

Solve the following

①

$$\frac{d^2 y}{dx^2} = xy$$

②

$$\frac{d^2 y}{dx^2} + xy = 1$$

Solⁿ(1)

$$\underline{\underline{y'' = xy}}$$

problems by finite difference

$$y(1) = 1 \quad \checkmark$$

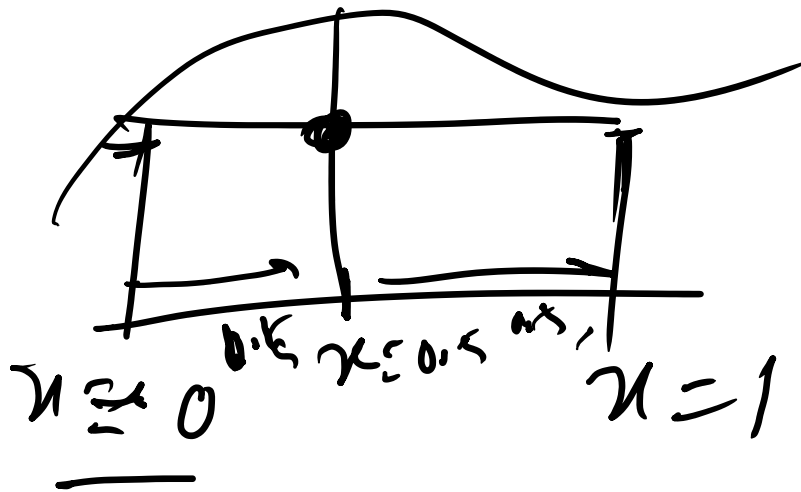
$$y(0) + y'(0) = 1 \quad \checkmark \quad h = 0.5$$

$$, \quad y(0) = 1, \quad y'(1) = 1 \quad \checkmark$$

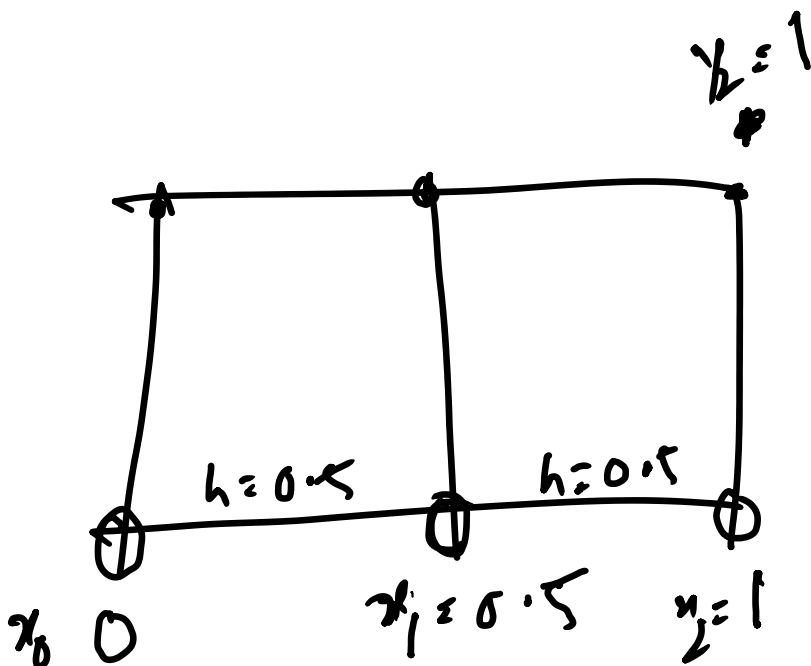
$$y'_i \approx \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y''_i(x_i) = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} \quad \checkmark$$

Runge Method



th $h = 0.5$





$$y_i'' = x_i y_i'$$

$$\Rightarrow \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$

$$\Rightarrow 4(y_{i-1} - 2y_i + y_{i+1})$$

$$\underline{i=1}$$

$$4(y_0 - 2y_1 + y_2)$$

$$\Rightarrow 4y_0 - 8y_1 + 4y_2$$

$$\Rightarrow \underline{4y_0} - 8y_1 + 4y_2$$

$$1 - x_i y_i = 0$$

$$h =$$

$$h^2.$$

$$y_i(x_i) - x_i y_i = 0$$



$$2) - x_1 y_1 = 0$$

$$y_2 - x_1 y_1 = 0$$

$$1 \quad y_2 = 1, \quad , \quad y$$

$$1 - \frac{1}{2} y_1 = 0$$

$$-\frac{1}{2}$$

$$-\frac{1}{4}$$



$$b_0 = 0, \quad x_2 = 1$$

$$\Rightarrow 4y_0 - 8.5$$

$$\Rightarrow \boxed{4y_0 - 8.5}$$

$$y(0) + y'(0) = 1, \quad y_2 =$$

$$\Rightarrow y_0 + y'_0 = 1 \Rightarrow y'_0 =$$

$$\Rightarrow y'_0 =$$

Putting $x=0$ in (2)

$$4(y_{-1} - 2y_0 + y_1)$$

$$y_1 + 4 = 0$$

$$y_1 = -4$$

1

$$\frac{y_{i+1}^* - y_{i-1}^*}{2h}$$

$$\frac{y_1 - y_{-1}}{2 \times 0.5} \Rightarrow y_0' - y_1 + y_{-1} = 0$$

$$\Rightarrow 1 - y_0 - y_1 + y_{-1} = 0$$

$$\Rightarrow \underbrace{y_0 + y_{-1}} = 1$$

$$y_1) - x_0 y_0 = 0$$

$$-\frac{1}{2}$$

$$\underline{\hspace{2cm}} \quad \textcircled{1}$$

✓

$$= 0$$

$$\boxed{-4_1 - 4 - 1 = 1} \quad \text{✓}$$

$$4(y_1 - 2y_0)$$

$$\Rightarrow y_1 - 2y_0 =$$

\Rightarrow

$$\boxed{-y_0 + 2y_1 = 1}$$

$$\boxed{\begin{array}{l} 4y_0 - 8.5y_1 = \\ -y_0 + 2y_1 = \end{array}}$$

$$\Rightarrow \left| \begin{array}{l} y_0 = -1 \quad \leftarrow \\ y_1 \geq 0 \quad \leftarrow \\ y_2 = 1 \quad \leftarrow \end{array} \right.$$

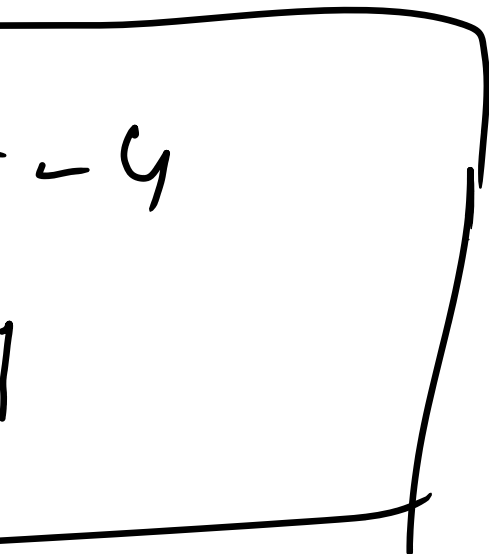
11) ... 0

$$+y_1 = 0$$

✓

✓

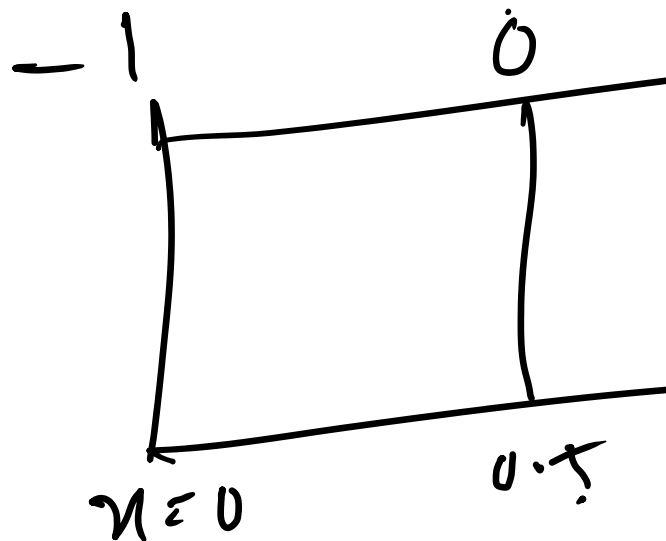
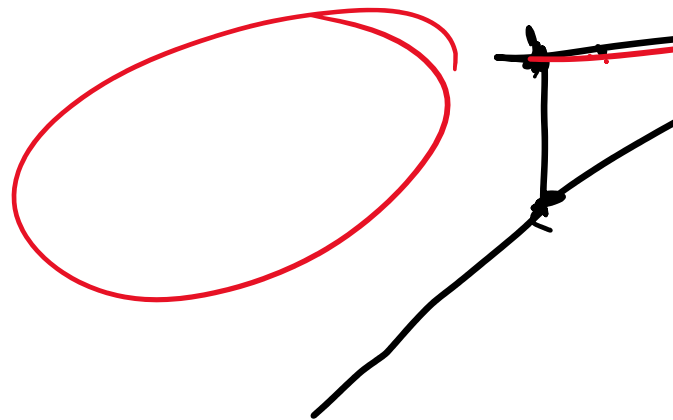
2

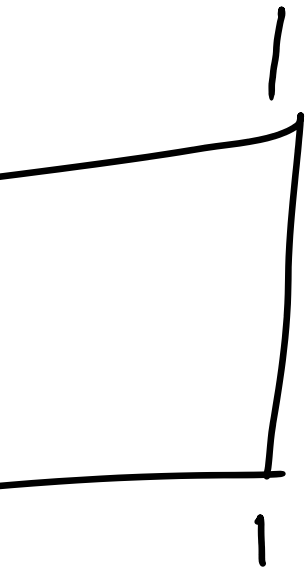
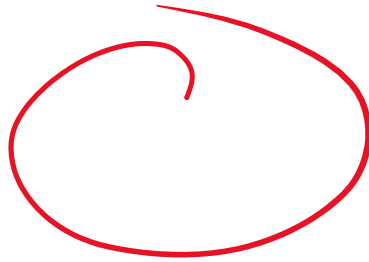
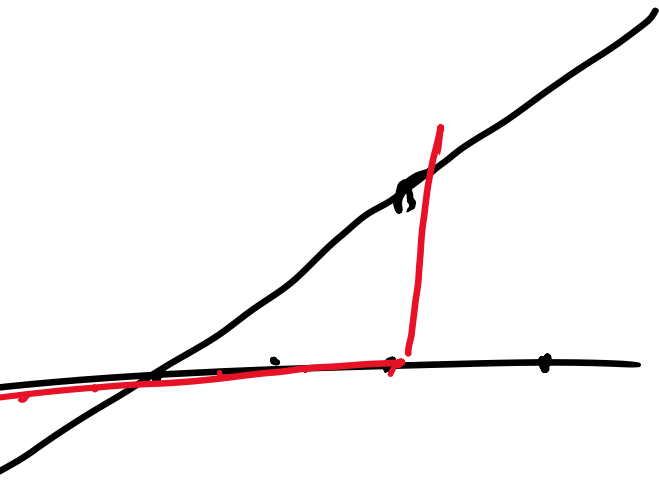


$$x_0 = 0$$

$$x_1 = 0.5$$

$$x_2 = 1$$





$$y'' - \pi y' + y = 0$$

$$\begin{array}{l} h = 0.3 \\ x_0 = 0 \\ \hline y_0 = 1 \end{array}$$

✓

let

$$\left\{ \begin{array}{l} f = z \\ g = -y + \pi z \\ x_0 = 0 \\ y_0 = 1 \end{array} \right.$$

$$\begin{array}{l} \checkmark \\ \checkmark \end{array} \left[\begin{array}{l} y' = \\ z' = \end{array} \right.$$

for $y(0.3)$ & $y(0.6)$

$$\left. \begin{array}{l} y(0) = 1 \\ y'(0) = -1 \end{array} \right\}, \quad h = \underline{0.3}$$

$$y'' - xy' + y = 0$$

$$\underline{y' = z}$$

$$z' - xz + y = 0$$

$$\left. \begin{array}{l} z, \quad \underline{y(0) = 1} \\ -y + xz, \quad z(0) = -1 \end{array} \right\} \begin{array}{l} x \text{ ind} \\ y, z: \text{df} \end{array}$$

pend

endi



$$\begin{cases} y_0 = 1 \\ z_0 = -1 \end{cases}$$

$$\text{let } y' = f(x, y, z) = z$$

$$z' = g(x, y, z) = -$$

$$k_1 = hf(x, y, z) = z =$$

$$k_2 = hf\left(x + \frac{h}{2}, y + \frac{k_1}{2}, z + \frac{k_2}{2}\right) =$$

$$y'' = 4(y - x), \quad 0$$

F.D.M

$$S_0^m x = 0.$$

$$y + xz$$

$$z + \frac{m_1}{2}$$

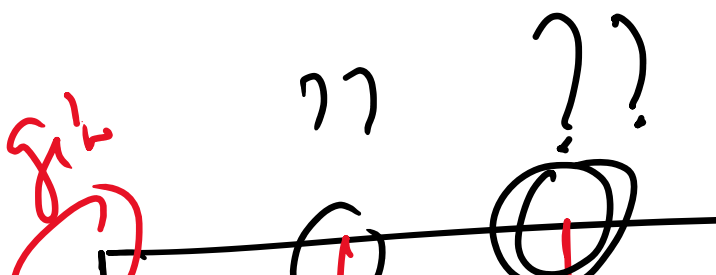
$$y_{n+1} = y_n + \frac{1}{6} (k_1$$

$$m_1 = f(m, y, z)$$

$$0 \leq x \leq 1, \quad y(0) = 0 \quad \& \quad y(1)$$

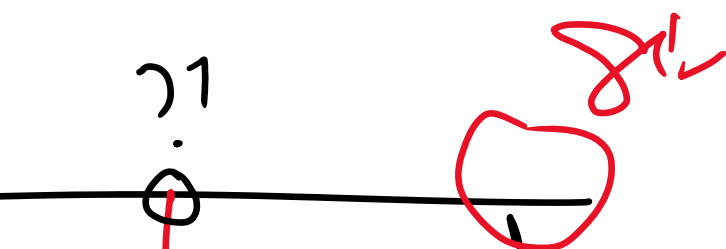
5

$$h = \frac{1}{4}$$

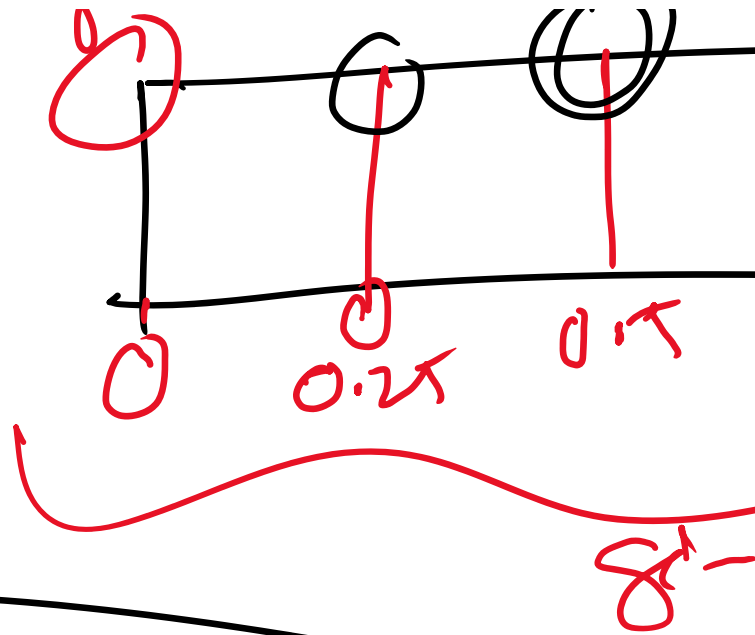


$$+ k_4 + 2(k_2 + k_3))$$

$$) = 2$$



$$y_{i+1} - 2y_i \left(\frac{9}{8} \right)$$



$$+ y_{l-1} = -\frac{1}{2} x_l^0$$

