NCERT-10.4.1.2.2

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January 23, 2025

Problem Statement

Find out whether the lines $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ intersect at a point, parallel or coincident

Input Parameters

Variable	Description
A	Matrix consisting of coefficients in the linear equation
L	Lower triangular matrix
U	Upper triangular matrix
$\mathbf{x}$	Solution to the linear equation

Theoritical Soln

Let a_1,b_1, and c_1 and a_2,b_2, and c_2 be the coefficents of x,y, and 1 in lines 1 and 2 respectively.
We get:

$$\frac{a_1}{a_2} = \frac{5}{7} \tag{3.1}$$

$$\frac{b_1}{b_2} = \frac{-2}{3} \tag{3.2}$$

$$\frac{c_1}{c_2} = \frac{-8}{9} \tag{3.3}$$

$$m_1 = \frac{-a_1}{b_1} = \frac{5}{4} \tag{3.4}$$

$$m_2 = \frac{-a_2}{b_2} = \frac{7}{6} \tag{3.5}$$

As all the ratios aren't equal to each other neither are m_1 and m_2 equal
$\therefore$ The lines intersect at a point

Computational Soln

Computational Solution:
The set of linear equations $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ can be represented by the following equation

$$\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{3.6}$$

Any on-singular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U:

$$A = L \cdot U \tag{3.7}$$

1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.
2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.
3. Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix **U** by eliminating the lower triangular portion of the matrix.
For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix **L**, where each entry in the column is determined by the values in the rows above it.
Using a code we get L,U as

Step-by-Step Process:

1. Initial Matrix:

$$A = \begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \tag{3.8}$$

2. Compute U (Upper Triangular Matrix):

Using the update equation for U:

$$U_{11} = A_{11} = 5, \quad U_{12} = A_{12} = -4 \tag{3.9}$$

For U_{22}:

$$U_{22} = A_{22} - L_{21} \cdot U_{12} = 6 - \frac{7}{5} \cdot (-4) = \frac{58}{5} \tag{3.10}$$

3. Compute L (Lower Triangular Matrix):

Using the update equation for L:

$$L_{21} = \frac{A_{21}}{U_{11}} = \frac{7}{5} \tag{3.11}$$

The final L matrix is:

$$L = \begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \tag{3.12}$$

4. Solving the System:

Using the equations $L\mathbf{y} = \mathbf{b}$ and $U\mathbf{x} = \mathbf{y}$:

Forward Substitution:

$$\begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{3.13}$$

Solving gives:

$$y_1 = -8, \quad y_2 = \frac{101}{5} \tag{3.14}$$

Backward Substitution:

$$\begin{pmatrix} 5 & -4 \\ 0 & \frac{58}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -8 \\ \frac{101}{5} \end{pmatrix} \tag{3.15}$$

Solving gives:

$$x_2 = \frac{101}{58}, \quad x_1 = \frac{-6}{29} \tag{3.16}$$

Thus, the solution is:

$$\mathbf{x} = \begin{pmatrix} \frac{-6}{29} \\ \frac{101}{58} \end{pmatrix} \tag{3.17}$$

Plot

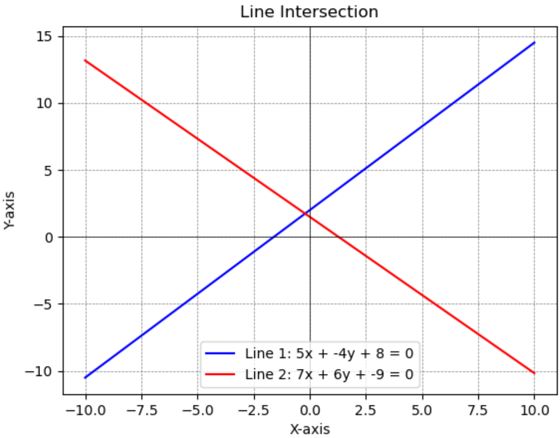

Figure: Solution to set of linear equations