Wednesday, 2 February 2022 12:02 PM

1. $\Delta^2 y_0 = \Delta(\Delta y_0) = \Delta(y_1 - y_0) = \Delta y_1 - \Delta y_0$
$= (y_2 - y_1) - (y_1 - y_0)$
$= y_2 - 2y_1 + y_0$

2. $\nabla^2 y_n = \nabla(\nabla y_n) = \nabla(y_n - y_{n-1})$
$= \nabla y_n - \nabla y_{n-1}$
$= (y_n - y_{n-1}) - (y_{n-1} - y_{n-2})$
$= y_n - 2y_{n-1} + y_{n-2}$

3. $E^2 y_0 = E(E y_0) = E y_1 = y_2$

4. $\delta^2 y_x = \delta\left[y\left(x+\frac{h}{2}\right) - y\left(x-\frac{h}{2}\right)\right] = \delta y_{x+\frac{h}{2}} - \delta y_{x-\frac{h}{2}}$
$= (y_{x+h} - y_x) - (y_x - y_{x-h})$
$= y_{x+h} - 2y_x + y_{x-h}$

$\Delta y_0 = y_1 - y_0 \Rightarrow \Delta y_r = y_{r+1} - y_r$

$\nabla y_n = y_n - y_{n-1} \Rightarrow \nabla y_1 = y_1 - y_0$

$E y_0 = y_1 \qquad E y_r = y_{r+1}$ etc

$\delta(f(x)) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$

$\delta = [E^{1/2} - E^{-1/2}]$

So, $\Delta f(x_r) = f(x_r + h) - f(x_r)$

Relation

$E = 1 + \Delta$

$\Delta = E - 1$

$E = (1 - \nabla)^{-1}$

$\delta = E^{1/2} - E^{-1/2}$

$\Delta\nabla = \Delta - \nabla$

& $\Delta\nabla = \Delta - \nabla = \delta^2$

Suppose we have arrived at x from x_0 by p jumps.
Same number of jumps, we must taken to come to y from y_0

Therefore, $y = E^p y_0$ and $x = x_0 + ph \Rightarrow p = \frac{x - x_0}{h}$

We know,
$y = (1+\Delta)^p y_0 \qquad$ where $x = x_0 + ph$
$= \left[1 + p\Delta + \frac{p(p-1)}{2!}\Delta^2 + \frac{p(p-1)(p-2)}{3!}\Delta^3 + \cdots\right] y_0$

or $y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$

The values of y_0, Δy_0, $\Delta^2 y_0$, ... etc. will be available from forward table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
x_1	y_1	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_1$	
x_2	y_2	Δy_2	$\Delta^2 y_2$		
x_3	y_3	Δy_3			
x_4	y_4				

Newton Forward and Backward interpolation Will give good results when interpolations are done near to the beginning and at end of the table respectively.

For better result near to the middle of the table, the Central formulae
1. Gauss Forward
2. Gauss Backward
3. Stirling
4. Bessel
5. Everett's

are suitable.

Gauss Forward formula as

$$y = y_0 + G_1\Delta y_0 + G_2\Delta^2 y_{-1} + G_3\Delta^3 y_{-1} + G_4\Delta^4 y_{-2} + \cdots \quad (*)$$

x	y	Δ	Δ^2	Δ^3	
x_{-2}	y_{-2}	Δy_{-2}	$\Delta^2 y_{-2}$	$\Delta^3 y_{-2}$	
x_{-1}	y_{-1}	Δy_{-1}	$\Delta^2 y_{-1}$	$\Delta^3 y_{-1}$	$\Delta^4 y_{-2}$
x_0	y_0	Δy_0	$\Delta^2 y_0$		
x_1	y_1	Δy_1			
x_2	y_2				

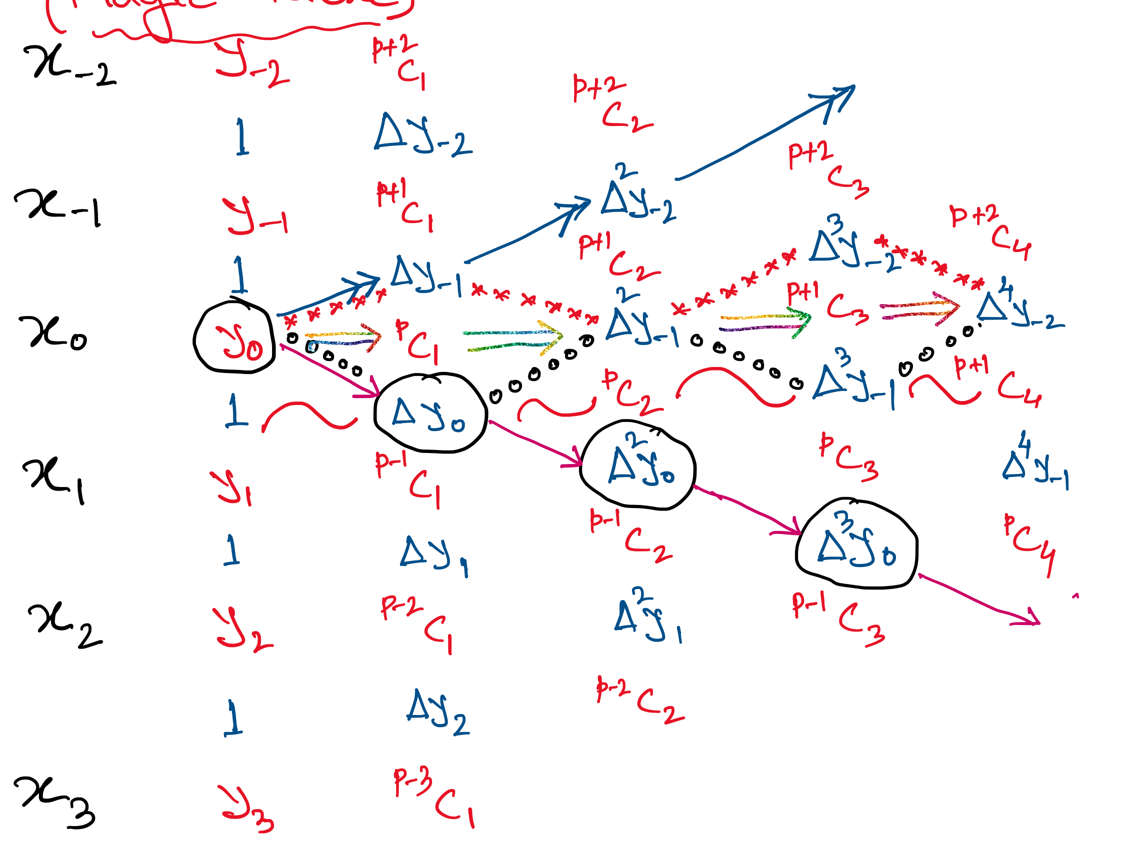
By introducing Newton forward formula one can write as

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-1} + \frac{(p+1)p(p-1)(p-2)}{4!}\Delta^4 y_{-2} + \cdots$$

Here $p = \frac{x - x_0}{h}$,
$\Delta^2 y_{-1} = \Delta^2 E^{-1} y_0 = \Delta^2(1+\Delta)^{-1} y_0$
$= \Delta^2(1 - \Delta + \Delta^2 - \Delta^3 + \Delta^4 \cdots) y_0$
$= \Delta^2 y_0 - \Delta^3 y_0 + \Delta^4 y_0 - \Delta^5 y_0 + \cdots$

Geophy Indian

Lozenge Diagram
(Magic Table)



1. Newton Forward
2. Newton Backward
3. Gauss Forward (Zigzag ·······)
4. Gauss Backward (Zigzag *****)
5. Stirling's formula
6. Bessel's formula

For N.F :
$y_p = y_0 + {}^pC_1\Delta y_0 + {}^pC_2\Delta^2 y_0 + {}^pC_3\Delta^3 y_0 + \cdots$

N.B. : $y_p = y_0 + {}^pC_1\Delta y_{-1} + {}^{(p+1)}C_2\Delta^2 y_{-2} + \cdots$

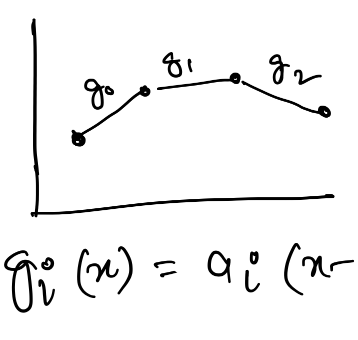
G.F. $y_p = y_0 + {}^pC_1\Delta y_0 + {}^pC_2\Delta^2 y_{-1} + {}^{(p+1)}C_3\Delta^3 y_{-1} + \cdots$

G.B. $y_p = y_0 + {}^pC_1\Delta y_{-1} + {}^{(p+1)}C_2\Delta^2 y_{-1} + {}^{(p+1)}C_3\Delta^3 y_{-2} + \cdots$

Stirling: $y_p = y_0\left(\frac{1+1}{2}\right) + \frac{\Delta y_1 + \Delta y_0}{2}\,{}^pC_1 + \Delta^2 y_{-1}\left({}^{p+1}C_3 + {}^pC_2\right) + {}^{p+1}C_3\left(\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2}\right) + \cdots$

Bessel $y_p = \frac{y_0 + y_1}{2} + \Delta y_0\left(\frac{{}^pC_1 + {}^{p+1}C_1}{2}\right) + {}^pC_2\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right) + \Delta^3 y_{-1}\left({}^{p+1}C_3 + {}^pC_3\right) + \cdots$

Piecewise linea interpolation



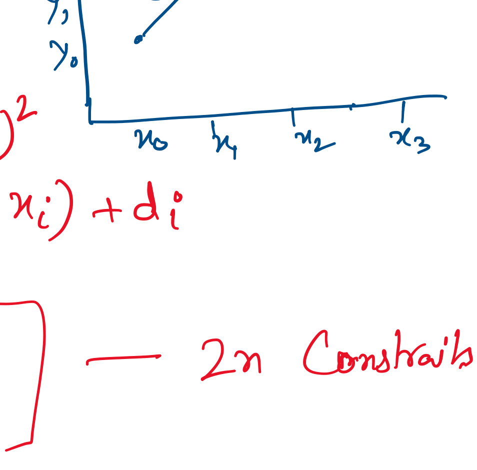
$g_i(x) = a_i(x - x_i) + b_i$

$g_i(x_i) = y_i$
$g_i(x_{i+1}) = y_{i+1}$

$\therefore b_i = y_i$
$a_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$

$\Rightarrow g_i(x) = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i) + y_i$

In Case of Cubic Spline interpolation
We have $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$
We want to find some value of x, which is not in the list.



$g_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_2(x - x_i) + d_i$

$\left.\begin{array}{l} g_i(x_i) = y_i \\ g_i(x_{i+1}) = y_{i+1} \end{array}\right]$ — $2n$ Constraints

$g_i'(x_{i+1}) = g_{i+1}'(x_{i+1})$ — $(n-1)$ Constraints

$g_i''(x_{i+1}) = g_{i+1}''(x_{i+1})$ $(n-1)$ Constraints