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Mathematical Modeling

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Problem Statement

A 200 ft x 400 ft field needs to be watered by circular spigots which move down the length of the field. The spigots need to be arranged in such a way that the amount of water each plant receives is roughly equal, while minimizing the amount of spigots and pipe used.

Model

Assumptions

We assume the following regarding this situation:

- Our sprinkler system is consistently moving at the same speed.
- The ground is flat and the apparatus will not be bouncing.
- Water pressure stays constant so the radius of sprinkler flow is the same.
- The sprinkler heads will all operate without mechanical issues and stay functional.
- Plants should not be watered by more than three spigots.
- Every point within the sprinkler's radius receives the same amount of water per unit of time.
- The plants are at a height that does not interfere with the operation of the sprinklers.

Parameters

The parameters of our problem are as follows:

- The field is 200x400 feet.
- We have 220 feet of pipe, with a maximum of 19 sprinkler heads to be distributed across it.
- Each sprinkler head has a radius of 22 ft.

Solution

Our solution is to place the sprinkler heads approximately 34.6 feet from each other. Therefore, we will only need 6 sprinkler heads to cover the width of the field. To ensure a fair distribution of water flow across all crops, we generated four functions: *q*, *w*, *l, and r*. These functions represent the trajectory of four sprinkler flows. They were defined by rearranging the equation for a circle and using the variable *k* to represent the space between each sprinkler on the pipe.

$$q(x) = \{-22 < x < 22: 2\sqrt{(22^2 - x^2, 0}\}$$

$$w(x) = \{-22+k < x < 22+k: 2\sqrt{(22^2 - (x - k)^2, 0}\}$$

$$l(x) = \{-22+2k < x < 22+2k: 2\sqrt{(22^2 - (x - 2k)^2, 0}\}$$

$$r(x) = \{-22+3k < x < 22+3k: 2\sqrt{(22^2 - (x - 3k)^2, 0}\}$$

Our strategy was to create a new function, j(x), that represents the sum of the sprinkler functions.

$$j(x) = q(x) + w(x) + l(x) + r(x)$$

Once this was achieved, we aimed to formulate a function that represents the average of j(x), yielding the average coverage of water for each spot of soil. This average function was named a(p) (Fig.1).

$$a(p) = \frac{\left(\int_0^{2k} j(x)\, dx\right)}{2k}$$

Figure 1. The average of a function can be found by taking the integral of the function over the interval and dividing this by the length of the interval.

Afterwards, we utilized the slider feature of Desmos to play around with the value of *k*, aiming to match the average function with the peak of the sprinkler functions, which is 44. A *k* value of 34.6 yields the closest match to the line y = 44 (Fig.2).

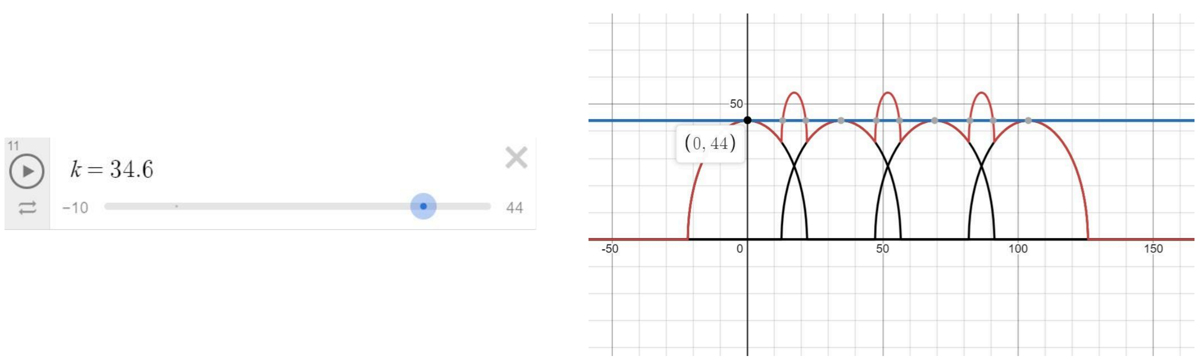

Fig 2. Desmos was used to find the value of *k* that matches the average of the sprinkler coverage to the peak.

https://www.desmos.com/calculator/g43ybfaj2c

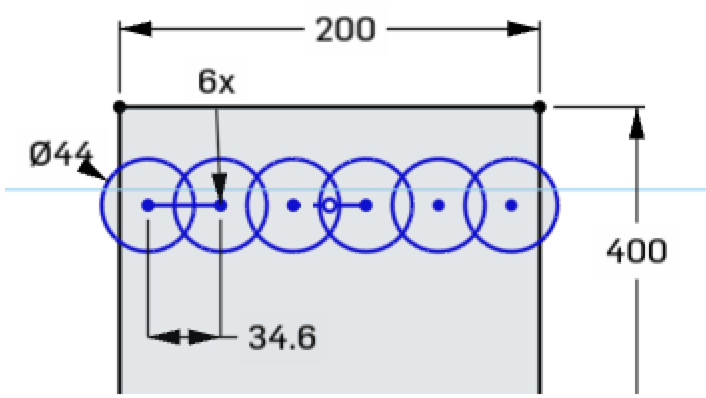

Figure 3. Spigot Model in OnShape

https://www.desmos.com/calculator/j6pil270zf

Results

The results of our model show that we will need six sprinkler heads, with a gap of approximately 34.6 feet to ensure the uniform distribution of water across the width of the field.

Analysis

Appropriateness & Robustness

This solution uses only 6 sprinklers, saving cost and minimizing land used by the tracks on the sprinkler system. Still, it ensures that each spot of soil gets the same amount of water on average, meaning that with some propagation of water from the richer areas to the slightly poorer ones, each plant should be similarly watered. Our solution is based on the average of the sum of the sprinkler functions. Therefore, any modifications to the sprinkler system, such as the radius of water trajectory, will lead to a skewed model and distribution of water.

References

"Average Values and Lengths of Functions - Calculus 2." *Varsitytutors.com*, 2024, www.varsitytutors.com/calculus_2-help/average-values-and-lengths-of-functions#:~:text=Correct%20answer%3A. Accessed 13 Sept. 2024.