Indian Institute of Technology Indore
Semester: Spring 2024
Numerical Methods (MA 204): Numerical Integration
Tutorial-1 (SM) : 03-02-2024

---

1. Determine a, b, c such that the formula

$$\int_0^h f(x)dx = h\left\{af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right\}$$

is exact for polynomials of as high order as possible, and determine the order of the truncation error.

2. Evaluate the integral $I = \int_1^2 \frac{2xdx}{1+x^4}$, using the Gauss-Legendre 1-point, 2-point and 3-point quadrature rules. Compare with the exact solution $I = \tan^{-1}(4) - (\pi/4)$.

3. Evaluate the integral $I = \int_0^\infty \frac{e^{-x}}{1+x^2}dx$, using the Gauss-Legendre 2-point and 3-point formulas.

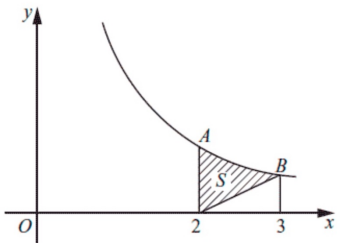
4. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ by subdividing the interval [0,1] into two equal parts and then applying the Gauss-Lengendra 3-point formula

$$\int_{-1}^1 f(x)dx = \frac{1}{9}\left[5f\left(-\sqrt{\frac{3}{5}}\right) + 8f(0) + 5f\left(\sqrt{\frac{3}{5}}\right)\right].$$

5. Given $f(x) = \frac{5}{3x^2-2}$, $x > 1$. (a) Complete the table below, giving the values of $f(x)$ to 2 decimal places:

x	2	2.25	2.5	2.75	3
f(x)	0.5	0.38	?	?	0.2

(b) Use the trapezium rule, with all the values of $f(x)$ from the table, to find an approximate value for $\int_2^3 f(x)dx$.



(c) Use your answer to part (b) to find an approximate value for the area of S.

6. Use Simpson's 1/3 rule to integrate $f(x) = 0.2 + 25x + 3x^2 + 2x^4$ from 0 to 2. Also find the relative error.

7. Evaluate the following interval by Trapezoidal rule, Simpson's 1/3 rule, and Simpson's 3/8 rule respectly, $\int_0^2 (e^{x^2} - 1)dx$ correct up to three decimal places.

8. Using Simpson's one-third rule find the mess of plane lamina bounded by the curve $y^2 = 4x$ and $y = x$. The density ρ of plane lamina at any point is given by $\sqrt{x^2 + y^2}$.

9. Find the remainder of the Simpson 3/8 rule

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

for equally spaced points $x_i = x_0 + ih, \quad i = 1, 2, 3$. Use this rule to approximate the value of the integral

$$I = \int_0^1 \frac{dx}{1+x}.$$

Also, find a bound on the error.

10. Evaluate $\int_0^1 \int_0^2 \frac{2xy}{(1+x^2)(1+y^2)} dxdy$ using Simpson's 1/3 rule with step size h=k=0.25.

11. Evaluate

$$\int\int_D \frac{1}{(x^2 + y^2)} dxdy$$

where D is the square with corners at $(1, 1), (2, 2), (1, 2)$ and $(2, 1)$.

12. Write a MATLAB code for solving question no. (7).

13. Write a code to find $\int_a^b f(x)dx$, $a = 0$, $b = 0.8$ using composite (a) trapezium rule, (b) Simpson's 1/3 rule and (c) Simpson's 3/8 rule, where $f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$.