Credit & Credibility

Arnav Sood

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Model

Big Picture This is a model of strategic behavior by a credit rater. In particular, the agency both chooses a strategy for assigning ratings to firms, and a single price p for the rating service. The idea is that p is a tool by which the reporter can control the "ground truth" of the client pool it has to rate, in the same way that college tuition can serve as a filter on applicants.

Global "Time 0" Objects

- A unit mass of firms $i \in [0, 1]$, with types $\theta_i \in \{H, L\}$
- Two type-dependent distributions of outside offers $F_H(\cdot)$ and $F_L(\cdot)$, on the same bounded support $[\underline{\omega}, \bar{\omega}]$, such that $\mathbb{E}F_L \leq \mathbb{E}F_H$. These can be interpreted as ratings offers from other reporters, private investment, a job-market outcome for an entrepreneur, etc.
- A credit market with a prior belief λ about the distribution of good firms in the market (i.e., the distributions of good firms that <u>exist</u>)
- A mechanistic confidence threshold $\underline{\lambda} > \lambda$ for investment which assigns investment outcomes to firms by

$$\mu : \tilde{\lambda} \mapsto \begin{cases} R & \tilde{\lambda} \geq \underline{\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Sequence of Moves

1. Credit reporter chooses a ratings strategy $\phi : \theta \mapsto \Delta(\{h, l\})$ and a price $p \in \mathbb{R}^+$.
   (a) Conjecture that $\Pr[h|H] = 1$ (why hide your peaches?)
   (b) So gives us really a choice of $\phi : L \mapsto \Delta(\{h, l\})$. Since there are only two types, this is just the choice of a scalar $\Pr[h|L]$. For simplicity, from now on we'll call that ϕ.

(c) Call those whole policy $m \in M$, where M is a space of mechanisms (??)

2. Firms, observing the reporter's moves, choose their entry policies.

$$e : \theta_i \times \omega_i \times m \to [0, 1] \tag{2}$$

3. The credit market updates its beliefs and assigns investment outcomes to firms

   (a) The market calculates a signal-independent <u>inflow prior</u> $\lambda^I(m)$, based only on the mechanism, which is a confidence rating conditioned solely on the entry decision.

   (b) The market calculates signal-dependent posteriors $\lambda^o(h, m), \lambda^o(l, m)$.

   (c) The market assigns investment outcomes.

Equilibrium Definition An equilibrium is:

1. A mechanism $m \equiv (\phi, p) \in M$
2. An entry map $e : \theta_i \times \omega_i \times m \to [0, 1]$
3. A set of updated beliefs $\lambda^I(m), \lambda^o(h, m), \lambda^o(l, m)$ and investment outcomes $\mu(\lambda)$

such that

1. The mechanism m satisfies the credit reporter's optimality condition

$$\max_{m \in M} p\mathbb{E}|\Gamma(m)|$$

   where $\Gamma(m)$ is the set of firms who choose to transact with the reporter.

2. The entry map satisfies firms' optimality:

   (a) $\lambda^o(h, m) < \underline{\lambda} \implies e = 0$ (don't enter if the market outcome is deterministically zero)

   (b) Otherwise,

$$e(\theta, m) = F_\theta(\phi_\theta R - p) \tag{3}$$

3. The posterior beliefs are compatible with Bayes' theorem

Model Solution

Market's Problem

Start with the investors' beliefs given $m \in M$ and e.

The inflow prior is:

$$\lambda^I(m) = \frac{e(H,m)\lambda}{e(H,m)\lambda + e(L,m)(1-\lambda)} \tag{4}$$

The signal-dependent prior is (afer applying our $\Pr[h|h] = 1$ conjecture):

$$\lambda^o(h,m) = \frac{\lambda^I(m)}{\lambda^I(m) + (1-\lambda^I(m))\phi} \tag{5}$$

Note that this is the only belief which matters, because if $\Pr[h|H] = 1$, then $\lambda^o(l) = 0$ (i.e., if high firms are always sorted correctly, then a low signal means you're worthless)

Firms' Problem

Look for a simultaneous Nash equilibrium $(e_H, e_L) \in \mathbb{R}^2$.

As above, in an interior equilibrium we have the entry rules:

- High types: $e_H = F_H(R-p)$
- Low types: $e_L = F_L(\phi R - p)$

subject to the constraint that $\mathbb{1}_{\lambda^o(h) \geq \underline{\lambda}} = 1$.

Rewrite that prior in terms of model primitives:

$$\lambda^o(h) = \frac{e_H\lambda}{e_H\lambda + \phi e_L(1-\lambda)} \geq \underline{\lambda} \tag{6}$$

We can rewrite this as a constraint on ϕ:

$$\phi \leq \frac{e_H\lambda(1-\underline{\lambda})}{e_L\underline{\lambda}(1-\lambda)} \tag{7}$$

This passes a sanity check, since it says that (a) the maximum allowable dishonesty ϕ is bounded above by some function of the right quantities, and (b) the bound is increasing in high entry, decreasing in low entry.

Rater's Problem

The rater wants to maximize $p\mathbb{E}|\Gamma(p,\phi)|$, as above. We know that entry flow is $F_L(\phi R - p) + F_H(R-p)$. So this gives us the following optimization problem:

$$\begin{aligned} \underset{p,\phi}{\text{maximize}} \quad & p\Big(F_H(R-p) + F_L(\phi R - p)\Big) \\ \text{subject to} \quad & 0 \leq \phi \leq \frac{e_H\lambda(1-\underline{\lambda})}{e_L\underline{\lambda}(1-\lambda)} \end{aligned} \tag{8}$$

We see that $\frac{\partial}{\partial \phi}$ of the objective is positive, which means that, whatever the price, the rater can benefit by lying a little more (within the constraint). Let τ be the ratio

$$\tau \equiv \frac{\lambda(1-\underline{\lambda})}{\underline{\lambda}(1-\lambda)} \tag{9}$$

Then we have

$$\begin{aligned} \underset{p,\phi}{\text{maximize}} \quad & p\Big(F_H(R-p) + F_L(\phi R - p)\Big) \\ \text{subject to} \quad & \frac{\phi}{\tau} = \frac{F_H(R-p)}{F_L(\phi R - p)} \equiv \frac{e_H}{e_L} \end{aligned} \tag{10}$$

We can sketch out this optimal curve for a simple example. (Would it help to interpret ϕ as the fixed point of a map $f : x \mapsto \frac{\tau e_H(p)}{e_L(\phi,p)}$?)