Sharpen your pencil Solution

What's the expectation and variance of the new probability distribution? How do these values compare to the previous payout distribution's expectation of -0.77 and variance of 2.6971?

y	-2	23	48	73	98
P(Y = y)	0.977	0.008	0.008	0.006	0.001

$$E(Y) = (-2) \times 0.977 + 23 \times 0.008 + 48 \times 0.008 + 73 \times 0.006 + 98 \times 0.001$$

$$= -1.954 + 0.184 + 0.384 + 0.438 + 0.098$$

$$= -0.85$$

$$Var(Y) = E(Y - \mu)^2$$

$$= \sum(y - \mu)^2 P(Y=y)$$

$$= (-2+0.85)^2 \times 0.977 + (23+0.85)^2 \times 0.008 + (48+0.85)^2 \times 0.008 + (73+0.85)^2 \times 0.006 + (98+0.85)^2 \times 0.001$$

$$= (-1.15)^2 \times 0.977 + (23.85)^2 \times 0.008 + (48.85)^2 \times 0.008 + (73.85)^2 \times 0.006 + (98.85)^2 \times 0.001$$

$$= 1.3225 \times 0.977 + 568.8225 \times 0.008 + 2386.3225 \times 0.008 + 5453.8225 \times 0.006 + 9771.3225 \times 0.001$$

$$= 1.2920825 + 4.55058 + 19.09058 + 32.722935 + 9.7713225$$

$$= 67.4275$$

The expectation is slightly lower, so in the long term, we can expect to lose $0.85 each game. The variance is much larger. This means that we stand to lose more money in the long term on this machine, but there's less certainty.