**INDIAN INSTITUTE OF TECHNOLOGY INDORE**

MA 203: Complex Analysis and Differential Equations-II

Autumn Semester 2023

Tutorial −3 (Complex Analysis)

1. The following functions are defined in the punctured plane $\mathbb{C} - \{0\}$. Is it possible to suitably define a value to the function at $z = 0$, so that they become continuous?

   (a) $f(z) = \frac{|z|^2}{z}$ yes

   (b) $f(z) = \frac{\overline{z}}{z}$ no

   (c) $f(z) = \frac{\text{Re } z}{z}$ no

   (d) $f(z) = \frac{z}{|z|}$ no

   (e) $f(z) = \frac{z\text{Re } z}{|z|}$ yes

2. Prove that the function $f(z) = \overline{z}$ is not differentiable anywhere.

3. Prove that the function $f(z) = z\text{Re}(z)$ is differentiable only at $z = 0$. Compute $f'(0)$.

4. Prove that the function $f(z) = |z|^2$ is differentiable only at $z = 0$.

5. Determine the points where the function $f(z) = x^2 + iy^2$ is differentiable. Find the value of $f'(z)$. Ans. Points $z = x + ix$, $f'(x + ix) = 2x$

6. Derivation of C-R equations.

7. Using $x = r\cos\theta, y = r\sin\theta$ and using chain rule, prove that C-R euqations are equivalent to
$$u_r = \frac{1}{r}v_\theta, v_r = -\frac{1}{r}u_\theta.$$
   The above pair of equations are the C-R equations in polar form.

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