INDIAN INSTITUTE OF TECHNOLOGY INDORE
MA-204 NUMERICAL METHODS
Assignment -1-System of Linear Equations

1. Solve the following systems of equations by converting the coefficient matrix to row reduced echelon form.

(a) $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & -2 \\ 3 & 1 & 2 \\ 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 3 \\ 8 \\ 7 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix}$

2. Solve the following systems of equations by Gauss elimination method:

(a) x+2y+z=0, 2x+2y+3z=3, x+3y= -2 (b) x+2y+3z=14, 2x+3y+4z=20, 3x+4y+z=14 (c) 2x+3y+z=9, x+2y+3z=6, 3x+y+2z=8

3. Find the inverse of the following matrices by the Gauss-Jordan elimination method:

(a) $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{pmatrix}$

4. Solve the following linear systems by Gauss-Jordan elimination method, with **partial pivoting** if necessary (but without scaling):

(a) 4x+y+z=4, x+4y-2z=4, 3x+2y-4z= 6 (b) x+y-z=2, 2x+3y+5z=-3, 3x+2y-3z=6 (c) 2x+3y+z=9, x+2y+3z=6, 3x+y+2z=8

5. Solve the following linear systems by Gauss elimination method, with **total pivoting** if necessary (but without scaling):

(a) 3x+5y+2z=8, 8y+2z=-7, 6x+2y+8z=26 (b) 2x+y-z=0, x+y+z=9, 2x+5y+7z=52 (c) x+y+z=2, 2x+2y+3z=7, 5x-y+13z=0

6. Solve the following systems of equations by Doolittle's and Crout's methods:

(a) 10x+y+z=12, x+10y+z=12, x+y+10z= 12 (b) x+y=0, y+z=1, x+z= 3 (c) x+y+z=2, 2x+2y+3z=7, 5x-y+13z=0

7. Verify whether the following matrices are positive definite:

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 15 & 4 & 2 & 9 & 0 \\ 4 & 7 & 1 & 1 & 1 \\ -2 & 1 & 18 & 6 & 6 \\ 9 & 1 & 6 & 19 & 3 \\ 0 & 1 & 6 & 3 & 11 \end{pmatrix}$

8. Solve the following systems of equations by Cholesky's method, if the method is applicable. If it is not applicable, give the reason.

(a) $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix}$