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1) $$\frac{d}{dx}\left(x^{-n} J_n(x)\right) = -x^{-n} J_{n+1}(x)$$

$$x^{-n} J_n(x) = \int -x^{-n} J_{n+1}(x)\,dx + c$$

$$\therefore \int J_n(x)\,dx = \int \underbrace{x^{n-1}}_{I}\ \underbrace{x^{-(n-1)} J_n(x)}_{II}$$

$$= x^{n-1}\int x^{-(n-1)} J_{n+1}(x) - \int (n-1)x^{n-2}\int x^{-(n-1)} J_n(x)$$

$$= x^{n-1}\left(-x^{-(n-1)} J_{n-1}(x)\right) + \int (n-1)x^{n-2} x^{-n} J_{n-1}(x)\,dx$$

$$= -J_n(x) + (n-1)\int x^{-2} J_{n-1}(x)\,dx + C$$

2) $$x^2 J_0 J_1 = xJ_0\ xJ_1$$

$$= (xJ_1)\frac{d}{dx}(xJ_1)$$

$$xJ_0 = \frac{d}{dx}(xJ_1)$$

3) From (I), y_1 can be expressed as

$$P_n(x) = \frac{1.3.5...(2n-1)}{\lfloor n}\left[x^n - \frac{n(n-1)}{2(2n-1)}x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2.4.(2n-1)(2n-3)}x^{n-4} - \cdots\right]$$

This is a Legendre polynomial of order n.

$$y_2 = b\left[x^{-n-1} + \frac{(n+1)(n+2)}{2(2n+3)}x^{-n-3} + \frac{(n+1)(n+2)(n+3)(n+4)}{2.4(2n+3)(2n+5)}x^{-n-5} + \cdots\right]$$

this is $Q_n(x)$ Lengendre eq^n of the Second Kind

$$Q_n(x) = \frac{n!}{1.3.5.\cdots(2n-1)} x^{-(n+1)} \left[1 + \frac{(n+1)(n+2)}{1.2.(2n+3)} + \cdots \right]$$