INDIAN INSTITUTE OF TECHNOLOGY INDORE

MA 203: Complex Analysis and Differential Equations-II

Autumn Semester 2023

Tutorial -1 (Complex Analysis)

1. Find the real numbers p and q such that the complex numbers $z = p + iq, \quad w = p + i\frac{1}{q}$ be equal.

2. (a) If Arg(z)=θ, determine Arg$(\overline{z})$.
   (b) If Arg(z)=θ, determine Arg(z^{-1}).
   (c) How are the complex numbers z_1 and z_2 related if $\arg(z_1) = \arg(z_2)$?
   Suggestion: Check the meaning/sense of equality here.
   (d) How are the arguments $\arg(z_1)$ and $\arg(z_2)$ related if $z_1 = z_2$?
   (e) Find two complex numbers z_1 and z_2 so that $\text{Arg}(z_1z_2) \neq \text{Arg}(z_1) + \text{Arg}(z_2)$.
   (f) Find two complex numbers z_1 and z_2 so that $\text{Arg}(z_1z_2) = \text{Arg}(z_1) + \text{Arg}(z_2)$.
   (g) When is $\text{Arg}(z_1z_2) \neq \text{Arg}(z_1) + \text{Arg}(z_2)$?
   (h) When is $\text{Arg}(z_1/z_2) \neq \text{Arg}(z_1) - \text{Arg}(z_2)$?

3. Find the principal argument of the following numbers:
   (a) $5i$
   (b) π
   (c) $\frac{1}{2} + \frac{i}{2}$
   (d) $-\frac{1}{2} - \frac{i}{2}$
   (e) $1 + \frac{i}{\sqrt{3}}$
   (f) $\frac{1}{1+i\sqrt{3}}$
   (g) $-\frac{1}{4} + \frac{i\sqrt{3}}{4}$
   (h) $\left(\frac{1}{1+i\sqrt{3}}\right)^3$
   (i) $-\frac{1+i}{1-i}$

4. Use De-Moiver's formula to derive the following trigonometric identities
   (a) $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$
   (b) $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$

5. Find all the values of the following roots
   (a) $\sqrt[3]{1}$
   (b) $\sqrt{3+4i}$ Ans: $\pm(2+i)$
   (c) $\sqrt[4]{(-1)}$ Ans: $\pm\frac{\sqrt{2}}{2}(1 \pm i)$
   (d) $\left(\frac{1}{1+i\sqrt{3}}\right)^3$ Ans: $-\frac{1}{2^3} + i0$
   (e) $\sqrt[3]{i}$

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