Credit and Credibility

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- **Empirics** 2008 Recession?

Our Model (English)

One-shot CRA choosing prices and ratings for a continuum of firms, with a lower bound on ratings informativeness.

- Firms: $i \in [0,1]$, with $\theta_i \in \{H, L\}$. Have outside options contingent on type: $\omega_i \sim F_\theta$.

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- Capital Market: Invests in any firm which gets a good signal (WLOG, it can be shown there are two signals), so long as there's a minimum "precision."
- CRA: Forward-looking to market behavior and firm entry decisions, chooses a public mechanism (p, ϕ), where $\phi \in \mathbb{R}$ summarizes the ratings policy.

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- Mechanistically solve:

$$V_i \equiv \max\{\omega_i, \mathbb{E}U(\theta) - p\} \tag{1}$$

Our Model (Capital Market)

- Assigns outcomes to firms according to a simple cutoff rule:

$$U : \theta \mapsto R \qquad \underbrace{\mathbb{1}_{\lambda^{o}(\theta) \geq \underline{\lambda}, \theta = h}}_{\text{Y/N investment decision}} \tag{2}$$

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- What's required for investment?

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- Objective is then:

$$\max_{p \geq 0, \phi \in [0,1]} p\{F_L(\phi R - p)(1-\lambda) + F_H(R - p)\lambda\} \qquad (3)$$

s.t. the constraint in (2) is satisfied.

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This objective is **forward-looking**, in that it incorporates optimality conditions for firms and the market.

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3. The market observes $\{(p, \phi), e_H, e_L\}$, and mechanistically executes the investment rule in (2).

Equilibrium Ingredients

A CRA policy (p, ϕ), entry decisions e_H, e_L, and a market ratings policy (call it $\pi : \xi \to \{0, R\}$) s.t. ...

1. $\pi(\cdot)$ solves the market's problem, (2).
2. Entry decisions e_H, e_L maximize the firm objective (1).
3. CRA actions (p, ϕ) maximize the CRA objective (3).
4. CRA actions are optimal in expectation, policies e_H, e_L are forward-looking best-responses, and market outcomes $\pi(\cdot)$ are updated in a Bayes-plausible way.

Our Model (Caveats)

Before proceeding with an analysis of this model, it's worth examining its implicit assumptions.

1. **No Transfer Payments:** Maybe L-type firms (who may have more to gain from a high credit rating) might pay more for a good rating. This is ruled out by our flat price p.

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3. **No Inside Information:** Likewise, we do not allow firms to generate their own information (e.g., in reality a high-type firm may decide to seek debt financing instead of equity financing.)
4. **No CRA Competition:** In practice, the credit-ratings industry is an oligopoly and not a monopoly.

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Backwards Induction

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3. Given $\uparrow$, solve for p, ϕ.

Market's Posterior

1. Start with *inflow prior*, $\Pr[\theta_i = H|\text{entry}]$:

$$\lambda^I \equiv \frac{e_H \lambda}{e_H \lambda + e_L (1 - \lambda)} \tag{4}$$

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3. Which can be rewritten in terms of raw policies:

$$\lambda^o(h) = \frac{e_H \lambda}{e_H \lambda + \phi e_L(1-\lambda)} \tag{6}$$

Entry Policies

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- In an interior solution, H types can expect reward R with probability 1, and L types can expect it with probability ϕ.
- Gives us the following:

$$e_H = F_H(R - p) \tag{7}$$

$$e_L = F_L(\phi R - p) \tag{8}$$

Market Credibility Constraint

Can rewrite $\lambda^o(h) \geq \underline{\lambda}$...

$$\begin{aligned}
\lambda^o(h) &\geq \underline{\lambda} \\
\frac{e_H \lambda}{e_H \lambda + \phi e_L (1-\lambda)} &\geq \underline{\lambda} && \text{by (6)} \\
e_H \lambda (1-\underline{\lambda}) &\geq \phi e_L \underline{\lambda}(1-\lambda) \\
\phi &\leq \frac{e_H \lambda (1-\underline{\lambda})}{e_L \underline{\lambda}(1-\lambda)} \qquad (9)
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This last constraint (which we call the **Market Credibility** constraint, or (MC)) can be interpreted as a sort of discipline on the CRA's dishonesty ϕ. It says that the CRA cannot lie so much that the market ceases to invest in any firms.

Raters' Lagrangian

We wish to solve (3) subject to (9).
Write the Lagrangian:

$$\begin{aligned}\mathscr{L} \equiv\ & p\{F_H(R-p)\lambda + F_L(\phi R - p)(1-\lambda)\} \\ & - \mu(\log\phi + \log F_L(\phi R - p) - \log F_H(R-p) - \log\tau)\end{aligned} \tag{10}$$

FOC p

$$
\begin{aligned}
0 = {} & F_H(R-p)\lambda + F_L(\phi R - p)(1-\lambda) \\
& - p\left(f_H(R-p)\lambda + f_L(\phi R - p)(1-\lambda)\right) \\
& + \underbrace{\mu\left(\frac{f_L(\phi R - p)}{F_L(\phi R - p)} - \frac{f_H(R-p)}{F_H(R-p)}\right)}_{\textit{Composition} \text{ of customer pool changes!}}
\end{aligned} \tag{11}
$$

FOC ϕ

$$\underbrace{\phi R(1-\lambda) f_L(\phi R - p)}_{\text{Marginal return to } L\text{-type firms}} = \mu \left(\frac{1}{p} + \underbrace{\frac{\phi R}{p} \cdot \frac{f_L(\phi R - p)}{F_L(\phi R - p)}}_{\text{something like } \Delta\%} \right) \tag{12}$$

Analytical Propositions

Note: Full proofs are up on GitHub.

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- **No Truth-Telling:** Will always be some entry. Since constraint is continuous, can always do a little better without exhausting slack.
- **Constraint Always Binds:** By Fermat, need (a) non-differentiable point, (b) a stationary point, or (c) a boundary point. Assume $F_H, F_L \in C^1$, so no (a). $\frac{\partial}{\partial \phi}$ of objective (3) is (subject to assumptions) nonzero. So, must be a boundary.

Vanilla Model

- $R = \frac{1}{2}$
- $\lambda = \frac{1}{3}$.
- $\underline{\lambda} = \frac{1}{2}$.
- $\omega_i \sim U[0, 1]$.
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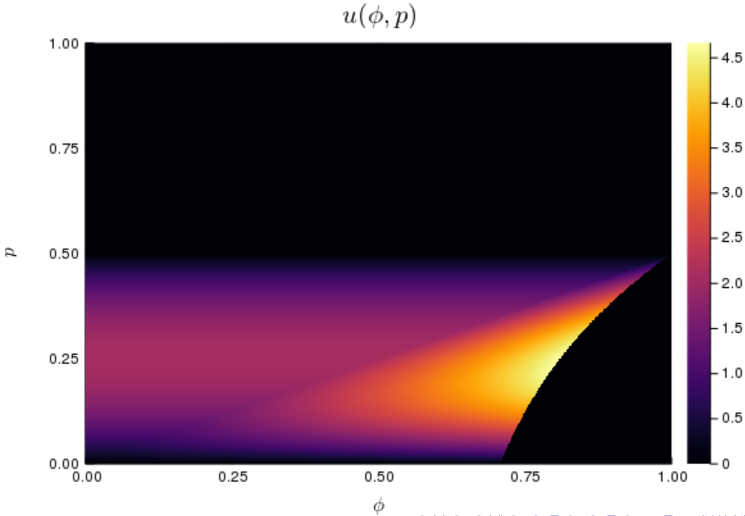
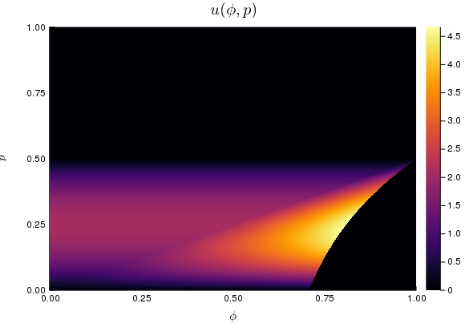
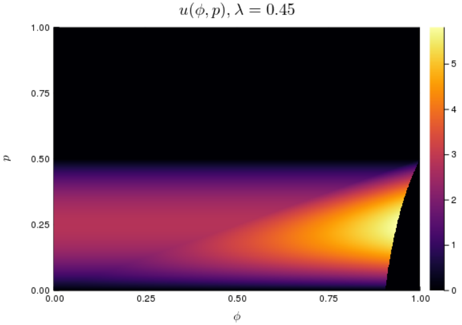
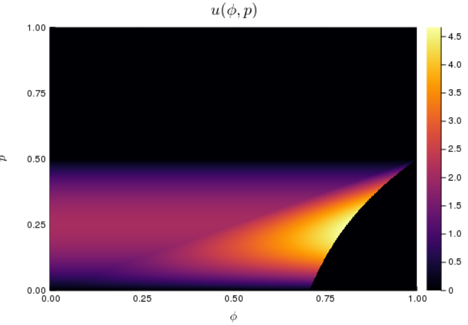


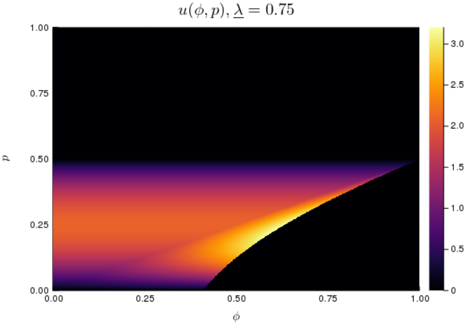
Adjusting λ



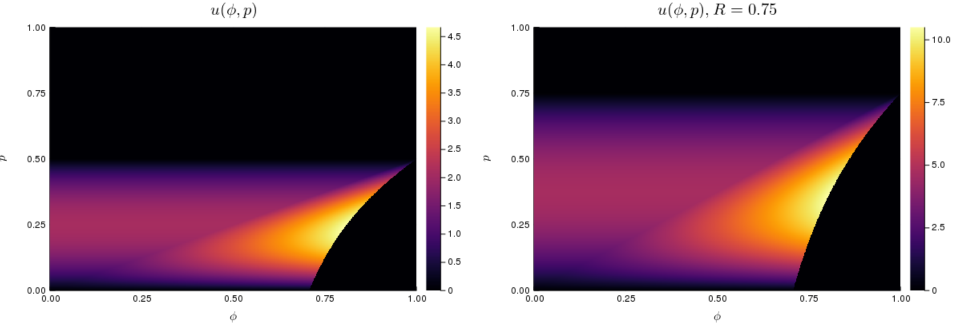


Adjusting $\underline{\lambda}$

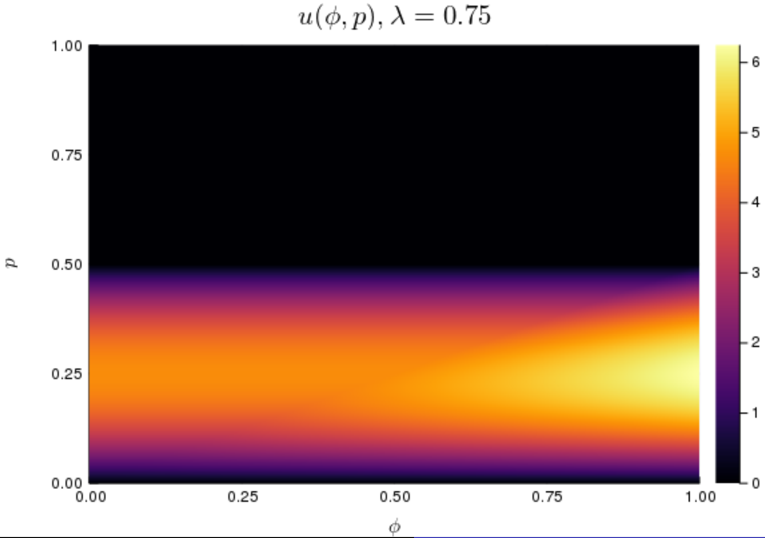




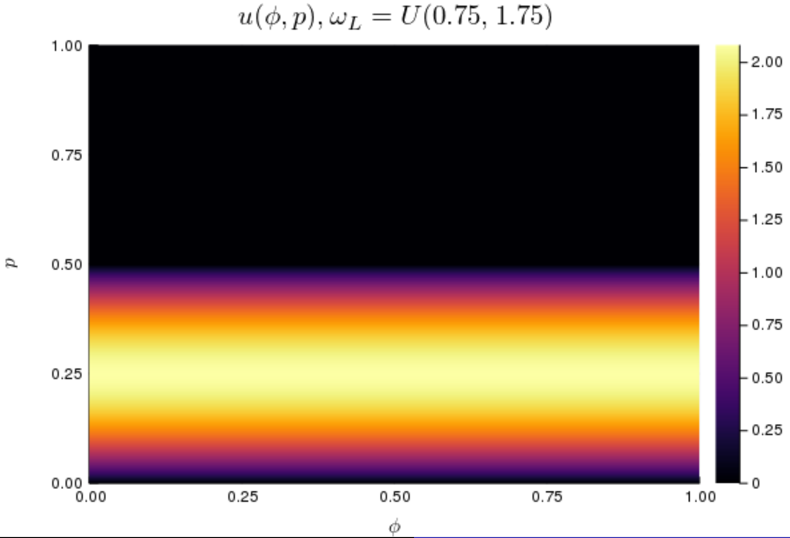
Adjusting R


$u(\phi, p)$
$u(\phi, p)$, $R = 0.75$
1.00
0.75
0.50
0.25
0.00
0.00
0.25
0.50
0.75
1.00
p
ϕ
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
10.0
7.5
5.0
2.5

Adjusting λ (Degenerate)



Adjusting F_L (Degenerate)



Wrap-Up

- Solving the one-shot (pricing, ratings) problem gives us standard monopoly FOCs, with an additional term.
- There is a clean closed form for the **market credibility constraint** (9), which always binds.
- Comparative statics reveal a number of outcomes, depending on parameters.
- Code up at `https://github.com/arnavs/credit-pricing`

Questions

FAQ...

- *Isn't your model unrealistic/overly stylized? Why is it useful?*
  A: The model helps illustrate the simultaneous role of CRA prices (as prices, and also as a "lever" to impact the ratings pool.)
- *You mentioned some caveats. Can these be relaxed?*
  A: The (no outside information) and (CRA is a monopoly) ones can be jointly loosened by modeling CRAs as symmetric price-makers. Firms can also produce their own signals, which would replace $\pi : \xi \to \{0, R\}$ with $\pi : \xi \times \Gamma \to \{0, R\}$.
- *How would one generalize beyond the one-shot binary case?*
  A: We could make the model a finite or infinite horizon model by incorporating a reputational term to the CRA objective (3), as we've seen in class. And/or partition the *n*-dimensional ratings space.
- *Data?*
  A: We haven't used any, but there is lots of data on credit ratings and outcomes, which could conceivably be used to calibrate a model of strategic behavior by CRAs. This isn't a *sui generis* area of research[2]

[2] See, e.g., Xia (2011), who studied over 70K ratings.