## Exercise Solution

The local diner has started selling fortune cookies at \$0.50 per cookie. Hidden within each cookie is a secret message. Most messages predict a good future for the buyer, but others offer money off at the diner. The probability of getting \$2 off is 0.1, the probability of getting \$5 off is 0.07, and the probability of getting \$10 off is 0.03.

If X is the net gain, what's the probability distribution of X? What are the values of E(X) and Var(X)?

Here's the probability distribution of X:

| x | −0.5 | 1.5 | 4.5 | 9.5 |
|---|---|---|---|---|
| P(X = x) | 0.8 | 0.1 | 0.07 | 0.03 |

$$E(X) = (-0.5)\times0.8 + 1.5\times0.1 + 4.5\times0.07 + 9.5\times0.03$$
$$= -0.4 + 0.15 + 0.315 + 0.285$$
$$= 0.35$$

$$Var(X) = E(X - \mu)^2$$
$$= \sum(x - \mu)^2 P(X=x)$$
$$= (-0.5-0.35)^2\times0.8 + (1.5-0.35)^2\times0.1 + (4.5-0.35)^2\times0.07 + (9.5-0.35)^2\times0.03$$
$$= (-0.85)^2\times0.8 + (1.15)^2\times0.1 + (4.15)^2\times0.07 + (9.15)^2\times0.03$$
$$= 0.7225\times0.8 + 1.3225\times0.1 + 17.2225\times0.07 + 83.7225\times0.03$$
$$= 0.578 + 0.13225 + 1.205575 + 2.511675$$
$$= 4.4275$$

The diner decides to put the price of the cookies up to \$1. What are the new expectation and variance?

The diner puts the price of the cookies up by \$0.50, which means that the new net gains are modelled by X − 0.5

$$E(X - 0.5) = E(X) - 0.5$$
$$= 0.35 - 0.5$$
$$= -0.15$$

$$Var(X - 0.5) = Var(X)$$
$$= 4.4275$$