

## Solved: The Case of the Moving Expectation.

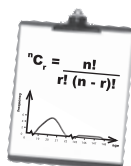
*How can the contestant figure out the new expectation in record time?*

The contestant looks around in panic for a brief moment and then relaxes. The change in values isn't such a big problem after all.

The contestant has already spent time calculating the expectation of the original values of all the boxes, and this has given him an idea of how much money is available for him to win.

The producer has told him that the new prizes are ten dollars less than twice the original prizes. In other words, this is a linear transform. If  $X$  represents the original prize money and  $Y$  the new, the values are transformed using  $Y = 2X - 10$ .

The contestant finds  $E(Y)$  using  $E(2X - 10) = 2E(X) - 10$ . This means that all he has to do to find the new expectation is double his original expectation and subtract 10.



### Vital Statistics Linear Transforms

If you have a variable  $X$  and numbers  $a$  and  $b$ , then:

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2\text{Var}(X)$$

## BULLET POINTS

- **Probability distributions** describe the probability of all possible outcomes of a given variable.
- The **expectation** is the expected average long-term outcome. It's represented as either  $E(X)$  or  $\mu$ , and is calculated using  $E(X) = \sum xP(X=x)$ .
- The **expectation of a function of  $X$**  is given by  $E(f(X)) = \sum f(x)P(X=x)$
- The **variance of a probability distribution** is given by  $\text{Var}(X) = E(X - \mu)^2$
- The **standard deviation of a probability distribution** is given by  $\sigma = \sqrt{\text{Var}(X)}$
- **Linear transforms** are when a variable  $X$  is transformed into  $aX + b$ , where  $a$  and  $b$  are constants. The expectation and variance are given by:
  - $E(aX + b) = aE(X) + b$
  - $\text{Var}(aX + b) = a^2\text{Var}(X)$