10.3.2.2.1

EE24BTECH11006 - Arnav Mahishi

Question:
Find out whether the lines $5x-4y+8=0$ and $7x+6y-9=0$ intersect at a point, parallel or coincident

input	Description	value
μ	Size of step	0.01
$p(x)$	The profit function	$41-72x-18x^2$
Tolerance	Maximum gradient until we can say x_n has converged	1e-5
x_0	Inital Guess of maxima	0

TABLE 0: Variables Used

Theoretical Solution:
Let a_1,b_1, and c_1 and a_2,b_2, and c_2 be the coefficents of x,y, and 1 in lines 1 and 2 respectively.
We get:

$$\frac{a_1}{a_2}=\frac{5}{7} \tag{0.1}$$

$$\frac{b_1}{b_2}=\frac{-2}{3} \tag{0.2}$$

$$\frac{c_1}{c_2}=\frac{-8}{9} \tag{0.3}$$

$$m_1=\frac{-a_1}{b_1}=\frac{5}{4} \tag{0.4}$$

$$m_2=\frac{-a_2}{b_2}=\frac{7}{6} \tag{0.5}$$

As all the ratios aren't equal to each other neither are m_1 and m_2 equal
$\therefore$ The lines intersect at a point
Computational Solution:
The set of linear equations $5x-4y+8=0$ and $7x+6y-9=0$ can be represented by the following equation

$$\begin{pmatrix}5 & -4\\7 & 6\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-8\\9\end{pmatrix} \tag{0.6}$$

Any non-singular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U:

$$A=L\cdot U \tag{0.7}$$

1. Initialization: - Start by initializing **L** as the identity matrix **L** = **I** and **U** as a copy of **A**.

2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.

3. Result: - After completing the iterations, the matrix **A** is decomposed into **L**·**U**, where **L** is a lower triangular matrix with ones on the diagonal, and **U** is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix **U** by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix **L**, where each entry in the column is determined by the values in the rows above it.
Using a code we get L,U as

Step-by-Step Process:

1. Initial Matrix:

$$A = \begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \tag{0.8}$$

2. Compute U (Upper Triangular Matrix):
Using the update equation for U:

$$U_{11} = A_{11} = 5, \quad U_{12} = A_{12} = -4 \tag{0.9}$$

For U_{22}:

$$U_{22} = A_{22} - L_{21} \cdot U_{12} = 6 - \frac{7}{5} \cdot (-4) = \frac{58}{5} \tag{0.10}$$

3. Compute L (Lower Triangular Matrix):
Using the update equation for L:

$$L_{21} = \frac{A_{21}}{U_{11}} = \frac{7}{5} \tag{0.11}$$

The final L matrix is:

$$L = \begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \tag{0.12}$$

4. Solving the System:

Using the equations $L\mathbf{y} = \mathbf{b}$ and $U\mathbf{x} = \mathbf{y}$:

- **Forward Substitution:**

$$\begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{0.13}$$

  Solving gives:

$$y_1 = -8, \quad y_2 = \frac{101}{5} \tag{0.14}$$

- **Backward Substitution:**

$$\begin{pmatrix} 5 & -4 \\ 0 & \frac{58}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -8 \\ \frac{101}{5} \end{pmatrix} \tag{0.15}$$

  Solving gives:

$$x_2 = \frac{101}{58}, \quad x_1 = \frac{-6}{29} \tag{0.16}$$

Thus, the solution is:

$$\mathbf{x} = \begin{pmatrix} \frac{-6}{29} \\ \frac{101}{58} \end{pmatrix} \tag{0.17}$$

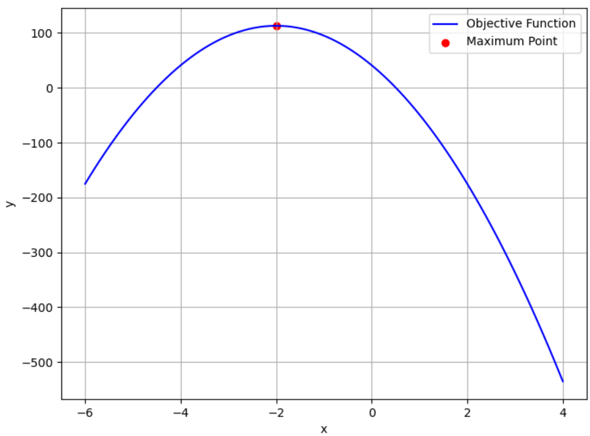

Fig. 0.1: Solution to set of linear equations