10.3.2.2.1

EE24BTECH11006 - Arnav Mahishi

Question:
Find out whether the lines $5x-4y+8=0$ and $7x+6y-9=0$ intersect at a point, parallel or coincident

Variable	Description
A	Matrix consisting of coefficients in the linear equation
L	Lower triangular matrix
U	Upper triangular matrix
$\mathbf{x}$	Solution to the linear equation

TABLE 0: Variables Used

Theoretical Solution:
Let a_1,b_1, and c_1 and a_2,b_2, and c_2 be the coefficents of x,y, and 1 in lines 1 and 2 respectively.
We get:

$$\frac{a_1}{a_2} = \frac{5}{7} \tag{0.1}$$

$$\frac{b_1}{b_2} = \frac{-2}{3} \tag{0.2}$$

$$\frac{c_1}{c_2} = \frac{-8}{9} \tag{0.3}$$

$$m_1 = \frac{-a_1}{b_1} = \frac{5}{4} \tag{0.4}$$

$$m_2 = \frac{-a_2}{b_2} = \frac{7}{6} \tag{0.5}$$

As all the ratios aren't equal to each other neither are m_1 and m_2 equal
$\therefore$ The lines intersect at a point
Computational Solution:
The set of linear equations $5x-4y+8=0$ and $7x+6y-9=0$ can be represented by the following equation

$$\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{0.6}$$

Any non-sigular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U

$$A\mathbf{x} = LU\mathbf{x} = \mathbf{b} \tag{0.7}$$

The upper triangular matrix U is found by row reducing A,

$$\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \xrightarrow{R_2 -> R_2 - \frac{7}{5}R_1} \begin{pmatrix} 5 & -4 \\ 0 & \frac{58}{5} \end{pmatrix} \tag{0.8}$$

Let

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \tag{0.9}$$

l_{21} is the multiplier used to zero a_{21}, so $l_{21} = \frac{7}{5}$.

Now,

$$A = \begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{0.10}$$

Now we can get the solution to our problem by the two step process,

$$L\mathbf{y} = \mathbf{b} \tag{0.11}$$

$$U\mathbf{x} = \mathbf{y} \tag{0.12}$$

Using forward substitution to solve the first equation,

$$\begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \tag{0.13}$$

$$\implies \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -8 \\ \frac{101}{5} \end{pmatrix} \tag{0.14}$$

$$\tag{0.15}$$

Now using back-substitution for the second equation,

$$\begin{pmatrix} 5 & -4 \\ 0 & \frac{58}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -8 \\ \frac{101}{5} \end{pmatrix} \tag{0.16}$$

$$\implies \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{-6}{29} \\ \frac{101}{58} \end{pmatrix} \tag{0.17}$$

Thus, align x= $\begin{pmatrix} \frac{-6}{29} \\ \frac{101}{58} \end{pmatrix}$

$\therefore$ The lines $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ intersect at $\left(\frac{6}{29}, \frac{101}{58}\right)$

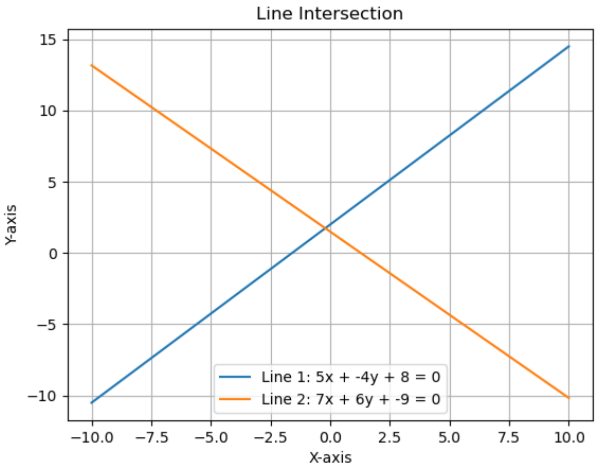

Fig. 0.1: Solution to set of linear equations