MA 204 Numerical Methods

Dr. Debopriya Mukherjee

Lecture-8

January 24, 2024

Contents

- Solution of a nonlinear equation, bisection and secant methods, Newton's method, rate of convergence.

Contents

- Solution of a nonlinear equation, bisection and secant methods, Newton's method, rate of convergence.
- Interpolation by polynomials, divided differences, error of the interpolating polynomial, piecewise linear and cubic spline interpolation.

Natural Cubic Spline

Given $t_0 < t_1 < \cdots < t_n$, we define the *cubic spline*, with

$$\mathcal{S}(x) = \mathcal{S}_i(x) \quad \text{for} \quad t_i \leq x \leq t_{i+1}.$$

Write

$$\mathcal{S}_i(x) = a_i x^3 + b_i x^2 + c_i x + d_i, \ \ i = 0, 1, \cdots, n-1.$$

Total number of unknowns=4.n

Natural Cubic Spline

Equations we have:

equation		number
$\mathcal{S}_i(t_i) = y_i,$	$i = 0, 1, \cdots, n-1$	n
$\mathcal{S}_{i+1}(t_{i+1}) = y_{i+1},$	$i = 0, 1, \cdots, n-1$	n
$\mathcal{S}'_i(t_{i+1}) = \mathcal{S}'_{i+1}(t_{i+1}),$	$i = 0, 1, \cdots, n-2$	$n-1$
$\mathcal{S}''_i(t_{i+1}) = \mathcal{S}''_{i+1}(t_{i+1}),$	$i = 0, 1, \cdots, n-2$	$n-1$
$\underbrace{\mathcal{S}''_0(t_0) = 0,\ \mathcal{S}''_{n-1}(t_n) = 0,}_{\text{Natural}}$		2

How to compute $\mathcal{S}_i(x)$? We know:

$$\mathcal{S}_i : \text{polynomial of degree } 3$$

$$\mathcal{S}_i' : \text{polynomial of degree } 2$$

$$\mathcal{S}_i'' : \text{polynomial of degree } 1$$

Procedure:

- Start with $\mathcal{S}_i''(x)$, they are all linear, one can use Lagrange form,
- Integrate $\mathcal{S}_i''(x)$ twice to get $\mathcal{S}_i(x)$, you will get two integration constants.
- Determine these constants.