Indian Institute of Technology Indore
Semester: Spring 2024
Numerical Methods (MA 204): Numerical Integration
Tutorial-2 (SM) : 10-02-2024

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1. Given the differential equation $\frac{dy}{dx} = x^3 + 2xy^2 + y^3$ with the initial condition $y(0) = 1$, use Taylor's series method to determine the value of $y(0.2)$.

2. Solve $y'' - xy' - y = 0$, $y(0) = 1, y'(0) = 0$ at $x = 0.1$, by Taylor's series method.

3. Solve $\frac{dy}{dx} = \frac{x^2}{y^2+1}$, $y(0) = 0$ at $x = 0.25$, $0.5, 1.0$, by Picard's method (correct upto three decimal places).

4. Using Euler's method compute y_1 and y_2 taking $h = 0.1$ from the following differential equation,
$$\frac{dy}{dx} = 1 + xy^2, \quad y(0) = 1.$$
Also compute the error therm for both y_1 and y_2.

5. Give the solution to the initial value problem $y' = 2x + y$, with $y(1) = 2$. Then create the approximation using improved/modified Euler's method with a step size of $h = 0.1$ and compare the results to the true solution on the interval [1,2].

6. Give the solution to the initial value problem $\frac{dy}{dx} = y^2 + yx$, $y(1) = 1$ at $x = 1.2$ and 1.4, by improved/modified Euler's method. Also calculate the error in the improved/modified Euler's method for those values of x.

7. Solve $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ at $x = 0.1$ with $h = 0.05$, by modified Euler's method.

8. Consider the Runge-Kutta second order method
$$y_{n+1} = y_n + (1 - \frac{1}{2\alpha})k_1 + \frac{1}{2\beta}k_2 \quad \text{with} \quad k_1 = hf(x_n, y_n), \;\; k_2 = hf(x_n + \alpha h, y_n + \beta k_1)$$
Find the region of absolute stability.

9. Solve for $y(03)$ and $y(0.6)$ using Runge-Kutta method of order 4 when $y(x)$ is the solution of the second order equation.
$$y'' - xy' + y = 0, \quad \text{with} \quad y(0) = 1, \quad y'(0) = -1$$
by taking $h = 0.3$.

10. Derive an implicit Runge-Kutta method of the form
$$y_{n+1} = y_n + w_1 k_1$$
$$\text{where } k_1 = hf(x_n + \alpha h, y_n + \beta k_1)$$

11. Solve $y' = -2xy^2$, $y(0) = 1$ with $h = 0.3$ second order implicit Runge-Kutta method.

12. Solve $2y''(t) - 5y'(t) - 3y(t) = 45e^{2t}$ at $t = 0$ with $x(0) = 2$, $x'(0) = 1$ and compare the solution with true solution $y(t) = 4e^{-t/2} + 7e^{3t} - 9e^{2t}$. (If the method is not mentioned then one should use a method which has the best accuracy). ?

13. Use Milne's predictor-corrector method to obtain the value of $y(0.3)$ of the system: $y' = x^2 + y^2 - 2$ with $(-0.1,\ 1.09),\ (0,\ 1),\ (0.1,\ 0.89),\ (0.2,\ 0.7605)$.

14. Given $y' = 1 + y^2$, where $y(0) = 0$, $, h = 0.2$ compute $y(0.8)$ using Adams-Moulton predictor-corrector method.

15. Solve the following ordinary differential equations by finite difference method:
(i) $y''(x) - xy(x) = 0$, $y(0) + y'(0) = 1$, $y(1) = 1$, with $h = 0.5$.
(ii) $xy''(x) + xy'(x) - 2y(x) = 2(x + 1)$, $y(0) = 0$, $y'(1) = 0$, with $h = 1/3$.

16. Write a program for Modified Euler's method, Taylor's series method, RK method to implement the above problems.