Division

$$\frac{f(x)}{g(x)} = \frac{\sum_{n=0}^{\infty} \alpha_n (x-a)^n}{\sum_{m=0}^{\infty} \beta_m (x-a)^m} = \sum_{n=0}^{\infty} c_n (x-a)^n$$

where $\sum_{n=0}^{\infty} \alpha_n (x-a)^n = \left[\sum_{m=0}^{\infty} \beta_m (x-a)^m\right]\left[\sum_{n=0}^{\infty} c_n (x-a)^n\right]$

in which c_n can be obtained by expanding the right-hand side and comparing coefficient of $(x-a)^n$ $n=0,1,2,3,\ldots$

$|x-a|<R_1$

$$f'(x) = \sum_{n=1}^{\infty} n\alpha_n (x-a)^{n-1} \quad \text{for } |x-a|<R_1$$

$$f''(x) = \sum_{n=1}^{\infty} n(n-1)\alpha_n (x-a)^{n-2} \quad \text{for } |x-a|<R_1$$

$f(x) = \sum \alpha_n (x-a)^n$

Integration

$$\int f(x)\,dx = \sum_{n=0}^{\infty} \alpha_n \frac{(x-a)^{n+1}}{n+1} \quad \text{for } |x-a|<R_1$$

Analytic Function

I $x=a$

$$f(x) = \sum_{n=0}^{\infty} \alpha_n (x-a)^n$$

Note

If a function $f(x)$ in the interval I containing 'a' is said to be analytic at $x=a$ then

If a function ... to be analytic at $x = a$ then

$$\lim_{x \to a} f(x) \text{ exist and finite}$$

$$\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_0(x)\,y = 0 \qquad \frac{d^ny}{dx^n}.$$

Ordinary point :

Consider the n^{th} order linear O.D.E

$$y^n(x) + P_{n-1}(x)\,y^{n-1}(x) + P_{n-2}(x)\,y^{n-2}(x) + \cdots + P_0(x)\,y(x) = f(x) \qquad (1)$$

$$y^n = \frac{d^ny}{dx^n}$$

A point $x = x_0$ is called an ordinary point of the given differential equation (1) if each of the coefficients $P_0, P_1, \ldots, P_{n-1}(x)$ and $f(x)$ are analytic at $x = x_0$

i.e $P_i(x)$ for $i = 0, 1, 2, \ldots, n-1$ and $f(x)$ can be expressed as power series about $x = x_0$ that are convergent for $|x - x_0| < R$, $R > 0$ i.e.

$$\underline{P_i(x)} = \sum_{n=0}^{\infty} P_{i,n}\,(x - x_0)^n$$

$$f(x) = \sum_{n=0}^{\infty} f_n\,(x - x_0)^n$$