Indian Institute of Technology Indore
MA204 Numerical Methods
Instructor: Dr. Debopriya Mukherjee
Tutorial Sheet 2

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1. Let $f(x) = 3^x$ for every $x \in \mathbb{R}$.
   (a) Use Lagrange interpolation to find a polynomial $p(x)$ of degree at most two that agrees with this function at the points $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$.
   (b) Find a bound on $|f(x) - p(x)|$ for each $x \in [0, 2]$.

2. Let $f(x) = 3^x$ for every $x \in \mathbb{R}$. Let $p(x)$ be the polynomial of degree at most two that agrees with this function at the points $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$. Use divided differences to construct $p(x)$.

3. For a function f the Newton divided-difference table is

$$\begin{array}{cccc}
x_i & f[x_i] & f[x_i, x_{i+1}] & f[x_0, x_1, x_2] \\
0 & 0 & & \\
 & & 3 & \\
1 & ? & & 3 \\
 & & ? & \\
2 & ? & &
\end{array}$$

   (a) Determine the missing entries in the table.
   (b) Give the interpolating polynomial $p(x)$.

4. Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0)$, $(0.5, y)$, $(1, 3)$ and $(2, 2)$. Find y if the coefficient of x^3 in $P_3(x)$ is 6.

5. Let $f(x) = e^x$ for $x \in [0, 2]$. Approximate $f(0.25)$ using linear interpolation with $x_0 = 0$ and $x_1 = 0.5$.

6. Let $i_0, i_1, \cdots, i_n$ be a rearangement of the integers $0, 1, 2, \cdots, n$. Show that

$$f[x_{i_0}, x_{i_1}, \cdots, x_{i_n}] = f[x_0, x_1, \cdots, x_n].$$

7. For a function f the Newton divided-difference table is

$$\begin{array}{llllll}
x_0 & = & 0.0 & f[x_0] & & \\
 & & & & f[x_0, x_1] & \\
x_1 & = & 0.4 & f[x_1] & & f[x_0, x_1, x_2] = \dfrac{50}{7} \\
 & & & & f[x_1, x_2] = 10 & \\
x_2 & = & 0.7 & f[x_2] = 6 & &
\end{array}$$

   Determine the missing entries.

8. Define interpolating polynomial. State Weierstrass Approximation Theorem. Write the explicit form of

   - Newton form of interpolating polynomial;
   - Lagrange form of interpolating polynomial.

9. For the function $f(x) = \cos x$, let $x_0 = 0$, $x_1 = 0.6$, $x_2 = 0.9$.

   - Construct the Lagrange interpolation polynomial of degree at most two to approximate $f(0.45)$.
   - Find the actual error at 0.45. Can you say about the error bound for the error.

10. Construct the Lagrange interpolating polynomial of degree 2 for $f(x) = \sin(\ln x)$ on the interval $[2, 2.6]$ with the points $x_0 = 2$, $x_1 = 2.4$, $x_2 = 2.6$. Find a bound for the absolute error.