Q1

a) convergent
b) divergent
c) convergent
d) convergent
e) convergent
f) converges if $x<1$, diverges if $x \geq 1$

Q2

a) $R = 3/2$
b) $R = 1/3$
c) $R = 1$
d) $R = 1$
e) $R = 1/2$

Q3 Interval $[-1, 3)$

Q4 a) $\mathrm{Re}\{z(c\bar{d} - a\bar{b})\} > \dfrac{|b|^2 - |d|^2}{2}$

b) $R^{1/m}$

c) $b_n = \dfrac{1}{n+1}$, $b_n = \dfrac{1}{2n}$

d) $b_n = \begin{cases} \frac{1}{2^n} & ; n \text{ even} \\ \frac{1}{3^n} & ; n \text{ odd} \end{cases}$

Q5

$$y(x) = a_0\left[1 - 2x^2 + \frac{1}{3}x^4\right] + a_1\left[x + \sum_{k=1}^{\infty} \frac{(2k-5)\ldots(-3)}{(2k+1)!}\, x^{2k+1}\right]$$

Q6

a) $x = 0$ irregular singular point
b) If $a = p$, then $x = -p$ is regular singular pt.
If $a \neq p$, then $x = -p$ is irregular
c) $x = 1, -1$ irregular
d) $x = 0$ irregular, $x = n\pi$; $n \in \mathbb{Z}$ regular
e) $x = 1$ regular

Q7 $fg = 1 - 2x + \frac{5}{2}x^2 + \ldots$

Q8

$$y(x) = 2 + x^2 - \frac{5}{12}x^4 + \frac{11}{72}x^6 - \ldots$$

Q10

$$y = 1 + 2(x-1) - 2(x-1)^2 + \frac{2}{3}(x-1)^3 - \frac{1}{6}(x-1)^4 + \frac{1}{15}(x-1)^5 + \ldots$$