MANNA MISSING

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Now: Rectangular Rule, $n=1$

$$\int_{x_0}^{x_1} f(x)\,dx = y_0(x_1-x_0) = hy_0$$

$n=1$
$$\text{Error} = h\int_0^1 p\,\Delta y_0\,dp = h\left[\tfrac{1}{2}\Delta y_0\right] = \frac{h^2}{2}f'(\xi) \quad \because \frac{\Delta y_0}{h} = f'(\xi)$$

For Lagrange's formula: fit a poly passing through $(x_0, y_0)\ (x_1, y_0)$

$$P(x) = \frac{x-x_1}{x_0-x_1}y_0 + \frac{x-x_0}{x_1-x_0}y_0 = y_0$$

Error $R(x) = (x-x_0)f'(\xi) \qquad x_0 \le \xi \le x_1$

$$\int_{x_0}^{x_1} P(x) = y_0(x_1-x_0) = hy_0$$

$$f'(\xi)\int_{x_0}^{x_0+h}(x-x_0)\,dx = f'(\xi)\frac{(x-x_0)^2}{2} = \frac{h^2}{2}f'(\xi)$$

$$\int_{x_0}^{x_1} R(x)\,dx = \int_{x_0}^{x_0+h}(x-x_0)f'(\xi)\,dx = f'(\xi)\cdot\frac{h^2}{2} = \frac{h^2}{2}f'(\xi), \quad x_0 \le \xi \le x_1$$

Composite Trapezoidal Rule:

$[a, b] \xrightarrow{\text{divided}}$ in n equal parts of length h.

ie $h = \frac{b-a}{n}$, Nodes $x_0, x_0+h, x_0+2h, \dots, x_0+nh$

$$\therefore h = \frac{b-a}{n}$$

$$\therefore \underbrace{\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_n) + 2\{f(x_1) + f(x_2) \dots + f(x_{n-1})\}\right]}_{\text{Composite formula}}$$

Composite Formula for Simpson's 1/3 formula

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4[f(x_1) + f(x_3) + \dots f(x_{2n-1})] + 2[f(x_2) + f(x_4) + \dots f(x_{2n-2})] + f(x_{2n})\right]$$

$$h = \frac{b-a}{n}$$

Composite formula for Simpson's 3/8 Rule

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[f(x_0) + f(x_{3k}) + 3[f(x_1) + f(x_2) + f(x_4) \dots]\right.$$

$$\int_a f(x)\,dx = \frac{3h}{8}\Big[f(x_0) + f(x_{3k})$$

$$+ 3\,[\, f(x_1) + f(x_2) + f(x_4) \cdots]$$

$$+ 2\,[\, f(x_3) + f(x_6) + \cdots f(x_{3k-3})]\Big]$$