

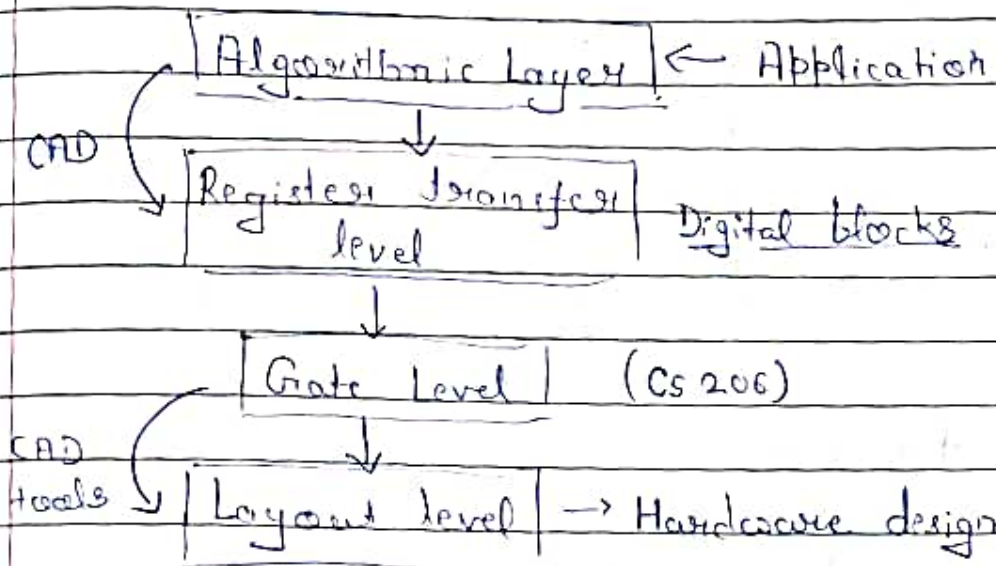
(Brown Verilog "VHDL")

Date:

P. No:

Digital Systems

(Mauris Mano "Digital Logic")



Name	NOT gate	AND gate	OR gate	XOR gate																																																			
Expression	\bar{A}	$A \cdot B$	$A + B$	$A\bar{B} + \bar{A}B$																																																			
Symbol																																																							
	<table><tr><th>A</th><th>\bar{A}</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	A	\bar{A}	0	1	1	0	<table><tr><th>A</th><th>B</th><th>$A \cdot B$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	$A \cdot B$	0	0	0	0	1	0	1	0	0	1	1	1	<table><tr><th>A</th><th>B</th><th>$A + B$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	$A + B$	0	0	0	0	1	1	1	0	1	1	1	1	<table><tr><th>A</th><th>B</th><th>$A \text{ XOR } B$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	$A \text{ XOR } B$	0	0	0	0	1	1	1	0	1	1	1	0
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$$\text{XNOR}(y) = \overline{\text{XOR}} \quad (AB + \bar{A}\bar{B})$$

$$y = \bar{A} \oplus B$$
$$= A \odot B$$

$$A \odot B = A\bar{B} + \bar{A}B$$

Basic Axioms:-

- | | |
|------------|--------------------|
| 1) $0+0=0$ | 5) $0 \cdot 0 = 0$ |
| 2) $0+1=1$ | 6) $0 \cdot 1 = 0$ |
| 3) $1+0=1$ | 7) $1 \cdot 1 = 1$ |
| 4) $1+1=1$ | 8) $\cancel{1}$ |

Boolean Laws:-

- 1) Commutative Law
 $A+B = B+A$
 $AB = BA$
- 2) Associative Law
 $A+(B+C) = (A+B)+C$
 $A(BC) = (AB)C$
- 3) Distributive Law
 $AB + A(C) = A(B+C)$

Rules:-

- | | |
|----------------------|---|
| 1) $A+0 = A$ | 7) $A\bar{A} = 0$ |
| 2) $A+1 = 1$ | 8) $A \cdot A = A$ |
| 3) $A \cdot 0 = 0$ | 9) $\bar{\bar{A}} = A$ |
| 4) $A \cdot 1 = A$ | 10) $A + AB = A(1+B)$
$= A$ |
| 5) $A + A = A$ | 11) $A + \bar{A}B = (A + \bar{A})(A + B)$
$= A + B$ (absorption) |
| 6) $A + \bar{A} = 1$ | 12) $(A+B)(A+C) = A + BC$ |

Combining Law:-

$$1) A(A+B) = A$$

$$2) AB + A\bar{B} = A$$

$$3) (A+B)(A+\bar{B}) = A$$

$$4) AC + B\bar{C} = AC + B\bar{C} + AB \quad (\text{consensus law})$$

$$\text{Consensus Law: } F = AC + B\bar{C} + AB$$

$$= AC + B\bar{C} + AB(C + \bar{C})$$

$$= AC + B\bar{C} + ABC + AB\bar{C}$$

$$= AC(1+B) + B\bar{C}(1+A)$$

$$= AC + B\bar{C}$$

$$F = (A+B)(\bar{A}+C)$$

DeMorgan's Law:-

$$1) \overline{AB} = \bar{A} + \bar{B}$$

$$2) \overline{A+B} = \bar{A} \cdot \bar{B}$$

v.3.1

Intel Quartus v13.1 (CAD tool)

Tutorial - 1

1. Convert from decimal to binary:
 - a) 0.188
 - b) 410
2. Hexadecimal to decimal from 257.7B
3. Hexadecimal to binary of 3ACF7
4. Octal to decimal of a) 531 b) 320.127
5. Decimal to octal of 316.32

Ques. (I) =

Subtract using Complement Method:-

- Q.1 → Subtract $(1000)_2 - (1110)_2$ using 2's complement method.
- Q.2 - Subtract $(13250)_{10} - (72532)_{10}$ Using 9's complement method.
- Q.3 - Subtract $(3250)_{10} - (72352)_{10}$ Using 10's complement method.

Sol. (I) • If there's a carry bit, drop the bit
 & the result is a +ve number.

- If no carry bit, take 2's complement of the result which is negative.

Code Conversion and BCD addition/subtraction:-

- Q.1- Convert $(101100011)_{BCD}$ to decimal.
- Q.2 Convert $(379)_{10}$ to BCD.
- Q.3 Convert Gray code $(11011)_{gray}$ to binary.
- Q.4- Convert binary $(10110)_{binary}$ to Gray code.
- Q.5- BCD addition of $(01100110)_{BCD}$ and $(01010011)_{BCD}$.
- Q.6- Add following using BCD addition method:
 $(956)_{10}$ and $(492)_{10}$.
- Q.7. Subtract following using BCD subtraction method: $(17)_{10}$ and $(12)_{10}$.

Ans:-

Minterm: In a f of 'n' variables, each term appears at least once.

Ex: $ABC + AC$ (Non-standard)

Product of Sum (PoS): $(A+B+C) \cdot (A+C)$
 \rightarrow non-standard

Maxterm

Canonical / Standard SOP / PoS :-

Ex: 1) Convert :-

$$\begin{aligned} X &= \bar{A}\bar{B} + ABC \\ &= \bar{A}\bar{B}(C + \bar{C}) + ABC \\ &= \underbrace{\bar{A}\bar{B}C} + \underbrace{\bar{A}\bar{B}\bar{C}} + \underbrace{ABC} \quad (\text{all are minterms}) \end{aligned}$$

Ex: 2) Convert :-

$$\begin{aligned} X &= (\bar{A} + \bar{B})(A + B + C) \\ &= (\bar{A} + \bar{B} + C\bar{C})(A + B + C) \\ &= \underbrace{(\bar{A} + \bar{B} + C)} \underbrace{(\bar{A} + \bar{B} + \bar{C})} \underbrace{(A + B + C)} \quad (\text{all are maxterms}) \end{aligned}$$

Combinational Circuits :-

- 1) Half-Adder (HA). \rightarrow (1-bit adder).
 \rightarrow Addition of 2 boolean variables

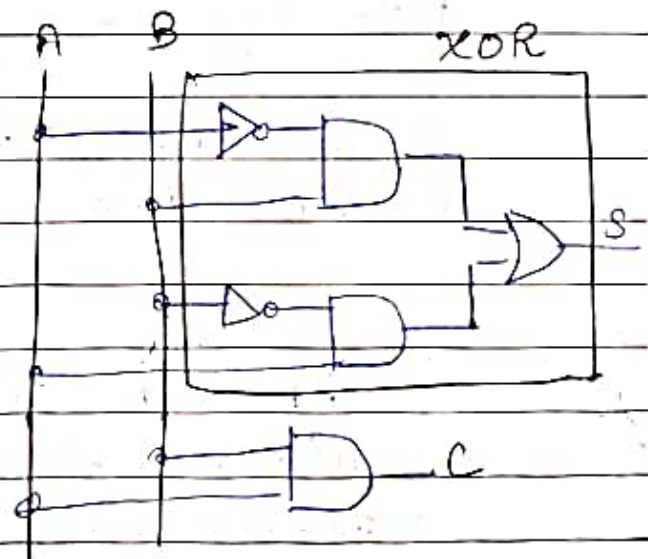


Truth Table :-

A	B	S	C
0	0	0	0
1	1	1	0
0	1	1	0
1	0	0	1

$$S = \bar{A}B + A\bar{B} = A \oplus B$$

$$C = AB$$



NAND logic: $S = \bar{A}B + A\bar{B}$

$$= A\bar{B} + A\bar{A} + \bar{A}B + B\bar{B}$$

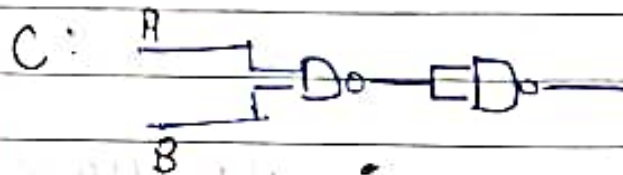
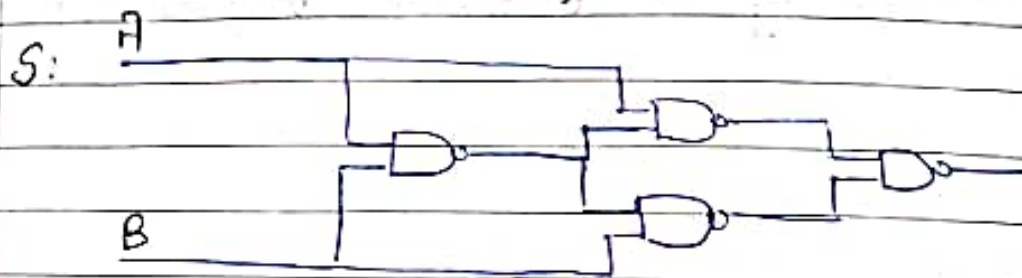
$$= A(\bar{A} + B) + B(\bar{A} + \bar{B})$$

$$= (A + B)(\bar{A} + \bar{B})$$

$$= (A + B)\overline{AB}$$

$$= A \cdot \overline{AB} + B \cdot \overline{AB}$$

$$\therefore \left. \begin{aligned} S &= (\overline{A \cdot \overline{AB}}) \cdot (\overline{B \cdot \overline{AB}}) \\ C &= \overline{AB} \end{aligned} \right\} \begin{aligned} \overline{XY} &= X + Y \\ \overline{XY} &= \overline{X} + \overline{Y} \end{aligned}$$

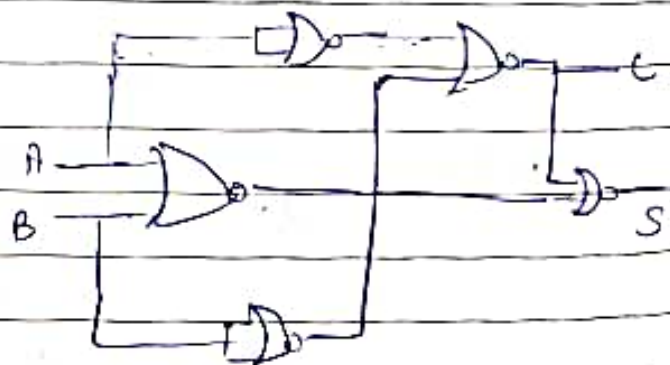


NOR logic:-

$$\begin{aligned} S &= \overline{A}B + A\overline{B} = \overline{A+B} + \overline{A+B} \\ C &= AB = \overline{\overline{A+B}} \end{aligned}$$

$$\begin{aligned} S &= \overline{A}B + A\overline{B} + \overline{A}B + AB \\ &= A(\overline{B} + B) + B(\overline{A} + A) \\ &= (A+B)(\overline{A} + \overline{B}) \\ &= \overline{A+B} + \overline{A+B} \end{aligned}$$

$$C = \overline{\overline{A+B}}$$



① - Full adder:-



	A	B	C	S	C _{out}
0	0	0	0	0	0
1	0	0	1	1	0
2	0	1	0	1	0
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	0	1
7	1	1	1	1	1

SOP :-

$$S = \sum_m (1, 2, 4, 7) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

$$C_{out} = \sum_m (3, 5, 6, 7)$$

$$\begin{aligned} S &= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC) \\ &= \bar{A}(B \oplus C) + A(\overline{B \oplus C}) \\ &= \bar{A}(B \oplus C) + A(\overline{B \oplus C}) \end{aligned}$$

$$S = A \oplus B \oplus C$$

$$\begin{aligned} [\bar{A}x + A\bar{x} &= A \oplus x \\ x &= B \oplus C \end{aligned}]$$

$$C = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$= C(A \oplus B) + AB$$

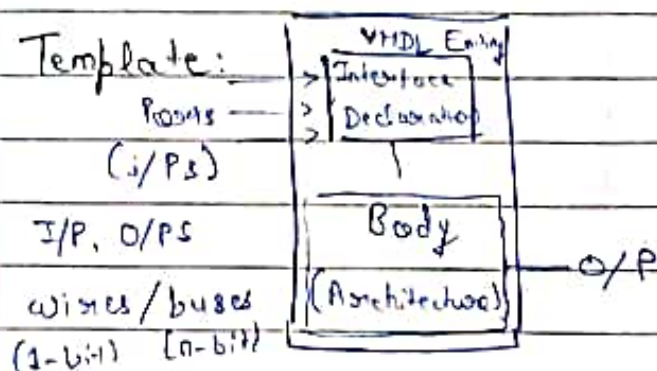
$$s. \quad C = AB + BC + CA$$

VHDL Fundamentals

VHDL: VHSIC Hardware Description Language

VHSIC: Very High Speed Integrated Circuit.

Other languages: Verilog, AHDL, etc.



Ports: Primary I/P, O/P.

Keywords:-

→ IS, IN, OUT, PORT MAP, END, START, INOUT, ENTITY, ARCHITECTURE

→ These are case insensitive

Descriptions:-

- 17 Signal names: Can be innumeration through command
- 22 Mode: IN - specifying that signal is an input
 OUT - specifying signal is an output
 INOUT - specifying signal is both i/p & o/p.
 Buffer -



- 37 Type: Built-in or user defined signal type:
examples:

bit: can have logical value 0 or 1.

std_logic: can have boolean values (1-bit info).

std_logic_vector: can have vector of bit values
 { n dec to 0 }
 { 3 dec to 0 }
 { 4-bit }

e.g.

ENTITY AND

e.g. ENTITY AND2 IS

Port (x: IN std_logic;

y: IN std_logic;

f: OUT std_logic);

end AND2;

architecture name

Architecture ~~AND2~~ AND2 of AND2 IS

Begin

$F \leftarrow x \text{ AND } y$;

end AND2.

Libraries:-

Library IEEE;

USE IEEE std_logic_1164.all;

USE IEEE numeric_std.all;

USE IEEE std_logic_arith.all;

Tutorial - 2

1. Express $F = A + B'C$ in canonical forms SOP and POS.
2. $\pi[B + C(AB + AC)']$, no. of NAND gates required to design the logic.
3. Is $(A \oplus B) \oplus B = A \oplus (B \oplus C) + (A \oplus A)$
4. $(A' + B) \cdot (A + B) = ?$
5. $F(A, B, C, D) = \pi(0, 1, 5, 7, 8, 9, 15)$
 $f(A, B, C, D) = \sum(2, 3, 4, 6, 12, 13)$
 $d(A, B, C, D) = (10, 13, 14)$

Ans. (1)

$$F = A + B'C$$

$$= A(BC + \bar{B}\bar{C}) + AB'C + A'B'C$$

$$= ABC + A\bar{B} + A\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

$$= ABC + A\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C}$$

$$F = ABC + A\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

$$F = A + B'C + (C')$$

$$= A + (B + \bar{C})$$

$$(A + C + \bar{C})$$

$$F = (F')' = (A' \cdot (B + C'))' = (A'B + A'C)$$

$$(A + B' + C) \cdot (A + B' + \bar{C}) = (A + B') \cdot (A + C)$$

$$2. \quad A[B + C(A \cdot (B+C))']$$

$$A[B + C(A' + B'C')]$$

$$A[B + A'C] + B$$

$$AB$$

$$3. \quad \frac{A}{B} = \frac{D}{C} \Rightarrow \frac{A}{B} = \frac{D}{C}$$

$$4. \quad (A' + B) \cdot (A + B) = B$$

5. a)

		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
		00	01	11	10
$\bar{A}\bar{B}$	00	1 ₁	1 ₂	0 ₃	0 ₄
$\bar{A}B$	01	0 ₅	1 ₆	1 ₇	0 ₈
$A\bar{B}$	11	0 ₉	1 ₁₀	1 ₁₁	1 ₁₂
AB	10	1 ₁₃	1 ₁₄	0 ₁₅	1 ₁₆

$$BD + \bar{B}\bar{C}$$

$$(B + \bar{D}) + B + C$$

$$\text{essential} \rightarrow 2$$

$$\text{non-essential}$$

B

$$6. \quad f = A + B + C + D + E$$

$$= A + B + C + D + E$$

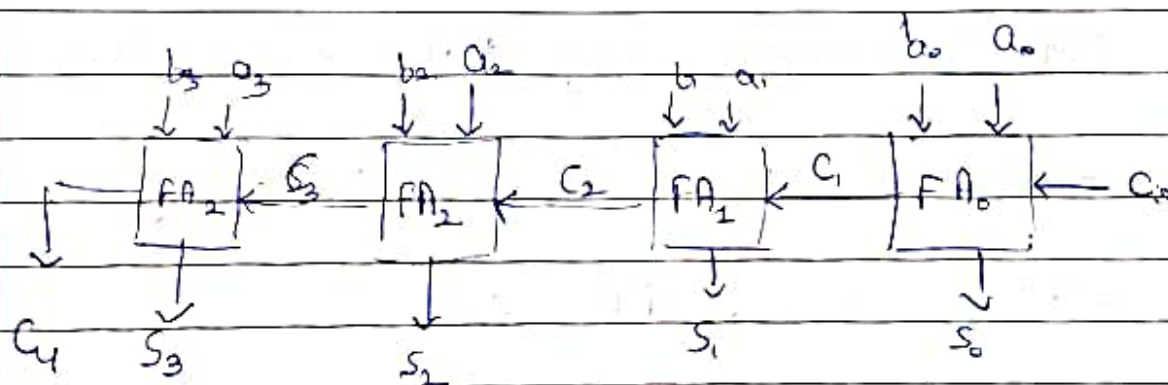
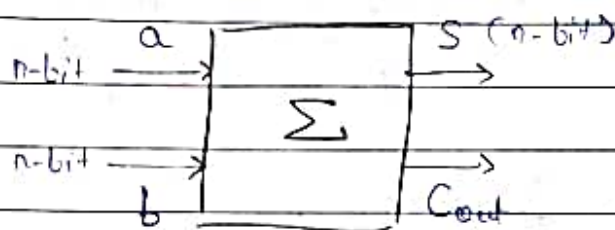
$$= A + B + C + D + E$$

$$A'B'C'D + A'B'C'E$$

$$A'B'C'(D + E)$$

Parallel Adders

- n -bit addition
- variable size i/p binary string
- unrestricted bit length



Parallel adder
(4-bit ripple carry adder)
(RCA)

$$[S_3 S_2 S_1 S_0] \rightarrow S$$

1) Area

2) Delay (high delay \rightarrow low performance)

For higher order bits, the delay is large.

(Carry bit has to travel all the way to cout)

Delay of RCA :-

$$T = (n-1)T_c + T_s$$

T_c : delay of carry being ripple through the previous stages

T_s : Delay of producing the final stage sum

Area of RCA: $n \times A_{FA}$

(can be found using no. of gates
→ expressed using universal logic)

1 inverter → 1 nmos
1 cmos

CMOS → Complementary Metal Oxide Semiconductor

$$n \times A_{FA} = (n-1) A_{FA} + A_{FA} \text{ (if } C_{in} = 0)$$

$$\text{e.g. } C_{in} = 0 \rightarrow FA_0 \rightarrow FA_1$$

Entity FA is

port (a, b, C_{in} : IN std. logic;
S, C: OUT std. logic);

end FA;

Architecture FA behaviour of FA is

Begin

$$S \leftarrow (a \text{ XOR } b) \text{ XOR } C_{in};$$

$$C \leftarrow (a \text{ AND } b) \text{ OR } (C_{in} \text{ AND } (a \text{ XOR } b));$$

End behaviour;

→ Library IEEE;
Using IEEE.std_logic_1164.all;

Entity FOURBIT IS

PORT (a, b : IN std_logic_vector (3 downto 0);

Cin : IN std_logic;

S : OUT std_logic_vector (3 downto 0);

Count : OUT std_logic);

END FOURBIT;

Architecture FOUR_struct of FOURBIT IS

Signal c : std_logic_vector (4 downto 1);

Component FA

port (a, b, Cin : IN std_logic;

S, C : OUT std_logic);

end component;

Begin

keypressed

FA(0) : FA portmap (a(0), b(0), Cin, S(0), c(1));

FA(1) : FA portmap (a(1), b(1), c(1), S(1), c(2));

FA(2) : FA portmap (a(2), b(2), c(2), S(2), c(3));

FA(3) : FA portmap (a(3), b(3), c(3), S(3), c(4));

Count = c(4);

END FOUR_struct;

(Schematic capture) → design entry modes

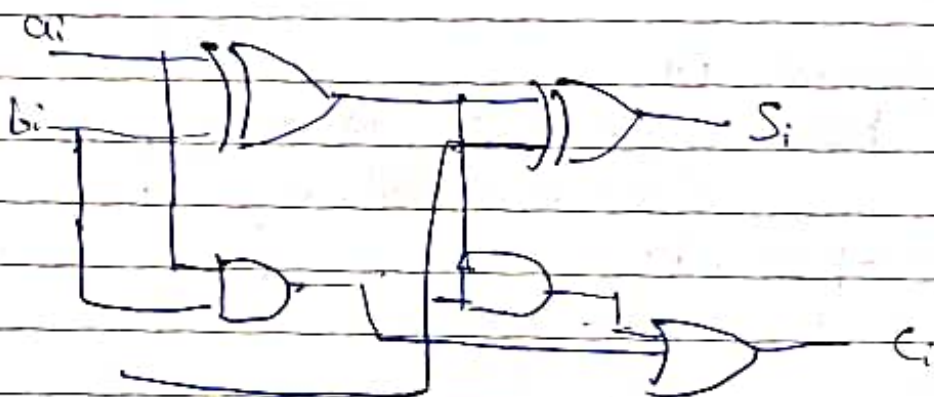
adders
(contd.)

$$C_{i+1} = a_i b_i + (a_i \oplus b_i) \cdot C_i$$

$$S_i = (a_i \oplus b_i) \oplus C_i$$

$$C_{i+1} = G_i + P_i (C_i)$$

where: $G_i = a_i b_i$ } carry generate
 $P_i = a_i \oplus b_i$ } carry propagate.



$$S_0 = P_0 \oplus C_0$$

$$C_1 = G_0 + P_0 C_0 \quad \text{--- (1)}$$

$$S_1 = P_1 \oplus C_1$$

$$\begin{aligned} C_2 = C_{1+1} &= G_1 + P_1 C_1 \\ &= G_1 + P_1 (G_0 + P_0 C_0) \\ &= G_1 + P_1 G_0 + P_1 P_0 C_0 \end{aligned} \quad \text{--- (2)}$$

$$S_2 = P_2 \oplus C_2$$

$$\begin{aligned} C_3 = C_{2+1} &= G_2 + P_2 C_2 \\ &= G_2 + P_2 (G_1 + P_1 G_0 + P_0 P_1 C_0) \\ &= G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_0 P_1 C_0 \end{aligned} \quad \text{--- (3)}$$

$$S_3 = P_3 \oplus C_3$$

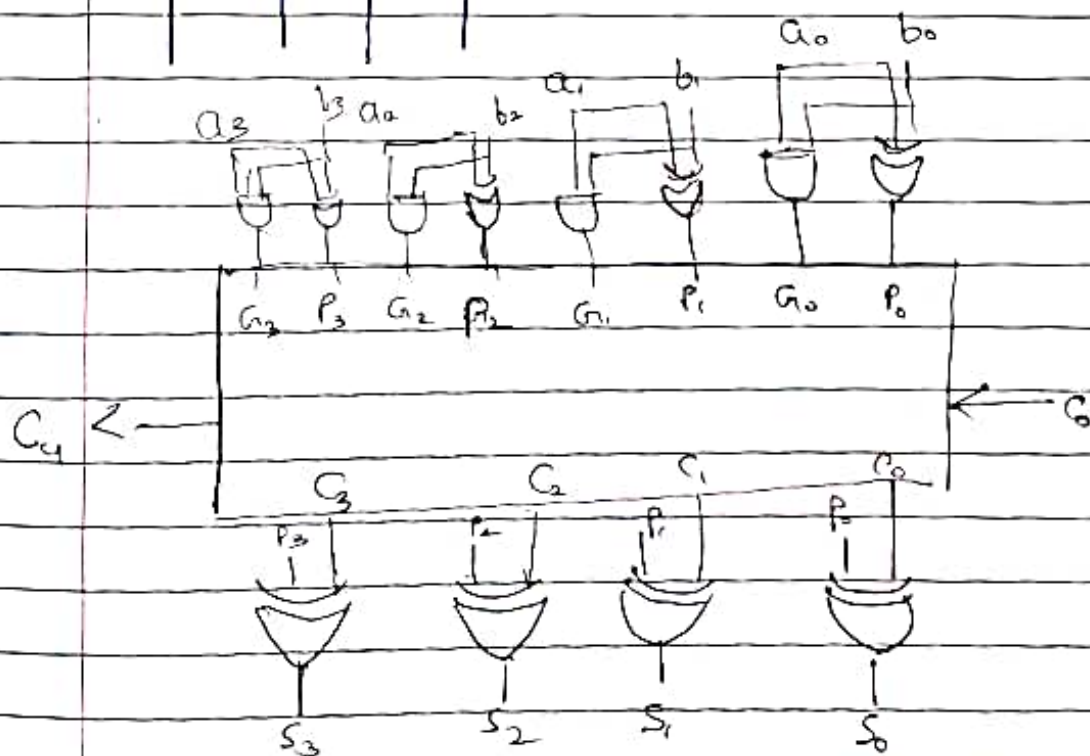
$$C_4 = C_3$$

$$= G_3 + P_3 C_3$$

$$= G_3 + P_3 (G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0)$$

$$= G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 C_0 \quad \text{--- (4)}$$

$P_0 \quad P_1 \quad P_2 \quad P_3$



BCD Adder

- Adds 2 binary digits
- Each binary digit is represented as 4-bit number
- Produces sum between 0-9
- e.g. 526 is represented as

$$\begin{array}{r}
 \begin{array}{ccc}
 5 & 2 & 6 \\
 \downarrow & \downarrow & \downarrow \\
 0101 & 0010 & 0110
 \end{array} \\
 \hline
 010100100110
 \end{array}$$

Case 1) Sum equals to 9 or less with carry 0 :-

$$\begin{array}{r}
 6 \quad 0110 \\
 + 3 \quad 0011 \\
 \hline
 9 \quad 1001
 \end{array}$$

Case 2) Sum greater than 9, with carry 1 :-

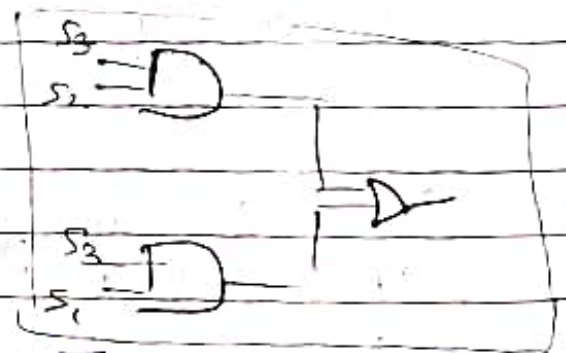
$$\begin{array}{r}
 7 \quad 0111 \\
 + 9 \quad 1000 \\
 \hline
 \text{Carry} \leftarrow 1 \quad 0000
 \end{array}$$

Sum the invalid o/p with '6'.

$$\begin{array}{r}
 0000 \\
 0110 \\
 \hline
 \text{Carry} \rightarrow 1 \quad 0110
 \end{array}$$

$$\begin{array}{r}
 6 \\
 + 8 \\
 \hline
 14
 \end{array}
 \begin{array}{r}
 0110 \\
 1000 \\
 \hline
 1110 \\
 0110 \\
 \hline
 00010100
 \end{array}$$

	S_3	S_2	S_1	S_0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
Invalid	1	0	1	0
	1	0	1	1
	1	1	0	0
	1	1	0	1
	1	1	1	0
	1	1	1	1
	1	1	1	1

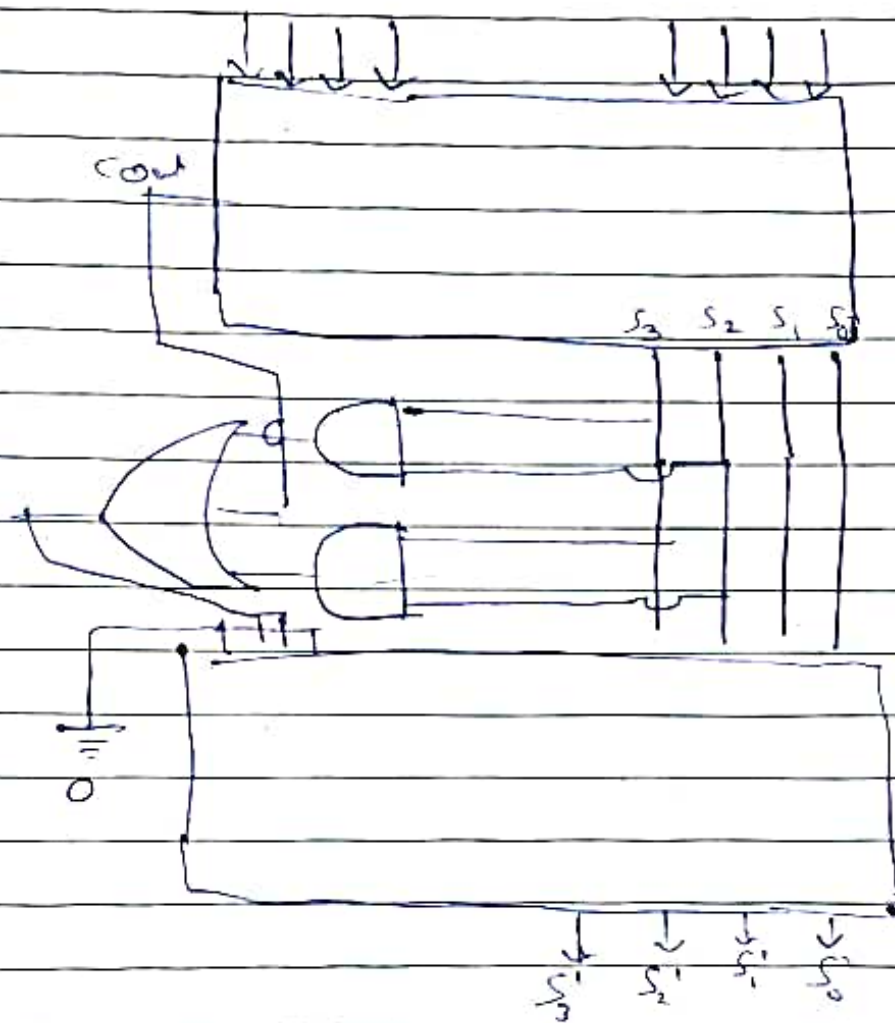
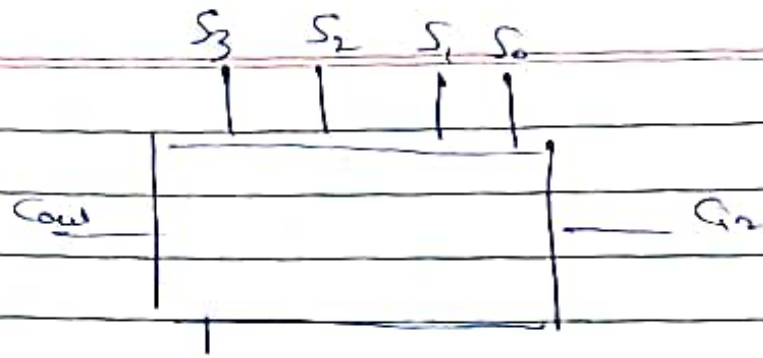


Invalid BCD
detection circuit.

$$\text{Invalid} \rightarrow S_3 S_2 + S_3 S_1$$

Date:

P. No:



Tutorial-4

$$1.a) - F = bc + bcd + ac' + ab$$

$$= bc + ac' + ab$$

$$= a(b+c) + bc$$

$$= a\bar{b}\bar{c} + bc$$

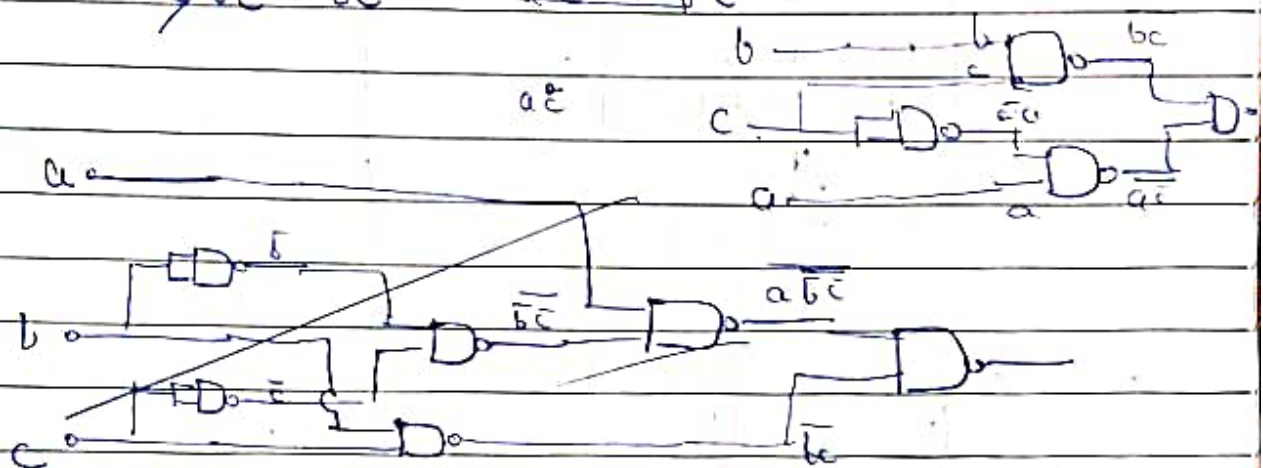
$$= \overline{a\bar{b}\bar{c} \cdot bc}$$

$$bc + ac' + ab$$

$$bc + ac'$$

$$\overline{bc \cdot ac'}$$

ac



$$bc + ac' + ab$$

$$bc + b\bar{c}$$

$$b(c + \bar{c}) + c'(c + a)$$

$$(c + a)(c' + b)$$

$$\bar{c}\bar{a} + bc + ac' + ab$$

$$\bar{c}\bar{a} \cdot \bar{c}\bar{b}$$

$$\bar{c}\bar{a} \cdot \bar{c} + \bar{c}\bar{a} \cdot b$$

b) → library IEEE;

use IEEE.std_logic_1164.all;

entity circuit is

port (a, b, c, d : IN std_logic;
F : OUT std_logic);

circuit;

architecture circuit of circuit is

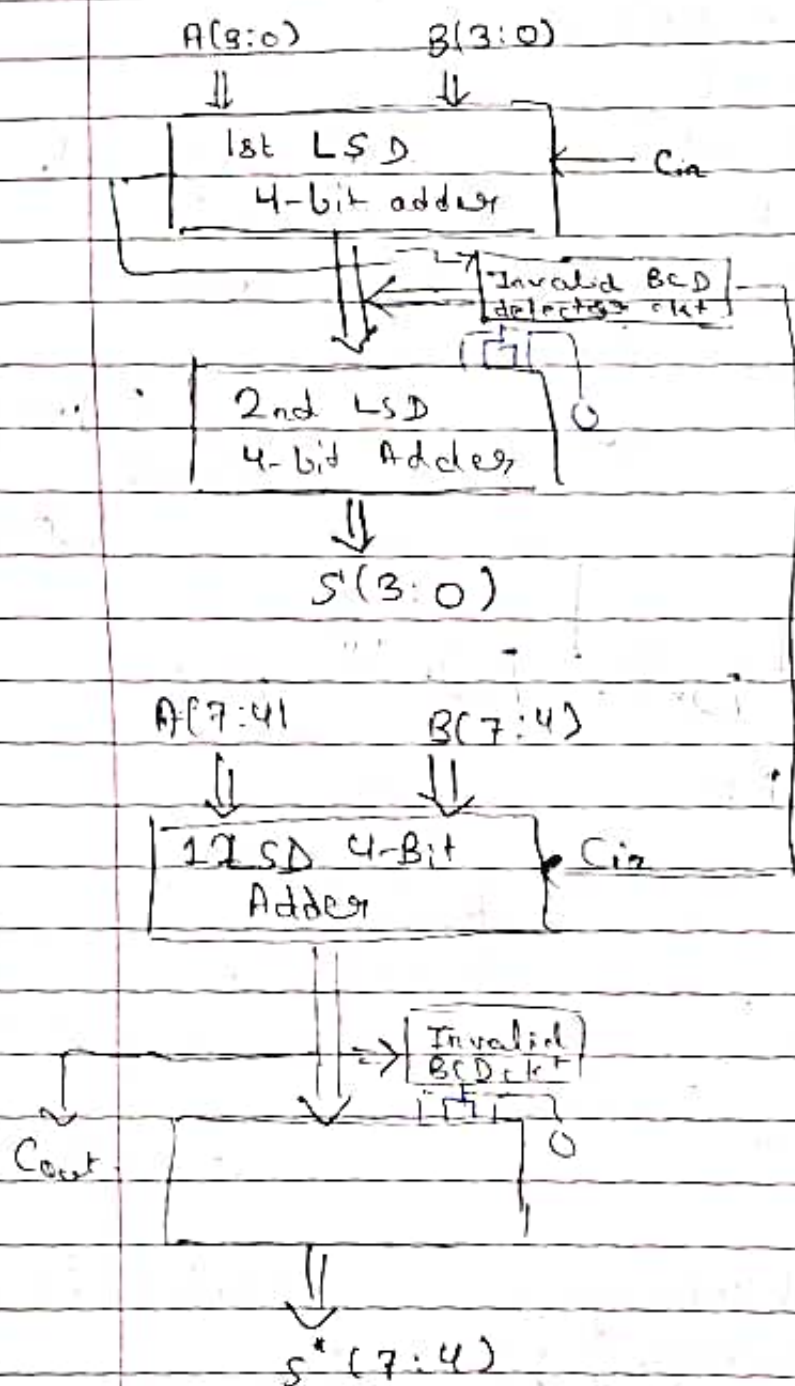
begin

F <= (c OR a) AND ((NOT c) OR b);

end circuit;

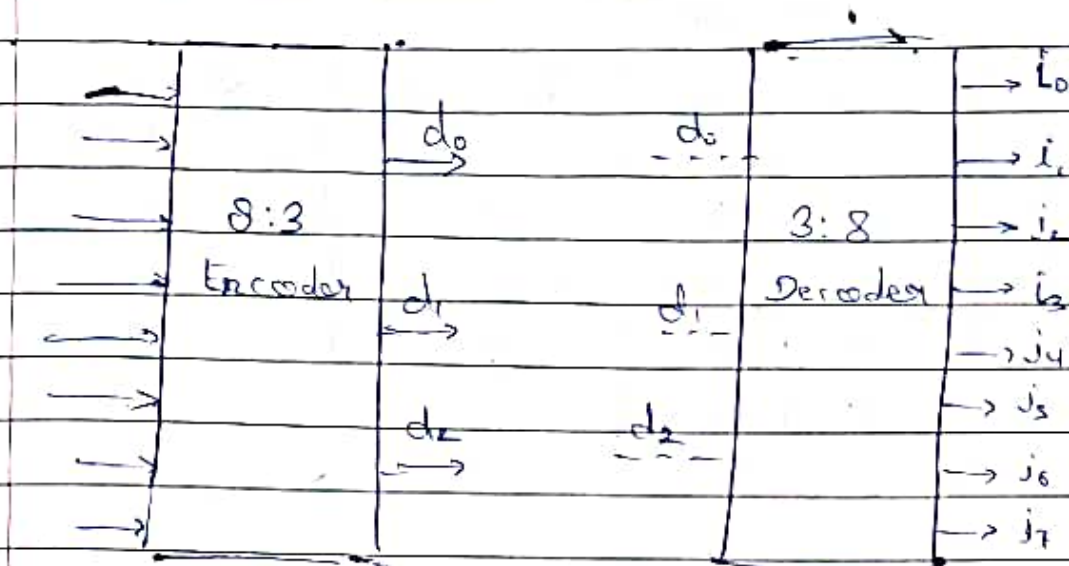


2-digit BCD adder:-

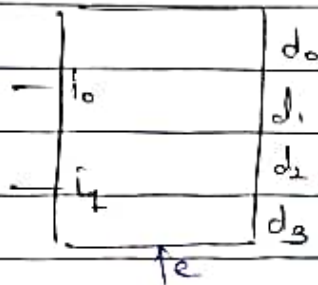


Components: IC 74LS 283 (4x)

Digital Transmission



2:4 Decoder:-



Decoder with

enabler: Decoder is responsible for converting an I/P binary no into a high o/p line.

e	I_1	I_0	d_0	d_1	d_2	d_3
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1

It is a combinational ckt. that converts binary input from n i/p lines to 2^n o/p lines.

@ $e = 0$, the decoder ckt is disabled OFF
 @ $e = 1$, the decoder ckt works normally

26:1 I/P 4-bit I/P.

Priority
Encoder:-

	D_3	D_2	D_1	D_0	A_1	A_0	V
	0	0	0	0	0	0	0
4:2	0	0	0	1	0	0	1
P.E	0	0	1	0	0	1	1
	0	1	x	x	1	0	1
	1	x	x	x	1	1	1

e.g.: $D_3 > D_2 > D_1 > D_0$

$$A_0 = \bar{D}_3 \bar{D}_2 D_1 + D_3$$

$$A_1 = \bar{D}_3 D_2 + D_3$$

Tutorial-6

①. Structural VHDL for full adder:-

```
library IEEE;
use IEEE.std_logic_1164.all;
```

```
entity eq_2 is
    PORT (a, b: IN std_logic;
          x: OUT std_logic);
end eq_2;
```

```
architecture eq_2 of eq_2 is
    begin
        x <= a OR b;
    end eq_2;
```

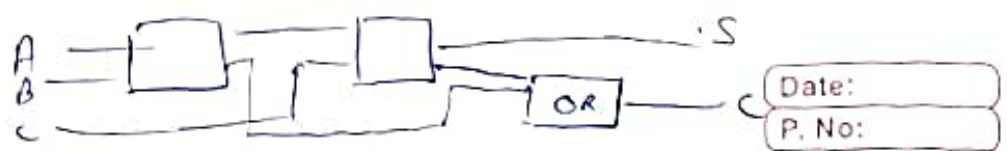
```
entity half-adder is
    PORT (x, y: IN std_logic;
          s, c: OUT std_logic);
```

```
architecture half-adder of half-adder is
```

```
    component eq_2
```

```
        PORT (a, b: IN std_logic;
              x: OUT std_logic);
```

```
    end component;
```



```

begin
    S <= x XOR y;
    C <= x AND y;
end half_adder;

```

```

entity full_adder is
    PORT (A, B, C_in : IN std_logic;
          S, C : OUT std_logic);
end full_adder;

```

signal s1, s2, s3 : std_logic;

architecture full_adder of full_adder is

```

    component half_adder is
        PORT (x, y : IN std_logic;
              s, c : OUT std_logic);
    end component;

```

```

end component;

```

component or_2 is (.) end component;

```

begin

```

```

    FA1: half_adder port map (A, B, s1, s3);

```

```

    FA2: half_adder port map (s1, C_in, S, s2);

```

```

    or_2: or_2 port map (s2, s3, C);

```

```

end full_adder;

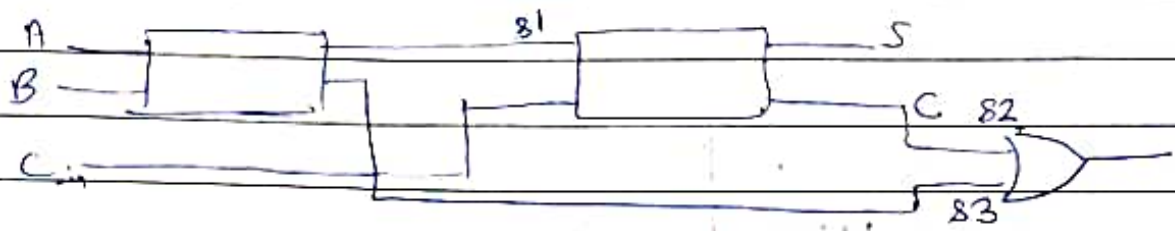
```


$$A \oplus B + (AB) \oplus$$

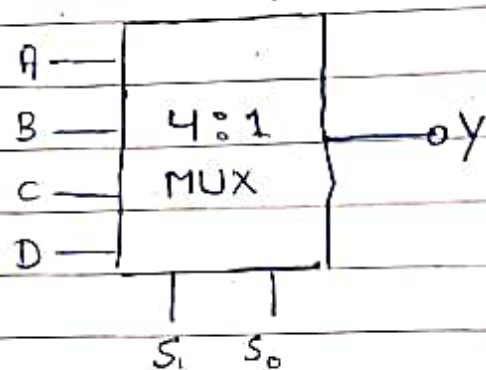
$$AB + C(A \oplus B)$$

Date:

P. No:

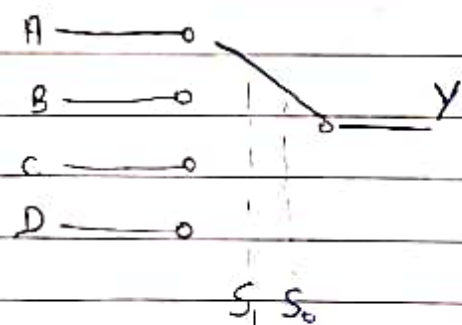


MULTIPLEXER



Def. A digital ckt. with 2^n inputs & one output line ($2^n:1$)

n : # select lines/control lines.



4 input Analog
MUX (switch)

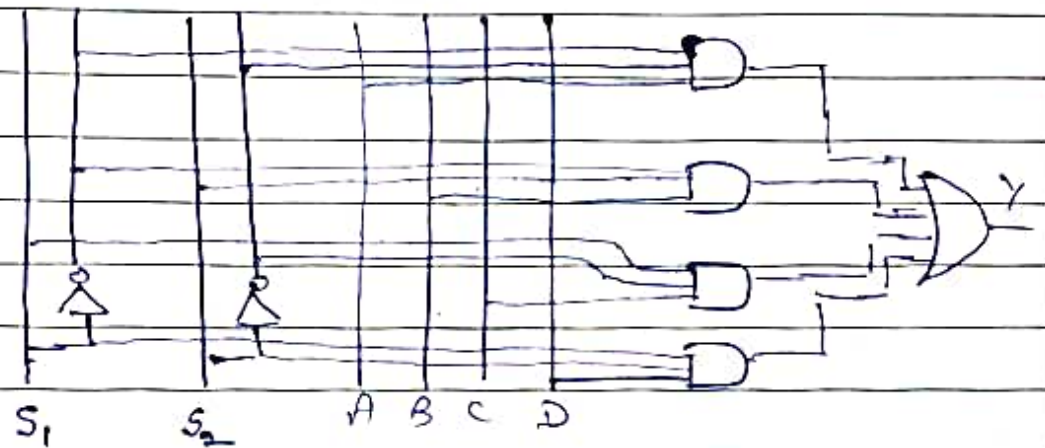
Encoder: $2^n \rightarrow n$

Decoder: $n \rightarrow 2^n$

MUX: $2^n \rightarrow 1$

Truth Table :-

S_1	S_0	Y	\longrightarrow	$Y = S_1' S_0' A + S_1' S_0 B + S_1 S_0' C + S_1 S_0 D$
0	0	A		
0	1	B		
1	0	C		
1	1	D		



S_2	S_1	S_0	Y
0	0	0	A
0	0	1	B
0	1	0	C
0	1	1	D
1	0	0	E
1	0	1	F
1	1	0	G
1	1	1	H

$$Y = \bar{S}_2 \bar{S}_1 \bar{S}_0 A + \bar{S}_2 \bar{S}_1 S_0 B + \bar{S}_2 S_1 \bar{S}_0 C + \bar{S}_2 S_1 S_0 D \\ + S_2 \bar{S}_1 \bar{S}_0 E + S_2 \bar{S}_1 S_0 F + S_2 S_1 \bar{S}_0 G + S_2 S_1 S_0 H$$

$$y = x_1 \times x_2 + m_1 + m_2 + R_1 \times R_2 + Z_1 \times Z_2$$

Multiplexer
(1M)

Address
(1A)

Tri-State Buffers
(TSB)

A TSB has one i/p, one o/p and one control line (e).

e	x	f
0	0	Z
0	1	Z
1	0	0
1	1	1

Active-high
non-inverted
TSB

'f' can have 3 o/p states
'Z', '0', '1'

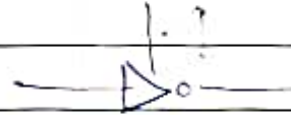
Z \rightarrow high-temperature (open-ckt)

If $e = 0$, $y = Z$
else $y = x$

Symbols :-



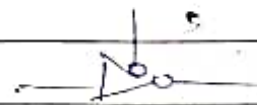
a) Active high
non-inverted
TSB



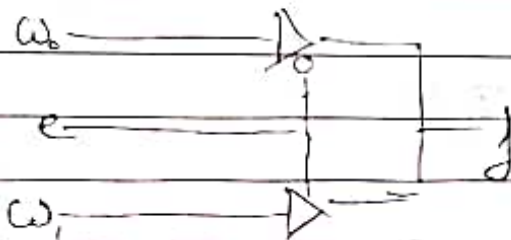
b) Active-high
inverted TSB



c) Active low
non-inverted
TSB



d) Active low
inverted TSB

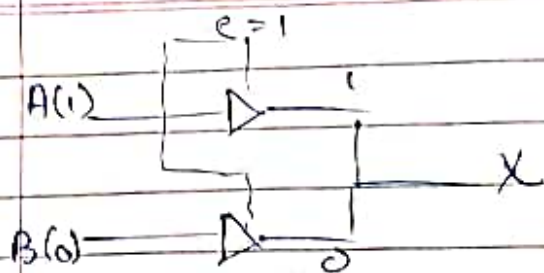


0, z → z
1, z → 1

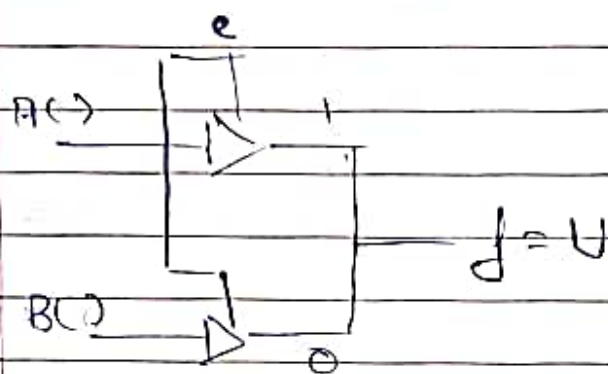
'0', '1', 'z', 'x' → 'x': don't care

'x': conflict of output

'u': uninitialized i/p value.



$X \rightarrow 1$ and 0 gives conflict output.



5-valued logic: 1, 0, X, Z, U

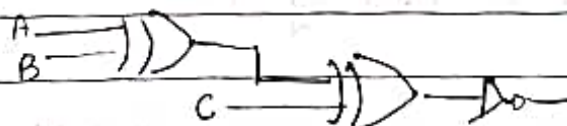
Parity Generator Function

data word			
A	B	C	O/P (P ₂) → code word.
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Concept: Adding a parity bit to the data word, such that the code-word has odd no. of 1s.

$$P(A, B, C) = \Sigma(0, 3, 5, 6)$$

$$\begin{aligned}
 P &= A'B'C' + A'BC + AB'C + ABC' \\
 &= C(A'B + AB') + C'(A'B + AB) \\
 &= C \oplus A \oplus B
 \end{aligned}$$



Single event upset (SEU) → We send an extra-bit to realize if bit flip occurred during transmission.

Detects but does not rectify. Can't detect multiple bit flips.

Odd Parity Checker:-

	A	B	C	P	PEC
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	1

Concept: Checks the 4-bit code word, to check for odd number of 1's.

If odd # 1's are present, then $PEC = 0$
else $PEC = 1$.

$$PEC = \sum_m (0, 3, 5, 6, 9, 10, 12, 15)$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 ₀	0 ₁	1 ₂	0 ₃
$\bar{A}B$	0 ₄	1 ₅	0 ₆	1 ₇
AB	1 ₁₂	0 ₁₃	1 ₁₄	0 ₁₅
$A\bar{B}$	0 ₈	1 ₉	0 ₁₁	1 ₁₀

$$PEC: \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABCD$$

$$\rightarrow \bar{A}(\bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} + B\bar{C}\bar{D} + BC\bar{D}) + A(\bar{B}\bar{C}D + \bar{B}C\bar{D} + B\bar{C}D + BC\bar{D})$$

$$\rightarrow \bar{A}(B \oplus C \oplus D) + A(\bar{B} \oplus \bar{C} \oplus \bar{D})$$

$$\rightarrow A \oplus B \oplus C \oplus D \quad (D = P)$$

$$\rightarrow A \oplus B \oplus C \oplus P$$

	A	B	C	D	E ₃	E ₂	E ₁	E ₀
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	0	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0

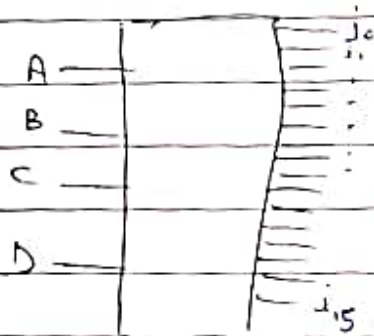
$$E_3 = \sum m(5, 6, 7, 8, 9)$$

$$E_2 = \sum m(1, 2, 3, 4, 9)$$

$$E_1 = \sum m(0, 3, 4, 7, 8)$$

$$E_0 = \sum m(0, 2, 4, 6, 8)$$

BCD to XS-3 converted



$$E_3 = j_4 + j_5 + j_6 + j_7 + j_8 - j_5 + j_6 + j_7 + j_8 + j_9$$

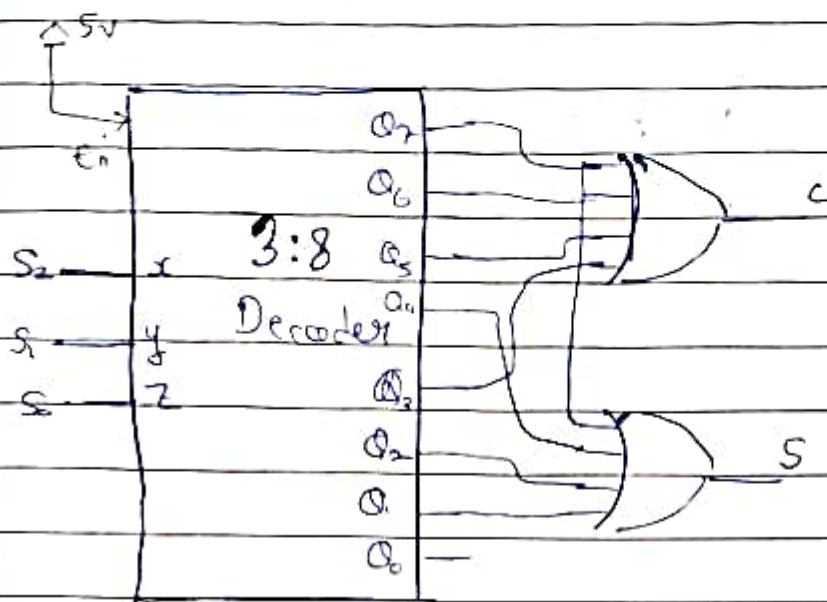
$$E_2 = j_4 + j_5 + j_6 + j_7 + j_8$$

$$E_1 = j_0 + j_3 + j_4 + j_7 + j_8$$

$$E_0 = j_0 + j_2 + j_4 + j_6 + j_8$$

Digital Design

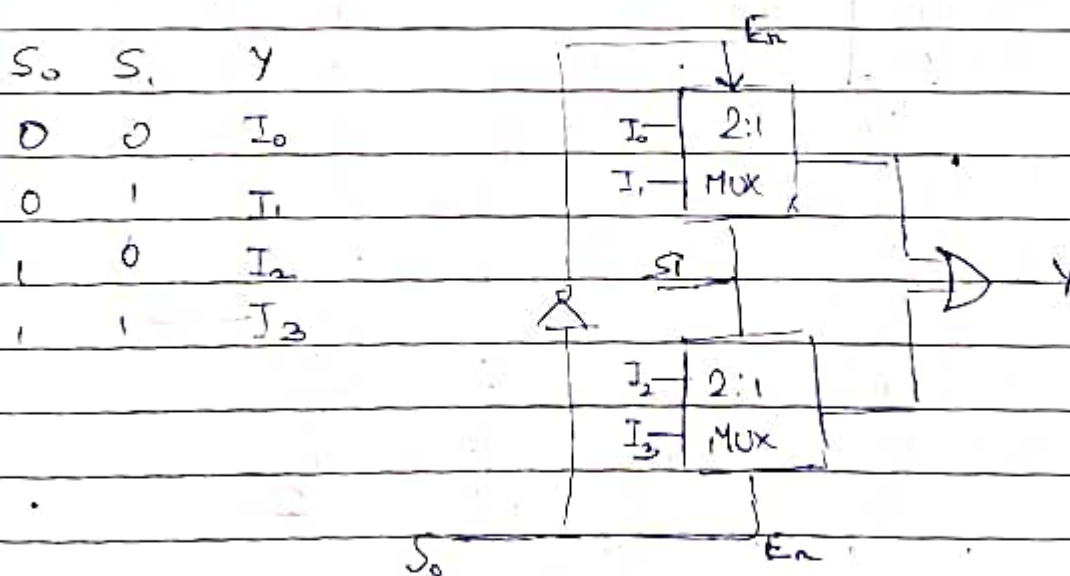
Implementing Address Using Decoder:-



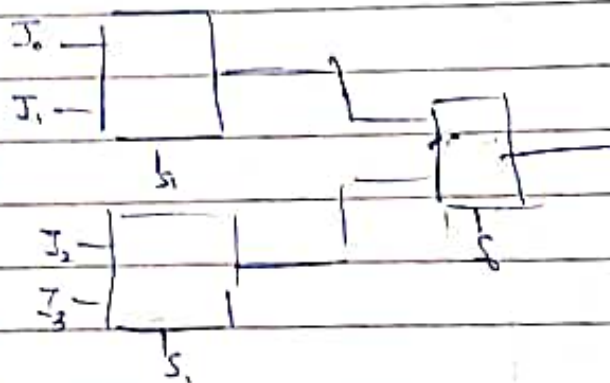
$$C = \sum m(3, 5, 6, 7)$$

$$S = \sum m(1, 2, 4, 7)$$

Designing 4:1 MUX:-

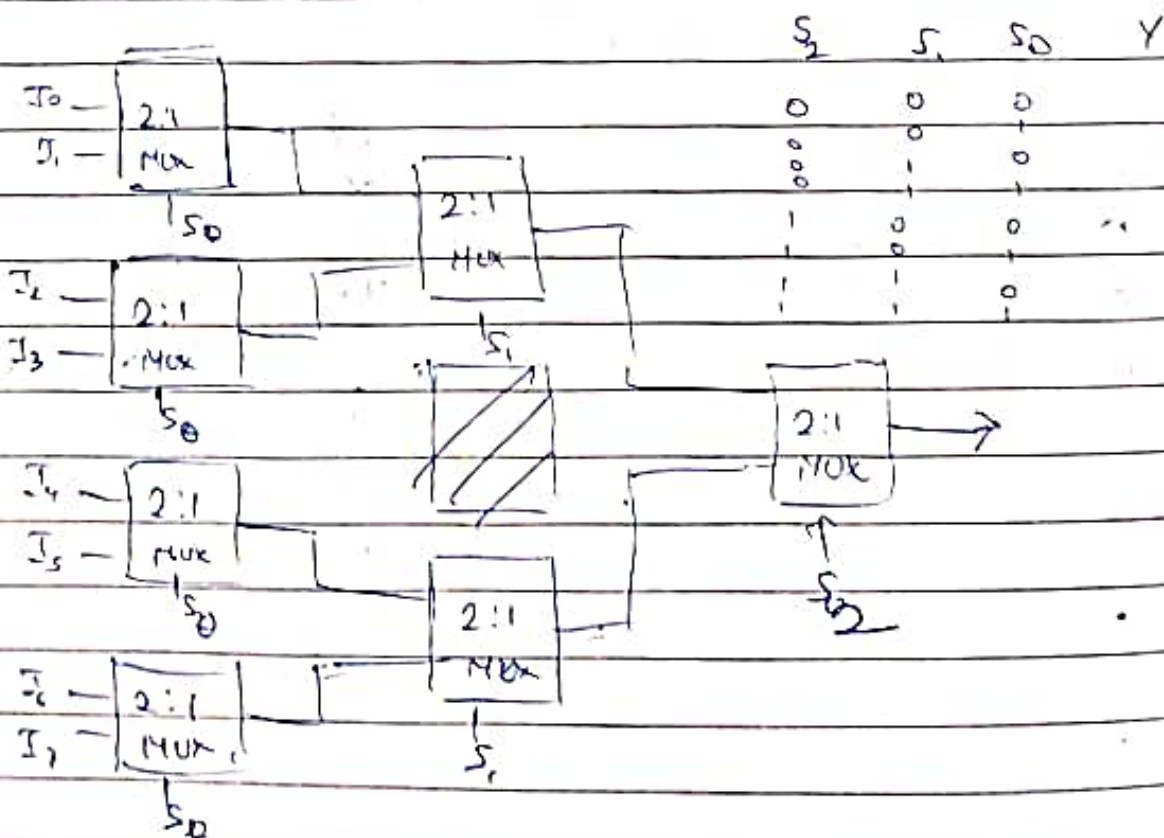


Designing 4:1 MUX using 2:1 MUX only:-

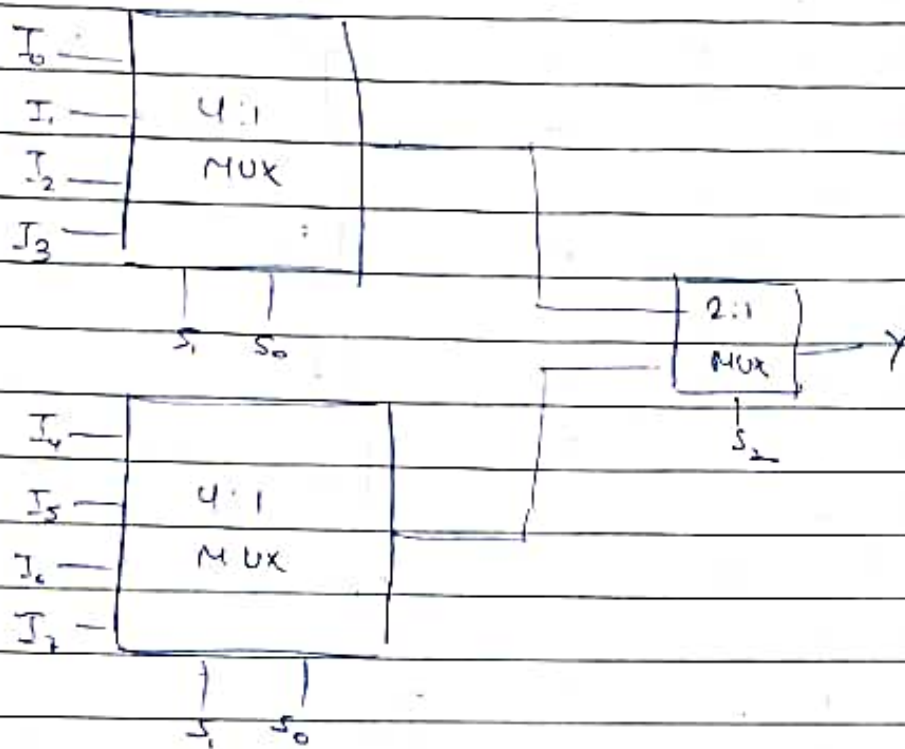


Design 32:1 MUX using 16:1 MUX & OR gate:-

Cascading of MUXES:-

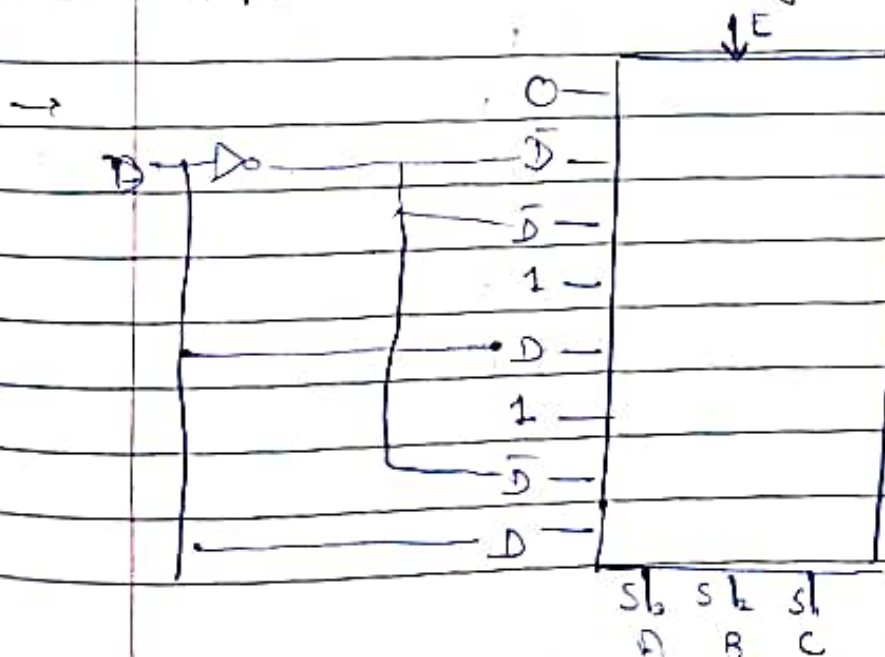


Design 8:1 MUX using 4:1 MUX and 2:1 MUX



Ques: $f(A, B, C, D) = \sum m(2, 4, 6, 7, 9, 10, 11, 12, 15)$

Implement the boolean function using 8:1 MUX.



S_3	S_2	S_1	S_0	Y	
0	0	0	0	0	} 0
0	0	0	1	0	
0	0	1	0	1	} 5
0	0	1	1	0	
0	1	0	0	1	} 5
0	1	0	1	0	
0	1	1	0	1	} 1
0	1	1	1	1	
1	0	0	0	0	} 0
1	0	0	1	1	
1	0	1	0	1	} 1
1	0	1	1	1	
1	1	0	0	1	} 5
1	1	0	1	0	
1	1	1	0	0	} 0
1	1	1	1	1	

000

001

011

010

100

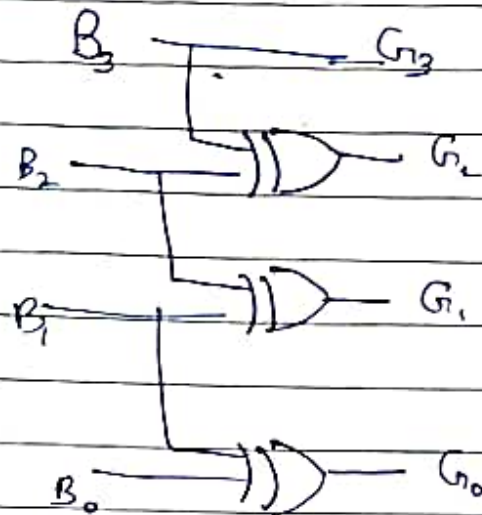
101

111

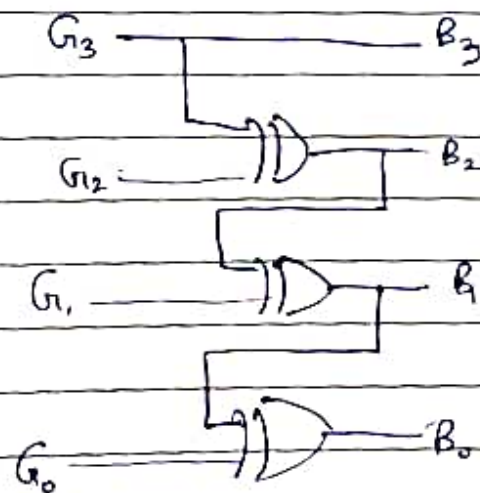
110

Tutorial-8

(1). Binary to Gray :-



(2). Gray to binary :-



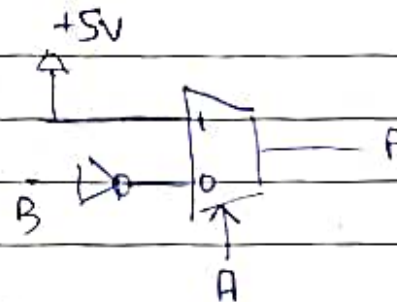
Implementation of Boolean functions

1) Implementing NOR gate using MUX:-

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

\Rightarrow

A	F
0	B'
1	0

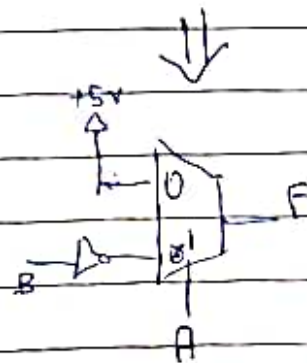


2) Implementing NAND gate using MUX:-

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

\Rightarrow

A	F
0	1
1	B'



3. Implementing ^{min} INVERTER using MUX:-

Shannon's expansion \rightarrow

$$F = x f_x + x' f_{x'}$$

where F is any function
 f_x and $f_{x'}$ are the positive and negative Shannon
 co-factors.

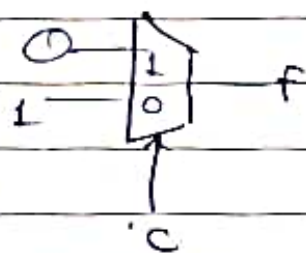
the cofactor is evaluated @ $x=1$

neg. cofactor is evaluated @ $x=0$

$$\rightarrow F = C'$$

$x = \cancel{0} 1$ $f_x = C'$	$x = 0$ $f_{x'} = C$
----------------------------------	-------------------------

$$\Rightarrow F = 1 C' + 0 C \text{ (or) } 0 C + 1 C'$$



$$A \times A' = 0$$

$$AB + A'B = A + B$$

Date:
 P. No:

$$\rightarrow F = AB + AC + BC$$

Use Shannon's expression w.r.t. A.

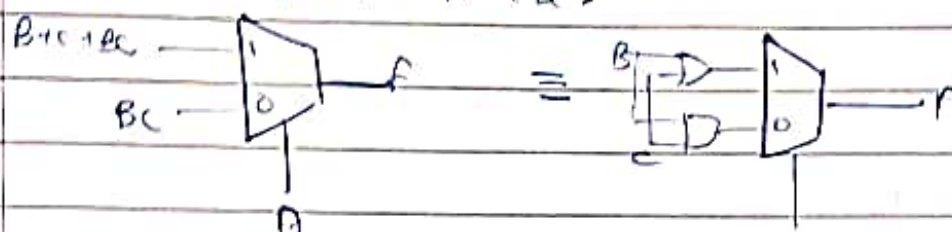
$$F = A(B+C) + BC$$

$$= A(B+C) + (A+A')BC$$

$$= A(B+C+BC) + A'(BC)$$

$$= A(1B+1C+1BC) + A'(0B+0C+1BC)$$

$$= A(B+C) + A'(BC)$$



$$\text{Sol}^2 - F = AF_A + A'F_A'$$

$$F = AG + A'H \quad \begin{cases} G = B+C \\ H = BC \end{cases}$$

$$G = BG_B + B'G_B'$$

$$= B(1+C) + B'(0+C)$$

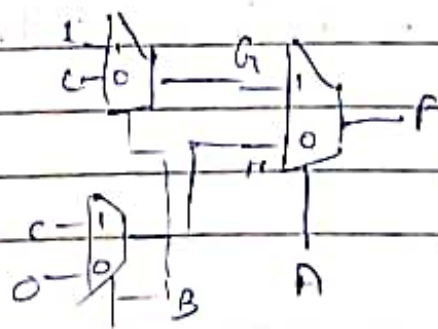
$$= B(1) + B'(C)$$

$$H = BC$$

$$H = BH_B + B'H_B'$$

$$= B(1.C) + B'(0.C)$$

$$= B(C) + B'(0)$$



Digital Comparators:-

1-Bit Comparators:-

R_i	B_i	E_i	L_i	G_i
0	0	1	0	0
0	1	0	1	0
1	0	0	0	1
1	1	1	0	0

$$\rightarrow L_i = \bar{R}_i B_i$$

$$\rightarrow G_i = A_i \bar{B}_i$$

$$\begin{aligned} \rightarrow E_i &= \bar{R}_i \bar{B}_i + A_i B_i \\ &= A_i \oplus B_i \end{aligned}$$

$$\text{Let } A = A_3 A_2 A_1 A_0$$

$$B = B_3 B_2 B_1 B_0$$

$$\rightarrow E = (A_3 \oplus B_3)(A_2 \oplus B_2)(A_1 \oplus B_1)(A_0 \oplus B_0)$$

$$\begin{aligned} \rightarrow L &= \bar{A}_3 B_3 + (A_3 \oplus B_3)(\bar{A}_2 B_2) + (A_3 \oplus B_3)(A_2 \oplus B_2)(\bar{A}_1 B_1) \\ &\quad + (A_3 \oplus B_3)(A_2 \oplus B_2)(A_1 \oplus B_1)(\bar{A}_0 B_0) \end{aligned}$$

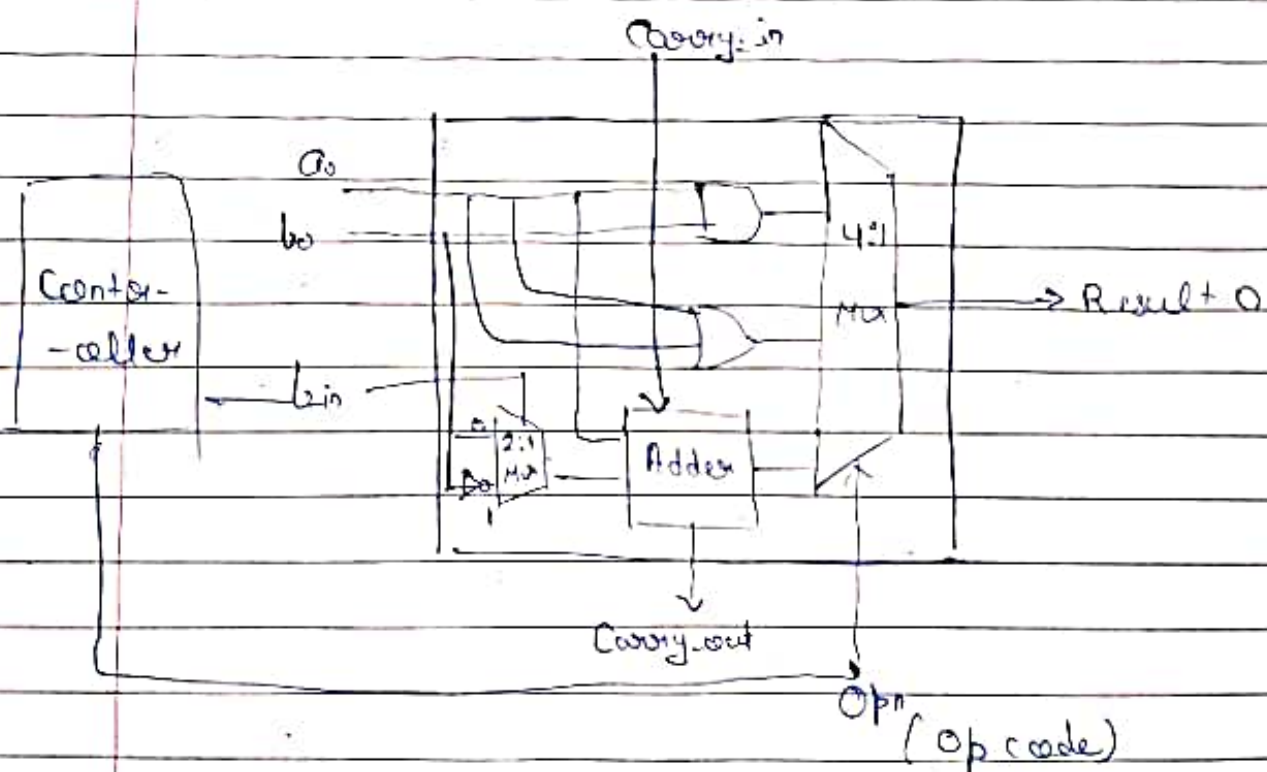
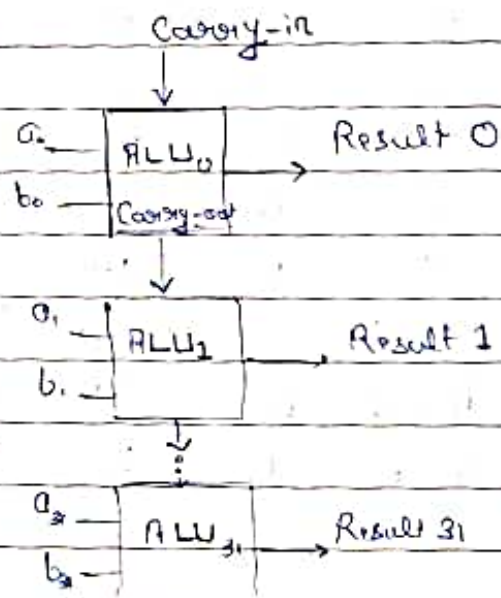
$$\begin{aligned} \rightarrow G &= A_3 \bar{B}_3 + (A_3 \oplus B_3)(A_2 \bar{B}_2) + (A_3 \oplus B_3)(A_2 \oplus B_2)(A_1 \bar{B}_1) \\ &\quad + (A_3 \oplus B_3)(A_2 \oplus B_2)(A_1 \oplus B_1)(A_0 \bar{B}_0) \end{aligned}$$

Date:

P. No:

A_3, B_3	A_2, B_2	A_1, B_1	A_0, B_0	$A > B$	$A < B$	$A = B$
$A_3 > B_3$	X	X	X	H	L	L
$A_3 < B_3$	X	X	X	L	H	L
$A_3 = B_3$	$A_2 > B_2$	X	X	H	L	L
$A_3 = B_3$	$A_2 < B_2$	X	X	L	H	L
$A_3 = B_3$	$A_2 = B_2$	$A_1 > B_1$	X	H	L	L
$A_3 = B_3$	$A_2 = B_2$	$A_1 < B_1$	X	L	H	L
$A_3 = B_3$	$A_2 = B_2$	$A_1 = B_1$	$A_0 > B_0$	H	L	L
$A_3 = B_3$	$A_2 = B_2$	$A_1 = B_1$	$A_0 < B_0$	L	H	L
$A_3 = B_3$	$A_2 = B_2$	$A_1 = B_1$	$A_0 = B_0$	L	L	H

Arithmetic Logic Unit (ALU) :-



Controller + ALU = Datapath

Control Lines :- (Signals)

b _{inv}	op _n	
0	00	AND
0	01	OR
0	10	ADD
1	11	SUBTRACT

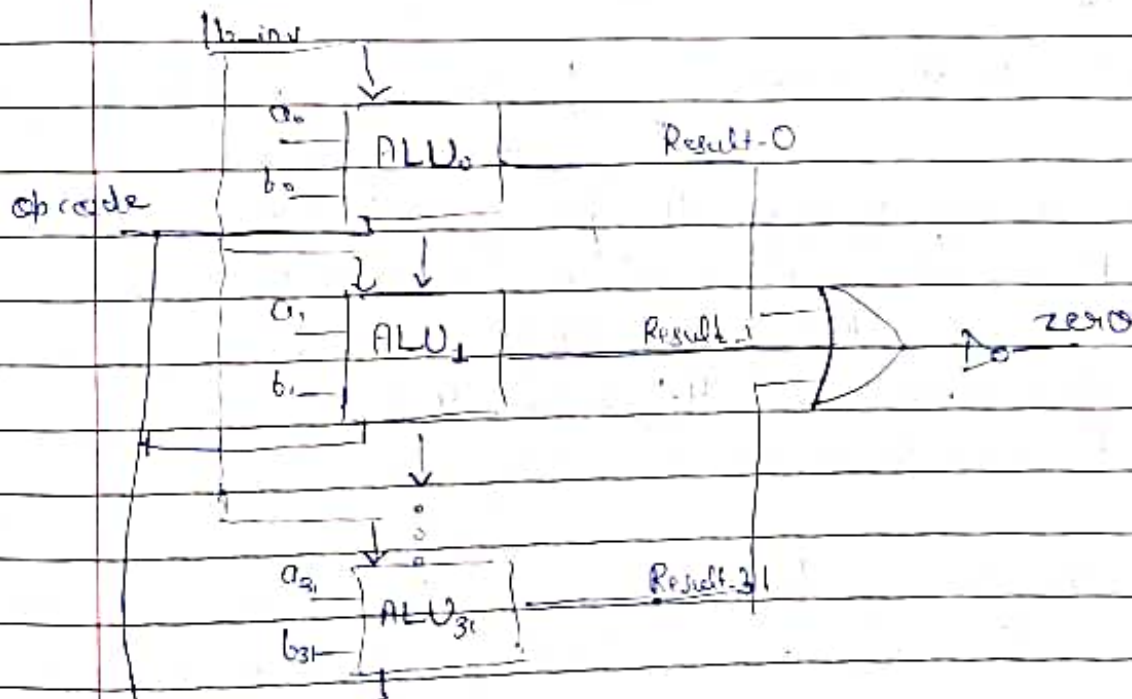
Test of Equality :-

If $A - B = 0$

then zero(flag) = 1

else

zero = 0



Hamming Code :-

① ② 3 ④ 5 6 7
 P_1 P_2 D_1 P_3 D_2 D_3 D_4

4-bit data word

7-bit code word

3-bit parity

Rules for Hamming Code Generation

Step 1: Mark all bits that are power of 2 as parity bits. (1, 2, 4, ...)

Step 2: Mark all the remaining positions for data encoding.

Step 3: Each parity calculates the parity of some bits.

Position 1: ^{check 1 bit} Skip 1 bit, Check 1 bit, Skip 1-bit, ...

Position 2: ^{check 2 bit} Check 2-bit, Skip 1-bit, ...

Position 3: Check 4-bits, Skip 4-bits, ...

Position 8: Check 8-bits, Skip 8-bits, ...

Step 4: Set the parity bit to '1' if there are odd no. of '1' (because - we're checking for even parity).

Data word :-

→ 1 0 0 1 1 0 1 0

1 2 3 4 5 6 7 8 9 10 11 12

0 1 ~~0~~ 1 ~~0~~ 0 1 0 1 0 1 0

P_1 P_2 D_1 P_3 D_2 D_3 D_4 P_4 D_5 D_6 D_7 D_8

Pos 1 → 1, 3, 5, 7, 9, ...

Pos 2 → 2, 3¹², 6, 7¹², 10, 11, ...

Pos 4 → 4, 5, 6, 7, 12, 13, 14, 15, ...

Transmitted Code Word → 0 1 1 0 0 1 0 1 0 1 0

Received Code Word → 0 1 1 1 0 0 1 0 1 1 1 0

(1) Check for even parity @ pos's: 8, 9, 10, 11, 12
→ fails
→ $x_1 = 1$

(2) Check for even parity @ pos's: 4, 5, 6, 7, 12
→ passes
→ $x_2 = 0$

(3) Check for even " " " : 2, 3, 6, 7, 10, 11
→ fails
→ $x_3 = 1$

→ Check for even parity @ pos's: 1, 3, 5, 7, 9, 11

→ passes

→ $x_4 = 0$

x_1	x_2	x_3	x_4
1	0	1	0

posⁿ 10