NCERT-11.16.3.11

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Problem Statement

In a lottery, a person chooses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers aldready fixed by the lotttery committee, he wins a prize. What is the probability of winning the prize in the game?

Theoritical Soln

The sample space is

$$\Omega = [\text{All possible collections of 6 numbers from 1 to 20}] \quad (3.1)$$

$$\implies |\Omega| = \binom{20}{6} \quad (3.2)$$

Assuming equally likely outcomes,

$$\Pr(\omega \in \Omega) = \frac{1}{|\Omega|} = \frac{1}{\binom{20}{6}} \quad (3.3)$$

CDF and PMF

Let X represent the event:

X=1, The person wins the prize (All the numbers match)

X=0: The person does not win (numbers don't match).

The Probability Mass Function (PMF) for the given random variable is

$$P(X = k) = \begin{cases} 1 - \frac{1}{\binom{20}{6}}, & k = 0 \\ \frac{1}{\binom{20}{6}}, & k = 1 \end{cases} \tag{3.4}$$

The Cumulative Distribution Function (CDF) for the given random variable is

$$F_X(k) = P(X \leq k) = \begin{cases} 0, & k < 0 \\ 1 - \frac{1}{\binom{20}{6}}, & 0 \leq k < 1 \\ 1, & k \leq 1 \end{cases} \tag{3.5}$$

The probability of winnig the lottery is

$$\Pr(X = 1) = \frac{1}{\binom{20}{6}} \tag{3.6}$$

Simulation

To simulate the lottery the process follows these steps:

Random Number Generation: Generate random numbers uniformly from 1 to 20 for both the player and the committee using a uniform random number generator.

Simulate Multiple Trials: Run the simulation for a large number of trials (e.g., 1,000,000), where in each trial the player's six numbers are compared to the committee's six numbers.

Match Calculation: If all six numbers match, the player wins. The number of wins is tracked.

Probability Calculation: Calculate the probability of winning as the ratio of winning trials to total trials. The true probability is $\frac{1}{\binom{20}{6}}$

Relative Frequency: Track and plot the relative frequency of winning over time to observe convergence to the true probability.

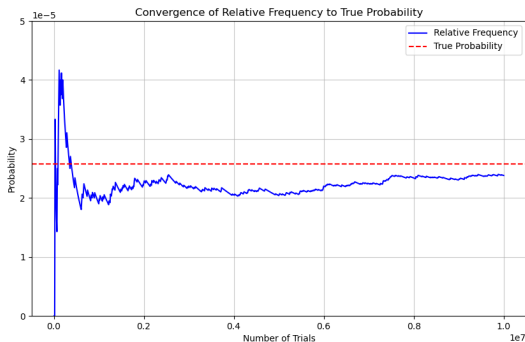# Convergence of relative frequency to true probability

Figure: Relative Frequency tends to True Probability
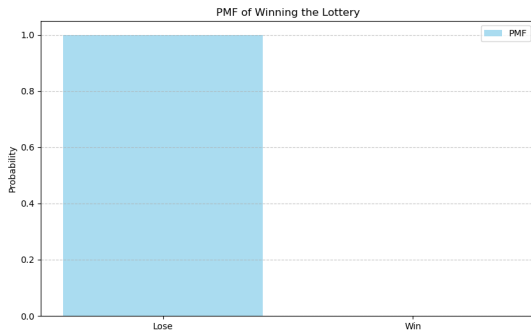
Probability Mass Function

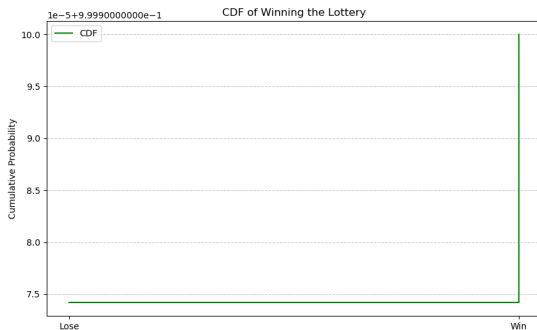Figure: Probability Mass Function of given Random variable

Cumulative Distribution Function

Figure: Cumulative Distribution Function of given Random variable