10.3.2.2.1

EE24BTECH11006 - Arnav Mahishi

Question:
Find out whether the lines $5x-4y+8=0$ and $7x+6y-9=0$ intersect at a point, parallel or coincident

Variable	Description
A	Matrix consisting of coefficients in the linear equation
L	Lower triangular matrix
U	Upper triangular matrix
$\mathbf{x}$	Solution to the linear equation

TABLE 0: Variables Used

Theoretical Solution:
Let a_1,b_1, and c_1 and a_2,b_2, and c_2 be the coefficents of x,y, and 1 in lines 1 and 2 respectively.
We get:

$$\frac{a_1}{a_2} = \frac{5}{7} \tag{0.1}$$
$$\frac{b_1}{b_2} = \frac{-2}{3} \tag{0.2}$$
$$\frac{c_1}{c_2} = \frac{-8}{9} \tag{0.3}$$
$$m_1 = \frac{-a_1}{b_1} = \frac{5}{4} \tag{0.4}$$
$$m_2 = \frac{-a_2}{b_2} = \frac{7}{6} \tag{0.5}$$

As all the ratios aren't equal to each other neither are m_1 and m_2 equal
$\therefore$ The lines intersect at a point
Computational Solution:
The set of linear equations $5x-4y+8=0$ and $7x+6y-9=0$ can be represented by the following equation

$$\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{0.6}$$

Any non-singular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U:

$$A = L \cdot U \tag{0.7}$$

The update equations for L and U are as follows:

1) **For U_{ij}:**

$$U_{ij} = A_{ij} - \sum_{k=1}^{i-1} L_{ik} U_{kj}, \quad i \leq j \tag{1.1}$$

2) **For L_{ij}:**

$$L_{ij} = \frac{1}{U_{jj}} \left(A_{ij} - \sum_{k=1}^{j-1} L_{ik} U_{kj} \right), \quad i > j \tag{2.1}$$

3) **Diagonal of L:**

$$L_{ii} = 1 \tag{3.1}$$

Step-by-Step Process:

1. Initial Matrix:

$$A = \begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \tag{3.2}$$

2. Compute U (Upper Triangular Matrix):
Using the update equation for U:

$$U_{11} = A_{11} = 5, \quad U_{12} = A_{12} = -4 \tag{3.3}$$

For U_{22}:

$$U_{22} = A_{22} - L_{21} \cdot U_{12} = 6 - \frac{7}{5} \cdot (-4) = \frac{58}{5} \tag{3.4}$$

3. Compute L (Lower Triangular Matrix):
Using the update equation for L:

$$L_{21} = \frac{A_{21}}{U_{11}} = \frac{7}{5} \tag{3.5}$$

The final L matrix is:

$$L = \begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \tag{3.6}$$

4. Solving the System:
Using the equations $L\mathbf{y} = \mathbf{b}$ and $U\mathbf{x} = \mathbf{y}$:

- **Forward Substitution:**

$$\begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{3.7}$$

  Solving gives:

$$y_1 = -8, \quad y_2 = \frac{101}{5} \tag{3.8}$$

- **Backward Substitution:**

$$\begin{pmatrix} 5 & -4 \\ 0 & \frac{58}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -8 \\ \frac{101}{5} \end{pmatrix} \tag{3.9}$$

  Solving gives:

$$x_2 = \frac{101}{58}, \quad x_1 = \frac{-6}{29} \tag{3.10}$$

Thus, the solution is:

$$\mathbf{x} = \begin{pmatrix} \frac{-6}{29} \\ \frac{101}{58} \end{pmatrix} \tag{3.11}$$

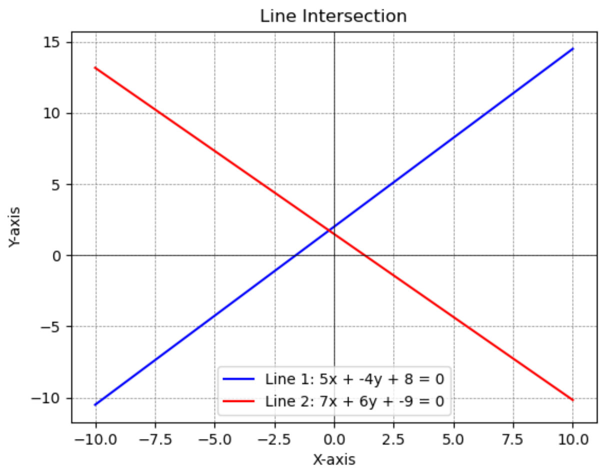

Fig. 3.1: Solution to set of linear equations