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EE24BTECH11006 - Arnav Mahishi

1) Let y_1 and y_2 be two linearly independent solutions of $y'' + (\sin x) y, 0 \leq x \leq 1$. Let $g(x) = W(y_1, y_2)(x)$ be the Wronkskian of y_1 and y_2.Then

   a) $g'>0$ on $[0, 1]$
   b) $g'<0$ on $[0, 1]$
   c) g' vanishes at only one point of $(0, 1)$
   d) g' vanishes at all points of $[0, 1]$

2) One particular solution of $y''' - y'' - y' + y = -e^x$ is constant multiple of

   a) xe^{-x}
   b) xe^x
   c) x^2e^{-x}
   d) x^2e^x

3) Let $a, b \in \mathbb{R}$. Let $y = (y_1, y_2)'$ be a solution of the system of equations $y_1 = y2, y_2' = ay_1 + by_2$. Every solution $y(x) \to 0$ as $x \to \infty$ if

   a) $a<0, b>0$
   b) $a<0, b>0$
   c) $a>0, b>0$
   d) $a>0, b<0$

4) Let G be a group of order 45. Let H be a 3-Sylow subgroup of G and K be a 5-Sylow subgroup of G. Then

   a) both H and K are normal in G
   b) H is normal in G but K is not normal in G
   c) H is not normal in G but K is normal in G
   d) both H and K are not normal in G

5) The ring $\mathbb{Z}\left[\sqrt{-11}\right]$ is

   a) a Euclidean Domain
   b) a Principal Ideal Domain, but not a Euclidean Domain
   c) a Unique Factorization Domain, but not a Principal Ideal Domain
   d) not a Unique Factorization Domain

6) Let R be a Principal Ideal Domain and a, b any two non-unit elements of R. Then the ideal generated by a and b is also generated by

   a) $a + b$
   b) ab
   c) $gcd(a, b)$
   d) $lcm(a, b)$

7) Consider the action of S_4, the symmetric group of order 4, on $\mathbb{Z}[x_1, x_2, x_3, x_4]$ given by $\sigma \cdot (x_1, x_2, x_3, x_4) = p(x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}, x_{\sigma(4)})$ for $\sigma \in S_4$. Let $HsubseteqS_4$ denote the cyclic subgroup generated by $(1, 4, 2, 3)$. Then the cardinality of the orbit $O_H(x_1x_3 + x_2x_4)$ of H on the polynomial $x_1x_3 + x_2x_4$ is

a) 1 b) 2 c) 3 d) 4

8) Let $f : l^2 \to \mathbb{R}$ be defined by $f(x_1, x_2, \cdots) = \sum_{n=1}^{\infty} \frac{x_n}{2^{\frac{n}{2}}}$ for $(x_1, x_2, \cdots) \in l^2$.Then $\|f\|$ is equal to

a) $\frac{1}{2}$ b) 1 c) 2 d) $\frac{1}{\sqrt{2}-1}$

9) Consider $\mathbb{R}^3$ with norm $\| \|_1$, and the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by the 3×3 matrix $\begin{pmatrix} 1 & 1 & 3 \\ 2 & 2 & 2 \\ 1 & 3 & -3 \end{pmatrix}$. Then the operator norm $\|T\|$ of T is equal to

a) 6 b) 7 c) 8 d) 42

10) Consider $\mathbb{R}^2$ with norm $\| \|_\infty$, and let $Y = \left\{(y_1, y_2) \in \mathbb{R}^2 : y_1 + y_2 = 0\right\}$. If $g : Y \to \mathbb{R}$ is defined by $g(y_1, y_2) = y2$ for $(y_1, y_2) \in Y$, then

a) g has no Hahn-Banach extension to $\mathbb{R}^2$
b) g has a unique Hahn-Banach extension to $\mathbb{R}^2$
c) every linear functional $f : \mathbb{R}^2 \to \mathbb{R}$ satisfying $f(-1, 1) = 1$ is a Hahn-Banach extension of g to $\mathbb{R}^2$
d) the functionals $f_1, f_2 : \mathbb{R}^2 \to \mathbb{R}$ given by $f_1(x_1, x_2) = x_2$ and $f_2(x_1, x_2) = -x_1$ are both Hahn-Banach extensions of g to $\mathbb{R}^2$

11) Let X be a Banach space and Y be a normed linear space. Consider a sequence (F_n) of bounded linear maps from X to Y such that for each fixed $x \in X$, the sequence $(F_n(x))$ is bounded in Y.Then

a) for each fixed $x \in X$, the sequence $(F_n(x))$ is convergent in Y
b) for each fixed $n \in N$, the set $\{F_n(x) : x \in X\}$ is bounded in Y
c) the sequence $(\|F_n\|)$ is bounded in $\mathbb{R}$
d) the sequence (F_n) is uniformly bounded on X

12) Let $H = L^2([0, \pi])$ with the usual inner product. For $n \in \mathbb{N}$, let $u_n(t) = \frac{\sqrt{2}}{\sqrt{\pi}} \sin nt, t \in [0, \pi]$, and $E = \{u_n : n \in \mathbb{N}\}$. Then

a) E is not a linearly dependent subset of H
b) E is a linearly independent subset of H, but is not a orthonormal subset of H.
c) E is a orthonormal subset of H, but is not an orthonormal basis for H.
d) E is an orthonormal basis for H.

13) Let $X = \mathbb{R}$ and let $\tau = \{U \subseteq X : X - U \text{ is finite}\} \cup \{\phi, X\}$. The sequence $1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}, \cdots$ in (X, τ)

a) converges to 0 and not to any other point of X
b) does not converge to 0
c) converges to each point of X
d) is not convergent to X

14) Let $E = \left\{(x, y) \in \mathbb{R}^2 : |x| \leq 1, |y| \leq 1\right\}$. Define $f : E \to \mathbb{R}$ by $f(x, y) = \frac{x+y}{1+x^2+y^2}$. Then the range of f is a

a) connected open set
b) connected closed set
c) bounded open set
d) closed and unbounded set

15) Let $X = \{1, 2, 3\}$ and $\tau = \{\phi, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$. The topological space (X, τ) is said to have the property P if for any two proper disjoint closed subsets Y and Z of X, there exist disjoint oper sets U, V such that $Y \subseteq U$ and $Z \subseteq V$. Then the topological space (X, τ)

a) is T_1 and satisfies P
b) is T_1 and does not satisfy P
c) is not T_1 and satisfies P
d) is T_1 and does not P

16) Which one of the follwing subsets of $\mathbb{R}$ (with the usual metric) is NOT complete?

a) $[1, 2] \cup [3, 4]$
b) $[0, \infty]$
c) $[0, 1)$
d) $\{0\} \cup \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$

17) Consider the function
$f(x) = \begin{cases} k(x - [x]), 0 \leq x < 2 \\ 0, \text{otherwise} \end{cases}$ where $\{x\}$ is the integral part of x. The value of k for which the above function is a probability density function of some random variable is

a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) 1
d) 2