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EE24BTECH11006 - Arnav Mahishi

1) Let $f(x) = \left|2x^2 + 5|x| - 3\right|, x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then $m + n$ is equal to:

a) 5 b) 2 c) 0 d) 3

2) Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p,q, and r be the consecutive terms of a non-constant G·P and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is

a) $\frac{80}{9}$ b) 9 c) $\frac{20}{3}$ d) 8

3) The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0; x \in [-2\pi, 2\pi]$ is:

a) 1 b) 3 c) 2 d) 0

4) The value of $\int_0^1 \left(2x^3 - 3x^2 - x + 1\right)^{1/3} dx$ is equal to

a) 0 b) 1 c) 2 d) -1

5) Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let the line passing through P and parallel to the y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that $PR : RQ = 4 : 3$ as P moves along the ellipse,is:

a) $\frac{11}{19}$ b) $\frac{13}{21}$ c) $\frac{\sqrt{139}}{23}$ d) $\frac{\sqrt{13}}{7}$

6) Let m and n be the coefficient of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{2}{3}}}\right)^{18}$ .Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is:

a) $\frac{4}{9}$ b) $\frac{1}{9}$ c) $\frac{1}{4}$ d) $\frac{9}{4}$

7) Let α be a non-zero real number. Suppose $f : R \to R$ is a differentiable function such that $f(0) = 2$ and $\lim_{x\to\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(\log_e 2)$ is equal to:

a) 3 b) 5 c) 9 d) 7

8) Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{8} = \frac{z+1}{2}$ which are at a distance of 6 units from the point $R(1,2,3)$. If the centroid of the triangle PQR is (α,β,γ), then $\alpha^2+\beta^2+\gamma^2$ is:

a) 26 b) 36 c) 18 d) 24

9) Consider a $\triangle ABC$ where $A(1,3,2)$, $B(-2,8,0)$, and $(3,6,7)$. If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is:

a) $\frac{37}{2\sqrt{38}}$ b) $\frac{\sqrt{38}}{2}$ c) $\frac{39}{2\sqrt{38}}$ d) $\sqrt{19}$

10) Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_n = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

a) 800 b) 890 c) 790 d) 690

11) If $\int_0^{\frac{\pi}{3}} \cos^4 x\,dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then $9a+8b$ is equal to:

a) 2 b) 1 c) 3 d) $\frac{3}{2}$

12) If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left|z + \frac{1}{2}(3+4i)\right|$

a) $\frac{5}{2}$ b) 2 c) 3 d) (3) 2

13) If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}\left(x^2+2x-15\right)$ is $(-\infty,\alpha)\cup[\beta,\infty)$, then $\alpha^2+\beta^3$ is equal to:

a) 140 b) 175 c) 150 d) 125

14) Consider the relations R_1 and R_2 defined as $aR_1b \iff a^2+b^2 = 1$ for all $a,b \in R$ and $(a,b)R_2(c,d) \iff a+d = b+c$ for all $(a,b),(c,d) \in N \times N$. Then
   a) Only R_1 is an equivalence relation
   b) Only R_2 is an equivalence relation
   c) R_1 and R_2 both are equivalence relations
   d) Neither R_1 nor R_2 is an equivalence relation

15) If the mirror image of the point $P(3,4,9)$ in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α,β,γ), then $14(\alpha+\beta+\gamma)$ is:

a) 102 b) 138 c) 108 d) 132