PROBLEM SET II FOR NUMERICAL PARTIAL DIFFERENTIAL EQUATIONS
COURSE TITLE: NUMERICAL METHODS,
COURSE CODE: MA 204
IIT INDORE

(1) The four sides of a square plate of side 12 cm made of homogeneous material are kept at constant temperature 0^o C and 100^o C as shown in the following figure Using a grid of 4 cm (as is shown in

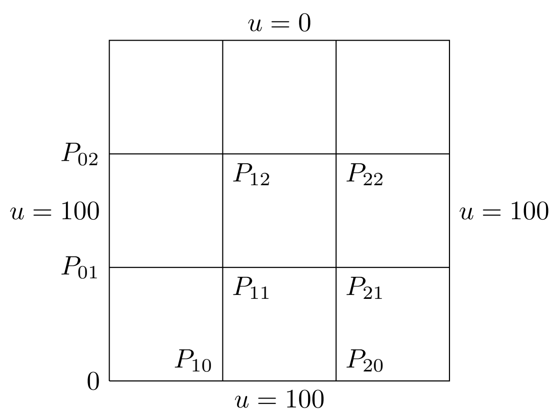

FIGURE 1. Grid and mesh points, domain 1.

the figure above) and applying ADI scheme (Alternate direction implicit scheme) find the stedy-state temperature at the mesh points P_{11}, P_{21}, P_{12} and P_{22}. Calculate upto six iterates *i.e.* upto u^6_{ij}, (recall that the first two iterates were already calculated during lecture session) after considering the initial guess $u^0_{11} = u^0_{12} = u^0_{21} = u^0_{22} = 100$.

(2) Solve the Laplace equation $\partial_{xx} + \partial_{yy} = 0$ at the mesh points of the 'Domain 2' (see the figure in the next page) using ADI scheme. Find upto u^2_{ij} after considering the initial guess $u^0_{12} = u^0_{22} = 1$ and $u^0_{11} = u^0_{21} = 0$.

(3) Solve the mixed boundary value problem for the Poisson equation

$$\Delta u = \partial_{xx} u + \partial_{yy} u = f(x, y) = 12xy$$

with boundary conditions

$$u(x, y) = 0 \text{ on } \overline{OA}, \overline{OC}, \; u(x, y) = 3y^3 \text{ on } \overline{AB} \text{ and } \partial_y u(x, y) = 6x \text{ on } \overline{BC}$$

shown in the 'Domain 3' (next page) by using domain extension technique as discussed in the class. Recall that we have already derived the system of linear equations during one of the lectures, you just need to apply Gauss elimination method to solve the same.

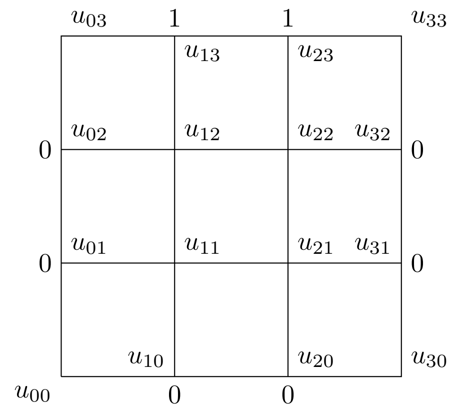

FIGURE 2. Grid and mesh points, Domain 2.

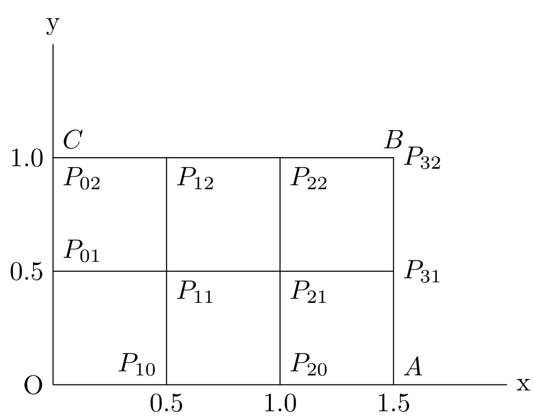

FIGURE 3. Grid and mesh points, Domain 3.

(4) Find the potential u (*i.e.* u solves Laplace equation) in the region in 'Domain 4' (next page) that has the boundary value given in that figure; here the curved portion of the boundary is an arc of the circle of radius 10 about $(0,0)$. Use the grid in the figure. After achieving a set of linear equations, use Gauss elimination to solve the same.
(Recall that we have derived a five point formulae in class which can be used to approximate the potential at P_{22} and P_{21}).

(5) Solve the following one dimensional heat equation in a rod of length l by separation of variables:

$$\begin{cases} \partial_t u(x,t) = \alpha^2 \partial_{xx} u(x,t), & \text{where } x \in (0,l),\ t > 0, \\ u(0,t) = u(l,t) = 0 & \\ u(x,0) = f(x), & \text{where } x \in (0,l), \end{cases} \tag{1}$$

where α is a non-zero constant and the function f is not identically zero.

(6) (*Inhomogeneous boundary condition*) Solve the following one dimensional heat equation in a rod of

length l by separation of variables:

$$\begin{cases} \partial_t u(x,t) = 9\partial_{xx} u(x,t), & \text{where } x \in [0,2],\ t \geqslant 0, \\ u(0,t) = 0,\ u(2,t) = 8, & \text{for all } t \geqslant 0, \\ u(x,0) = 2x^2, & \text{where } x \in [0,2]. \end{cases} \tag{2}$$

Figure corresponding to question no. 4:

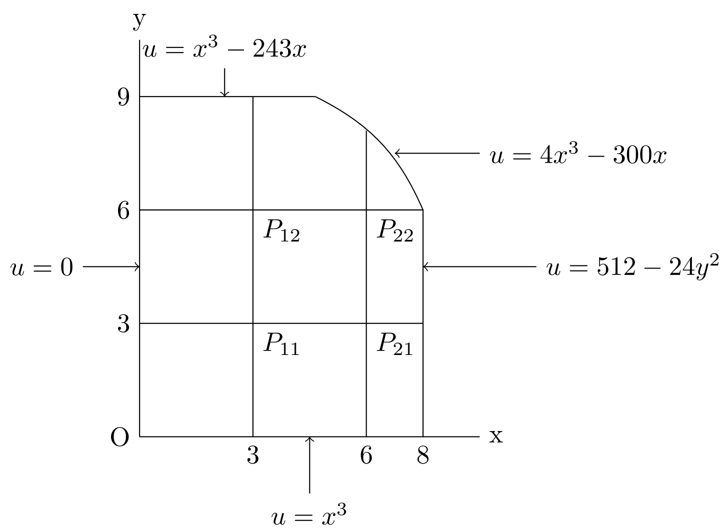

FIGURE 4. Grid and mesh points, Domain 4.