

11.16.3.11

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Question:- In a lottery, a person chooses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins a prize. What is the probability of winning the prize in the game?

Solution:

The sample space is

$$\Omega = [\text{All possible collections of 6 numbers from 1 to 20}] \quad (0.1)$$

$$\Rightarrow |\Omega| = \binom{20}{6} \quad (0.2)$$

Assuming equally likely outcomes,

$$\Pr(\omega \in \Omega) = \frac{1}{|\Omega|} = \frac{1}{\binom{20}{6}} \quad (0.3)$$

Let X represent the event:

$X=1$, The person wins the prize (All the numbers match)

$X=0$: The person does not win (numbers don't match).

The Probability Mass Function (PMF) for the given random variable is

$$P(X = k) = \begin{cases} 1 - \frac{1}{\binom{20}{6}}, & k = 0 \\ \frac{1}{\binom{20}{6}}, & k = 1 \end{cases} \quad (0.4)$$

The Cumulative Distribution Function (CDF) for the given random variable is

$$F_X(k) = P(X \leq k) = \begin{cases} 0, & k < 0 \\ 1 - \frac{1}{\binom{20}{6}}, & 0 \leq k < 1 \\ 1, & k \leq 1 \end{cases} \quad (0.5)$$

The probability of winning the lottery is

$$\Pr(X = 1) = \frac{1}{\binom{20}{6}} \quad (0.6)$$

Simulation

To simulate the lottery, the process follows these steps:

- 1) **Random Number Generation:** Generate random numbers uniformly from 1 to 20 for both the player and the committee using a uniform random number generator.
- 2) **Simulate Multiple Trials:** Run the simulation for a large number of trials (e.g., 1,000,000), where in each trial the player's six numbers are compared to the committee's six numbers.

- 3) **Match Calculation:** If all six numbers match, the player wins. The number of wins is tracked.
- 4) **Probability Calculation:** Calculate the probability of winning as the ratio of winning trials to total trials. The true probability is $\frac{1}{\binom{20}{6}}$.
- 5) **Relative Frequency:** Track and plot the relative frequency of winning over time to observe convergence to the true probability.
- 6) **Final Results:** After all trials, generate:
 - Probability Mass Function (PMF)
 - Cumulative Distribution Function (CDF)
 - Convergence of relative frequency to the true probability

The following shows how the relative frequency reaches true probability with increasing number of trials of the event.

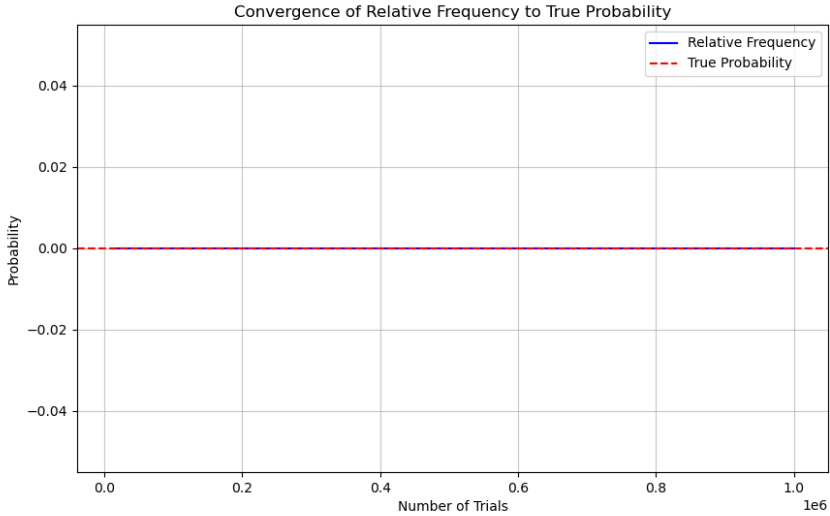


Fig. 6.1: Relative Frequency tends to True Probability

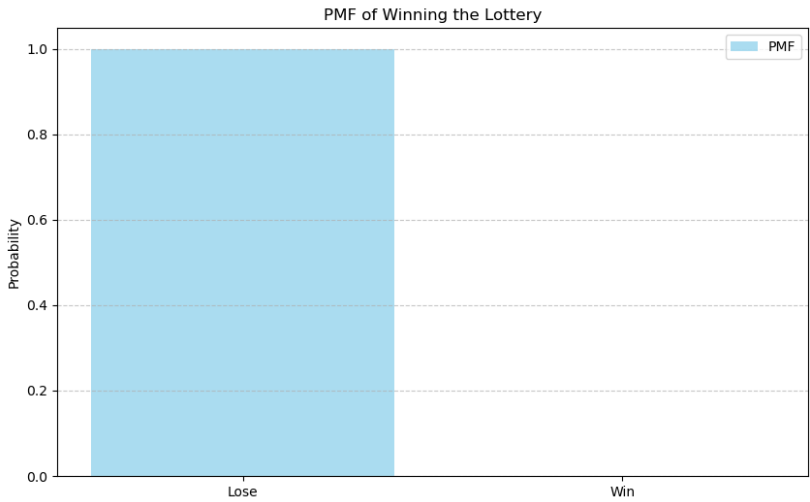


Fig. 6.2: Probability Mass Function of given Random variable

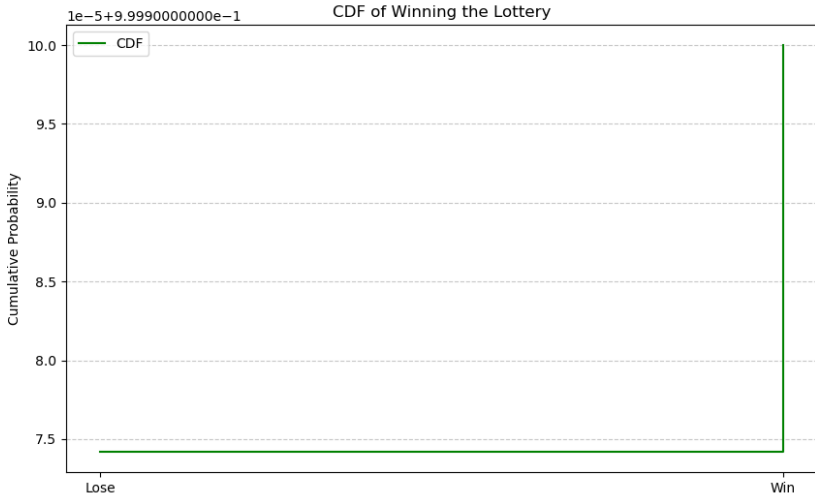


Fig. 6.3: Cumulative Distribution Function of given Random variable