04-8-2023-shift-1

EE24BTECH11006 - Arnav Mahishi

1) The area of the region $\{(x,y) : x^2 \leq y \leq 8 - x^2, y \leq 7\}$

   a) 24 b) 21 c) 20 d) 18

2) Let $P = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$, $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $Q = PAP^T$. If $P^T Q^{2007} P = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $2a + b - 3c - 4d$ equal to

   a) 2004 b) 2007 c) 2005 d) 2006

3) Negation of $(p \to q) \to (q \to q)$ is

   a) $(\neg q) \wedge p$ b) $p \vee (\neg q)$ c) $(\neg p) \vee q$ d) $q \wedge (\neg p)$

4) Let $C(\alpha, \beta)$ be the circumcenter of the triangle formed by the lines

$$\begin{aligned} 4x + 3y &= 69 \\ 4y - 3x &= 17 \\ x + 7y &= 61 \end{aligned}$$

   Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

   a) 18 b) 15 c) 16 d) 17

5) Let α, β, γ, be the three roots of the equation $x^3 + bx + c = 0$. If $\beta y = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to

   a) $\frac{155}{8}$ b) 21 c) 19 d) $\frac{169}{8}$

6) Let the number of elements in set A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is:

   a) 752 b) 772 c) 782 d) 792

7) If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:5:20, then the coefficient of the fourth term is

a) 5481 b) 3654 c) 2436 d) 1817

8) Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q. Let the point $G(10, 10)$ be the centroid of the triangle PQR. If $c - m = 6$ then $(PQ)^2$ is

a) 325 b) 346 c) 296 d) 317

9) Let $S_K = \frac{1+2+...+K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}\left(Bn^2 + Cn + D\right)$ where $A, B, C, D \in N$ and A has least value. Then

a) $A + B$ is divisible by D
b) $A + B = 5(D - C)$
c) $A + C + D$ is not divisible by B
d) $A + B + D$ is divisible by 5

10) The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$

a) $2\sqrt{6}$ b) $3\sqrt{6}$ c) $6\sqrt{3}$ d) $6\sqrt{2}$

11) The number of arrangements of the letters of the word "*INDEPENDENCE*" in which all the vowels always occur together is

a) 16800 b) 14800 c) 18000 d) 33600

12) If the points with position vectors $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear then, $(19\alpha - 6\beta)^2$ is equal to

a) 49 b) 36 c) 25 d) 16

13) In a bolt factory, machines A,B, and C manufacture respectively 20%, 30%, and 50% of the total bolts. Of their output 3,4, and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, then the probabilty that it is manufactured by the machine C.

a) $\frac{5}{14}$ b) $\frac{3}{7}$ c) $\frac{9}{28}$ d) $\frac{2}{7}$

14) If for $z = \alpha + \imath\beta$,$|z + 2| = z + (4 + \imath)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation

a) $x^2 + 3x - 4$ b) $x^2 + 7x + 12$ c) $x^2 + x - 12$ d) $x^2 + 2x - 3$

15) $\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{sin^3(4x)}{\log_e(2x+1)^5}\right)\right)$ is equal to

a) 24 b) 19 c) 18 d) 15