Matgeo-7-7.3-5

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Problem Statement

If a circle passes through the points $(0,0)$,$(a,0)$,and $(0,b)$ then find the coordinates of its centre.

Input Parameters

input	**value**
x_1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
x_2	$\begin{pmatrix} a \\ 0 \end{pmatrix}$
x_3	$\begin{pmatrix} 0 \\ b \end{pmatrix}$

Table: Input Parameters

Equation relating centre with points

Given pts x_1,x_2,x_3 on circle:

$$\begin{pmatrix} 2x_1 & 2x_2 & 2x_3 \\ 1 & 1 & 1 \end{pmatrix}^T \begin{pmatrix} u \\ f \end{pmatrix} = - \begin{pmatrix} \|x_1\|^2 \\ \|x_2\|^2 \\ \|x_3\|^2 \end{pmatrix} \tag{3.1}$$

$$\implies \begin{pmatrix} 2x_1^T & 1 \\ 2x_2^T & 1 \\ 2x_3^T & 1 \end{pmatrix} \begin{pmatrix} u \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ -a^2 \\ -b^2 \end{pmatrix} \tag{3.2}$$

$$\implies \begin{pmatrix} 0 & 0 & 1 \\ 2a & 0 & 1 \\ 0 & 2b & 1 \end{pmatrix} \begin{pmatrix} u \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ -a^2 \\ -b^2 \end{pmatrix} \tag{3.3}$$

Row Reduction

The augemented matrix for this

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 2a & 0 & 1 & -a^2 \\ 0 & 2b & 1 & -b^2 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 2a & 0 & 0 & -a^2 \\ 0 & 2b & 1 & -b^2 \end{pmatrix} \tag{3.4}$$

$$\Longrightarrow \xleftrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 2a & 0 & 0 & -a^2 \\ 0 & 2b & 0 & -b^2 \end{pmatrix} \tag{3.5}$$

Finding Centre

Thus Let $u = \begin{pmatrix} -x \\ -y \end{pmatrix}$ then

$$\begin{pmatrix} 2a & 0 \end{pmatrix} u = -a^2 \text{ and } \begin{pmatrix} 0 & 2b \end{pmatrix} u = -b^2 \tag{3.6}$$

$$\implies \begin{pmatrix} 2a & 0 \end{pmatrix} \begin{pmatrix} -x \\ -y \end{pmatrix} = -a^2 \tag{3.7}$$

$$\implies -2ax = -a^2 \implies x = \frac{a}{2} \tag{3.8}$$

$$\begin{pmatrix} 0 & 2b \end{pmatrix} \begin{pmatrix} -x \\ -y \end{pmatrix} = -b^2 \tag{3.9}$$

$$\implies -2by = -b^2 \implies y = \frac{b}{2} \tag{3.10}$$

$$\implies u = \begin{pmatrix} -x \\ -y \end{pmatrix} = -\begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \end{pmatrix} \implies c = -u = \begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \end{pmatrix} \tag{3.11}$$

C Code

```
#include <stdio.h>
#include <math.h>

#define MAX_POINTS 3

struct Point {
  double x, y;
};

void printMatrix(const struct Matrix *matrix) {
  for (int i = 0; i < MAX_POINTS; i++) {
    for (int j = 0; j < MAX_POINTS; j++) {
      printf("%.2f␣", matrix->data[i][j]);
    }
    printf("\n");
  }
}
```

```
void rowReduction(struct Matrix *A) {
  for (int i = 0; i < MAX_POINTS - 1; i++) {
    if (fabs(A->data[i][i]) < 1e-6) {
      printf("Matrix␣may␣be␣singular.␣Results␣might␣be␣inaccurate.\n");
      return;
    }
    for (int j = i + 1; j < MAX_POINTS; j++) {
      double factor = A->data[j][i] / A->data[i][i];
      for (int k = 0; k < MAX_POINTS; k++) {
        A->data[j][k] -= factor * A->data[i][k];
      }
    }
  }
}
```

```
void findCenterAndRadius(const struct Point *points, struct Point *
    center, double *radius) {
  struct Matrix A = {
    {{2 * (points[1].x - points[0].x), 2 * (points[1].y - points[0].y),
        points[1].x * points[1].x - points[0].x * points[0].x + points[1].y
        * points[1].y - points[0].y * points[0].y}},
    {{2 * (points[2].x - points[0].x), 2 * (points[2].y - points[0].y),
        points[2].x * points[2].x - points[0].x * points[0].x + points[2].y
        * points[2].y - points[0].y * points[0].y}}
  };
  rowReduction(&A);
  center->x = -A.data[0][2] / (2 * A.data[0][0]);
  center->y = (-A.data[1][2] - 2 * A.data[1][0] * center->x) / (2 * A
      .data[1][1]);
  *radius = sqrt((points[0].x - center->x) * (points[0].x - center->x)
      + (points[0].y - center->y) * (points[0].y - center->y));
}
```

```
int main() {
  struct Point points[MAX_POINTS] = {{0, 0}, {2, 0}, {0, 2}};
  struct Point center;
  double radius;

  findCenterAndRadius(points, &center, &radius);

  printf(”Center:␣(%.2f,␣%.2f)\n”, center.x, center.y);
  printf(”Radius:␣%.2f\n”, radius);

  return 0;
}
}
```

Python Code

```
#Code by GVV Sharma
#September 12, 2023
#Revised July 21, 2024
#released under GNU GPL
#Point Vectors
import sys #for path to external scripts
sys.path.insert(0, '/home/arnav/matgeo/codes/CoordGeo’) #path to my scripts
import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
from mpl_toolkits.mplot3d import Axes3D
```

```
#local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen
import ctypes
from ctypes import Structure, c_double
# Read data from the text file
with open(”circle_data.txt”, ”r”) as f:
     lines = f.readlines()

# Extract the points, center, and radius
x1, y1 = map(float, lines[0].split(”,”))
x2, y2 = map(float, lines[1].split(”,”))
x3, y3 = map(float, lines[2].split(”,”))
center_x, center_y = map(float, lines[3].split(”,”))
radius = float(lines[4])
```

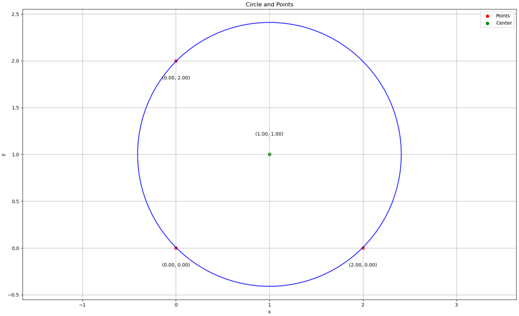
```
# Generate points for the circle
theta = np.linspace(0, 2 * np.pi, 100)
x_circle = center_x + radius * np.cos(theta)
y_circle = center_y + radius * np.sin(theta)

# Plot the circle and points
plt.plot(x_circle, y_circle, color='blue')
plt.scatter([x1, x2, x3], [y1, y2, y3], color='red', label='Points')
plt.scatter(center_x, center_y, color='green', label='Center')

# Label the points with coordinates
plt.text(x1, y1 - 0.2, f"({x1:.2f},␣{y1:.2f})", ha='center')
plt.text(x2, y2 - 0.2, f"({x2:.2f},␣{y2:.2f})", ha='center')
plt.text(x3, y3 - 0.2, f"({x3:.2f},␣{y3:.2f})", ha='center')
plt.text(center_x, center_y + 0.2, f"({center_x:.2f},␣{center_y:.2f})", ha='
     center')
```

```
plt.xlabel('x')
plt.ylabel('y')
plt.title('Circle␣and␣Points')
plt.legend()
plt.grid(True)
plt.axis('equal') # Ensure x and y axes have the same scale
plt.show()
```

Plot of circle and points



Figure