Indian Institute of Technology Indore
MA203 Complex Analysis and Differential Equations-II
(Autumn Semester 2023)
Instructor: Dr. Debopriya Mukherjee
Tutorial Sheet 1

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1. What are the *fundamental periods* of the following functions ($k \neq 0$)?
   $(a)\ \sin 2\pi x$ $(b)\ \cos\dfrac{x}{2}$ $(c)\ \sin\dfrac{2\pi n x}{k}$ $(d)\ |\sin 3x|$ $(e)\ \cos\left|\dfrac{x}{2}\right|$

2. Show that the function $f(x) = c$, with c being a constant, is periodic with any period but has no fundamental period.

3. If $f(x)$ and $g(x)$ have period p, show that $h(x) = a\,f(x) + b\,g(x)$ (with a and b being constants) has a period p.

4. If $f(x)$ has period p, show that $f(ax)$, $a \neq 0$, and $f(x/b)$, $b \neq 0$, are periodic functions of x of periods p/a and bp, respectively.

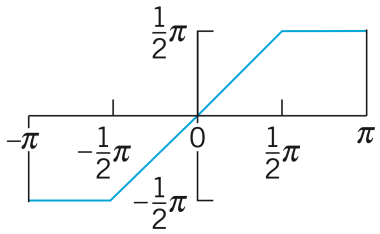
5. Find the Fourier series for the functions—given or shown in the figures—which are assumed to have the period 2π.

   $(a)\ x^2, \quad -\pi < x < \pi$

   $(b)\ x^2, \quad 0 < x < 2\pi$

   $(c)\ f(x) = \begin{cases} x & \text{if} \quad -\pi < x < 0 \\ \pi - x & \text{if} \quad 0 < x < \pi \end{cases}$

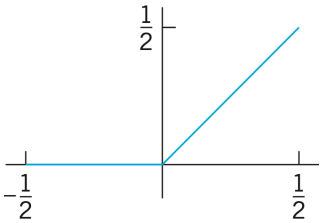
   (d)



6. Are the following functions (which are assumed to have period p) even or odd or neither even nor odd? Find their Fourier series.

   $(a)\ f(x) = x\,|x|, \quad -1 < x < 1 \quad (p = 2)$

   (b)



   (c)