Example 3: Write Newton's form of interpolation polynomial for the data

x_i	0	1	2/3	1/3
y_i	1	0	1/2	0.866

Answer: Set up the triangular table for computation

0	$\boxed{1}$			
1	0	$\boxed{-1}$		
2/3	0.5	-1.5	$\boxed{-0.75}$	
1/3	0.8660	-1.0981	-0.6029	$\boxed{0.4413}$

So we have

$$a_0 = 1, \; a_1 = -1, \; a_2 = -0.75, \; a_3 = 0.4413.$$

Then,

$$P_3(x) = \boxed{1} + \boxed{-1}x + \boxed{-0.75}\, x(x-1) + 0.4413\, x(x-1)(x-2/3)$$

Flexibility of Newton's form: easy to add additional points to interpolate

The recursion is initiated with

$$f[x_i] = y_i, \quad i = 0, 1, 2, \ldots$$

Then, $$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0}$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1}, \ldots$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_1, x_0]}{x_2 - x_0},$$

$$f[x_1, x_2, x_3] = \frac{f[x_3, x_2] - f[x_2, x_1]}{x_3 - x_1}$$

For a general step, we have

$$f[x_0, x_1, \ldots, x_k] = \frac{f[x_1, x_2, \ldots, x_k] - f[x_0, x_1, \ldots, x_{k-1}]}{x_k - x_0}$$

The constants a_k's in the Newton's form are computed as:

$$a_0 = f[x_0],\quad a_1 = f[x_0, x_1],\ \ldots,\ a_k = f[x_0, x_1, \ldots, x_k]$$

We compute the $f[\ldots]$'s through the following table:

x_0	$f[x_0] = \boxed{y_0} \leftarrow a_0$				
		a_1			
x_1	$f[x_1] = y_1$	$f[x_0, x_1] = \dfrac{f[x_1] - f[x_0]}{x_1 - x_0}$	a_2		
x_2	$f[x_2] = y_2$	$f[x_1, x_2] = \dfrac{f[x_2] - f[x_1]}{x_2 - x_1}$	$f[x_0, x_1, x_2]$	$\ddots$	
$\vdots$		$\vdots$	$\vdots$		a_n
			$f[x_{n-2}, x_{n-1}, x_n]$	$\ddots$	$(*)$
x_n	$f[x_n] = y_n$	$f[x_{n-1}, x_n] = \dfrac{f[x_n] - f[x_{n-1}]}{x_n - x_{n-1}}$			

$(*) \to f[x_0, x_1, \ldots, x_n]$.

The diagonal elements give us the a_i's.