

**Indian Institute of Technology Indore**  
**Semester: Spring 2024**  
**Numerical Methods (MA 204): Numerical Integration**  
**Tutorial-2 (SM) : 10-02-2024**

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1. Given the differential equation  $\frac{dy}{dx} = x^3 + 2xy^2 + y^3$  with the initial condition  $y(0) = 1$ , use Taylor's series method to determine the value of  $y(0.2)$ .
2. Solve  $y'' - xy' - y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 0$  at  $x = 0.1$ , by Taylor's series method.
3. Solve  $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ ,  $y(0) = 0$  at  $x = 0.25, 0.5, 1.0$ , by Picard's method (correct upto three decimal places).
4. Using Euler's method compute  $y_1$  and  $y_2$  taking  $h = 0.1$  from the following differential equation,

$$\frac{dy}{dx} = 1 + xy^2, \quad y(0) = 1.$$

Also compute the error term for both  $y_1$  and  $y_2$ .

5. Give the solution to the initial value problem  $y' = 2x + y$ , with  $y(1) = 2$ . Then create the approximation using improved/modified Euler's method with a step size of  $h = 0.1$  and compare the results to the true solution on the interval  $[1, 2]$ .
6. Give the solution to the initial value problem  $\frac{dy}{dx} = y^2 + yx$ ,  $y(1) = 1$  at  $x = 1.2$  and  $1.4$ , by improved/modified Euler's method. Also calculate the error in the improved/modified Euler's method for those values of  $x$ .
7. Solve  $\frac{dy}{dx} = x^2 + y$ ,  $y(0) = 1$  at  $x = 0.1$  with  $h = 0.05$ , by modified Euler's method.
8. Consider the Runge-Kutta second order method

$$y_{n+1} = y_n + \left(1 - \frac{1}{2\alpha}\right)k_1 + \frac{1}{2\beta}k_2 \quad \text{with} \quad k_1 = hf(x_n, y_n), \quad k_2 = hf(x_n + \alpha h, y_n + \beta k_1)$$

Find the region of absolute stability.

9. Solve for  $y(0.3)$  and  $y(0.6)$  using Runge-Kutta method of order 4 when  $y(x)$  is the solution of the second order equation.

$$y'' - xy' + y = 0, \quad \text{with} \quad y(0) = 1, \quad y'(0) = -1$$


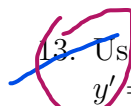


by taking  $h = 0.3$ .

10. Derive an implicit Runge-Kutta method of the form

$$y_{n+1} = y_n + w_1 k_1$$

$$\text{where } k_1 = hf(x_n + \alpha h, y_n + \beta k_1)$$

11. Solve  $y' = -2xy^2$ ,  $y(0) = 1$  with  $h = 0.3$  second order implicit Runge-Kutta method.

12. Solve  $2y''(t) - 5y'(t) - 3y(t) = 45e^{2t}$  at  $t = 0$  with  $x(0) = 2$ ,  $x'(0) = 1$  and compare the solution with true solution  $y(t) = 4e^{-t/2} + 7e^{3t} - 9e^{2t}$ . (If the method is not mentioned then one should use a method which has the best accuracy). 
-  13. Use Milne's predictor-corrector method to obtain the value of  $y(0.3)$  of the system:  
 $y' = x^2 + y^2 - 2$  with  $(-0.1, 1.09)$ ,  $(0, 1)$ ,  $(0.1, 0.89)$ ,  $(0.2, 0.7605)$ .
-  14. Given  $y' = 1 + y^2$ , where  $y(0) = 0$ ,  $h = 0.2$  compute  $y(0.8)$  using Adams-Moulton predictor-corrector method.
-  15. Solve the following ordinary differential equations by finite difference method:  
(i)  $y''(x) - xy(x) = 0$ ,  $y(0) + y'(0) = 1$ ,  $y(1) = 1$ , with  $h = 0.5$ .  
(ii)  $xy''(x) + xy'(x) - 2y(x) = 2(x + 1)$ ,  $y(0) = 0$ ,  $y'(1) = 0$ , with  $h = 1/3$ .
16. Write a program for Modified Euler's method, Taylor's series method, RK method to implement the above problems.