

## Matgeo-7-7.3-5

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## 1 Problem

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## Problem Statement

If a circle passes through the points  $(0, 0)$ ,  $(a, 0)$ , and  $(0, b)$  then find the coordinates of its centre.

# Input Parameters

input	value
$x_1$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$x_2$	$\begin{pmatrix} a \\ 0 \end{pmatrix}$
$x_3$	$\begin{pmatrix} 0 \\ b \end{pmatrix}$

Table: Input Parameters

## Equation relating centre with points

Given pts  $x_1, x_2, x_3$  on circle:

$$\begin{pmatrix} 2x_1 & 2x_2 & 2x_3 \\ 1 & 1 & 1 \end{pmatrix}^T \begin{pmatrix} u \\ f \end{pmatrix} = - \begin{pmatrix} \|x_1\|^2 \\ \|x_2\|^2 \\ \|x_3\|^2 \end{pmatrix} \quad (3.1)$$

$$\implies \begin{pmatrix} 2x_1^T & 1 \\ 2x_2^T & 1 \\ 2x_3^T & 1 \end{pmatrix} \begin{pmatrix} u \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ -a^2 \\ -b^2 \end{pmatrix} \quad (3.2)$$

$$\implies \begin{pmatrix} 0 & 0 & 1 \\ 2a & 0 & 1 \\ 0 & 2b & 1 \end{pmatrix} \begin{pmatrix} u \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ -a^2 \\ -b^2 \end{pmatrix} \quad (3.3)$$

## Row Reduction

The augmented matrix for this

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 2a & 0 & 1 & -a^2 \\ 0 & 2b & 1 & -b^2 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 2a & 0 & 0 & -a^2 \\ 0 & 2b & 1 & -b^2 \end{pmatrix} \quad (3.4)$$

$$\implies \xleftrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 2a & 0 & 0 & -a^2 \\ 0 & 2b & 0 & -b^2 \end{pmatrix} \quad (3.5)$$

## Finding Centre

Thus, Let  $u = \begin{pmatrix} -x \\ -y \end{pmatrix}$  then

$$(2a \ 0) u = -a^2 \text{ and } (0 \ 2b) u = -b^2 \quad (3.6)$$

$$\implies (2a \ 0) \begin{pmatrix} -x \\ -y \end{pmatrix} = -a^2 \quad (3.7)$$

$$\implies -2ax = -a^2 \implies x = \frac{a}{2} \quad (3.8)$$

$$(0 \ 2b) \begin{pmatrix} -x \\ -y \end{pmatrix} = -b^2 \quad (3.9)$$

$$\implies -2by = -b^2 \implies y = \frac{b}{2} \quad (3.10)$$

$$\implies u = \begin{pmatrix} -x \\ -y \end{pmatrix} = -\begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \end{pmatrix} \implies c = -u = \begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \end{pmatrix} \quad (3.11)$$

## C Code

```
#include <stdio.h>
#include <math.h>

#define MAX_POINTS 3

struct Point {
    double x, y;
};

struct Matrix {
    double data[MAX_POINTS][MAX_POINTS];
};
```



```

void rowReduction(struct Matrix *A, struct Matrix *b) {
    for (int i = 0; i < MAX_POINTS - 1; i++) {
        for (int j = i + 1; j < MAX_POINTS; j++) {
            double factor = A->data[j][i] / A->data[i][i];
            for (int k = 0; k < MAX_POINTS; k++) {
                A->data[j][k] -= factor * A->data[i][k];
            }
            b->data[j][0] -= factor * b->data[i][0];
        }
    }
}

```

```

void findCenterAndRadius(const struct Point *points, struct Point *
    center, double *radius) {
    struct Matrix A = {{ {2 * points[0].x, 2 * points[0].y, points[0].x *
        points[0].x + points[0].y * points[0].y},
        {2 * points[1].x, 2 * points[1].y, points[1].x * points[1].x +
        points[1].y * points[1].y},
        {2 * points[2].x, 2 * points[2].y, points[2].x * points[2].x +
        points[2].y * points[2].y}}} };
    struct Matrix b = {{ {points[0].x * points[0].x + points[0].y * points
        [0].y}, {points[1].x * points[1].x + points[1].y * points[1].y}, {
        points[2].x * points[2].x + points[2].y * points[2].y}}} };
    rowReduction(&A, &b);
    center->x = A.data[0][2] / (2 * A.data[0][0]);
    center->y = A.data[1][2] / (2 * A.data[1][1]);
    *radius = sqrt((points[0].x - center->x) * (points[0].x - center->
        x) + (points[0].y - center->y) * (points[0].y - center->y));
}

```

```

int main() {
    struct Point points[MAX_POINTS] = {{0, 0}, {2, 0}, {0, 2}};
    struct Point center;
    double radius;
    findCenterAndRadius(points, &center, &radius);
    FILE *fp = fopen("circle_data.txt", "w");
    if (fp == NULL) {
        printf("Error opening file.\n");
        return 1;
    }
    fprintf(fp, "Center: (%.2f, %.2f)\n", center.x, center.y);
    fprintf(fp, "Radius: %.2f\n", radius);
    fclose(fp);
    return 0;
}

```

# Python Code

```
#Code by GVV Sharma
#September 12, 2023
#Revised July 21, 2024
#released under GNU GPL
#Point Vectors
import sys #for path to external scripts
sys.path.insert(0, '/home/arnav/matgeo/codes/CoordGeo') #path to my
    scripts
import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
from mpl_toolkits.mplot3d import Axes3D
```

```
#local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen
import ctypes
from ctypes import Structure, c_double

# Read data from the text file
with open("circle_data.txt", "r") as f:
    lines = f.readlines()

# Extract the points, center, and radius
x1, y1 = map(float, lines[0].split(","))
x2, y2 = map(float, lines[1].split(","))
x3, y3 = map(float, lines[2].split(","))
center_x, center_y = map(float, lines[3].split(","))
radius = float(lines[4])
```

```
# Generate points for the circle
```

```
circle=circ_gen(np.array([center_x,center_y]),radius)
```

```
# Plot the circle and points
```

```
plt.plot(circle[0],circle[1], color='blue')
```

```
plt.scatter([x1, x2, x3], [y1, y2, y3], color='red', label='Points')
```

```
plt.scatter(center_x, center_y, color='green', label='Center')
```

```
# Label the points with coordinates
```

```
plt.text(x1, y1 - 0.2, f'({x1:.2f},{y1:.2f})', ha='center')
```

```
plt.text(x2, y2 - 0.2, f'({x2:.2f},{y2:.2f})', ha='center')
```

```
plt.text(x3, y3 - 0.2, f'({x3:.2f},{y3:.2f})', ha='center')
```

```
plt.text(center_x, center_y + 0.2, f'({center_x:.2f},{center_y:.2f})', ha='center')
```

```
plt.xlabel('x')  
plt.ylabel('y')  
plt.title('Circle and Points')  
plt.legend()  
plt.grid(True)  
plt.axis('equal') # Ensure x and y axes have the same scale  
plt.show()
```

# Plot of circle and points

