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EE24BTECH11006 - Arnav Mahishi

1) The subspace $\mathbb{Q} \times [0,1]$ of $\mathbb{R}^2$ (with the usual topology) is

   a) dense in $\mathbb{R}^2$ b) connected c) separable d) compact

2) $\frac{\mathbb{Z}_2[x]}{<x^3+x^2+1>}$ is

   a) a field having 8 elements
   b) a field having 9 elements
   c) an infinite field
   d) NOT a field

3) The number of elements in a principal ideal domain can be

   a) 15 b) 25 c) 35 d) 36

4) Let F, G and H be pairwise independent events such that $P(F) = P(G) = P(H) = \frac{1}{3}$ and $(F \cap G \cap H) = \frac{1}{4}$. Then the probability that at least one event among F, G and H occurs is

   a) $\frac{11}{12}$ b) $\frac{7}{12}$ c) $\frac{5}{12}$ d) $\frac{3}{4}$

5) Let X be a random variable such that $E\left(X^2\right) = E(X) = 1$. Then $E\left(X^{100}\right) =$

   a) 0 b) 1 c) 2^{100} d) $2^{100} + 1$

6) For which of the following distributions, the weak law of large numbers does NOT hold?

   a) Normal b) Gamma c) Beta d) Cauchy

7) If $D \equiv \frac{d}{dx}$ then the value of $\frac{1}{xD+1}\left(x^{-1}\right)$

   a) $\log x$ b) $\frac{\log x}{x}$ c) $\frac{\log x}{x^2}$ d) $\frac{\log x}{x^3}$

8) The equation $\left(\alpha xy^3 + y\cos x\right)dx + \left(x^2y^2 + \beta\sin x\right)dy = 0$ is exact for

   a) $\alpha = \frac{3}{2}, \beta = 1$ b) $\alpha = 1, \beta = \frac{3}{2}$ c) $\alpha = \frac{2}{3}, \beta = 1$ d) $\alpha = 1, \beta = \frac{2}{3}$

9) If $A = \begin{pmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{pmatrix}$ then the trace of A^{102} is

a) 0 b) 1 c) 2 d) 3

10) Which of the following matrices are NOT diagonalizable?

a) $\begin{pmatrix} 11 & \\ 1 & 2 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ c) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

11) Let V be the column space of the matrix $A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -1 \end{pmatrix}$. Then the orthogonal projection of $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ on V is

a) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ c) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

12) Let $\sum_{n=-\infty}^{\infty} a_n (z+1)^n$ be the Laurent series expansion of $f(z) = \sin\left(\frac{z}{z+1}\right)$. Then $a_{-2} =$

a) 1 b) 0 c) $\cos(1)$ d) $\frac{-1}{2}\sin(1)$