DSA ESE

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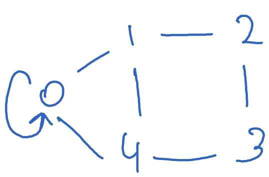
Types of graph:

1) Null : No vertex
2) Trivial: single vertex
3) Complete: direct path b/w any two nodes
4) Regular : All nodes have same degree
5) Bipartite: divided into two graphs with vertices set V_1 & V_2 st $V_1 \cap V_2 = \emptyset$ and there are no direct paths b/w vertices within V_1 & V_2
6) Weighted & Unweighted
7) Directed & Undirected
8) Cyclic: Atleast one cycle in it
9) Connected & Disconnected

Representation.

1) Adjancy List.



0 → 0 1 4
1 → 0 2 4
2 → 1 3
3 → 2 4
4 → 0 1 3

2) Matrix representation.

	0	1	2	3	4
0	1	1	0	0	1
1	1	0	1	0	1
2	0	1	0	1	0
3	0	0	1	0	1
4	1	1	0	1	0

* replace by weights if given

TRAVERSALS

BFS (Breadth First Search)

→ traverses at the same height first, then goes to next depth.

→ uses Queue & visited array

→ used for getting shortest distance from the starting node to any node.

→ $O(V+E)$ V = vertices E = edges

colours : white → not visited

gray → visited

black → no more use.

π → parent

d → dist from source

BFS (G, S)

```
for each vertex u in G - {s}
    u. colour = white
    u. dist = ∞
    u. π = nill

s. colour = gray
s. d = 0
s. π = nill
enqueue (Q, S)
while Q != empty
    u = dequeue (Q)
    for each v in G. adj [u]
        if v. colour = white
```

v. colour = gray
v.d = u.d+1
v.π = u
enqueue(Q,v)
u. colour = black.

DFS (Depth First Search)

- uses stack & visited array
- first searches the entire depth of a node, then moves on to another node at same depth.
- explore neighbours of neighbours before sibling
- maynot give shortest path.
- O(V+E)

```
DFS(G)                          // Main program
{   for each u in V             // Initialize
    {   color[u]=W; pred[u]=NIL; }
    time=0;
    for each u in V
        if(color[u]==W) DFSVisit(u); // Start new tree
}

DFSVisit(u)                     // Process vertex u
{   color[u]=G;                 // Vertex discovered
    d[u]=++time;                // Time of discovery
    for each v in adj[u]
        if(color[v]==W)         // Visit undiscovered
        {   pred[v]=u;  DFSVisit(v);} //    neighbours
    color[u]=B;                 // Vertex finished
    f[u]=++time;                // Time of finish
}
```

BFS

DFS

Web crawlers
topological sort
Shortest dist
GPS navigation

Detecting cycles in graph
finding a path
checking if graph is bipartite
back tracking
topological sort

if branches are many. BFS stuck.

if vertexes are may. DFS stuck

BFS gives a guarenteed sol^n. DFS doesn't.

Spanning Tree.

A tree which contains all the vertices of the graph but has no cycles in it.

edges = # vertices −1 $E = V-1$
spanning trees are acyclic
connected and contains all vertices

props

Minimum Spanning tree = path with least weights. May not be unique.

KRUSKALS ALGO (greedy approach)

→ sort all edges in increasing order
→ Pick the lowest cost edge and add it to the MST such that it doesnt create a cycle

Time complexity: Sorting $E \log E$
union find algo for a edge = $\log V$ to detect cycles

for E edges UFA = E log V

E log V > E log E

$\therefore O(E \log V)$

Make-set (u) ⇒ makes a set with element u.

find-set (u) ⇒ finds a set with u as an element

union (x, y) ⇒ union of sets x & y

Disjoint set DS

```
KRUSKAL(G):
A = ∅
For each vertex v ∈ G.V:
    MAKE-SET(v)
For each edge (u, v) ∈ G.E ordered by increasing order by weight(u, v):
    if FIND-SET(u) ≠ FIND-SET(v):
    A = A ∪ {(u, v)}
    UNION(u, v)
return A
```

PRIMS ALGO (greedy approach)

start with any random edge.

See to which all vertices the edge is connected to.

Add the min possible weight which doesn't create a cycle.

Now again repeat. Now select the min wt

Main idea: Always add the edge which will add a new vertice to the MST and has the least weight. At all times the tree is connected but not cyclic.

→ $O(E \log V)$

SEARCHING ALGO

Linear Search.

Linear Search.

best : $O(1)$

worst : $O(n)$

Binary Search

→ requires sorted algo

best : $O(1)$

worst : $O(\log n)$

Ternary Search

Fibonacci Search

Exponential Search.

SORTING ALGO

Selection Sort.

find min element in array

put it on the first index

decrease size of array by one and repeat

```
for i=1 to i=n-1
  for j=i+1 to j=n
    if ( a[i] > a[j] )
      Swap( a[i], a[j] )
```

time = $O(n^2)$ best / worst / avg

space = $O(1)$

swaps = $O(n)$

Stable = No

inplace = yes.

Bubble Sort.

Time: worst $O(n^2)$
best $O(n)$

Space: $O(1)$

inplace: yes

stable: yes

```
for i=1 to i=n
    for j=0 to j=n-i-1
        if (a[j] > a[j+1])
            swap(a[j], a[j+1])
```

INSERTION SORT

Time: worst $O(n^2)$
best $O(n)$

Space: $O(1)$

inplace: yes

stable: yes

```
for i=2 to n
    key = A[i]
    j = i-1
    while (j>0 && A[j] > key)
        a[j+1] = a[j]
        j--
    a[j+1] = key
```

if our input is close to the best case, then we can use this

if our input data is small.

Basically our array is divided into two parts sorted & unsorted.

in each iteration we take an element from unsorted part and put it in its correct position of sorted array.

MERGE SORT

→ based on DNC

→ Divide array then combine in sorted manner

→ merge sort better than heap sort

time: best $O(n\log n)$
avg $O(n\log n)$
worst $O(n\log n)$

space: $O(n)$

→ Divide array then combine in sorted manner

→ merge sort better than heap sort

worst $O(n \log n)$

space: $O(n)$

inplace: No

stable: yes.

```
Merge ( A, start, mid, end)

    len1 = mid-start -1
    len2 = end-mid
    leftArr [len1]        rightArr [len2]

    for i=0 to i=len1-1
        LeftArr[i]= A[start+i]

    for j=0 to j=len2-1
        rightArr[j] = A[mid+1+j]

    i=0  j=0  k=start
    while i<len1 && j<len2
        if leftArr[i] <= rightArr[j]
            A[k]= leftArr[i]
            i++
        else
            A[k]= rightArr[j]
            j++
        k++

    while (i<len1)
        A[k]= leftArr[i]
```

i++
k++
while (j < len2)
A[k] = right Arr[j]
j++
k++

Merge Sort (A, start, end)
if start < end
$$mid = start + \frac{(end - start)}{2}$$
Merge Sort (A, start, mid)
Merge Sort (A, mid+1, end)
Merge (A, start, mid, end)

Quick Sort

Time: $O(n \log n)$ best
$O(n^2)$ worst

Space: $O(1)$ for non stable
$O(n)$ for stable

in-place = depending on version
Stable = No
pivot → first element
last element
median element

* An element (pivot) is put to its correct place in a single pass.

Partion (A, p, r)
i = p-1   j = p   pivot = A[r]

Quick Sort (A, p, r)
if (p < r)

Partition (...)

```
i = p-1    j = p    pivot = A[r]
while (j < r)
    if ( A[j] < pivot)
        i++
        swap (A[i], A[j])
    j++
i++
swap (A[i], pivot)
return i
```

Quick Sort (A, p, r)

```
if (p < r)
    return
i = Partition (A, p, r)
Quick Sort (A, p, i-1)
Quick sort (A, i+1, r)
```

p = Start
r = end

General Stuff

→ comparison based algo have lower bound of $O(n \log n)$

because no. of permutations of input = $n!$

$2^h > n!$ $n! = c\,n^n$ Sterling approx

$\therefore h \geq n \log n$

h = height of BT . h = no. of comparisons to be made.

Time Complexity of Quick Sort.

K = pivots correct position.

K-1 K n-K

Sub1 Sub2

$T(n) = T(n-K) + N$ → for splitting the array (traversing in partition func^n)

How to check whether a graph is bipartite using DFS

→ Assign a colour to a node

0 = not visited
1 = colour 1
2 = colour 2

→ Assign a colour to the node

1 = colour 1

2 = colour 2

→ if node colour = 0 ; give correct colour

→ if node colour ≠ 0 ; its parent & its own colour need to be diff

else its not bipartite.

SORTING Rev

	Time Best	Avg	Worst	Space	in place	stable
selection	n^2	n^2	n^2	1	Yes	No
bubble	n	n^2	n^2	1	Yes	Yes
insertion	n	n^2	n^2	1	yes	yes
Quick	nlogn	nlogn	n^2	1	Yes	No
Merge	nlogn	nlogn	n^2	n	No	Yes
Heap	nlogn	n logn	nlogn	1	Yes	No

Time complexity of Merge Sort.

$$T(N) = 2\,T\left(\frac{N}{2}\right) + O(N)$$

↳ for traversing

$$O(\underbrace{n}_{\text{for merging}}\ \underbrace{\log n}_{\text{height of tree.}})$$

unstable = S Q H

not inplace = Merge.

unstable = S Q H
↙ ↓ ↘
selection Quick Heap

not inplace = Merge.

Dijkstra Algorithm

```
int minDist ( int dist [ ], bool fixed [ ])

    int min_dist = INT_MAX
    int min_index
    for ( int i = 0 ; i < V ; i++)
    {    if ( fixed [ i ] = = false && dist [ i ] < = min_dist )
          {       min_dist = dist[i] ;
                  min_index = i
          }
    }
  return min_index;

void Dijkstra ( int graph [ ] [ ] , int source )

    int dist [v];
    bool fixed [v];
    for ( int i=0 ; i< v ; i++)
    {
         dist [ i ] = INT_MAX;
         fixed [ i ] = false;
    }
    dist [source] = 0;
```

```
for ( int count=0 ; count < V-1; count++)
{
    int u= minDist ( dist, fixed);
    fixed[u] = true;
    for ( int i=0 ; i<V ; i++)
    {
        if ( graph[u][i] !=0 && fixed [i] == false
             && dist[u] != INT_MAX &&
             dist[u] + graph [u] [i] < dist [i] )
            dist [i] = dist[u] + graph[u] [i];
    }
}
```

```cpp
// Returns the value of maximum profit
int knapSackRec(int W, int wt[], int val[], int index, int** dp)
{
    // base condition
    if (index < 0)
        return 0;
    if (dp[index][W] != -1)
        return dp[index][W];

    if (wt[index] > W) {

        // Store the value of function call
        // stack in table before return
        dp[index][W] = knapSackRec(W, wt, val, index - 1, dp);
        return dp[index][W];
    }
    else {
        // Store value in a table before return
        dp[index][W] = max(val[index]
                           + knapSackRec(W - wt[index], wt, val,
                                         index - 1, dp),
                       knapSackRec(W, wt, val, index - 1, dp));

        // Return value of table after storing
        return dp[index][W];
    }
}
```

```cpp
// Returns length of LCS for X[0..m-1],
// Y[0..n-1]
int lcs(char* X, char* Y, int m, int n,
        vector<vector<int> >& dp)
{
    if (m == 0 || n == 0)
        return 0;
    if (X[m - 1] == Y[n - 1])
        return dp[m][n] = 1 + lcs(X, Y, m - 1, n - 1, dp);

    if (dp[m][n] != -1) {
        return dp[m][n];
    }
    return dp[m][n] = max(lcs(X, Y, m, n - 1, dp),
                          lcs(X, Y, m - 1, n, dp));
}
```

```cpp
bool Graph::isCyclicUtil(int v, bool visited[],
                         bool* recStack)
{
    if (visited[v] == false) {
        // Mark the current node as visited
        // and part of recursion stack
        visited[v] = true;
        recStack[v] = true;

        // Recur for all the vertices adjacent to this
        // vertex
        list<int>::iterator i;
        for (i = adj[v].begin(); i != adj[v].end(); ++i) {
            if (!visited[*i]
                && isCyclicUtil(*i, visited, recStack))
                return true;
            else if (recStack[*i])
                return true;
        }
    }

    // Remove the vertex from recursion stack
    recStack[v] = false;
    return false;
}
```

```cpp
bool Graph::isCyclic()
{
    // Mark all the vertices as not visited
    // and not part of recursion stack
    bool* visited = new bool[V];
    bool* recStack = new bool[V];
    for (int i = 0; i < V; i++) {
        visited[i] = false;
        recStack[i] = false;
    }

    // Call the recursive helper function
    // to detect cycle in different DFS trees
    for (int i = 0; i < V; i++)
        if (!visited[i]
            && isCyclicUtil(i, visited, recStack))
            return true;

    return false;
}
```

```cpp
// function to check whether a graph is bipartite or not
bool isBipartite(vector<int> adj[], int v,
                 vector<bool>& visited, vector<int>& color)
{

    for (int u : adj[v]) {

        // if vertex u is not explored before
        if (visited[u] == false) {

            // mark present vertices as visited
            visited[u] = true;

            // mark its color opposite to its parent
            color[u] = !color[v];

            // if the subtree rooted at vertex v is not bipartite
            if (!isBipartite(adj, u, visited, color))
                return false;
        }

        // if two adjacent are colored with same color then
        // the graph is not bipartite
        else if (color[u] == color[v])
            return false;
    }
    return true;
}
```

```cpp
bool solveNQUtil(int board[N][N], int col)
{
    // base case: If all queens are placed
    // then return true
    if (col >= N)
        return true;

    // Consider this column and try placing
    // this queen in all rows one by one
    for (int i = 0; i < N; i++) {

        // Check if the queen can be placed on
        // board[i][col]
        if (isSafe(board, i, col)) {

            // Place this queen in board[i][col]
            board[i][col] = 1;

            // recur to place rest of the queens
            if (solveNQUtil(board, col + 1))
                return true;

            // If placing queen in board[i][col]
            // doesn't lead to a solution, then
            // remove queen from board[i][col]
            board[i][col] = 0; // BACKTRACK
        }
    }

    // If the queen cannot be placed in any row in
    // this column col  then return false
    return false;
}
```

```
int BFS(int mat[][COL], Point src, Point dest)
{
    if (!mat[src.x][src.y] || !mat[dest.x][dest.y])
        return -1;

    bool visited[ROW][COL];
    memset(visited, false, sizeof visited);

    visited[src.x][src.y] = true;

    queue<queueNode> q;
    queueNode s = {src, 0};
    q.push(s);  // Enqueue source cell

    while (!q.empty())
    {
        queueNode curr = q.front();
        Point pt = curr.pt;
        if (pt.x == dest.x && pt.y == dest.y)
            return curr.dist;
        q.pop();

        for (int i = 0; i < 4; i++)
        {
            int row = pt.x + rowNum[i];
            int col = pt.y + colNum[i];
            if (isValid(row, col) && mat[row][col] &&
               !visited[row][col])
            {
                visited[row][col] = true;
                queueNode Adjcell = { {row, col},curr.dist + 1 };
                q.push(Adjcell);
            }
        }
    }
    return -1;
```

```
int solveKTUtil(int x, int y, int movei, int sol[N][N],
                int xMove[8], int yMove[8])
{
    int k, next_x, next_y;
    if (movei == N * N)
        return 1;

    /* Try all next moves from
    the current coordinate x, y */
    for (k = 0; k < 8; k++) {
        next_x = x + xMove[k];
        next_y = y + yMove[k];
        if (isSafe(next_x, next_y, sol)) {
            sol[next_x][next_y] = movei;
            if (solveKTUtil(next_x, next_y, movei + 1, sol,
                            xMove, yMove)
                == 1)
                return 1;
            else

               // backtracking
                sol[next_x][next_y] = -1;
        }
    }
    return 0;
}
```

```
bool solveSudoku(int grid[N][N], int row, int col)
{
    if (row == N - 1 && col == N)
        return true;

    if (col == N) {
        row++;
        col = 0;
    }

    if (grid[row][col] > 0)
```

```
// Function to get all valid paths
void findPath(int r, int c, vector<vector<int> >& maze,
              int n, vector<string>& ans,
              string& currentPath)
{

    // if we reach the bottom right cell of the matrix,
    // add the current path to ans and return
    if (r == n - 1 && c == n - 1) {
        ans.push_back(currentPath);
        return;
    }

    // Mark the current cell as blocked
    maze[r][c] = 0;

    for (int i = 0; i < 4; i++) {
        // find the next row based on the current row
        // (r) and the dr[] array
        int nextr = r + dr[i];
        // find the next column based on the current
        // column (c) and the dc[] array
        int nextc = c + dc[i];
        // check if the next cell is valid or not
        if (isValid(nextr, nextc, n, maze)) {
            currentPath += direction[i];
            // Recursively call the solve function for
            // the next cell
            findPath(nextr, nextc, maze, n, ans,
                     currentPath);
            currentPath.pop_back();
        }
    }
    // Mark the current cell as unblocked
    maze[r][c] = 1;
}
```

```
bool solveSudoku(int grid[N][N], int row, int col)
{
    if (row == N - 1 && col == N)
        return true;

    if (col == N) {
        row++;
        col = 0;
    }

    if (grid[row][col] > 0)
        return solveSudoku(grid, row, col + 1);

    for (int num = 1; num <= N; num++)
    {

        if (isSafe(grid, row, col, num))
        {

            grid[row][col] = num;

            if (solveSudoku(grid, row, col + 1))
                return true;
        }

        grid[row][col] = 0;
    }
    return false;
}
```

```
}
```