INDIAN INSTITUTE OF TECHNOLOGY INDORE

MA 203 Complex Analysis and Differential Equations-II

Autumn Semester

Tutorial – 7 (Complex Analysis)

1. Find the values of the integrals (i) $\int_C \operatorname{Re} z\, dz$ (ii) $\int_C z^2\, dz$ from the point $z_1 = 1$ to the point $z_2 = i$ in the positive direction on the following curves C:

   (a) the boundary of the square: $0 \leq x \leq 1,\ 0 \leq y \leq 1$.

   Ans: (i) $1/2 + i$ (ii) $\frac{-1-i}{3}$

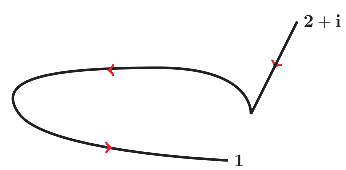
   (b) the part of the circle: $z = \mathrm{e}^{it}$, $0 \leq t \leq \frac{\pi}{2}$.

   Ans: (i) $i\pi/4 - 1/2$ (ii) $\frac{-1-i}{3}$

   (c) the straight line segment $z = (1-t) + it\ 0 \leq t \leq 1$.

   Ans: (i) $\frac{-1+i}{2}$ (ii) $\frac{-1-i}{3}$

2. Find $I = \int_C z\, dz$, where C is the contour given below:



3. Evaluate the following integrals:

   (a) $\int_C |z|^2\, dz$, where C is an arc of unit circle $|z| = 1$ traversed in the clockwise direction with initial point -1 and final point i.

   (b) $\int_C |z|\overline{z}\, dz$, where C consists of the half-circle $z = R\mathrm{e}^{it}$, $0 \leq t \leq \pi$, and the straight line segment: $-R \leq \operatorname{Re} z \leq R$, $\operatorname{Im} z = 0$ and traversed in the anticlockwise direction. Ans: $R^3\pi i$

   (c) $\int_C \overline{z}\, dz$, where C consists of the semi circle $z = \mathrm{e}^{it}$, $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$, and the straight line segment: $z = it,\ \ -1 \leq t \leq 1$, and traversed in the anticlockwise direction. Ans: πi

4. Explain why the integrals in question 3 cannot be evaluated by the method of indefinite integral.

5. Evaluate the following integrals using the method of indefinite integral.

   (a) $\int_C \sin^2 z\, dz$, C from $-\pi i$ along $|z| = \pi$ to πi in the right half plane
   Ans: $\pi i - \frac{1}{4}[\sin(2\pi i) - \sin(-2\pi i)]$

   (b) $\int_C z\mathrm{e}^{z^2}\, dz$, C from 1 along the axis to i Ans: $\frac{1}{2}[\mathrm{e} - \mathrm{e}^{-1}]$

6. Explain why the integral $\int_C \frac{1}{z-3i}\, dz$, where C is the circle $|z| = \pi$ traversed in counter clockwise direction, cannot be evaluated by the method of indefinite integral.

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