Q10 $xy'' + y' + 2y = 0$ ——(1) $y(1) = 1 \quad y'(1) = 2$

$x = 1$ $\quad P_1 = \frac{1}{x} \quad P_2 = \frac{2}{x}$ analytic at $x = 1$

$(x - x_0)$

Let $y = \sum_{n=0}^{\infty} (x-1)^n c_n = c_0 + c_1(x-1) + c_2(x-1)^2 + c_3(x-1)^3 + \cdots$ $\quad (x - x_0)$

$y' = \sum_{n=1}^{\infty} n c_n (x-1)^{n-1} \qquad y'' = \sum_{n=2}^{\infty} n(n-1) c_n (x-1)^{n-2}$

Substituting y, y', y'' in eq (1)

$\Rightarrow$. — - —

$\rightarrow$ - ——— —

$$\Rightarrow \sum_{n=1}^{\infty} (n+1) n \, c_{n+1} (x-1)^n + \sum_{n=0}^{\infty} (n+2)(n+1) c_{n+2} (x-1)^n$$
$$+ \sum_{n=0}^{\infty} (n+1) c_{n+1} (x-1)^n + 2 \sum_{n=0}^{\infty} c_n (x-1)^n = 0$$

$\Rightarrow$ - - - - - - .

$2c_2 + c_1 + 2c_0 = 0 \Rightarrow c_2 = -\frac{c_1 + 2c_0}{2}$

$(n+1) n \, c_{n+1} + (n+2)(n+1) c_{n+2} + (n+1) c_{n+1} + 2 c_n = 0$ $\quad \forall n \geq 1$

$\Rightarrow (n+1)(n+2) c_{n+2} + (n+1)^2 c_{n+1} + 2 c_n = 0 \quad \forall n \geq 1$

$$\Rightarrow c_{n+2} = -\frac{(n+1)^2 c_{n+1} + 2 c_n}{(n+1)(n+2)} \quad \forall n \geq 1 \qquad (*)$$

give $y = 1$ & $y' = 2$ at $x = 1$

$c_0 = 1$, & $c_1 = 2$

$c_2 = -\frac{2+2}{2} = -2$

put $n = 1, 2, 3, \ldots$ in $(*)$

$c_2 = \ldots = 2$

Putting $n = 1, 2, 3, \ldots$ in (*)

$$c_3 = -\frac{2^2 c_2 + 2c_1}{2\cdot 3} = -\frac{4\times(-2) + (2\times 2)}{2\cdot 3} = \frac{2}{3}$$

$$c_4 = -\frac{3^2 c_3 + 2c_2}{3\cdot 4} = \frac{-9\times(2/3) + 2\times(-2)}{3\cdot 4} = -\frac{1}{6}$$

$$c_5 = \frac{1}{15}$$

$$y = 1 + 2(x-1) + 2(x-1)^2 + \frac{2}{3}(x-1)^3 - \frac{1}{6}(x-1)^4 + \frac{1}{15}(x-1)^5 + \ldots$$

Frobenius Method (F.M.)

$$x^2 y'' + x\,p(x)\,y' + Q\,y = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{p(x)}{x}\frac{dy}{dx} + \frac{Q(x)}{x^2}y = 0$$

Here $x=0$ is a R.S.P. & $p(x)$ & $Q(x)$ are analytic for all $|x|<R$, $R>0$

Then the series solution about $x=0$ can be found using F.M.

Let us assume a trial solution

$$y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}, \quad a_0 \neq 0, \quad 0<x<R$$

In order to find a_n's we take r is any parameter

$$y'(x) = \sum_{n=1}^{\infty} a_n (n+r)\, x^{n+r-1}$$

$$y''(x) = \sum_{n=2}^{\infty} a_n (n+r)(n+r-1)\, x^{n+r-2}$$

$$y = a_0 + a_1 x + a_2 x^2 + \ldots$$

$$\sqrt{x}$$

$$\frac{1}{x(1-x)} = \frac{1}{x} + $$

$$\Rightarrow \frac{1}{x}(1-x)^{-1} = \frac{1}{x}\left[1 + x + x^2 + \ldots\right]$$

$$= \frac{1}{x} + 1 + x + x^2 + x^3 + \ldots$$

$$= \frac{1}{x}\cdot\frac{1}{1-x}$$

$$= x^{-1}(1 + x + x^2 + x^3 + \ldots)$$

$$= \frac{1}{x}\cdot\frac{1}{1-x}$$

$$y = x^r(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots)$$

$$= \sum_{n=0}^{\infty} a_n x^{n+r}$$