Credit & Credibility

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Overivew We study a model of strategic behavior by a profit-maximizing credit rating agency (CRA). The rater jointly chooses the ratings and the (flat) price of a rating, so as to influence the size and composition of the set of ratings-seeking firms. We derive equilibrium conditions for the static case, and present some numerical results.

Of course, this is not a *sui generis* area of research. Bolton, Freixas, and Shapiro (2010) model CRAs as players in a competitive market. Bar-Isaac and Shapiro (2010) analyze the time-varying nature of their incentives and behavior over the business cycle, and Kamenica and Gentzow (2011) establish the seminal "Bayesian Persuasion" setting for strategic information transmission.

We are indebted to all of this work (and more). The chief contribution of this note is to jointly solve the pricing problem and "decision problem" for ratings in one shot.

JEL Codes G24, D82, D83

Contents

1 Setting

1.1 Agents

The model has three classes of agent:

- A mass-one continuum of firms, who have types θ_i distributed i.i.d. in $\{H, L\}$ ("High," "Low.") Each firm also has an associated *outside option* ω_i, which is drawn i.i.d. from a (differentiable, etc.) type-contingent distribution F_θ with compact support $[\underline{\omega_\theta}, \overline{\omega_\theta}]$

  Firms solve a static decision problem about market entry:

  $$V_i \equiv \max\{\omega_i, \mathbb{E}U(\theta) - p\} \tag{1}$$

  where $U(\theta)$ is the payoff from obtaining a rating as a firm of type θ, and p is the price of a rating.

- A single *credit rater* (CRA). The CRA's goal is to choose a ratings scheme m (in practice, summarized by a dishonesty parameter ϕ and a price p), to solve

  $$\max_{p\geq 0, \phi\in[0,1]} p\{F_L(\phi R - p)(1-\lambda) + F_H(R-p)\lambda\} \tag{2}$$

  In (2), λ is the common prior about the share of H-type firms.

- A *public debt market*, which maps signals to investment outcomes according to a simple cutoff rule:

  $$U : \theta \mapsto R\mathbb{1}_{\lambda^o(\theta)\geq\underline{\lambda}, \theta=h} \tag{3}$$

  That is: if the market's belief that a firm in question is of type H (written $\lambda^o(h)$) exceeds some exogenous cutoff ($\underline{\lambda}$), then the firm is assigned a flat investment $R \in \text{supp}F_H$.

  Note that we've implicitly assumed the set of firm signals is $\{h, l\}$ such that $m : H \mapsto h$ always. We will defend this later.

1.2 Timing

1. The CRA chooses a scheme $m \equiv (p, \phi)$ according to the common prior λ about the share of high-type firms, in order to solve (2).

2. Firms observe (p, ϕ), and make entry decisions according to (1). Denote the proportion of firms of type θ which decide to buy ratings as e_θ (i.e., the mass of high entrants is $e_H\lambda$.)

3. The market observes $\{(p, \phi), e_H, e_L\}$, and mechanistically executes the investment rule in (3).

1.3 Caveats

Before proceeding with an analysis of this model, it's worth examining its implicit assumptions.

1. **No Transfer Payments:** It's conceivable that firms of the low type (who may have more to gain from a high credit rating, say if $\mathbb{E}F_L < \mathbb{E}F_H$) might pay more to the CRA for a good rating. This is ruled out by our flat price p.

2. **No Outside Information:** In this simple model, the only source of information available to the market is the single CRA. In practice, there is a competitive market for information[1], which may discipline the information put out by a CRA.

3. **No Inside Information:** Likewise, we do not allow firms to generate their own information (e.g., in reality a high-type firm may decide to seek debt financing instead of equity financing, so as to signal its type.)

4. **No CRA Competition:** In practice, the credit-ratings industry is an oligopoly and not a monopoly (i.e., it is dominated by Fitch, Moody's, and Standard & Poor's.) So the monopoly pricing section of this note is likely not generalizable.

Of course, there are many other unrealistic aspects of this simple static model, but these are perhaps the most significant.

[1] See, e.g., Veldkamp (2005)

2 General Case

In this section, we derive objects such as equilibrium conditions which are relevant to most cases of the model (i.e., we impose relatively loose restrictions on parameters.)

2.1 Equilibrium Conditions

2.1.1 Market's Problem

Begin with the market's problem; implementing the investment rule (3). The key is to construct an expression for $\lambda^o(h)$, or market confidence in the validity of an h signal.

First, define the *inflow prior*, or the confidence that $\theta = H$ conditional merely on seeking a rating.

$$\lambda^I \equiv \frac{e_H \lambda}{e_H \lambda + e_L(1-\lambda)} \tag{4}$$

This is the raw proportion of high types in the ratings pool, ignoring the actual ratings.

Using this object, and we can give an expression for:

$$\lambda^o(h) \equiv \Pr[\theta = H|h] \tag{5}$$

Which is:

$$\lambda^o(h) = \frac{\lambda^I}{\lambda^I + (1-\lambda^I)\phi} \tag{6}$$

Note that (as mentioned earlier, and to be justified later) $m : H \mapsto h$ always, so dishonesty is strictly about ratings $L \mapsto h$. These happen at rate ϕ.

After substitution:

$$\lambda^o(h) = \frac{e_H \lambda}{e_H \lambda + \phi e_L(1-\lambda)} \tag{7}$$

2.1.2 Firms' Problem

As above, firms are mechanically making entry decisions based on (1).

Assume that $\lambda^o(h) \geq \underline{\lambda}$ (otherwise, all firms would face market return 0.) Assume also that there is an interior solution (so $F_\theta \in (0,1)$.) Then, we have:

$$e_H = F_H(R - p) \tag{8}$$

$$e_L = F_L(\phi R - p) \tag{9}$$

In other words, given continuua of firms, the share of firms with outside options leading to entry is exactly given by the CDF of the relevant distribution.

2.1.3 Rater's Problem: Credibility Constraint

The problem here is to solve (2) in a forward-looking way (i.e., anticipating (8), (9), (7).)

However, there is a constraint. To motivate entry, we assumed that $\lambda^o(h) \geq \underline{\lambda}$ (that is, the market is confident enough in the h signal to invest.) That is:

$$\begin{aligned}
\lambda^o(h) &\geq \underline{\lambda} \\
\frac{e_H \lambda}{e_H \lambda + \phi e_L (1-\lambda)} &\geq \underline{\lambda} \qquad\qquad \text{by (7)} \\
e_H \lambda (1-\underline{\lambda}) &\geq \phi e_L \underline{\lambda} (1-\lambda) \\
\phi &\leq \frac{e_H \lambda (1-\underline{\lambda})}{e_L \underline{\lambda} (1-\lambda)} \qquad (10)
\end{aligned}$$

This last constraint (which we call the **Market Credibility** constraint, or (MC)) can be interpreted as a sort of discipline on the CRA's dishonesty ϕ. It says that the CRA cannot lie so much that the market ceases to invest in any firms.

2.1.4 Raters' Problem: Equilibrium Conditions

We wish to solve (2) subject to (10). As before, we assume an interior solution, $F_H(R-p) \in (0,1), F_L(\phi R - p) \in (0,1)$.

Write the Lagrangian:

$$\begin{aligned}
\mathscr{L} \equiv\ & p\{F_H(R-p)\lambda + F_L(\phi R - p)(1-\lambda)\} \qquad (11) \\
& - \mu(\log \phi + \log F_L(\phi R - p) - \log F_H(R-p) - \log \tau)
\end{aligned}$$

Where:

$$\tau \equiv \frac{\lambda(1-\underline{\lambda})}{\underline{\lambda}(1-\lambda)} \qquad (12)$$

Note that τ is a constant function of exogenous parameters.

We can then derive relevant FOCs:

- **FOC (ϕ)**

$$\phi p R (1-\lambda) f_L(\phi R - p) = \mu \left(1 + \frac{\phi R f_L(\phi R - p)}{F_L(\phi R - p)}\right) \qquad (13)$$

The LHS of this equation is the payoff accruing to L-types for any ϕ. The RHS is something like a percentage shift in the entrance of L-types, scaled by payoff ϕR.

- **FOC** p

$$\begin{aligned} 0 = & F_H(R-p)\lambda + F_L(\phi R - p) \\ & - p\left(f_H(R-p)\lambda + f_L(\phi R - p)(1-\lambda)\right) \\ & + \mu\left(\frac{f_L(\phi R - p)}{F_L(\phi R - p)} - \frac{f_H(R-p)}{F_H(R-p)}\right) \end{aligned} \tag{14}$$

  The last term is the most interesting for this paper; it captures the CRA's sensitivity to distributional effects (i.e., the fact that the CRA cares not only *how many* firms are buying ratings, but about the types of those firms, since that impacts the allowable amount of lying, etc.)

  The first two terms are standard monopoly pricing objects (the first term says that raising prices gets us more per transaction, the second that we lose firms at the margins by doing so.)

- **Complementary Slackness**

$$\mu(\log\phi + \log F_L(\phi R - p) - \log F_H(R-p) - \log\tau) = 0 \tag{15}$$

  This is (10), but log-linearized for convenience.

2.2 Expression for Lagrange Multiplier

It's possible to rewrite the FOC (ϕ), (13), to get an expression for μ:

$$\mu = \phi R(1-\lambda)F_L(\phi R - p) \cdot \frac{f_L(\phi R - p)}{F_L(\phi R - p) + \phi R f_L(\phi R - p)} \tag{16}$$

The left-hand multiplier is the market payoff accruing to low-type firms. The fractional term is the new share of L-type entrants (i.e., after taking into account new entry at the margin.)

The canonical interpretation of this multiplier is that it corresponds to the value of slightly relaxing the market credibility constraint (10) (i.e., of lowering $\underline{\lambda}$.)

What's worth noting is that, in an interior solution, the following conditions are equivalent to the binding of (MC) (i.e., of a nonzero μ):

1. Market reward R is nonzero.
2. The price p is nonzero.
3. $\lambda \neq 1$ (i.e., there is a nonzero mass of L-type firms.)
4. The dishonesty parameter ϕ is nonzero.
5. The distribution $f_L(\cdot)$ of L-type firms is not locally stationary (i.e., $f_L(\phi R - p) \neq 0$.) This amounts to saying that a small perturbation will motivate some low entry.

2.3 Distribution-Contingency of Equilibria

2.3.1 Existence and Uniqueness

Since so many of the equilibrium objects (i.e., (13), (14), (15), (16))) are based on densities and CDFs, it's reasonable to ask to what extent the equilibria are distribution-dependent.

Proposition 1. If F_H and F_L are differentiable functions $F_\theta : \mathbb{R} \to \mathbb{R}$ with compact supports $\omega_\theta \equiv \text{supp} F_\theta$, and the following parameter restrictions hold, then there is a unique equilibrium.

- Some restriction on R.
- Some restriction on λ.
- Some restriction on $\underline{\lambda}$.

Proof. □

2.3.2 Dispersion in Distributions

In practice, it's reasonable to expect that $\mathbb{E}F_L < \mathbb{E}F_H$. One interpretation of the outside option ω_i is that it corresponds to outcomes on the private debt market, so this simply means that high types do strictly better there, in expectation, than low types.

In this section, we examine the mileage we can derive from this assumption.

3 Numerical Implementation

4 Uniform Case

This section studies a highly specialized case of the model. We assume that:

$$\omega \equiv \omega_H = \omega_L$$
$$\omega \sim U[\underline{\omega}, \underline{\omega} + 1]$$

Such that $R \in \text{supp}\{\omega_\theta\}$.

Note that this assumption implies that the **private debt market** (one interpretation for the type-contingent outside options firms are assigned generally) is perfectly uninformed. That is, outside options are not type-contingent.

4.1 Analytical Results

4.2 Numerical Results and Comparative Statics

Here, we go over numerical results which are specific to the case above.

5 Comparative Statics

6 References

Appendices

A Analytical Appendix

B Computational Appendix