

7-7.3-5

EE24BTECH11006 - Arnav Mahishi

Question: If a circle passes through the point $(0,0)$, $(a,0)$, and $(0,b)$ then find the coordinates of its centre.

input	value
x_1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
x_2	$\begin{pmatrix} a \\ 0 \end{pmatrix}$
x_3	$\begin{pmatrix} 0 \\ b \end{pmatrix}$

TABLE 0: Input Parameters

Solution:

$$\text{Given pts } x_1, x_2, x_3 \text{ on circle: } \begin{pmatrix} 2x_1 & 2x_2 & 2x_3 \\ 1 & 1 & 1 \end{pmatrix}^T \begin{pmatrix} u \\ f \end{pmatrix} = - \begin{pmatrix} \|x_1\|^2 \\ \|x_2\|^2 \\ \|x_3\|^2 \end{pmatrix} \quad (0.1)$$

$$\Rightarrow \begin{pmatrix} 2x_1^T & 1 \\ 2x_2^T & 1 \\ 2x_3^T & 1 \end{pmatrix} \begin{pmatrix} u \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ -a^2 \\ -b^2 \end{pmatrix} \quad (0.2)$$

$$\Rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 2a & 0 & 1 \\ 0 & 2b & 1 \end{pmatrix} \begin{pmatrix} u \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ -a^2 \\ -b^2 \end{pmatrix} \quad (0.3)$$

The augmented matrix for this

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 2a & 0 & 1 & -a^2 \\ 0 & 2b & 1 & -b^2 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 2a & 0 & 0 & -a^2 \\ 0 & 2b & 1 & -b^2 \end{pmatrix} \quad (0.4)$$

$$\Rightarrow \xleftrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 2a & 0 & 0 & -a^2 \\ 0 & 2b & 0 & -b^2 \end{pmatrix} \quad (0.5)$$

Let $u = \begin{pmatrix} -x \\ -y \end{pmatrix}$ then

$$(2a \ 0)u = -a^2 \text{ and } (0 \ 2b)u = -b^2 \quad (0.6)$$

$$(0.7)$$

$$\Rightarrow \begin{pmatrix} 2a & 0 \end{pmatrix} \begin{pmatrix} -x \\ -y \end{pmatrix} = -a^2 \quad (0.8)$$

$$\Rightarrow -2ax = -a^2 \Rightarrow x = \frac{a}{2} \quad (0.9)$$

$$\begin{pmatrix} 0 & 2b \end{pmatrix} \begin{pmatrix} -x \\ -y \end{pmatrix} = -b^2 \quad (0.10)$$

$$\Rightarrow -2by = -b^2 \Rightarrow y = \frac{b}{2} \quad (0.11)$$

$$\Rightarrow u = \begin{pmatrix} -x \\ -y \end{pmatrix} = -\begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \end{pmatrix} \quad (0.12)$$

$$\therefore c = -u = \begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \end{pmatrix} \quad (0.13)$$

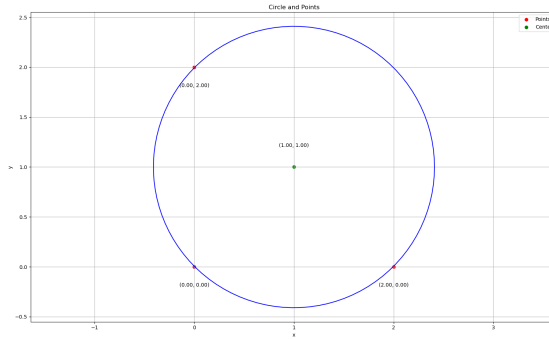


Fig. 0.1: Plot of circle