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EE24BTECH11006 - Arnav Mahishi

1) Consider the following statements:
   I: $\log(|z|)$ is harmonic on $\mathbb{C}\backslash\{0\}$
   II:$\log(|z|)$ has a harmonic conjugate on $\mathbb{C}\backslash\{0\}$

   a) both I and II are true
   b) I is true but II is false
   c) I is false but II is true
   d) both I and II are false

2) Let G and H be defined by
   $G = \mathbb{C}\backslash\{z = x + iy \in \mathbb{C} : x \leq 0, y = 0\}$,
   $H = \mathbb{C}\backslash\{z = x + iy \in \mathbb{C} : x \in \mathbb{Z}, x \leq 0, y = 0\}$,
   Suppose $f : G \to \mathbb{C}$ and $g : H \to \mathbb{C}$ are analytical functions. Consider the following statements:
   I: $\int_\gamma f dz$ is independent of paths γ in G joining $-i$ and i
   II: $\int_\gamma g dz$ is independent of paths γ in H joining $-i$ and i

   a) both I and II are true
   b) I is true but II is false
   c) I is false but II is true
   d) both I and II are false

3) Let $f(z) = e^{\frac{1}{z}}, z \in \mathbb{C}\backslash\{0\}$ and let, for $n \in \mathbb{N}$,
   $R_n = \left\{z = x + iy \in \mathbb{C} : |x| < \frac{1}{n}, |y| < \frac{1}{n}\right\}\backslash\{0\}$.
   If for a subset of S of $\mathbb{C}$, $\overline{S}$ denotes the closure of S in $\mathbb{C}$, then

   a) $\overline{f(R_{n+1})} \neq \overline{f(R_n)}$
   b) $\overline{f(R_n)}\backslash\overline{f(R_{n+1})} = \overline{f(R_n\backslash R_{n+1})}$
   c) $\overline{f\left(\cap_{n=1}^{\infty} R_n\right)} = \cap_{n=1}^{\infty}\overline{f(R_n)}$
   d) $\overline{f(R_n)} = \overline{f(R_{n+1})}$

4) Suppose that $U = \mathbb{R}^2\backslash\left\{(x, y) \in \mathbb{R}^2 : x, y \in \mathbb{Q}\right\}, V = \mathbb{R}^2\backslash\left\{(x, y) \in \mathbb{R}^2 : x>0, y = \frac{1}{x}\right\}$. Then with repsect to the Euclidean metric on $\mathbb{R}^2$,

   a) both U and V are disconnected
   b) U is disconnected but V is connected
   c) U is connected but V is disconnected
   d) both U and V are connected

5) If $(D1)$ and $(D2)$ denote the dual probems of the linear programming problems $(P1)$ and $(P2)$, respectively, where
   $(P1)$: minimize $x_1 - 2x_2$, subject to $-x_1 + x_2 = 10, x_1, x_2 \geq 0$
   $(P2)$: minimize $x_1 - 2x_2$, subject to $-x_1 + x_2 = 10, x_1 - x_2 = 10, x_1, x_2 \geq 0$, then
   a) both $(D1)$ and $(D2)$ are infeasible
   b) $(P2)$ is infeasible and $(D2)$ is feasible
   c) $(P2)$ is infeasible and $(D2)$ is feasible but unbounded

d) $(P1)$ is feasible but unbounded and $(D1)$ is feasible

6) If $(4,0)$ and $\left(0,-\frac{1}{2}\right)$ are the critical points of the function $f(x,y) = 5-(\alpha+\beta)x^2 + \beta y^2 + (\alpha+1)y^3 + x^3$, where $\alpha,\beta \in \mathbb{R}$, then

a) $\left(4,-\frac{1}{2}\right)$ is point of local maxima of f
b) $\left(4,-\frac{1}{2}\right)$ is a saddle point of f
c) $\alpha = 4, \beta = 2$
d) $\left(4,-\frac{1}{2}\right)$ is a point of local minima of f

7) Consider the iterative scheme $x_n = \frac{x_{n-1}}{2} + \frac{3}{x_{n-1}}, n \geq 1$ with initial point $x_0>0$. Then the sequence $\{x_n\}$

a) converges only if $x_0>1$
b) converges only if $x_0>3$
c) converges for any x_0
d) does not converge for any x_0

8) Let $C[0,1]$ denote the space of all real-valued continous functions on $[0,1]$ equipped with the supremum norm $||.||_\infty$. Let $T: C[0,1] \to C[0,1]$ be the linear operator defined by $T(f)(x) = \int_0^x e^{-y} f(y)\,dy$ Then

a) $||T|| = 1$
b) $I-T$ is not invertible
c) T is surjective
d) $||I+T|| = 1 + ||T||$

9) Suppose that M is a 5×5 matrix with real entries and $p(x) = det(xI-M)$. Then
a) $p(0) = det(M)$
b) every eigen value of M is real if $p(1)+p(2) = 0 = p(2)+p(3)$
c) M^{-1} is necessarily a polynomial in M in degree 4 if M is invertible
d) M is not invertible if $M^2 - 2M = 0$

10) Let $C[0,1]$ denote the space of all real-valued continous functions on $[0,1]$ equipped with the supremum norm $||.||_\infty$. Let $f \in \mathbb{C}[0,1]$ be such that $|f(x)-f(y)| \leq M|x-y|$, for all $x,y \in [0,1]$ and for some $M>0$. For $n \in \mathbb{N}$, let $f_n(x) = f\left(x^{1+\frac{1}{n}}\right)$. If $S = \{f_n : n \in \mathbb{N}\}$, then

a) the closure of S is compact
b) S is closed and bounded
c) S is bounded but not totally bounded
d) S is compact

11) Let $K: \mathbb{R}\times(0,\infty) \to \mathbb{R}$ be a function such that the solution of the initial value problem $\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta x^2}, u(x,0) = f(x), x \in R, t>0$, is given by $u(x,t) = \int_{\mathbb{R}} K(x-y,t) f(y)\,dy$ for all bounded continous functions f, Then the value of $\int_{\mathbb{R}} K(x,t)\,dx$ is ____________

12) The number of cyclic subgroups of the quaternion group $Q_8 = \langle a,b | a^4 = 1, a^2 = b^2, ba = a^3 b\rangle$ is ____________

13) The number of elements of order 3 in the symmetric group S_6 is ____________