
Sam likes to eat out at two restaurants. Restaurant A is generally more expensive than restaurant B, but the food quality is generally much better.

Below you'll find two probability distributions detailing how much Sam tends to spend at each restaurant. As a general rule, what would you say is the difference in price between the two restaurants? What's the variance of this?

**Restaurant A:**

| **x** | **20** | **30** | **40** | **45** |
|---|---|---|---|---|
| **P(X = x)** | 0.3 | 0.4 | 0.2 | 0.1 |

**Restaurant B:**

| **y** | **10** | **15** | **18** |
|---|---|---|---|
| **P(Y = y)** | 0.2 | 0.6 | 0.2 |

Let's start by finding the expectation and variance of X and Y.

$$E(X) = 20\times0.3 + 30\times0.4 + 40\times0.2 + 45\times0.1$$
$$= 6 + 12 + 8 + 4.5$$
$$= 30.5$$

$$Var(X) = (20-30.5)^2\times0.3 + (30-30.5)^2\times0.4 + (40-30.5)^2\times0.2 + (45-30.5)^2\times0.1$$
$$= (-10.5)^2\times0.3 + (-0.5)^2\times0.4 + 9.5^2\times0.2 + 14.5^2\times0.1$$
$$= 110.25\times0.3 + 0.25\times0.4 + 90.25\times0.2 + 210.25\times0.1$$
$$= 33.075 + 0.1 + 18.05 + 21.025$$
$$= 72.25$$

$$E(Y) = 10\times0.2 + 15\times0.6 + 18\times0.2$$
$$= 2 + 9 + 3.6$$
$$= 14.6$$

$$Var(Y) = (10-14.6)^2\times0.2 + (15-14.6)^2\times0.6 + (18-14.6)^2\times0.2$$
$$= (-4.6)^2\times0.2 + 0.4^2\times0.6 + 3.4^2\times0.2$$
$$= 21.16\times0.2 + 0.16\times0.6 + 11.56\times0.2$$
$$= 4.232 + 0.096 + 2.312$$
$$= 6.64$$

The difference between X and Y is modeled by X – Y.

$$E(X - Y) = E(X) - E(Y)$$
$$= 30.5 - 14.6$$
$$= 15.9$$

$$Var(X - Y) = Var(X) + Var(Y)$$
$$= 72.25 + 6.64$$
$$= 78.89$$