Indian Institute of Technology Indore
MA204 Numerical methods
Instructor: Dr. Debopriya Mukherjee
Tutorial Sheet 1

---

1. Show that the error in approximation of the root at the n^{th} step ($n = 1, 2, 3, \dots$) in the bisection method is bounded by $(b-a)/2^n$, where $[a, b]$ is the interval containing the root that is being approximated.

2. Find an approximation to $\sqrt[3]{25}$ correct to within 10^{-4} using the bisection method.

3. (a) Show that the order of convergence of Newton's (or also called the Newton–Raphson) method is 2, i.e. the method is quadratically convergent.

   (b) Show that the order of convergence of the secant method is $(1 + \sqrt{5})/2 \approx 1.618$.

4. Use Newton's method to find solutions accurate to within 10^{-5} to the following problems

   (a) $\ln(x-1) + \cos(x-1) = 0, \quad \text{for} \quad 1.3 \leq x \leq 2.$

   (b) $e^x + 2^{-x} + 2\cos x = 6, \quad \text{for} \quad 1 \leq x \leq 2.$

   You may use the mid point of the interval as the initial guess in each case.

5. Repeat problem 5 with the secant method. Take x_0 same as that taken in problem 5 and x_1 from the first iteration of Newton's method but with only three significant digits after rounding off.

6. Let $f(x) = e^x - x - 1$. Show that $f(x)$ has a zero of multiplicity 2 at $x = 0$. Find this root accurate to within 10^{-5} by (i) Newton's method and (ii) by modified Newton's method. [Realize the rate of convergence of the two methods to the root.]

7. A calculator is defective: it can only add, subtract, and multiply. Use the equation $1/x = 1.37$, the Newton Method, and the defective calculator to find $1/1.37$ correct to 8 decimal places.

8. Implement the bisection method, Newton's method and the secant method in your favorite programming language to find the root of $x^6 - x - 1 = 0$ that lies in the interval $[1, 2]$. The tolerance for the error you may take as 10^{-5}. For Newton's method, you may take $x_0 = 1.5$ and for the secant method you may take $x_0 = 1.5$ and $x_1 = 1$.

9. Show that $x^3 + 4x^2 = 10$ has a solution in $[1, 2]$, and use the bisection method to determine an approximation to this solution that is accurate to at least within 10^{-5}.

10. Use the bisection method to find solutions accurate to within 10^{-4} for the following problems

    (a) $e^x - x^2 + 3x = 2 \quad \text{for} \quad 0 \leq x \leq 1,$

    (b) $x + 1 - 2\sin(\pi x) = 0 \quad \text{for} \quad 0.5 \leq x \leq 1.$