

04-8-2023-shift-1

EE24BTECH11006 - Arnav Mahishi

1) The area of the region $\{(x, y) : x^2 \leq y \leq 8 - x^2, y \leq 7\}$

a) 24

b) 21

c) 20

d) 18

2) Let $P = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$, $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $Q = PAP^T$. If $P^T Q^{2007} P = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $2a + b - 3c - 4d$ equal to

a) 2004

b) 2007

c) 2005

d) 2006

3) Negation of $(p \rightarrow q) \rightarrow (q \rightarrow q)$ is

a) $(\neg q) \wedge p$

b) $p \vee (\neg q)$

c) $(\neg p) \vee q$

d) $q \wedge (\neg p)$

4) Let $C(\alpha, \beta)$ be the circumcenter of the triangle formed by the lines

$$4x + 3y = 69$$

$$4y - 3x = 17$$

$$x + 7y = 61$$

Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

a) 18

b) 15

c) 16

d) 17

5) Let α, β, γ , be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to

a) $\frac{155}{8}$

b) 21

c) 19

d) $\frac{169}{8}$

6) Let the number of elements in set A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is:

a) 752

b) 772

c) 782

d) 792

7) If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:5:20, then the coefficient of the fourth term is

- a) 5481 b) 3654 c) 2436 d) 1817

8) Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q . Let the point $G(10, 10)$ be the centroid of the triangle PQR . If $c - m = 6$ then $(PQ)^2$ is

- a) 325 b) 346 c) 296 d) 317

9) Let $S_K = \frac{1+2+\dots+K}{K}$ and $\sum_{j=1}^n S_j^2 = \frac{n}{A} (Bn^2 + Cn + D)$ where $A, B, C, D \in N$ and A has least value. Then

- a) $A + B$ is divisible by D c) $A + C + D$ is not divisible by B
b) $A + B = 5(D - C)$ d) $A + B + D$ is divisible by 5

10) The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$

- a) $2\sqrt{6}$ b) $3\sqrt{6}$ c) $6\sqrt{3}$ d) $6\sqrt{2}$

11) The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is

- a) 16800 b) 14800 c) 18000 d) 33600

12) If the points with position vectors $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear then, $(19\alpha - 6\beta)^2$ is equal to

- a) 49 b) 36 c) 25 d) 16

13) In a bolt factory, machines A, B , and C manufacture respectively 20%, 30%, and 50% of the total bolts. Of their output 3, 4, and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, then the probability that it is manufactured by the machine C .

- a) $\frac{5}{14}$ b) $\frac{3}{7}$ c) $\frac{9}{28}$ d) $\frac{2}{7}$

14) If for $z = \alpha + i\beta$, $|z + 2| = z + (4 + i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation

- a) $x^2 + 3x - 4$ b) $x^2 + 7x + 12$ c) $x^2 + x - 12$ d) $x^2 + 2x - 3$

15) $\lim_{x \rightarrow 0} \left(\left(\frac{(1 - \cos^2(3x))}{\cos^3(4x)} \right) \left(\frac{\sin^3(4x)}{\log_e(2x+1)^5} \right) \right)$ is equal to

- a) 24 b) 19 c) 18 d) 15