Numerical Methods (MA 204)

Module-II: Numerical Integration and ODE

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Date	Lecture	Topic
30 Jan	Lecture-1	Numerical integration, composite rules, error formulae
31 Jan	Lecture-2	Rectangular Rule Quadrature Formula
01 Feb	Lecture-3	Trapezoidal Rule Simpson's 1/3 Simpson's 3/8
06 Feb	Lecture-4	Numerical solution of ordinary differential equations Linear, Non-linear, IVP, BVP, order of convergence Picard's method of successive approximation
07 Feb	Lecture-5	Taylor's method Euler and Modified Euler's Methods
08 Feb	Lecture-6	Runge-Kutta (RK) methods up to 4th order
13 Feb	Lecture-7	
14 Feb	Lecture-8	Multi-step methods: Predictor-corrector methods,
15 Feb	Lecture-9	Euler's predictor-corrector method, Error in Euler's predictor-corrector method,

1. E. Kreyszig, ***Advanced Engineering Mathematics***, John Wiley & Sons, 2020, ISBN: 9781119455929.

2. S. S. Sastry, ***Introductory Methods of Numerical Analysis***, PHI Learning,
ISBN-978-81-203-4592-8, 2012.

3. S. D. Conte and Carle de Boor, ***Elementary Numerical Analysis - An Algorithmic Approach***, SIAM, 2018, ISBN: 978161197520

4. S. Dey and S. Gupta, Numerical Methods, McGraw Hill, 2013, ISBN: 9781259062582

What is the integration?

Integration is the process of evaluating an indefinite integral or a definite integral.

$\underbrace{\int}_{\text{Integral sign}} \underbrace{f(x)}_{\text{Integrand}}\, dx$

x is called the variable of integration

Definite Integral → $\int_a^b f(x)\,dx$

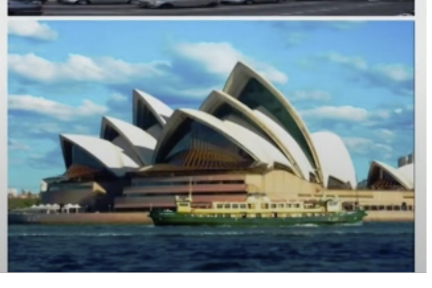
Indefinite Integral → $\int_a^{\infty} f(x)\,dx$

Integral sign

Integrand

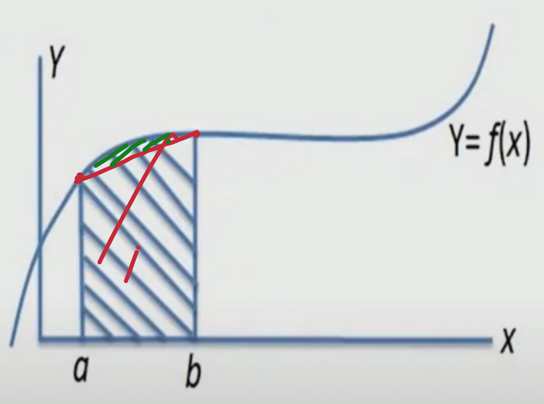
How integration applies to the real world?

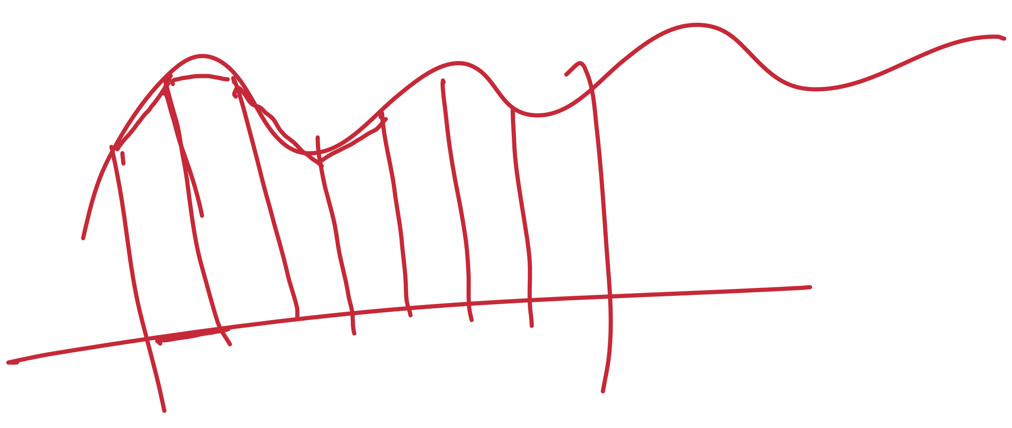
- Integration was used to design the PETRONAS Towers making it stronger
- Many differential equation were used in the designing of the Sydney Opera House
- Finding areas under curved surface, Centers of mass, displacement and velocity, and fluid flow are other uses of integration



What is Definite Integral ?

- The definite integral of the function $f(x)$ from a to b is the area bounded by $y=f(x)$, $y=0$, $x=a$ and $x=b$.





Why Numerical Integration ?

- If the function $f(x)$ is not given explicitly but the values are given at discrete points.
- Definite integral of some complicated functions e.g. $\int_0^1 e^{-x^2}dx$ is very difficult to carry out.

Difficulties in definite integral

Methodology For Numerical Integration

- Let us suppose that the functional values are known at $x=a$, $x=b$ and $(n-1)$ internal points in (a, b), namely $x=x_i$, $i=1(1)n-1$.
- Let us assume that $a=x_0<x_1<x_2\ldots<x_n=b$. These points on the x-axis are called pivotal or nodal points.
- Thus there are (n+1) nodal points, and n sub-intervals $[x_i, x_{i+1}]$, $i=1(1)n-1$.
- Evaluation of the integral to approximate the function $f(x)$ by a polynomial and integrate it.

- Approximating the function by a single polynomial globally over the entire domain $a\le x \le b$, it is approximated in piecewise manner.
- We fit the polynomial P(x) over k sub-intervals passing through the points (x_i, y_i), $i=0(1)k$ and evaluate the integral

$$I_1=\int_{x_0}^{x_k} f(x)dx=\int_{x_0}^{x_k} P(x)dx.$$

Obviously it covers k intervals, (x_i, x_{i+1}), $i=0(1)k$. The process is repeated for next k intervals and so on until the entire domain is covered.

$$I=\int_{x_0}^{x_K} f(x)\,dx=\int_{x_0}^{x_K} P(x)\,dx$$

The approximating