ES221 :Mechanics of Solids

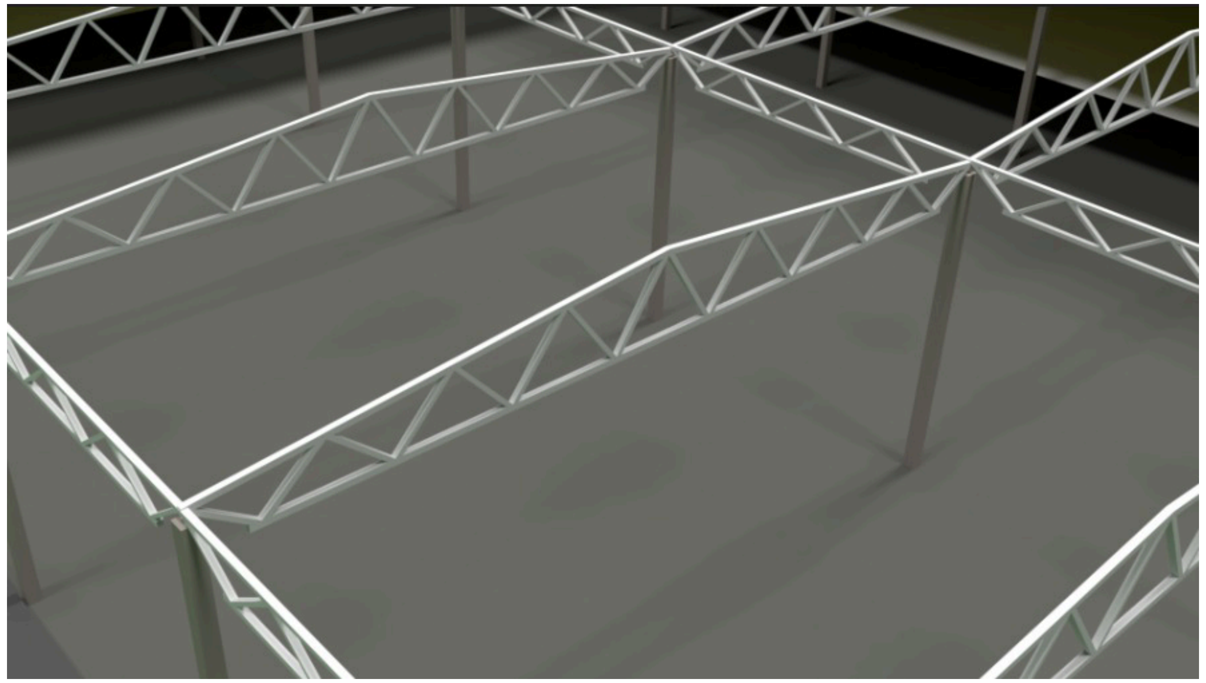
Project : Modeling of Internal Forces and Deflections in Roof Trusses Using FEM

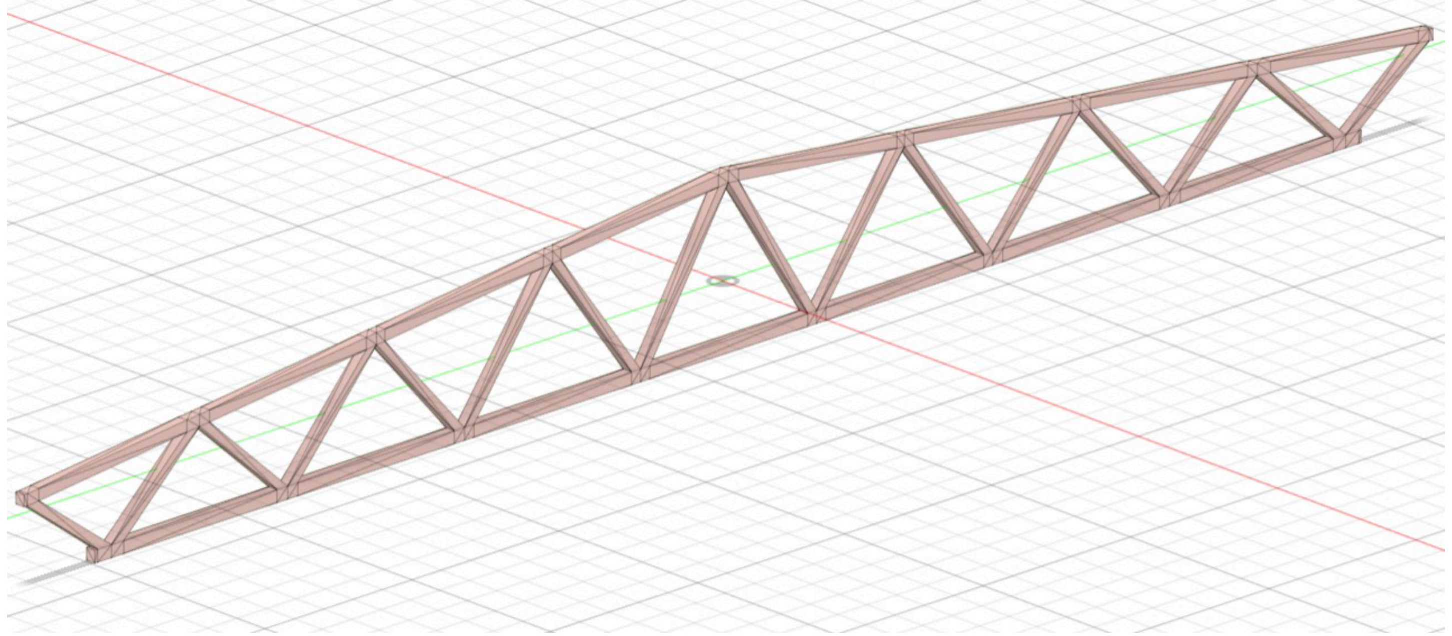
Prof. Harmeet Singh

Kavya Shah (23110163)
Apoorv Rane (23110036)
Arnav Gogate (23110043)
Goraksh Bendale (23110118)

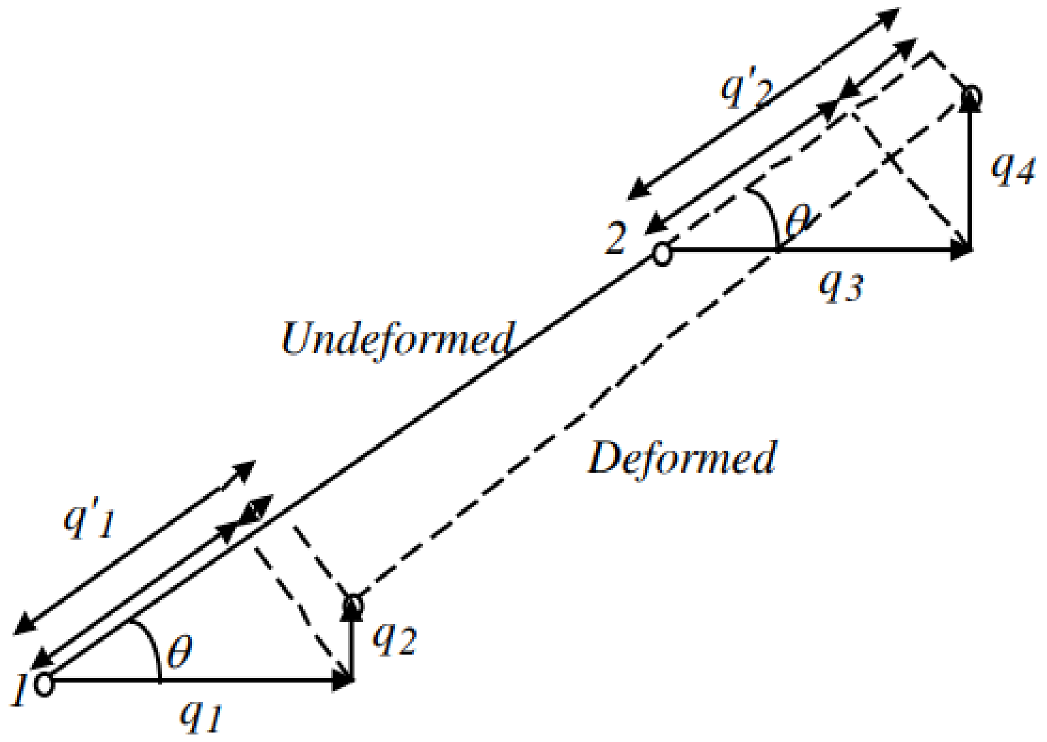
Project Objectives

- Perform 2D truss analysis using FEM in Python
- Compute nodal displacements and axial stresses
- Validate results with ANSYS Workbench
- Understand structural behaviour under static loads





FEM Background



$$\boldsymbol{q}' = \begin{bmatrix} q_1' \\ q_2' \end{bmatrix}, \quad \boldsymbol{k}' = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Global displacements:

$$\boldsymbol{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Transformation matrix:

$$\boldsymbol{M} = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}, \quad \text{with } c = \cos\theta, \quad s = \sin\theta$$

So final global stiffness matrix for a truss element is:

$$\boldsymbol{k} = \frac{AE}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

Solve this equation to get the unknown displacements

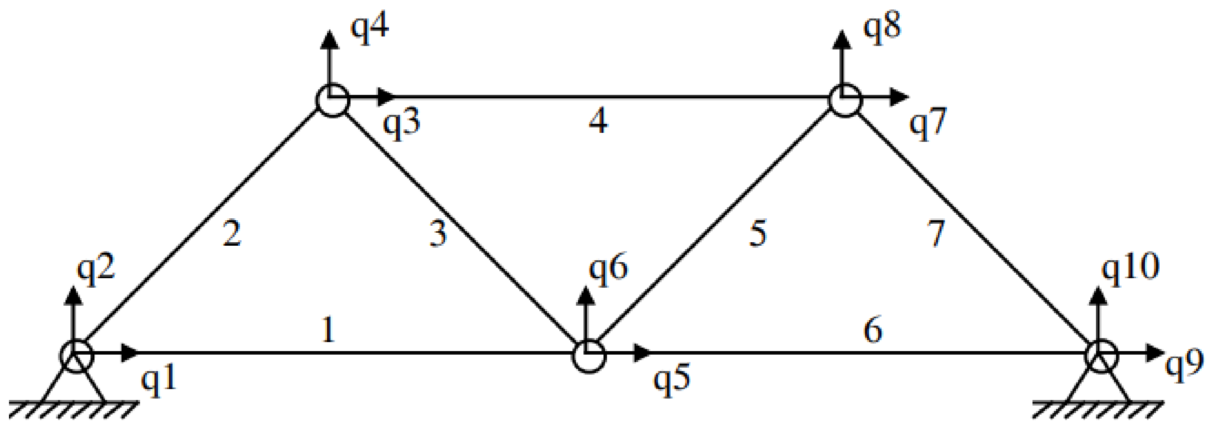
$$\boldsymbol{KQ} = \boldsymbol{F}$$

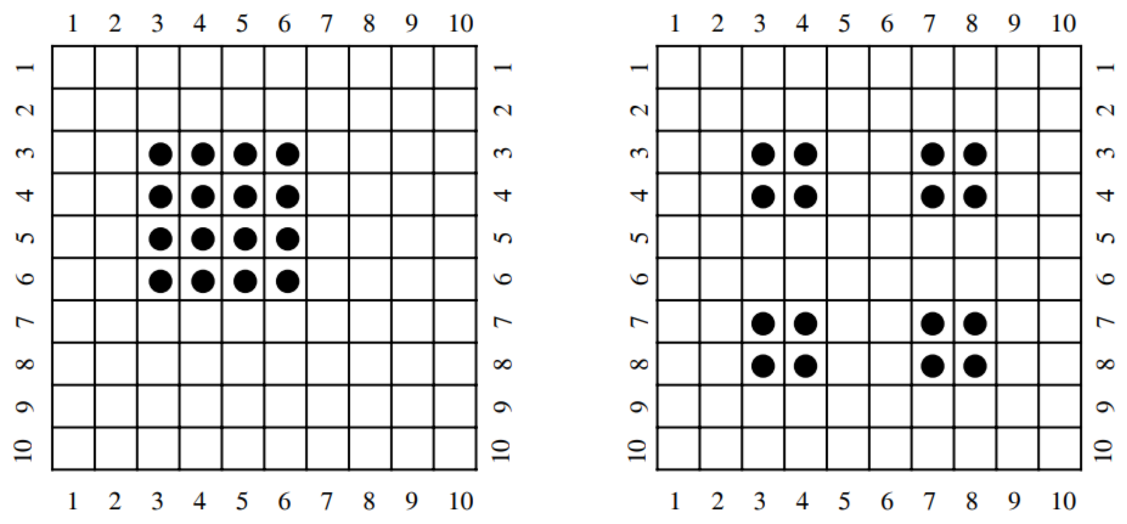
For stresses:

$$\sigma = \frac{E}{L}[-1 \quad 1]\,\boldsymbol{q}' = \frac{E}{L}[-1 \quad 1]\,\boldsymbol{Mq}$$

Methodology

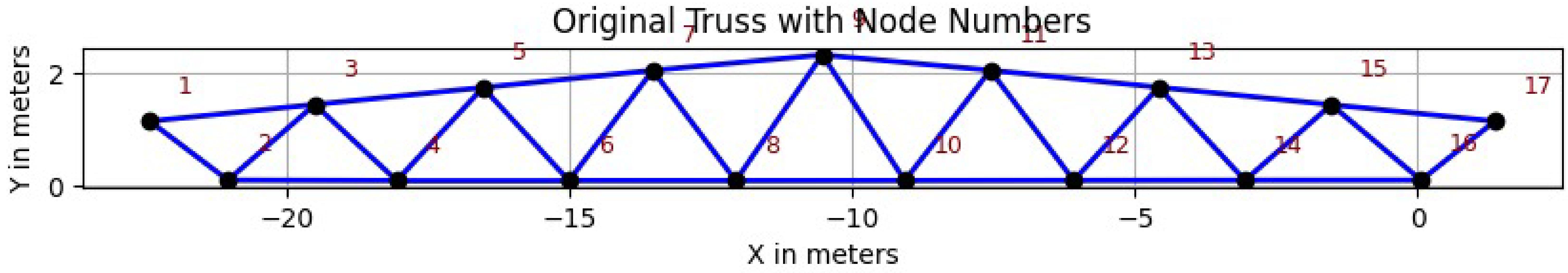
- Compute element stiffness matrices
- Assemble global stiffness matrix
- Apply boundary conditions
- Input external forces
- Solve for displacements
- Compute stresses and internal forces





Contribution of element 3 to the system stiffness matrix

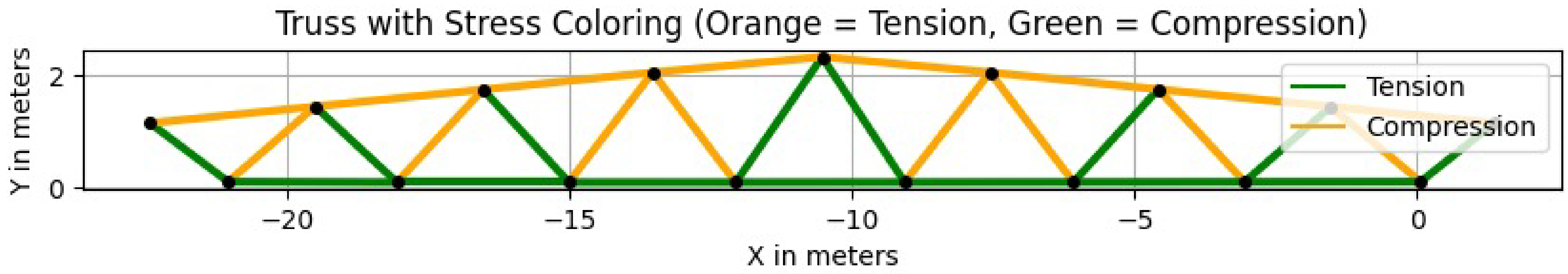
Contribution of element 4 to the system stiffness matrix

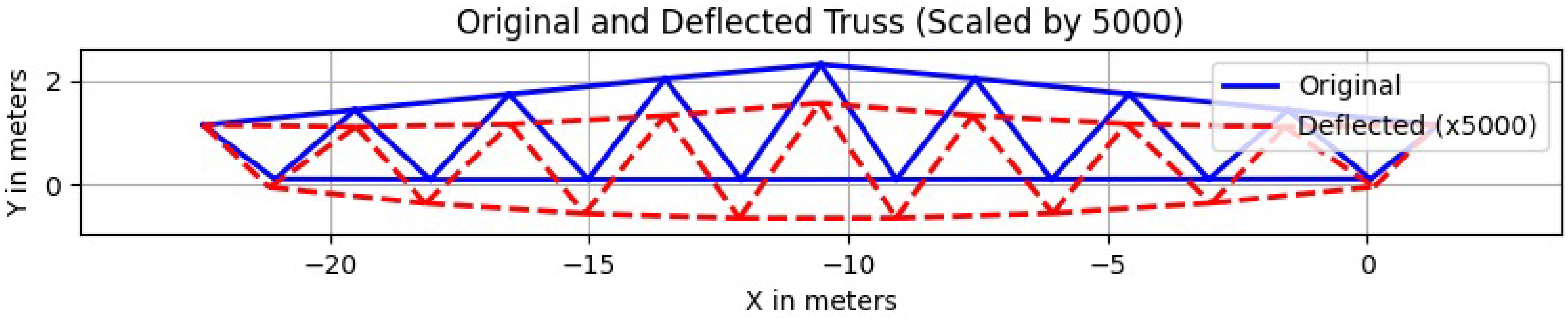


Results

Node	Force (in N)
3	1000
5	2000
7	2000
9	2000
11	2000
13	1000
15	2000

Youngs Modulus: 200E9 GPa
Cross Section Area: 0.05 m^2
Boundary Conditions : Node 1 (Fixed Support)
Node 17 (Roller Support)



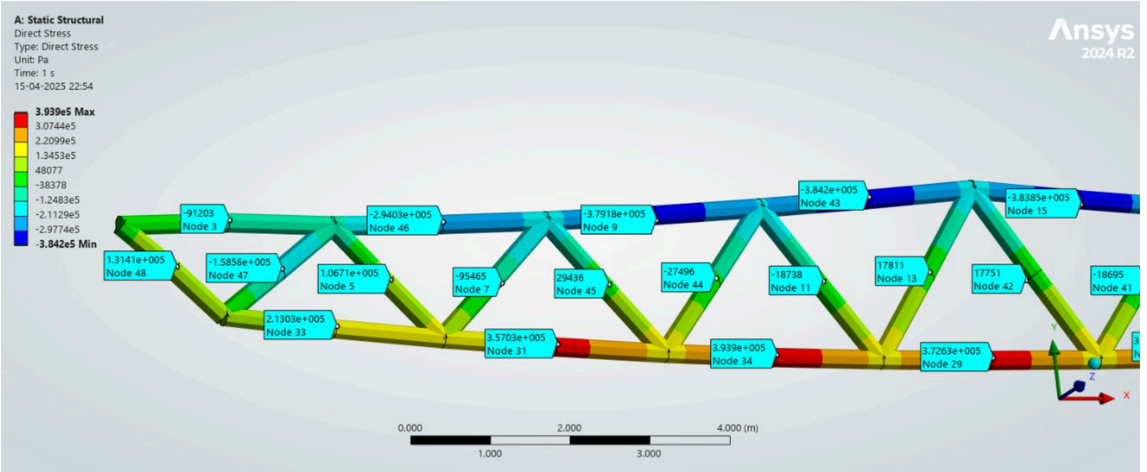


Results

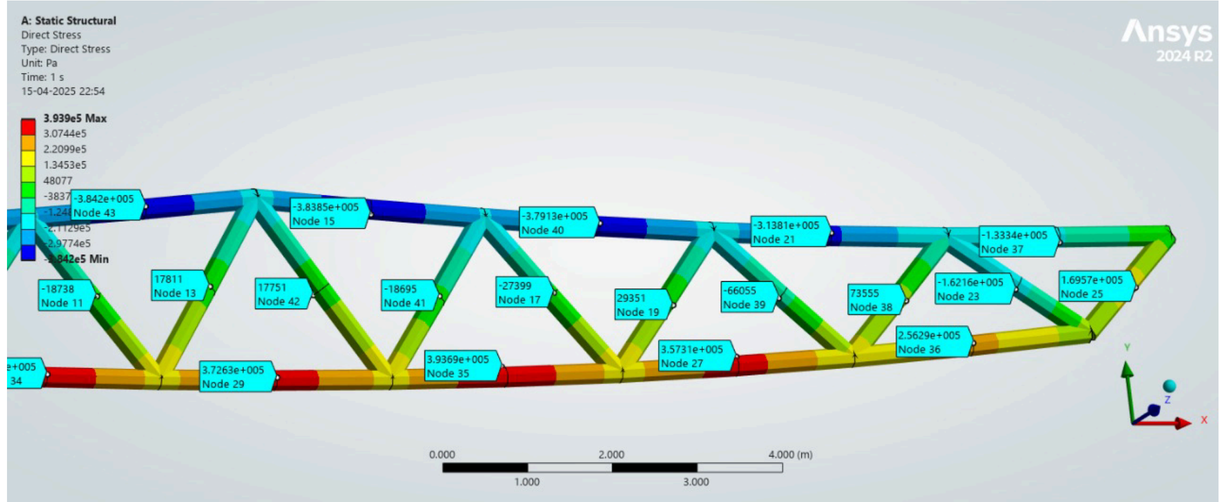
Node	X Displacement (m)	Y Displacement (m)
1	0.000000	0.000000
2	-0.000022	-0.000032
3	0.000005	-0.000066
4	-0.000019	-0.000092
5	0.000005	-0.000115
6	-0.000013	-0.000132
7	0.000001	-0.000143
8	-0.000007	-0.000149
9	-0.000004	-0.000150
10	-0.000001	-0.000149
11	-0.000009	-0.000143
12	0.000005	-0.000131
13	-0.000012	-0.000115
14	0.000011	-0.000093
15	-0.000012	-0.000065
16	0.000015	-0.000030
17	-0.000008	0.000000

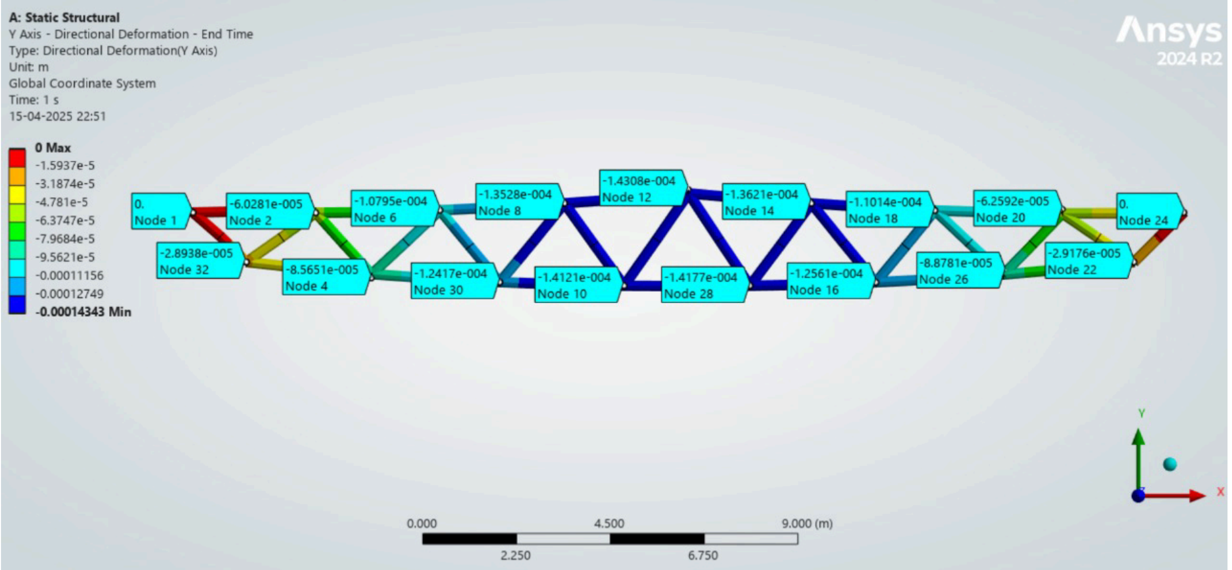
Element	Connection	Stress (Pa)	Force (N)
1	$1 \leftrightarrow 2$	172240.70	8612.04
2	$2 \leftrightarrow 3$	-156878.21	-7843.91
3	$3 \leftrightarrow 4$	94232.89	4711.64
4	$4 \leftrightarrow 5$	-88320.51	-4416.03
5	$5 \leftrightarrow 6$	23308.06	1165.40
6	$6 \leftrightarrow 7$	-21320.65	-1066.03
7	$7 \leftrightarrow 8$	-24346.03	-1217.30
8	$8 \leftrightarrow 9$	23107.48	1155.37
9	$9 \leftrightarrow 10$	17057.89	852.90
10	$10 \leftrightarrow 11$	-18053.61	-902.68
11	$11 \leftrightarrow 12$	-28685.83	-1434.29
12	$12 \leftrightarrow 13$	30784.35	1539.22
13	$13 \leftrightarrow 14$	-67793.40	-3389.67
14	$14 \leftrightarrow 15$	75662.34	3783.12
15	$15 \leftrightarrow 16$	-168758.69	-8437.94
16	$16 \leftrightarrow 17$	174156.58	8707.83
17	$17 \leftrightarrow 15$	-136746.24	-6837.31
18	$15 \leftrightarrow 13$	-323845.93	-16192.30
19	$13 \leftrightarrow 11$	-391111.35	-19555.57
20	$11 \leftrightarrow 9$	-397086.47	-19854.32
21	$9 \leftrightarrow 7$	-400835.32	-20041.77
22	$7 \leftrightarrow 5$	-402923.84	-20146.19
23	$5 \leftrightarrow 3$	-326663.86	-16333.19
24	$3 \leftrightarrow 1$	-138265.69	-6913.29
25	$2 \leftrightarrow 4$	255929.28	12796.46
26	$4 \leftrightarrow 6$	385060.45	19253.02
27	$6 \leftrightarrow 8$	413798.89	20689.94
28	$8 \leftrightarrow 10$	386044.04	19302.20
29	$10 \leftrightarrow 12$	406467.67	20323.38
30	$12 \leftrightarrow 14$	368400.48	18420.02
31	$14 \leftrightarrow 16$	265270.69	13263.54

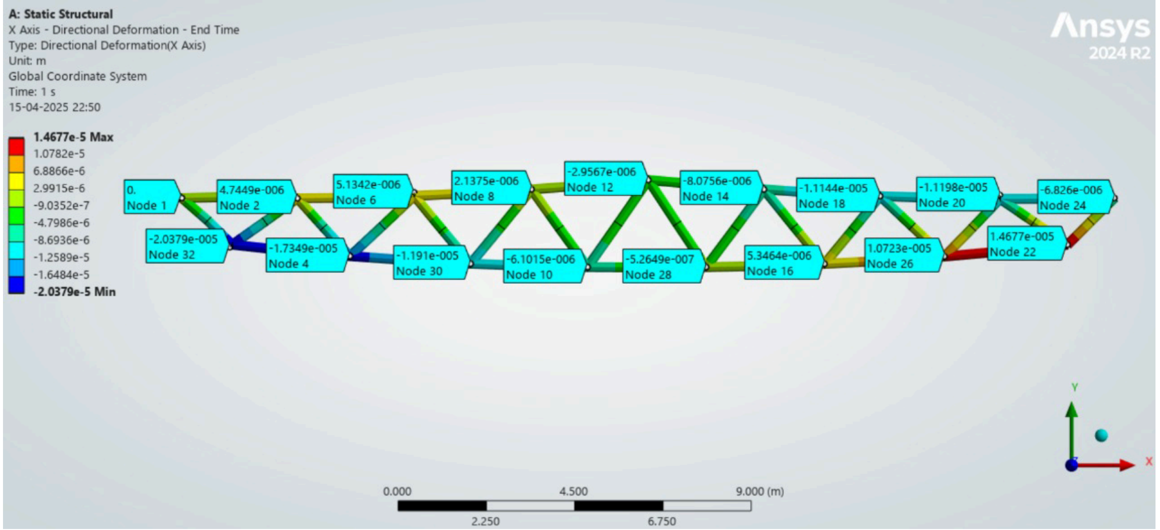
Validation with ANSYS



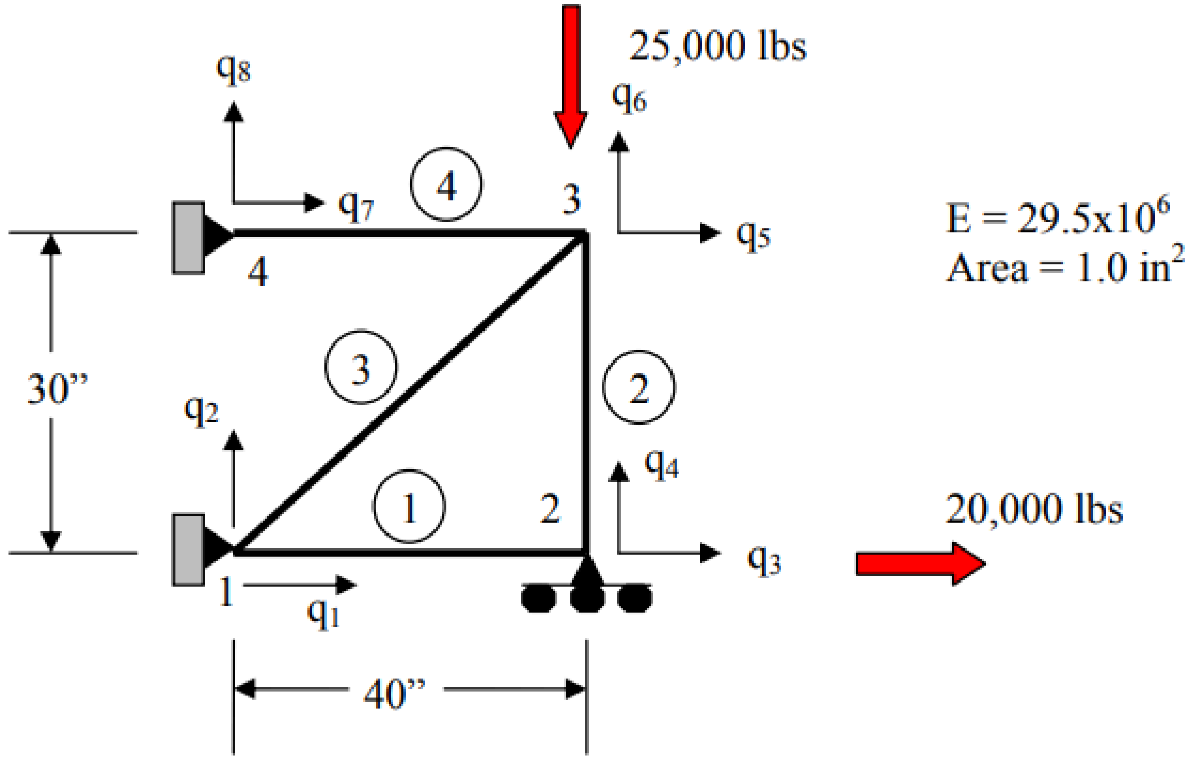
X- deformation deviation: ~15.6%
Y- deformation deviation: ~5.2%
Stress deviation: ~10.1%







Demo



$$\begin{Bmatrix} q_3 \\ q_5 \\ q_6 \end{Bmatrix} = \begin{Bmatrix} 27.12\times10^{-3} \\ 5.65\times10^{-3} \\ -22.25\times10^{-3} \end{Bmatrix} \quad \text{inches}$$

Questions?

Source Code

Project Report