1D Heat $\rightarrow$ $u_t = c\,u_{xx}$

1D Laplace $\rightarrow$ $\frac{\partial u}{\partial t} \rightarrow 0$ (measured in equilibrium condⁿ)

Tut - 1

3) $h(x+p) = af(x+p) + bg(x+p)$

$h(x+p) = af(x) + bg(x) = h(x)$

4) $f(x) \rightarrow p$

$f(ax + p) = f(ax)$

$f\left(a\left(x + \frac{p}{a}\right)\right) = f(ax)$

$f(x) \rightarrow p$

$$h(x) = f\left(\frac{x}{b}\right)$$

$$= f\left(\frac{x}{b} + p\right)$$

$$= f\left(\frac{1}{b}(x + pb)\right)$$

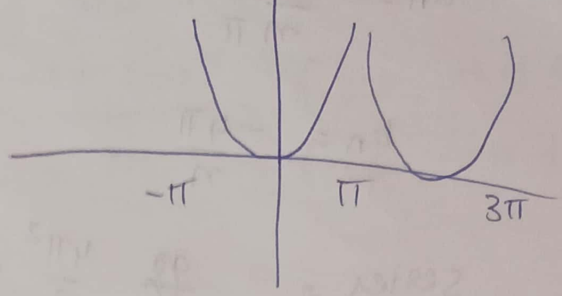
$$= h(x+pb)$$

pb is period

5) a) $f(x) = x^2$, $-\pi < x < \pi$

$\downarrow$

even



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$a_n = \frac{2\pi^2}{3}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx = 0$$

sin

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cos nx \, dx = \frac{1}{\pi} \left[x^2 \left(\frac{\sin nx}{n} \right) \Big|_{-\pi}^{\pi} - \int 2x \left(\frac{\sin nx}{n} \right) dx \right.$$

$$= \frac{1}{\pi} \left[-2 \left[\frac{x(-\cos nx)}{n^2} \Big|_{-\pi}^{\pi} + \int (+\cos nx) \, dx \right.\right.$$

$$\downarrow$$

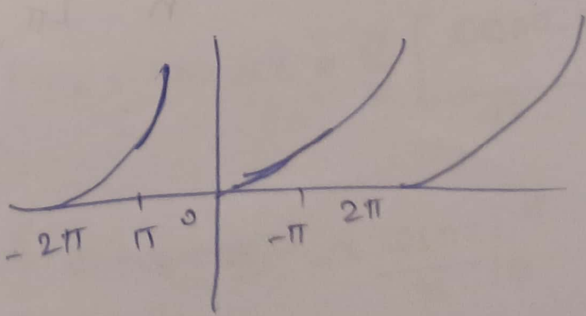
$$\frac{\sin nx}{n^2} \Big|_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left[+2 \left[\frac{2\pi \quad (-1)^n}{n^2} \right] \right]$$

$$= (-1)^n \frac{4}{n^2}$$

b) $f(x) = x^2$, $0 < x < 2\pi$

Not even



Method -1

$$a_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \cos nx \, dx$$

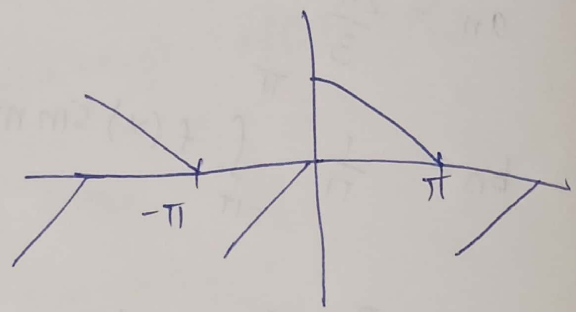
$$a_0 = \frac{1}{\pi} \left[\frac{x^3}{3} \right] = \frac{8\pi^2}{3}$$

$$a_n = \frac{2}{n\pi} \times 2\pi \times \frac{\cos 2n\pi}{n} = \frac{4}{n^2}$$

$$b_n = -\frac{4\pi}{n}$$

$$\text{series} = \frac{4\pi^2}{3} + \sum_{n=1}^{\infty} \left(\frac{4}{n^2} \cos nx - \frac{4\pi}{n} \sin nx \right)$$

(c) $$a_n = \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$



$$a_0 = \int_{-\pi}^{0} x \, dx + \int_{0}^{\pi} (\pi - x) \, dx$$

$$= \left[0 - \frac{\pi^2}{2} \right] + \int_0^\pi \pi x - \frac{x^2}{2}$$

$$= \left[0 - \frac{\pi^2}{2} \right] + \left[\pi^2 - \frac{\pi^2}{2} \right] = 0$$

$$a_n = \frac{1}{\pi} \left[\int_{-\pi}^{0} x \cos nx \, dx + \int_0^\pi (\pi - x) \cos nx \, dx \right]$$

$$= \frac{x \sin nx}{n} \Big|_{-\pi}^{0} + \int_{-\pi}^{0} \frac{\cos nx}{n^2} + \frac{\pi \sin nx}{n} \Big|_0^\pi$$

$$- \oint \frac{x \sin nx}{n} \Big|_0^\pi$$

$$- \frac{\cos nx}{n^2} \Big|_0^\pi$$

$$= 0 + \left[\frac{1}{n^2} - \frac{\cos n\pi}{n^2} \right]$$

$$+ 0 - 0 - \left[\frac{\cos n\pi}{n^2} - \frac{1}{n^2} \right]$$

$$= \frac{2}{n^2\pi}\left[1-(-1)^n\right] = \begin{cases} 0 & ; n \text{ is even} \\ \frac{4}{n^2\pi} & ; n \text{ is odd} \end{cases}$$

$$b_n = \int_{-\pi}^{\pi} f(x) \sin nx \, dx = \frac{1-(-1)^n}{n} = \begin{cases} 0, & n \text{ is even} \\ \frac{2}{n} ; & n \text{ is odd} \end{cases}$$

$$\Rightarrow \frac{4}{\pi}\left[\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right]$$

$$+ 2\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)$$

(d)
$$\begin{cases} -\frac{\pi}{2}, & -\pi < x < -\frac{\pi}{2} \\ x, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2} < x < \pi \end{cases}$$

$a_n = 0$, $n = 0, 1, 2 \ldots$

$$b_n = \frac{1}{\pi}\left[\frac{2}{n^2}\sin\frac{n\pi}{2} - \frac{\pi(-1)^n}{n}\right] \quad n = 1, 2 \ldots$$

$$b_n = \int_{-\pi}^{-\pi/2} \frac{-\frac{\pi}{2}\sin nx \, dx}{n} + \int_{-\pi/2}^{\pi/2} x \sin nx \, dx + \int_{\pi/2}^{\pi} \frac{\frac{\pi}{2}\sin nx \, d}{n}$$

$$= \frac{+\pi}{2}\left[\frac{\cos\frac{n\pi}{2} - \cos n\pi}{n}\right] + \int_0^{\pi} 2x \sin nx \, dx - \frac{\pi}{2}\left[\frac{\cos n\pi - \cos\frac{n\pi}{2}}{n}\right.$$

$$= \frac{\pi}{n}\left[\cos\frac{n\pi}{2} - \cos n\pi\right] + 2\left[\cancel{0 + \int \cos} - x\frac{\cos nx}{n}\Big|_0^{\pi} - \left[-\frac{\sin nx}{n^2}\Big|_0^{\pi}\right]\right.$$

$=$

6) (a) $f(x) = x|x|$, $-1 < x < 1$

$f(-x) = -x(+x) = -x^2$ } odd fx^n $\quad \frac{n\pi}{L}x$

$f(x) = x^2$

period $= 2 \Rightarrow L = 1$ $\quad \int_{-L}^{L} \left(\frac{n\pi}{L}x\right)$

$a_n = \frac{1}{\pi}\int_{-1}^{1} f(x) \cos(n\pi x) = 0$

$f(x)$ → odd, $\cos$ → even

$b_n = \int_{-1}^{1} f(x) \sin(n\pi x) = 2\int_0^1 x^2 \sin n\pi x$

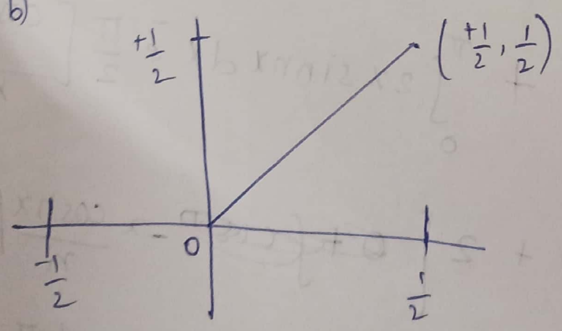
$= 2\left[-x^2 \frac{\cos n\pi x}{n\pi}\Big|_0^1 + \int_0^1 2x \frac{\cos n\pi x}{n\pi}\right]$

$= 2\left[\frac{-\cos n\pi}{n\pi} - 2x \frac{\sin n\pi x}{n^2\pi^2}\Big|_0^1 + 2\frac{\cos n\pi x}{n^3\pi^3}\Big|_0^1\right]$

$= 2\left[\frac{-\cos n\pi}{n\pi} + \frac{2}{n^3\pi^3}[\cos n\pi - 1]\right] \Rightarrow \frac{-2}{n\pi}(-1)^n$

$= \frac{2\cos n\pi}{n\pi}\left[-1 - \frac{9}{n^2\pi^2}\right] + \frac{4}{n^3\pi^3}$ $\quad -\frac{4}{n^3\pi^3}[1-(-1)^n]$

(b)



$x, \quad (0, \frac{1}{2})$

$a_n = \frac{1}{L}\int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$

$= 2\int_0^{\frac{1}{2}} x \cos(2n\pi x)\, dx$

$= 4$

$= 8\left[x \frac{\sin(2n\pi x)}{2n\pi}\Big|_0^{\frac{1}{2}} + \frac{\cos(2n\pi x)}{4n^2\pi^2}\Big|_0^{\frac{1}{2}}\right.$

$$a_n = \frac{\cos n\pi - 1}{2n^2\pi^2}$$

$$b_n = \frac{-\cos n\pi}{2n\pi}$$

(c) $b_n = \frac{4}{n^2\pi} \sin \frac{n\pi}{2}$, $\left\{ \begin{array}{ll} 0 ; & n \text{ is even} \\ \frac{4}{n^2\pi} ; & n = 1, 5, 9 \ldots \\ \frac{-4}{n^2\pi} ; & n = 3, 7, 11 \ldots \end{array} \right.$

04/10

Ex: $\frac{\partial u}{\partial t} = u\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^3 u}{\partial z^3} \longrightarrow$ order = 3

Semi Linear $u_{xx} + uu_y + u_{yz} = x^2 + y^2 + u$