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EE24BTECH11006 - Arnav Mahishi

1) Let L be the line of intersection of planes $r \cdot (i - j + 2k) = 2$ and $r \cdot (2i + j - k) = 2$. If $P(\alpha, \beta, \gamma)$ is the foot of perpendicular on L from the point $(1, 2, 0)$, then the value of $35(\alpha + \beta + \gamma)$ is equal to

   a) 101 b) 119 c) 143 d) 134

2) Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to:

   a) 1862 b) 1842 c) 1852 d) 1872

3) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3 & \text{if } x > 0 \\ 3xe^x & \text{if } x \leq 0 \end{cases}$$

   Then f is an increasing function in the interval.

   a) $\left(-\frac{1}{2}, 2\right)$ b) $(0, 2)$ c) $\left(-1, \frac{3}{2}\right)$ d) $(-3, -1)$

4) Let $y = y(x)$ be the solution of the differential equation $\csc^2 x dy + 2dx = (1 + y\cos 2x)\csc^2 x dx$, with $y(\pi/4) = 0$. Then, the value of $(y(0) + 1)^2$ is equal to:

   a) $e^{\frac{1}{2}}$ b) $e^{-\frac{1}{2}}$ c) e^{-1} d) e

5) Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non-singular, is:

   a) $\frac{45}{162}$ b) $\frac{23}{81}$ c) $\frac{22}{81}$ d) $\frac{43}{162}$

6) Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\mathbf{a}| = 10$. Then a possible value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right] + \left[\vec{a}\ \vec{b}\ \vec{d}\right] + \left[\vec{a}\ \vec{c}\ \vec{d}\right]$ is equal to :

   a) -42 b) -40 c) -29 d) -38

7) If

$$\int_0^{100\pi} \frac{\sin^2 x}{e^{\frac{x}{\pi} - \left[\frac{x}{\pi}\right]}} dx = \frac{\alpha\pi^3}{1 + 4\pi^2}$$

   where $[x]$ is the greatest integer less than or equal to x, then the value of α is:

a) $200(1-e^{-1})$ b) $100(1-e)$ c) $50(e-1)$ d) $150(e^{-1}-1)$

8) Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is not true?

a) $\vec{a} \times \left(\left(\vec{b}+\vec{c}\right) \times \left(\vec{b}-\vec{c}\right)\right) = 0$

b) Projection of $\vec{a}$ on $\left(\vec{b} \times \vec{c}\right)$ is 2

c) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] + \left[\vec{c}\ \vec{a}\ \vec{b}\right] = 8$

d) $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$

9) The values of λ and μ such that the system of equations

$$x + y + z = 6,$$
$$3x + 5y + 5z = 26,$$
$$x + 2y + \lambda z = \mu$$

has no solution, are :

a) $\lambda = 3, \mu = 5$ b) $\lambda = 3, \mu \neq 10$ c) $\lambda \neq 2, \mu = 10$ d) $\lambda = 2, \mu \neq 10$

10) If the shortest distance between the straight lines $3(x-1) = 6(y-2) = 2(z-1)$ and $4(x-2) = 2(y-\lambda) = (z-3)$, $\lambda \in \mathbb{R}$ is $\frac{1}{\sqrt{38}}$ then the integral value of λ is equal to:

a) 3 b) 2 c) 5 d) -1

11) Which of the following Boolean expressions is not a tautology ?

a) $(p \Rightarrow q) \vee (\neg q \Rightarrow p)$

b) $(q \Rightarrow p) \vee (\neg q \Rightarrow p)$

c) $(p \Rightarrow \neg q) \vee (\neg q \Rightarrow p)$

d) $(\neg p \Rightarrow q) \vee (\neg q \Rightarrow p)$

12) Let $A = \left[a_{ij}\right]$ be a real matrix of order 3×3, such that $a_{i1}+a_{i2}+a_{i3} = 1$, for $i = 1, 2, 3$. Then, the sum of all the entries of the matrix A^3 is equal to:

a) 2 b) 1 c) 3 d) 9

13) Let $[x]$ denote the greatest integer less than or equal to x. Then, the values of $x \in \mathbb{R}$ satisfying the equation $[e^x] + 2 + [e^x + 1] - 3 = 0$ lie in the interval:

a) $[\,0, \frac{1}{e}\,)$ b) $[\,\log_e 2, \log_e 3\,)$ c) $[\,1, e\,)$ d) $[\,0, \log_e 2\,)$

14) Let the circle $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

a) $\frac{25}{9} < C < \frac{13}{3}$ b) $100 < C < 165$ c) $81 < C < 156$ d) $100 < C < 156$

15) Let n denote the number of solutions of the equation $z^2 + 3z = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to:

a) 1 b) $\frac{4}{3}$ c) $\frac{3}{2}$ d) 2