INDIAN INSTITUTE OF TECHNOLOGY INDORE

MA 203: Complex Analysis and Differential Equations-II

September 02, 2023 (Autumn Semester)

Tutorial −2 (Differential Equations-II)

1. Use method of Frobenius to solve the following differential equations:

    (a) $2x^2y''(x) + xy'(x) - (x+1)y(x) = 0$

    (b) $2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1-x^2)y = 0$

    (c) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1+x^2)y = 0$

    (d) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-1)y = 0$

    (e) $(x^2-1)^2y''(x) + (x+1)y'(x) - y(x) = 0$

2. Write the general solution of the following equations (Bessel's Equations):

    (a) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-25)y = 0$

    (b) $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + (1 - 1/6.25x^2)y = 0$

    (c) $z\frac{d^2y}{dz^2} + \frac{dy}{dz} + zy = 0$

    (d) $16x^2y'' + 16xy' + (16x^2-1)y = 0$

3. Prove that

    (a) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$

    (b) $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}(\frac{\sin x}{x} - \cos x)$

    (c) $J_2'(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0(x)$

    (d) $[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2 = \frac{2}{\pi x}$

    (e) $\lim\limits_{z\to 0}\frac{J_n(z)}{z^n} = \frac{1}{2^n\Gamma(n+1)^n}$, where $n > -1$

    (f) $\int_0^1 \frac{uJ_0(xu)}{(1-u^2)^{1/2}}du = \frac{\sin x}{x}$

    (g) $J_nJ_{-n}' - J_n'J_{-n} = -\frac{2\sin\pi n}{\pi x}$

    (h) $\frac{d}{dx}\left(\frac{J_{-n}}{J_n}\right) = -\frac{2\sin\pi n}{\pi x J_n^2}$

4. Prove that $J_n(x)$ is the coefficient of z^n in the expansion of e^x

5. Show that the Bessel's function $J_n(x)$ is an even function when n is even and is odd when n is odd.

****END****