leq 1\}$ connected?
5. Which of the following subsets of $\mathbb{C}$ are domain?
   (a) $\{z : |z - 1| < 1\} \cup \{z : |z - 3| < 1\}$.
   (b) $\{z : |z - 1| < 1\} \cup \{z : |z - 3| < 1\} \cup \{2\}$.
   (c) $\{z : \mathrm{Re}\ z \times \mathrm{Im}\ z > 0\} \cup \{0\}$.
   (d) $\{z : \mathrm{Re}\ z \times \mathrm{Im}\ z > 0\}$.
6. (a) Is the interval $(0, 1)$ open (subset) of $\mathbb{R}$?.
   (b) Is the subset $D = \{z : x \in (0, 1), y = 0\}$ open (subset) of $\mathbb{C}$?.
7. What are the boundary point(s) of a deleted neighborhood of a point $z_0 \in \mathbb{C}$?
8. Is the subset $D = \{z : 0 \leq \mathrm{Arg}(z) \leq \pi/4, z \neq 0\}$ open?
9. Is the subset $D = \{z : |z| < 1\} \cup \{z : |z + 2| \leq 1\}$ connected?
10. Prove that the subset $D = \{z : |z| < 1\}$ is an open set.
11. Let c_0 be a limit point of S. Then prove that every neighbourhood of c_0 contains infinitely many points of S.

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