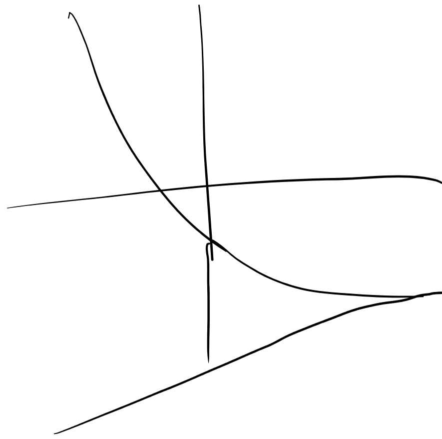
$$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\cdots$$

term	Sum
$\frac{1}{3}$	$\frac{1}{3} \approx 0.33$
$\frac{1}{3}+\frac{1}{9}$	$\frac{4}{9} \approx 0.44$
$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}$	$\frac{13}{27} \approx 0.48$
$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}$	$\frac{40}{81} \approx 0.49$
$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}$	$\frac{121}{243} \approx 0.50$



Power Series (P.S.)

A power series about a point $x=a$ (or a centered at $x=a$) is an expression of the

$$\sum_{n=0}^{\infty} c_n (x-a)^n$$

where a and c_n are real number and c_n's are called the coefficients of the series.

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + \cdots \quad \text{——(1)}$$

$a=0$ then the power series becomes

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

→ The power series may converge for some values of x and diverge for other values of x.

→ We shall that there is a number R so that the power series will converge for $|x-a|<R$ and will diverge for $|x-a|>R$ this number R is called the radius of convergence of the series.

→ This number R is called the radius of convergence of the series.

→ If $|x-a|=R$ then the power series may or may not converge. But this will not change the radius of convergent. That means at $x=a-R$ and $x=a+R$. We have to check the convergency of the power series

But this will not change the radius of convergence. ...
and $x = a + R$. We have to check the convergence of the power series separately.

Interval of Convergence of the power Series

The interval of Convergence must contain the interval $a - R < x < a + R$, since the power series converges for these values

$\Rightarrow$ The interval of all x's including the end points for which the power series converge is called the interval of Convergence of the series

$$\boxed{a - R < x < a + R}$$

Note that

P.S. Converges for $x = a$ be?

$$\sum_{n=0}^{\infty} c_n (x-a)^n = \sum c_n (a-a)^n = \underline{\underline{c_0}}$$

Thus even if the series may not converges for any other values of x, it is guaranted that it will converge for $x = a$

Determine the radius of Convergence of a power Series.

Ratio test

If $\lim_{n \to \infty} \left| \frac{c_{n+1}}{c_n} \right| = L$ then the power Series

when $L > 0$ OR $L = 0$ OR $L = \infty$

$\sum_{n=0}^{\infty} c_n (x-a)^n$ has a radius of Convergence $R = \frac{1}{L}$

Here if $L = 0$ then $R = \infty$

if $L = \infty$ then $R = 0$

Root test

If $\lim_{n \to \infty} |c_n|^{1/n} = L$

ex $\sum_{n=0}^{\infty} \frac{(x-6)^n}{n^n}$

ex

$$\sum_{n=0}^{\infty} \frac{(x-6)}{n^n}$$

$$\sum_{n=0}^{\infty} \frac{(x-6)^n}{n^n} = \sum_{n=0}^{\infty} C_n (x-a)^n$$

$$C_n = \frac{1}{n^n} \qquad a = 6$$

$$\Rightarrow \lim_{n\to\infty} |C_n|^{1/n} = \lim_{n\to\infty} \left(\frac{1}{n^n}\right)^{1/n} = \lim_{n\to\infty} \frac{1}{n} = 0 \quad \text{So, } L=0, \quad R = \infty$$

So, the series is converges for all values of x i.e

$$-\infty < x < \infty \Rightarrow a - R < x < a + R$$

$$\Rightarrow 6 - \infty < x < 6 + \infty$$

$$\lim_{n\to\infty} \left|\frac{C_{n+1}}{C_n}\right| = \lim_{n\to\infty} \frac{1}{(n+1)^{n+1}} n^n = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} \cdot \frac{1}{n+1}$$

$$= 0 = L$$

$$R = \infty$$

H.T. ① $\displaystyle\sum \frac{n!\,(5x+3)^n}{(n+1)^2+4n}$ ② $\displaystyle\sum_{n=0}^{\infty} \frac{(3x+4)^n}{(n^3+2)\,3^n}$