# NCERT-11.16.3.11

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# Problem Statement

In a lottery, a person chooses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers aldready fixed by the lotttery committee, he wins a prize. What is the probability of winning the prize in the game?

# Theoritical Soln

The sample space is

$$\Omega = [\text{All possible collections of 6 numbers from 1 to 20}] \tag{3.1}$$

$$\implies |\Omega| = \binom{20}{6} \tag{3.2}$$

Assuming equally likely outcomes,

$$\Pr(\omega \in \Omega) = \frac{1}{|\Omega|} = \frac{1}{\binom{20}{6}} \tag{3.3}$$

# CDF and PMF

Let X represent the event:
X=1, The person wins the prize (All the numbers match)
X=0: The person does not win (numbers don't match).
The Probability Mass Function (PMF) for the given random variable is

$$P(X=k)=\begin{cases}1-\frac{1}{\binom{20}{6}}, & k=0\\ \frac{1}{\binom{20}{6}}, & k=1\end{cases} \tag{3.4}$$

The Cumulative Distribution Function (CDF) for the given random variable is

$$F_X(k)=P(X\leq k)=\begin{cases}0, & k<0\\ 1-\frac{1}{\binom{20}{6}}, & 0\leq k<1\\ 1, & k\leq 1\end{cases} \tag{3.5}$$

The probability of winnig the lottery is

$$\Pr(X=1)=\frac{1}{\binom{20}{6}} \tag{3.6}$$

# Simulation

To simulate the lottery the process follows these steps:
**Random Number Generation:** Generate random numbers uniformly from 1 to 20 for both the player and the committee using a uniform random number generator.
**Simulate Multiple Trials:** Run the simulation for a large number of trials (e.g., 1,000,000), where in each trial the player's six numbers are compared to the committee's six numbers.
**Match Calculation:** If all six numbers match, the player wins. The number of wins is tracked.**Probability Calculation:** Calculate the probability of winning as the ratio of winning trials to total trials. The true probability is $\frac{1}{\binom{20}{6}}$
**Relative Frequency:** Track and plot the relative frequency of winning over time to observe convergence to the true probability.

# Convergence of relative frequency to true probability

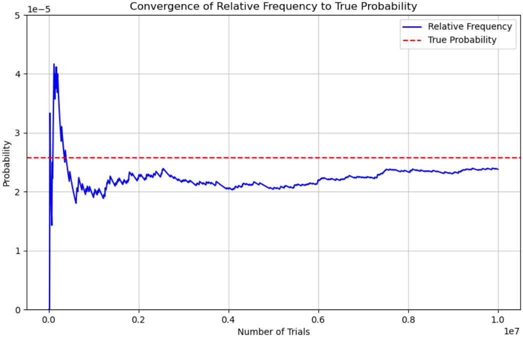


Figure: Relative Frequency tends to True Probability

# Probability Mass Function

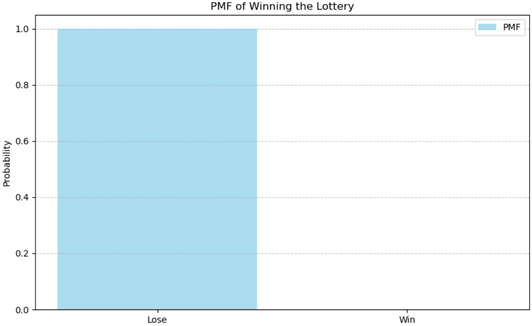


Figure: Probability Mass Function of given Random variable

# Cumulative Distribution Function

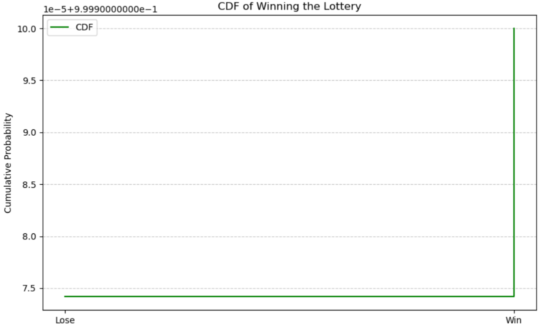


Figure: Cumulative Distribution Function of given Random variable