

INDIAN INSTITUTE OF TECHNOLOGY INDORE

MA 203: Complex Analysis and Differential Equations-II

Autumn Semester

Tutorial -5 (Complex Analysis)

1. Let u and v be two harmonic functions. Prove that a linear combination $\alpha u + \beta v$, $\alpha, \beta \in \mathbb{R}$, is also a harmonic function.
2. Is u^2 a harmonic function if u is a non-constant harmonic function? Give reason for your answer.
3. Let u be any harmonic function. For what functions f is the function $f(u)$ also harmonic?
Ans: $Au + B$
4. Are $|f(z)|$ harmonic functions if $f(z)$ is an analytic function? Ans: No
5. Explain whether harmonic functions of the given form (different from a constant) exist and if they exist find them.
 - (a) $u = \phi(x)$ Ans: $u = Ax + B$
 - (b) $u = \phi(ax+by)$, (a, b are real numbers) Ans: $u = A(ax+by) + B$
 - (c) $u = \phi(xy)$ Ans: $u = Axy + B$
 - (d) $u = \phi(x^2+y^2)$ Ans: $u = A \log(x^2+y^2) + B$
6. Is it possible to have an analytic function F in a domain D such that $F'(z) = |z|^2$ for all $z \in D$? Give reason for your answer. (**NOTE:** The same question is available in Tutorial 4. Here you are advised to use different concept to address the question.)
7. If u and v are harmonic conjugate to each other in some domain, then show that u and v must be constant.

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